Continuous Tensor Network Renormalization for Quantum Fields

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Joint work with Qi Hu and Guifré Vidal

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YITP

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The renormalization group

[Kadanoff '66, Wilson '71,…]

Study and comparison of the behaviour of a physical system at different scales

Formalized as **RG flow:**

RG step = Coarse-graining + Rescaling

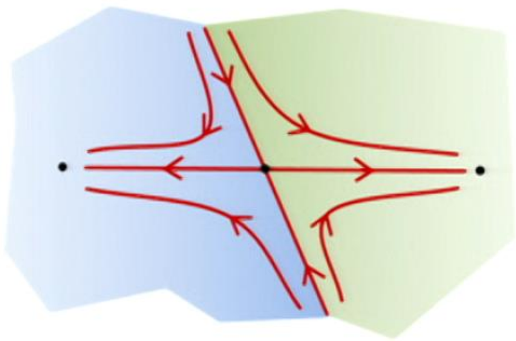
Fixed points of RG flow constitute **conformal field theories** (CFTs):

- Describe universality classes (second order phase transitions)
- Characterized by conformal data

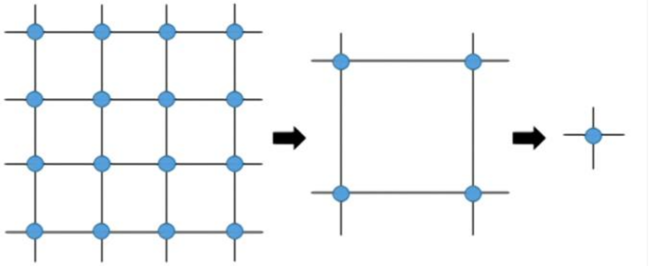
The renormalization group

Interesting problem: computational implementation of RG flow

Why?



Phase classification problems



Computational efficiency

Holography

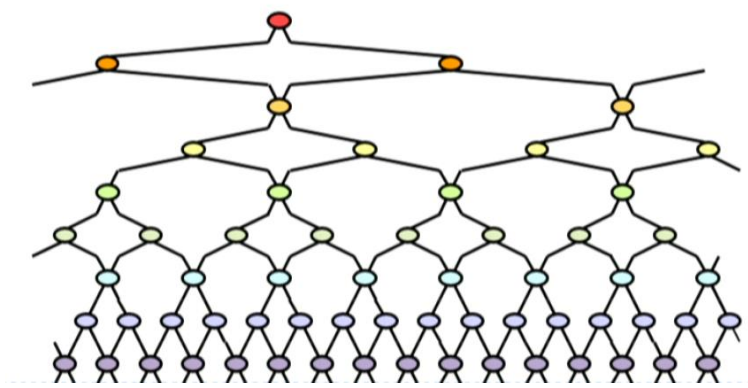
Tensor networks

Computational tools (+ physical insight!)

Allow for an **efficient representation and manipulation** of (e.g.)

quantum states

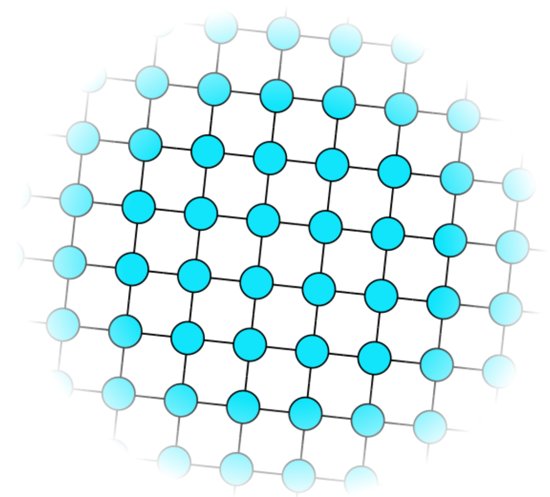
Including RG flow!



[MERA: Vidal '06]

statistical partition functions

Euclidean path integrals



[TRG: Levin, Nave '06] [TNR: Evenbly, Vidal '14]

…on the **lattice**.

Continuous tensor networks: A research program

Aim: import ideas and techniques from (lattice) tensor networks to be applied in the realm of **quantum field theory**.

