Signature of quantum chaos in operator entanglement in 2d CFTs

MASAHIRO NOZAKI(iTHEMS Riken and UCB)

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A faculty

in China

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Based on the collaboration with

Shinsei Ryu, Laimei Nie, Mao Tian Tan, Jonah Kudler-Flam, Eric Mascot, and Masaki Tezuka

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arXiv:19xx.xxxxx [hep-th]

Contents of my talk

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- Tripartite

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- Line tension picture

7. Local operator entanglement (work in progress)

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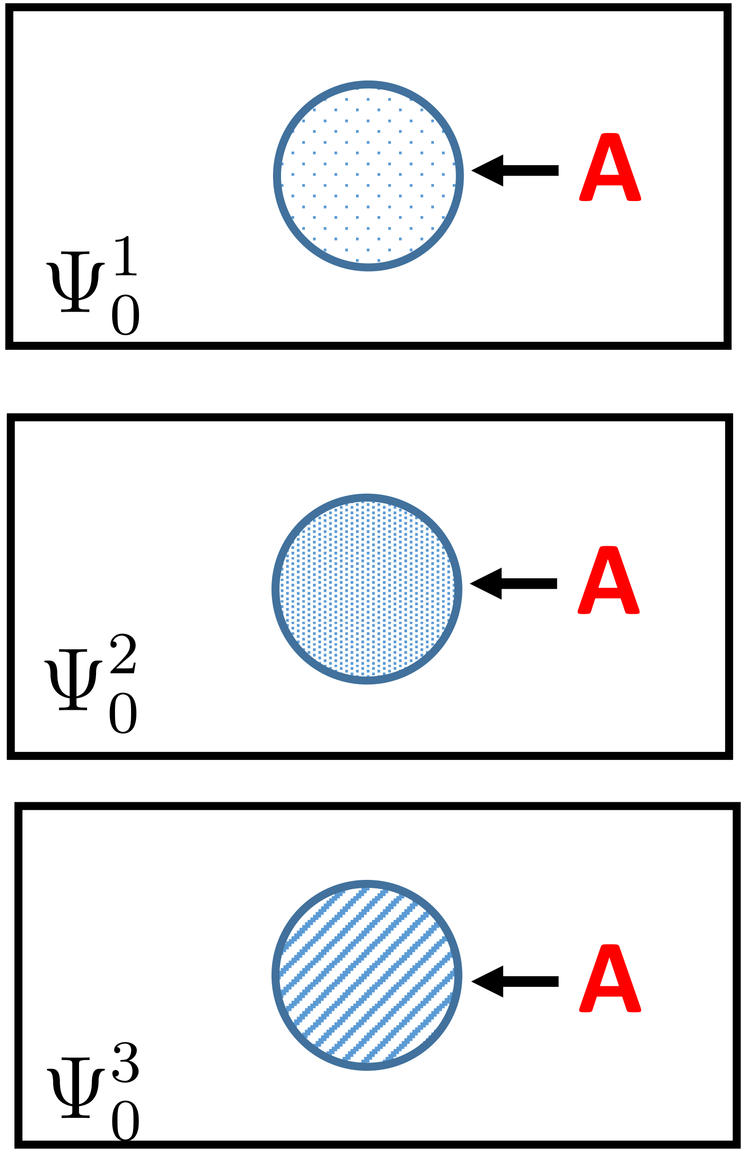
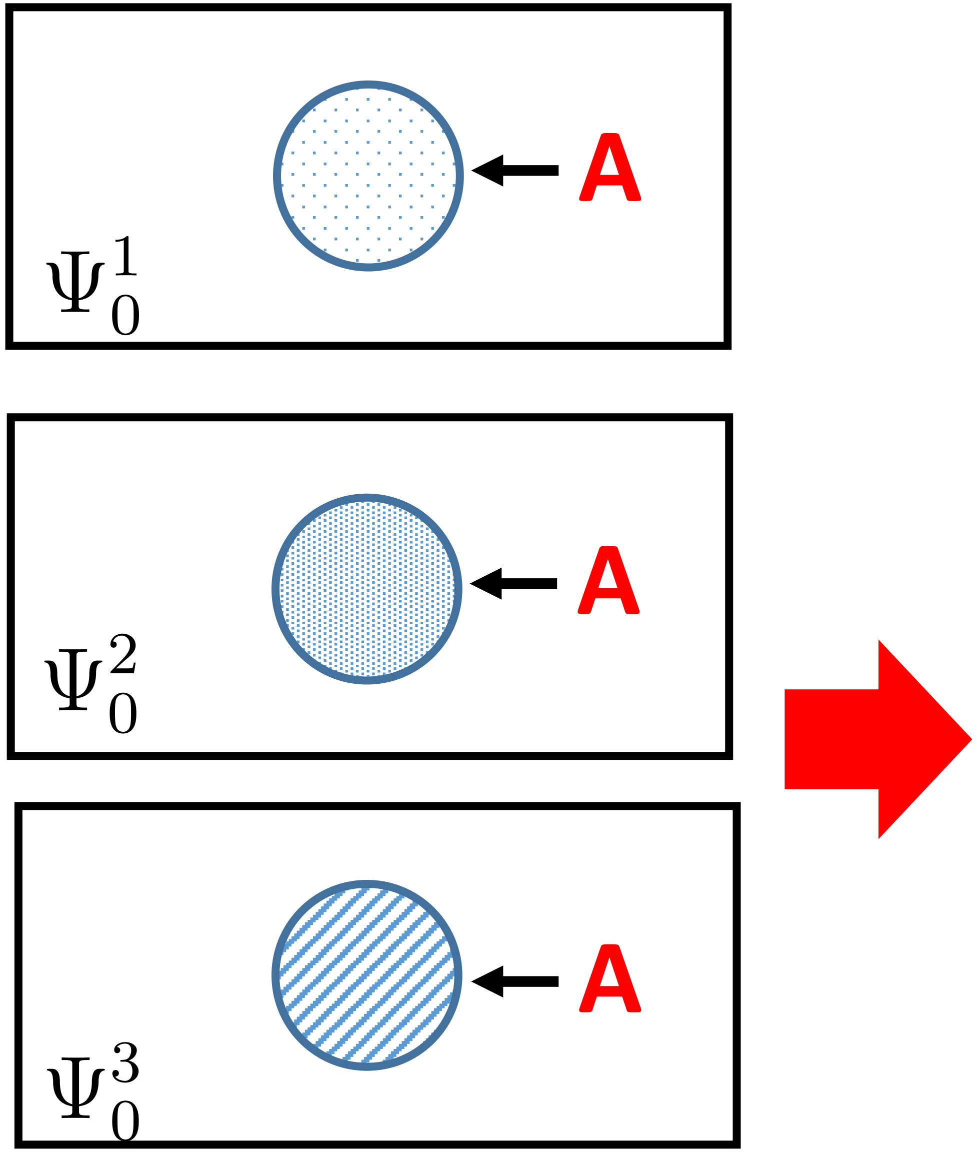
Thermalization



$\left|\Psi_0^i\right\rangle$ are initial states.

Thermalization

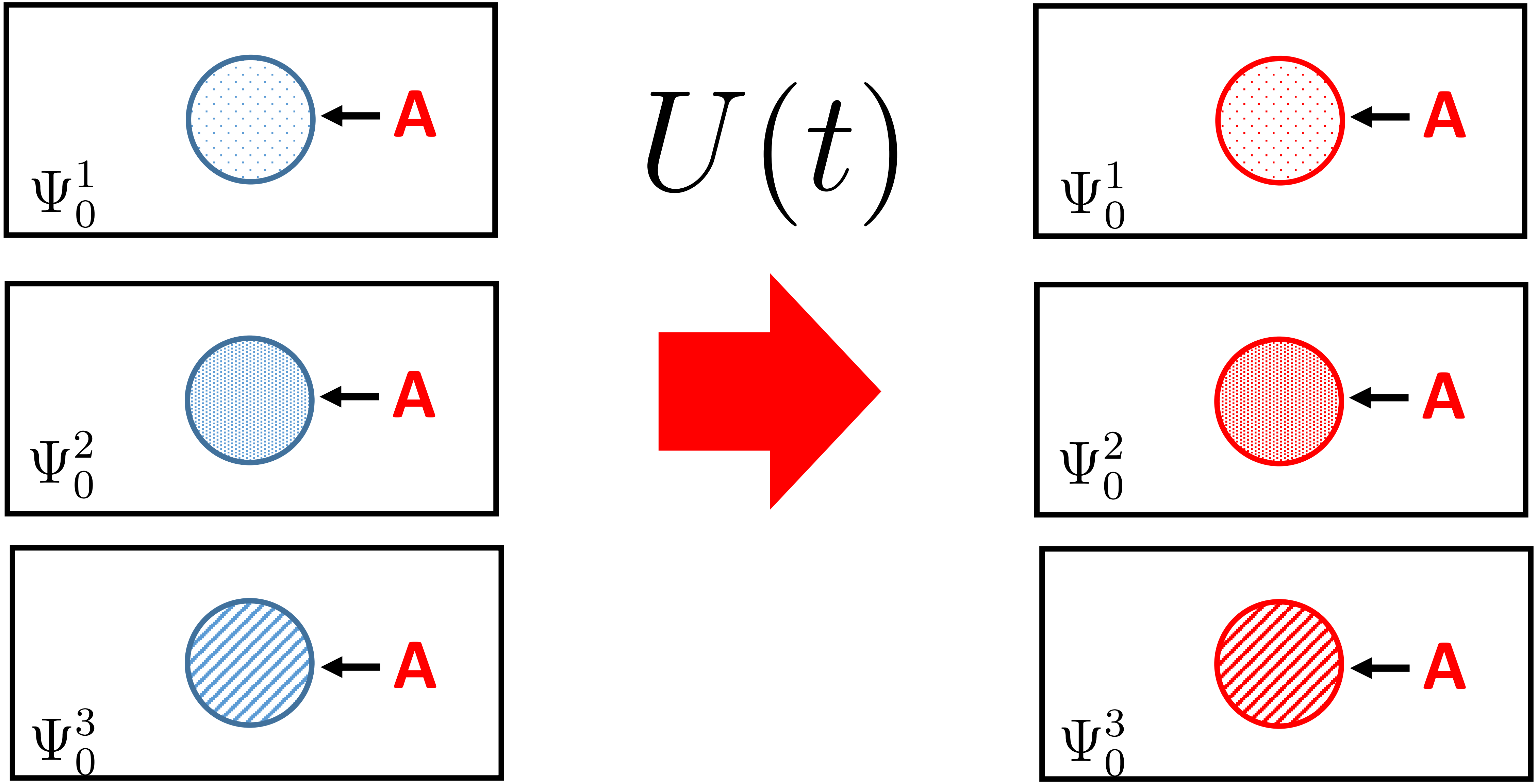


$\left|\Psi_0^i\right\rangle$ are initial states.

Local observables in A depend on initial condition.

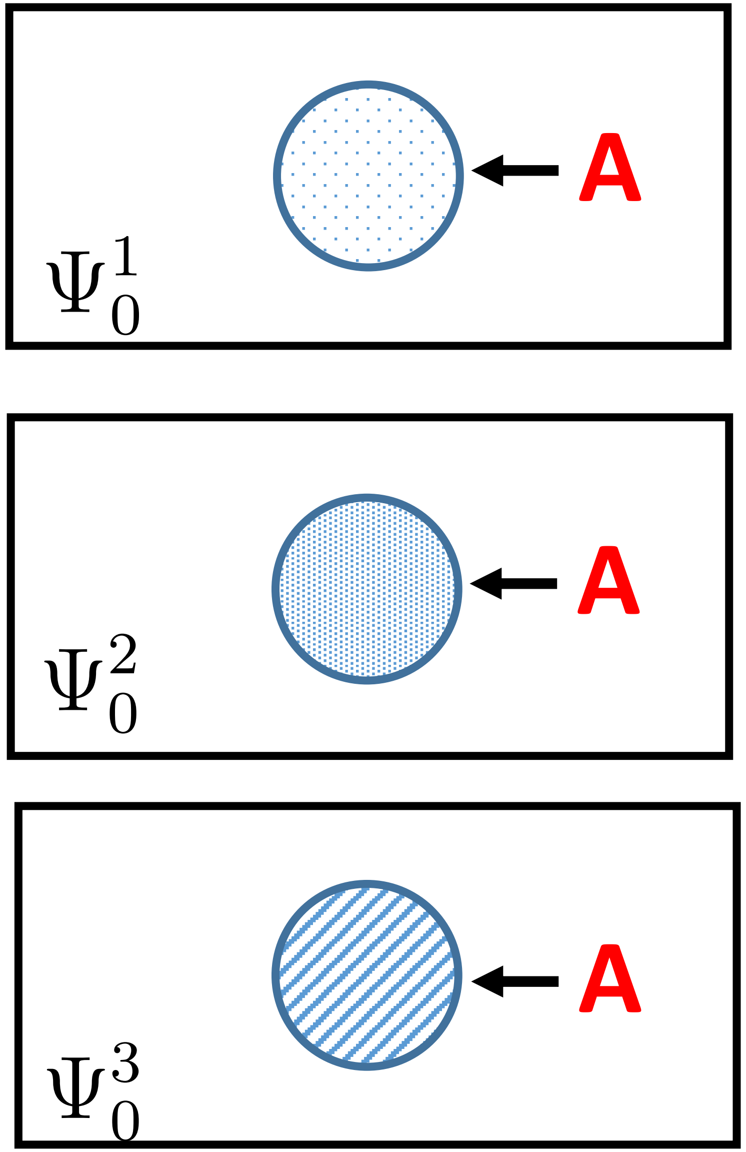
Thermalization

$$\left|\Psi^i(t)\right\rangle = U(t)\left|\Psi_0^i\right\rangle$$

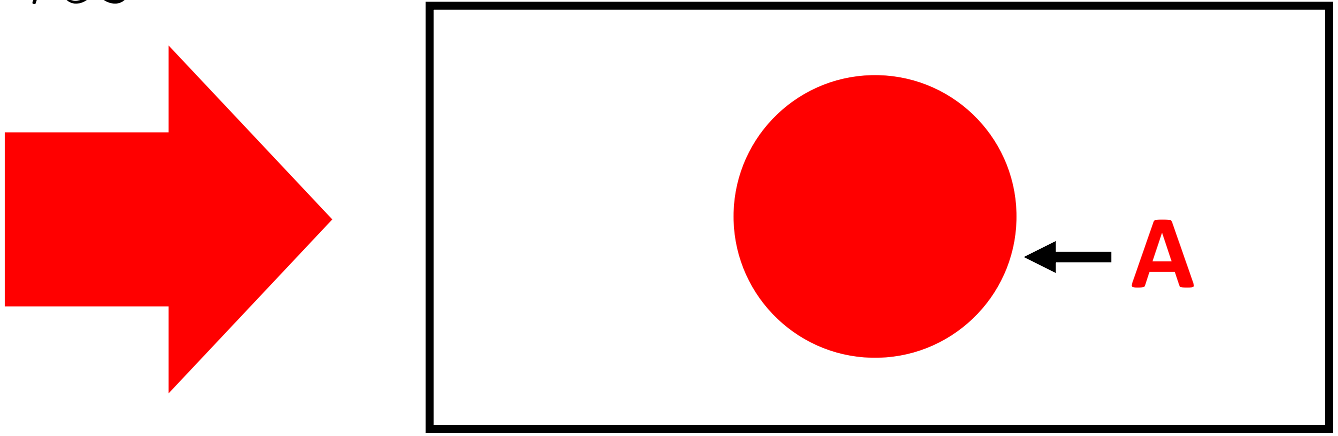


Thermalization

$$\left|\Psi^i_{\text{late}}\right\rangle = \lim_{t\to\infty} U(t)\left|\Psi^i_0\right\rangle$$



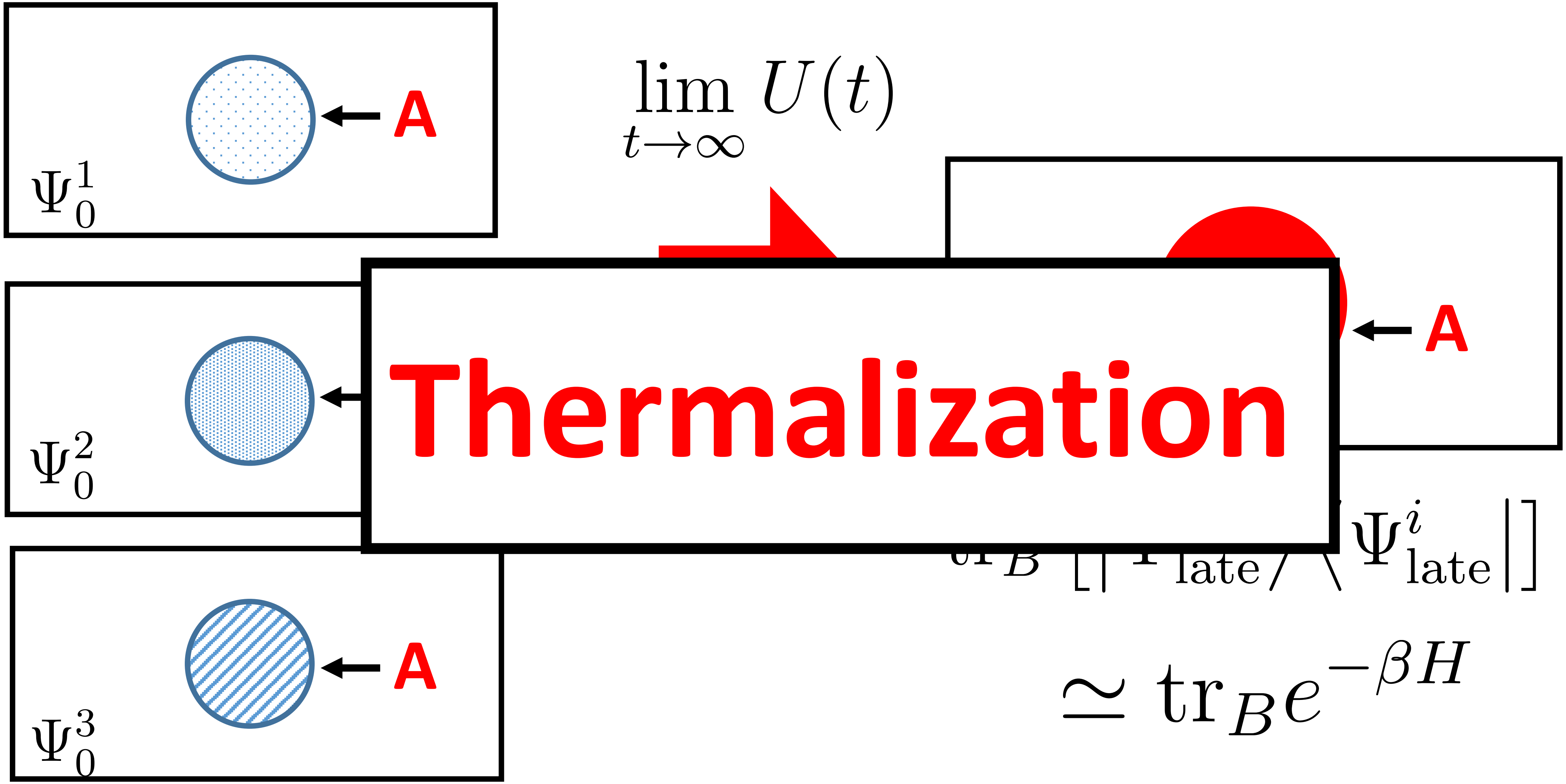
$$\lim_{t\to\infty} U(t)$$



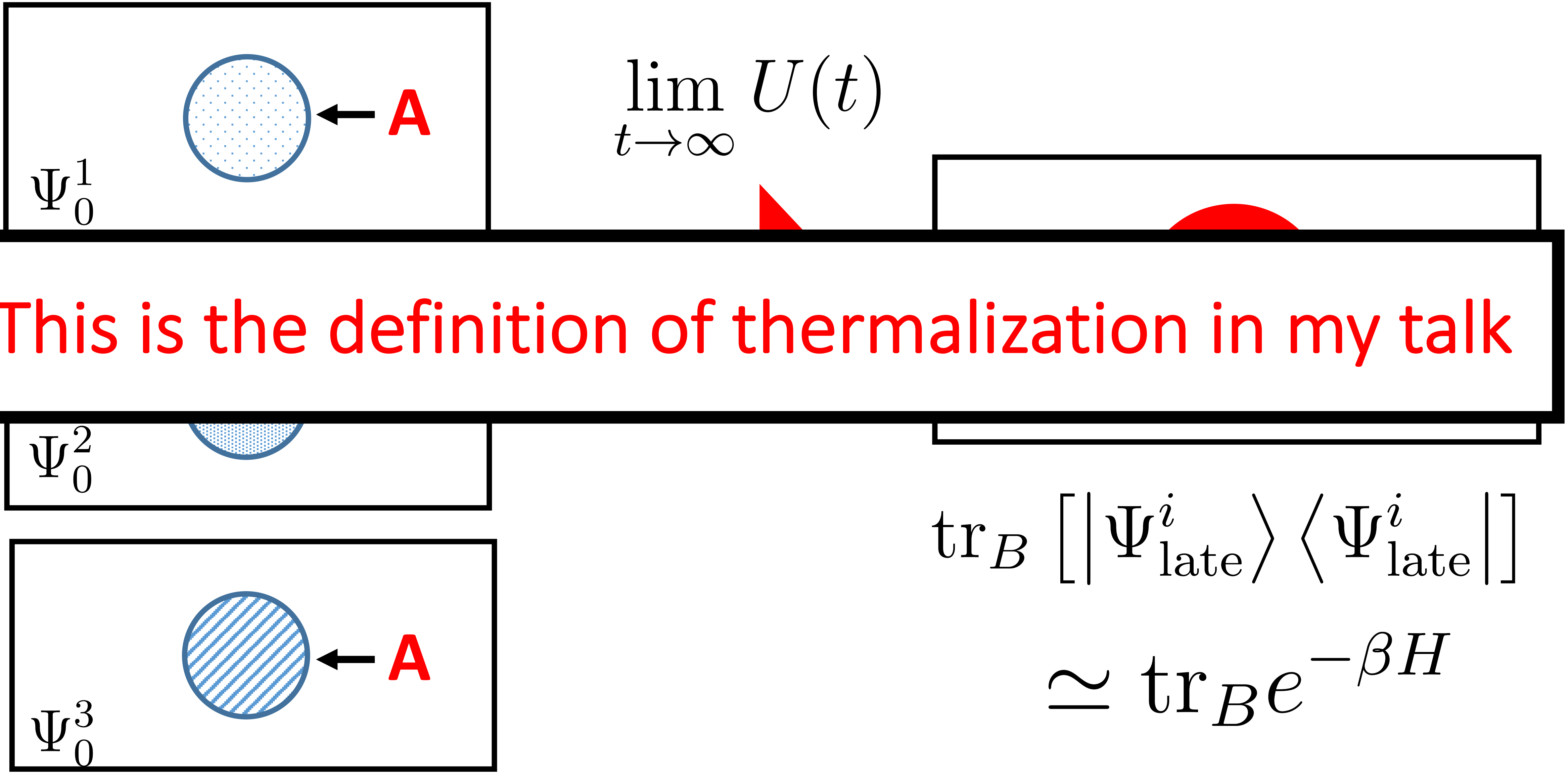
$$\mathrm{tr}_B\left[\left|\Psi^i_{\text{late}}\right\rangle\left\langle\Psi^i_{\text{late}}\right|\right]$$

$$\simeq \mathrm{tr}_B e^{-\beta H}$$

Thermalization $\left|\Psi^i_{\rm late}\right\rangle = \lim_{t\to\infty} U(t)\left|\Psi^i_0\right\rangle$



Thermalization $\left|\Psi^i_{\text{late}}\right\rangle = \lim_{t\to\infty} U(t)\left|\Psi^i_0\right\rangle$

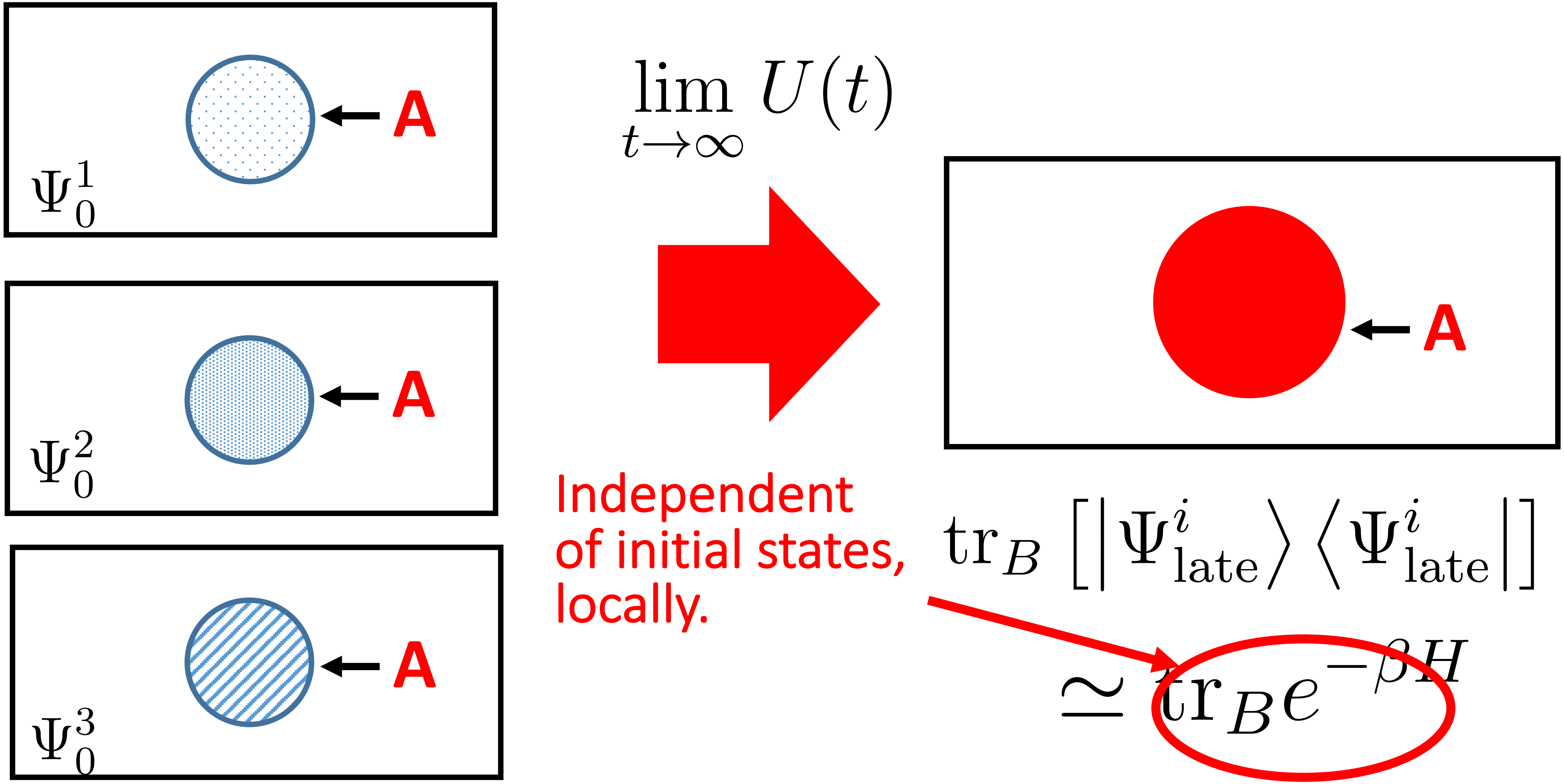


This is the definition of thermalization in my talk

$$\mathrm{tr}_B\left[\left|\Psi^i_{\text{late}}\right\rangle\left\langle\Psi^i_{\text{late}}\right|\right]$$

$$\simeq \mathrm{tr}_B e^{-\beta H}$$

Thermalization $\left|\Psi^i_{\text{late}}\right\rangle = \lim_{t\to\infty} U(t)\left|\Psi^i_0\right\rangle$



$$\lim_{t\to\infty} U(t)$$

Independent of initial states, locally.

$$\mathrm{tr}_B\left[\left|\Psi^i_{\text{late}}\right\rangle\left\langle\Psi^i_{\text{late}}\right|\right]$$

$$\simeq \mathrm{tr}_B e^{-\beta H}$$

Thermalization

Ex. $\mathrm{dis}(\rho_A(t), \rho_A^{th}) \to 0$

$$\langle \Psi(t) | \mathcal{O} | \Psi(t) \rangle \to \mathrm{tr}\left(\rho^{\mathrm{th}} \mathcal{O}\right)$$

States can be approximated by thermal state, locally .

Thermalization

Ex. $\mathrm{dis}(\rho_A(t), \rho_A^{th}) \to 0$

$\langle\Psi(t)|\,\mathcal{O}\,|\Psi(t)\rangle \to \mathrm{tr}\left(\rho^{\mathrm{th}}\mathcal{O}\right)$

States can be approximated by thermal state, locally .

➡ ***Thermalize!!***

Thermalization depends on

(1) Initial state, (2)Dynamics.

Thermalization

Ex. $\mathrm{dis}(\rho_A(t), \rho_A^{th}) \to 0$

$$\langle \Psi(t) | \mathcal{O} | \Psi(t) \rangle \to \mathrm{tr}\left(\rho^{\mathrm{th}} \mathcal{O}\right)$$

States can be approximated by thermal state, locally .

➡ ***Thermalize!!***

Key point:

Locally, states forget the initial conditions.

Scrambling

Ex. $\mathrm{dis}(\rho_A(t), \rho_A^{th}) \to 0$

$\langle \Psi(t) | \mathcal{O} | \Psi(t) \rangle \to \mathrm{tr}\left(\rho^{\mathrm{th}} \mathcal{O}\right)$

States can be approximated by thermal state, locally .

➡ ***Thermalize!!***

Key point:

Locally, states forget the initial conditions.

⬆

Scrambling effect

Key point:

Locally, states forget the initial condition.

Scrambling effect

$$|\Psi(t)\rangle = U(t)|\Psi\rangle$$

Scrambling effect depends on time evolution operator.

Key point:

Locally, states forget the initial condition.

Scrambling effect

$$|\Psi(t)\rangle = U(t)|\Psi\rangle$$

Scrambling effect depends on time evolution operator.

I would like to know ***how the scrambling effect depends on the unitary channels.***

Key point:

Locally, states forget the initial condition.

Scrambling effect

$$|\Psi(t)\rangle = U(t)|\Psi\rangle$$

Scrambling effect depends on time evolution operator.

I would like to quantify ***scrambling effect***.

Key point:

Locally, states forget the initial condition.

Scrambling effect

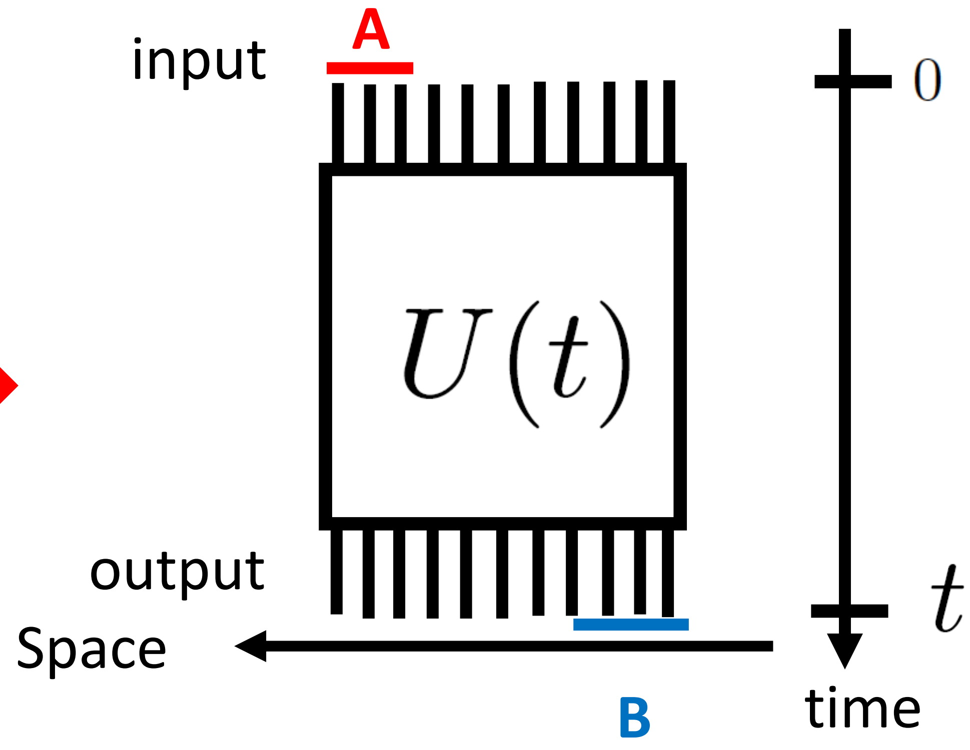
$$|\Psi(t)\rangle = U(t)|\Psi\rangle$$

Scrambling effect depends on time evolution operator.

To understand scrambling
leads to understanding thermalization.

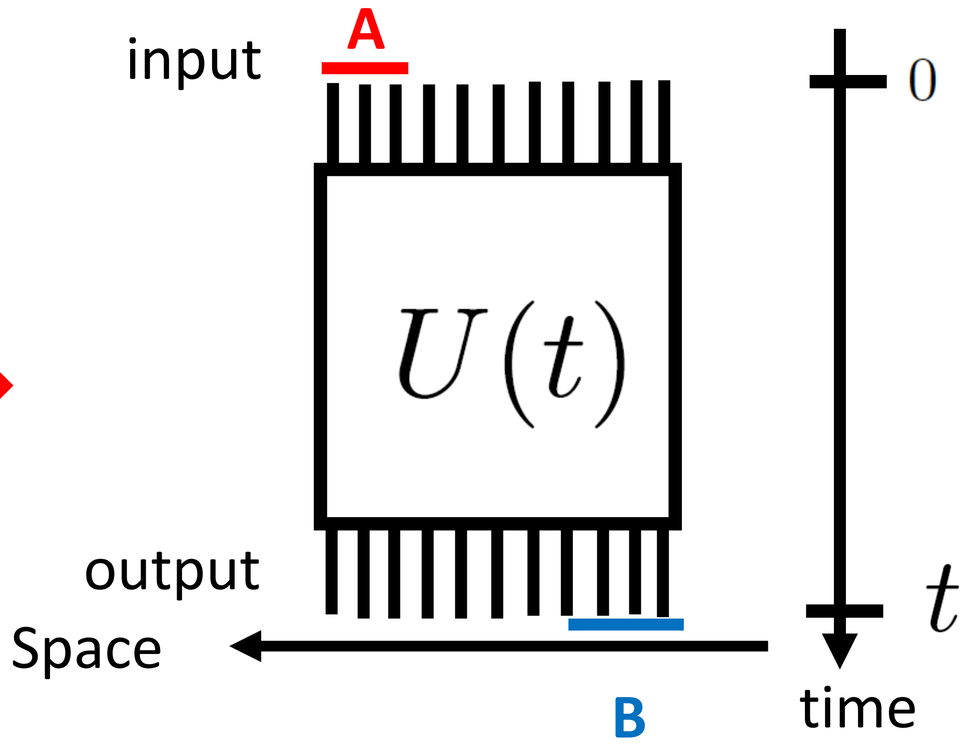
Scrambling

$$U(t) = e^{-itH}$$



Scrambling

$$U(t) = e^{-itH}$$



Expectation

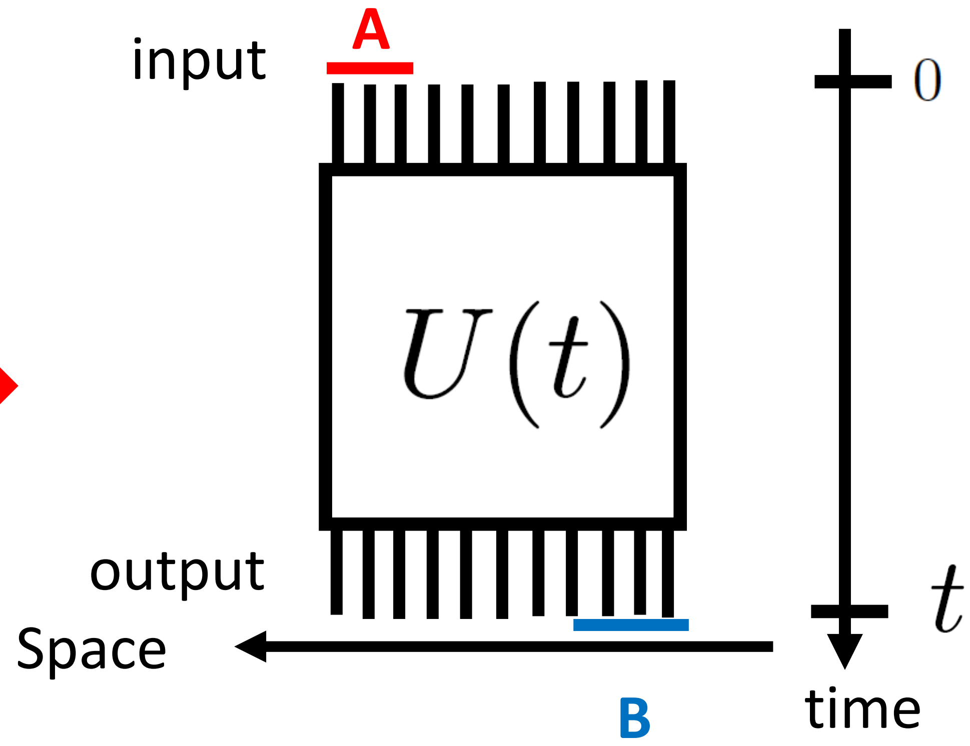
Correlation between A and B

How much information sent from A to B due to

$$U(t) = e^{-itH}$$

Scrambling

$$U(t) = e^{-itH}$$



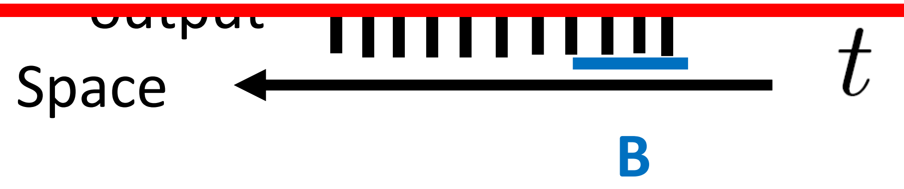
Expectation

No correlation between A and B

Signature of (maximally) scrambling

Concept

We would like to study the correlation between A and B by studying operator entanglement, which is independent of state.



