Holographic computation of quantum correction in the bulk

- Dual covariant perturbation -

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Ref. S.Aoki-J.Balog-SY PTEP 2019 (2019) no.4, 043

Covariant perturbation in gravity

Consider Einstein-Hilbert action in any dimensions

$$S = \frac{1}{16\pi G_N} \int d^D x \sqrt{|g|} R$$

$g = \mathrm{det} g_{\mu\nu}$ R: Ricci scalar, $G_N \sim \kappa^2$: Newton constant

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Dimension of the Newton constant:

$$[\hbar] = [\frac{1}{\kappa^2}] \times L^D \times L^{-2} \quad \rightarrow \quad [\kappa] = \sqrt{\frac{L^{D-2}}{[\hbar]}} \simeq M^{1-D/2}$$

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A: Yes, in holography!

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Boundary ⟷ **Bulk**

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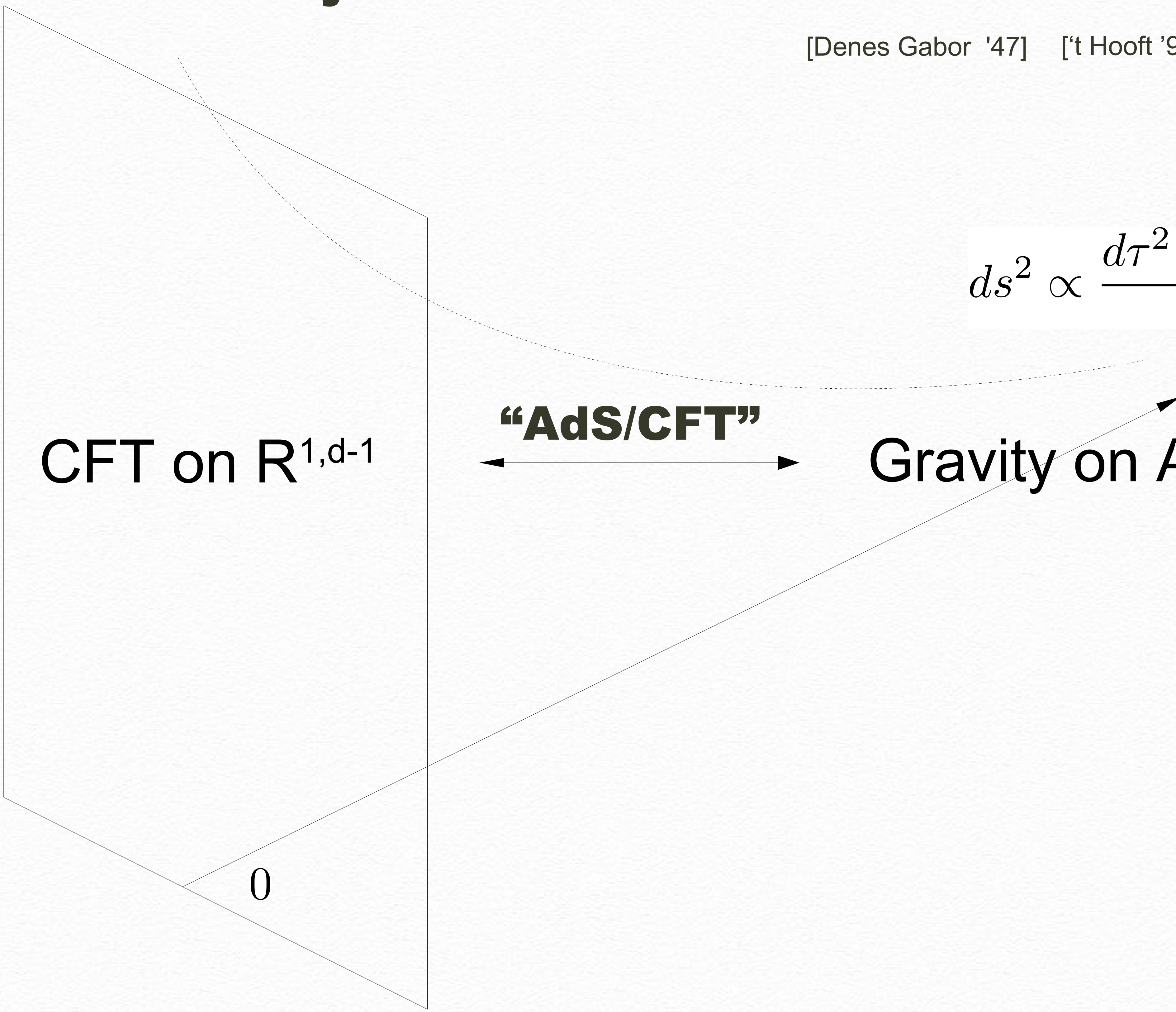
$$ds^2 \propto \frac{d\tau^2 + (dx^\mu)^2}{\tau^2}$$