Remark: Two main approaches to continuum limit

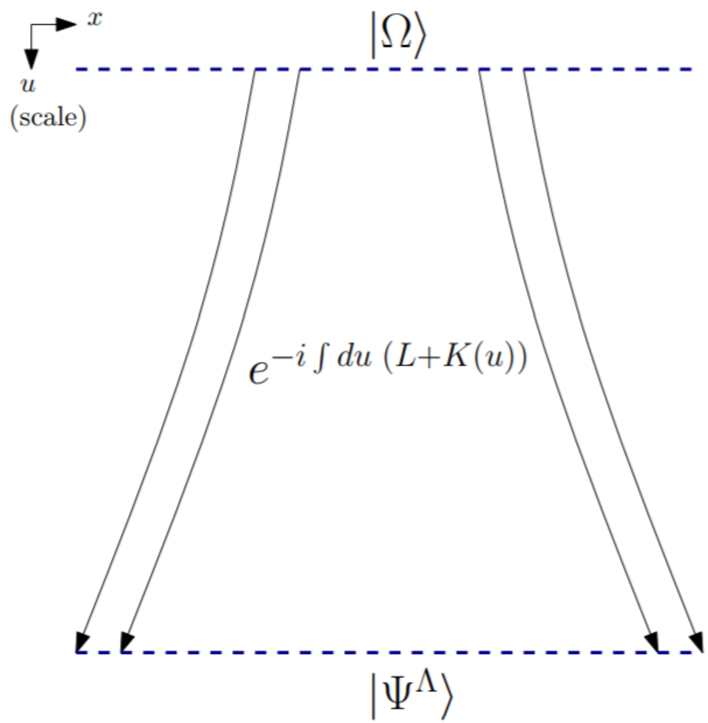
- "$N \to \infty$" limit: tensors become closer to a trivial tensor as their number diverges

- "Conceptual" limit: imitate tensor network constructions directly in a continuum setting

Continuous tensor networks: A research program

In particular, RG flow:

quantum states



[cMERA: Haegeman et al. ‘11]

statistical partition functions

Euclidean path integrals

This talk

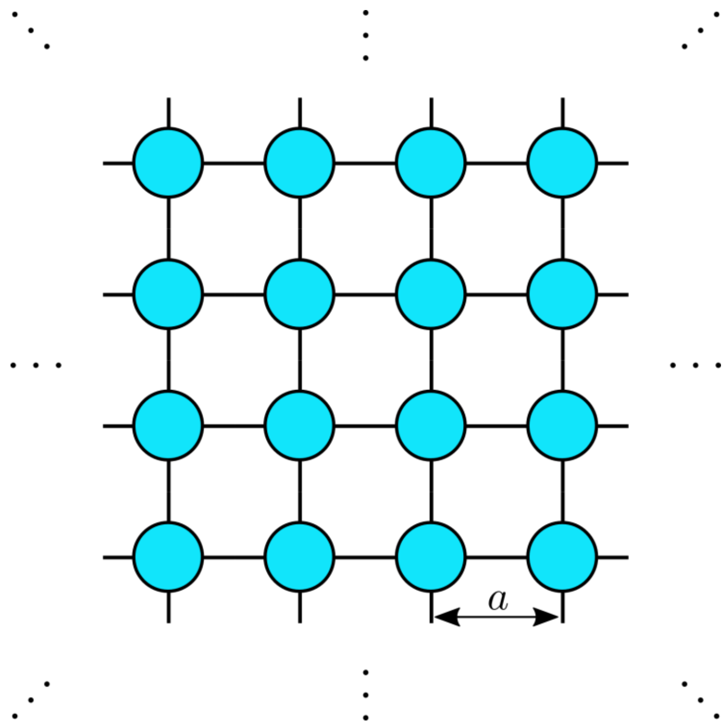
(See related work: Caputa et al., 2017, Bhattacharyya et al., 2018)

This talk

- ✓ Introduction and motivation
- Review of (lattice) Tensor Network Renormalization
- Our proposal: continuous Tensor Network Renormalization
- Proof-of-principle example: free boson
- Conclusion

