# Quantum Coarse-graining behind black holes

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Berkeley

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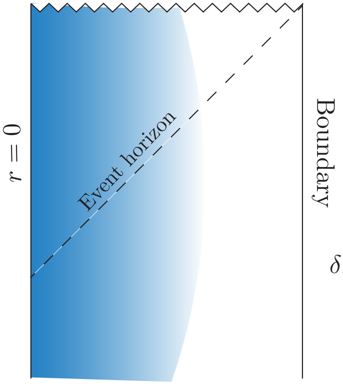
# Black hole second law

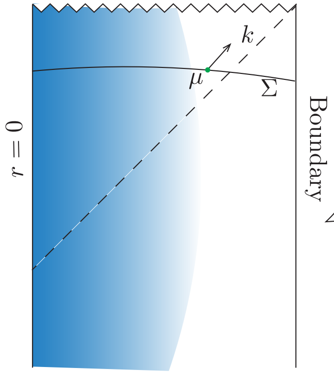
Black holes are thermodynamic objects with $S_{BH} = \frac{A}{4G\hbar}$

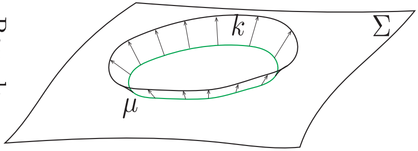



$\delta A_{EH} \geq 0$

# Black hole second law

More local definition of black holes



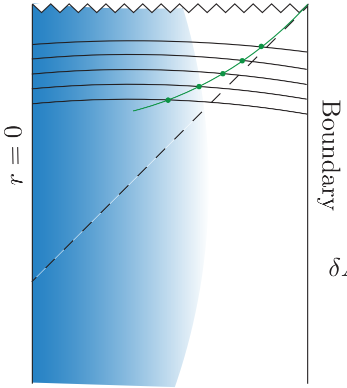




$$\theta_k = \frac{1}{\mathcal{A}} \frac{d\mathcal{A}}{d\lambda}$$

Marginally trapped surface ($\theta_k = 0, \theta_l < 0$)

# Black hole second law

Apparent horizons have an area law

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# Black hole second law

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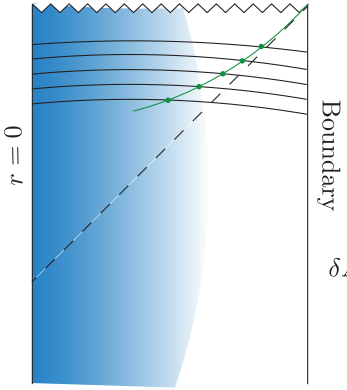


$\delta A \geq 0$

Is $A[\mu]/4G\hbar$ a coarse-grained entropy?

# Black hole second law

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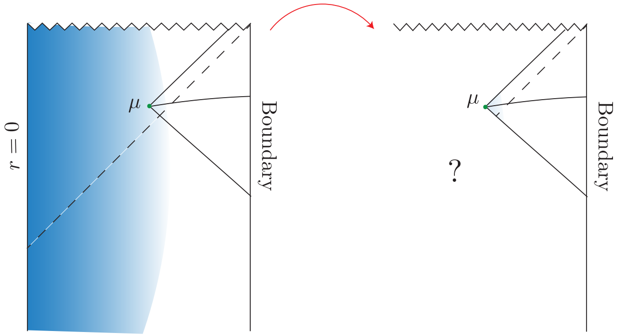
We need a microscopic theory+prescription for coarse-graining AdS/CFT!

