

The third ExU annual meeting

Sept. 11, 2023@Panasonic Auditorium
in Yukawa Hall

**Time and space issues
that appeared in my
quantum information
research**

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[quantum computer](#) [quantum communication](#)

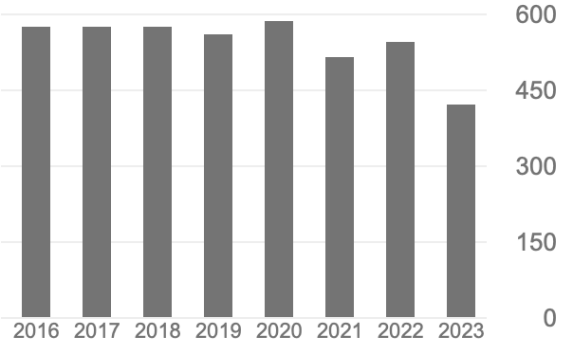
フォロー

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タイトル	引用先	年
Quantum entanglement for secret sharing and secret splitting A Karlsson, M Koashi, N Imoto Physical Review A 59 (1), 162	1135	1999
Quantum cryptography with coherent states B Huttner, N Imoto, N Gisin, T Mor Physical Review A 51 (3), 1863	806	1995
Quantum nondemolition measurement of the photon number via the optical Kerr effect N Imoto, HA Haus, Y Yamamoto Physical Review A 32 (4), 2287	772	1985
Amplitude squeezing in a semiconductor laser using quantum nondemolition measurement and negative feedback Y Yamamoto, N Imoto, S Machida Physical Review A 33 (5), 3243	339	1986
Direct observation of Hardy's paradox by joint weak measurement with an entangled photon pair K Yokota, T Yamamoto, M Koashi, N Imoto New Journal of Physics 11 (3), 033011	298	2009
Concentration and purification scheme for two partially entangled photon pairs T Yamamoto, M Koashi, N Imoto Physical Review A 64 (1), 012304	287	2001
Experimental extraction of an entangled photon pair from two identically decohered pairs T Yamamoto, M Koashi, ŞK Özdemir, N Imoto Nature 421 (6921), 343-346	272	2003

引用先 [すべて表示](#)

	すべて	2018 年以來
引用	11377	3206
h 指標	54	30
i10 指標	150	80



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0 件の論文 11 件の論文

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Quantum nondemolition measurement of the photon number via the optical Kerr effect

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(Received 30 April 1985)

This paper proposes a quantum nondemolition measurement scheme for the photon number. The signal and probe optical waves interact via the optical Kerr effect. The optical phase of the probe wave is selected as the readout observable for the measurement of the photon number of the signal wave. The measurement accuracy Δn and the imposed phase noise $\Delta\phi$ of the signal wave satisfy Heisenberg's uncertainty principle with an equality sign, $\langle(\Delta n)^2\rangle\langle(\Delta\phi)^2\rangle = \frac{1}{4}$.

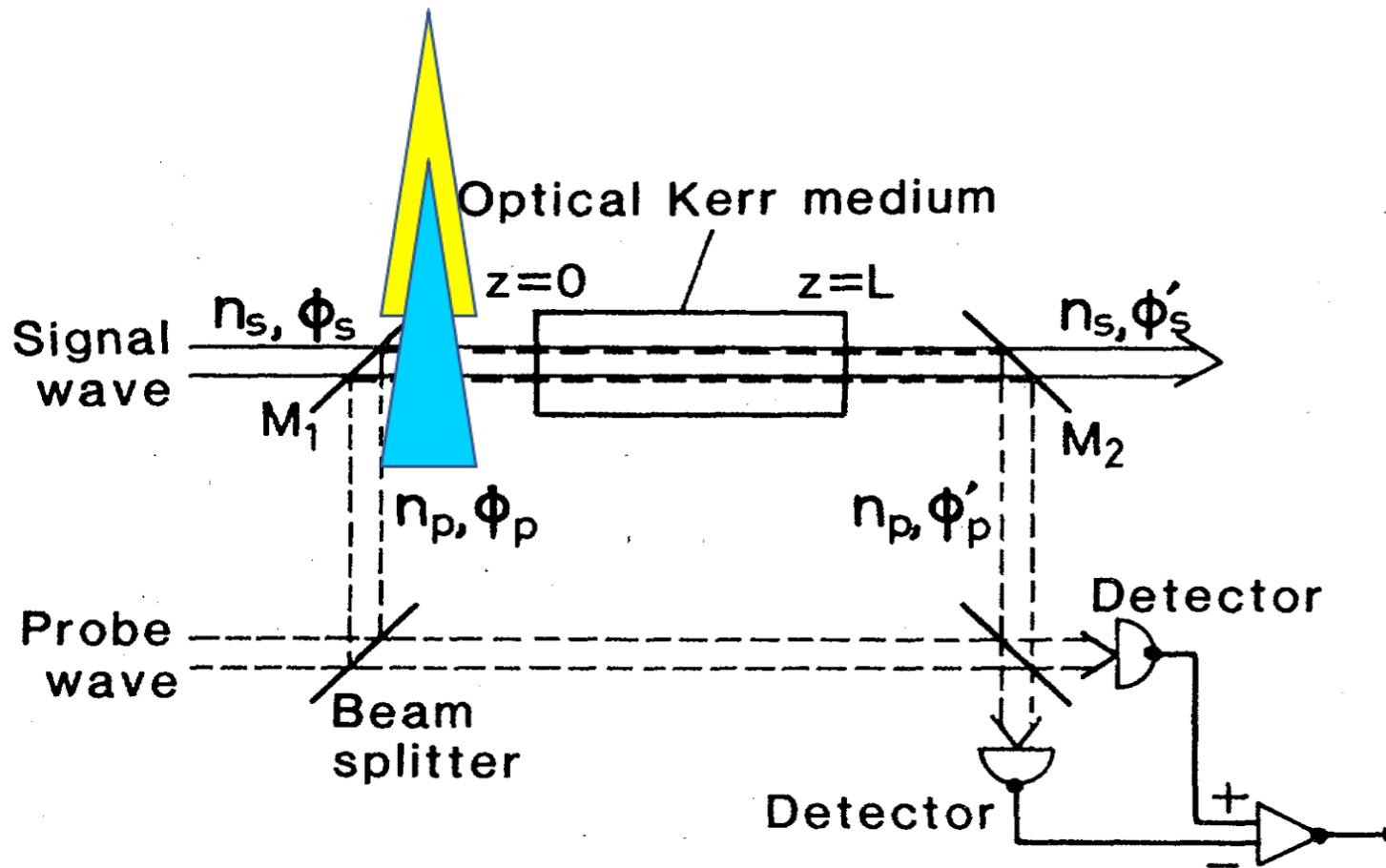


FIG. 1. Configuration for the QND measurement of the signal photon number. Transmissions of mirrors M_1 and M_2 are unity for signal frequency. Signal wave passes through the optical Kerr medium without changing its photon number. Phase of the probe wave is modulated by the signal photon number.

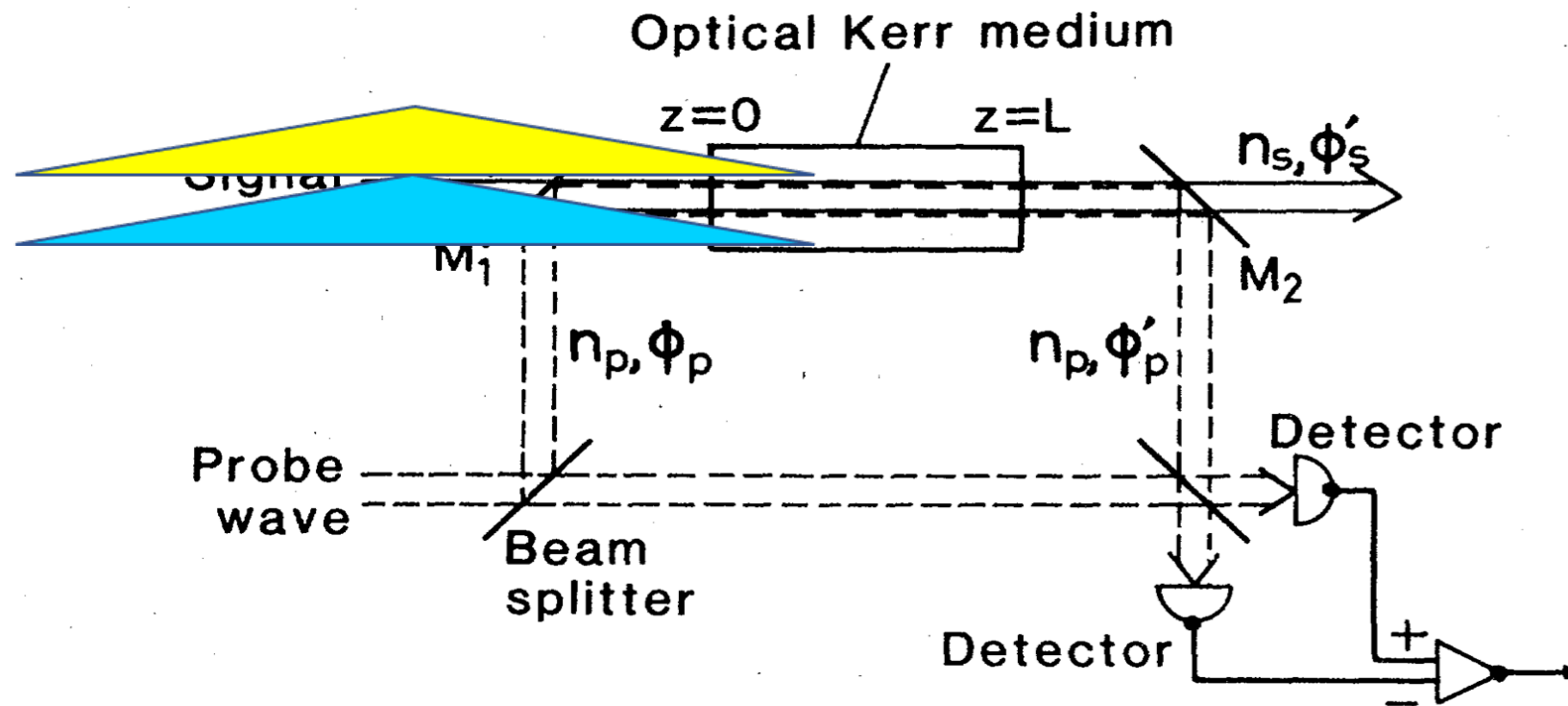


FIG. 1. Configuration for the QND measurement of the signal photon number. Transmissions of mirrors M_1 and M_2 are unity for signal frequency. Signal wave passes through the optical Kerr medium without changing its photon number. Phase of the probe wave is modulated by the signal photon number.