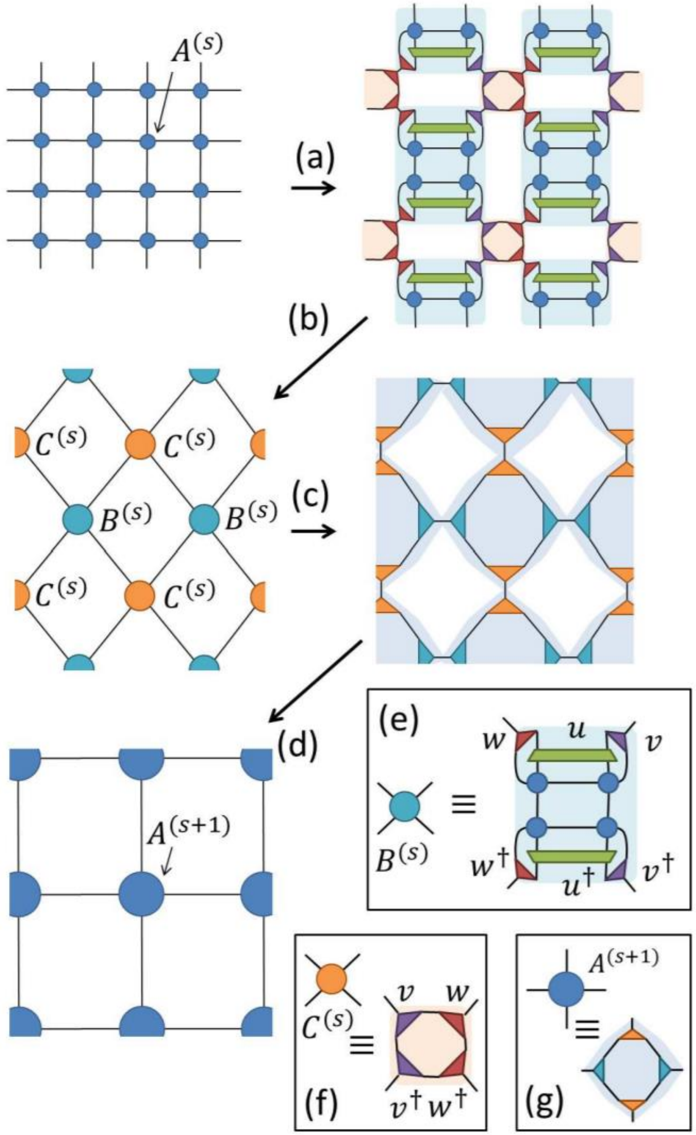
Lattice TNR - Setting

[Evenbly, Vidal '14]

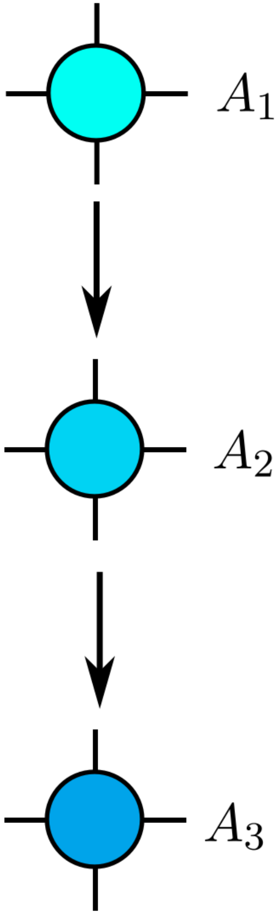


- Bonds represent degrees of freedom we sum over
- Tensors contain information of local Boltzmann weights
- Lattice spacing provides UV cutoff

Lattice TNR - Algorithm

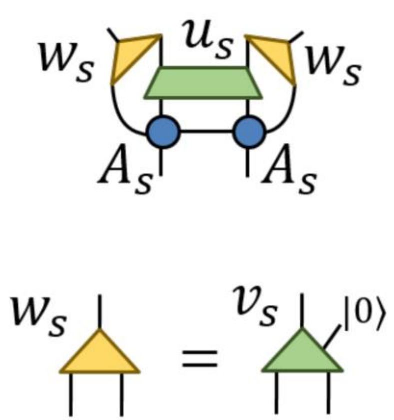


RG flow in the space of tensors!