CFT on $R^{1,d-1}$ ⟷ **"AdS/CFT"** ⟷ Gravity on AdS_{d+1}

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Lifshitz FT on $R^{1,d-1}$ ⟷ Gravity on $Lifs_{d+1}$

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[Kachru-Liu-Mulligan '08]

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Flow equation approach

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cf. [Aoki-SY-Yoshida '19]

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Consider a free O(n) vector model and smear a vector field $v^a(x)$ by a free **flow equation**:

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The solution is

flowed operator

$$\phi^a(x;\eta) = \int d^d y\, K(x-y;\eta) v^a(y).$$

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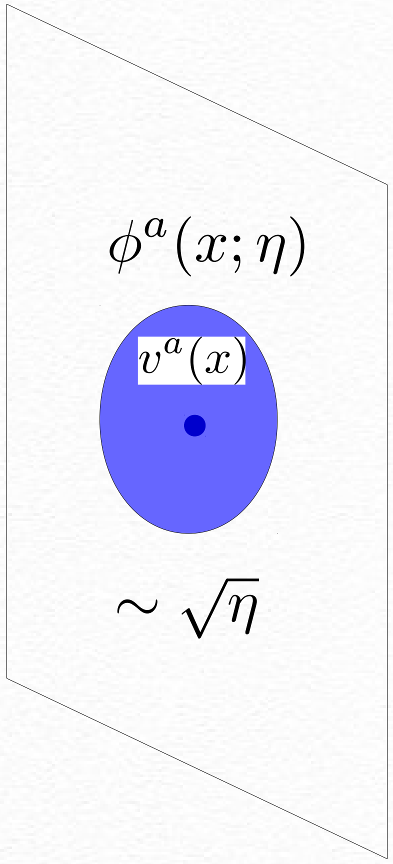

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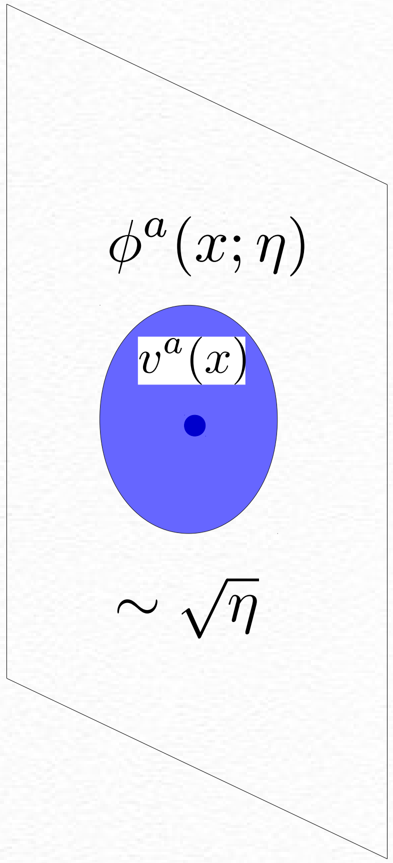

Claim: **UV singularity in the coincidence limit is resolved**.