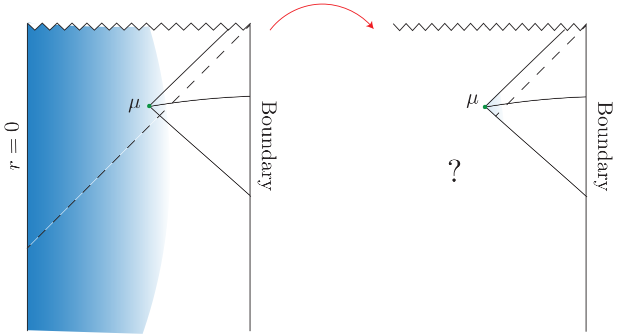
# Review of Engelhardt-Wall construction

Coarse-graining prescription in AdS/CFT



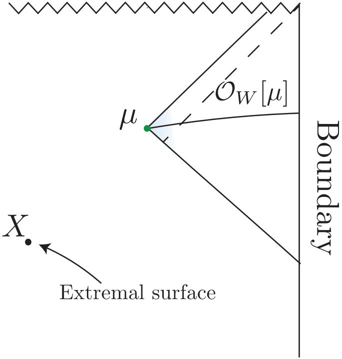
# Review of Engelhardt-Wall construction

Coarse-graining prescription in AdS/CFT




Ryu-Takayanagi prescription $S_{\mathsf{CFT}} = \frac{A[X]}{4G\hbar}$
where $X$ is an extremal surface ($\theta_k = \theta_l = 0$)

# Review of Engelhardt-Wall construction




$$\max\ A[X] : \text{holding } \mathcal{O}_W[\mu] \text{ fixed}$$

$$\Longrightarrow\ S_{\text{coarse}} = \frac{A[X]}{4G\hbar}$$

# Review of Engelhardt-Wall construction

Can show $A[X] \leq A[\mu]$

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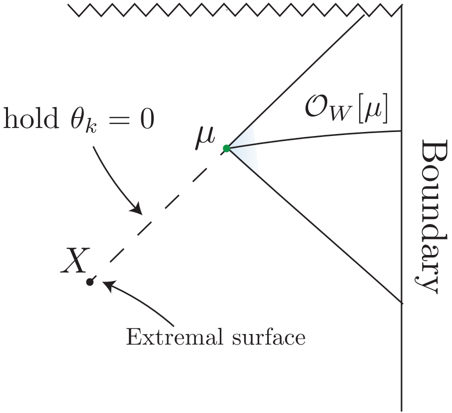
Engelhardt-Wall's explicit construction




X was found such that $A[X] = A[\mu] \implies S_{coarse} = \frac{A[\mu]}{4G\hbar}$!

# Generalized entropy

Area law can be violated quantum-mechanically! e.g. Hawking evaporation

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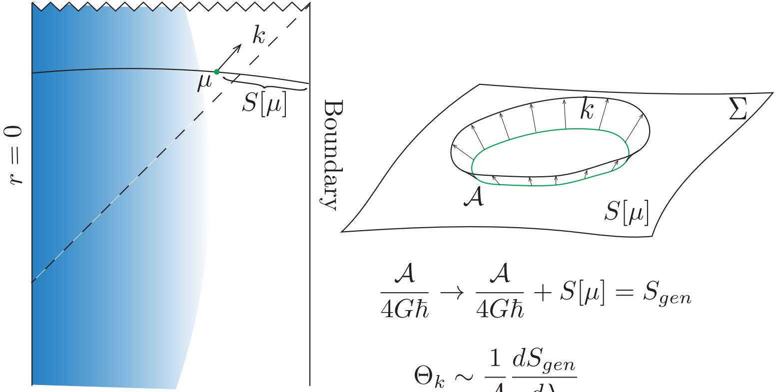
Area law can be violated quantum-mechanically! e.g. Hawking evaporation
Add to the area the entropy of matter outside $S_{gen}[\mu] = \frac{A}{4G\hbar} + S_{out}$
Generalized entropy!