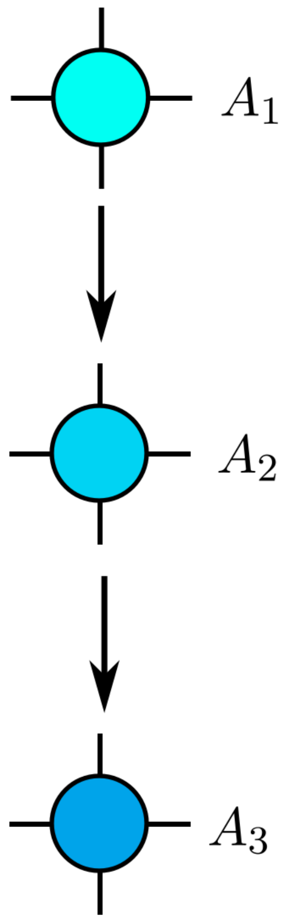
Lattice TNR - Algorithm

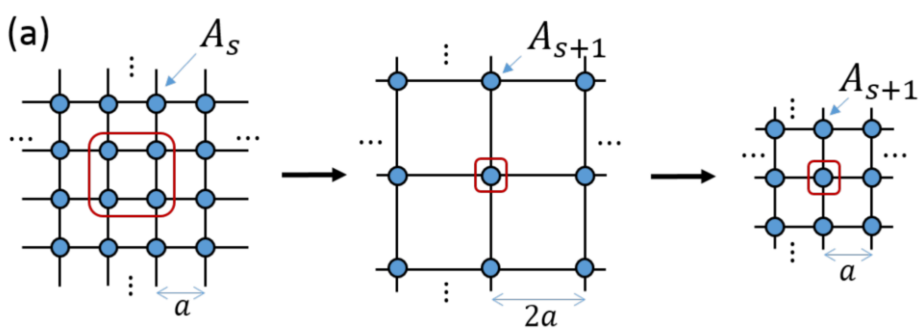
[Evenbly, Vidal '14]



Disentanglers and isometries (chosen variationally) provide a **local rearrangement of DOF**

RG flow in the space of tensors!





At each step, the lattice needs **rescaling**

Lattice TNR - Flow

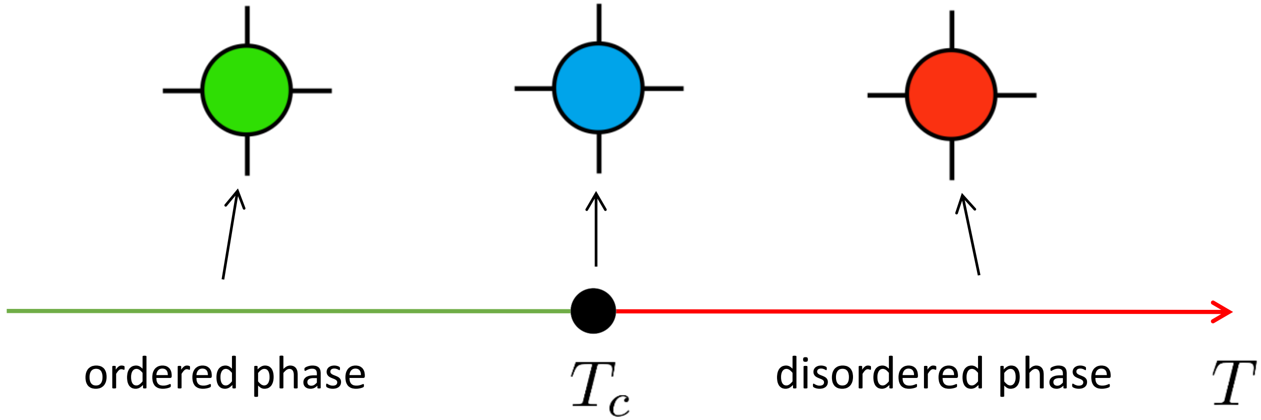
[Evenbly, Vidal '14]

Each phase flows to a fixed, **scale-invariant tensor**.

At criticality, **conformal data** are retrievable from the fixed point tensor!