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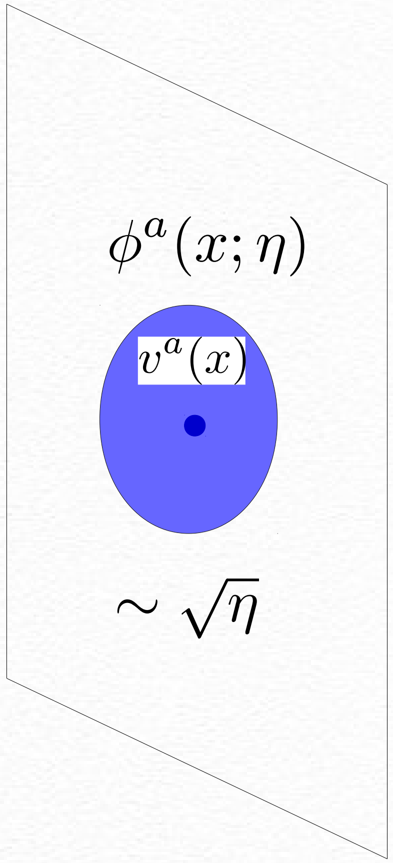

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$$\because \quad \langle \phi^a(x_1;\eta_1)\phi^b(x_2;\eta_2)\rangle = \delta^{ab} F(x_{12}^2;\eta_+) = \frac{\delta^{ab}}{\eta_+^{\Delta}} F\left(\frac{x_{12}^2}{\eta_+};1\right)$$

$$\eta_+ := \eta_1 + \eta_2$$

$$x_{12} := x_1 - x_2$$

$$F(x;1) = \frac{1}{(4\pi)^{\frac{d}{2}}} \int_0^1 du e^{-xu/4} u^{\Delta - 1}$$

$$\Delta = \frac{d-2}{2}$$

Construction of holographic space

[Aoki-Kikuchi-Onogi ’15]

[Aoki-Balog-Onogi-Weisz ’16,'17]

[Aoki-SY ’17]

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Def. (Dimensionless normalized operator)

$$\sigma^a(x;\eta) := \frac{\phi^a(x;\eta)}{\sqrt{\langle \sum_b \phi^b(x;\eta)^2 \rangle}}$$

"Operator renormalization"

NOTE: $\langle \sigma^a(x;\eta)\sigma^a(x;\eta)\rangle = 1$

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$$\hat{g}_{MN}(x;\eta) := \frac{\partial \sigma^a(x;\eta)}{\partial z^M} \frac{\partial \sigma^a(x;\eta)}{\partial z^N}$$