# Generalized entropy



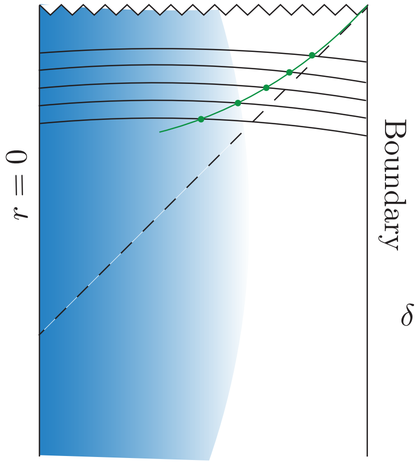

$$\frac{\mathcal{A}}{4G\hbar} \to \frac{\mathcal{A}}{4G\hbar} + S[\mu] = S_{gen}$$

$$\Theta_k \sim \frac{1}{\mathcal{A}} \frac{dS_{gen}}{d\lambda}$$

Quantum marginally trapped surface $\Theta_k = 0$

# Generalized entropy

Quantum apparent horizons satisfy $S_{gen}$ law

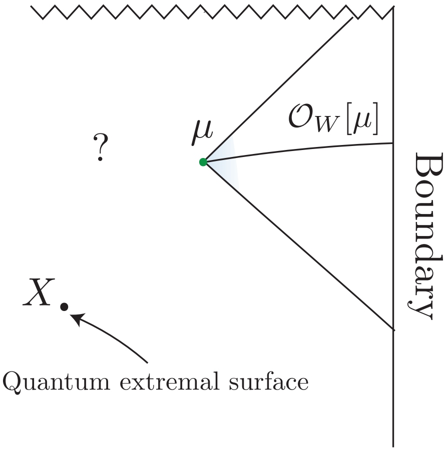



$$\delta S_{gen} \geq 0$$

# Quantum coarse-graining?

Quantum corrected RT formula: $S_{CFT} = S_{gen}[X]$
$X$ : quantum extremal surface

$$S_{gen}[X] \stackrel{?}{=} S_{gen}[\mu]$$



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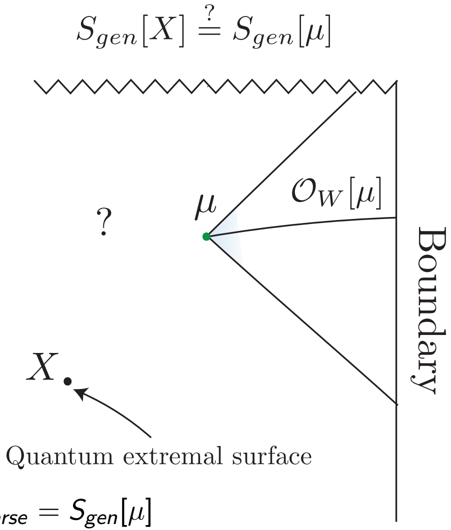
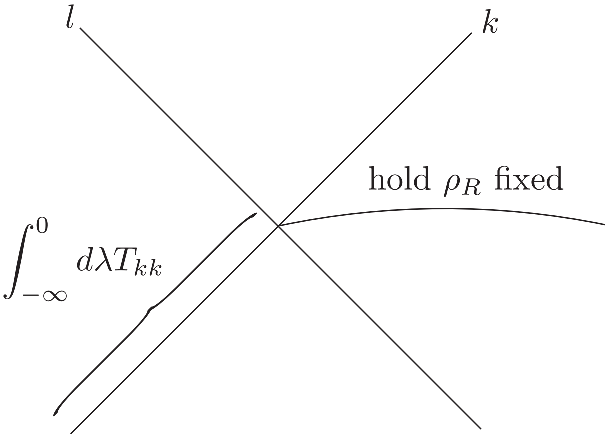



If so, then $S_{coarse} = S_{gen}[\mu]$

# Ant conjecture: Energy-minimizing states

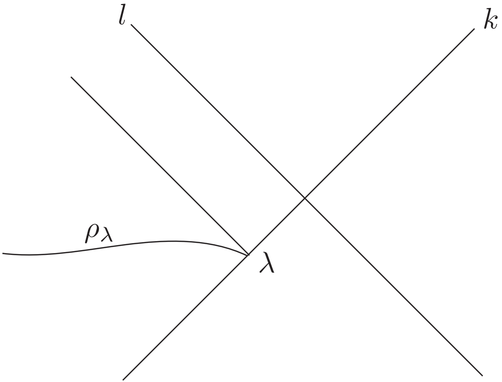
Aron Wall's thought experiment [1701.03196]




$$\underset{\text{all states}}{\text{minimum}} \int_{-\infty}^{0} d\lambda T_{kk} \ : \ \text{holding } \rho_R \text{ fixed}$$

