Phase transitions in the Rényi entropies of large-N interacting vector models in $2+1$ dimensions

- von Neumann entropy can be extracted from Renyi entropies $S_n(A)$ provided the limit $n \to 1$ exists.
- Renyi entropy of a disc-shaped region can be mapped to the free energy, evaluated as a path integral on $\mathbb{H}_2 \times S^1$. $\frac{1}{1-n} ln Tr \rho^n = \frac{2\pi Rn}{n-1} F(\beta = 2\pi nR)$ [CHM].
- We consider the disc to be in the interacting $O(N)$ vacuum and use the large N limit (saddle-point)+ Hubbard Stratonovich to solve.
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$$(-\partial^2 + m^2 + g\sigma)\phi = 0; \quad \phi \text{ is the vev}$$

$$\phi^2 - \sigma + \frac{1}{Vol(\mathbb{H}_2)} tr\left(\frac{1}{-\partial^2 + m^2 + g\sigma}\right) = 0.$$

- We find strong evidence for an ordering phase transition at $n = 1$. But, what does this imply for the replica trick?