<u>Example:</u> phase diagram of the 2D lattice Ising model

$$Z = \sum_{\{\sigma\}} e^{-H(\{\sigma\})/T}, \quad H(\{\sigma\}) = -\sum_{\langle i,j \rangle} \sigma_i \sigma_j$$



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- Proof-of-principle example: free boson
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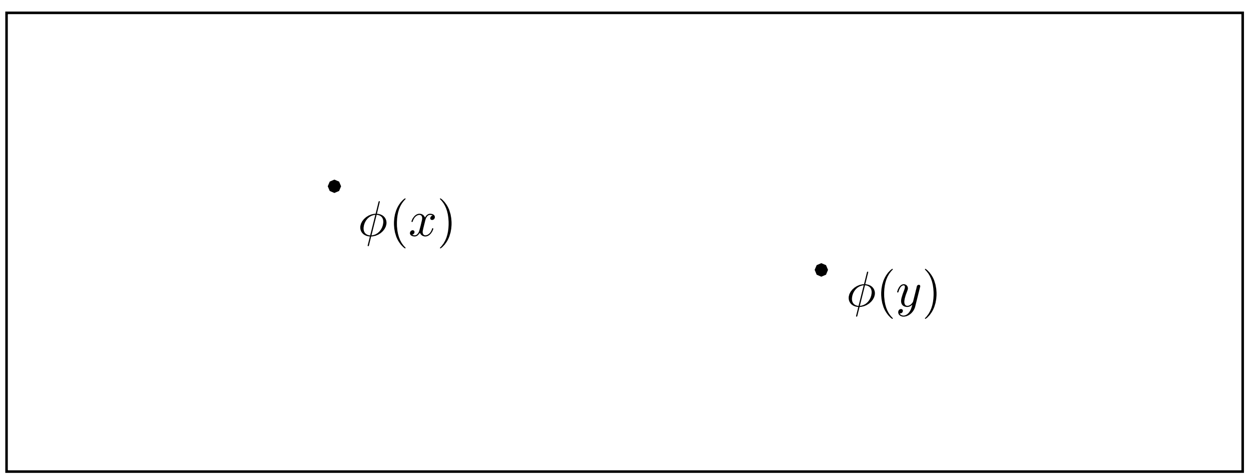
cTNR - Setting

[Hu, A.F.-R., Vidal ‘18]

Euclidean Path Integral…

$$Z = \int [D\phi]\, e^{-S[\phi(x)]}$$



cTNR - Setting

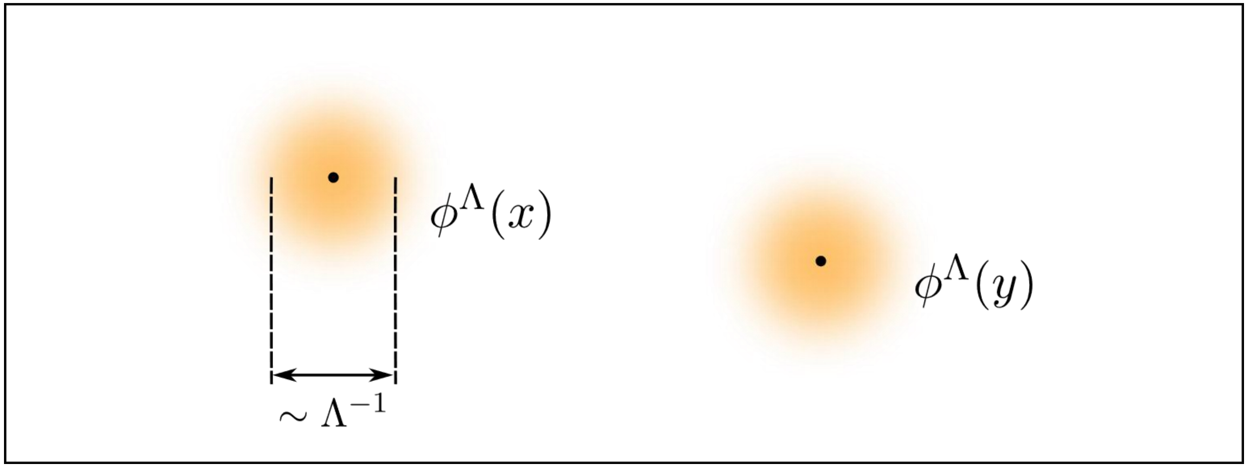
[Hu, A.F.-R., Vidal '18]

Euclidean Path Integral...

$$Z^{\Lambda} = \int [D\phi] \, e^{-S^{\Lambda}[\phi(x)]}$$

...with smeared fields

$$\phi^{\Lambda}(\mathbf{x}) := \int d\mathbf{y} \; \mu(|\mathbf{x}-\mathbf{y}|)\phi(\mathbf{y}) \qquad S^{\Lambda}[\phi] := S[\phi^{\Lambda}]$$



cTNR - Setting (Example)