IV. SELF-PHASE-MODULATION EFFECT

Equations (8)–(10) are idealized in the sense that they do not include the self-modulation of the phase caused by the signal and probe waves. In order to treat the Kerr medium more realistically, we must consider the full Hamiltonian. We shall then show that it is possible to arrive at a QND measurement arrangement which is describable in terms of the ideal Hamiltonians (8)–(10).

The perturbation energy due to the third-order non-linear effect is

$$\begin{aligned}
 H' &= \int \int \int \left[\int E dP_{\text{NL}} \right] dV \\
 &= \frac{3}{4} \int \int \int \sum \chi_{ijkl}^{(3)} E_i E_j E_k E_l dV .
 \end{aligned} \tag{26}$$

Here, $\chi^{(3)}$ is defined not only for the optical Kerr effect but also for every process in which four photons are emitted or absorbed. In contrast, it should be noted that $\chi^{(3)}$ in (6) is phenomenologically defined for the optical Kerr effect, especially for the phase modulation of the probe wave by the signal wave.

V. MEASUREMENT ACCURACY AND THE IMPOSED PHASE NOISE

In general quantum measurements, the product of the measurement accuracy and the additional uncertainty imposed on the conjugate observable is expected to satisfy the inequality of Heisenberg's uncertainty principle. However, whether the equality sign is achievable or not in a QND measurement has not yet been investigated. We will show that the proposed QND measurement scheme provides the minimum uncertainty product of measurement accuracy for photon number and imposed phase noise.

Consider the case without the self-phase-modulation effect for both the signal and probe waves. The output phase of the signal is, in analogy with (22),

$$\phi'_s = \phi_s + \sqrt{F} n_p , \quad (36)$$

APPENDIX

In this appendix the output of the proposed interferometer—balanced-mixer detector is derived. The observed photon number is defined as the output current divided by a normalized factor which changes the current into the photon number. Equations (23)—(25) are derived by the obtained formula for the observed photon-number operator.

Figure 3 shows the present scheme in which the annihilation operator for each part of the interferometer is specified. The probe laser output a is divided by beam splitter

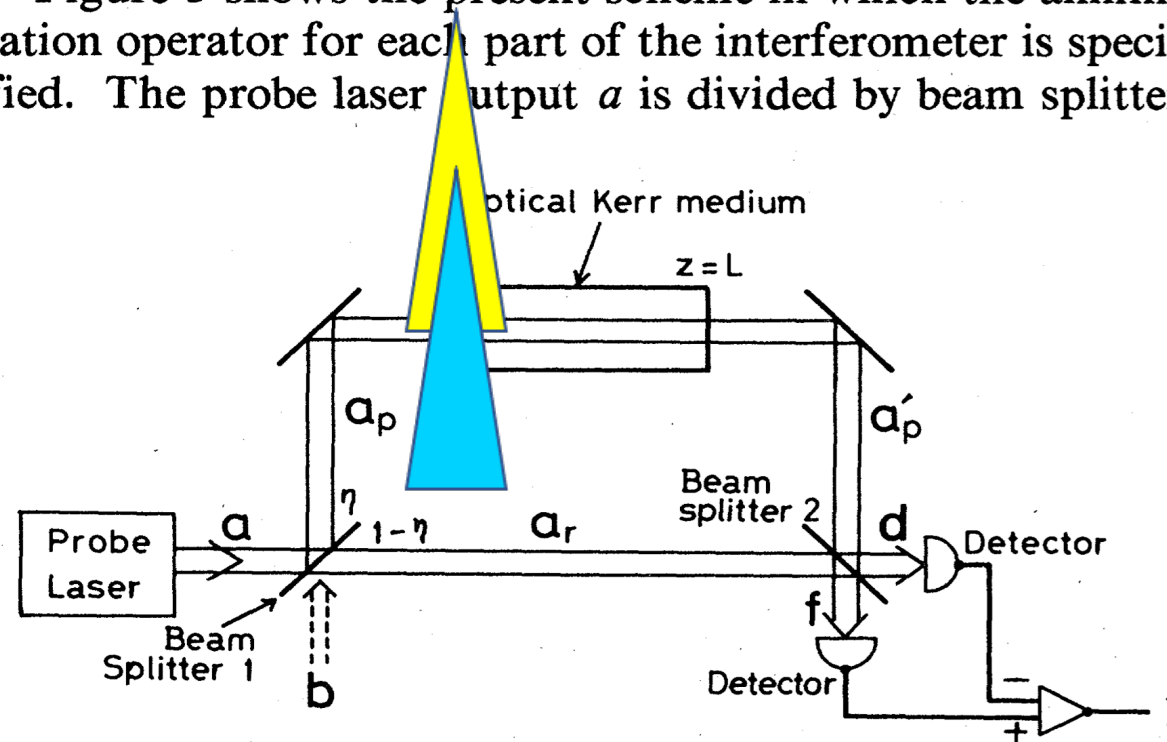


FIG. 3. Detailed description of the annihilation operators in the interferometer—balanced-mixer detector. Probe wave and reference wave are denoted as a_p and a_r , respectively. Zero-point fluctuation, b , is mixed at beam splitter 1.

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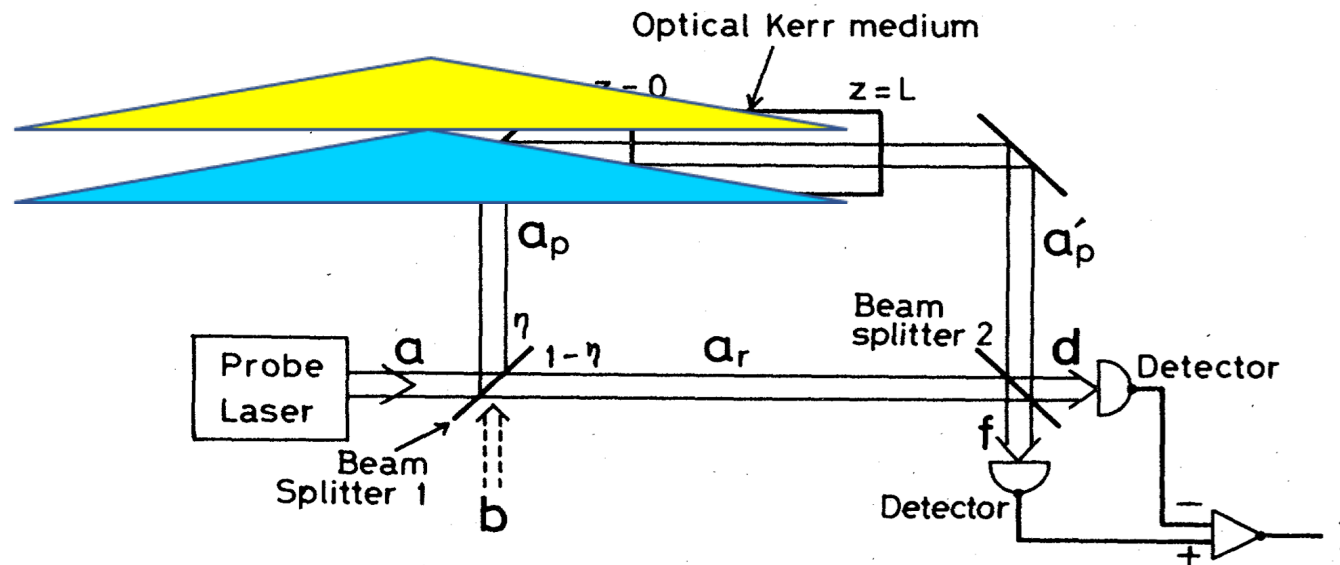
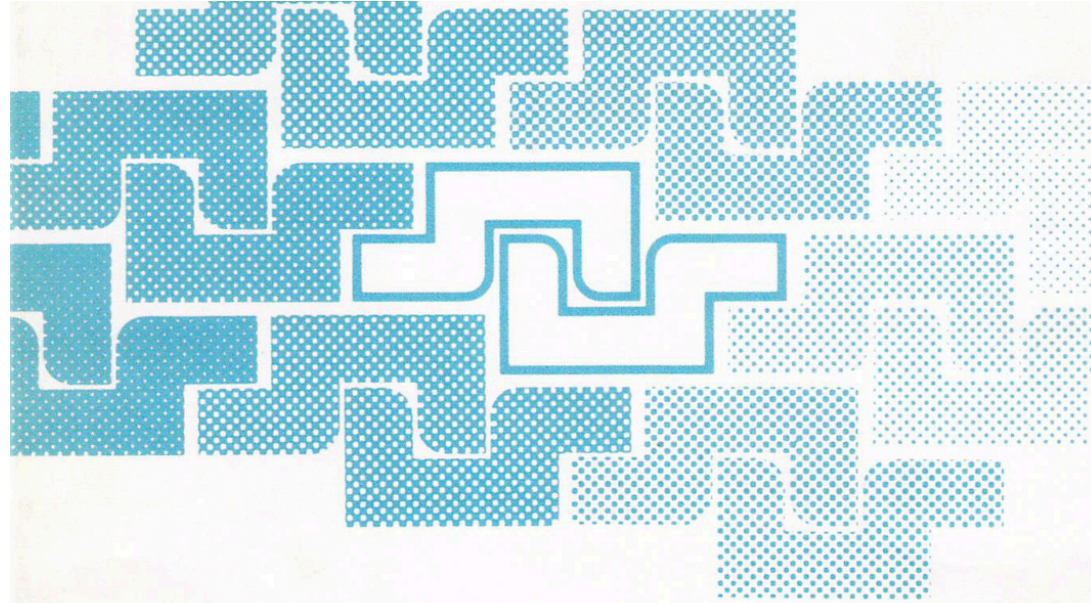