# Relative entropy in QFT

$$S_{rel}(\rho|\lambda) = tr(\rho \log \rho - \rho \log \sigma)$$




$$S_{rel}(\rho_\lambda|\sigma_\lambda) = \frac{2\pi}{\hbar} \int_{-\infty}^{\lambda} d\lambda' \lambda'(\lambda - \lambda') T_{kk} - \Delta S \geq 0$$

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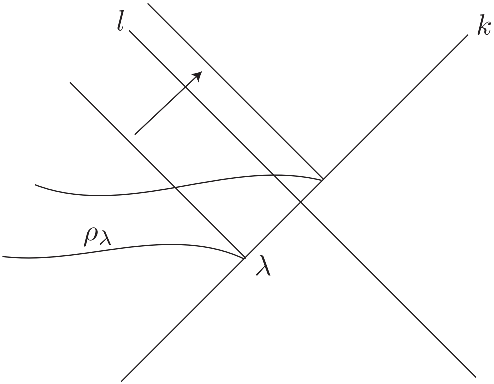
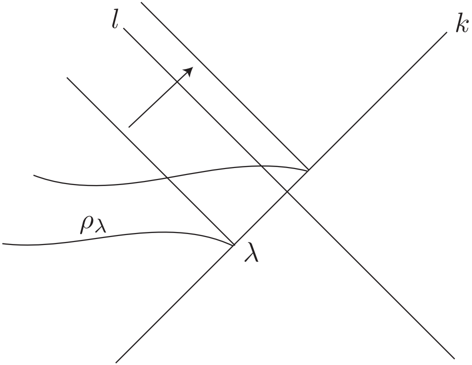



$$S_{rel}(\rho_\lambda|\sigma_\lambda) = \frac{2\pi}{\hbar}\int_{-\infty}^{\lambda} d\lambda' \lambda'(\lambda - \lambda') T_{kk} - \Delta S \ \geq 0$$

$$\partial_k S_{rel} = \frac{2\pi}{\hbar}\int_{\infty}^{\lambda} d\lambda' T_{kk} - \partial_k S \ \geq 0$$

# Relative entropy in QFT

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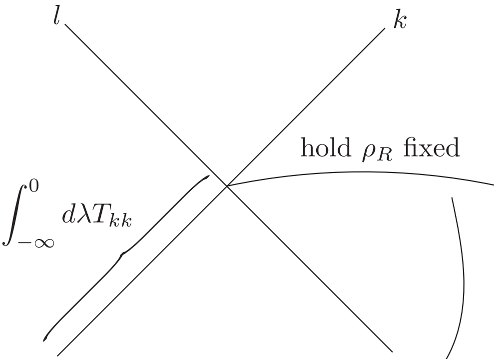
$$\partial_k S_{rel} = \frac{2\pi}{\hbar}\int_{\infty}^{\lambda} d\lambda' T_{kk} - \partial_k S \ \geq 0$$

$$\partial_k^2 S_{rel} = \frac{2\pi}{\hbar} T_{kk} - \partial_k^2 S \ \geq 0 \quad \text{(QNEC)}$$