$$g_{MN}(z) := \langle \hat{g}_{MN}(x;\eta) \rangle_{CFT}$$

$$z^M = (x^\mu, \tau) \text{ with } \tau \propto \sqrt{\eta}$$

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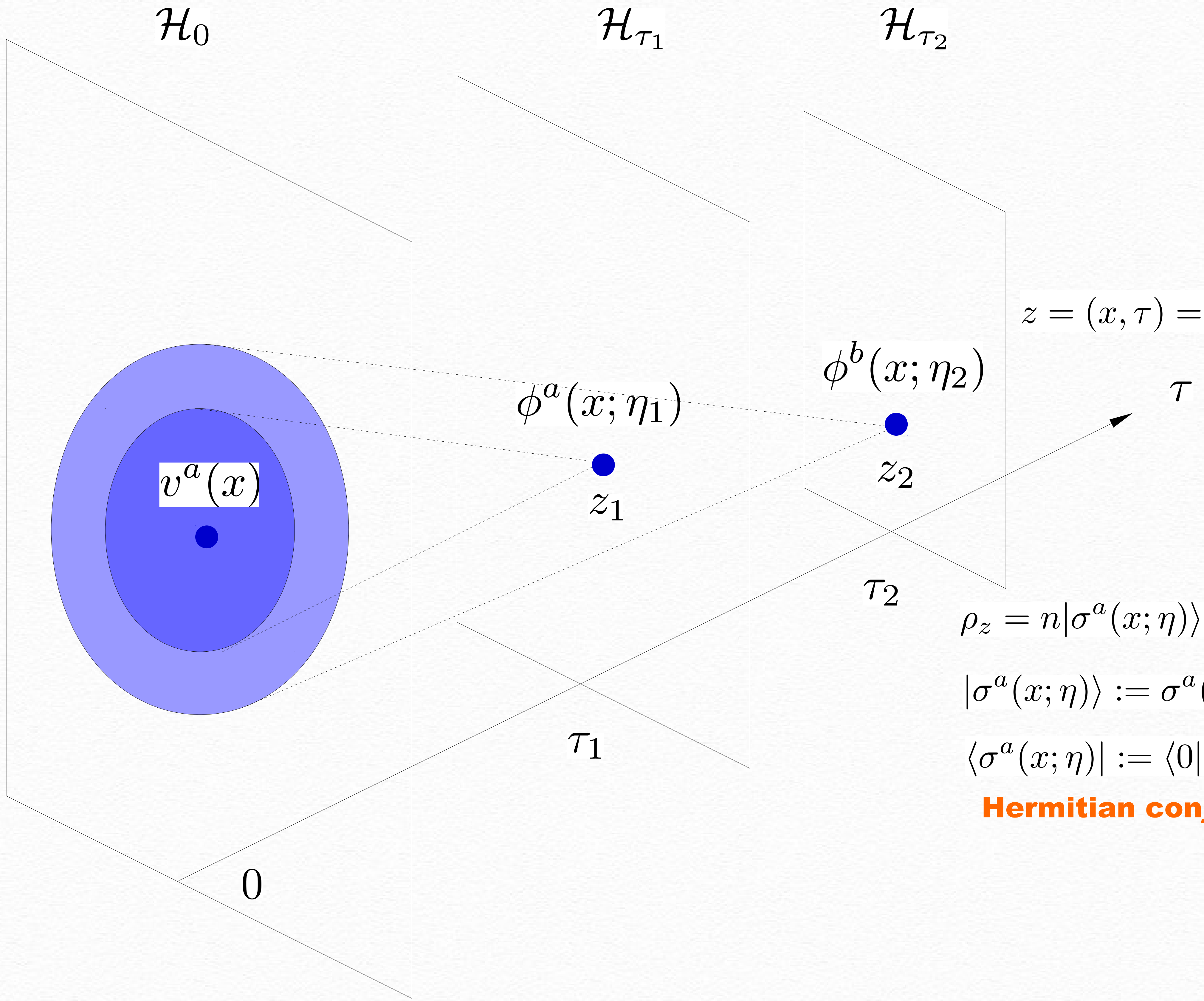
In the current case,

$$\langle \sigma^a(x_1;\eta_1)\sigma^b(x_2;\eta_2)\rangle = \delta^{ab}\left(\frac{2\sqrt{\eta_1\eta_2}}{\eta_+}\right)^{\Delta} G\left(\frac{x_{12}^2}{\eta_+}\right), \qquad G(u) := F(u;1)/F(0;1)$$

$$g_{\mu\nu}(z) = \delta_{\mu\nu}\frac{\Delta}{\tau^2}, \quad g_{\tau\tau}(z) = \frac{\Delta}{\tau^2} \qquad \tau := \sqrt{-\Delta\eta/G'(0)}$$

