Target Space EE

work in progress

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The Dream

$$S_{WS} = \frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{g}g^{\mu\nu}\partial_\mu X^a\partial_\nu X^b G_{ab}(X) + \dots$$

Spacetime <-> Values of X -fields

$$H_{BFSS} = \frac{1}{2p_{11}} tr_{N\times N}\left(P^iP^i + [X^i, X^j]^2 + fermions\right)$$

Space <-> Moduli Space of D0's

Spatial Subregion = Partition of Target Space

Today

Basic Idea:
Subregion of target space
$\Updownarrow$
Subalgebra of Observables

$$Tr\left(\rho\mathcal{O}\right) = Tr\left(\rho_{\mathcal{A}}\mathcal{O}\right) \quad \forall \mathcal{O} \in \mathcal{A}$$

-> Target Space EE = $S(\rho_{\mathcal{A}})$

Focus here: 1st Quantized QM Particles
+compare to 2nd Quantized QFT

Simplest Ex.: Particle on a Line

$$Z_{QM} = \int Dx(t) e^{\frac{i}{\hbar}\int dt \frac{1}{2}\dot{x}^2 - V(x)}$$

x(t) <-> QFT on a point (+ time)

Physical Space <-> Values of x

$A = \{x : x < x_0\}$ $\bar{A} = \{x : x \geq x_0\}$

$$\mathscr{H} = L^2(A \cup \bar{A}) = \mathscr{H}_A \oplus \mathscr{H}_{\bar{A}}$$

Subregion <-> Subalgebra

$$\mathscr{A} = \Big\langle \{ \, |x\rangle\langle x'| : x, x' \in A \} \cup \mathbb{I}_{\mathscr{H}} \Big\rangle \qquad \left(\begin{array}{c|c} \mathcal{O}_A & \\ \hline & c\mathbb{I}_{\bar{A}} \end{array}\right)$$

$$\mathscr{A}' = \Big\langle \{ \, |x\rangle\langle x'| : x, x' \in \bar{A} \} \cup \mathbb{I}_{\mathscr{H}} \Big\rangle \qquad \left(\begin{array}{c|c} c'\mathbb{I}_A & \\ \hline & \mathcal{O}_{\bar{A}} \end{array}\right)$$

$$\begin{aligned} \mathscr{Z} &= \mathscr{A} \cap \mathscr{A}' \\ &= \Big\langle \Pi_{A,1} = \int_A dx \, |x\rangle\langle x| \cup \Pi_{A,0} = \int_{\bar{A}} dx \, |x\rangle\langle x| \Big\rangle \end{aligned} \qquad \left(\begin{array}{c|c} c_1\mathbb{I}_A & \\ \hline & c_2\mathbb{I}_{\bar{A}} \end{array}\right)$$

Non-Trivial Center <-> $\mathscr{H} \neq \mathscr{H}_A \otimes \mathscr{H}_{\bar{A}}$

What Algebras Can Do For You

$$\mathscr{Z} = \langle \Pi_\alpha \rangle \longrightarrow \mathscr{H} = \oplus_\alpha \Pi_\alpha \mathscr{H} = \oplus_\alpha \mathscr{H}_{F_\alpha} \otimes \mathscr{H}_{\bar{F}_\alpha}$$

Here: $\Pi_{A,1}, \Pi_{A,0}$ $\quad \mathscr{H}_{QM} \cong \left(\mathscr{H}_A \otimes \mathbb{C}\right) \oplus \left(\mathbb{C} \otimes \mathscr{H}_{\bar{A}}\right)$

$$\rho_{\mathscr{A}} = \oplus_\alpha p_\alpha \hat{\rho}_\alpha \otimes \frac{1_{\bar{F}_\alpha}}{|\bar{F}_\alpha|} \longrightarrow S(\rho_{\mathscr{A}}) = -\sum_\alpha p_\alpha log p_\alpha + \sum_\alpha p_\alpha S(\hat{\rho}_\alpha)$$