Expectation

No correlation between A and B

Signature of (maximally) scrambling

Operator entanglement

Unitary channel:

$$U(t) = e^{-itH} = \sum_a e^{-iE_a t} \left|a\right\rangle_{out} \left\langle a\right|_{in}$$

Channel-state dual map: $\left\langle a\right|_{in} \to \left|a\right\rangle_{in}$

Operator entanglement

Unitary channel:

$$U(t) = e^{-itH} = \sum_a e^{-iE_a t} \left|a\right\rangle_{out} \left\langle a\right|_{in}$$

Channel-state dual map: $\left\langle a\right|_{in} \rightarrow \left|a\right\rangle_{in}$

Dual state:

$$\left|U(t)\right\rangle = \mathcal{N} \sum_a e^{-(it+\epsilon)E_a} \left|a\right\rangle_{out} \left|a\right\rangle_{in}$$

Hilbert space:

$$\mathcal{H} \rightarrow \mathcal{H}_{in} \otimes \mathcal{H}_{out}$$

Operator entanglement

Unitary channel:

$$U(t) = e^{-itH} = \sum_a e^{-iE_a t} \left|a\right\rangle_{out} \left\langle a\right|_{in}$$

Channel-state dual map: $\left\langle a\right|_{in} \rightarrow \left|a\right\rangle_{in}$

Dual state:

$$\left|U(t)\right\rangle = \mathcal{N} \sum_a e^{-(it+\epsilon)E_a} \left|a\right\rangle_{out} \left|a\right\rangle_{in}$$

A regulator for normalization.

Hilbert space:

$$\mathcal{H} \rightarrow \mathcal{H}_{in} \otimes \mathcal{H}_{out}$$

Operator entanglement

Unitary channel:

$$U(t) = e^{-itH} = \sum_a e^{-iE_a t} \left| a \right\rangle_{out} \left\langle a \right|_{in}$$

Channel-state dual map: $\left\langle a \right|_{in} \to \left| a \right\rangle_{in}$

Dual state:

$$\left| U(t) \right\rangle = \mathcal{N} \sum_a e^{-(it+\epsilon)E_a} \left| a \right\rangle_{out} \left| a \right\rangle_{in}$$

Only this depends on the initial state.

Hilbert space:

$$\mathcal{H} \to \mathcal{H}_{in} \otimes \mathcal{H}_{out}$$

Operator entanglement

Unitary channel:

$$U(t) = e^{-itH} = \sum_a e^{-iE_a t} \left|a\right\rangle_{out} \left\langle a\right|_{in}$$

Channel-state dual map: $\left\langle a\right|_{in} \rightarrow$

Our initial state

Energy eigenstate

$$\left|\text{Initial}\right\rangle = \mathcal{N} \sum_a^{\frac{1}{\epsilon}} C_a \left|a\right\rangle \approx \mathcal{N} \sum_a C_a e^{-\epsilon H} \left|a\right\rangle$$

Dual state:

$$\left|U(t)\right\rangle = \mathcal{N} \sum_a e^{-(it+\epsilon)E_a} \left|a\right\rangle_{ou}$$

Operator entanglement

Unitary channel:

$$U(t) = e^{-itH} = \sum_a e^{-iE_at} |a\rangle_{out} \langle a|_{in}$$

Channel-state dual map: $\langle a|_{in} \rightarrow$

Our initial state

Energy eigenstate

$$|\text{Initial}\rangle = \mathcal{N} \sum_a^{\frac{1}{\epsilon}} C_a |a\rangle \approx \mathcal{N} \sum_a C_a e^{-\epsilon H} |a\rangle$$

Dual state:

$$|U(t)\rangle = \mathcal{N} \sum_a e^{-(it+\epsilon)E_a} |a\rangle_{ou}$$

Operator entanglement

Unitary channel:

$$U(t) = e^{-itH} = \sum_a e^{-iE_a t} \left|a\right\rangle_{out} \left\langle a\right|_{in}$$

Channel-state dual map: $\left\langle a\right|_{in} \to \left|a\right\rangle_{in}$

Dual state:

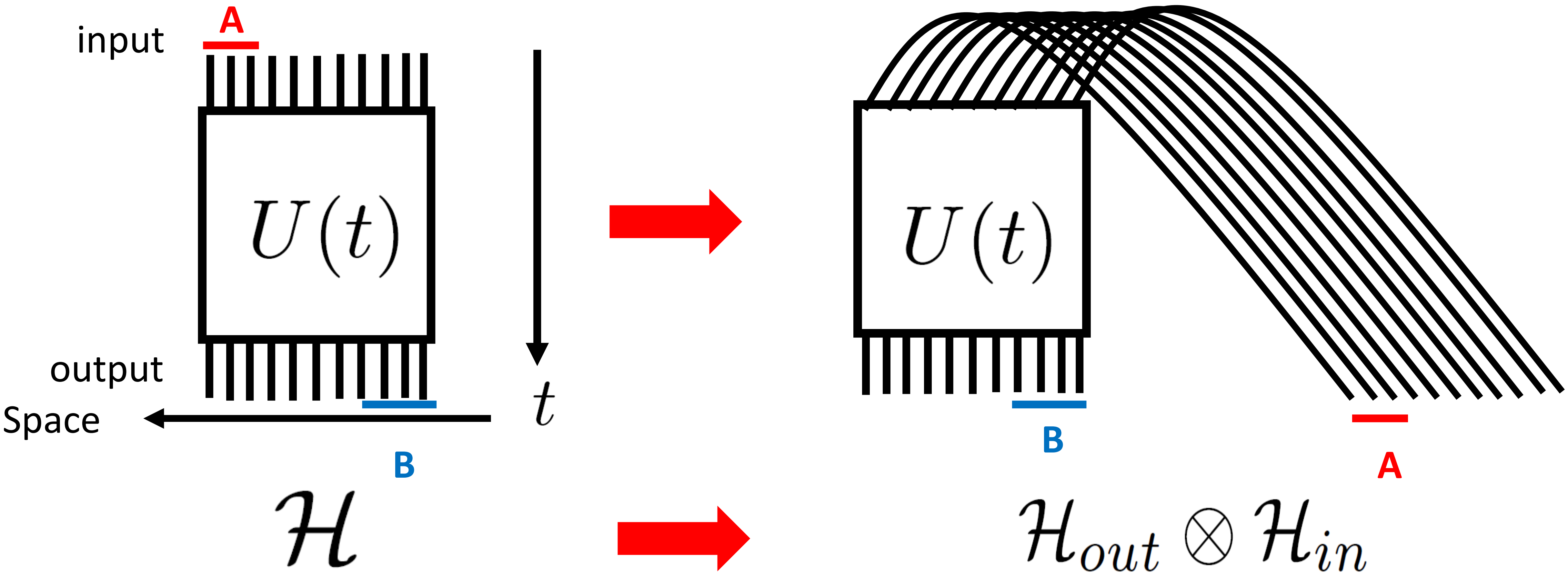
$$\left|U(t)\right\rangle = \mathcal{N} e^{-\frac{it}{2}(H_{in}+H_{out})} \left|TFD\right\rangle$$

Hilbert space:

$$\mathcal{H} \to \mathcal{H}_{in} \otimes \mathcal{H}_{out}$$

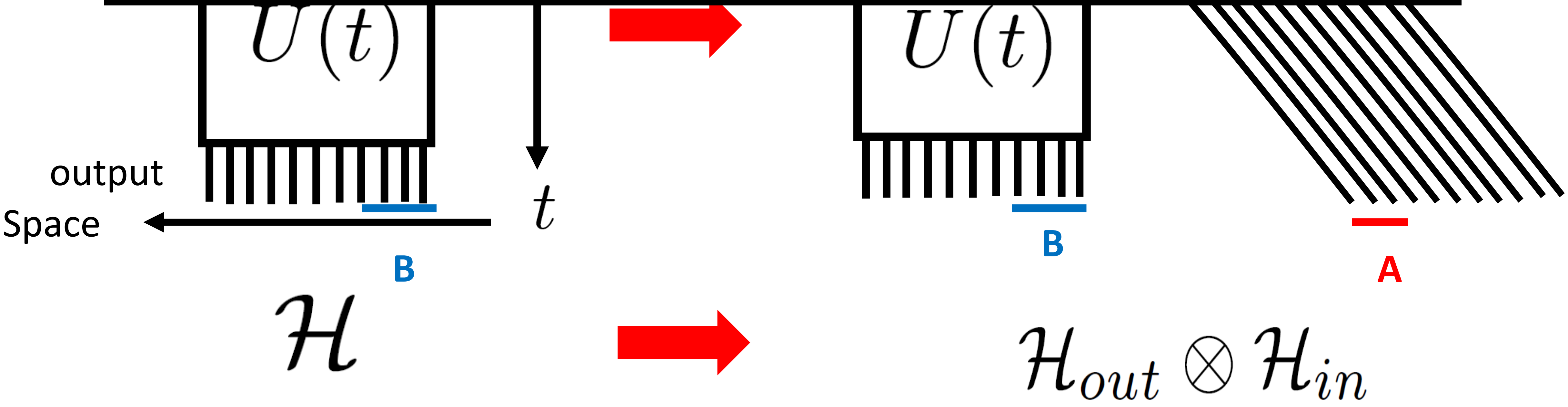
Scrambling

$$\langle \quad |_{in} \rightarrow | \quad \rangle_{in}$$

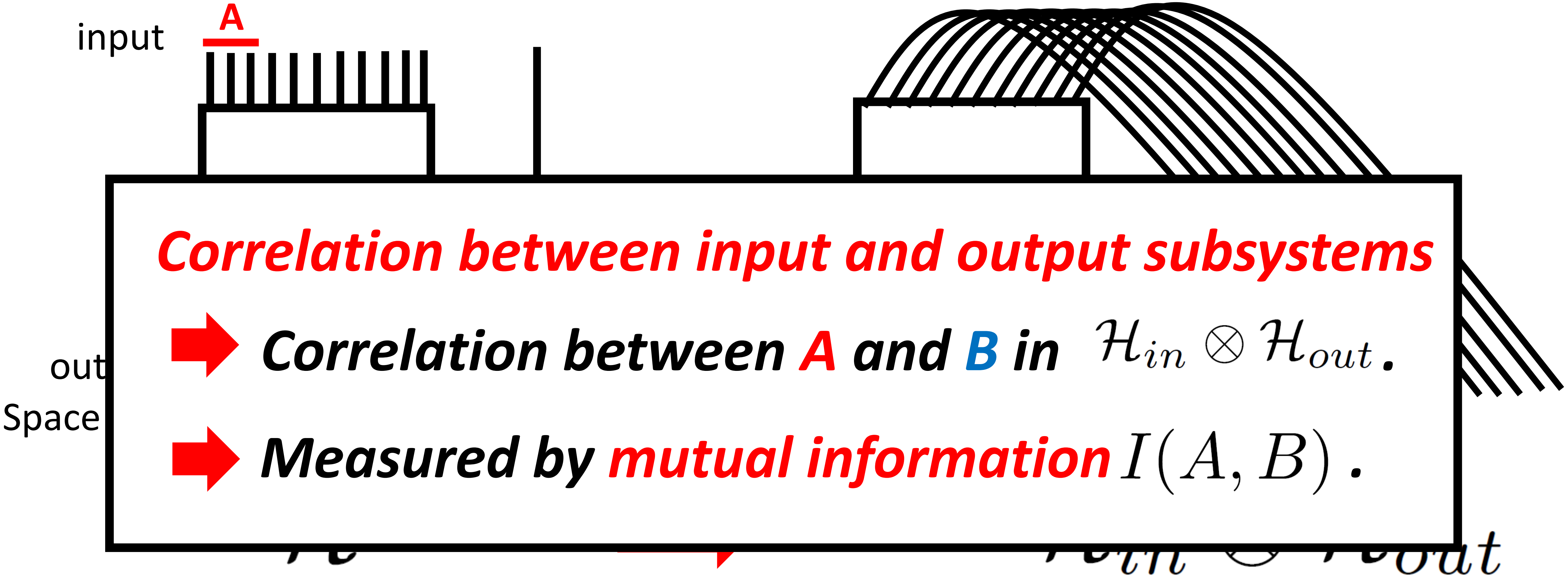


Correlation between input and output subsystems

➡ ***Correlation between A and B in $\mathcal{H}_{out} \otimes \mathcal{H}_{in}$.***



Concept

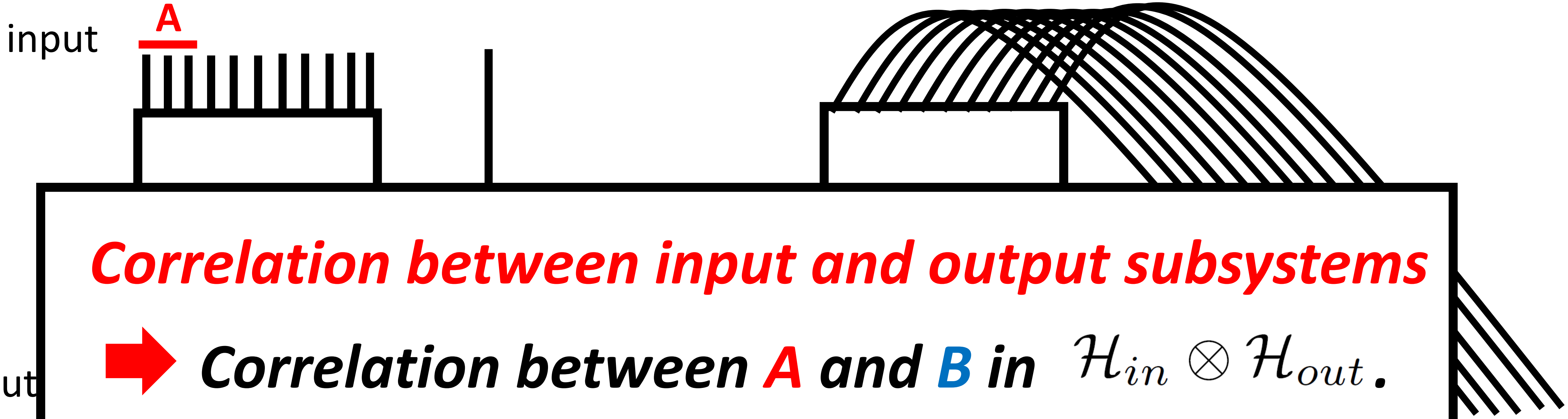


Correlation between input and output subsystems

➡ ***Correlation between A and B in*** $\mathcal{H}_{in} \otimes \mathcal{H}_{out}$ **.**

➡ ***Measured by mutual information*** $I(A, B)$ **.**

Concept



Correlation between input and output subsystems

- **Correlation between A and B in** $\mathcal{H}_{in} \otimes \mathcal{H}_{out}$ **.**
- **Measured by** $I(A,B)$ **,** $\mathcal{E}(A,B)$ **.**

Motivation

Which CFT (QFT) shows a signature of scrambling ?

Spin system: [Hosur-Qi-Roberts-Yoshida'16]

Motivation

How much information from A to B are scrambled due to channels in field theory?

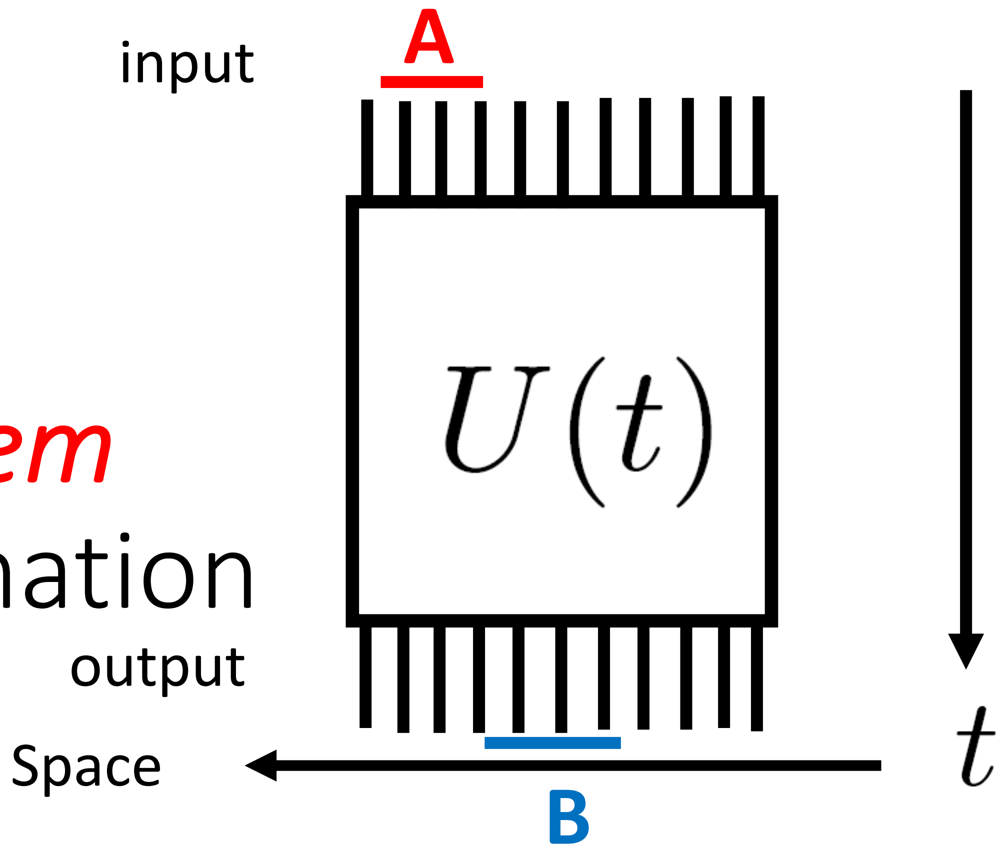
Spin system: [Hosur-Qi-Roberts-Yoshida’16]

Results (Main1)

No correlation
between A and any B

No one in output subsystem
can't get quantum information
locally.

Signature of
maximally scrambling



Results (Main1)

Holographic channel

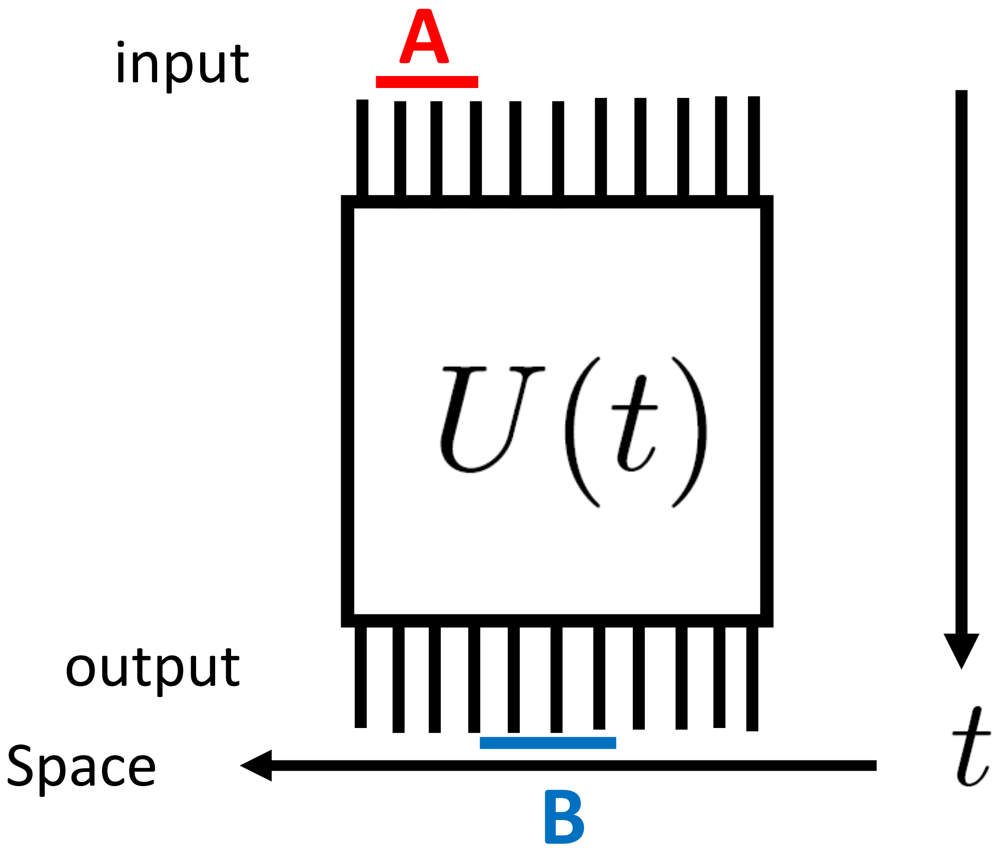
For ***disjoint*** or ***late-time*** case, ***for any B***,

$$I(A,B)=0, \mathcal{E}(A,B)=0$$

⇕

No correlation

between A and any B



Results (Main1)

Holographic channel

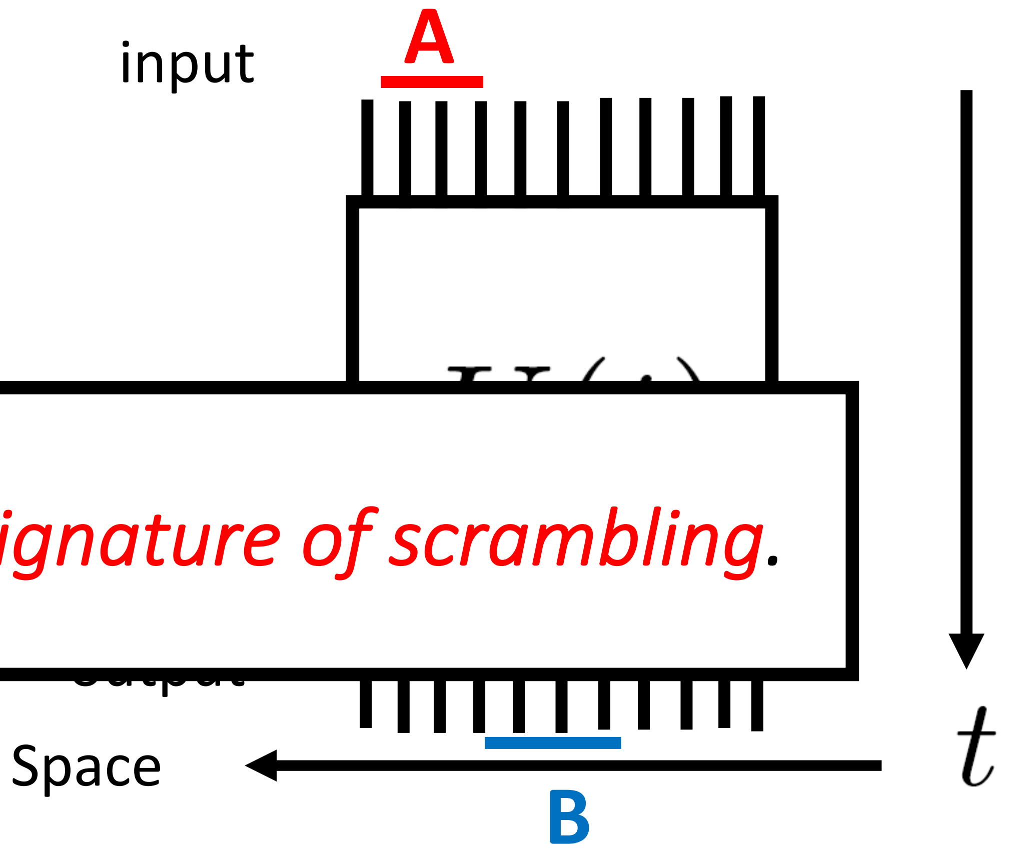
For ***disjoint*** or ***late-time*** case, ***for any B***,

$$I(A,B)=0, \mathcal{E}(A,B)=0$$

Holographic channel shows a signature of scrambling.

No correlation

between A and any B



Results (Main2)

$$I(A,B) = S_A + S_B - S_{A\cup B}$$

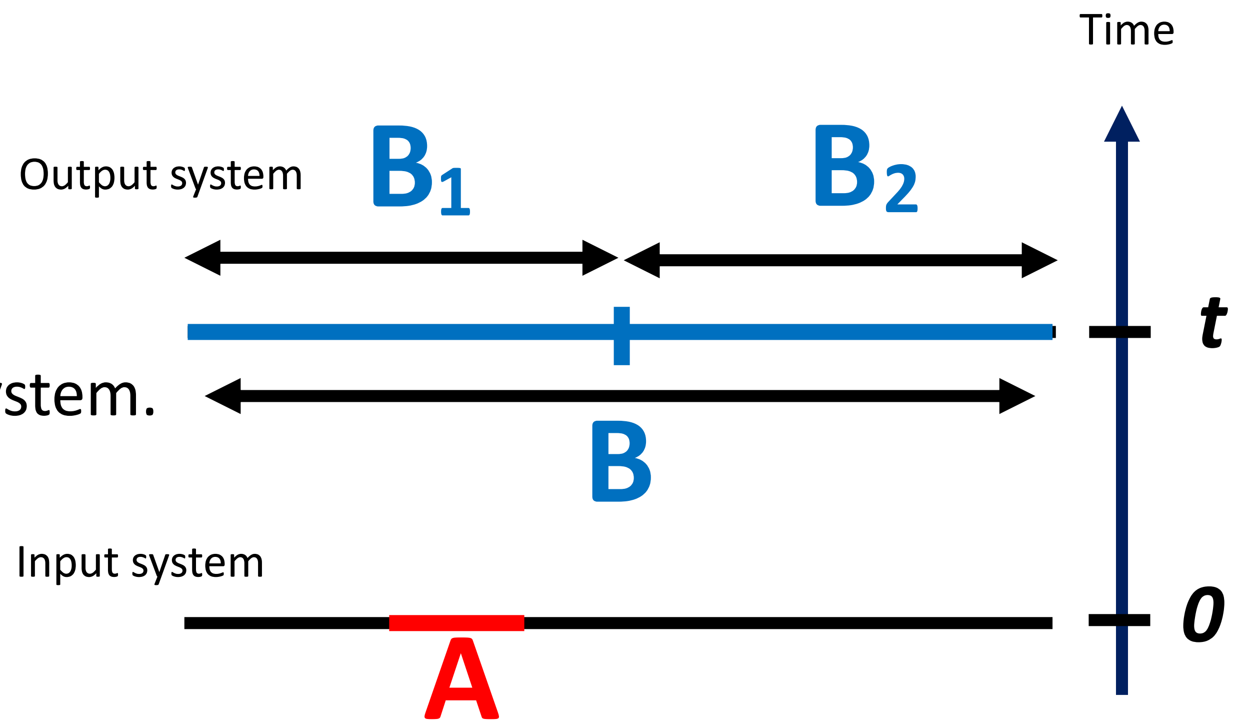
We have computed tri-partite operator mutual information:

$$\boxed{I(A,B_1,B_2) = I(A,B_1) + I(A,B_2) - I(A,B)}$$

B is the whole of output system.

B_1 and **B_2** are the halves of output system.

A is subsystem in input system.



Results (Main2)

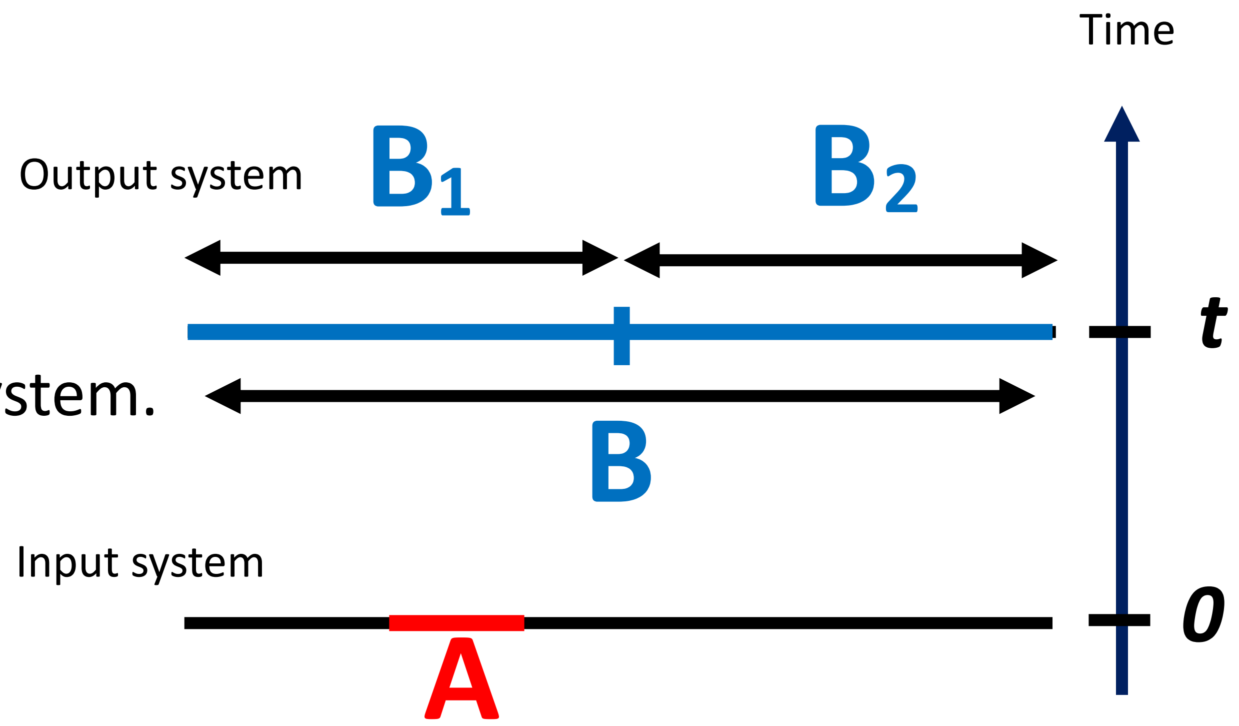
We have computed tri-partite operator logarithmic negativity:

$$\boxed{\mathcal{E}_3(A, B_1, B_2) \equiv \mathcal{E}(A, B_1) + \mathcal{E}(A, B_2) - \mathcal{E}(A, B)}$$

B is the whole of output system.

B_1 and **B_2** are the halves of output system.

A is subsystem in input system.



2d free fermion channel

$$\boxed{I(A, B_1, B_2) = 0, \mathcal{E}_3(A, B_1, B_2) = 0}$$

B is the whole of output system.

B_1 and **B_2** are the halves of output system.

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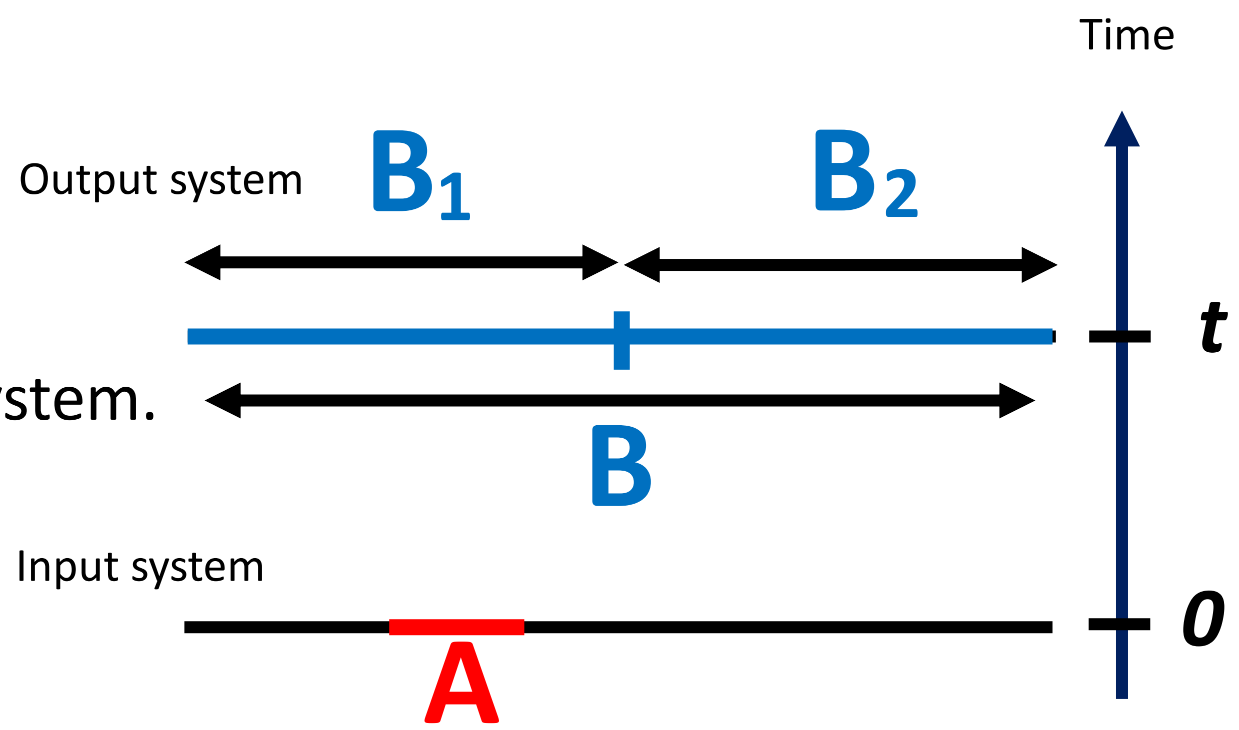
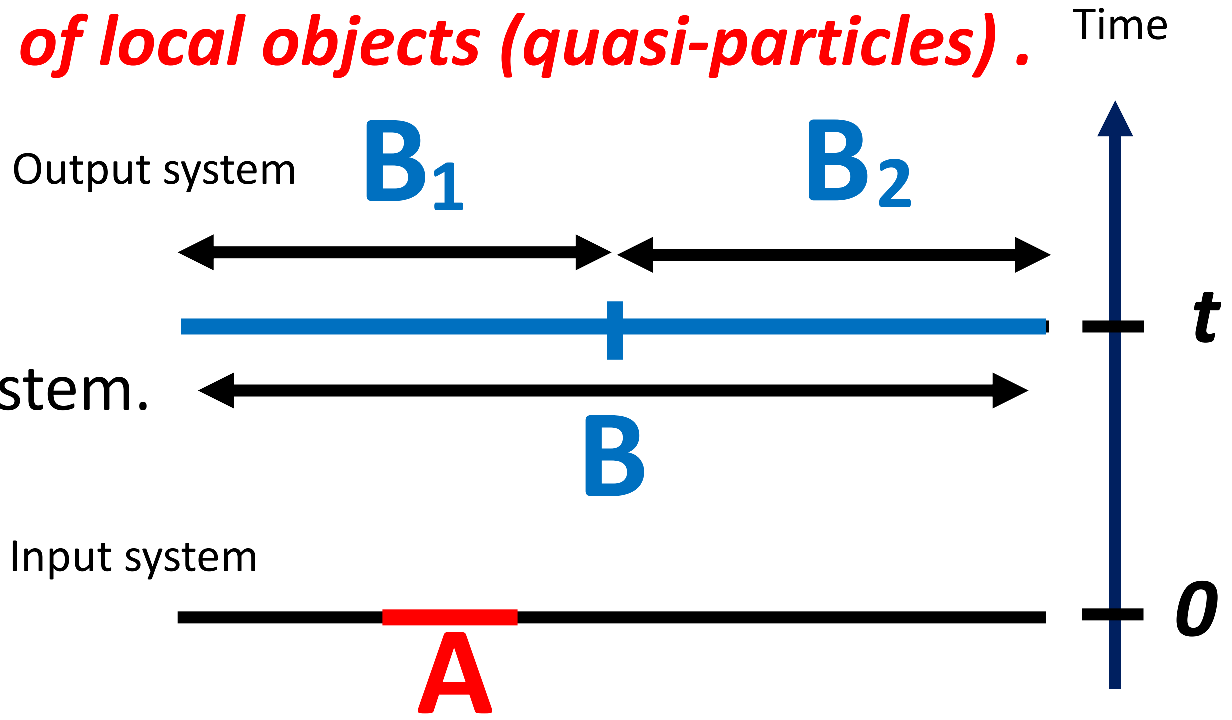


2d free fermion channel

$$\boxed{I(A, B_1, B_2) = 0, \mathcal{E}_3(A, B_1, B_2) = 0}$$

This can be interpreted in terms of

the relativistic propagation of local objects (quasi-particles) .



B is the whole of output system.

B$_1$ and **B**$_2$ are the halves of output system.

A is subsystem in input system.

2d chaotic channel (holographic channel)

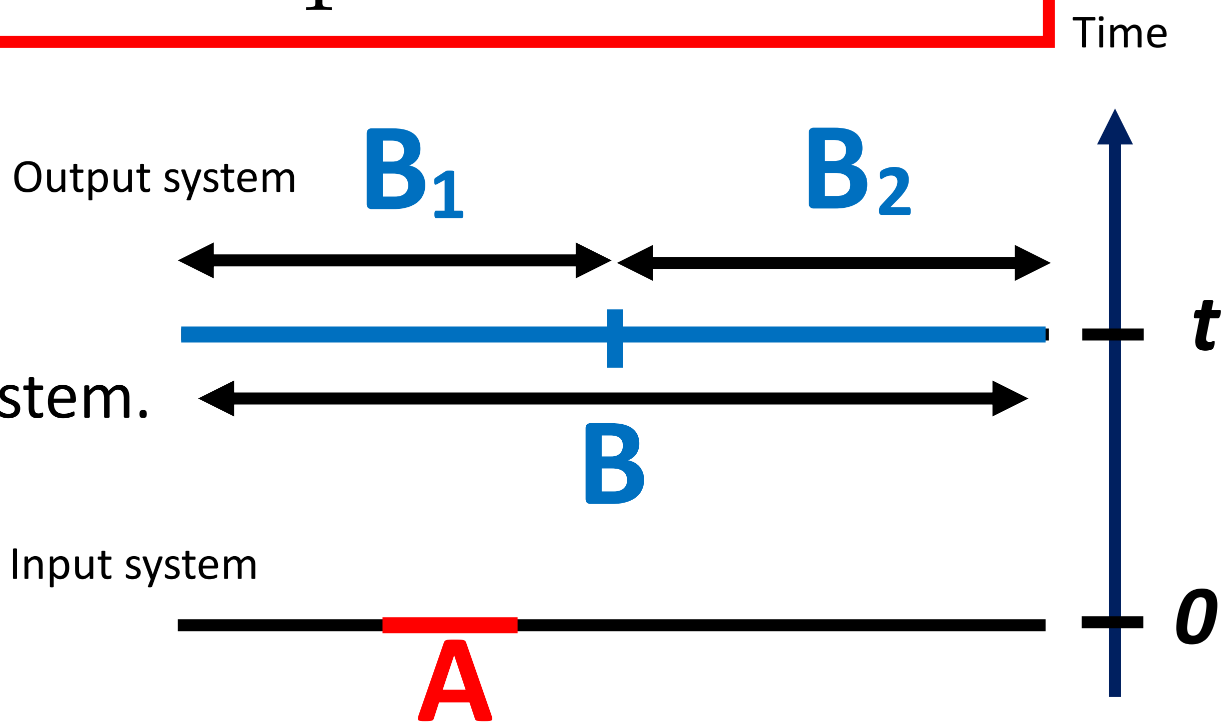
Late time:

$$I(A, B_1, B_2) \to -2S_A$$
$$\mathcal{E}_3(A, B_1, B_2) \to -2 \times \frac{3}{4} S_A^{(1/2)} = -2\mathcal{E}_{A,\bar{A}}$$

B is the whole of output system.

B₁ and **B₂** are the halves of output system.

