Holographic Entanglement Entropy renormalization through extrinsic counterterms

Based on 1803.04990, 1806.10708 and work in progress

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Holographic Entanglement Entropy

- EE is defined as the von Neumann Entropy of reduced density matrix for subsystem A:

$$S_{EE} = -tr\left(\widehat{\rho}_A \ln \widehat{\rho}_A\right).$$

- In AdS/CFT, for Einstein-AdS bulk gravity, EE can be computed using area prescription of Ryu-Takayanagi [hep-th/0603001]:

$$S_{EE} = \frac{Vol(\Sigma)}{4G}.$$

- Σ is minimal surface in AdS bulk. $\partial\Sigma$ at spacetime boundary B is required to be conformally cobordant to entangling surface ∂A at conformal boundary C.

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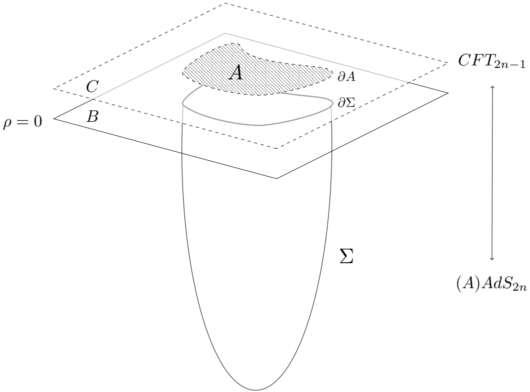
Ryu-Takayanagi Construction



Replica Trick

- Computation of S_{EE} reduced to evaluating Euclidean on-shell action I_E for gravity dual on conically singular manifold $\widehat{M}_D^{(\alpha)}$ with angular deficit of $2\pi(1-\alpha)$.
- $\widehat{M}_D^{(\alpha)}$ is the bulk gravity dual of the CFT replica orbifold defined in the replica-trick construction (Cardy and Calabrese [0905.4013]). It is sourced by codimension-2 cosmic brane with tension $T(\alpha) = \frac{(1-\alpha)}{4G}$, coupled through NG action for Einstein gravity. (Dong [1601.06788]; Lewkowycz and Maldacena [1304.4926]). Brane becomes RT surface in tensionless limit.
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Euclidean Einstein-Hilbert Action and Ryu-Takayanagi

- Consider Euclidean EH action evaluated in orbifold $\widehat{M}_D^{(\alpha)}$,

$$I_E^{EH} = \frac{1}{16\pi G}\left(\int\limits_{\hat{M}_D^{(\alpha)}} d^D x\sqrt{\mathcal{G}}\left(R^{(\alpha)} - 2\Lambda\right)\right).$$

- Using that

$$R^{(\alpha)} = R + 4\pi\left(1-\alpha\right)\delta_{\Sigma}$$

(Fursaev, Patrushev and Solodukhin [1306.4000]), S_{EE} is then given by area prescription of RT.

- EH action is divergent → S_{EE} is divergent. Use renormalized action to obtain universal part of HEE.

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Renormalization through extrinsic counterterms

- Scheme (Olea [hep-th/0504233]; [hep-th/0610230]) considers counterterms depending explicitly on both intrinsic $\mathcal{R}_{ijkl}$ and extrinsic curvatures K_{ij} of the boundary (FG foliation).

$$I_{ren} = I_{EH} + c_d \int_{\partial M} B_d \left(h, K, \mathcal{R} \right).$$

- Boundary term is fixed. Different form for even and odd-dimensional bulks. For odd d, B_d is Chern form of Euler theorem.

$$\int\limits_{M_{d+1}} \mathcal{E}_{d+1} = (4\pi)^{\frac{(d+1)}{2}} \left(\frac{(d+1)}{2} \right)! \chi \left(M_{d+1} \right) + \int\limits_{\partial M_{d+1}} B_d .$$

- Unique value of coupling constant c_d provides well defined (Asymptotically Dirichlet) variational principle and finite action, consistent with correct thermodynamics. Agreement with standard holographic renormalization discussed in Miskovic and Olea [0902.2082]; Miskovic, Tsoukalas and Olea [1404.5993].

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General formulation of Extrinsic counterterms

$$B_d\left(h,K,\mathcal{R}\right)=d^dx\sqrt{-h}\left(d+1\right)\int\limits_0^1 dt\delta_{[j_1\cdots j_d]}^{[i_1\cdots i_d]}K_{i_1}^{j_1}\left(\frac{1}{2}\mathcal{R}_{i_2i_3}^{j_2j_3}-t^2K_{i_2}^{j_2}K_{i_3}^{j_3}\right)$$

$$\ldots\times\left(\frac{1}{2}\mathcal{R}_{i_{d-1}i_d}^{j_{d-1}j_d}-t^2K_{i_{d-1}}^{j_{d-1}}K_{i_d}^{j_d}\right),\ (d=\text{odd})$$

$$=d^dx\sqrt{-h}d\int\limits_0^1 dt\int\limits_0^t ds\delta_{[j_1\cdots j_d]}^{[i_1\cdots i_d]}K_{i_1}^{j_1}\delta_{i_2}^{j_2}\left(\frac{1}{2}\mathcal{R}_{i_3i_4}^{j_3j_4}-t^2K_{i_3}^{j_3}K_{i_4}^{j_4}\right.$$