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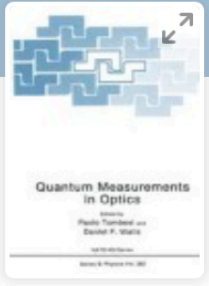


Quantum Measurements in Optics

Edited by
Paolo Tombesi and
Daniel F. Walls

NATO ASI Series

Series B: Physics Vol. 282



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Quantum Mechanical Treatment of a Propagating Optical Beam

[Nobuyuki Imoto](#), [John R. Jeffers](#) & [Rodney Loudon](#)

Chapter

370 Accesses | **1** [Citations](#)

Part of the [NATO ASI Series](#) book series (NSSB, volume 282)

Abstract

Quantum mechanics has been established on the basis of the Hamiltonian formula which describes the time evolution of the system. In any textbook, the quantization procedure starts from the box-quantization, in which spatial modes of a cavity are first defined, and then the time evolution of the modes is described. The Hamiltonian \hat{H} has a role of “time evolution generator,” which governs the time evolution of an operator \hat{a} as

$$\delta \iiint \int \int \mathcal{L} dt dx dy dz = 0$$

(\mathcal{L} : Lagrangian density)
Minimum action principle

Present theory

$$\delta \int_{z_0}^{z_1} L(z) dz = 0$$

where $L(z) \equiv \int \int_A \int_0^T \mathcal{L} dt dx dy$

⇓

$$\frac{\partial}{\partial z} \left[\frac{\partial L}{(\partial \Phi / \partial z)} \right] = \frac{\partial L}{\partial \Phi}$$

⇓ (Legendre transform)

$$\frac{dA}{dz} = \{A, I_z\}$$

where $I_z \equiv \int \int_A \int_0^T \mathcal{I}_z dt dx dy,$

$$\mathcal{I}_z \equiv \Pi \cdot \frac{\partial \Phi}{\partial z} - \mathcal{L}, \quad \text{and}$$

$$\Pi \equiv \frac{\partial \mathcal{L}}{\partial (\partial \Phi / \partial z)} \quad (\text{conjugate observable})$$

$$\begin{aligned} &\Rightarrow \{\Phi(t, x, y, z), \Pi(t', x', y', z)\} \\ &= \delta(t' - t) \delta(x' - x) \delta(y' - y) \end{aligned}$$

Usual theory

$$\delta \int_{t_0}^{t_1} L(t) dt = 0$$

where $L(t) \equiv \int \int \int_V \mathcal{L} dx dy dz$ (Lagrangian)

⇓

$$\frac{\partial}{\partial t} \left[\frac{\partial L}{(\partial \Phi / \partial t)} \right] = \frac{\partial L}{\partial \Phi} \quad (\text{Lagrange equation})$$

⇓ (Legendre transform)

$$\frac{dA}{dt} = \{A, H\}$$

where $H \equiv \int \int \int_V \mathcal{H} dx dy dz$ (Hamiltonian),

$$\mathcal{H} \equiv \Pi \cdot \frac{\partial \Phi}{\partial t} - \mathcal{L} \quad (\text{Hamiltonian density}), \quad \text{and}$$

$$\Pi \equiv \frac{\partial \mathcal{L}}{\partial (\partial \Phi / \partial t)} \quad (\text{conjugate observable})$$

$$\begin{aligned} &\Rightarrow \{\Phi(t, x, y, z), \Pi(t, x', y', z')\} \\ &= \delta(x' - x) \delta(y' - y) \delta(z' - z) \end{aligned}$$

$$\hat{E}(t, x, y, z) = \sum_{\omega} \sqrt{\frac{\hbar k_{\omega}}{2\epsilon_{\omega} AT}} e^{-i\omega t} [\hat{a}_{\omega}(z) + \text{H.c.}] , \quad (2)$$

where A is the cross-sectional area of the beam. This expression will be used in later sections.

The spatial evolution of $\hat{a}_{\omega}(z)$ is given by equation of evolution

$$\frac{d}{dz} \hat{a}_{\omega}(z) = \frac{1}{i\hbar} [\hat{a}_{\omega}(z), \hat{I}_z(z)] , \quad (3)$$

where \hat{I}_z is the *spatial evolution generator* for the z axis, which is defined as $\hat{I}_z = \iint dx dy \int_0^T dt T_{zz}$, where T_{zz} is the (z, z) component of the Maxwell energy-momentum tensor. \hat{I}_z is then expressed by the field components as

$$\hat{I}_z = \iint dx dy \int_0^T dt \left[\hat{E}_z \hat{D}_z + \hat{H}_z \hat{B}_z - \frac{1}{2} (\hat{\mathbf{E}} \cdot \hat{\mathbf{D}} + \hat{\mathbf{H}} \cdot \hat{\mathbf{B}}) \right] . \quad (4)$$

For a plane wave beam, the integral for (x, y) plane should be restricted within the cross-sectional area of the beam. When there is only dispersion but no perturbation, the unperturbed spatial evolution generator, \hat{I}_0 is of the form

$$\hat{I}_0 = - \sum_{\omega} \hbar k_{\omega} \left(\hat{a}_{\omega}^{\dagger} \hat{a}_{\omega} + \frac{1}{2} \right) , \quad (5)$$

which leads to the trivial propagation solution:

$$\hat{a}_{\omega}(z) = e^{ik_{\omega} z} \hat{a}_{\omega}(0) . \quad (6)$$

When there is an interaction, the slowly varying annihilation operator $\hat{A}_{\omega}(z)$ is defined by

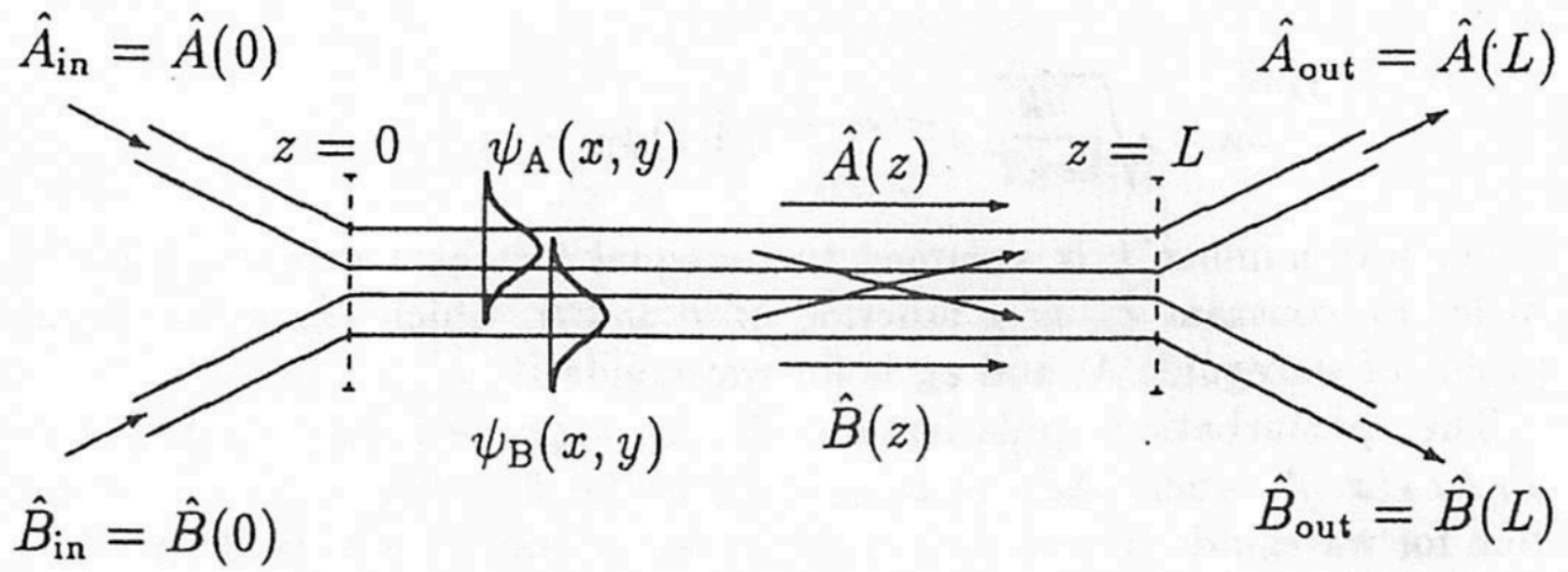


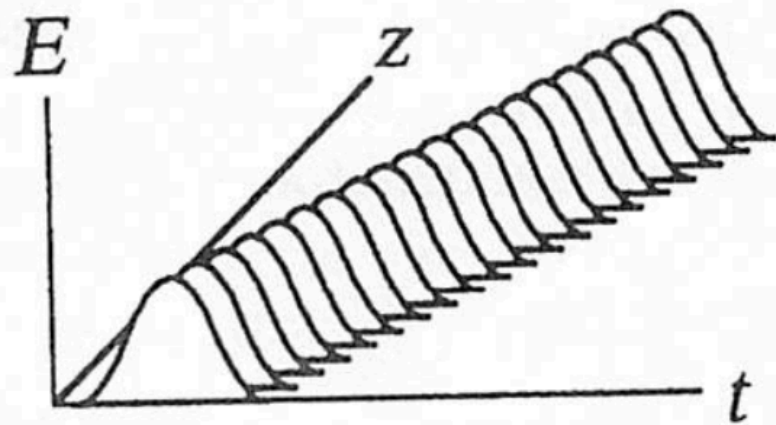
FIG. 1. Schematic view of a directional coupler

$$\frac{d}{dz} \hat{A}_\omega(z) = \frac{1}{i\hbar} [\hat{A}_\omega(z), \hat{I}_{\text{int}}(z)]. \quad (10)$$