[Hu, A.F.-R., Vidal '18]

We work with the free Klein-Gordon field:

$$S[\phi] = \frac{1}{2} \int d\mathbf{x} \left[\phi(\mathbf{x}) \nabla^2 \phi(\mathbf{x}) + m^2 \phi(\mathbf{x})^2 \right]$$

cTNR - Setting (Example)

[Hu, A.F.-R., Vidal '18]

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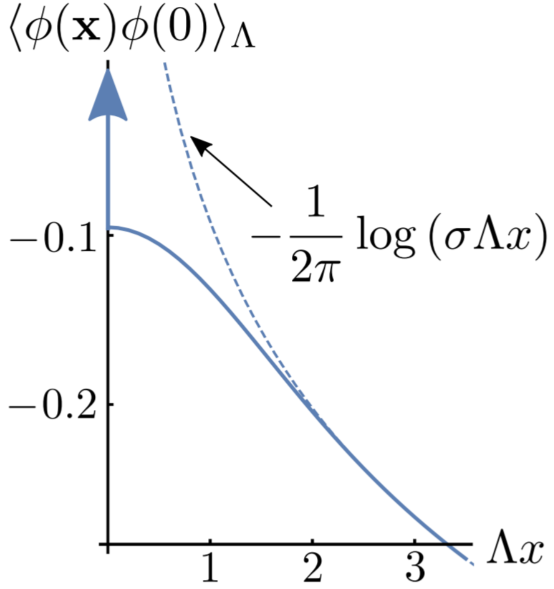
$$S^{\Lambda}[\phi] = \frac{1}{2}\int d\mathbf{x}\,\left[\phi^{\Lambda}(\mathbf{x})\nabla^2\phi^{\Lambda}(\mathbf{x}) + m^2\phi^{\Lambda}(\mathbf{x})^2\right]$$

The two-point function goes to a constant at scales smaller than the cutoff



cTNR - Algorithm

[Hu, A.F.-R., Vidal '18]

Don't forget goal: implement RG flow at the level of Euclidean path integrals

$$\phi(\mathbf{x}) \to \phi(\mathbf{x}) + \delta\phi(\mathbf{x}), \qquad \delta\phi(\mathbf{x}) = (L + K_s)\phi(\mathbf{x})$$

Don't forget the lattice: local rearrangement of DOF + rescaling

Scaling

$$L\phi(\mathbf{x}) = (-\mathbf{x}\cdot\nabla - \Delta_\phi)\phi(\mathbf{x})$$

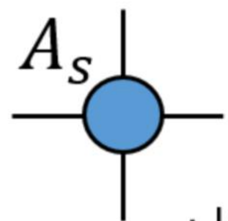
Disentangling

$$K_s\phi(\mathbf{x}) = F(s, \phi^\Lambda(\mathbf{x}), \nabla^2\phi^\Lambda(\mathbf{x}), \ldots)$$

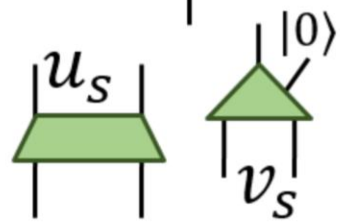
cTNR - Algorithm

Lattice – continuum analogy

lattice spacing $a \longleftrightarrow 1/\Lambda$ smearing length



$\longleftrightarrow \quad e^{-dx\,\mathcal{L}_s^{\Lambda}}$



$\longleftrightarrow \quad K_s$

RG flow of the Euclidean path integral

$$Z_s^{\Lambda} \equiv \mathcal{P}e^{\int_0^s du\ (L+K_u)} \qquad Z^{\Lambda} = \int [d\phi] e^{-S_s^{\Lambda}[\phi]}$$

cTNR – Algorithm (Example)

Linear Ansatz for the disentangler:

$$K_s\phi(\mathbf{x}) = \int d\mathbf{y}\; g(s, |\mathbf{x}-\mathbf{y}|)\phi(\mathbf{y})$$

Massless free boson is a CFT, so we impose an RG **fixed point condition**:

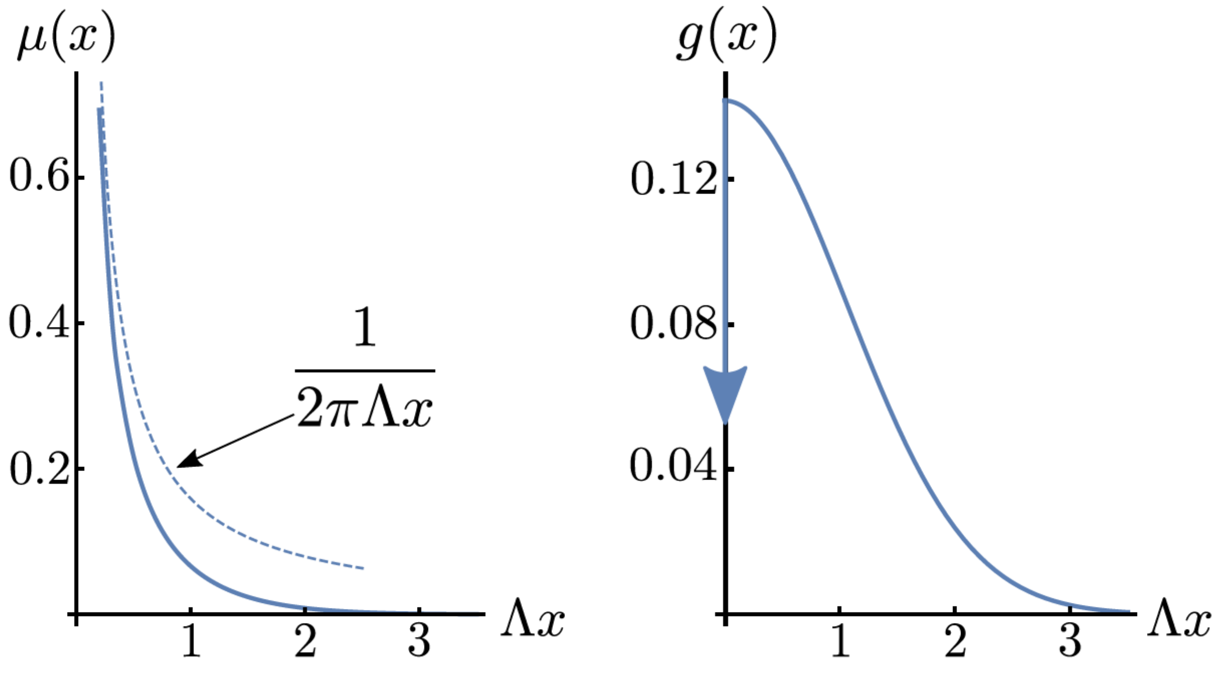
$$(L + K_\star)\; S_\star^\Lambda[\phi] = 0$$

(+ the disentangler *does not depend on scale*)

cTNR – Algorithm (Example)