$$\left.+\frac{s^2}{\ell^2}\delta_{i_3}^{j_3}\delta_{i_4}^{j_4}\right)\ldots\left(\frac{1}{2}\mathcal{R}_{i_{d-1}i_d}^{j_{d-1}j_d}-t^2K_{i_{d-1}}^{j_{d-1}}K_{i_d}^{j_d}+\frac{s^2}{\ell^2}\delta_{i_{d-1}}^{j_{d-1}}\delta_{i_d}^{j_d}\right),$$

$$(d=\text{even})$$

Extrinsic counterterms reproduce correct thermodynamics

- For Schwarzschild-AdS in 4D:

$$\beta^{-1} I_E^{B_3} = \frac{1}{2} M - \frac{Vol\left(\Sigma_{k,2}\right)}{16\pi G} \lim_{r\to\infty}\left[\frac{\pi r^3}{\ell^2}\right]$$
$$\beta^{-1} I_E^{EH} = \frac{1}{2} M + \frac{Vol\left(\Sigma_{k,2}\right)}{16\pi G} \lim_{r\to\infty}\left[\frac{\pi r^3}{\ell^2}\right] - TS_{BH}$$

- For Schwarzschild-AdS in 5D:

$$\beta^{-1} I_E^{B_4} = \frac{1}{3} M - \frac{Vol\left(\Sigma_{k,3}\right)}{16\pi G} \lim_{r\to\infty}\left[\frac{2 r^4}{\ell^2}\right] + E_0$$
$$\beta^{-1} I_E^{EH} = \frac{2}{3} M + \frac{Vol\left(\Sigma_{k,3}\right)}{16\pi G} \lim_{r\to\infty}\left[\frac{2 r^4}{\ell^2}\right] - TS_{BH}$$

- For Schwarzschild-AdS in any dimension:

$$\beta^{-1} I_E^{ren} = \beta^{-1} I_E^{EH} + \beta^{-1} I_E^{B_d} = M + (E_0) - TS$$

E_0 only for d even

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Euler density and extrinsic surface terms for cones

- The Euler density in even 2n-dimensional conically singular manifolds obeys

$$\int_{M_{2n}^{(\alpha)}} \mathcal{E}_{2n}^{(\alpha)} = \int_{M_{2n}} \mathcal{E}_{2n}^{(r)} + 4\pi n (1-\alpha) \int_{\Sigma} \mathcal{E}_{2(n-1)} + \mathcal{O}\left((1-\alpha)^2\right)$$

(Fursaev and Solodukhin [hep-th/9501127]; Fursaev, Patrushev and Solodukhin [1306.4000])

- By the Euler theorem, the n-th Chern form obeys

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Euclidean action on replica orbifold

- In particular, we find that

$$I_E^{ren}\left[\widehat{M}_D^{(\alpha)}\right] = I_E^{ren}\left[\widehat{M}_D^{(\alpha)} \setminus \Sigma\right] + \frac{(1-\alpha)}{4G} Vol_{ren}\left[\Sigma\right].$$

- $Vol_{ren}\left[\Sigma\right]$ is the renormalized area of the cosmic brane with tension T (the RT surface for T $\to$ 0). (Anastasiou, I.J.A., Arias and Olea [1806.10708])
- Then, S_{EE}^{ren} is given by

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Topological form of renormalized HEE for odd-dimensional CFTs

- Using Euler theorem, for d=2n-1, renormalized HEE can be written as

$$S_{EE}^{ren} = -\frac{\ell^2}{8G(2n-3)}\left(\int_\Sigma d^{2n-2}y\sqrt{\gamma}\ell^{2(n-2)}P_{2n-2}[\mathcal{F}] - c_{2n-2}(4\pi)^{n-1}(n-1)!\chi[\Sigma]\right),$$

$$\mathcal{F}_{cd}^{ab} = \mathcal{R}_{cd}^{ab} + \frac{\delta_{[cd]}^{[ab]}}{\ell^2}$$

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- EE is separated into a geometric part ($\int P_{2n-2}[\mathcal{F}]$) and a purely topological part ($\chi[\Sigma]$).
- Geometric part is zero when extremal surface has constant curvature.
- Topological part is robust against continuous deformations of the entangling surface.
- For ball-shaped entangling regions, renormalized EE agrees with computation of universal part by Kawano, Nakaguchi and Nishioka [1410.5973]. Related to the F-quantity in 3D.

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Renormalized HEE for even-dimensional CFTs

- For even-d CFTs, the renormalized EE is logarithmically divergent and it corresponds to the universal part.
- It contains the information about the conformal anomaly of the CFT.
- In particular, for ball-shaped entangling regions, we have

$$S_{EE}^{ren} = 2\left(-1\right)^{n} \log\left(\varepsilon\right) A$$

$$A = \frac{\ell^{(2n-1)} \pi^{(n-1)}}{8G\left(n-1\right)!},$$

in agreement with Myers and Sinha [1006.1263].

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Interpretation of Results

- Renormalized EE equal to the universal part of EE. Related to parameters of CFT, e.g., a^*-charge (odd-d CFT) or A-anomaly coefficient (even-d CFT).
- a^* and the A-anomaly coefficient are conjectured to be C-function candidates (e.g., Myers and Sinha [1006.1263]).
- For odd-d CFTs, renormalized EE can be written as sum of topological invariant and polynomial in contractions of $\mathcal{F}$.
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