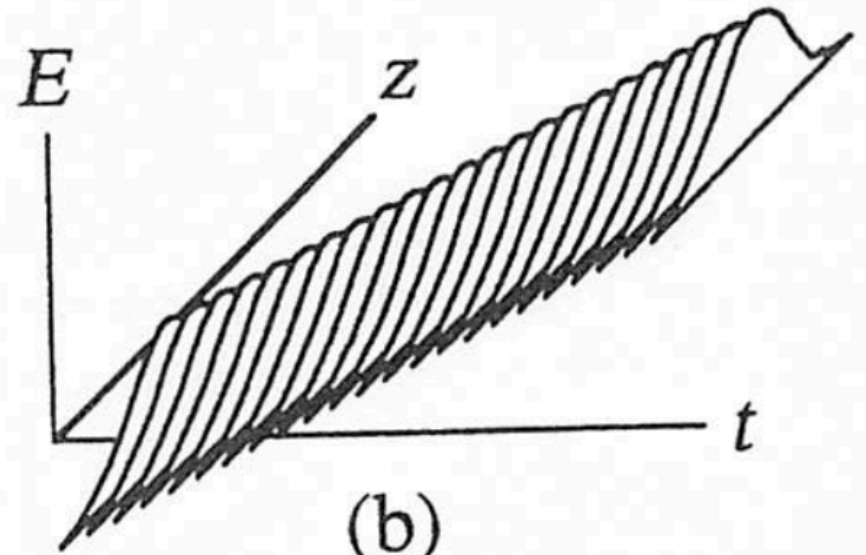
\hat{I}_{int} is expressed using the perturbation polarization $\hat{\mathbf{P}}$ as

$$\hat{I}_{\text{int}} = \iiint \int_0^T \left(E_z P_z - \frac{1}{2} \hat{\mathbf{E}} \cdot \hat{\mathbf{P}} \right) dt dx dy. \quad (11)$$

Since $\hat{\mathbf{P}}$ is a function of the field, \hat{I}_{int} is expressed by $\hat{A}_\omega(z)$'s and $\hat{A}_\omega^\dagger(z)$'s, using the quantized expression of the field. Equation of evolution (10) thus gives a set of coupled-mode equations for relevant $\hat{A}_\omega(z)$'s and $\hat{A}_\omega^\dagger(z)$'s. The above formula is consistently used in solving specific problems described hereafter.



(a)



(b)

FIG. 5. Stationary pulse propagation in a non-dispersive medium. (a) Spatial evolution of a temporal pulse mode. (b) Time evolution of a spatial pulse mode.

Anomalous commutation relation and modified spontaneous emission inside a microcavity

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(Received 7 February 1994)

Usual quantum-optical operator relations for a beam splitter are shown to lead to an anomalous commutation relation inside a microcavity. The physical origin of this anomaly is identified as self-interference of the mode whose coherence length is longer than the round-trip length of the cavity. Altered spontaneous emission of an excited atom is found to be a direct manifestation of this anomalous commutation relation. The anomalous Heisenberg uncertainty relations, which are derived from the commutation relation according to the Schwartz inequality, cannot be detected by probing the internal field with a beam splitter. The anomalous commutation relation, however, can be related to the change in the effective reflectivity of the beam splitter. The similarity and difference between an excited atom and a probe beam splitter are discussed.

PACS number(s): 03.65.Bz, 42.50.Dv, 42.50.Lc

The commutation relation can be $\cong 1$ inside a cavity
 Ueda & Imoto PRA50, 89(1994)

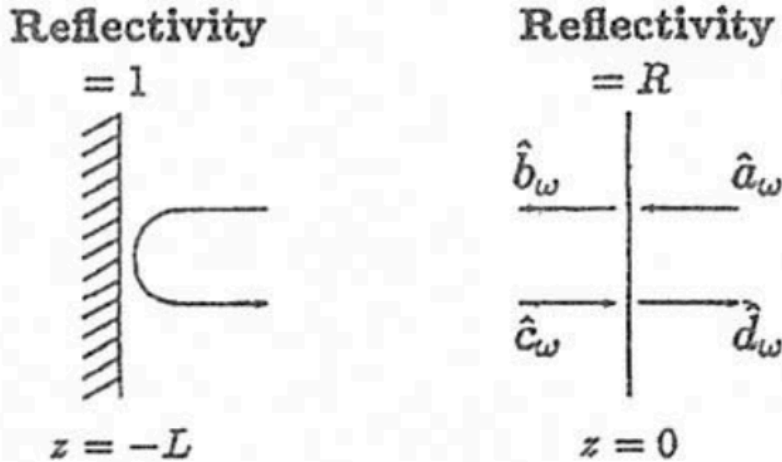


FIG. 1. Microcavity and field operators.

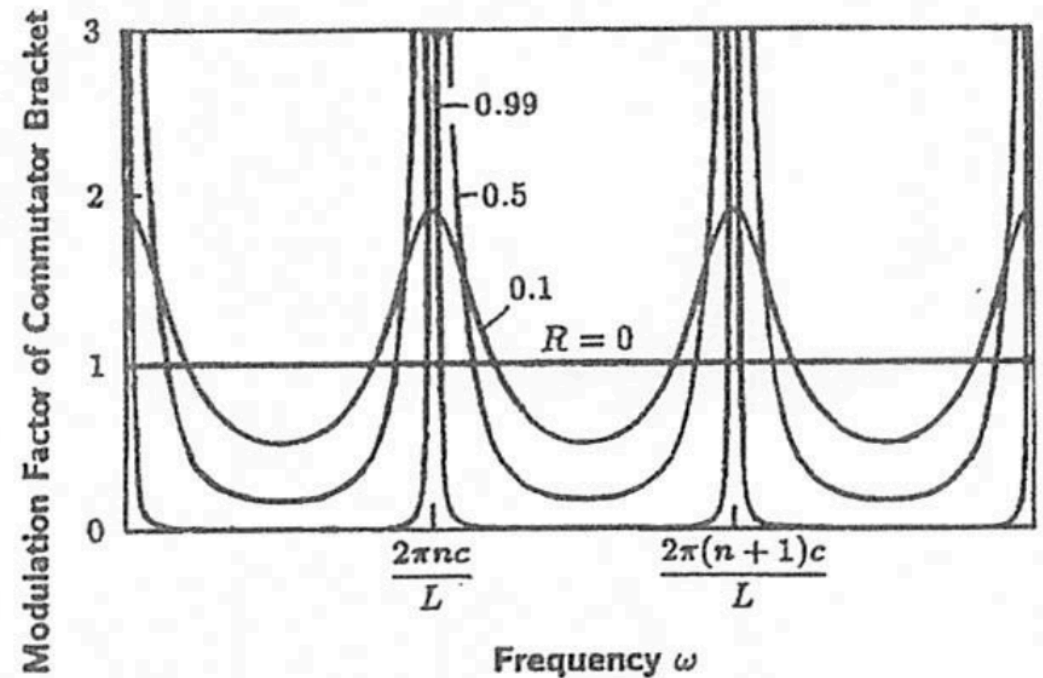


FIG. 2. The commutator-bracket value inside a microcavity as a function of the wave number k for several values of the reflectivity R .