A is subsystem in input system.



2d chaotic channel (holographic channel)

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B is the whole of output system.

B$_1$ and **B$_2$** are the halves of output system.

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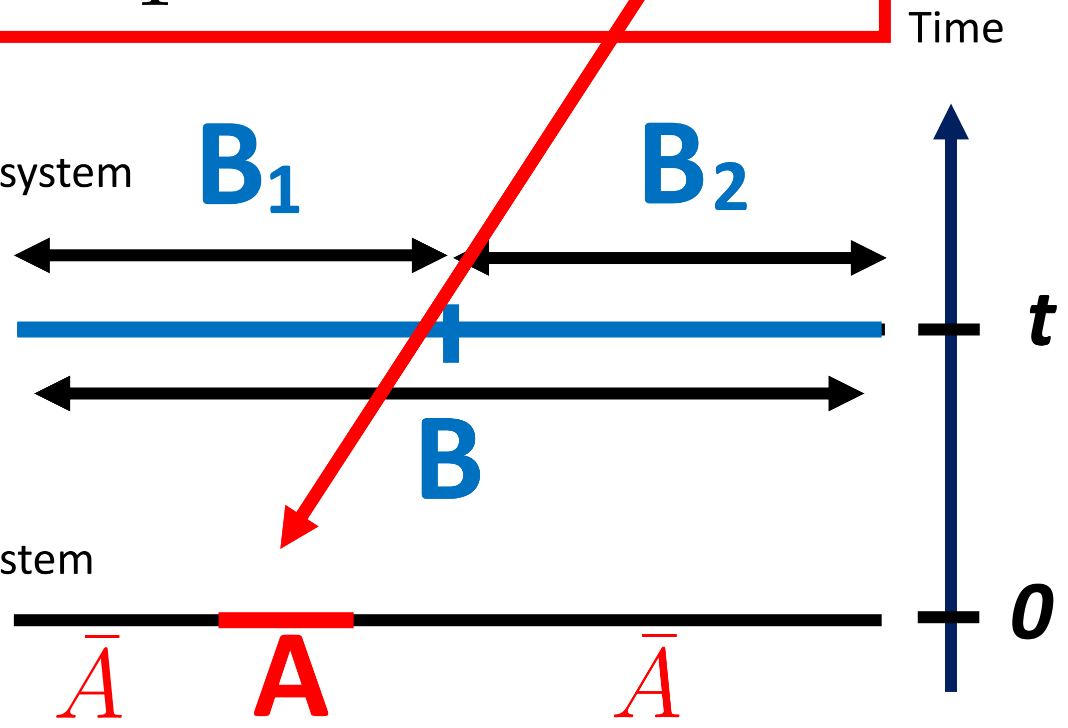

2d chaotic channel (holographic channel)

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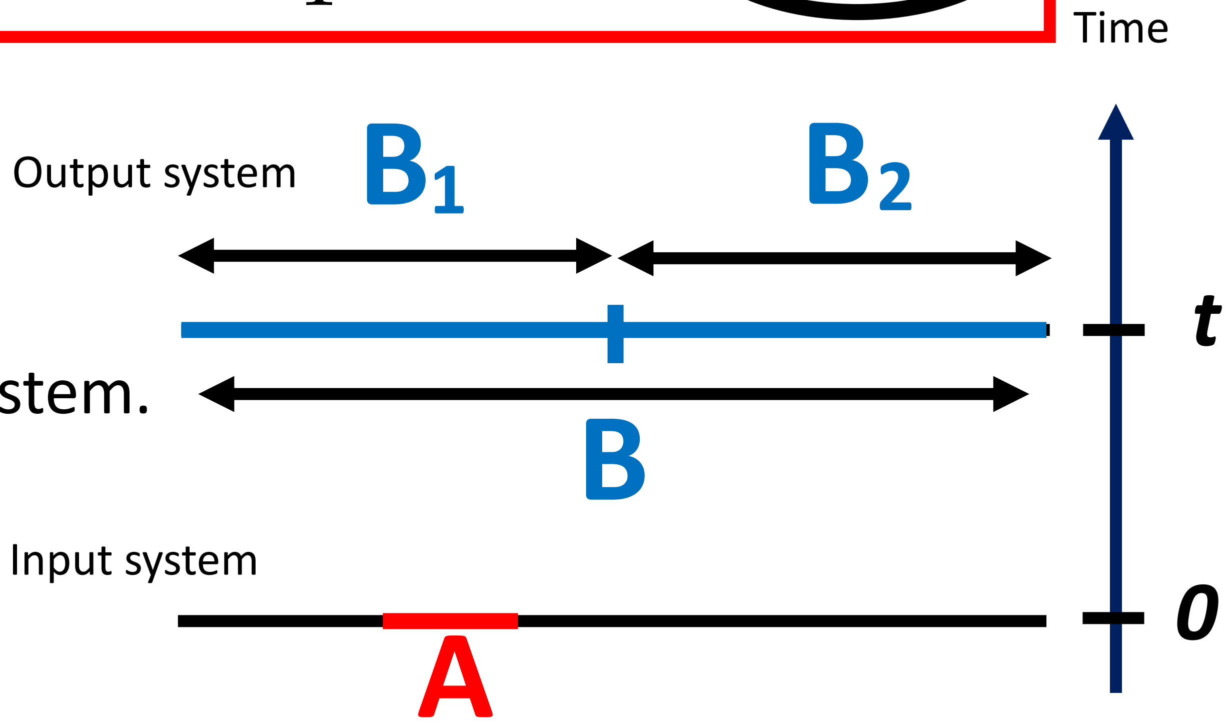
Lower bound

B is the whole of output system.

B₁ and **B₂** are the halves of output system.

A is subsystem in input system.



2d chaotic channel (holographic channel)

Lower bound

Late time:

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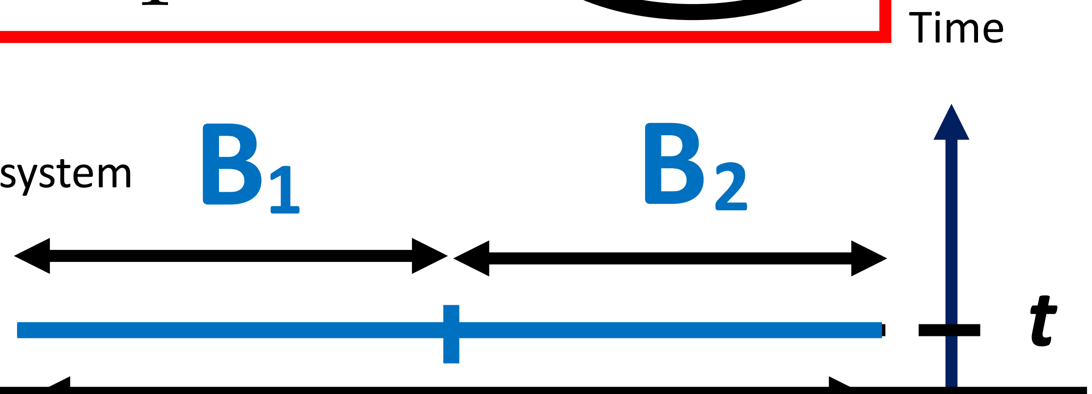


B is the whole of output system.

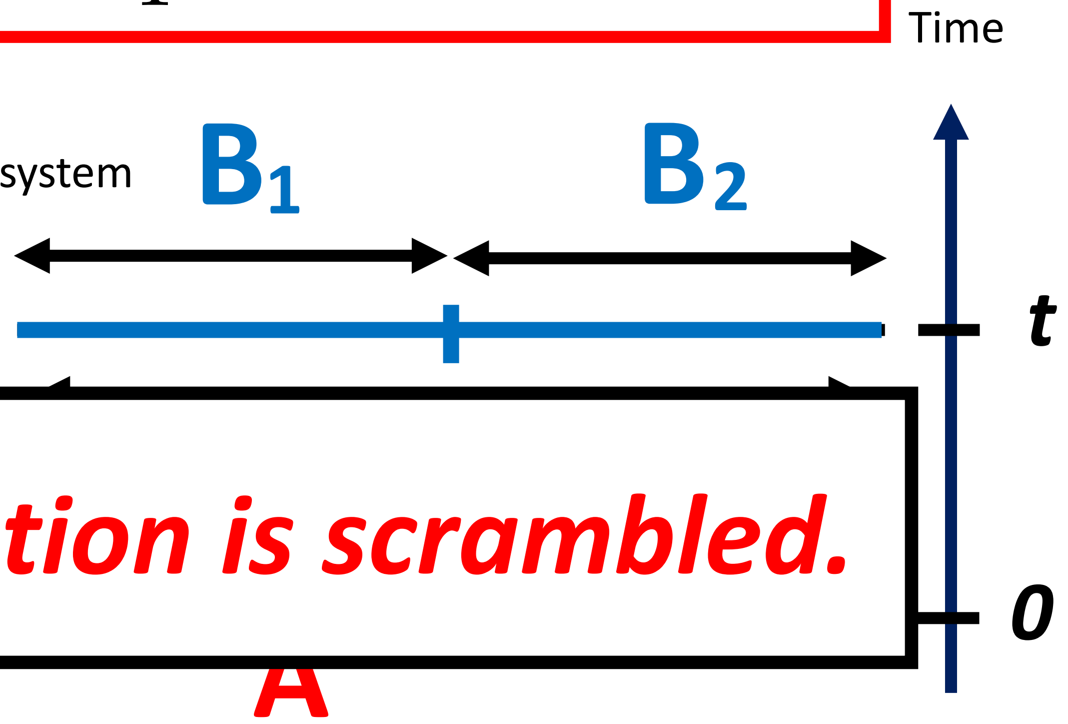
We expect ***QFT-channels with strong scrambling ability*** to ***satisfy this lower bound, eventually***.

2d chaotic channel (holographic channel)

Late time:

$$I(A, B_1, B_2) \rightarrow -2S_A$$

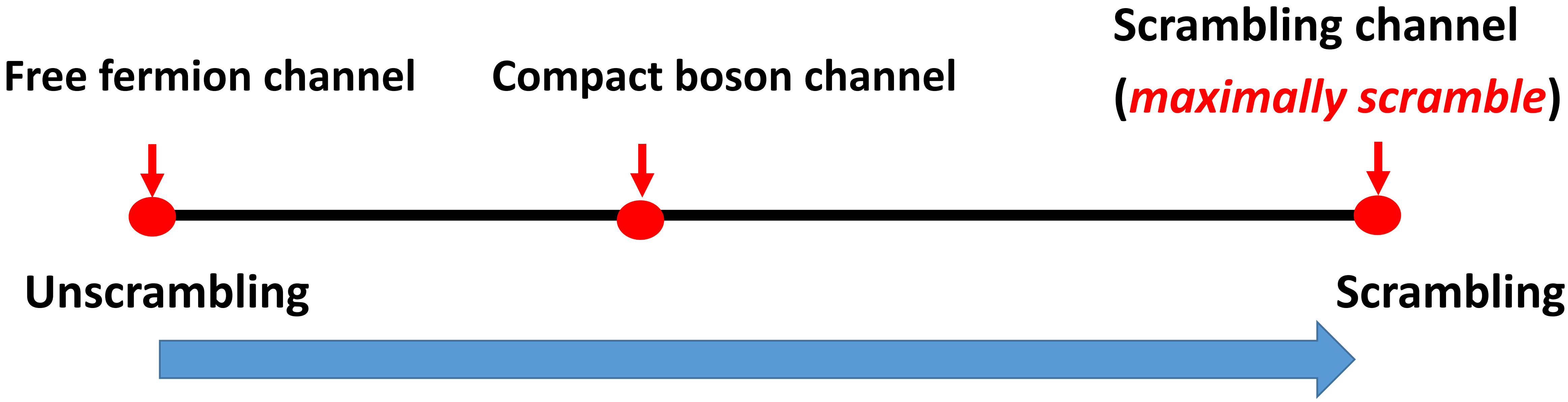
$$\mathcal{E}_3(A, B_1, B_2) \rightarrow -2 \times \frac{3}{4} S_A^{(1/2)} = -2\mathcal{E}_{A,\bar{A}}$$



B is the whole of output system.

This shows all information is scrambled.

How channels scrambles information



Bipartite operator mutual information in the replica trick

What we compute is

$$I(A,B) = S_A + S_B - S_{A\cup B} = \lim_{n\to 1}\left[S_A^{(n)} + S_B^{(n)} - S_{A\cup B}^{(n)}\right]$$

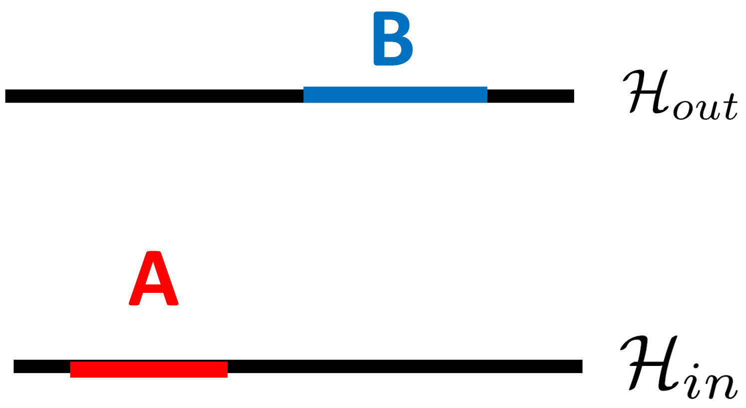
$$= \lim_{n\to 1}\frac{1}{1-n}\left[\log tr_A\,(\rho_A)^n + \log tr_B\,(\rho_B)^n - \log tr_{A\cup B}\,(\rho_{A\cup B})^n\right]$$

State:

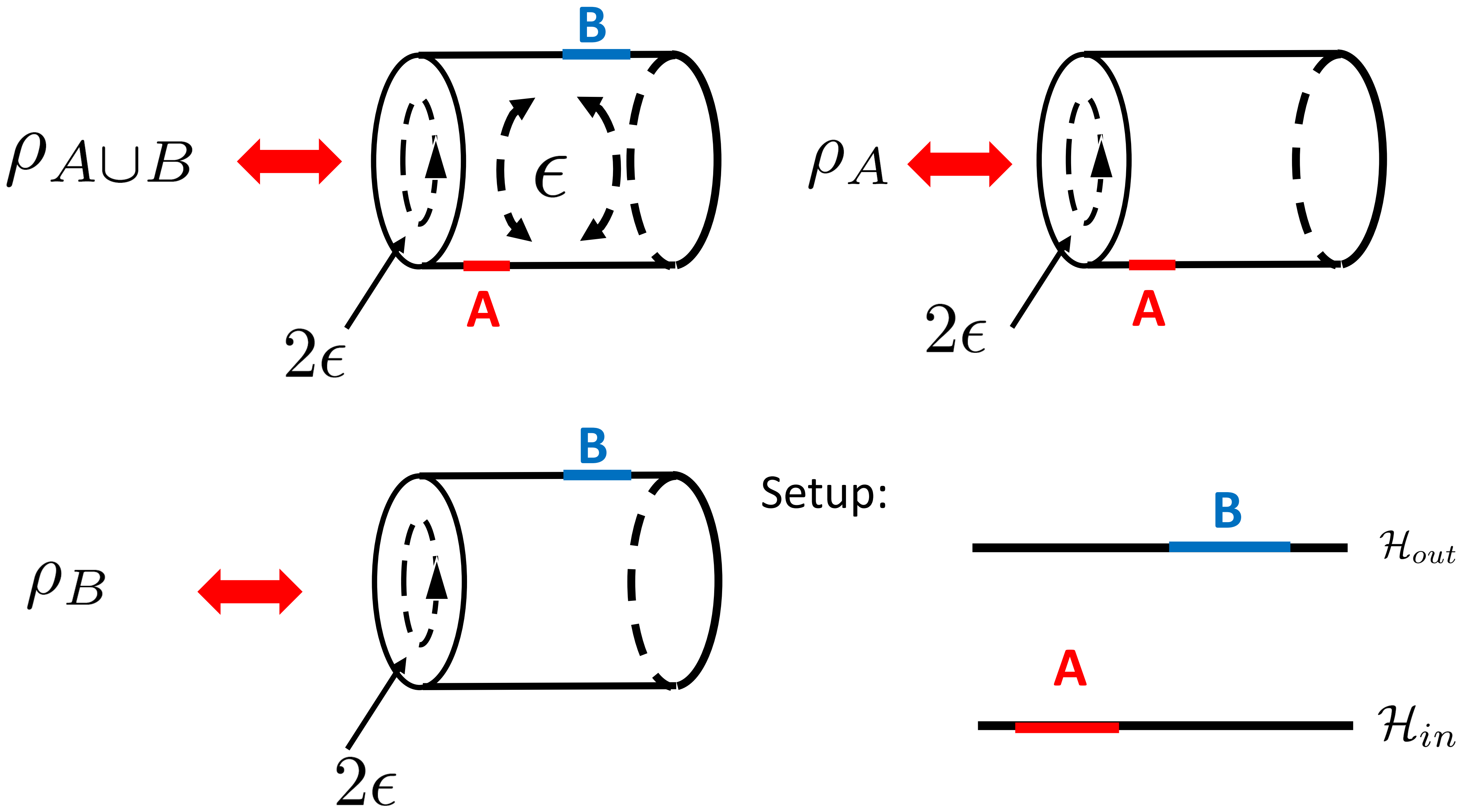
$$|U(t)\rangle = \mathcal{N} e^{-\frac{it}{2}(H_{in}+H_{out})}\,|TFD\rangle$$

$$\rho = |U(t)\rangle\,\langle U(t)|$$

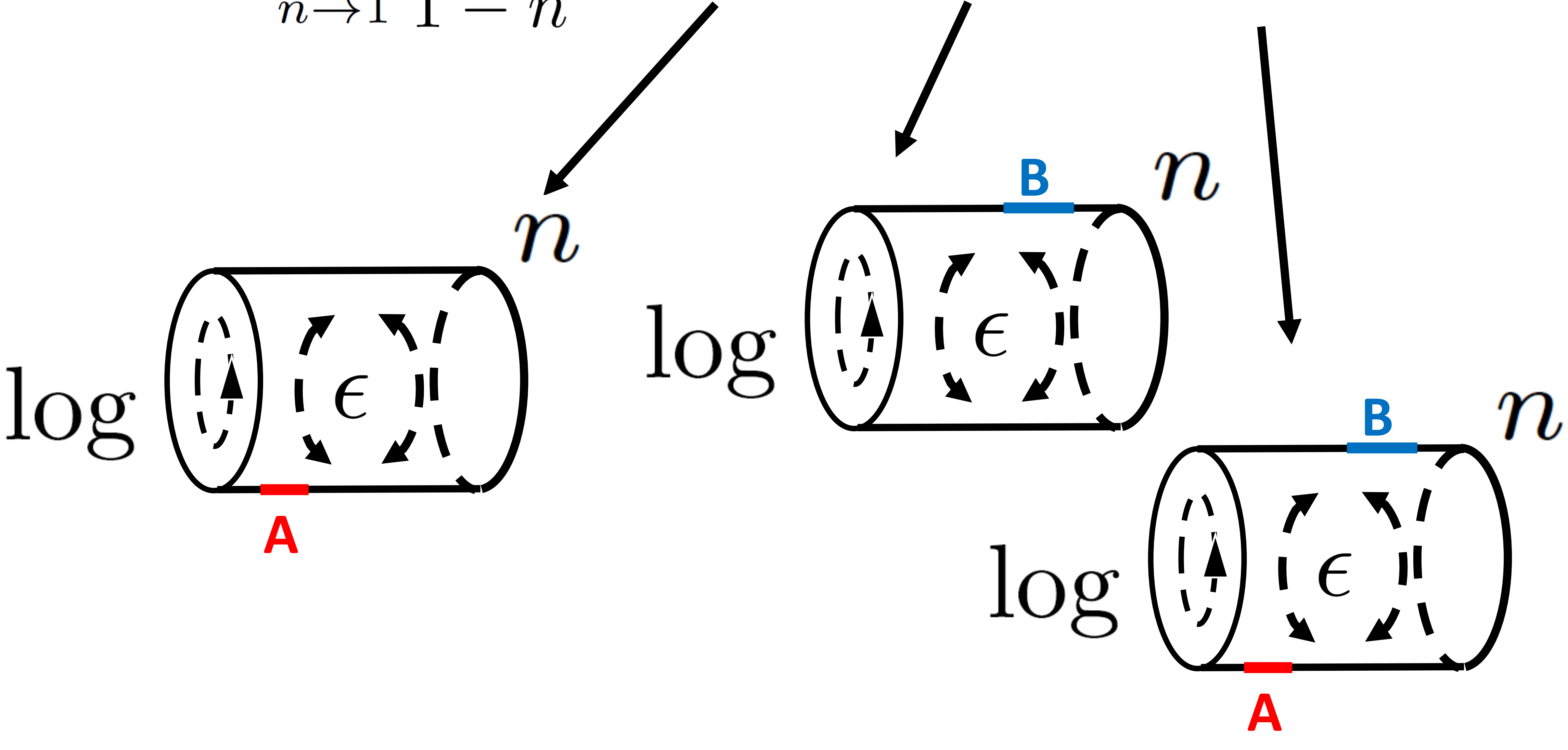
Setup:



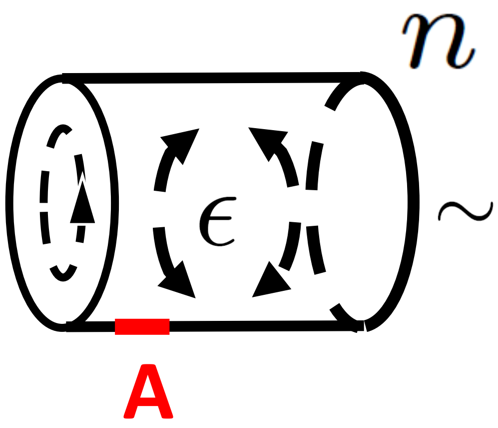
Bipartite operator mutual information in the replica trick



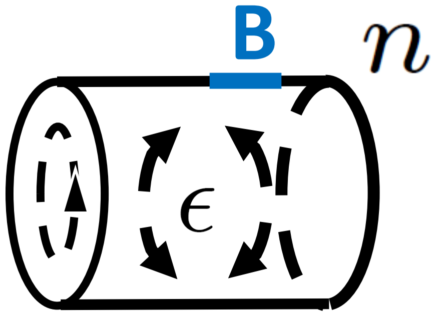
$$I(A,B) = \lim_{n\to 1} \frac{1}{1-n}\left[\log \rho_A^n + \log \rho_B^n - \log \rho_{A\cup B}^n\right]$$



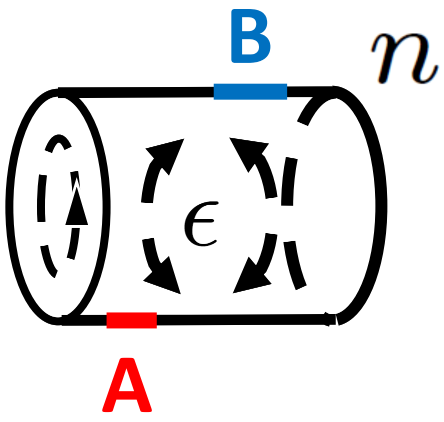
$$I(A,B)=\lim_{n\to 1}\frac{1}{1-n}\left[\log\rho_A^n+\log\rho_B^n-\log\rho_{A\cup B}^n\right]$$

log 

$\sim \log\left\langle \sigma_n^A\bar{\sigma}_n^A\right\rangle_{2\epsilon}$

log 

$\sim \log\left\langle \sigma_n^B\bar{\sigma}_n^B\right\rangle_{2\epsilon}$

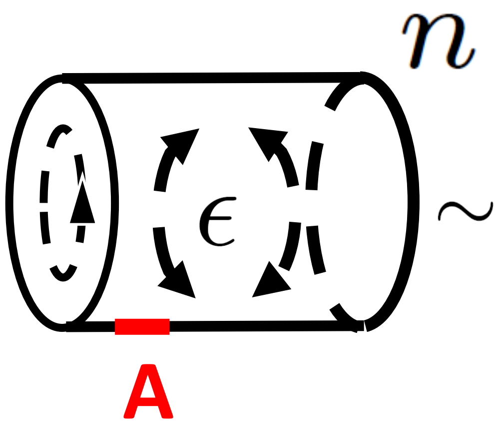
log 

$\sim \log\left\langle \sigma_n^A\bar{\sigma}_n^A\sigma_n^B\bar{\sigma}_n^B\right\rangle_{2\epsilon}$

σ_n^A : Twist operator in **A**

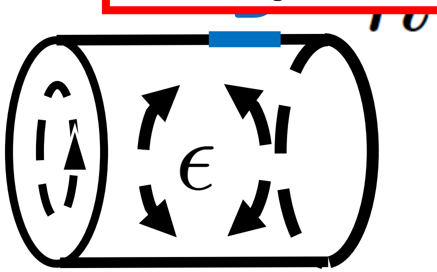
σ_n^B : Twist operator in ***B***

$$I(A,B)=\lim_{n\to1}\frac{1}{1-n}\left[\log\rho_A^n+\log\rho_B^n-\log\rho_{A\cup B}^n\right]$$

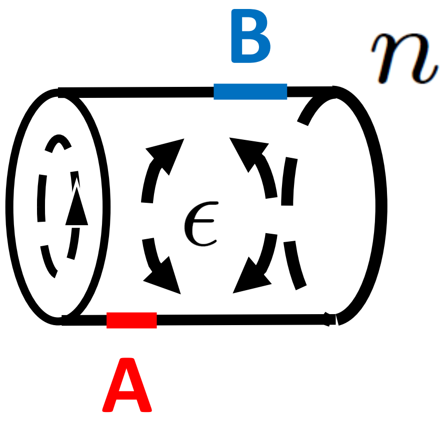
log 

$\sim \log\langle\sigma_n^A\bar{\sigma}_n^A\rangle_{2\epsilon}$

Independent of channels

log 

$\sim \log\langle\sigma_n^B\bar{\sigma}_n^B\rangle_{2\epsilon}$

log 

$\sim \log\langle\sigma_n^A\bar{\sigma}_n^A\sigma_n^B\bar{\sigma}_n^B\rangle_{2\epsilon}$

σ_n^A : Twist operator in **A**

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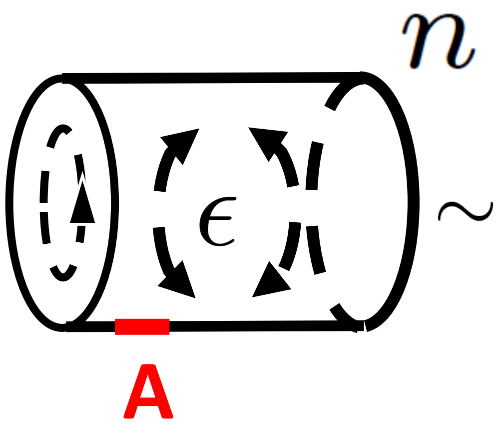
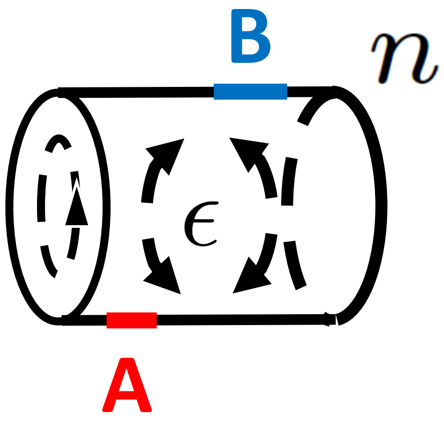
$$I(A,B)=\lim_{n\to 1}\frac{1}{1-n}\left[\log\rho_A^n+\log\rho_B^n-\log\rho_{A\cup B}^n\right]$$

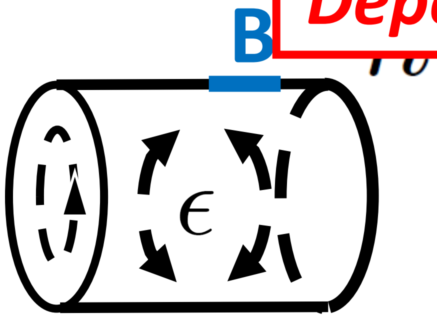
$\log$ 

$\sim \log\left\langle \sigma_n^A\bar{\sigma}_n^A\right\rangle_{2\epsilon}$

$\log$ 

$\sim \log\left\langle \sigma_n^A\bar{\sigma}_n^A\sigma_n^B\bar{\sigma}_n^B\right\rangle_{2\epsilon}$

Depend on channels!!

$\log$ 

$\sim \log\left\langle \sigma_n^B\bar{\sigma}_n^B\right\rangle_{2\epsilon}$

σ_n^A : Twist operator in **A**

σ_n^B : Twist operator in ***B***

Bipartite operator mutual information in the replica trick

$$I(A,B) = \lim_{n\to 1} \frac{1}{1-n} \left[\log tr_A \left(\rho_A\right)^n + \log tr_B \left(\rho_B\right)^n - \log tr_{A\cup B} \left(\rho_{A\cup B}\right)^n\right]$$

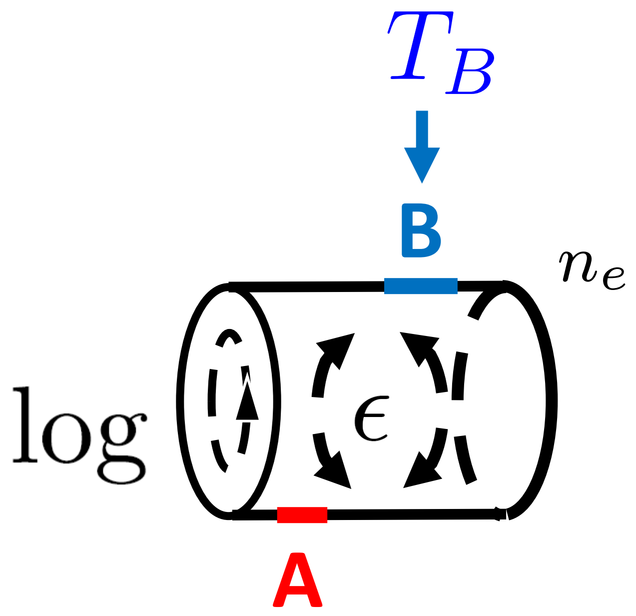
$$\sim \lim_{n\to 1} \frac{1}{1-n} \log\left[\frac{\left\langle \sigma_n^A \bar{\sigma}_n^A \right\rangle_{2\epsilon} \left\langle \sigma_n^B \bar{\sigma}_n^B \right\rangle_{2\epsilon}}{\left\langle \sigma_n^A \bar{\sigma}_n^A \sigma_n^B \bar{\sigma}_n^B \right\rangle_{2\epsilon}}\right]$$

Bipartite operator mutual information in the replica trick

$$\mathcal{E}_{A,B} = \lim_{n_e \to 1} \log \left[\mathrm{tr}_{A\cup B} \left(\rho_{A\cup B}^{T_B} \right)^{n_e} \right]$$

Bipartite operator mutual information in the replica trick

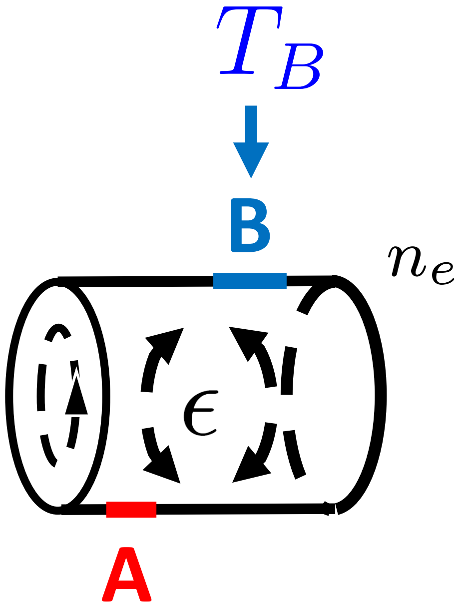
$$\mathcal{E}_{A,B} = \lim_{n_e \to 1} \log \left[\mathrm{tr}_{A\cup B} \left(\rho_{A\cup B}^{T_B} \right)^{n_e} \right]$$



Bipartite operator mutual information in the replica trick

$$\mathcal{E}_{A,B} = \lim_{n_e \to 1} \log \left[\mathrm{tr}_{A\cup B} \left(\rho_{A\cup B}^{T_B} \right)^{n_e} \right]$$

$\log$

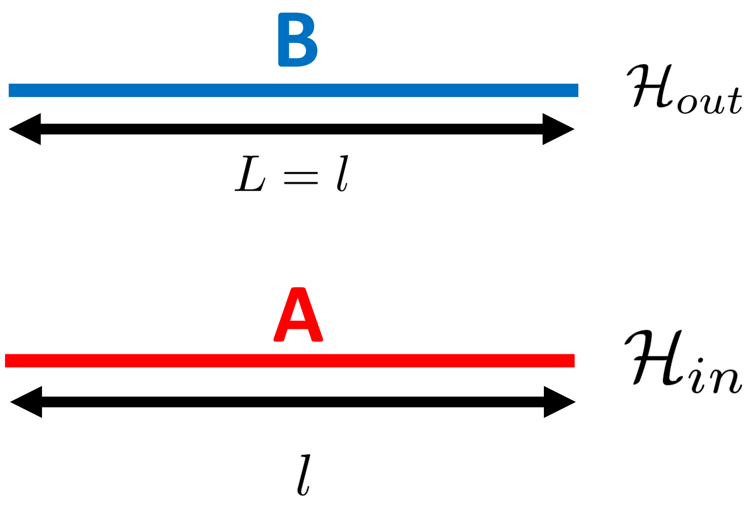


$$\sim \log \left\langle \sigma_{n_e}^{A} \bar{\sigma}_{n_e}^{A} \bar{\sigma}_{n_e}^{B} \sigma_{n_e}^{B} \right\rangle$$

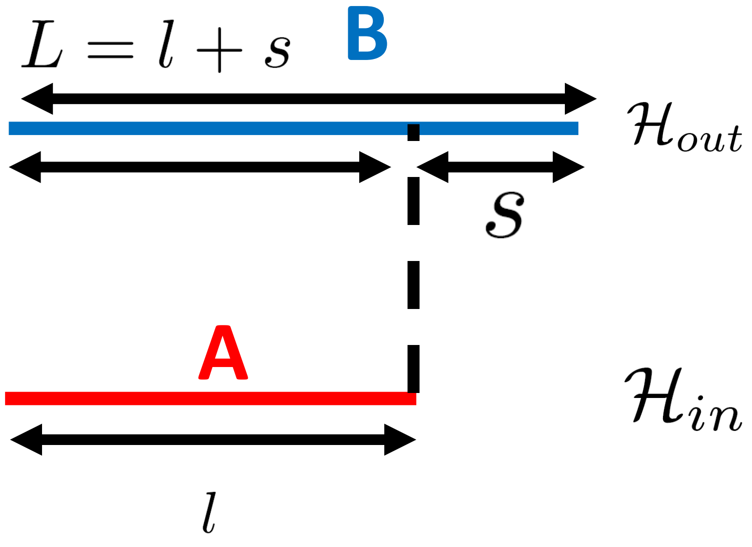
Free fermion channel

We consider the following setups to extract properties of free fermion channel:

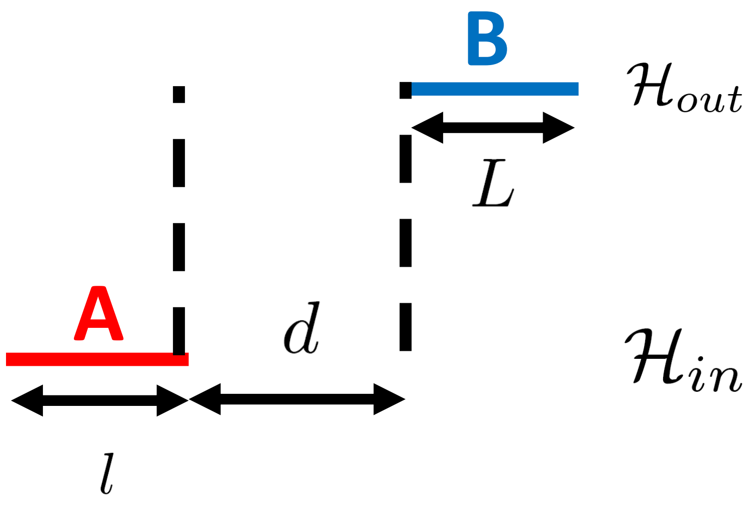
1. Fully overlapping case



2. Partially overlapping case



3. Disjoint case

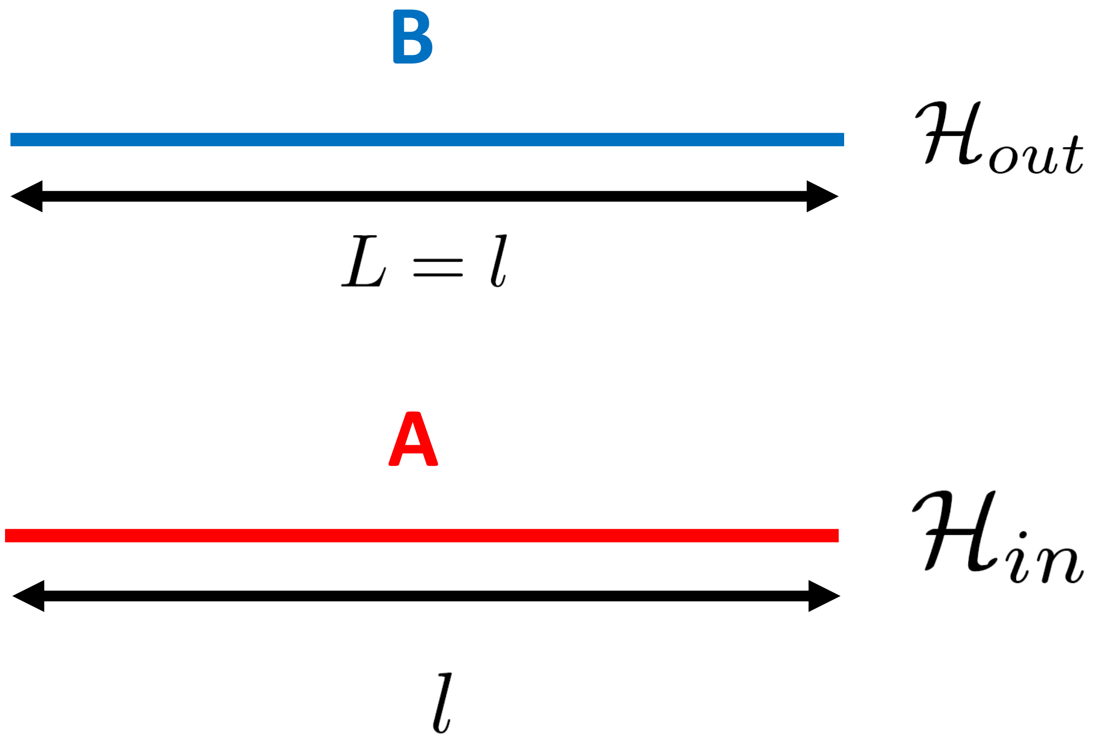


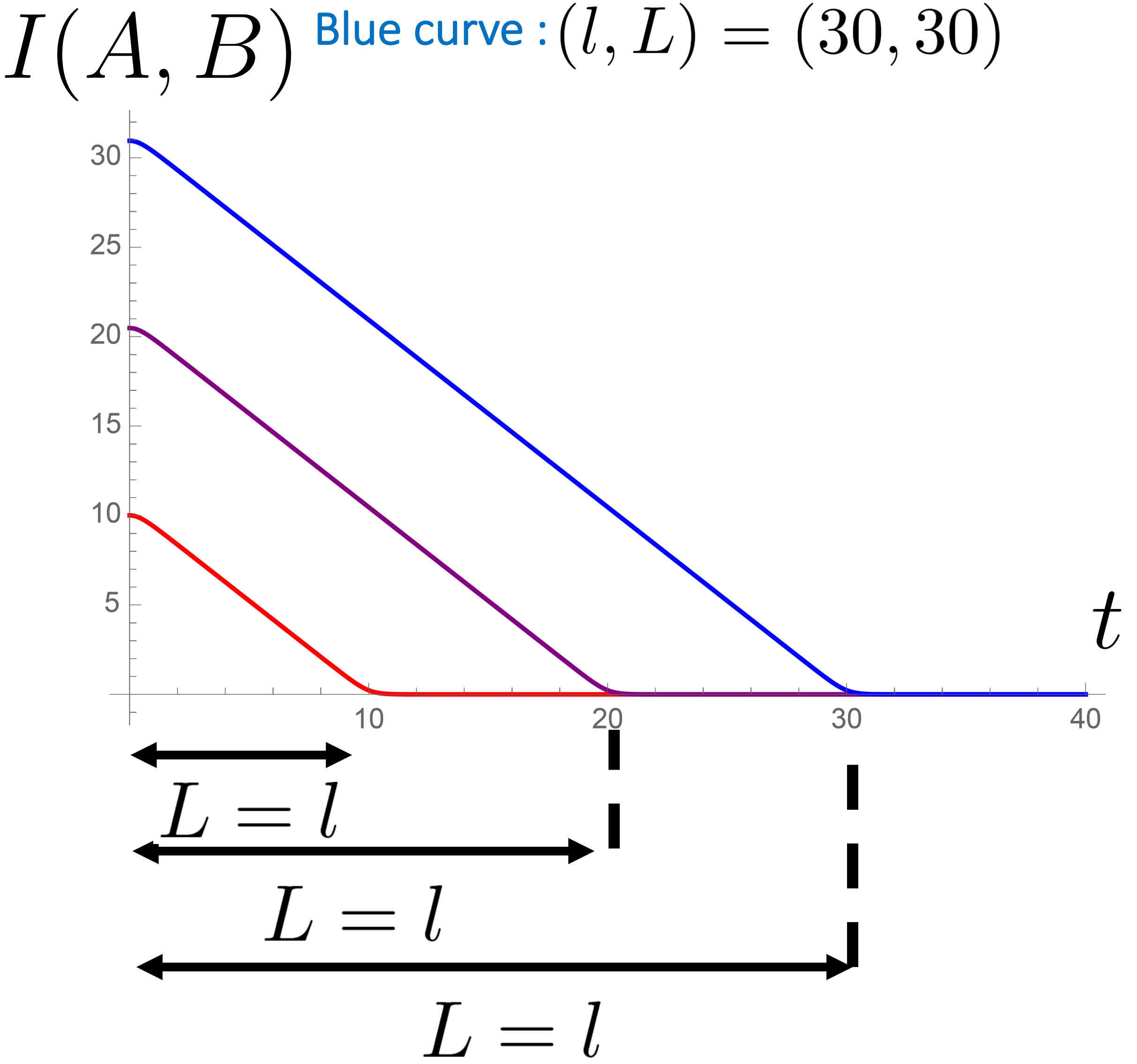
1. Fully overlapping case

Red curve : $(l, L) = (10, 10)$

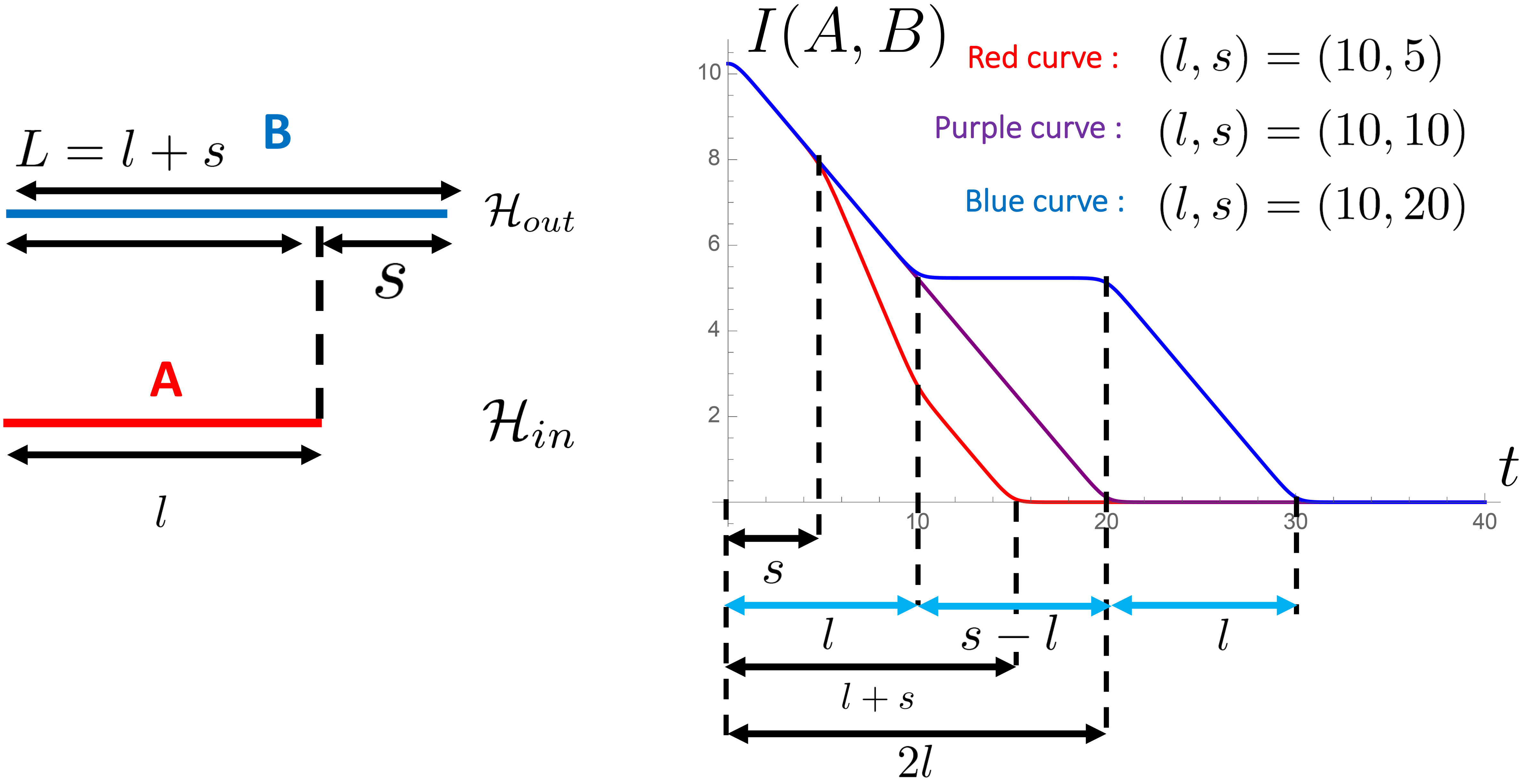
Purple curve : $(l, L) = (20, 20)$

Blue curve : $(l, L) = (30, 30)$

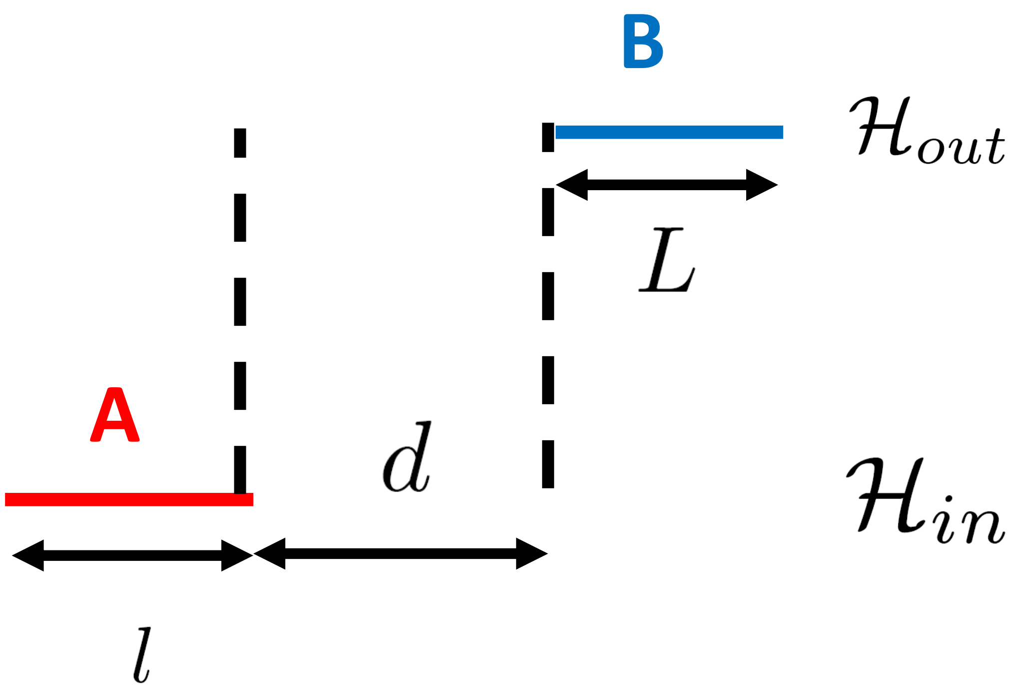




2. Partially overlapping case



3. Disjoint case

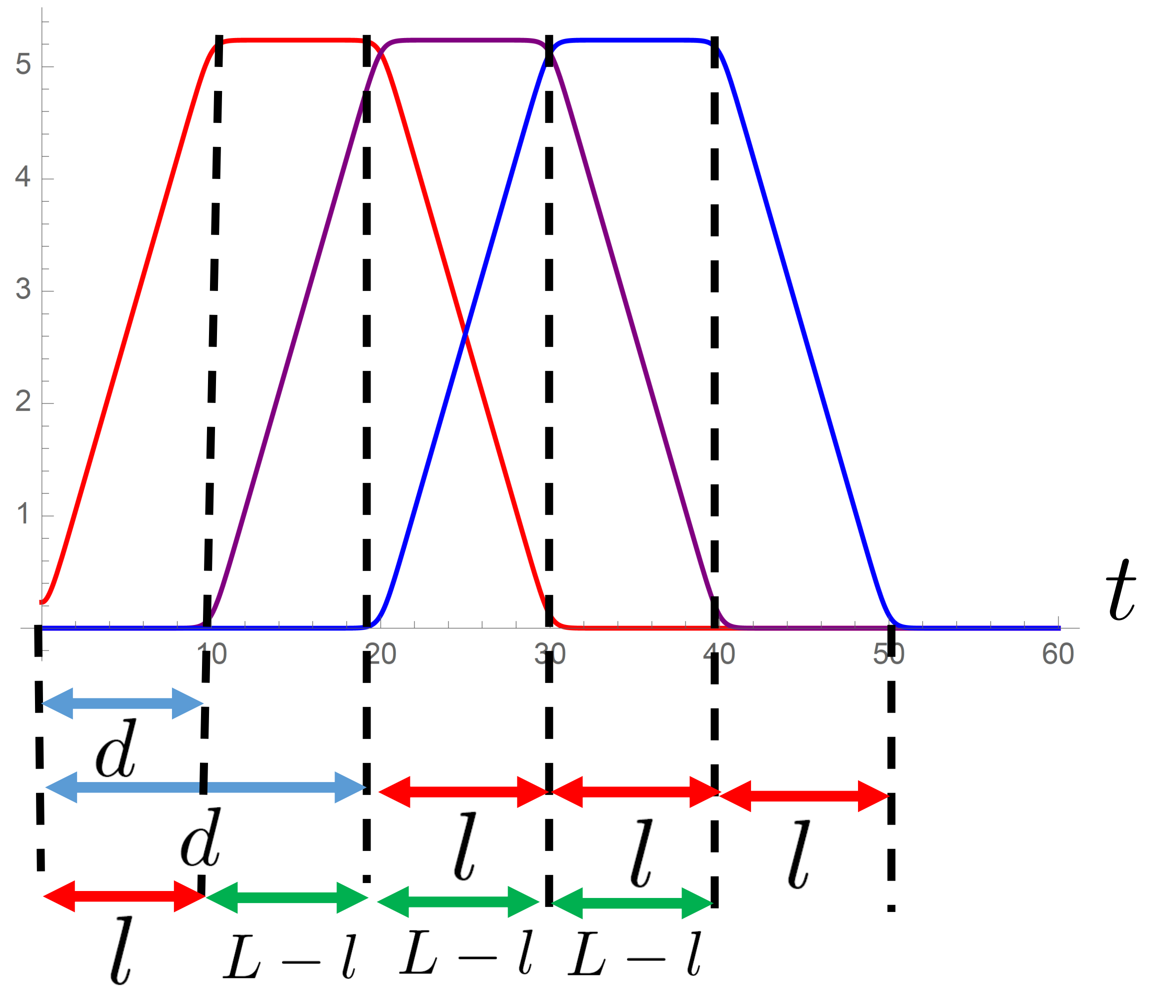


Red curve : $(l, L, d) = (10, 20, 0)$

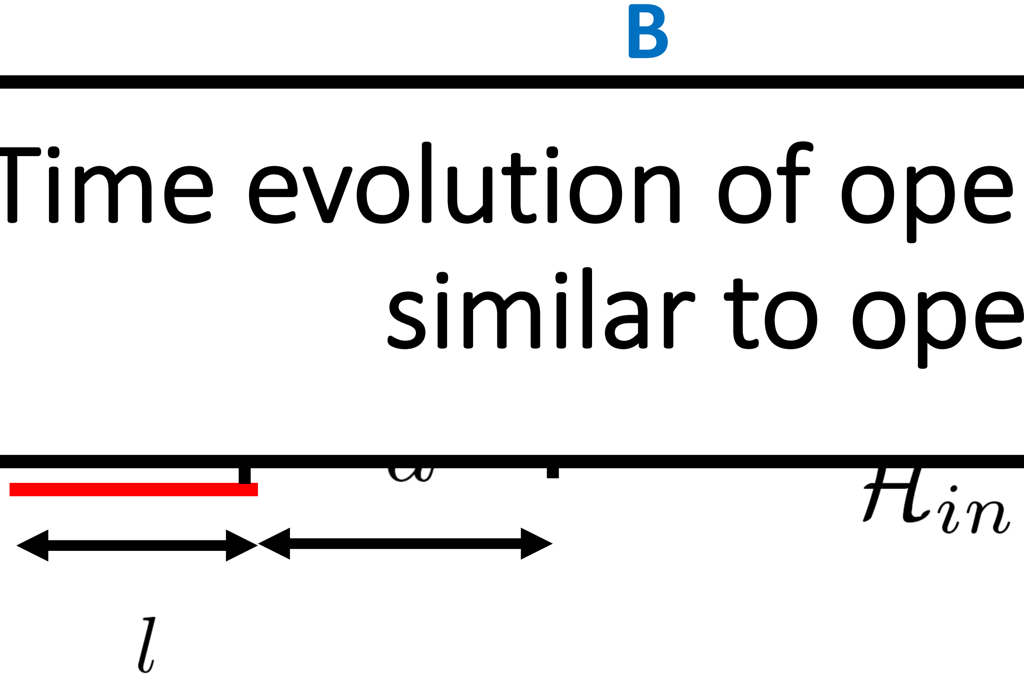
Purple curve : $(l, L, d) = (10, 20, 10)$

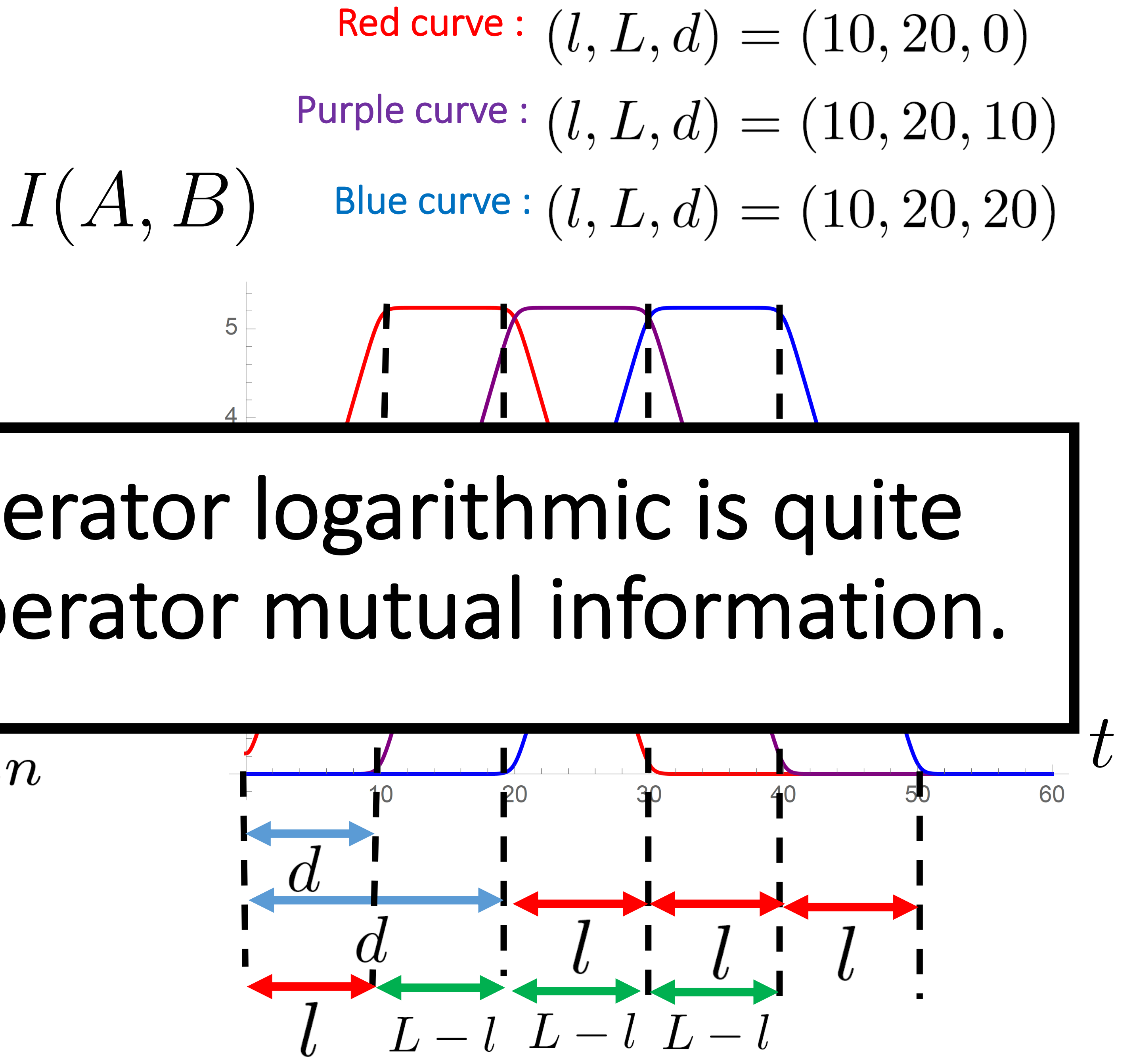
Blue curve : $(l, L, d) = (10, 20, 20)$

$I(A, B)$

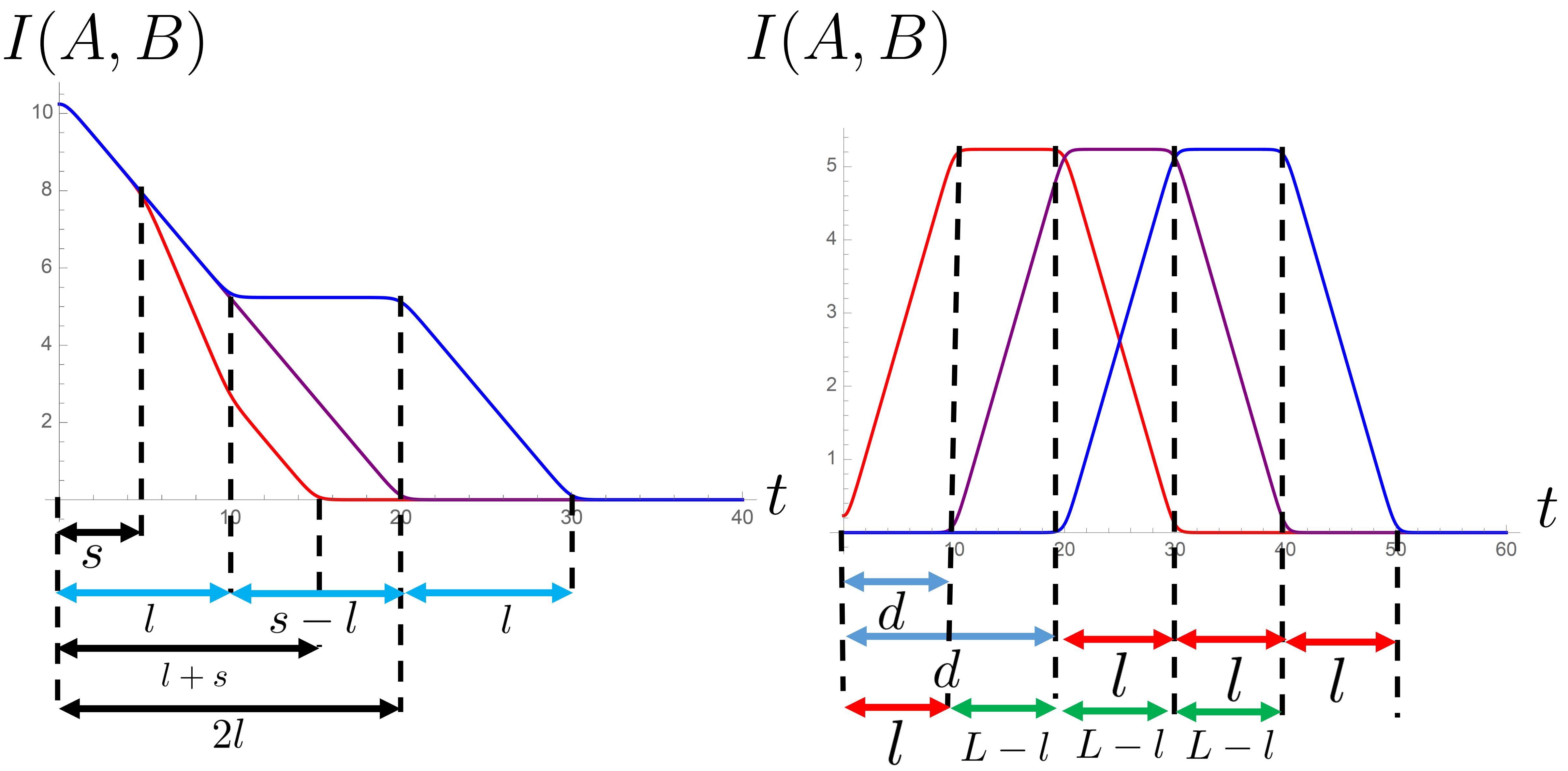


3. Disjoint case



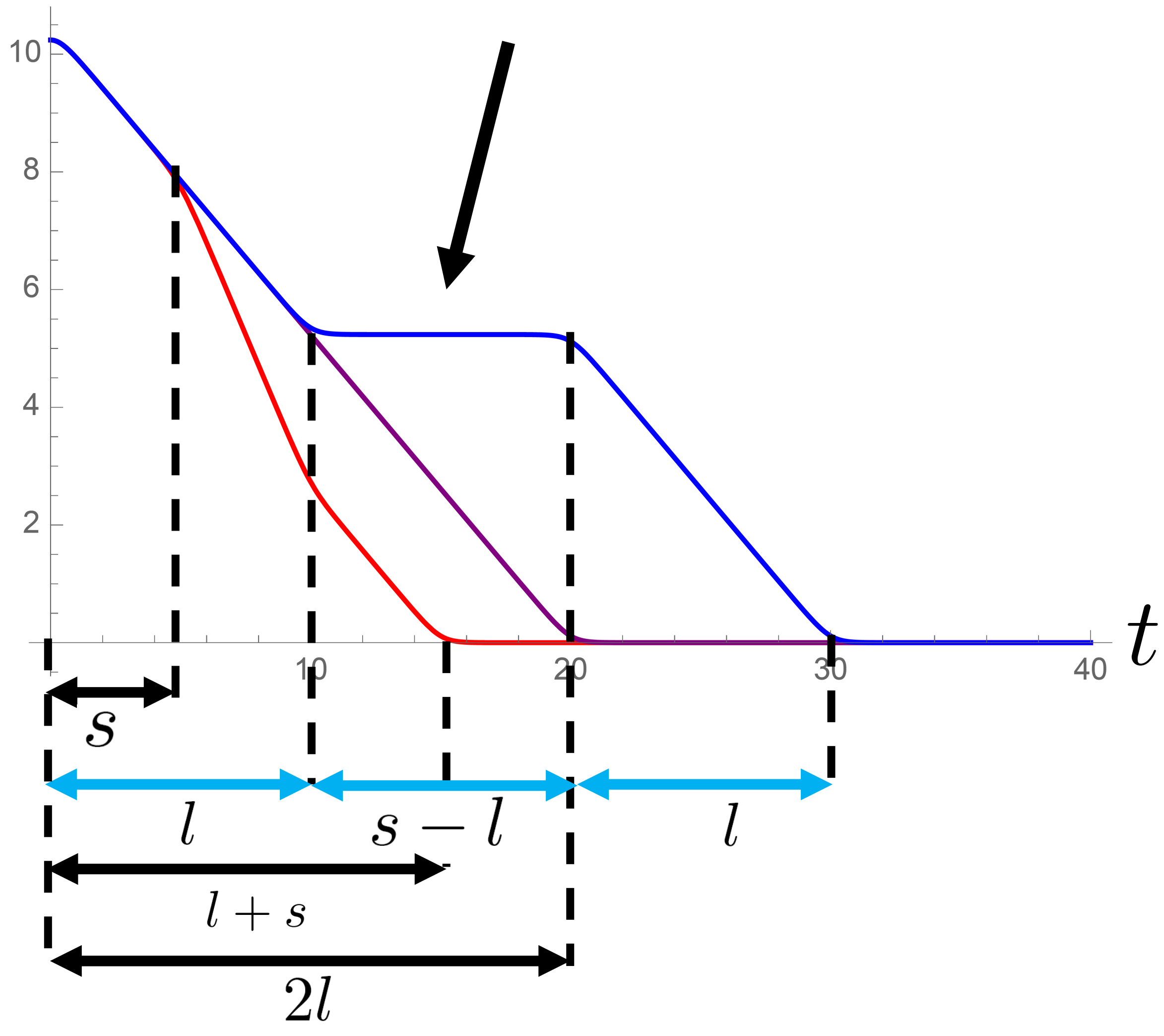


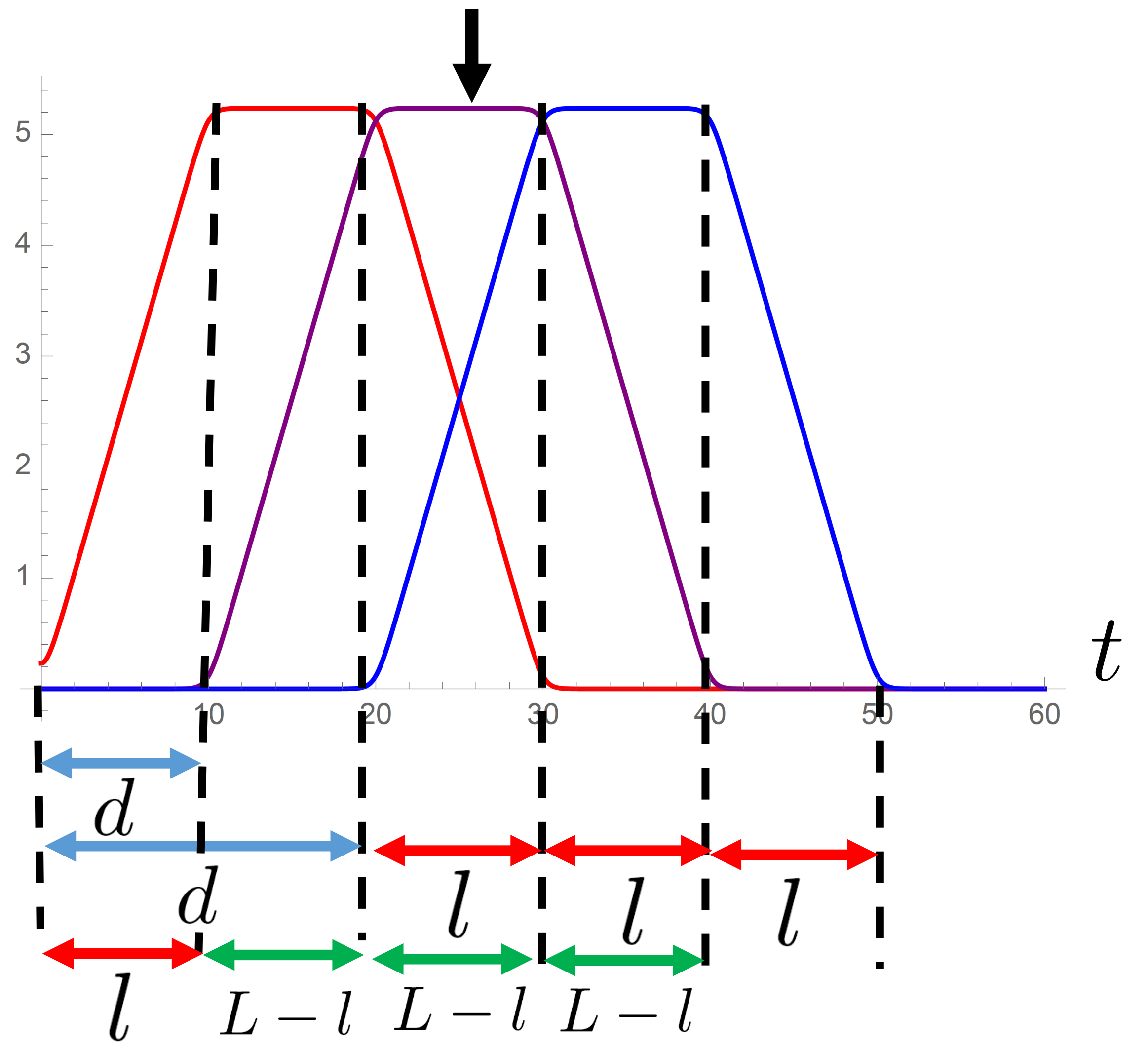
Time evolution of operator logarithmic is quite similar to operator mutual information.



$I(A,B)$
t
s
l
$s-l$
l
$l+s$
$2l$
$I(A,B)$
t
d
d
l
l
l
l
$L-l$
$L-l$
$L-l$

Slopes and bumps shows *properties of free fermion channel are interpreted in terms of the relativistic propagation of quasi-particles*.



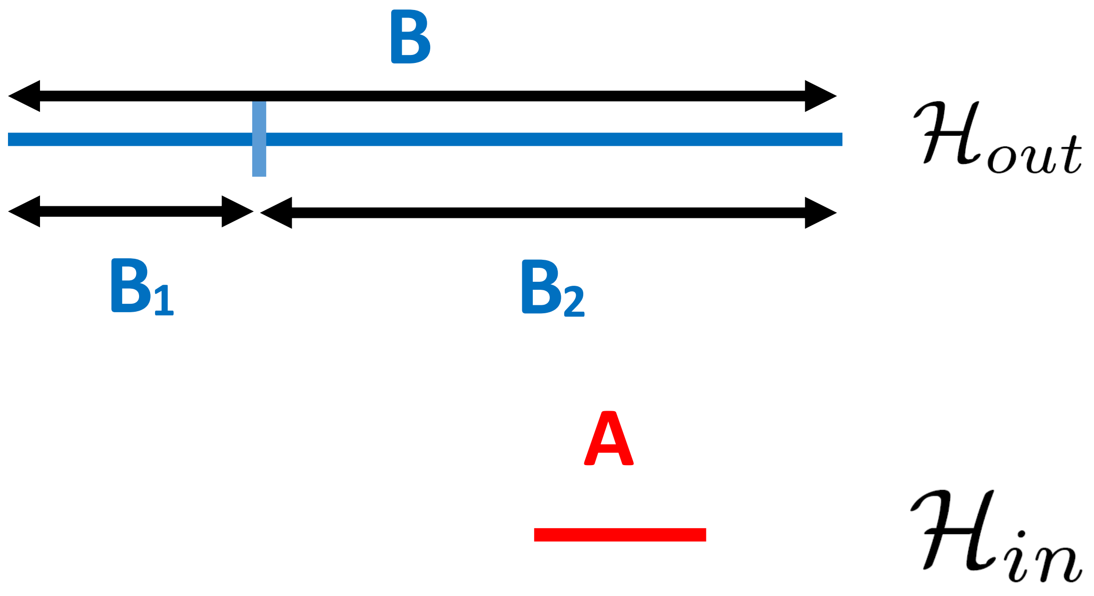


Tripartite operator mutual information

$$I(A, B_1, B_2) =$$

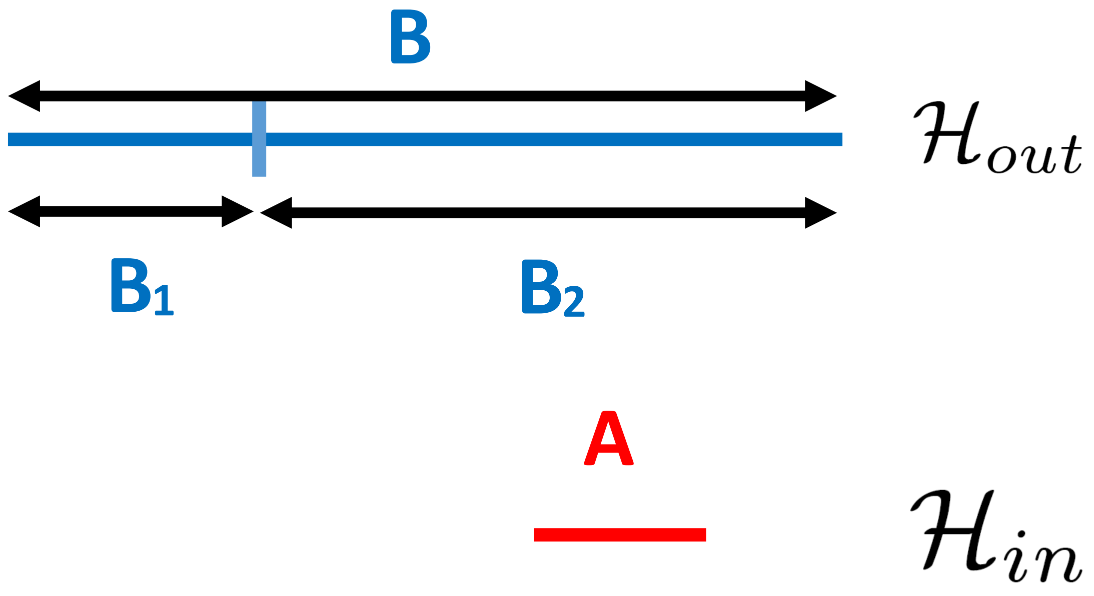
$$I(A, B_1) + I(A, B_2) - I(A, B)$$

$$= 0$$



Tripartite operator mutual information

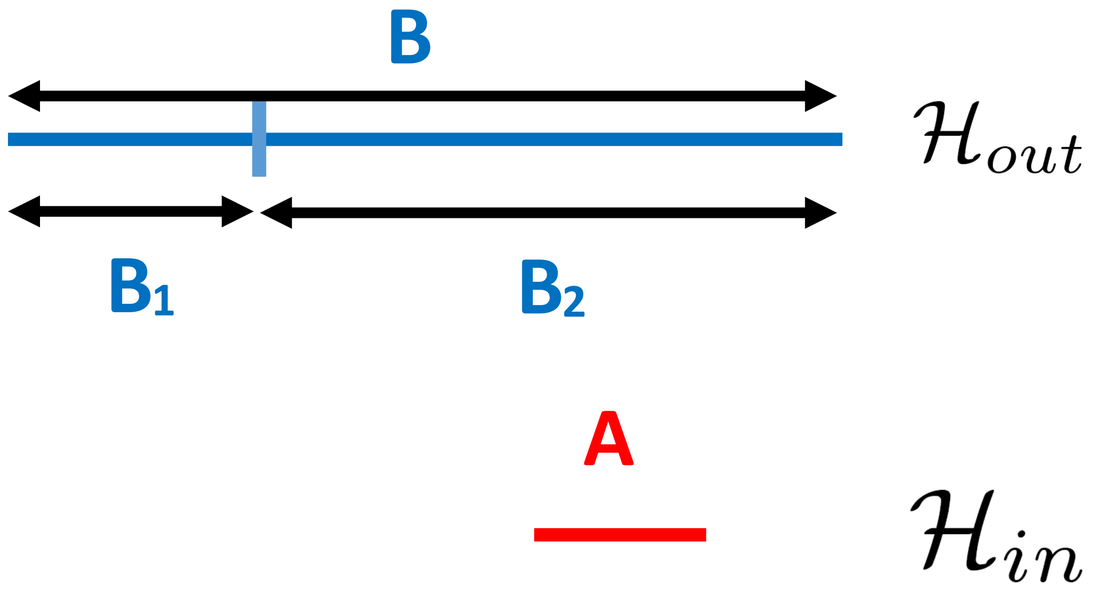
$$\boxed{\begin{aligned} &I(A, B_1, B_2) = \\ &I(A, B_1) + I(A, B_2) - I(A, B) \\ &\qquad = 0 \end{aligned}}$$



doesn't depend on the time and the choice for subsystems.

