# Universal Scaling in Fast Quenches Near Lifshitz-Like Fixed Points

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- If the system crosses / is driven to a critical point

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2. **Smooth** Quenches ($\delta t^{-1} \ll \Lambda$)
   - Fast quenches

   $$\delta t^{-1} \ll \lambda_{\mathrm{i}}^{\frac{1}{d-\Delta}}, \lambda_{\mathrm{f}}^{\frac{1}{d-\Delta}}, \cdots$$

   - Slow quenches

   $$\lambda_{\mathrm{i}}^{\frac{1}{d-\Delta}}, \lambda_{\mathrm{f}}^{\frac{1}{d-\Delta}}, \cdots \lesssim \delta t^{-1} \ll \Lambda$$

[Myers, Das, Galante, Nozaki, Das, Caputa, Heller, van Niekerk, $\cdots$]

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- Critical points often have $z \neq 1$ !
- How to model systems with Lifshitz-like $(z \neq 1)$ fixed points?

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$$[D, J_{ij}] = 0 \quad , \quad [D, P_i] = i\, P_i \quad , \quad [D, H] = i\, z\, H$$

where

$$H = -i\partial_t \quad , \quad J_{ij} = -i(x_i\partial_j - x_j p_i)$$
$$P_i = -i\partial_i \quad , \quad D = -i(z\, t\partial_t + x^i\partial_i)$$

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- Anisotropic scaling: $z\, T^0{}_0 + T^i{}_i = 0$

# Outline + Statement of Results

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  - In strongly coupled regime (via holographic models)
  - In free field theories

# Outline + Statement of Results

- How does a theory with Lifshitz-like fixed point respond to time dependent parameters in the Hamiltonian?
- As a first step I report results we have found in **fast quench** regime
- We study theories under **relevant deformations** in two distinct regimes
  - In strongly coupled regime (via holographic models)
  - In free field theories
- The respond of the system is **universal**:
  - only depends on $\Delta$
  - *independent* of (i) quench details (ii) state
  - free theory matches with holography

# Holographic Setup

- EMD theory

$$S = \frac{-1}{16\pi G_N} \int d^{d+1}x\sqrt{-g}\Big(\mathcal{R} + \Lambda - \frac{1}{2}(\partial\chi)^2 \\ - \frac{1}{4}e^{\lambda\chi}F^2 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - V(\phi)\Big)$$

where $m^2 = \Delta(\Delta - d_z)$ and $d_z := d + z - 1$.

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where $m^2 = \Delta(\Delta - d_z)$ and $d_z := d + z - 1$.

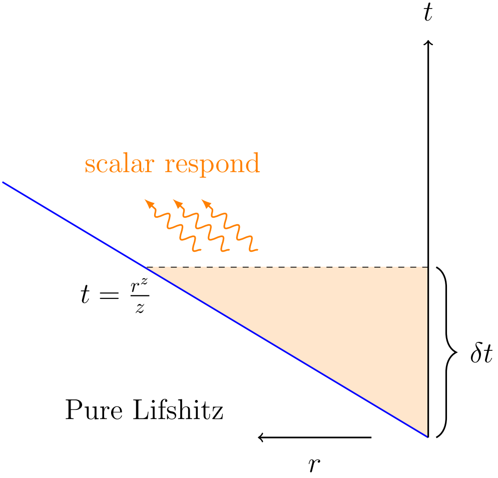
- Take th following solution $(z \geq 1)$ [Taylor '08]

$$ds^2 = -\frac{f(t,r)}{r^{2z-2}} dt^2 + \frac{dr^2}{r^4 f(t,r)} + g(t,r)^2 d\vec{x}^2$$

in pure Lifshitz background $\; f(t,r) = r^{-2} \;,\; g(t,r) = r^{-1}$

# Holographic Scenario

- define: $r = \delta t\,\hat{r}$ , $t = \delta t^z\,\hat{t}$ , $\cdots$ and take $\delta t \to 0$



# Holographic Scenario

- In terms of dimensionless parameters $r = \delta t\, \hat{r},\ t = \delta t^z\, \hat{t}$

$$\phi(\hat{t}, \hat{r}) = \delta t^{d_z - \Delta} \hat{r}^{d_z - \Delta} \left[p_s(\hat{t}) + \cdots\right] + \delta t^{\Delta} \hat{r}^{\Delta} \left[p_r(\hat{t}) + \cdots\right]$$

- Source profile

$$p_s(t) = \delta p \begin{cases} \hat{t}^{\kappa} & 0 < t < \delta t \\ 1 & \delta t \leq t \end{cases}$$

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- Response profile

$$p_r(\hat{t}) = a_\kappa \cdot \delta p \cdot \delta t^{d_z - 2\Delta} \cdot \hat{t}^{\frac{d_z - 2\Delta}{z} + \kappa}$$