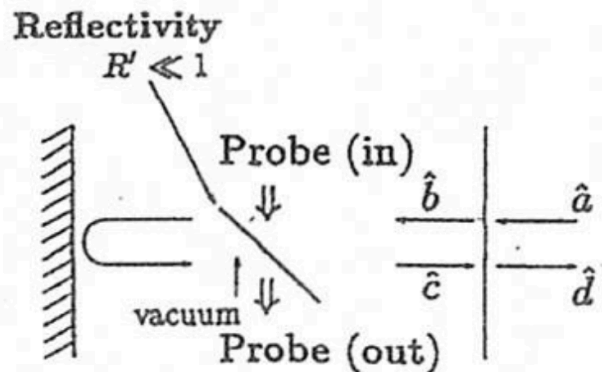
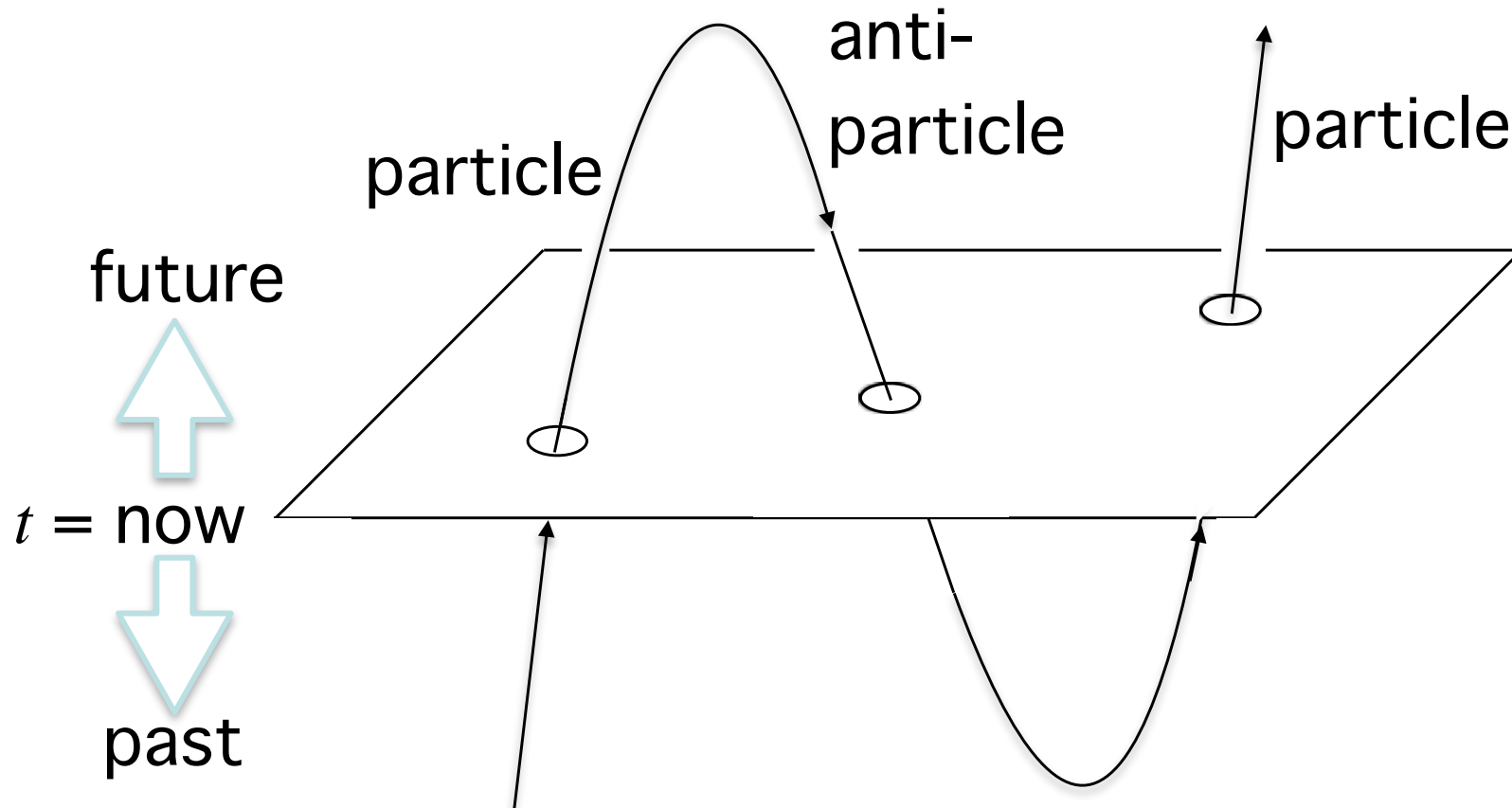


FIG. 3. Microcavity with a probe beam splitter inside.

Question:
 How about “cavity
 in time domain”?

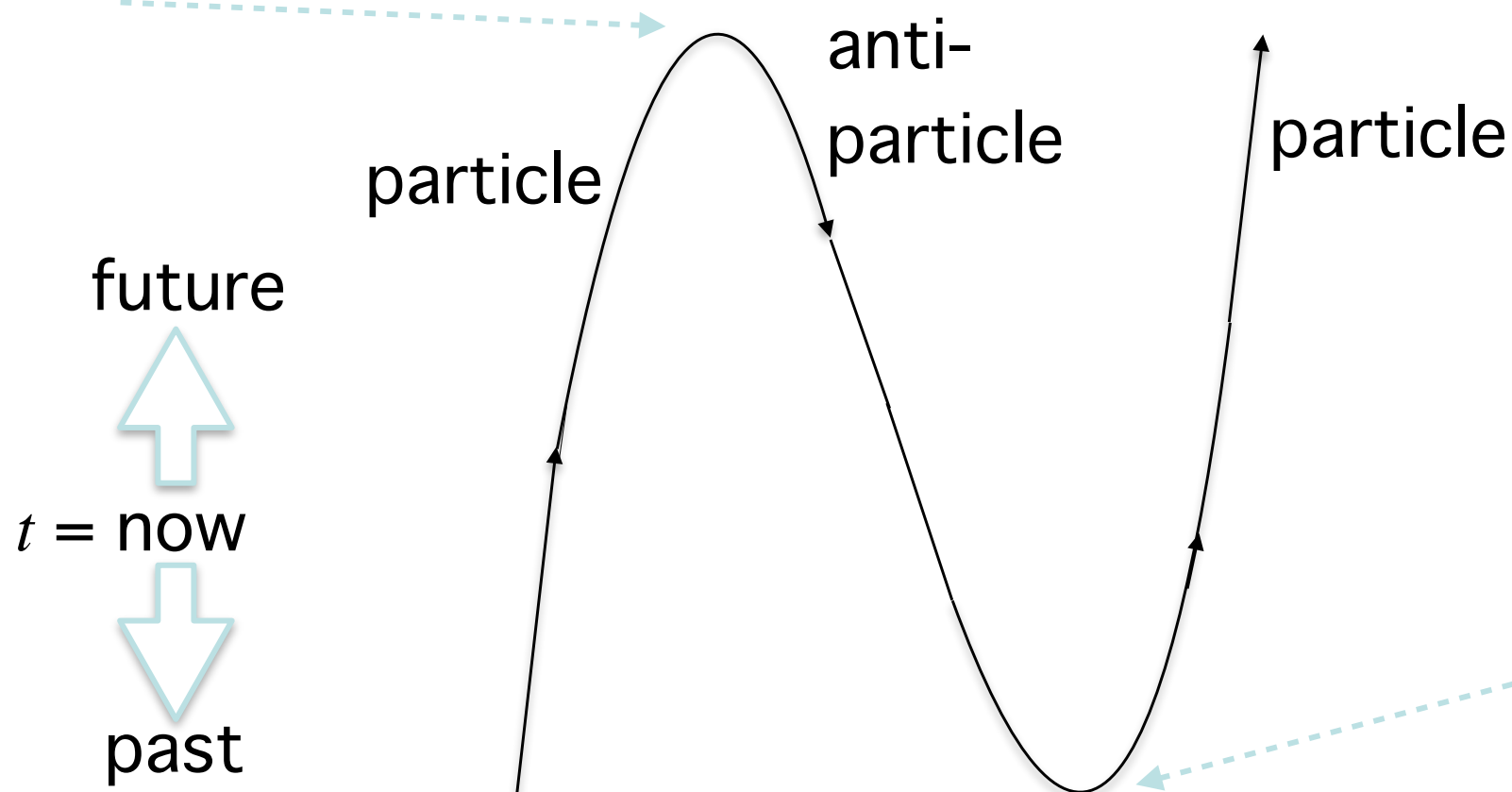
Consider the following situation:

It is a famous story that Wheeler said "Why are electrons indistinguishable? That's because they are single electron incarnate."



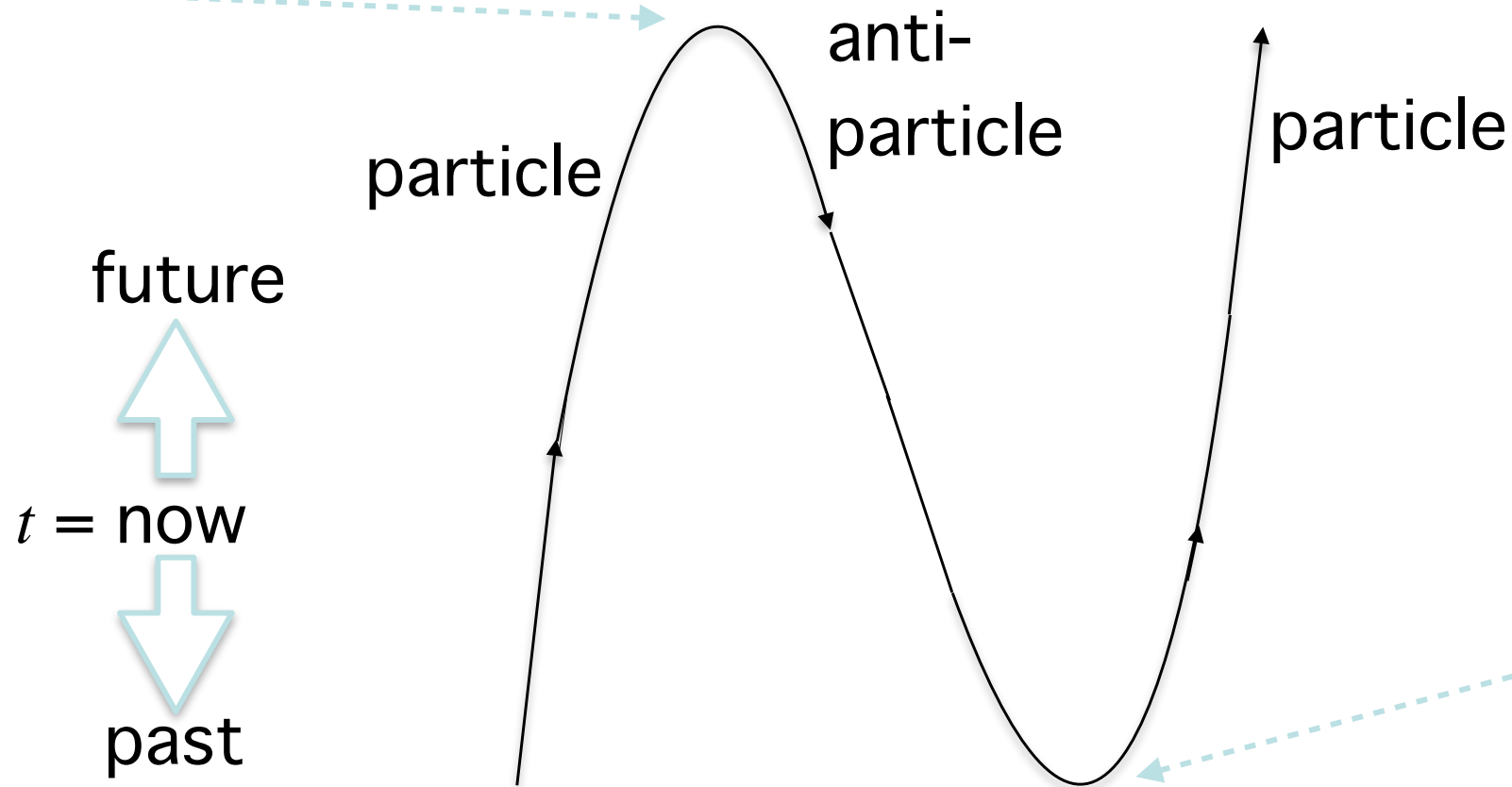
How can you observe this phenomena?