Tripartite operator logarithmic negativity

$$\boxed{\begin{aligned}&\mathcal{E}_3(A, B_1, B_2) = \\ &\quad \mathcal{E}(A, B_1) + \mathcal{E}(A, B_2) \\ &\qquad\qquad - \mathcal{E}(A, B_1 \cup B_2) = 0\end{aligned}}$$



$$\mathcal{E}_3(A, B_1, B_2) = 0$$
$$I(A, B_1, B_2) = 0$$

$\Rightarrow$ *Relativistic propagation of quasi-particle.*

Toy model

The time evolution of operator mutual information (logarithmic negativity) and tripartite operator mutual information (logarithmic negativity) for free fermion channel can be interpreted ***in terms of the relativistic propagation of local objects*** as follows:

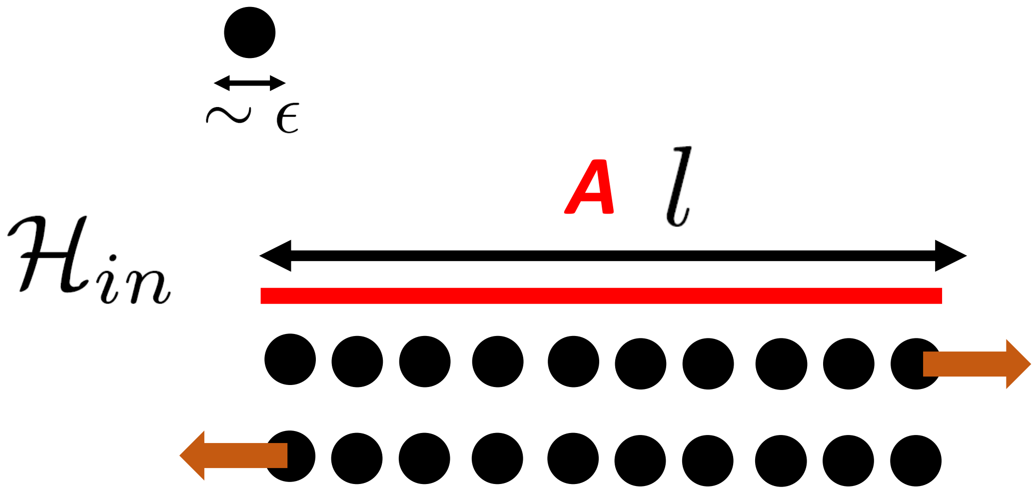
1. Each point in the input subsystem ***A*** has two particles.

   - One of them propagates in the right direction ( ) at speed of light.

   - The other ( ).

   - particle size $\sim \epsilon$

   -# of particles in ***A*** is proportional to ***the input subsystem size*** l

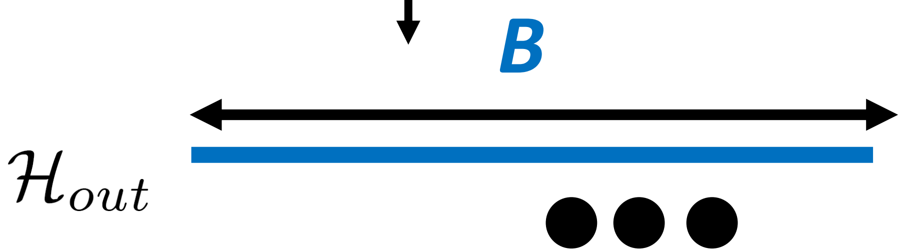


Toy model

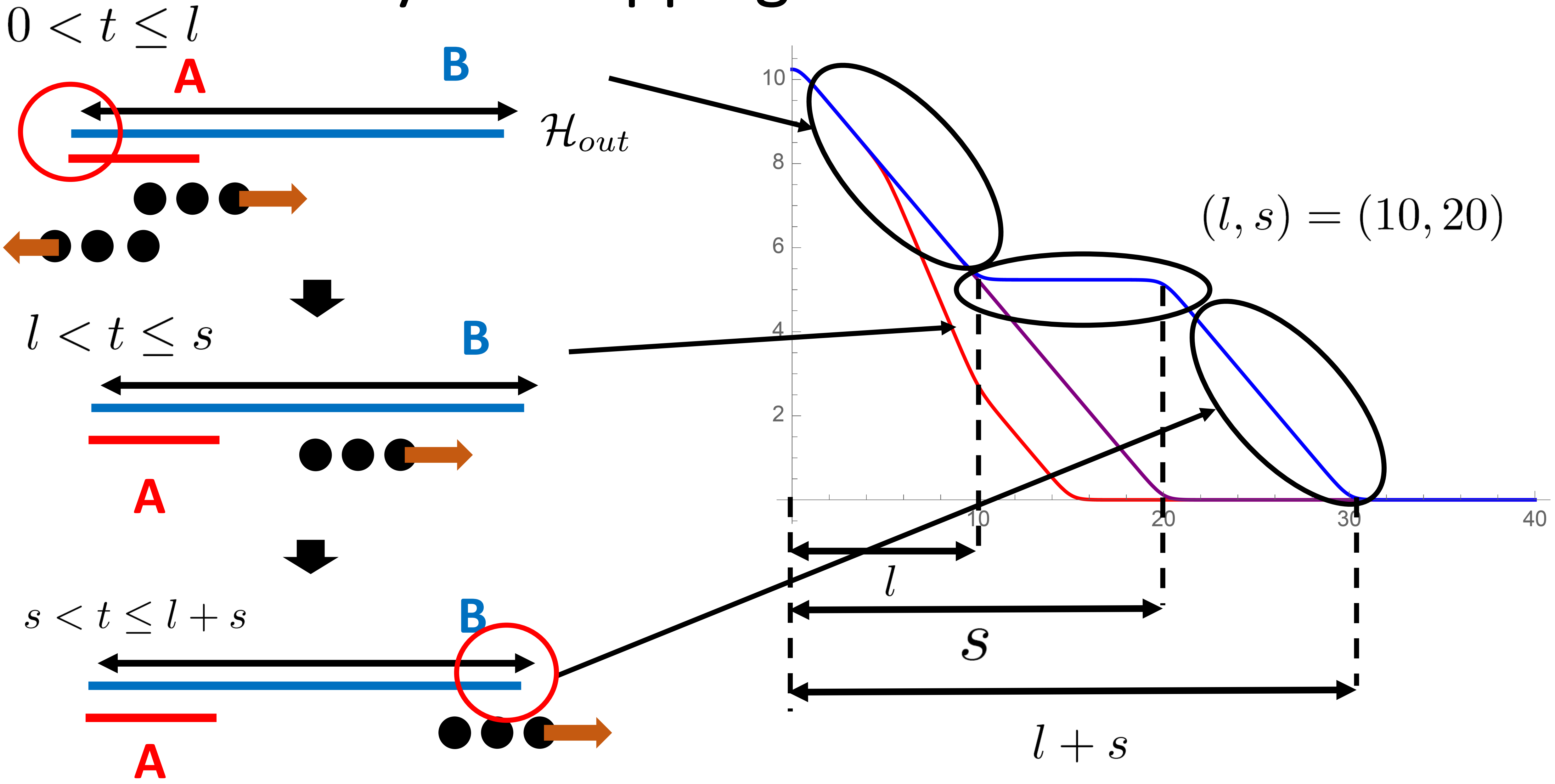
2. The particles in the output subsystem ***B*** contribute to $I(A,B)$.

- $I(A,B) \propto$ # of particles in ***B***.

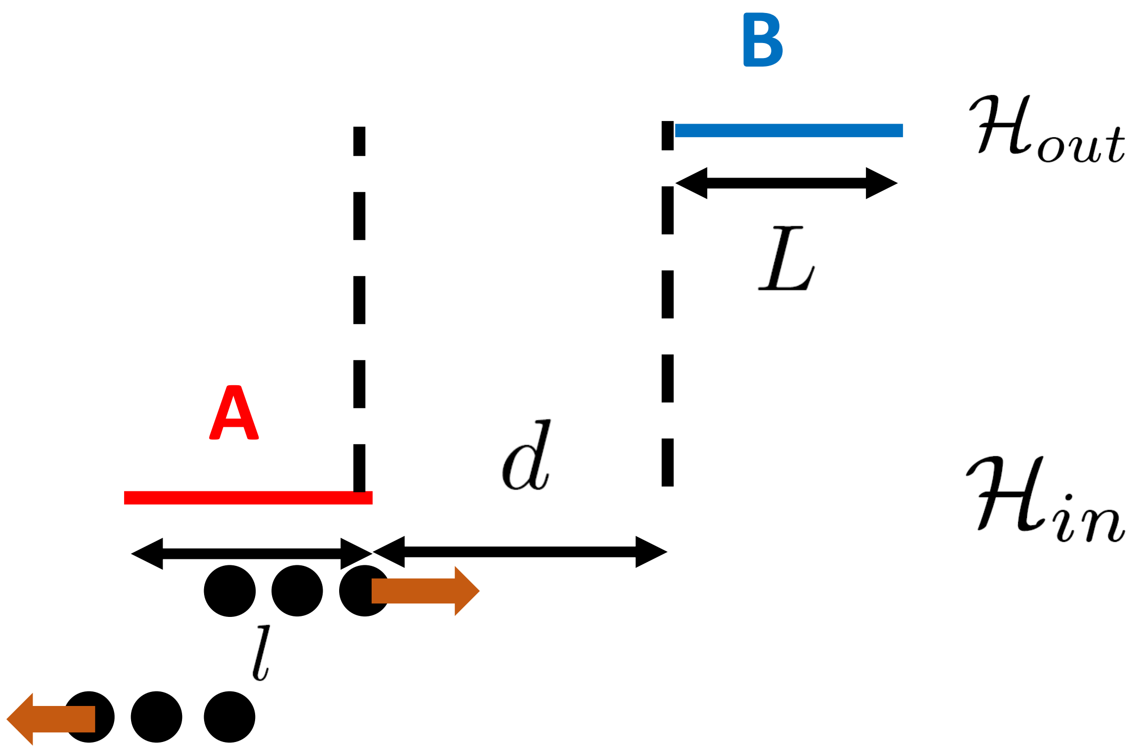
$$I(A,B) \propto 3$$



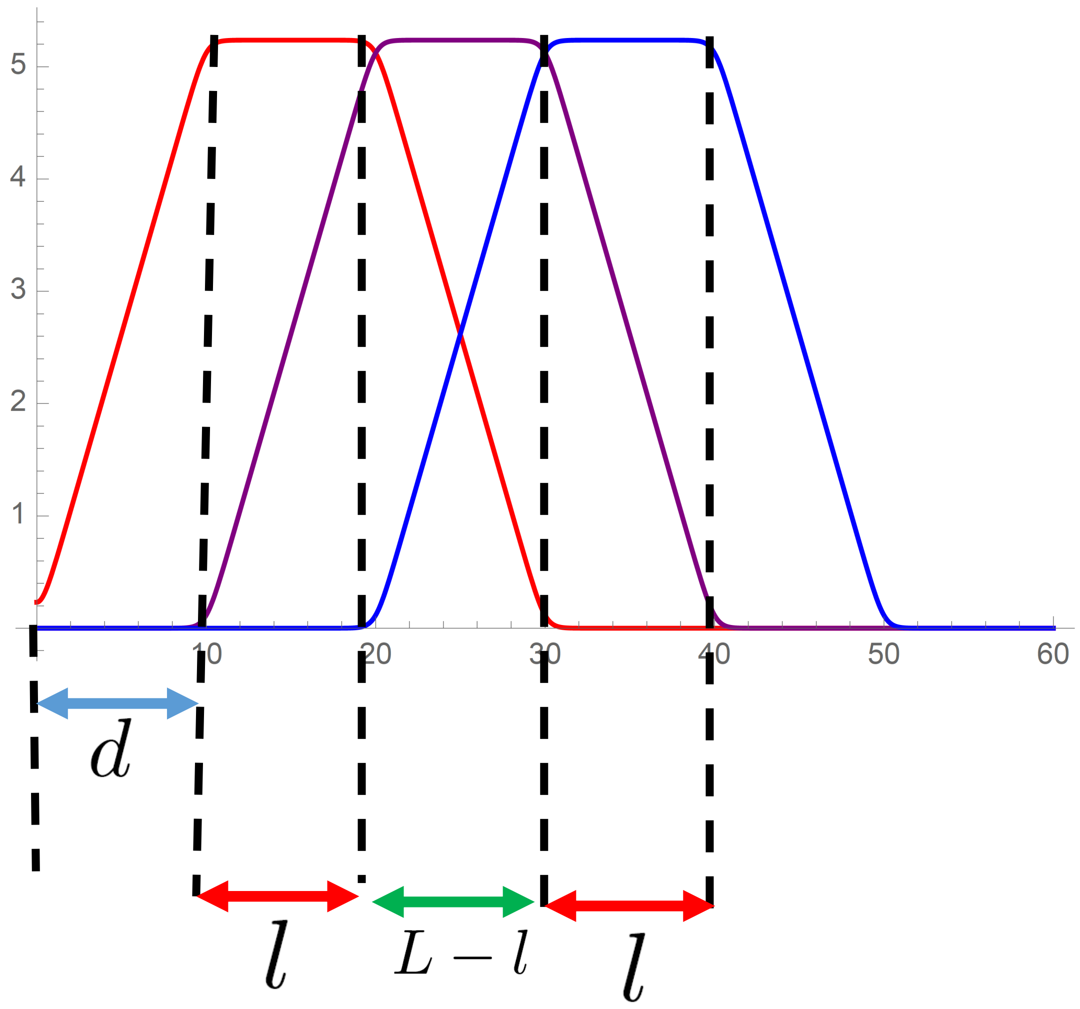
2. Partially overlapping case



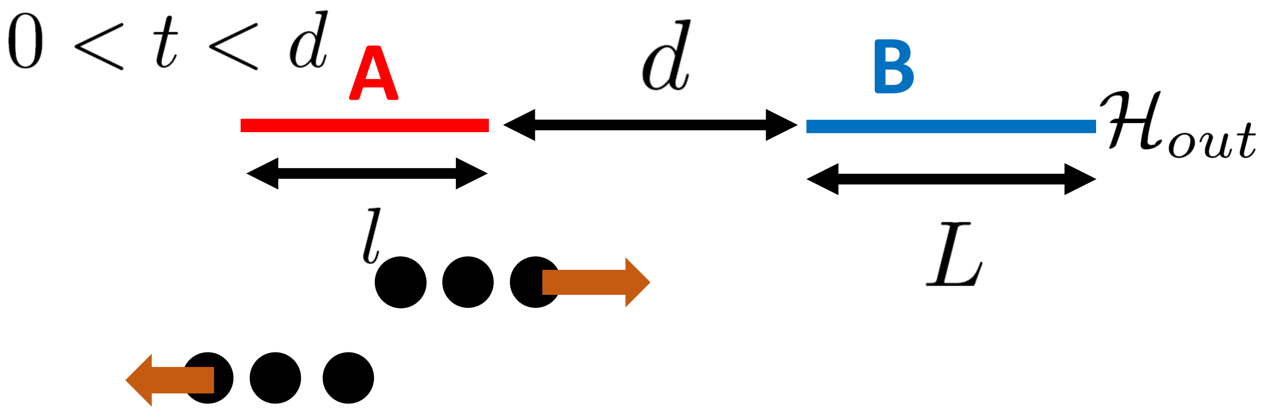
3. Disjoint case



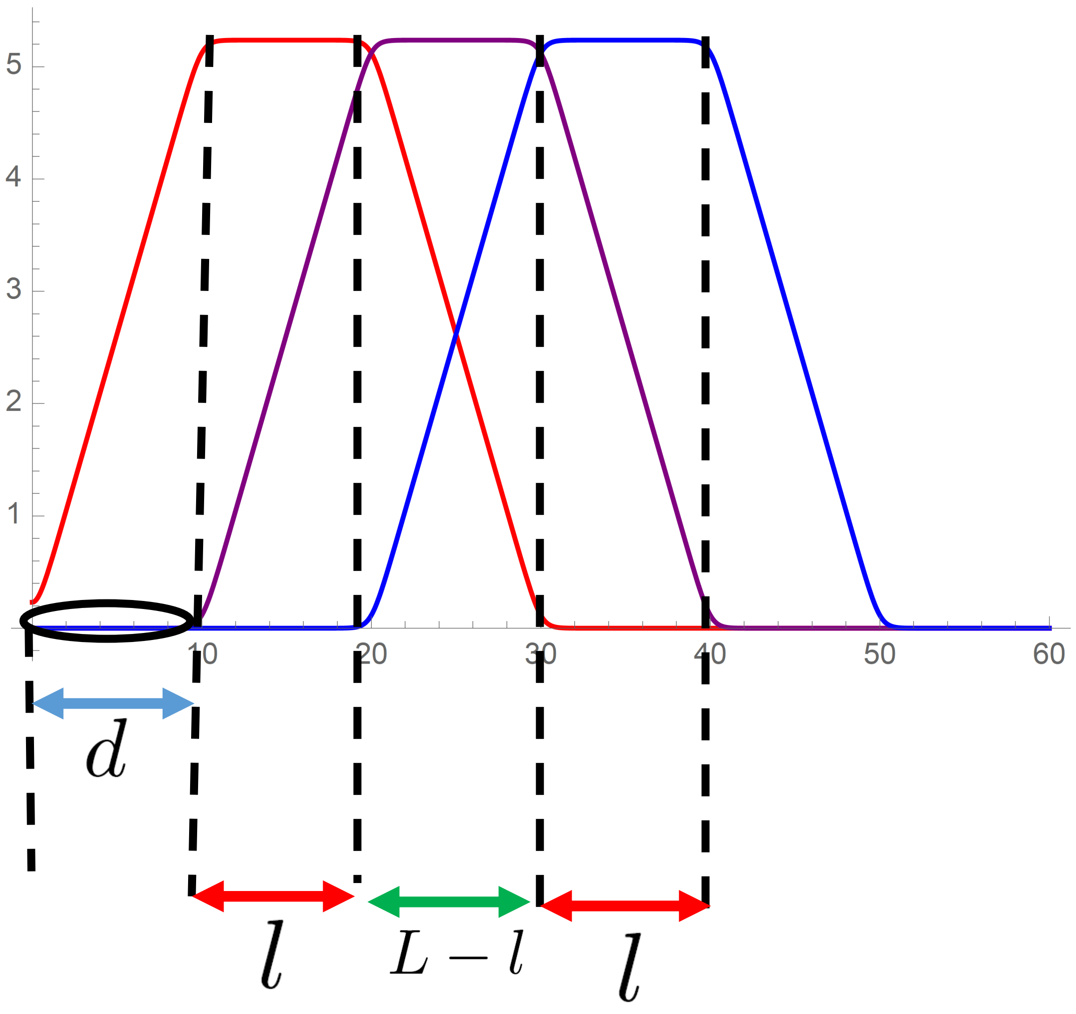
Purple curve : $(l, L, d) = (10, 20, 10)$



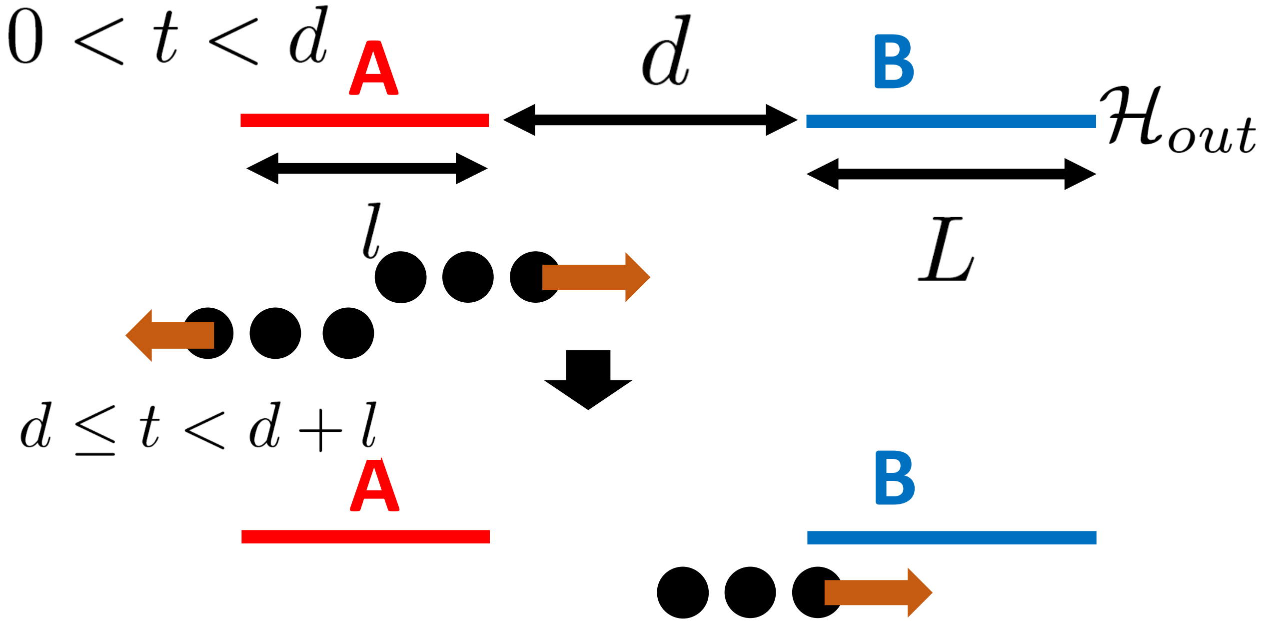
3. Disjoint case



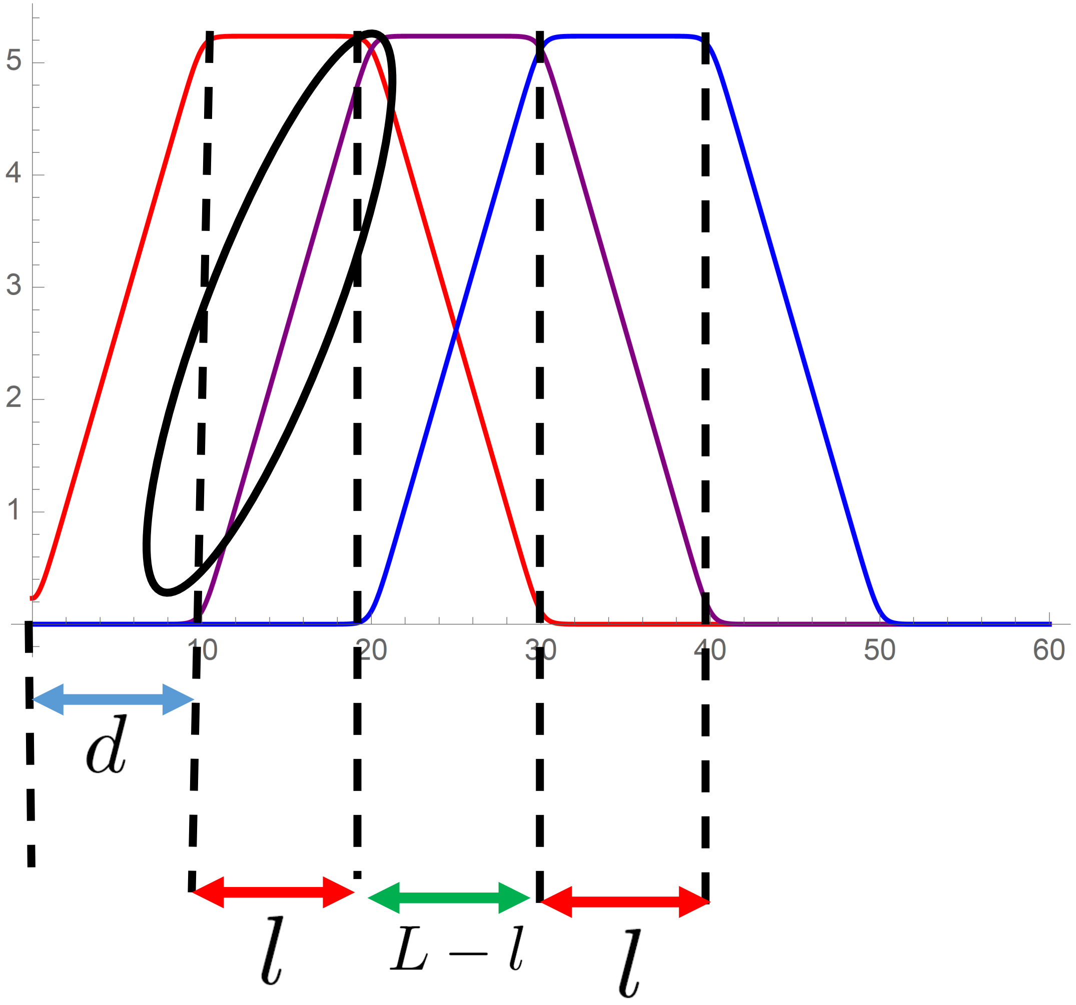
Purple curve : $(l, L, d) = (10, 20, 10)$



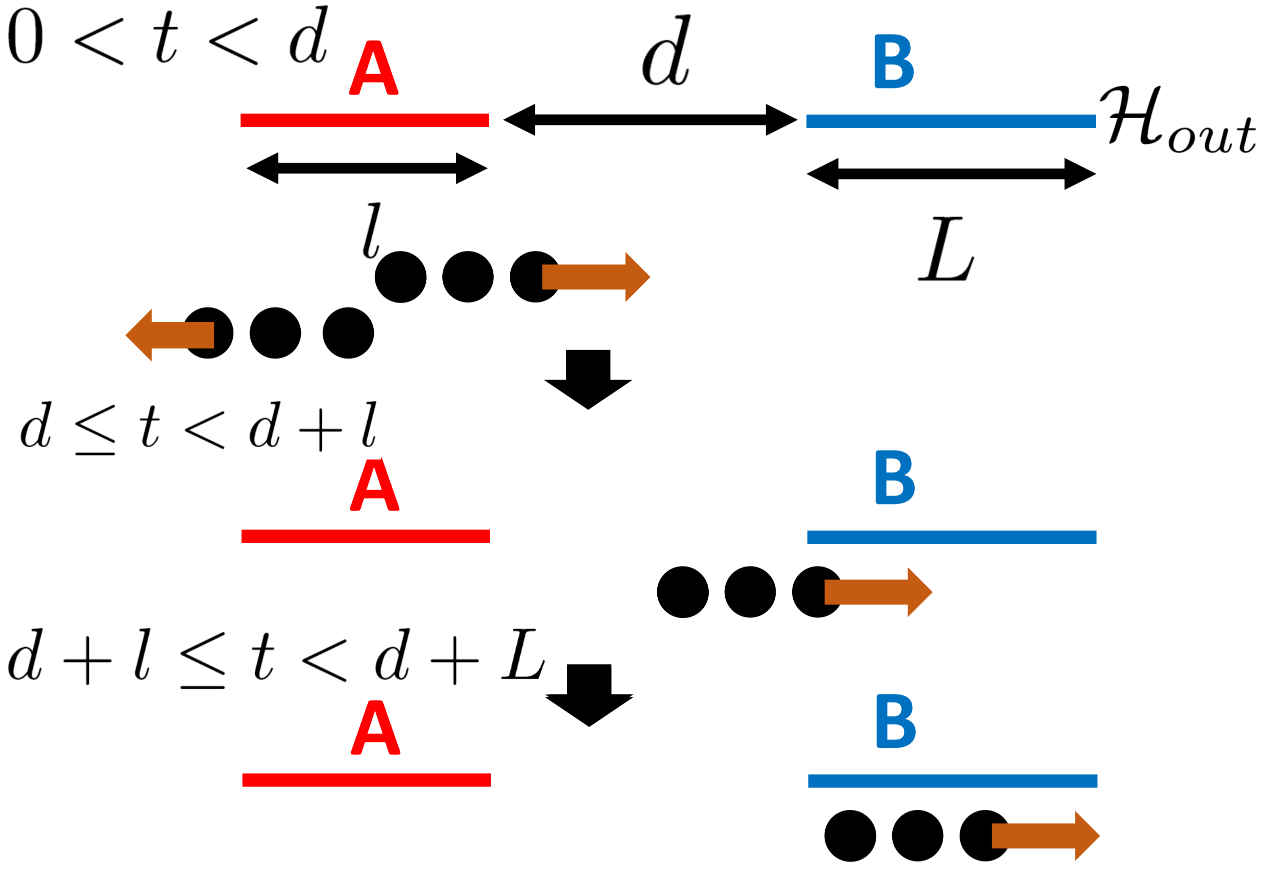
3. Disjoint case



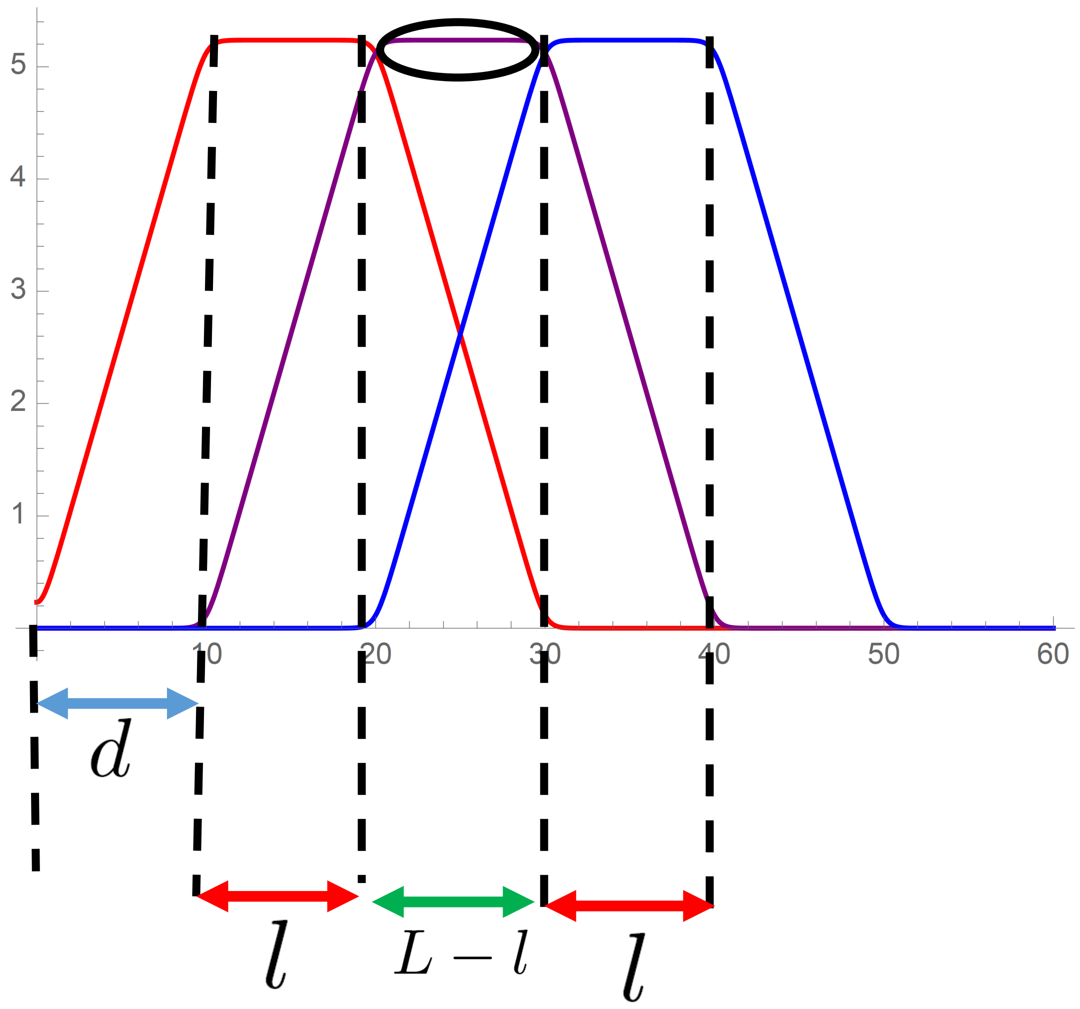
Purple curve : $(l, L, d) = (10, 20, 10)$



3. Disjoint case

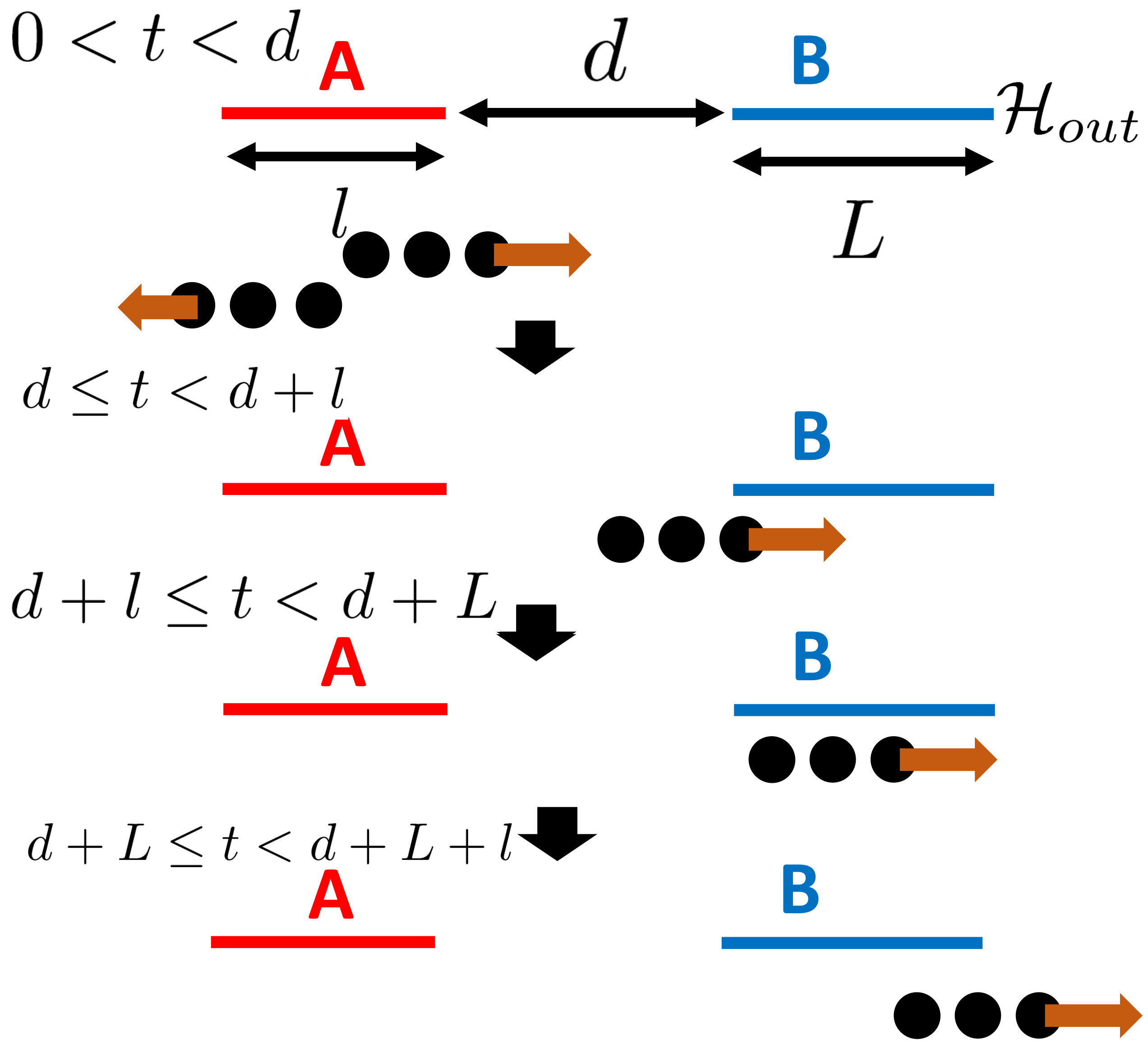


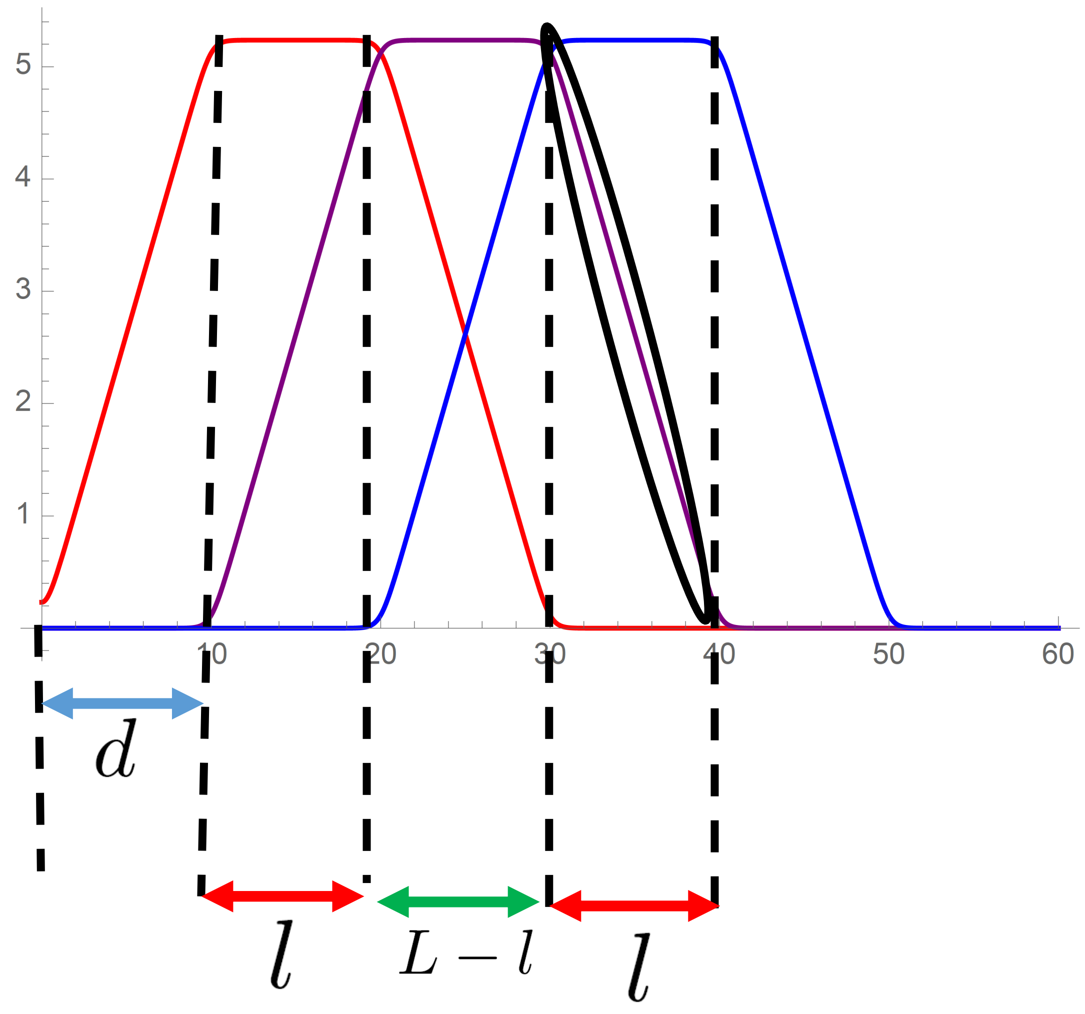
Purple curve : $(l, L, d) = (10, 20, 10)$