➡ $$ds^2 = \Delta\frac{dx^2 + d\tau^2}{\tau^2}.$$

Smearing and extra direction



$$z = (x, \tau) = (x, \sqrt{2d\eta})$$

$$\rho_z = n|\sigma^a(x;\eta)\rangle\langle\sigma^a(x;\eta)|,$$

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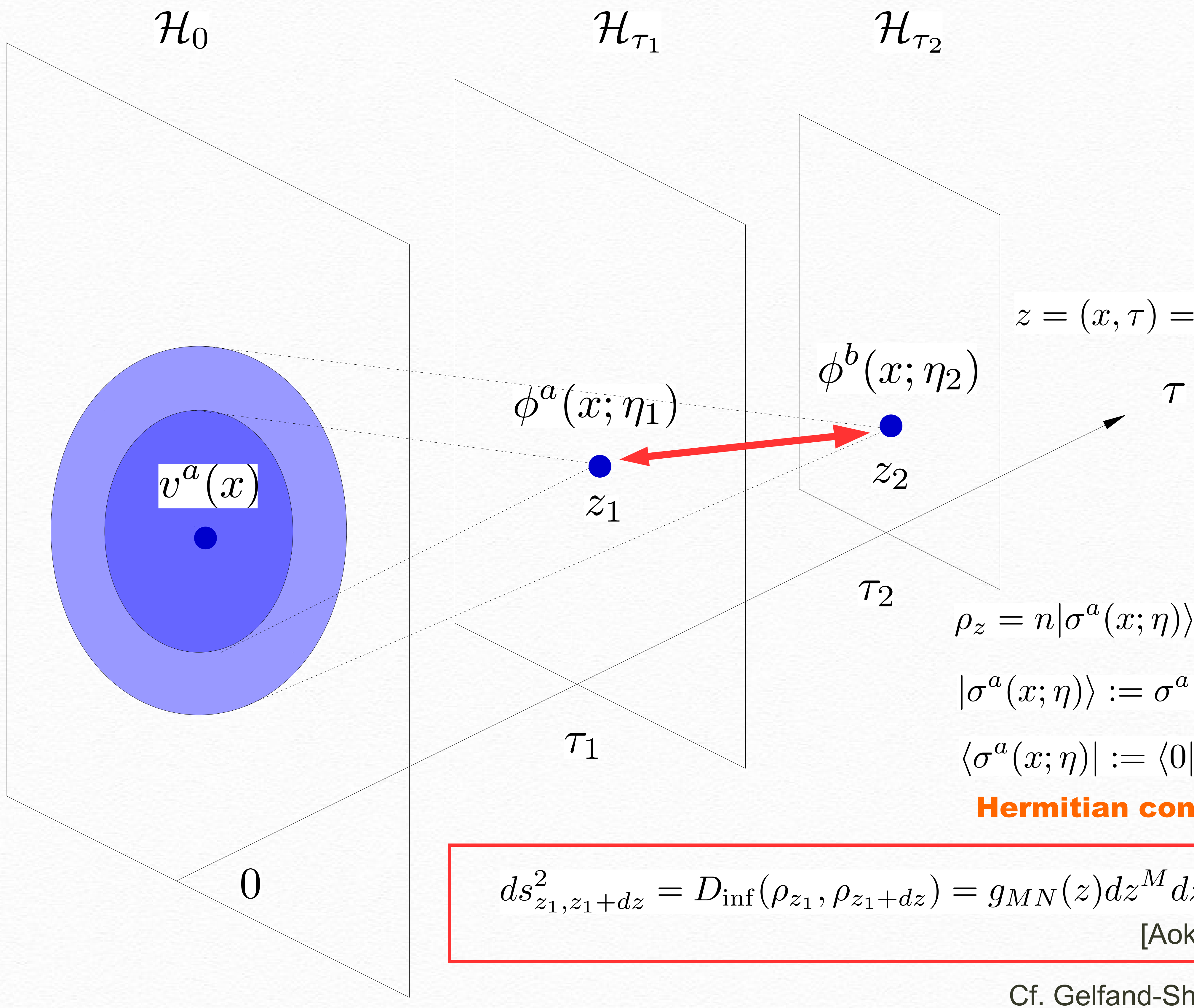

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$$ds^2_{z_1,z_1+dz} = D_{\mathrm{inf}}(\rho_{z_1}, \rho_{z_1+dz}) = g_{MN}(z)dz^M dz^N$$

[Aoki-SY '17]

Cf. Gelfand-Shirov thm

Holographic computation of quantum correction

[S.Aoki-J.Balog-SY '18]

Pregeometric operators

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Ex.

$$\begin{aligned}
\hat{\Gamma}^M_{LN}(x;\eta) =& \frac{1}{2}\hat{g}^{MP}(x;\eta)(\hat{g}_{P\{N,L\}}(x;\eta) - \hat{g}_{NL,P}(x;\eta)) \\
\hat{R}_{LP}{}^M{}_N(x;\eta) =& \partial_{[L}\hat{\Gamma}^M_{P]N}(x;\eta) + \hat{\Gamma}^M_{[LQ}(x;\eta)\hat{\Gamma}^Q_{P]N}(x;\eta) \\
\hat{R}_{PN}(x;\eta) =& \hat{R}_{MP}{}^M{}_N(x;\eta), \\
\hat{R}(x;\eta) =& \hat{g}^{PN}(x;\eta)\hat{R}_{PN}(x;\eta), \\
\hat{G}_{MN}(x;\eta) =& \hat{R}_{MN}(x;\eta) - \frac{1}{2}\hat{g}_{MN}(x;\eta)\hat{R}(x;\eta).
\end{aligned}$$