Usually, you just wait this (particle/anti-particle generation + recombination) spontaneously happens in the vacuum.

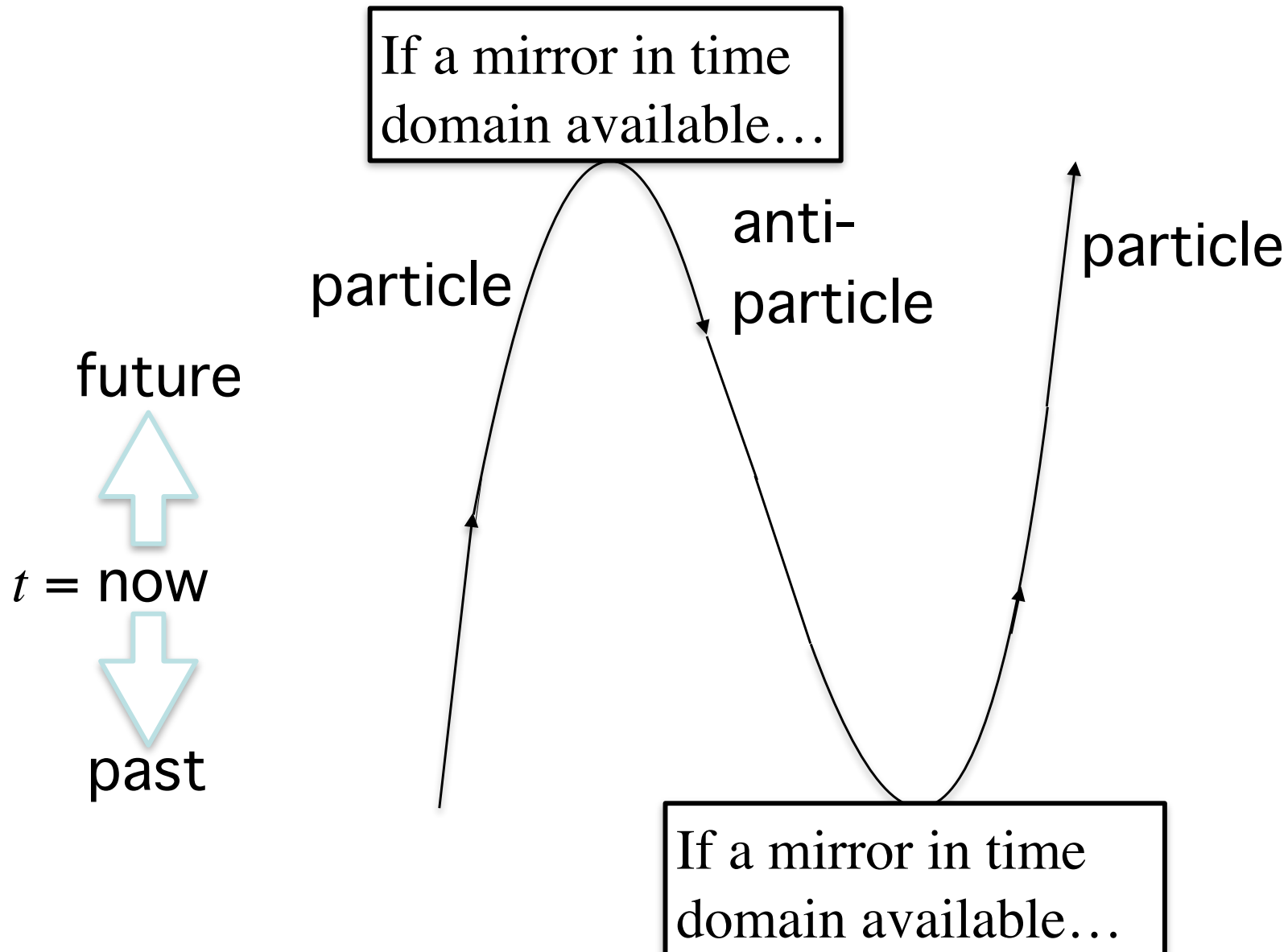


Usually, you just wait this (particle/anti-particle generation + recombination) happens from vacuum.

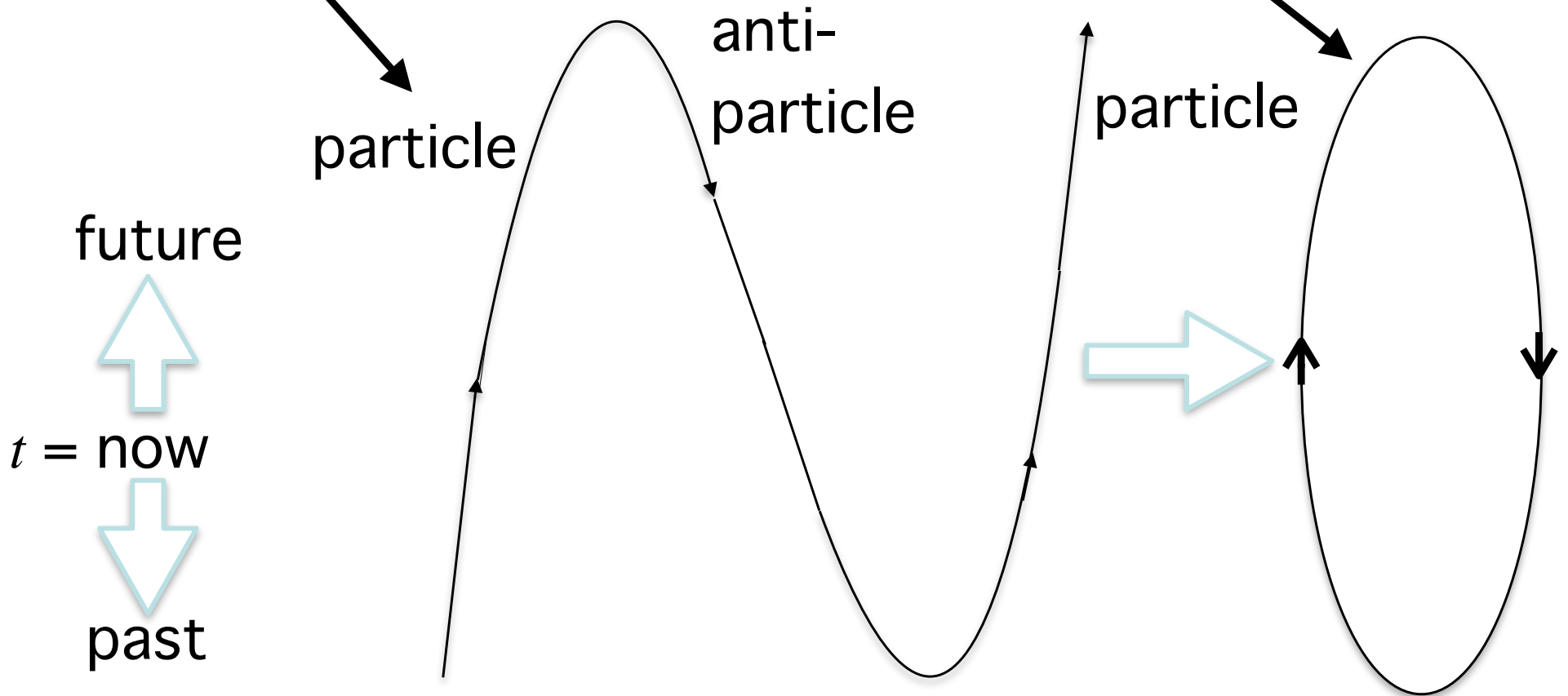
But I want to ask: isn't there any possibility of engineering this happen?