3. Disjoint case

Purple curve : $(l, L, d) = (10, 20, 10)$

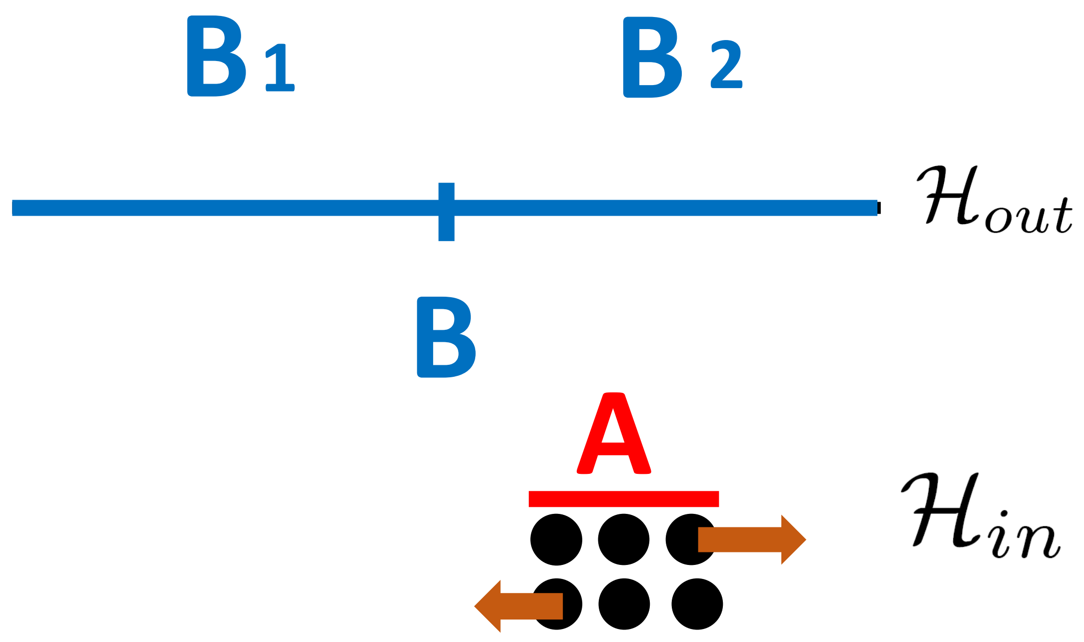




Tri-partite information

- Tri-partite information can be interpreted in terms of the relativistic propagation of local objects, too.

$$\boxed{\begin{aligned} I(A,B_1,B_2) = \\ I(A,B_1)+I(A,B_2)-I(A,B) \\ =0 \end{aligned}}$$

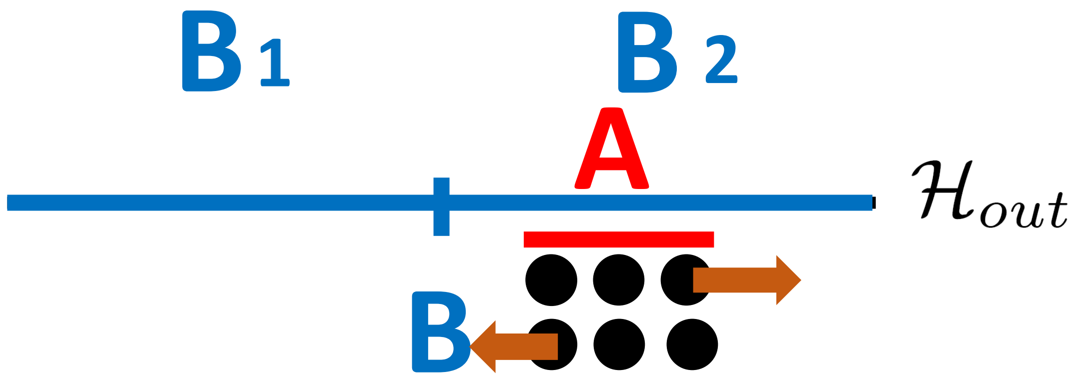


Tri-partite information

- Tri-partite information can be interpreted in terms of the relativistic propagation of local objects, too.

$$I(A, B_1, B_2) =$$
$$I(A, B_1) + I(A, B_2) - I(A, B)$$
$$= 0$$

@time =0



Tri-partite information

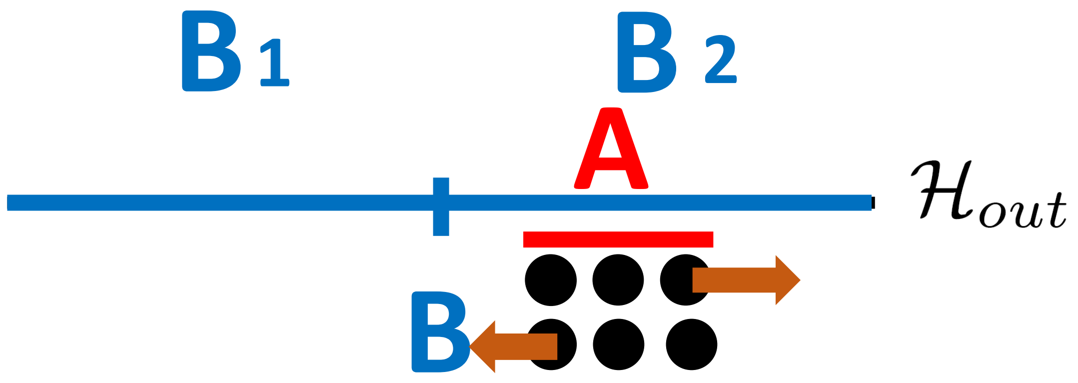
- Tri-partite information can be interpreted in terms of the relativistic propagation of local objects, too.

$$I(A, B_1, B_2) =$$

$$I(A, B_1) + \underline{I(A, B_2)} - \underline{I(A, B)}$$

$$= 0$$

@time =0



Tri-partite information

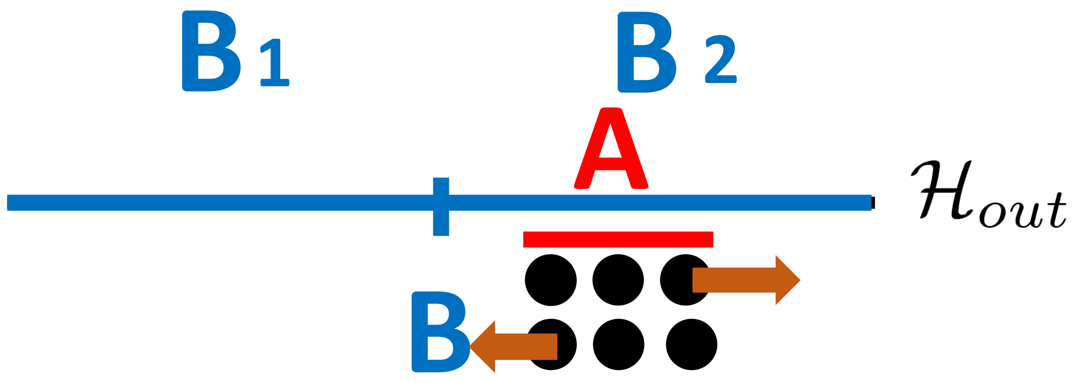
- Tri-partite information can be interpreted in terms of the relativistic propagation of local objects, too.

$$I(A, B_1, B_2) =$$

$$\underbrace{I(A, B_1)}_{=0} + \underbrace{I(A, B_2)}_{\propto 6} - \underbrace{I(A, B)}_{\propto 6}$$

$$= 0$$

@time =0



Tri-partite information

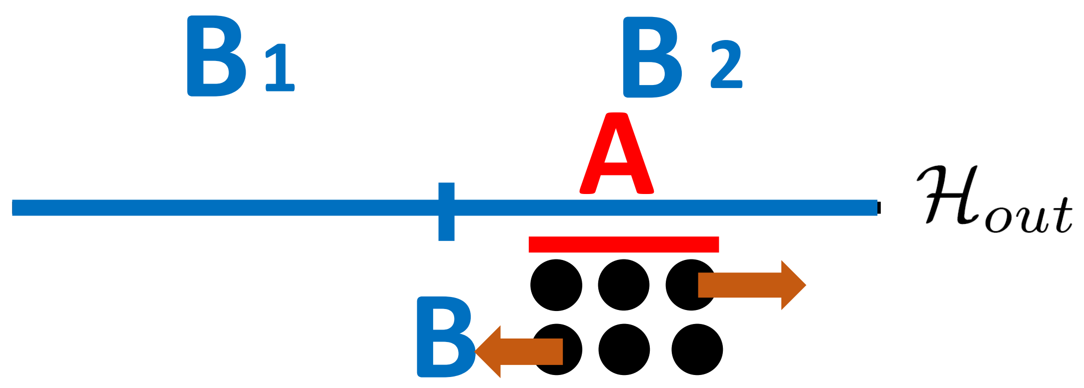
- Tri-partite information can be interpreted in terms of the relativistic propagation of local objects, too.

$$I(A, B_1, B_2) =$$

$$\underbrace{I(A, B_1)}_{=0} + \cancel{\underbrace{I(A, B_2)}_{\propto 6}} - \cancel{\underbrace{I(A, B)}_{\propto 6}}$$

$$= 0$$

@time =0



Tri-partite information

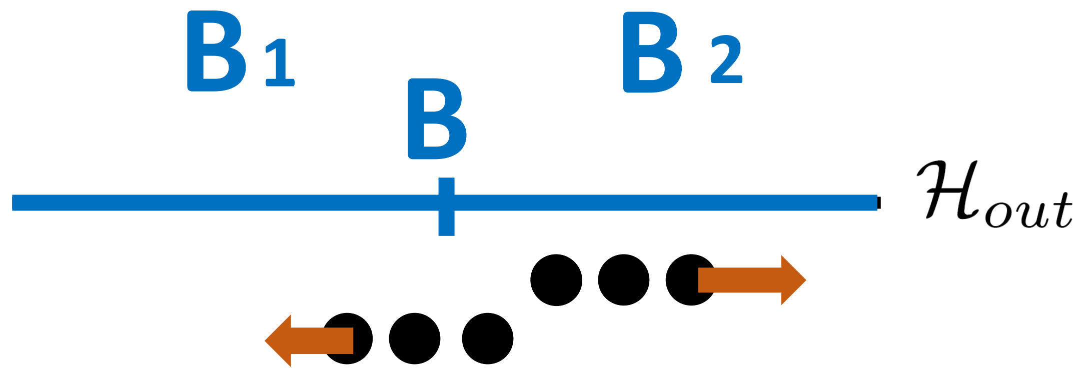
- Tri-partite information can be interpreted in terms of the relativistic propagation of local objects, too.

$$I(A, B_1, B_2) =$$

$$\underbrace{\underline{I(A, B_1)}}_{\propto 2} + \underbrace{\underline{I(A, B_2)}}_{\propto 4} - \underbrace{\underline{I(A, B)}}_{\propto 6}$$

$$= 0$$

@time = t



Tri-partite information

- Tri-partite information can be interpreted in terms of the relativistic propagation of local objects, too.

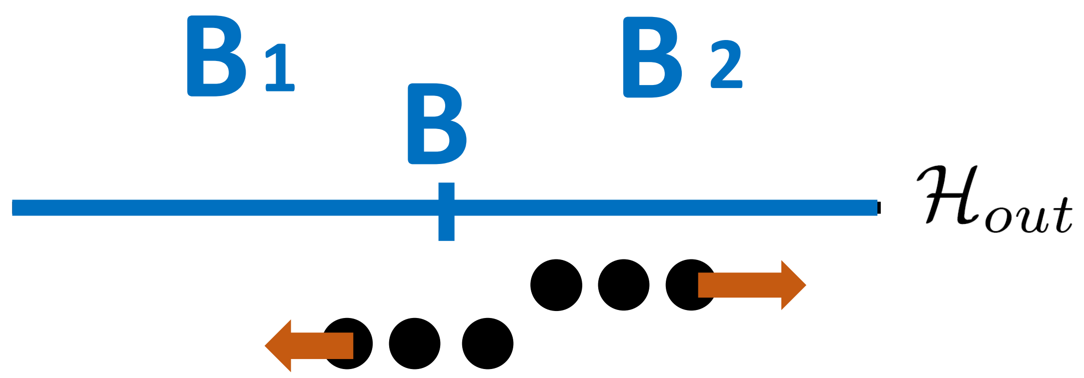
$$I(A,B_1,B_2) =$$

$$\cancel{I(A,B_1)} + \cancel{I(A,B_2)} - \cancel{I(A,B)}$$

$$\propto 2 \qquad \propto 4 \qquad \propto 6$$

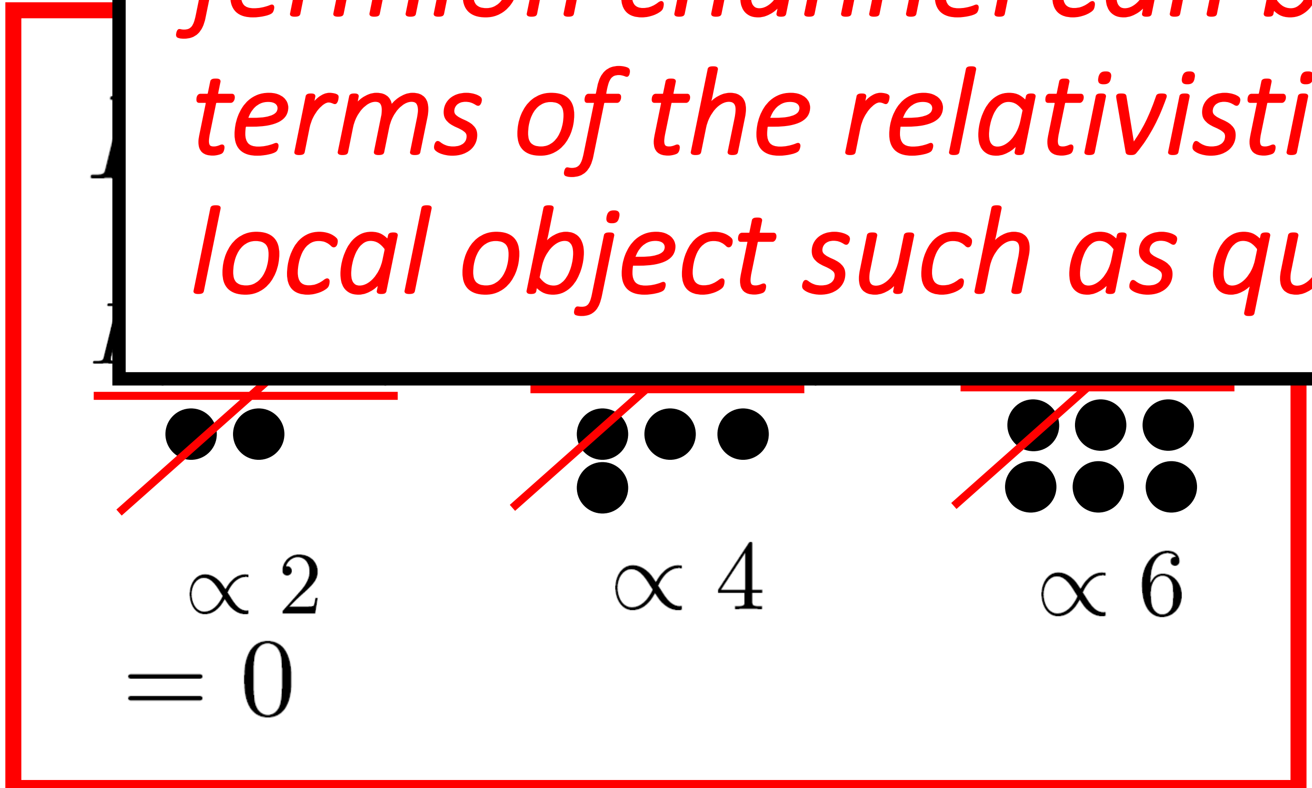
$$= 0$$

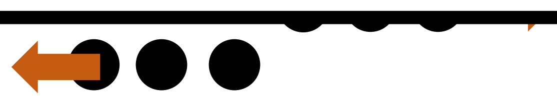
@time = t



Tri-partite information

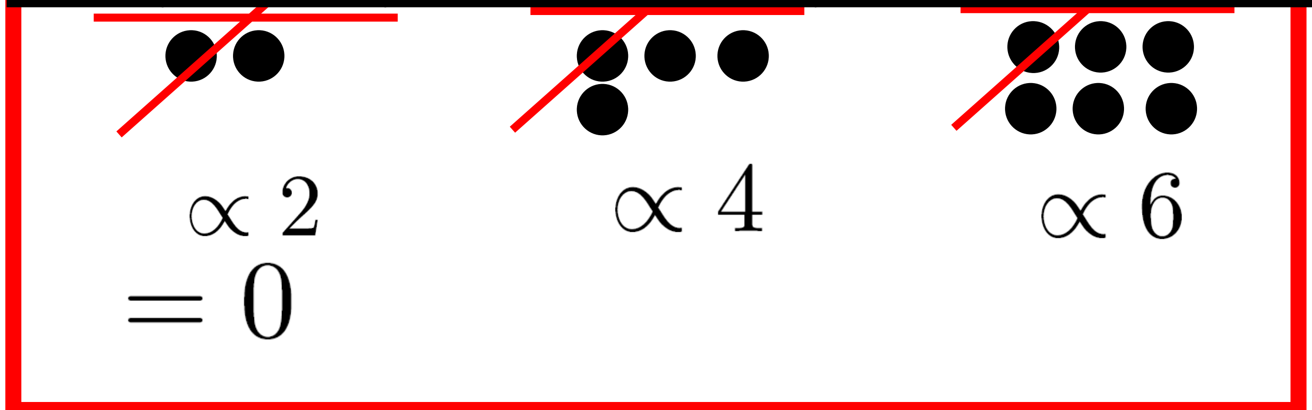
Operator mutual information for free fermion channel can be interpreted in terms of the relativistic propagation of local object such as quasi-particles.

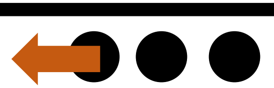




Tri-partite information

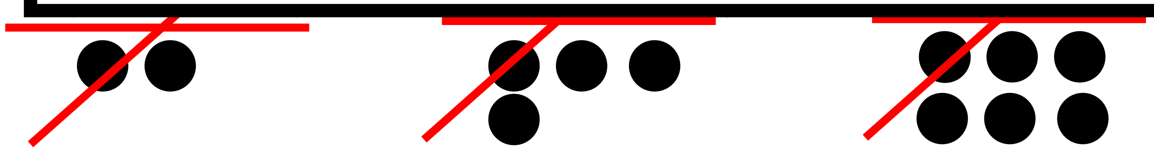
For free fermion channel, quantum correlation between input and output subsystems is explained by local object (quasi-particles)!!





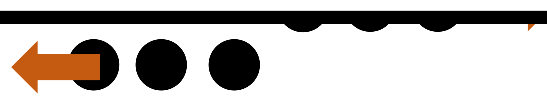
Tri-partite information

Quantum information for free fermion channel is carried by local object (quasi-particles)!!



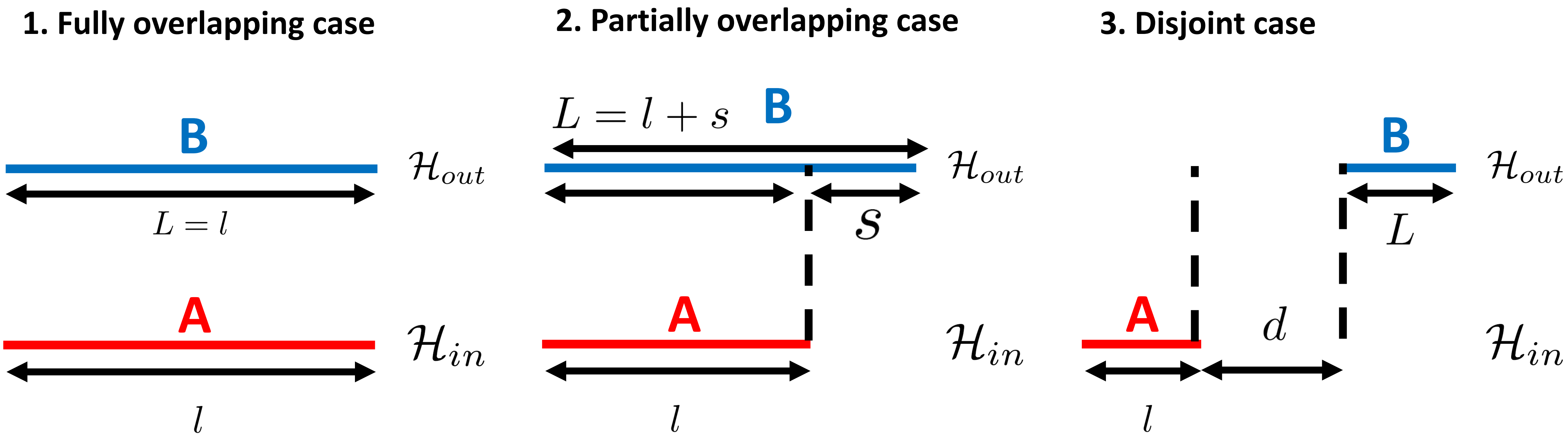
$\propto 2$ $\quad \propto 4$ $\quad \propto 6$

$= 0$



Comparison

We consider the following setups to extract properties of compact boson and holographic channels by comparing them to free fermion channel:



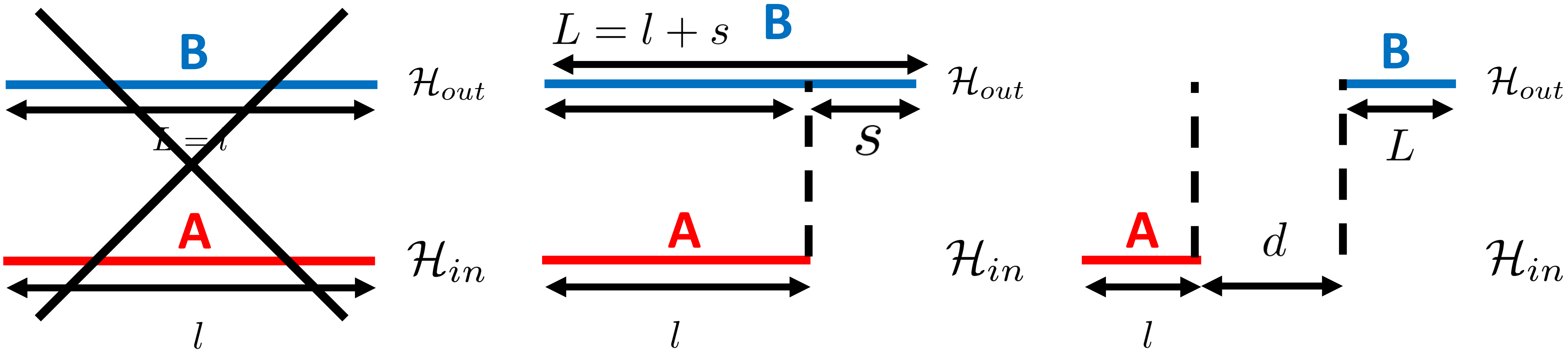
Comparison

We consider the following setups to extract properties of compact boson and holographic channels by comparing them to free fermion channel:

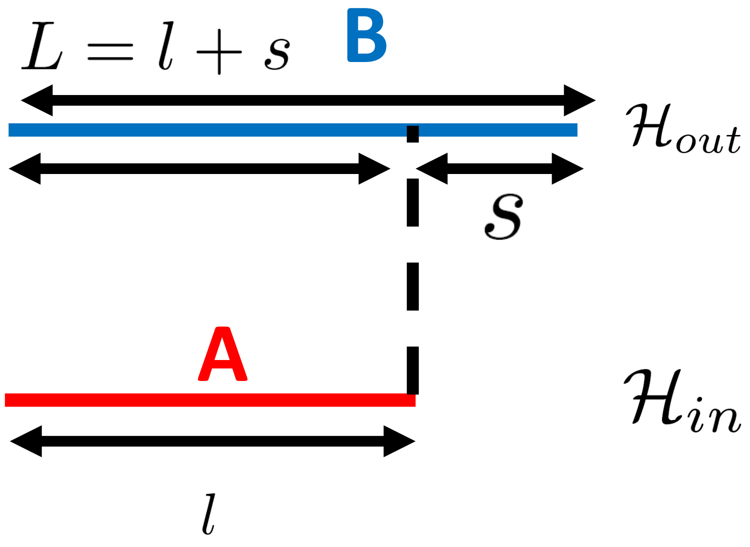
1. Fully overlapping case

2. Partially overlapping case

3. Disjoint case

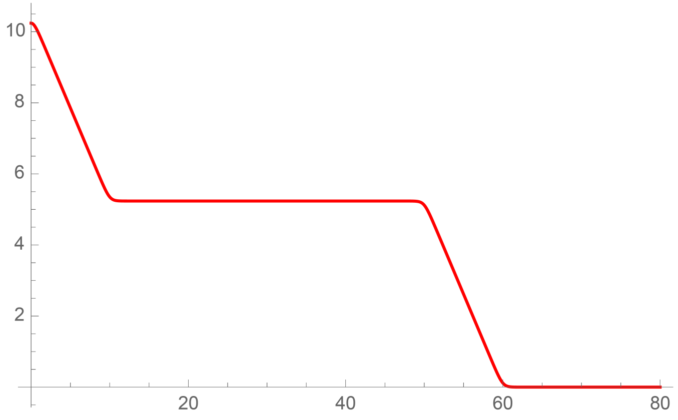


2. Partially overlapping case

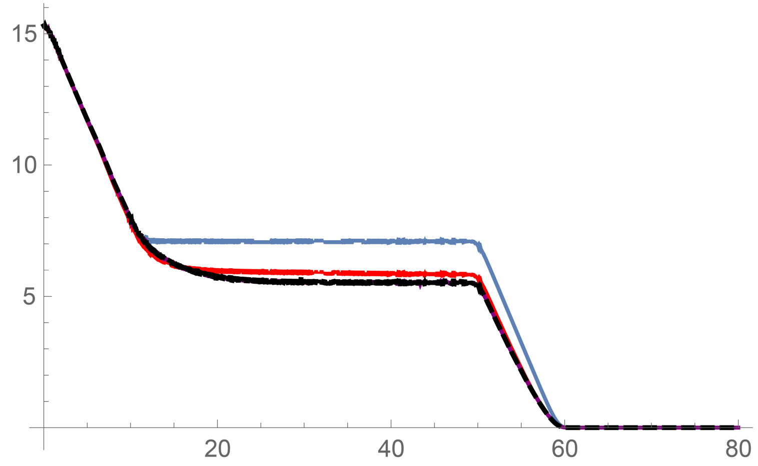


$$(\epsilon, l, s) = (1, 10, 50)$$

Free fermion channel:



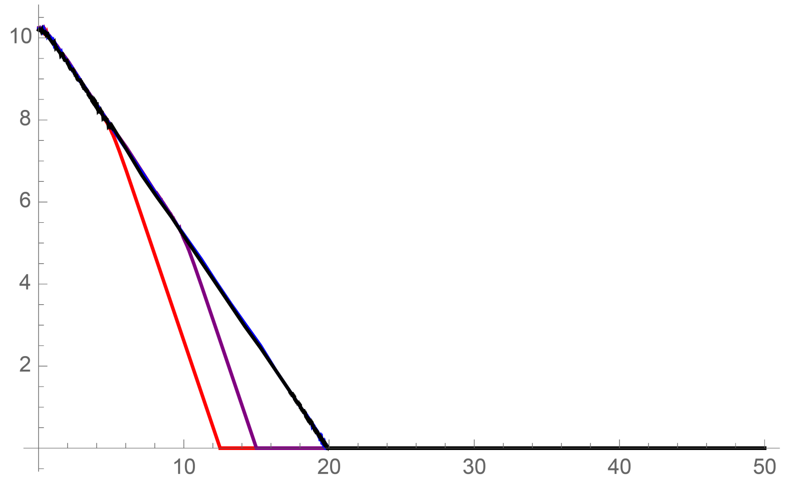
Compact boson channel:



$\eta = 1$:Blue $\eta = 6$:Purple

$\eta = \pi$:Red $\eta = \frac{1}{6}$:Black dash

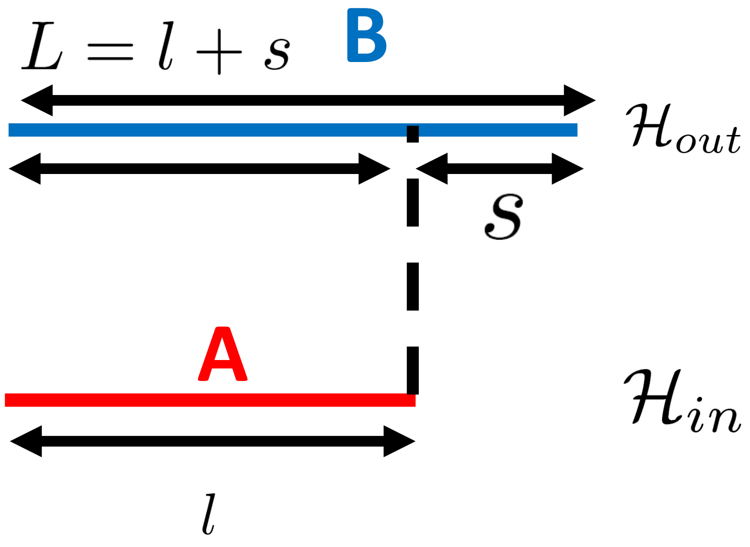
Holographic channel:



$(\epsilon, l, s) = \underline{(1, 10, 5)}$ Red, $\underline{(1, 10, 10)}$ Purple,

$\underline{(1, 10, 20)}$ Blue, $\underline{(1, 10, 50)}$ Black

2. Partially overlapping case

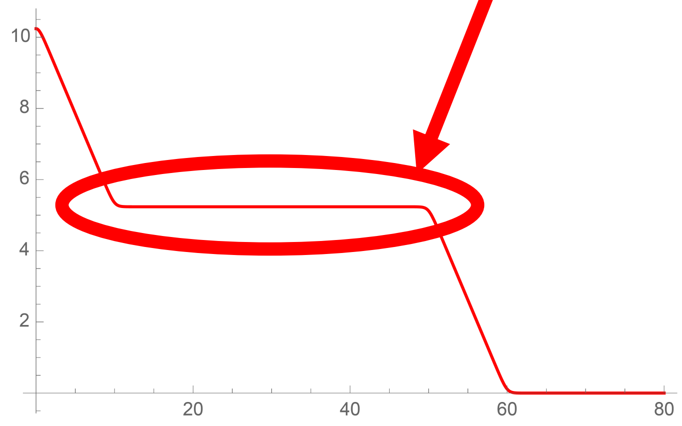


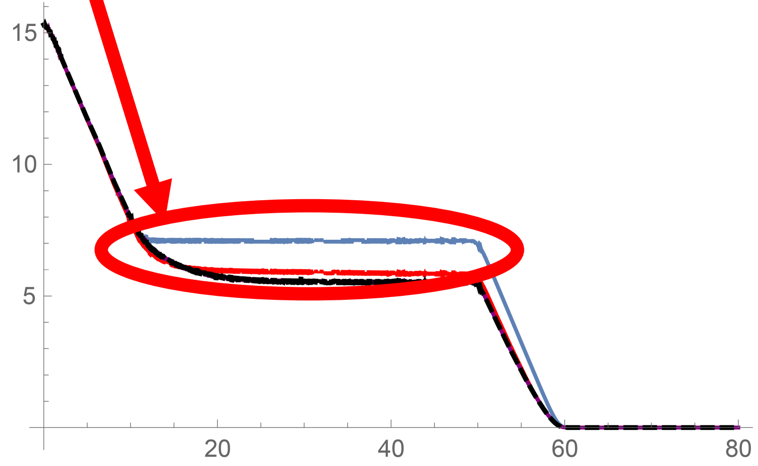
$(\epsilon, l, s) = (1, 10, 50)$

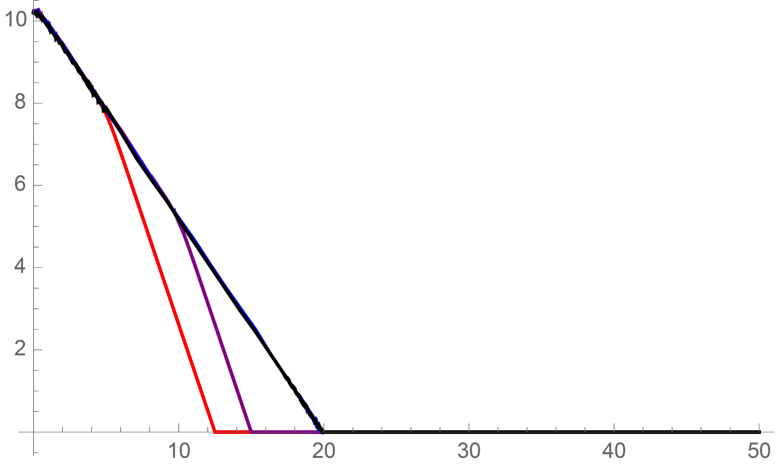
Quasi-particle picture works well.

Free fer

Holographic channel:







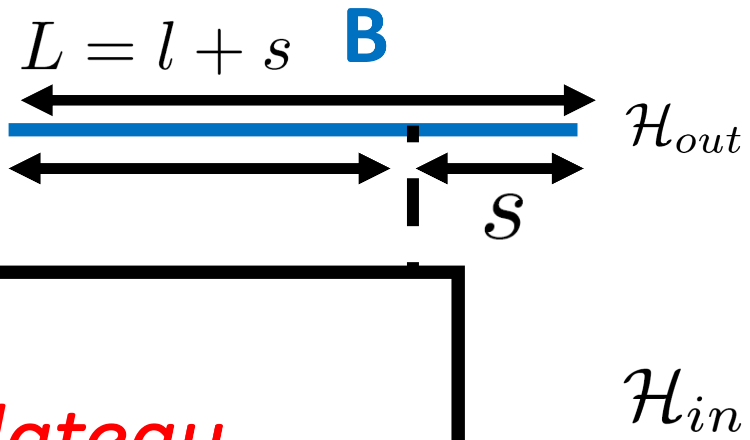
$\eta = 1$:Blue $\eta = 6$:Purple

$\eta = \pi$:Red $\eta = \frac{1}{6}$:Black dash

$(\epsilon, l, s) = \underset{\text{Red}}{(1, 10, 5)}, \underset{\text{Purple}}{(1, 10, 10)},$

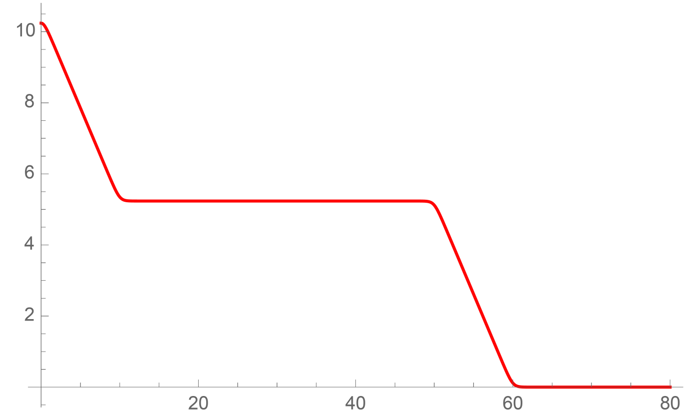
$\underset{\text{Blue}}{(1, 10, 20)}, \underset{\text{Black}}{(1, 10, 50)}$

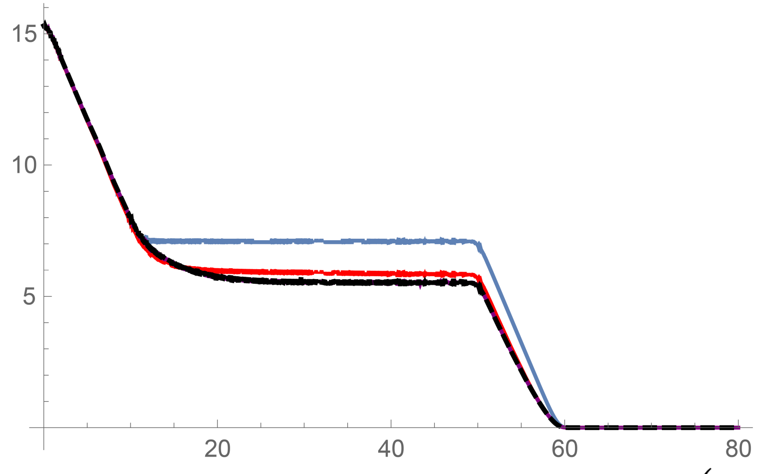
2. Partially overlapping case

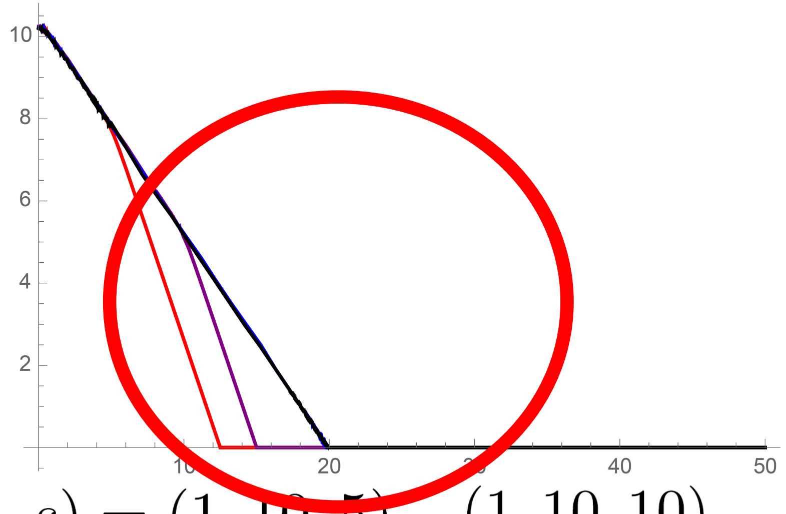


Holographic channel doesn’t show a plateau.

c channel:







$\eta = 1$:Blue $\eta = 6$:Purple

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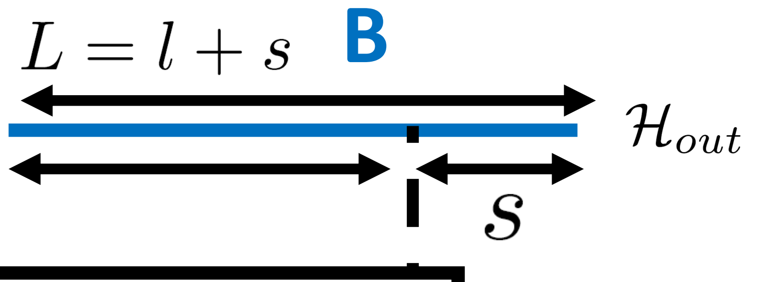
$(\epsilon, l, s) = \underbrace{(1, 10, 5)}_{\text{Red}}, \underbrace{(1, 10, 10)}_{\text{Purple}},$

$\underbrace{(1, 10, 20)}_{\text{Blue}}, \underbrace{(1, 10, 50)}_{\text{Black}}$