# Ant conjecture: Energy-minimizing states



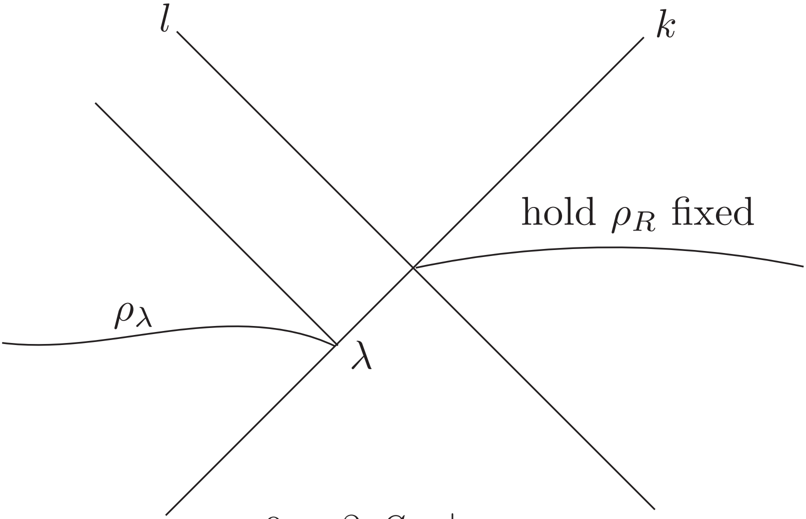

$$\frac{2\pi}{\hbar}\int_{-\infty}^{0} d\lambda T_{kk} \geq \partial_k S_R$$

$$\text{Aron's conjecture: } \partial_k S_R = \underset{\text{all states}}{\text{minimum}} \int_{-\infty}^{0} d\lambda T_{kk} \ : \ \text{holding } \rho_R \text{ fixed}$$

(There exists a state that satisfies it)

# Ant conjecture: Energy-minimizing states

Let's go to that state!




$$0 = \partial_k S_{rel}\big|_{\lambda=0}$$

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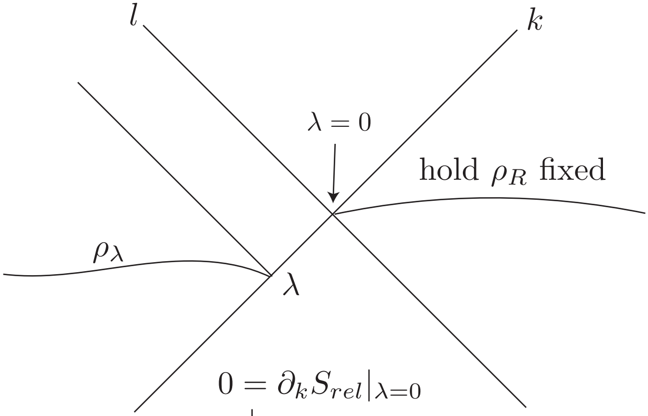


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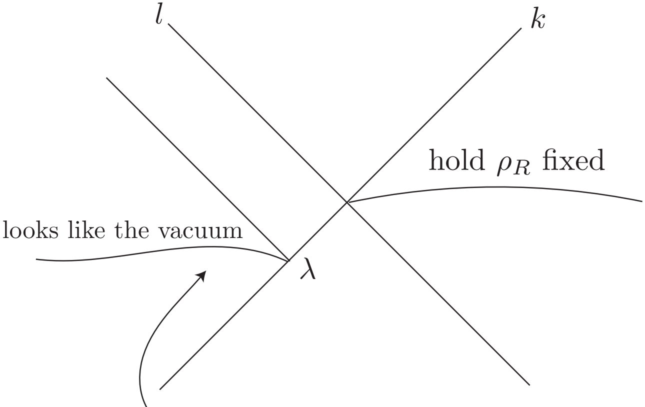
$$\partial_k S_{rel} \geq 0$$