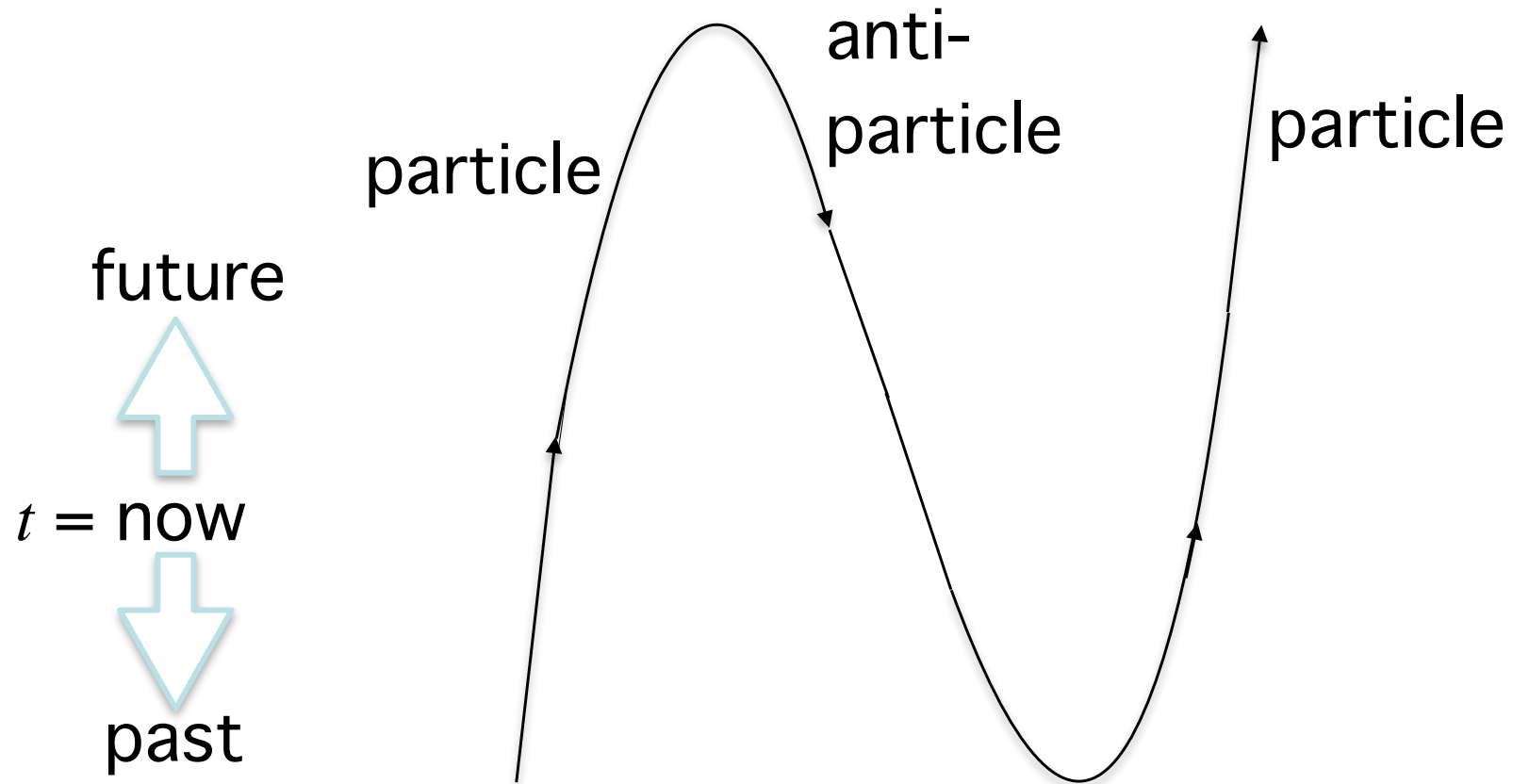
If you have mirrors in time domain, however, you may engineer this happen.



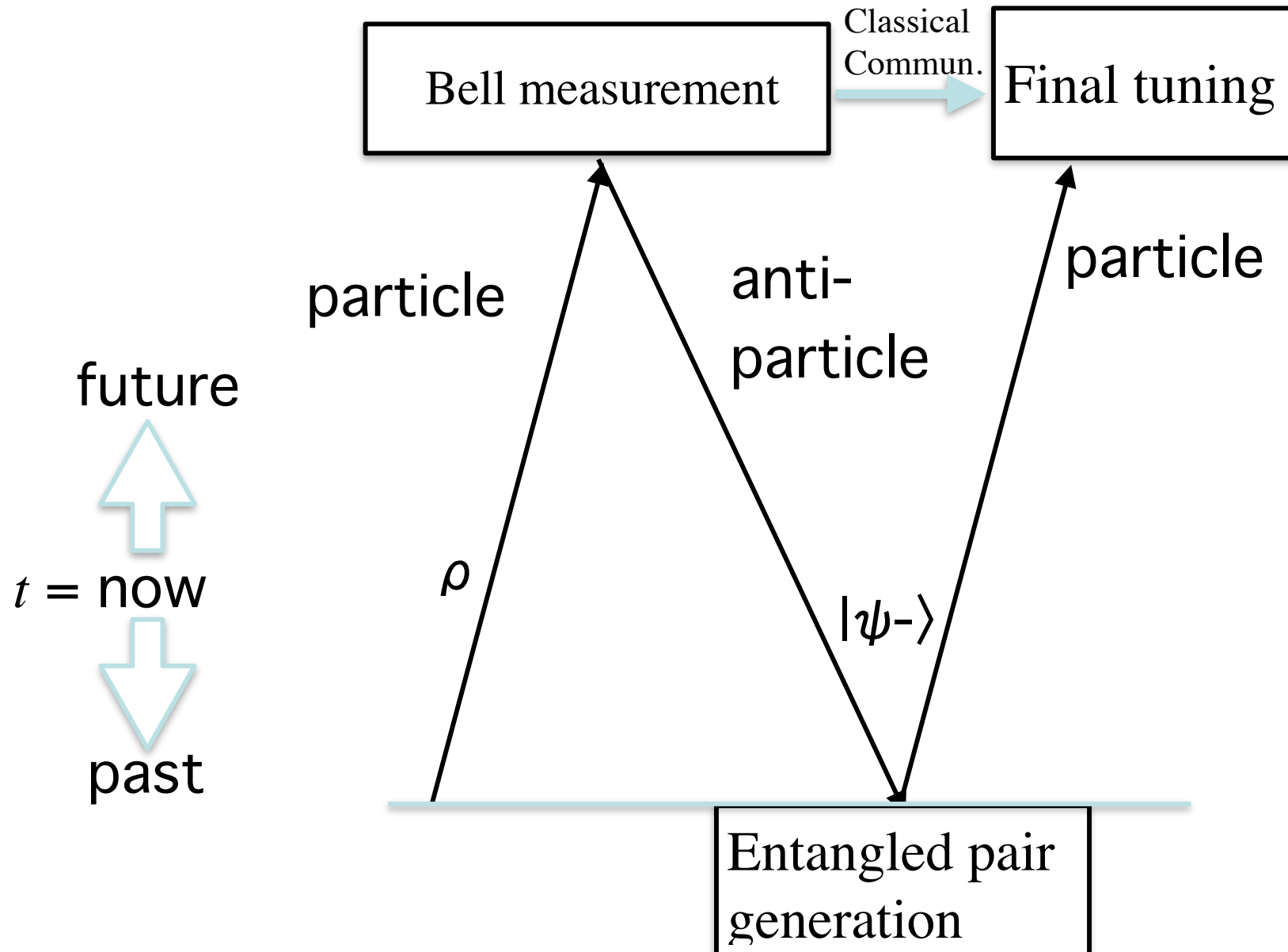
If this kind of thing could be reproduced controllably,
then, time-domain resonators also could be built.



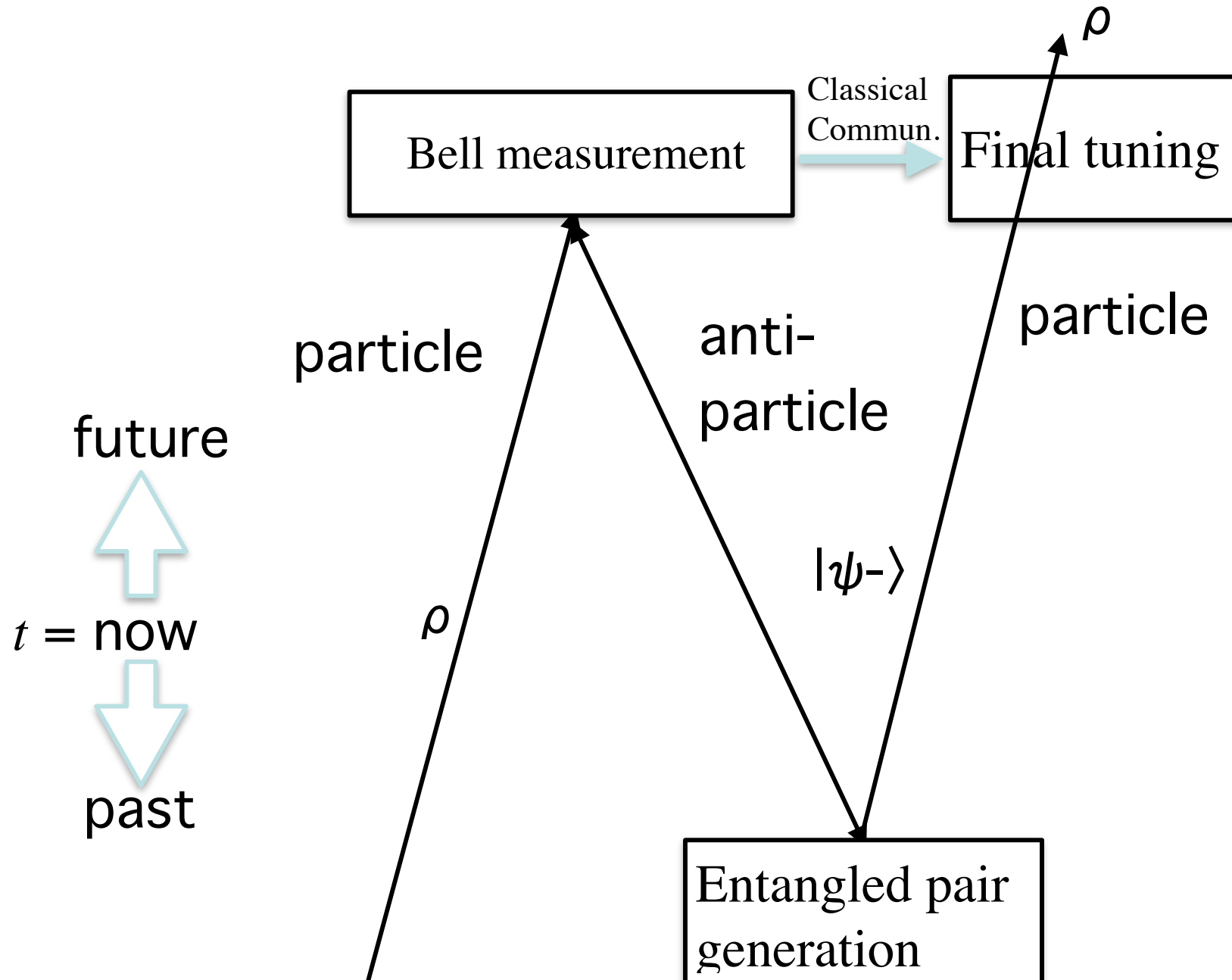
Quantum teleportation can do it!