2. Partially overlapping case

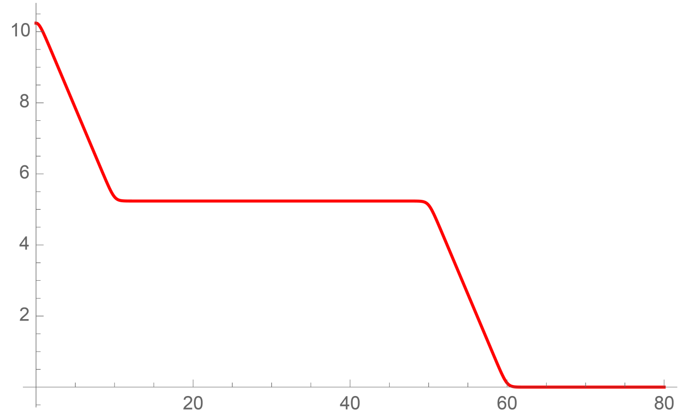


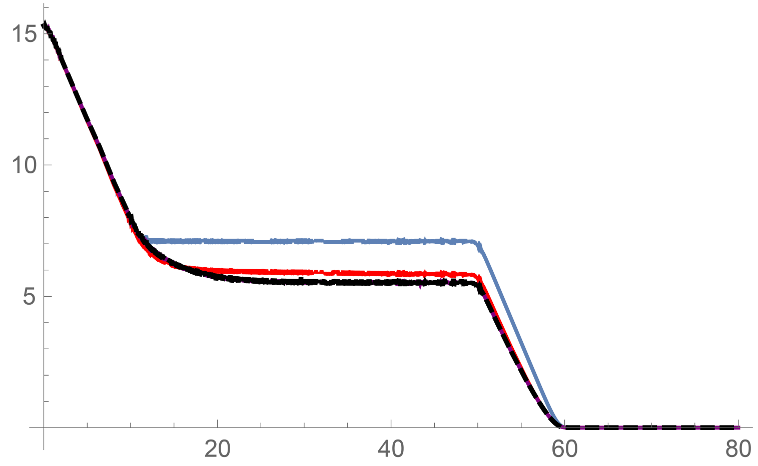
$\mathcal{H}_{in}$

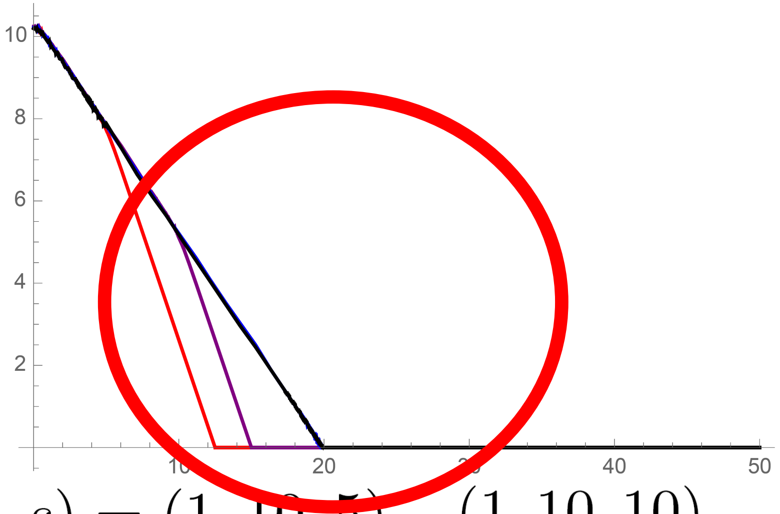
This property is beyond the particle interpretation.

c channel:

channel:







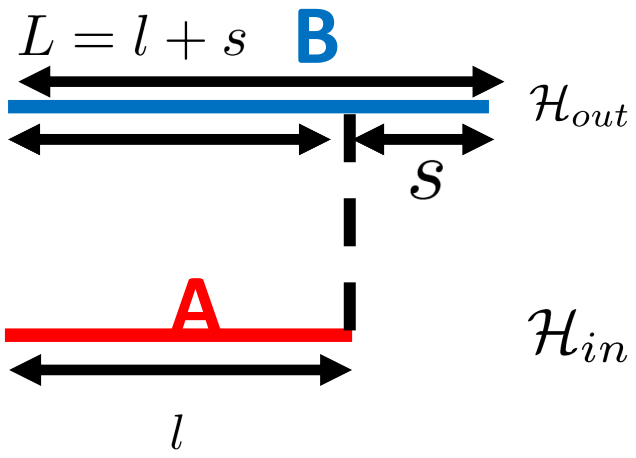
$\eta = 1$:Blue $\quad \eta = 6$:Purple

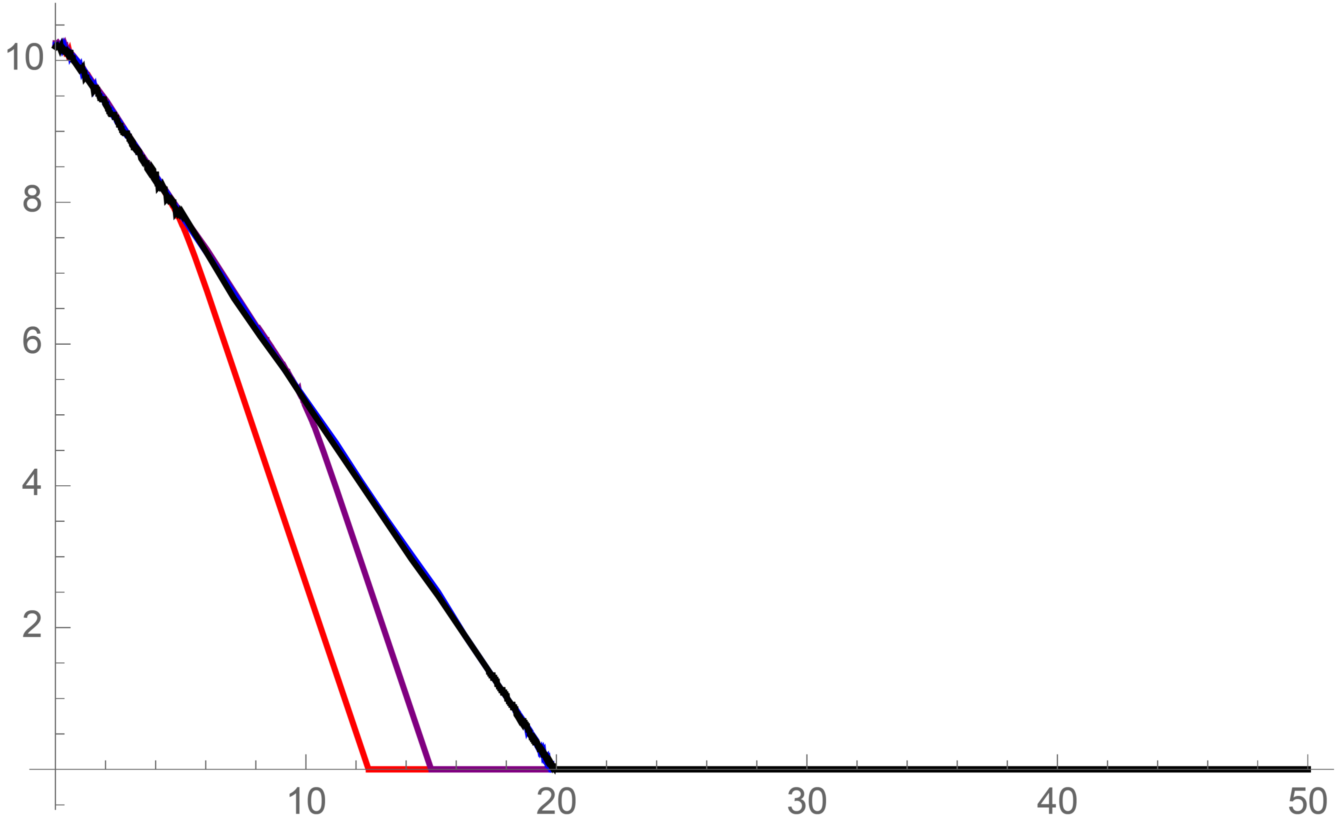
$\eta = \pi$:Red $\quad \eta = \frac{1}{6}$:Black dash

$(\epsilon, l, s) = \underline{(1, 10, 5)}$, $\underline{(1, 10, 10)}$,
Red Purple
$\underline{(1, 10, 20)}$, $\underline{(1, 10, 50)}$
Blue Black

Holographic channel

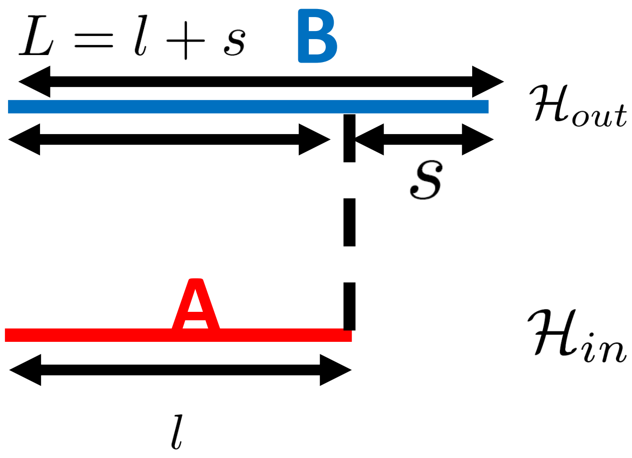
$$(\epsilon, l, s) = \underbrace{(1, 10, 5)}_{\text{Red}}, \underbrace{(1, 10, 10)}_{\text{Purple}}, \underbrace{(1, 10, 20)}_{\text{Blue}}, \underbrace{(1, 10, 50)}_{\text{Black}}$$



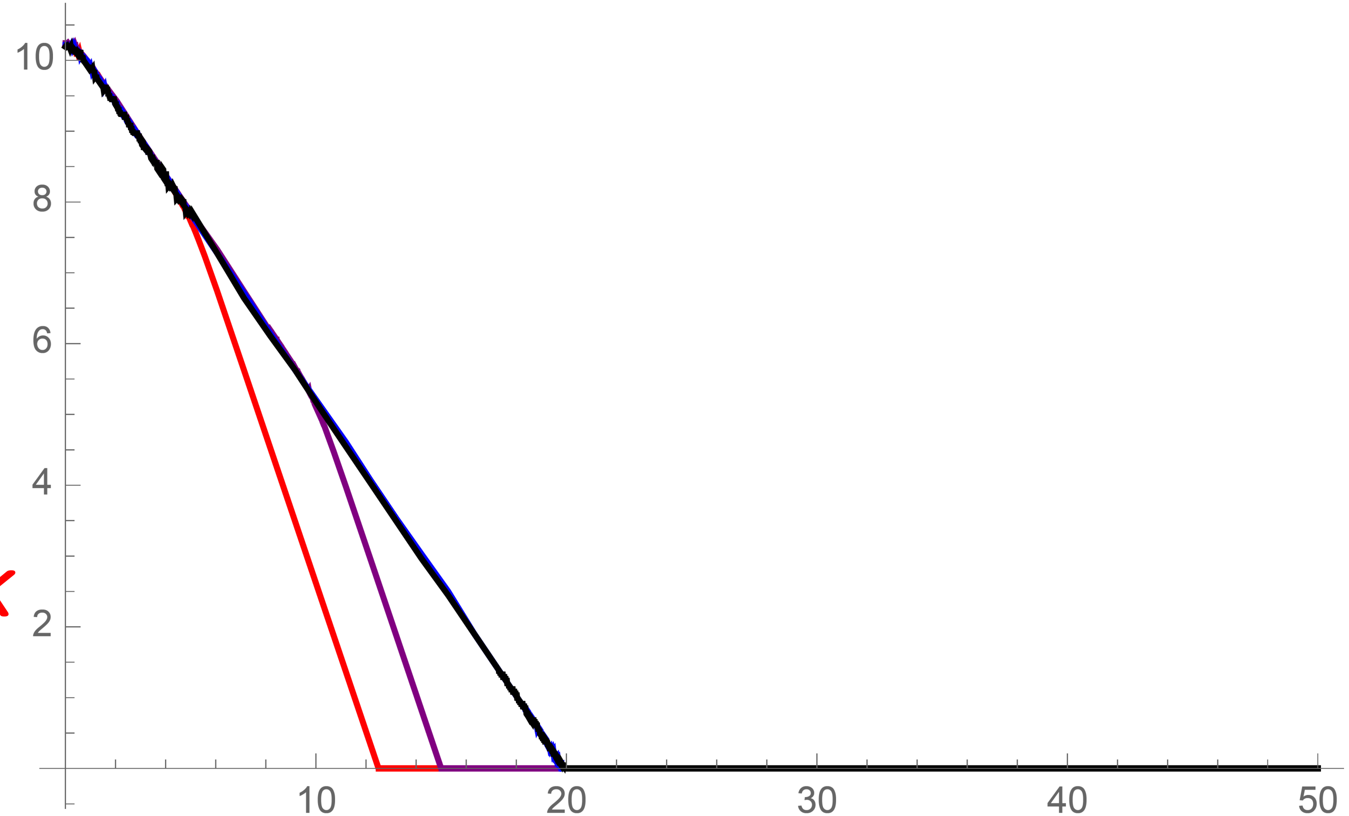


Holographic channel

$(\epsilon, l, s) = \underbrace{(1,10,5)}_{\text{Red}}, \underbrace{(1,10,10)}_{\text{Purple}}, \underbrace{(1,10,20)}_{\text{Blue}}, \underbrace{(1,10,50)}_{\text{Black}}$



Monotonically decreasing

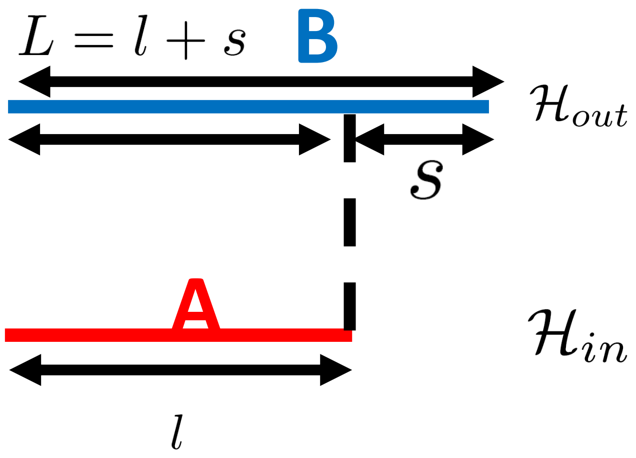


Once quantum information leaks from B, *it keeps to leak before all information leaks*.

Holographic channel

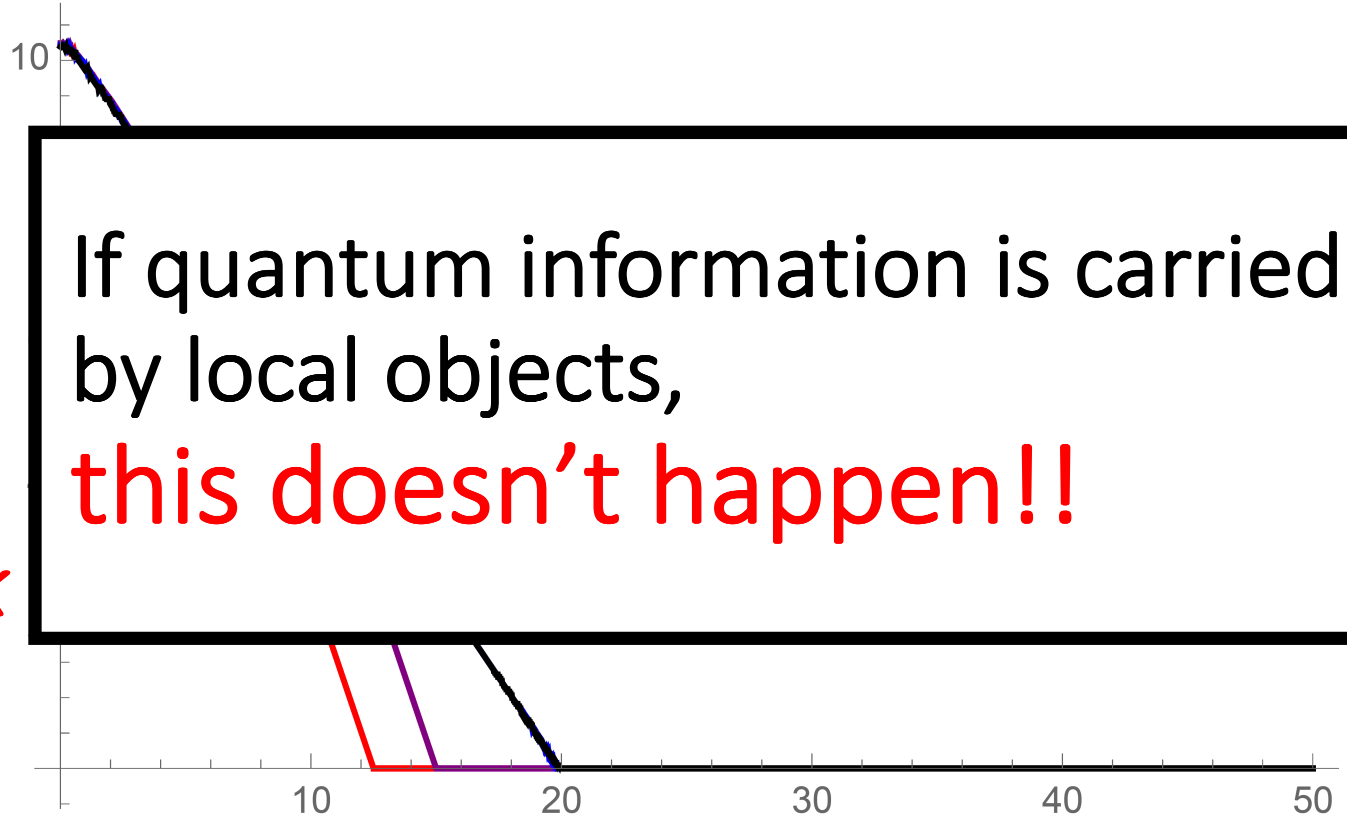
$(\epsilon, l, s) = \underset{\text{Red}}{\underline{(1,10,5)}}, \underset{\text{Purple}}{\underline{(1,10,10)}},$

$\underset{\text{Blue}}{\underline{(1,10,20)}}, \underset{\text{Black}}{\underline{(1,10,50)}}$



Monotonically decreasing

Once quantum information leaks from B, *it keeps to leak before all information leaks*.



If quantum information is carried by local objects,

this doesn't happen!!

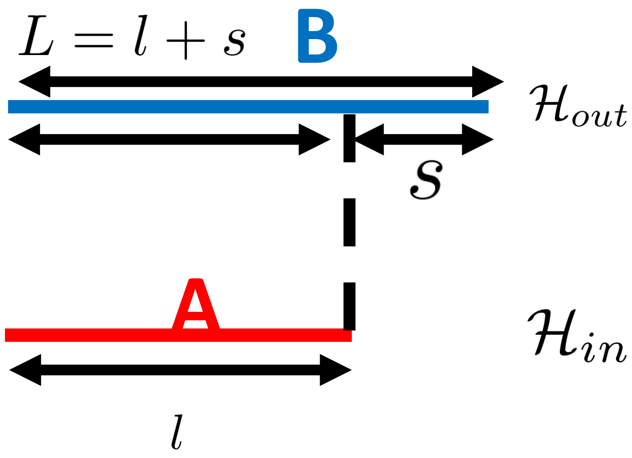
Holographic channel

$(\epsilon, l, s) = \underline{(1, 10, 5)}$, $\underline{(1, 10, 10)}$,

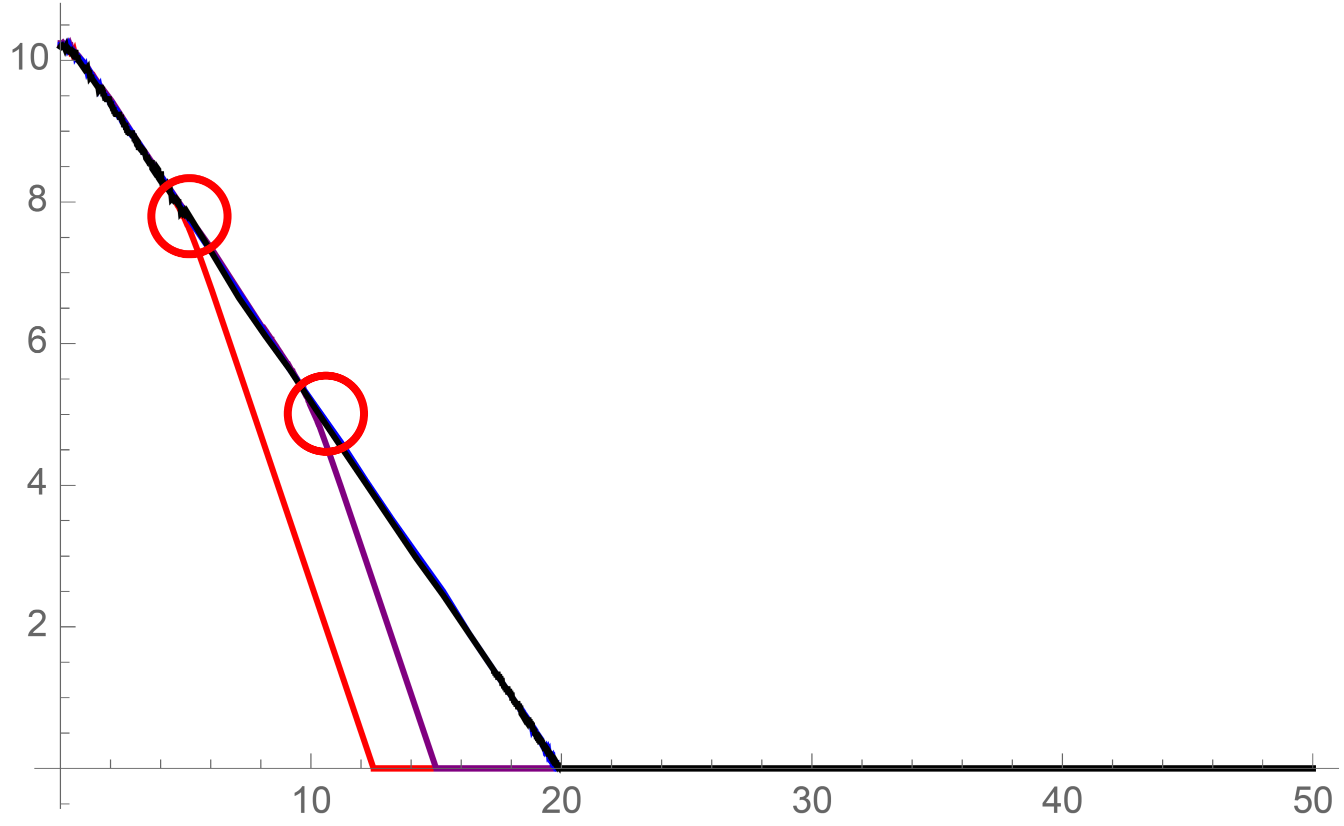
Red Purple

$\underline{(1, 10, 20)}$, $\underline{(1, 10, 50)}$

Blue Black

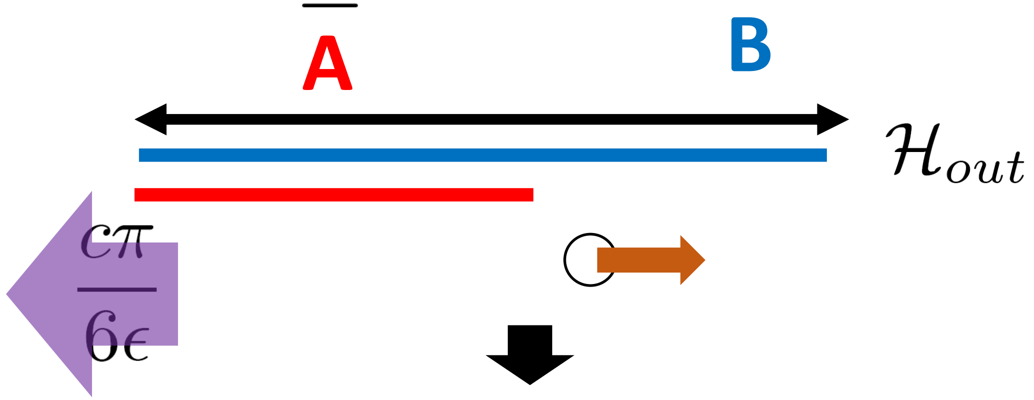


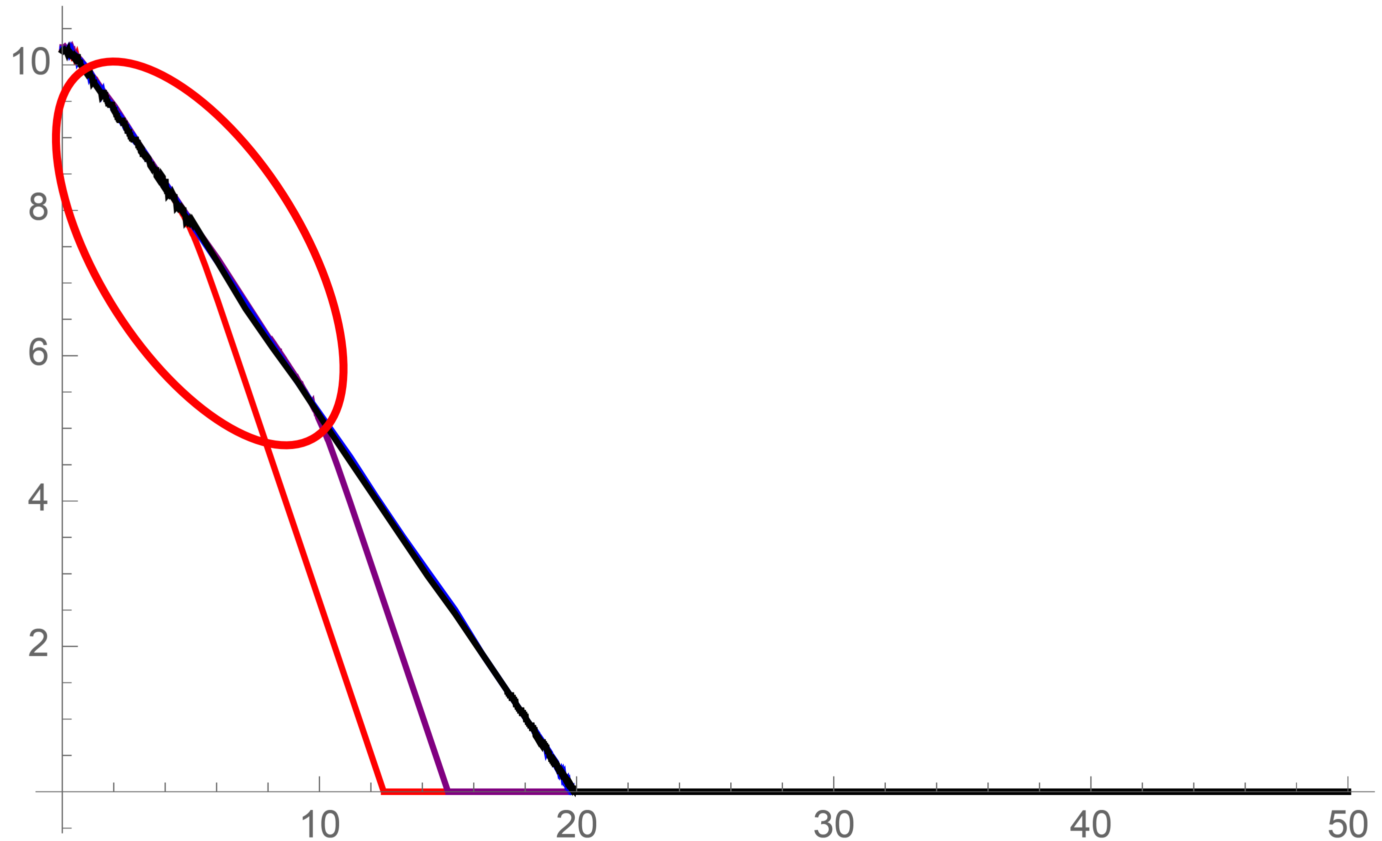
The slope changes at $t = s$.



Heuristic explanation

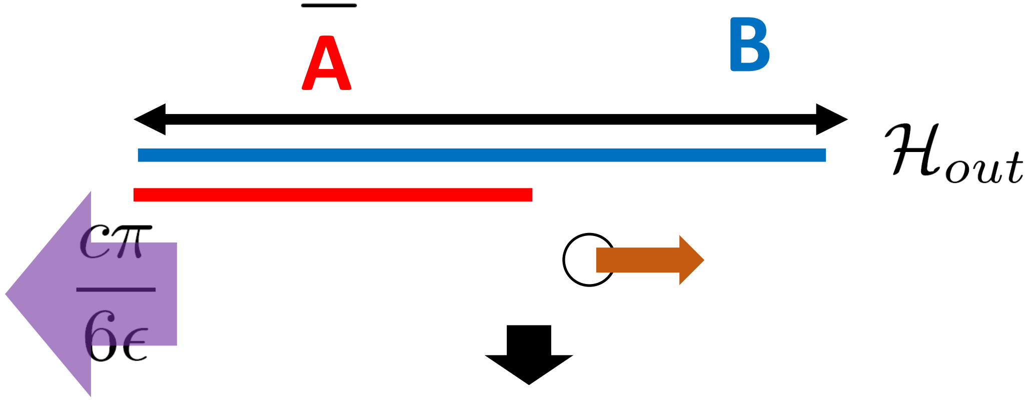
$$0 < t \leq s$$

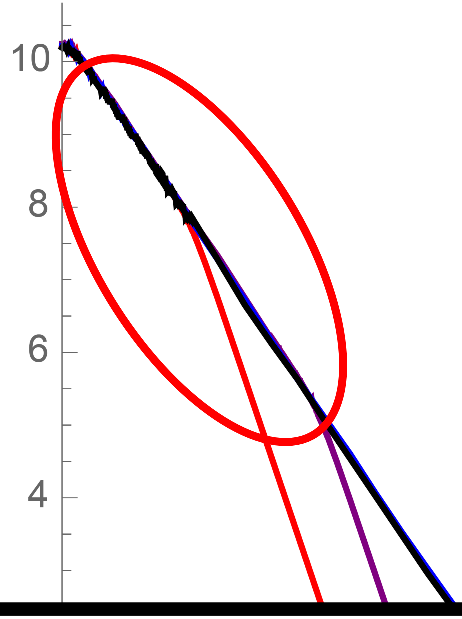




Heuristic explanation

$0 < t \leq s$





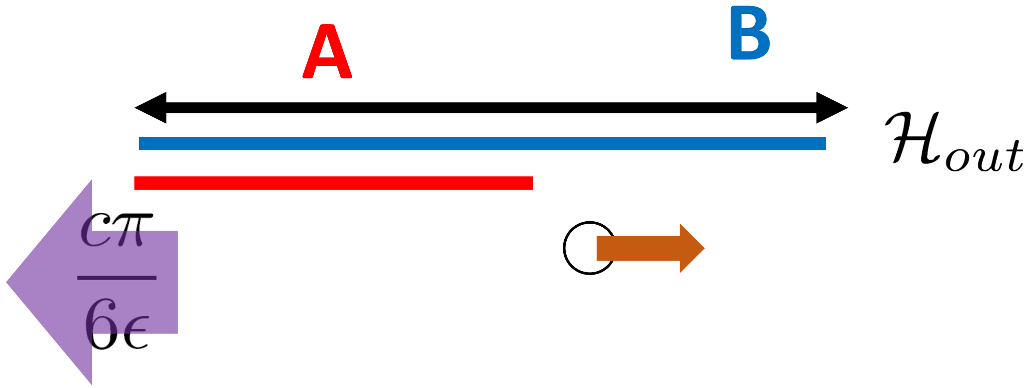
Quantum information keeps to go out from the left boundary with $\frac{c\pi}{6\epsilon}$.

At t=0, right-moving signal appears at the right boundary of A.

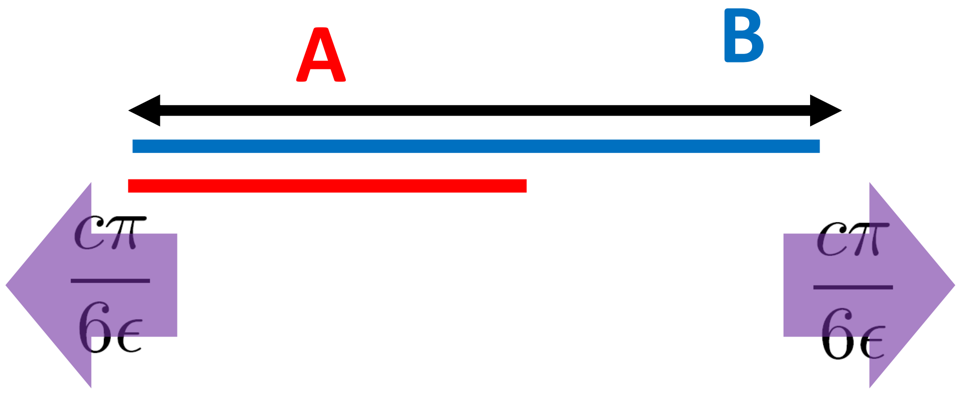
Its speed is the light's.