- From holographic renormalization for $2\Delta = d_z + 2nz$ there is logarithmic enhancement [Andrade-Ross '12]

# Lifshitz Free Scalar Theory

- Scalar Theory with Lifshitz symmetry $(m \to 0)$ [Alexandre '11]

$$I = \frac{1}{2} \int dt d\vec{x} \left[ \dot{\phi}^2 - \sum_{i=1}^{d-1} (\partial_i^z \phi)^2 - m^{2z}(t)\phi^2 \right],$$

where

$$[t] = -z, \quad [x_i] = -1, \quad [m] = 1, \quad [\phi] = \frac{d-z-1}{2}$$

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- A solvable mass profile

$$m^{2z}(t) = \frac{m_0^{2z}}{2} \left( 1 - \tanh \frac{t}{\delta t^z} \right)$$

$m(t \to -\infty) = m_0$ and $m(t \to +\infty) = 0$

# Lifshitz Free Scalar Theory

- Standard expansion

$$\phi(x,t) = \int d^{d-1}k \left( a_k u_k + a_k^\dagger u_k^* \right)$$

- the in-mode is solved as

$$u_k = \frac{1}{\sqrt{2\omega_{\text{in}}}} e^{i(k.x - \omega_+ t)} \left( 2\cosh \frac{t}{\delta t^z} \right)^{-i\omega_- \delta t^z} \times$$
$${}_2F_1 \left( 1 + i\omega_- \delta t^z, i\omega_- \delta t^z, 1 - i\omega_{\text{in}} \delta t^z; 1 - \frac{m^{2z}(t)}{m_0^{2z}} \right)$$

where $\omega_{\text{in}} = \sqrt{k^{2z} + m_0^{2z}}$ and $\omega_\pm = (|k|^z \pm \omega_{\text{in}})/2$

# Scaling of mass operator

- We look at

$$\langle \phi^2 \rangle_{\text{ren}} = \sigma_d \int dk \left( \frac{k^{d-2}}{\omega_{\text{in}}} \left| {}_2F_1 \right|^2 - f_{\text{ct}}^{(d)}(k, z, m(t)) \right)$$

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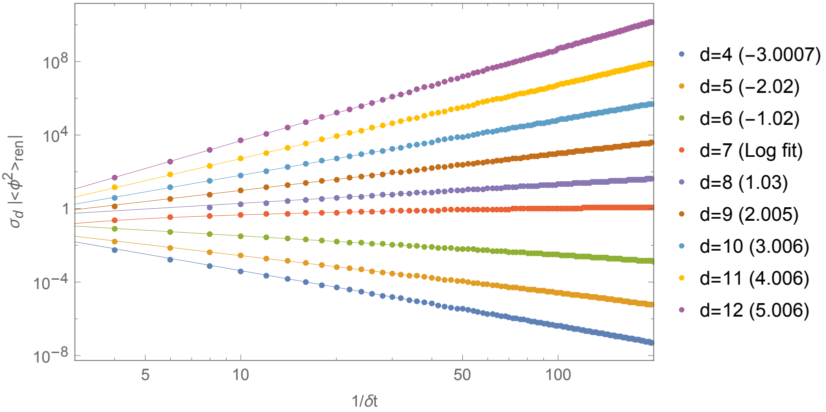
- Logarithmic enhancement at

$$d = z + 1 + 2nz$$

which matches with holographic renormalization condition $2\Delta = d_z + 2nz$ for relevant operators

# Numerical Result for $z = 2$



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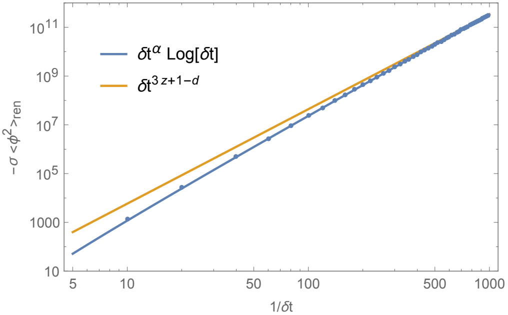

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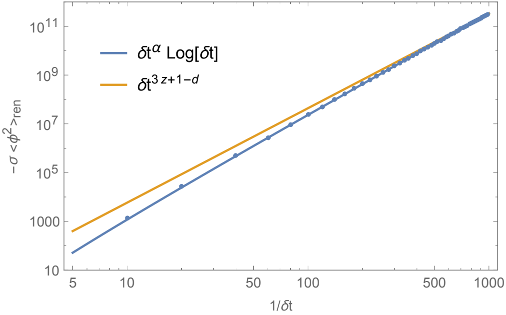


That's it!