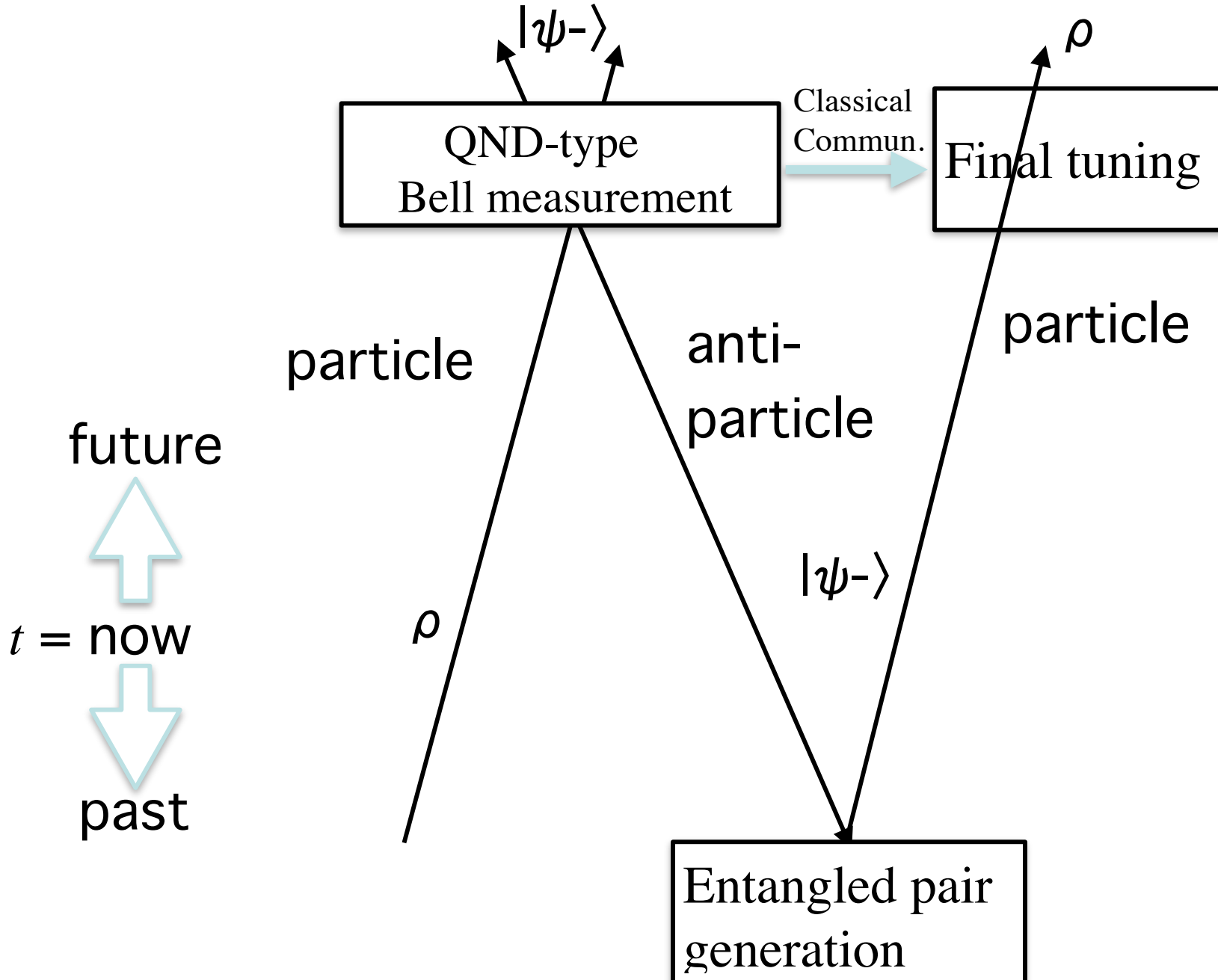
Quantum teleportation can do it!

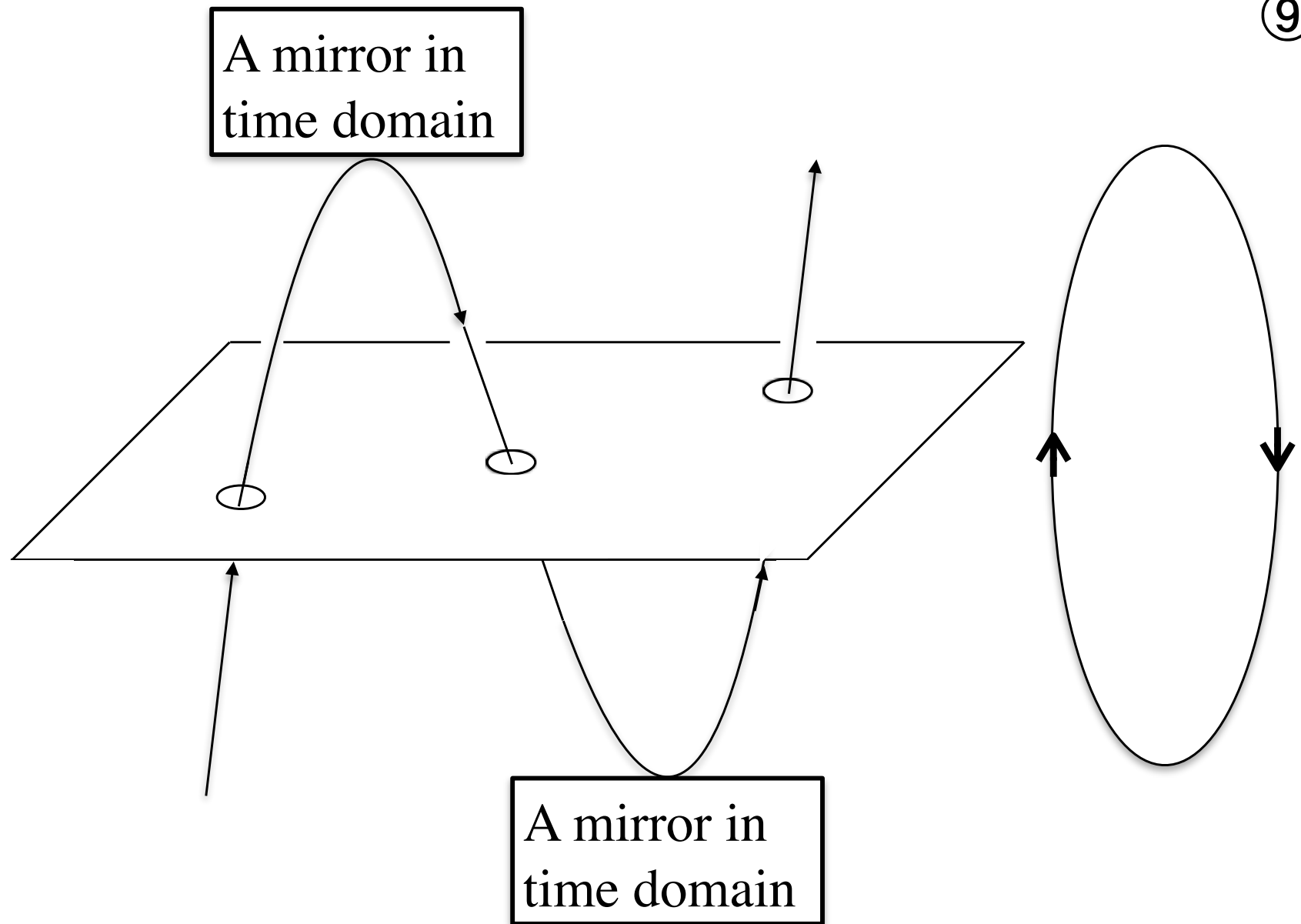


Quantum teleportation can do it!



Quantum teleportation can do it!





How to realize the “closed-loop time domain cavity” is the future problem.

Thank you for your attention!

Appendix: Field commutators are always normal even in cavities.

As discussed in slide ⑦, we derived that the commutator $[a(\omega), a^\dagger(\omega')]$ becomes anomalous, which is because the ω modes are incompatible with the resonator modes [PRA50,89 (1994)]. (This anomaly is related to the Purcell effect.)

In this appendix, we show that the field commutators are normal even inside the resonator. This was published in PRL77, 1739 (1996). (See below).

VOLUME 77, NUMBER 9

PHYSICAL REVIEW LETTERS

26 AUGUST 1996

Field Commutation Relations in Optical Cavities

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(Received 15 September 1995)*

We introduce a simple quantum theory of the lossy beam splitter. When applied to describe a Fabry-Pérot cavity this leads to apparently anomalous commutation relations for the intracavity operators. We show that these unfamiliar properties are nevertheless consistent with the fundamental canonical commutator for the vector potential and electric field operators. This result is derived as a consequence of causality as applied to the properties of mirror reflection coefficients. [S0031-9007(96)00953-2]

PACS numbers: 42.50.Dv, 03.65.Ca, 42.50.Lc