Here:

$$\rho_{\mathscr{A}} = \left(Tr(\Pi_{A,1}\rho\Pi_{A,1}) \frac{\Pi_{A,1}\rho\Pi_{A,1}}{Tr(\Pi_{A,1}\rho\Pi_{A,1})} \otimes \frac{1}{1} \right) \oplus \left(Tr(\Pi_{A,0}\rho\Pi_{A,0}) 1 \otimes \frac{\mathbb{I}_{\bar{A}}}{Tr(\mathbb{I}_{\bar{A}})} \right)$$

For $\rho = |\psi\rangle\langle\psi|$ $\quad p_A = \int_A dx |\psi(x)|^2$

$$S(\rho_{\mathscr{A}}) = -p_A log(p_A) - (1 - p_A) log(1 - p_A)$$

N Particles, General Target Space

$$\mathscr{A} = \left\langle \{P_{S_N}\left(| \overrightarrow{x} \rangle \langle \overrightarrow{x}' | \otimes 1 ... \otimes 1 \right) P_{S_N} : \overrightarrow{x}, \overrightarrow{x}' \in A\} \cup \mathbb{I}_{\mathscr{H}} \right\rangle$$

$$\mathscr{Z} = \left\langle \{P_{S_N}\left(\int_A d\overrightarrow{x} \, | \overrightarrow{x} \rangle \langle \overrightarrow{x} | \otimes 1 ... \otimes 1 \right) P_{S_N}\} \cup \mathbb{I}_{\mathscr{H}} \right\rangle$$

$$\frac{L^2(A \cup \bar{A})^{\otimes N}}{S_N} = \oplus_{k=0}^{N} \frac{L^2(A)^{\otimes k}}{S_k} \otimes \frac{L^2(\bar{A})^{\otimes N-k}}{S_{N-k}}$$

Same as QFT?

Embed State of Rel. Particle in QFT

$$|\psi\rangle_{QM} \to \int dx \psi(x) a_x^\dagger |vac\rangle$$

$$H_{QM} = \sqrt{\hat{p}^2 + m^2} \to H_{QFT} = \int \frac{dp}{2\pi} \sqrt{p^2 + m^2} a_p^\dagger a_p$$

2 notions of "spatial" locality in QFT

$$a_x = \int \frac{dp}{2\pi} e^{ipx} a_p$$

$$H_{QFT} = \int dx dy K(x, y) a_x^\dagger a_y$$

$$K(x, y) = \int dp \sqrt{p^2 + m^2} e^{ip(x-y)}$$

$$\omega_p = \sqrt{p^2 + m^2}$$

$$\phi_x = \int \frac{dp}{2\pi} \frac{1}{\sqrt{2\omega_p}} \left(a_p + a_{-p}^\dagger \right) e^{ipx}$$

$$H_{QFT} = \int dx \pi_x^2 + (\nabla \phi_x)^2 + m^2 \phi_x^2$$

$$[\hat{\phi}_x, a_{x'}] = \int \frac{dp}{2\pi} \frac{1}{\sqrt{2\omega_p}} e^{ip(x-x')} \neq \delta(x-x') \longrightarrow$$