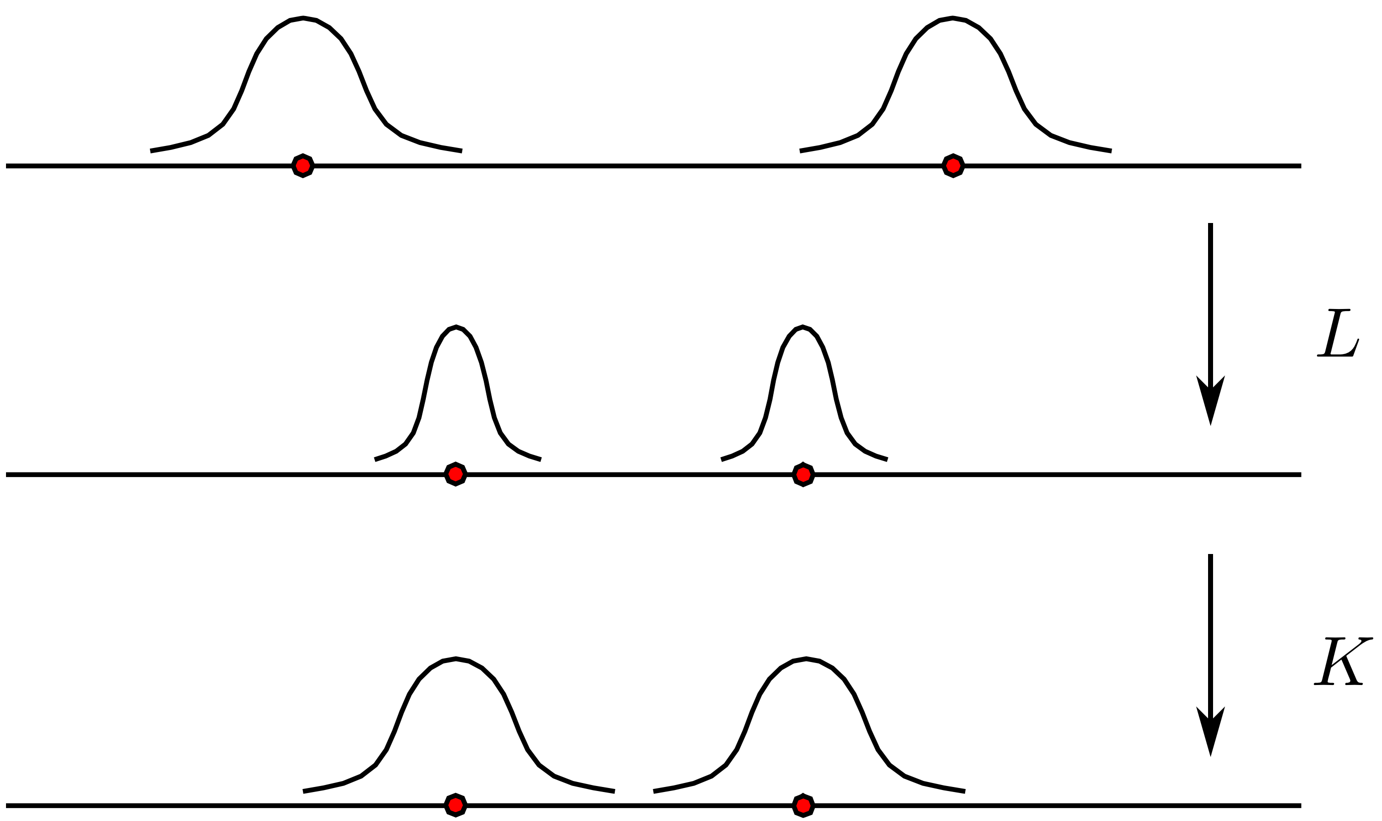
Success! The fixed point condition translates into $g(k) = \frac{k\partial_k \mu(k)}{\mu(k)}$

Choosing appropriately quasilocal functions, we find an explicit realization for which the regularized free boson CFT is a fixed point of cTNR:



cTNR – Algorithm (Example)

Understanding the transformations:



The net effect of both generators on the cutoff is to leave it **invariant**

cTNR – Algorithm (Example)

If we add a mass, we obtained the expected flow to a **massive fixed point**:

$$S_s^{\Lambda}[\phi] \equiv e^{s(L+K_\star)} S^{\Lambda}[\phi]$$
$$= \frac{1}{2} \int d\boldsymbol{x} \Big(- \phi^{\Lambda}(\boldsymbol{x}) \Delta \phi^{\Lambda}(\boldsymbol{x}) + m^2 e^{2s} \phi^{\Lambda}(\boldsymbol{x})^2 \Big)$$

We can also recover **correct conformal data** from a fixed point!

$$(L + K_\star)\ O_\alpha^{\Lambda}(\mathbf{0}) = -\Delta_\alpha\ O_\alpha^{\Lambda}(\mathbf{0}),$$

(rotation generator) $\rightarrow$ $$R\ O_\alpha^{\Lambda}(\mathbf{0}) = s_\alpha\ O_\alpha^{\Lambda}(\mathbf{0}),$$

Free boson case → Smeared version of the original CFT scaling operators

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Take-home messages

- Tensor networks are able to realize the **RG flow on the lattice**.

- We propose a general scheme for a similar implementation of the RG flow for **continuous** partition functions / Euclidean path integrals.

- This scheme can be applied to the free boson theory, yielding the **correct fixed point behavior**, and the correct conformal data.

Outlook

- Extension to fermions and gauge theories (19XX.XXXX)

- Move past analytic solutions: truly variational algorithm (probably in parallel to cMERA)

Thanks!

To know more: arXiv 1809.05176

(or let’s talk!)

Acknowledgements

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