Bulk interpretation

$$\boxed{\langle \hat{G}_{AB} \rangle_\psi = T^{\text{bulk}}_{AB} \Big|.}$$

$$\langle \hat{G}_{AB} \rangle_\psi := \langle \psi | \hat{G}_{AB} | \psi \rangle$$

CLAIM: This induced Einstein tensor is expected to describe **that of dual quantum gravity**.

$$\boxed{\frac{1}{n} \sim \frac{G_N}{L_{\text{ads}}^{D-2}}}$$

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$\therefore$ In particular, let us compute LHS in the **1/n expansion**:

$$\langle G_{AB}[\hat{g}] \rangle = G_{AB}[\langle \hat{g} \rangle] + \langle G_{AB}[\hat{g}] \rangle_c$$

Leading Order (LO) $\updownarrow$ Classical geometry

Next to Leading Order (NLO) $\updownarrow$ Quantum correction

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NOTE: This framework itself should be applicable to other formulation!

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$$\ddot{G}_{AB} = \ddot{R}_{AB} - \frac{1}{2}\hat{h}_{AB}\dot{R} - \frac{1}{2}g_{AB}\ddot{R},$$

$$\begin{aligned}
\langle \hat{h}_{AB}\dot{R} \rangle =& \langle \hat{h}_{AB}\{ g^{GH} g^{CF} (\hat{h}_{FG;HC} - \hat{h}_{HG;FC}) - \hat{h}^{CD} R_{CD} \} \rangle , \\
\langle \ddot{R} \rangle =& g^{AB} \langle \ddot{R}_{AB} \rangle - \frac{1}{2} g^{AE} g^{BF} g^{CD} \\
& \times \left\{ \left\langle \hat{h}_{EF} \left(\hat{h}_{D\{A;B\}C} - \hat{h}_{BA;DC} - \hat{h}_{DC;AB} \right) \right\rangle - 2R_{AB} \left\langle \hat{h}_{ED} \hat{h}_{CF} \right\rangle \right\} .
\end{aligned}$$

Quantum correction to bulk CC

Ex. Consider free O(n) vector model.

① Consider the vacuum state. ➡ The stress tensor is only cosmological constant.

$$\langle \hat{G}_{AB} \rangle_\psi = T^{\text{bulk}}_{AB}|. \qquad T^{\text{bulk}}_{AB}| = -\Lambda g_{AB}$$

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$$\Lambda = -\frac{d(d-1)}{2\Delta}\left(1 + \frac{d+4}{n}\right) + \mathcal{O}\left(\frac{1}{n^2}\right), \quad L^2_{\text{AdS}} \propto \Delta\left(1 - \frac{d+4}{n}\right) + \mathcal{O}\left(\frac{1}{n^2}\right)$$

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<u>Remark 1</u>: The induced metric does not receive the quantum correction.

<u>Remark 2</u>: The dual gravity theory is **renormalizable** in this framework.

Summary

- Demonstrated how to compute **quantum corrections in the bulk** via **flow equation approach** by **dual covariant perturbation**.

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Future directions

- Dynamics in the bulk? For excited states? working in progress [Aoki-Balog-SY]

- Locality in the bulk? Bulk local operator? cf. [Hamilton-Kabat-Lifshitz-Lowe '06]

- 1-loop calculation of dual gravity (higher-spin)? cf. [Giombi-Klebanov '02]...

- Finite temperature? BH?

$\vdots$

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Future directions

- Dynamics in the bulk? For excited states? working in progress [Aoki-Balog-SY]

- Locality in the bulk? Bulk local operator? cf. [Hamilton-Kabat-Lifshitz-Lowe '06]

- 1-loop calculation of dual gravity (higher-spin)? cf. [Giombi-Klebanov '02]...

- Finite temperature? BH?

$\vdots$

Thank you!