$$\partial_k^2 S_{rel} \geq 0$$

$$\partial_k S_{rel} = 0, \quad \lambda \leq 0$$

# Ant conjecture: Energy-minimizing states

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$$\Downarrow$$

$$T_{kk} = 0, \quad \lambda < 0$$

$$\partial_k S_R = 0, \quad \lambda < 0$$

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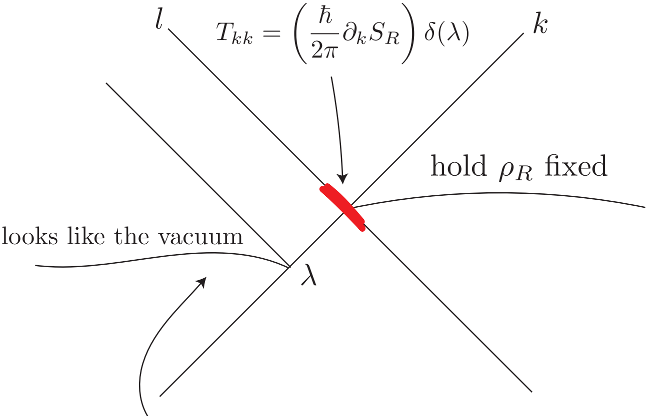


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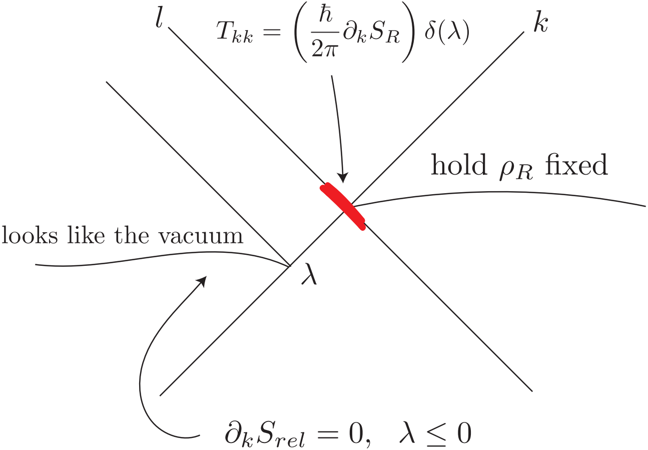


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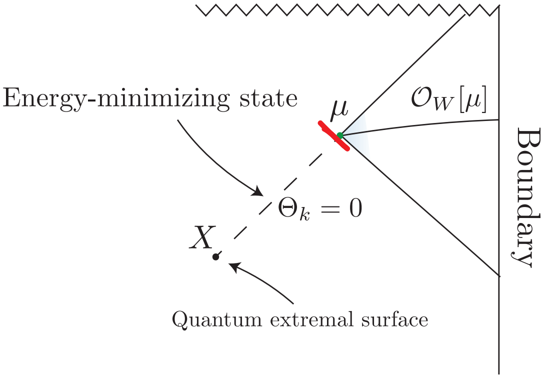
Faulkner-Ceyhan [1812.04683]

# Put them together: quantum coarse-graining

$$S_{gen}[X] \stackrel{?}{=} S_{gen}[\mu]$$




$$\theta_k[\mu] = -4G\hbar\partial_k S[\mu] \xrightarrow[\partial_k\theta_k \supset 8\pi G T_{kk}]{T_{kk} = \left(\frac{\hbar}{2\pi}\partial_k S[\mu]\right)\delta(\lambda)} \theta_k[\text{below } \mu] = 0$$

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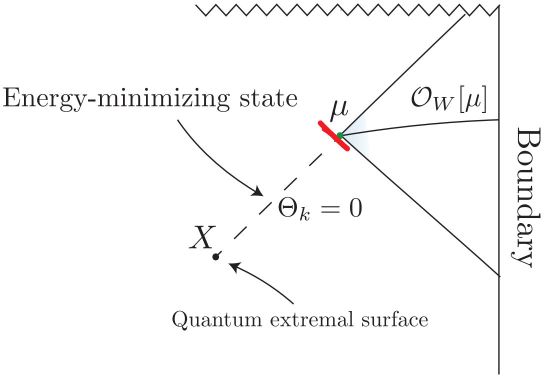


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$$\partial_k S[\text{below } \mu] = 0$$

$$S_{gen}[X] = S_{gen}[\mu] \implies S_{coarse} = S_{gen}[\mu]$$

Thank you!