Act locally in different tensor factorizations

$$\mathcal{H}_{QFT} = \bigotimes_{x_{(a)}} \mathcal{H}_{x_{(a)}}$$

$$|vac\rangle = \bigotimes_{x_{(a)}} |0\rangle_{x_{(a)}}$$

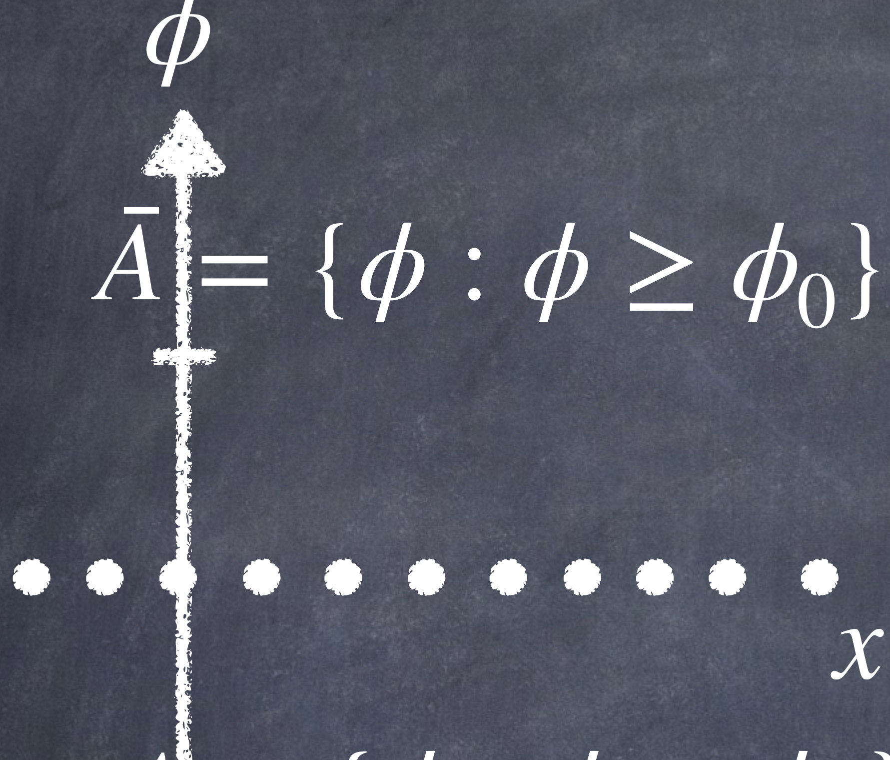
$$\mathcal{H}_{QFT} = \bigotimes_{x_{(\phi)}} \mathcal{H}_{x_{(\phi)}}$$

$$|vac\rangle \neq \bigotimes_{x_{(\phi)}} |0\rangle_{x_{(\phi)}}$$

$$\langle x | x' \rangle_{QM} = \delta(x-x') = \langle vac | a_x a^\dagger_{x'} | vac \rangle$$

Notion of spatial locality in 1st quantized theory not the usual one we consider in 2nd quantized QFT!

Not just about QM!



$$\bar{A} = \{\phi : \phi \geq \phi_0\}$$

$$A = \{\phi : \phi < \phi_0\}$$

$$\mathscr{H}_{QFT} = L^2(T)^{\otimes|\Lambda|}$$

$$H_{QFT} = \sum_{x\in\Lambda} \pi_x^2 + m^2\phi_x^2 + \left(\phi_{x+1} - \phi_x\right)^2$$

Equivalent to system of <u>distinguishable</u> particles labeled by "x"

$$\mathscr{A} = \left\langle \{\, |\phi_x\rangle\langle\phi_x'| \otimes 1_{rest} : \phi_x, \phi_x' \in A\} \,\forall x \right\rangle$$

Will match EE computed in "3rd" Quantized theory on (Target x Base)

Short-Term Goals

Worldline Formalism & Reparametrization Invariance

QFT Replica Trick as Periodic Wordline Computation (cf. Susskind Uglum) -> Reproduces area law ground state EE relative to ϕ_x tensor factorization

$$S_{EE} = (1 - n\partial_n) log(Z_n^{QFT})\Big|_{n=1}$$

$$log(Z_n^{QFT}) = \int_\epsilon^\infty \frac{ds}{s} \int d^d x \langle x^\mu | e^{-s(p^2+m^2)} | x^\mu \rangle$$

$$\langle \phi(x^\mu)\phi(y^\nu) \rangle = \int_\epsilon^\infty ds \langle x^\mu | e^{-s(p^2+m^2)} | y^\nu \rangle$$

Algebraic Def on Wordline?