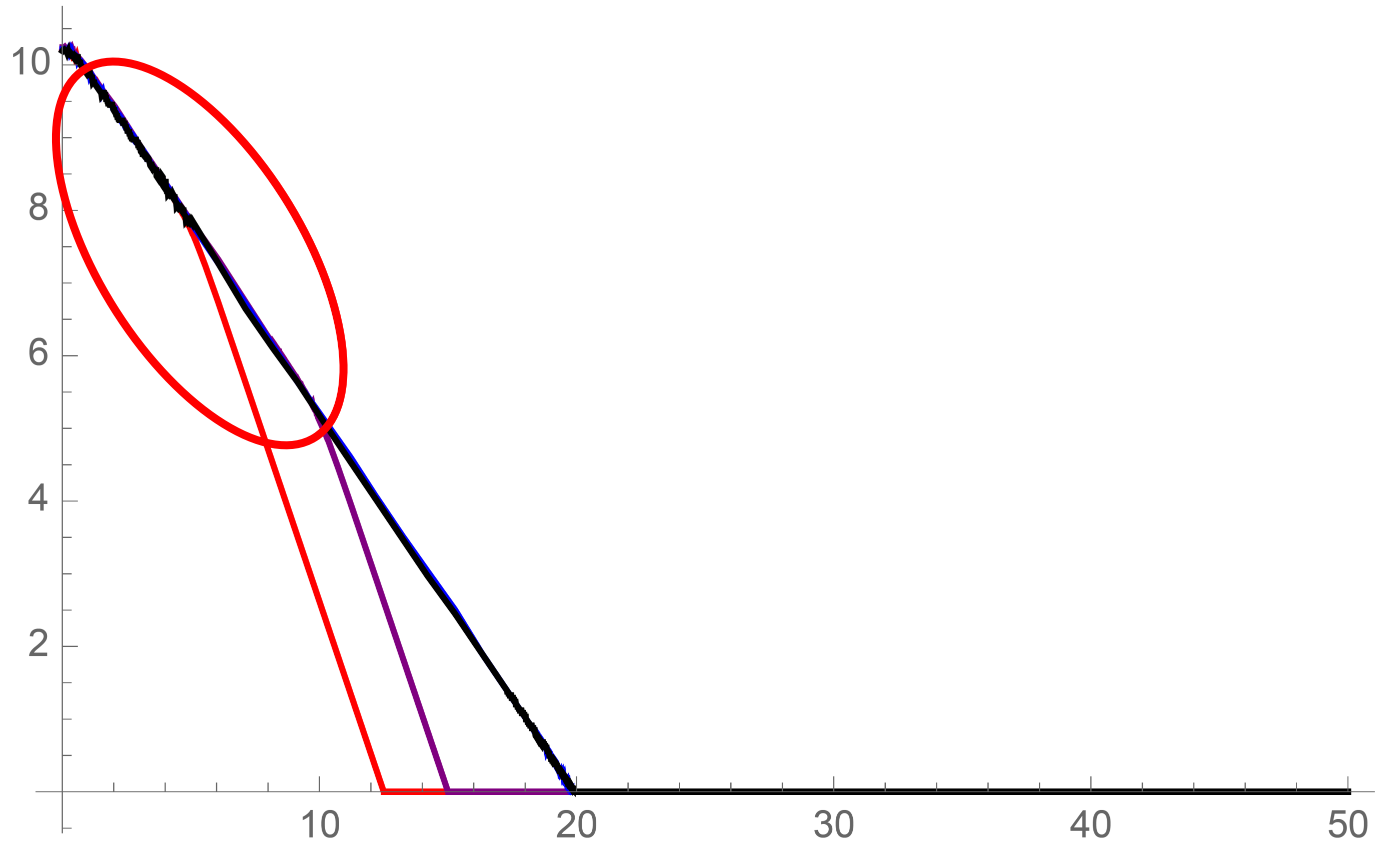
Heuristic explanation

$$0 < t \leq s$$



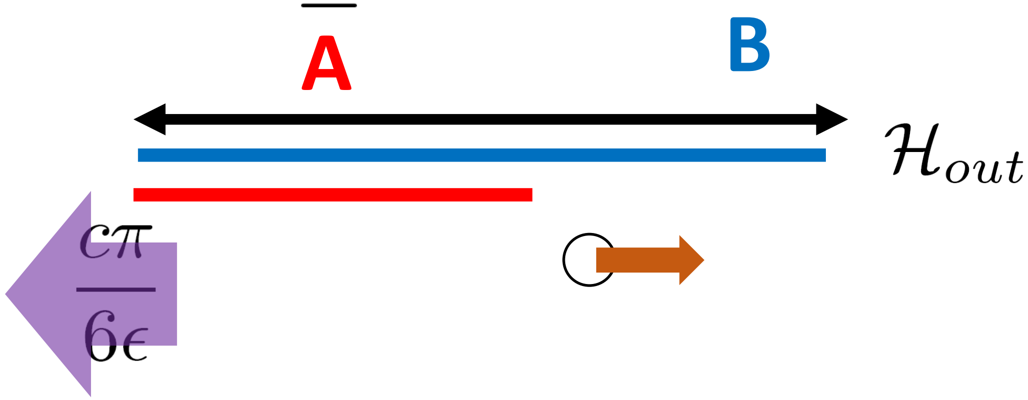
$$s < t \leq l + s/2$$



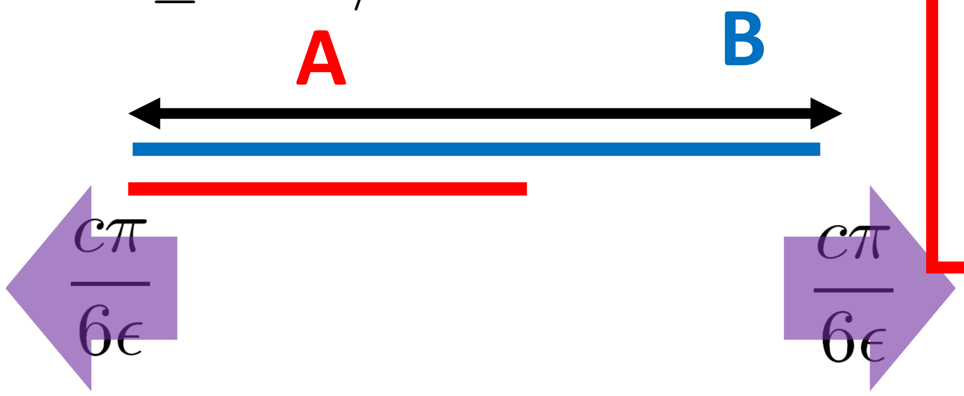


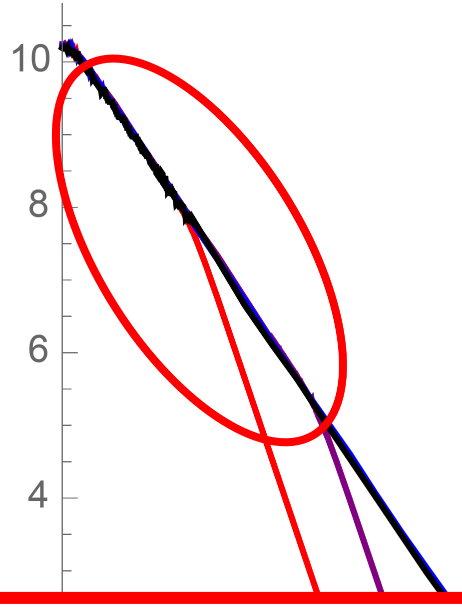
Heuristic explanation

$$0 < t \leq s$$



$$s < t \leq l + s/2$$

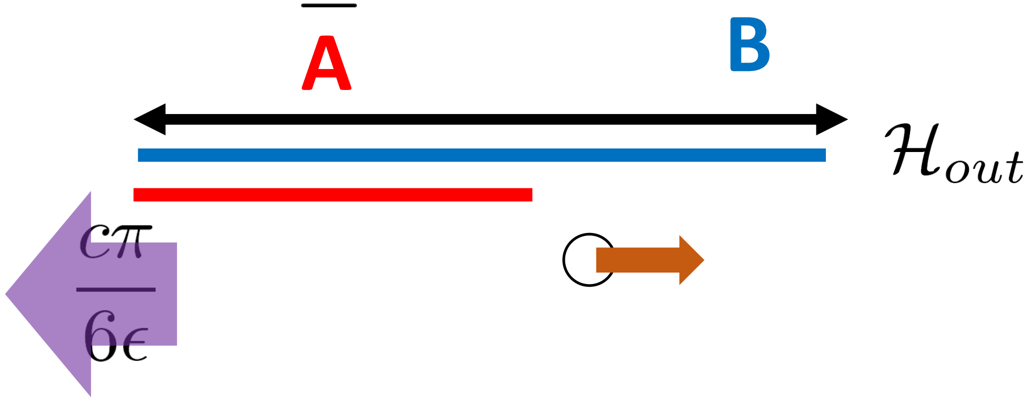


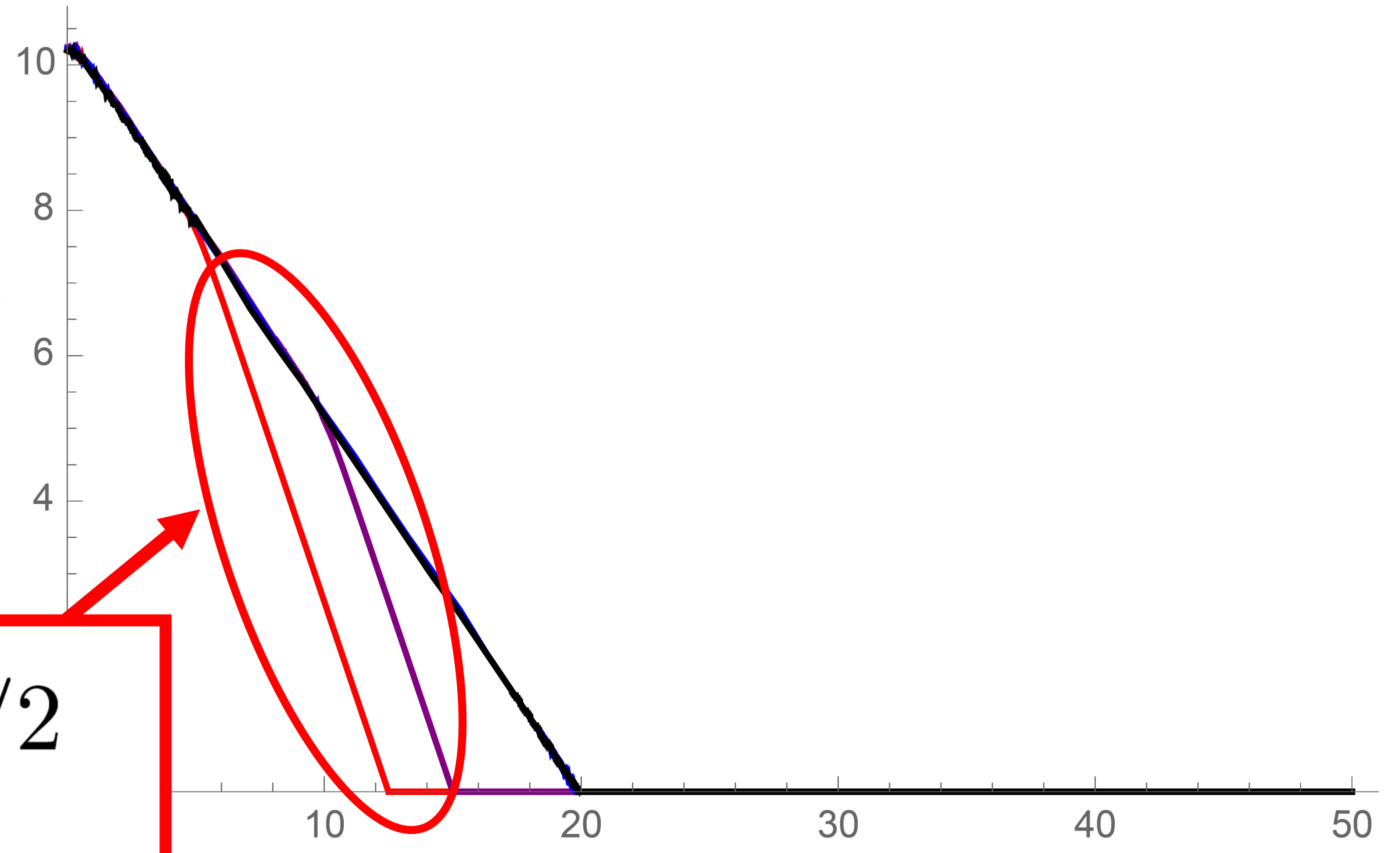


When the right-moving signal hits the right boundary of B, the information starts to go out from B with $\frac{c\pi}{6\epsilon}$.

Heuristic explanation

$0 < t \le s$



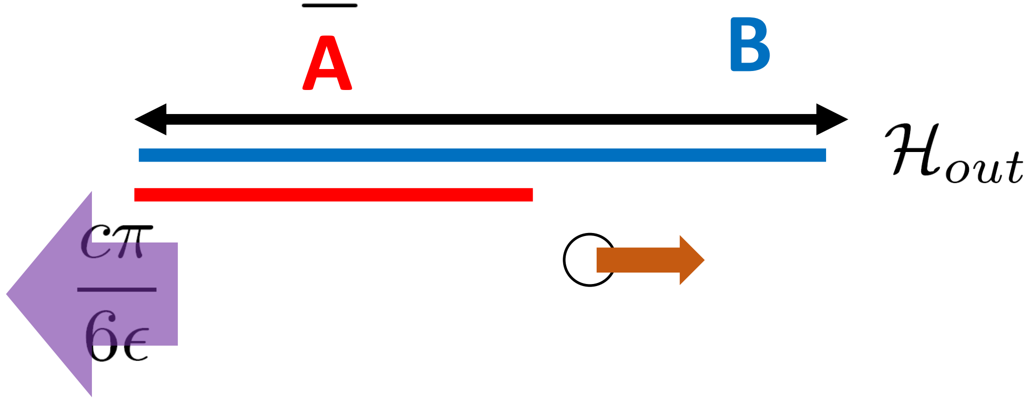


$I(A, B)$ in $s < t \le l + s/2$

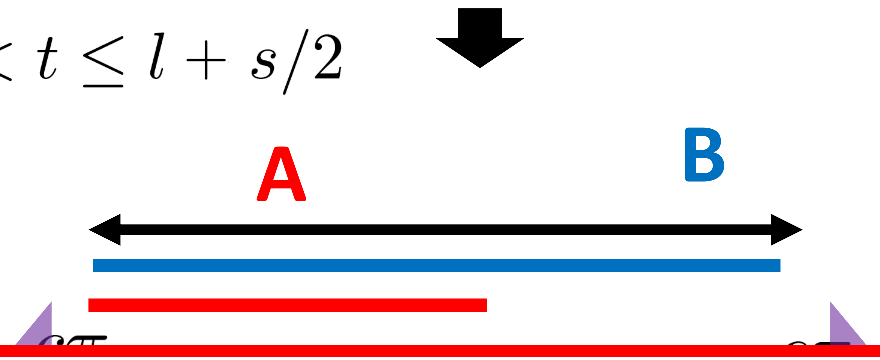
decreases twice faster than that in $0 < t \le s$.

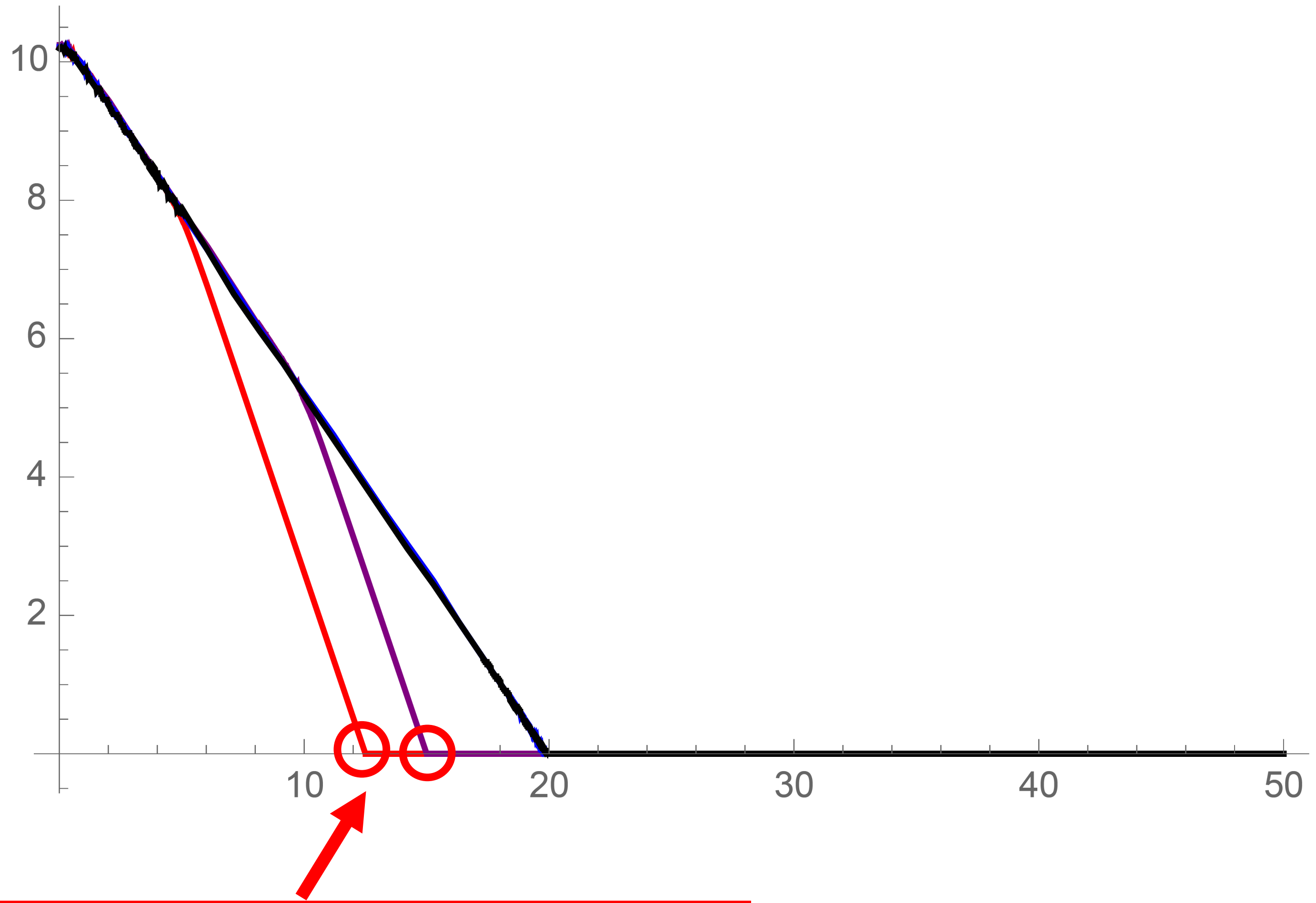
Heuristic explanation

$0 < t \le s$



$s < t \le l + s/2$



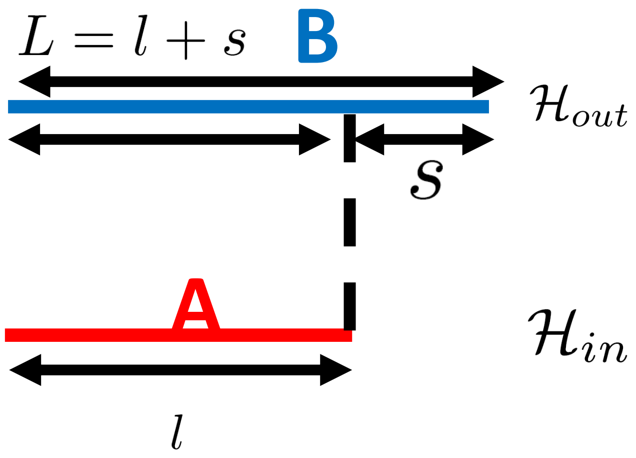


All the information of *A goes out from B @ t=l +s/2.*

Holographic channel

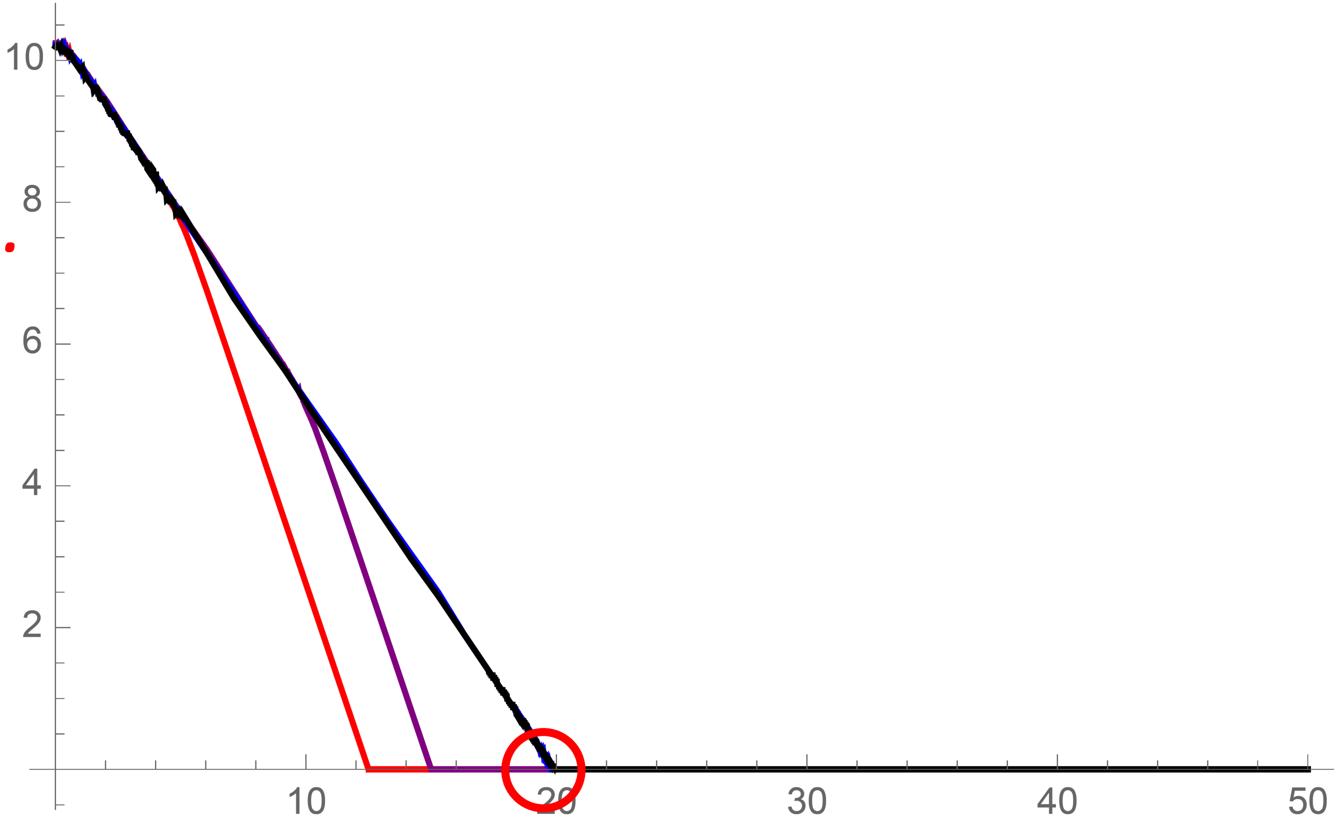
$(\epsilon, l, s) = (1, 10, 5)$ Red, $(1, 10, 10)$ Purple,

$(1, 10, 20)$ Blue, $(1, 10, 50)$ Black



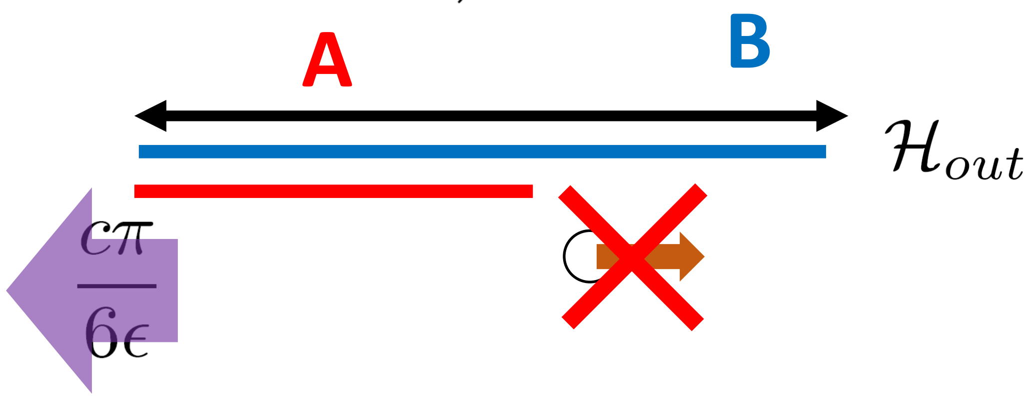
For $s \geq 2l$, $I(A, B)$ *linearly decreases*.

$I(A, B) = 0$ @ $t = 2l$.

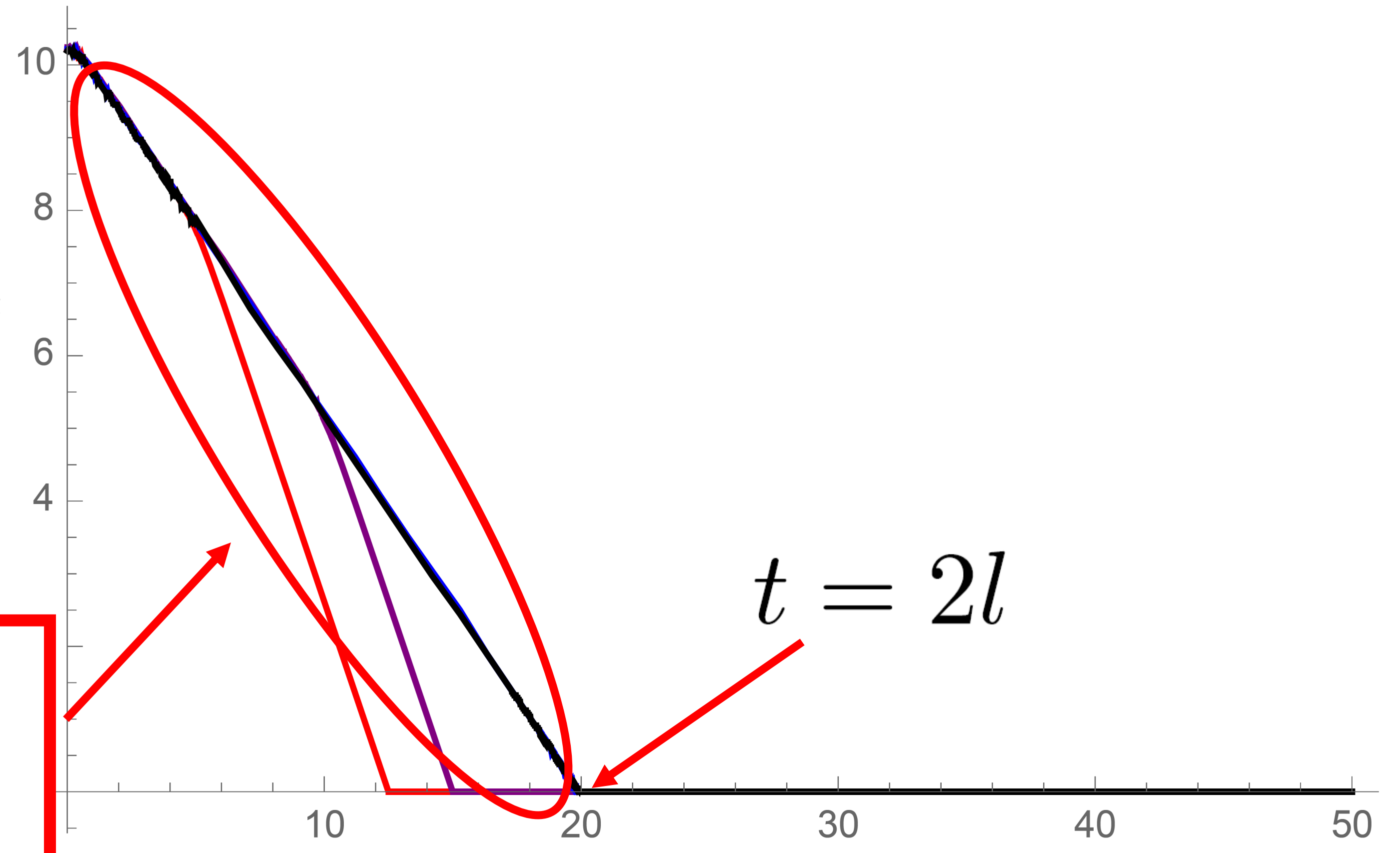


Heuristic explanation

$$0 < t \leq s\,,\, s \geq 2l.$$

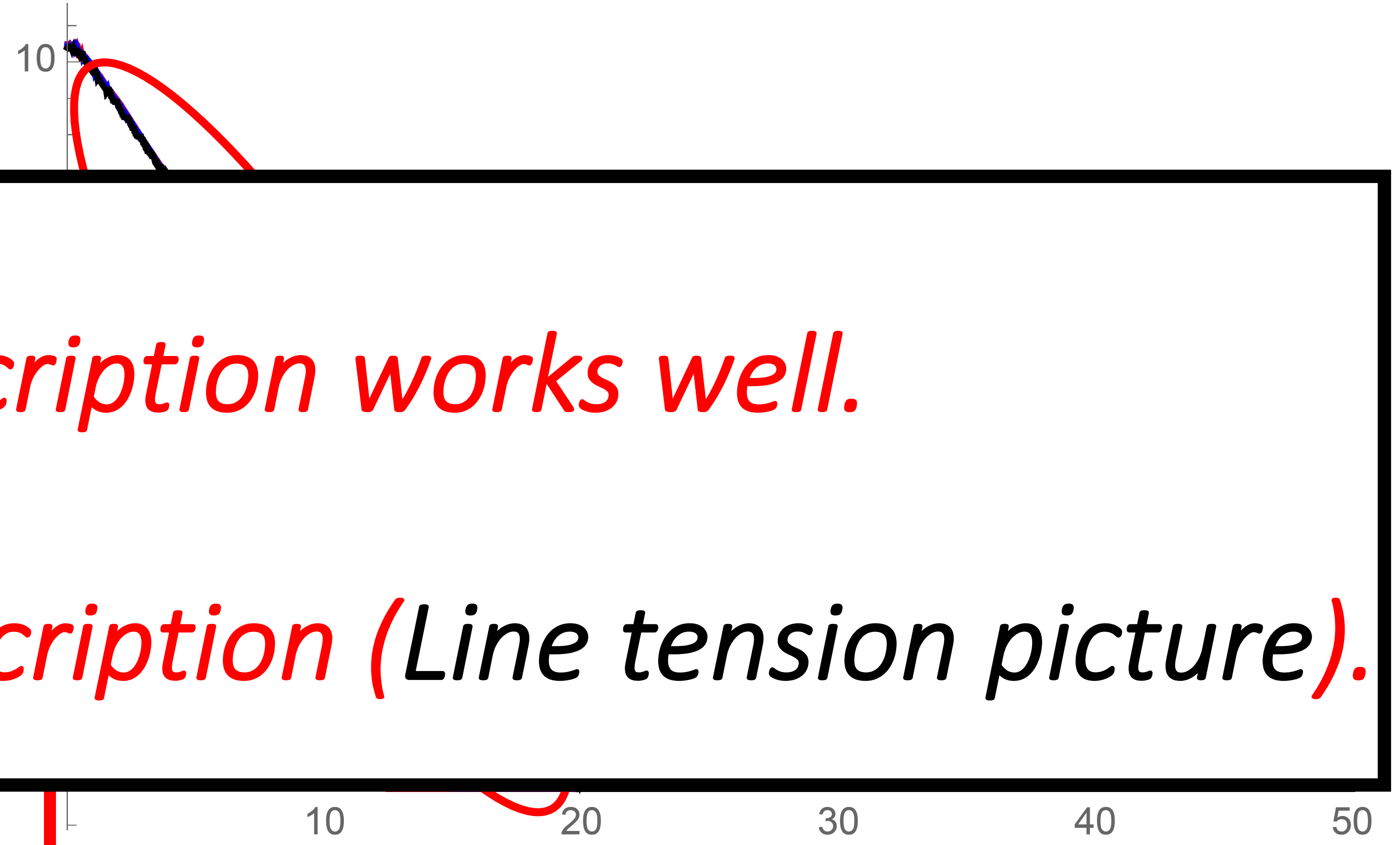


All the information of *A* goes out from the left boundary of *B* before the signal arrives at the right boundary. Once whole information has gone, the signal disappears.



Heuristic explanation

$$0 < t \leq s \, , s \geq 2l .$$



- Early time:

Quasi-particle description works well.

- Late time:

We need some description (Line tension picture).

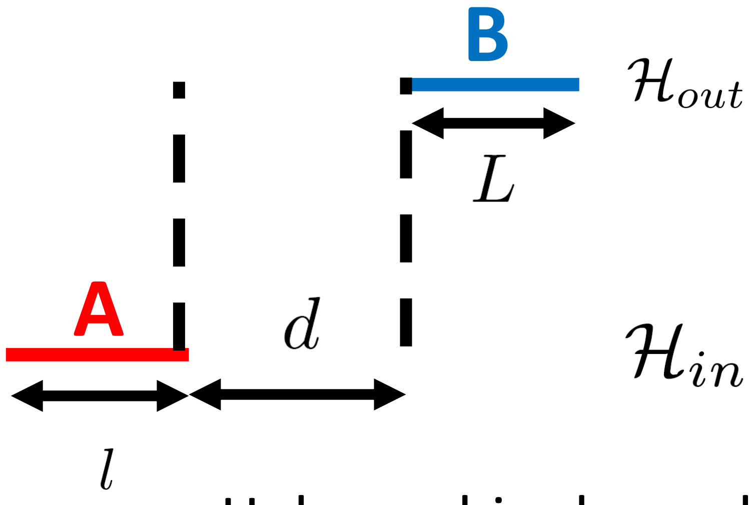
A

o

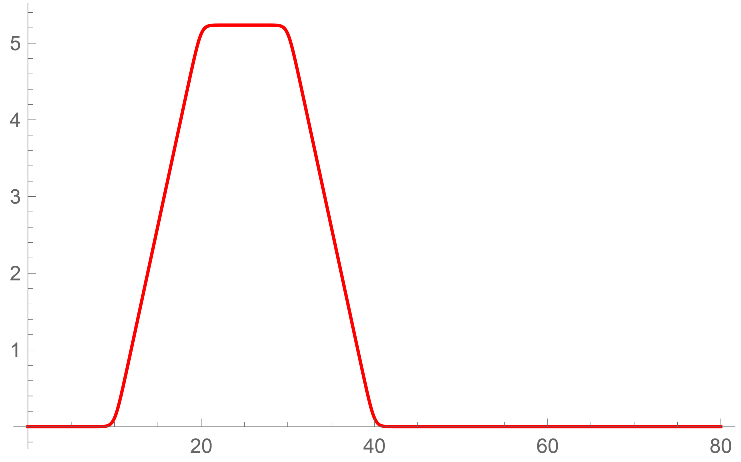
B before the signal arrives at the right boundary. The signal disappears.

3. Disjoint interval

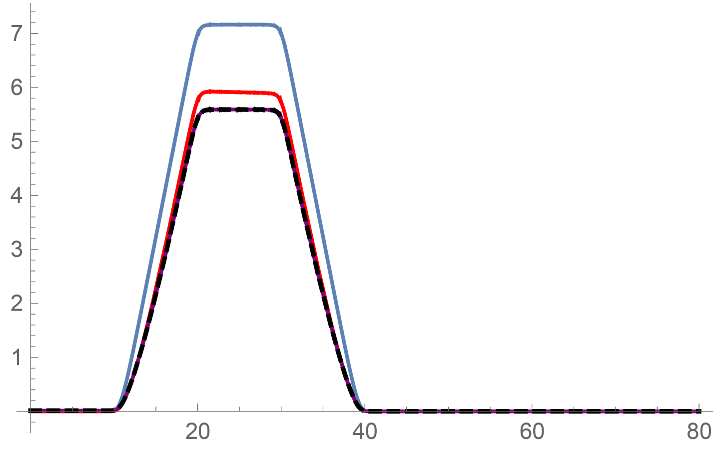
$$(\epsilon, l, L, d) = (1, 10, 20, 10)$$



Free fermion channel:



Compact boson channel:



$\eta = 1$:Blue $\quad \eta = 6$:Purple

$\eta = \pi$:Red $\quad \eta = \frac{1}{6}$:Black dash

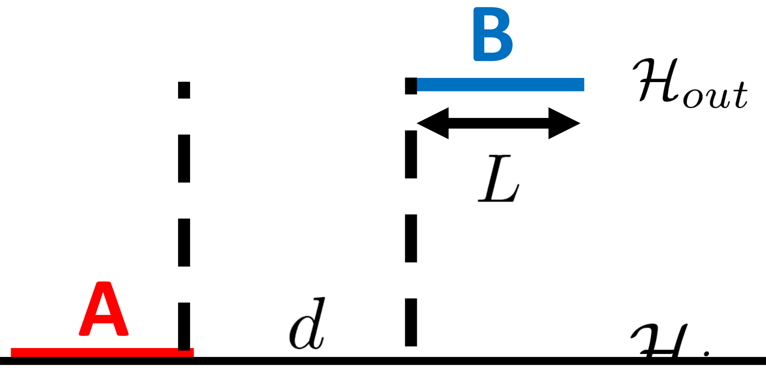
Holographic channel:

If A and B are disjoint($d \geq 0$),

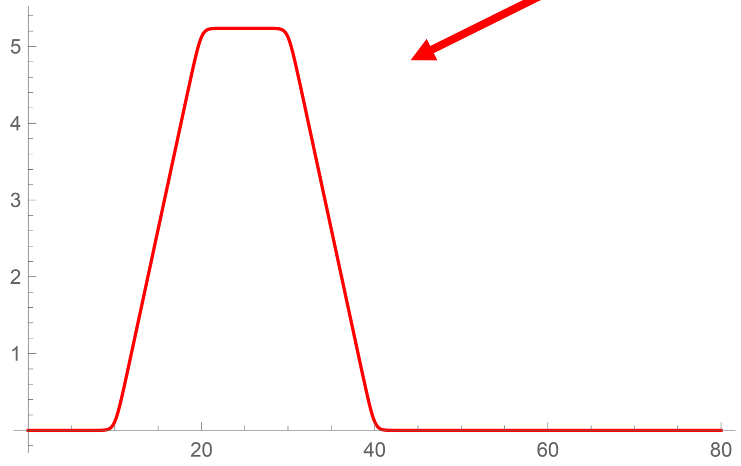
$$I(A, B) = 0$$

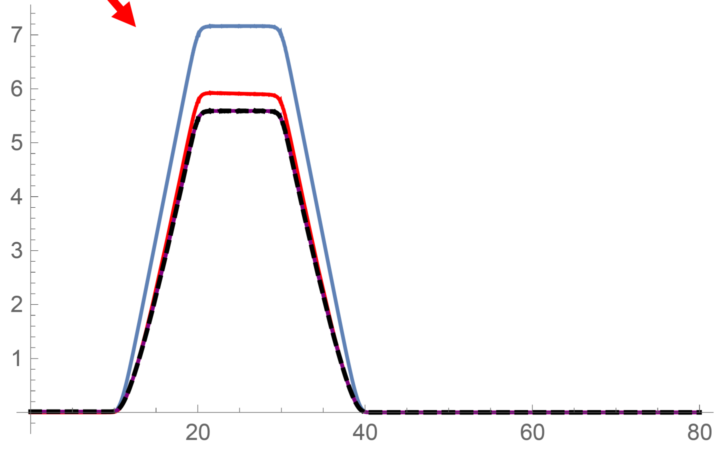
3. Disjoint interval

$$(\epsilon, l, L, d) = (1, 10, 20, 10)$$



Time evolution can be interpreted in terms of particles, almost.





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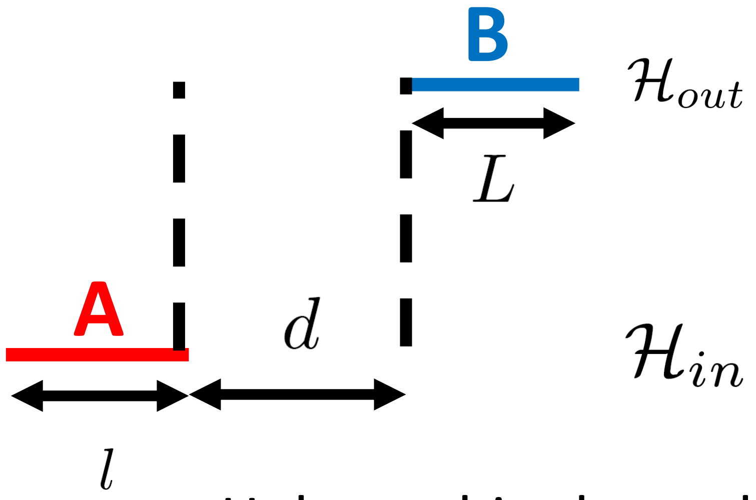
If A and B are disjoint($d \geq 0$),

$$I(A, B) = 0$$

3. Disjoint interval

$$(\epsilon, l, L, d) = (1, 10, 20, 10)$$

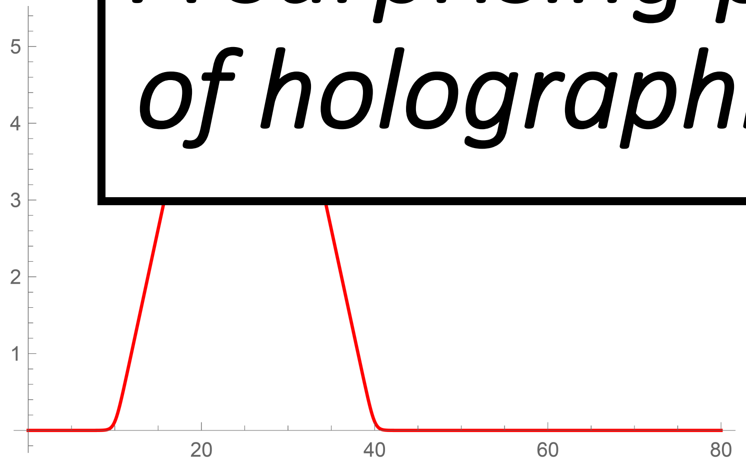


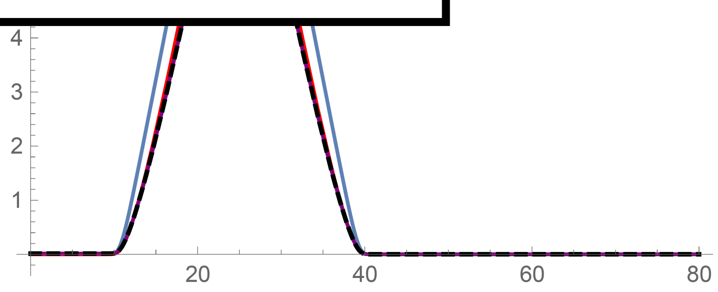
A surprising property of holographic channel.

Holographic channel:

If A and B are disjoint($d \geq 0$),

$$I(A,B) = 0$$





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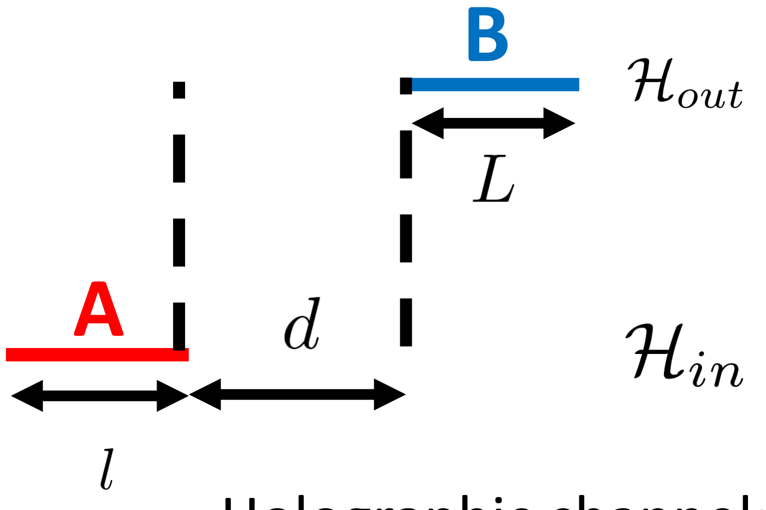
3. Disjoint interval

$$(\epsilon, l, L, d) = (1, 10, 20, 10)$$



Holographic channel:

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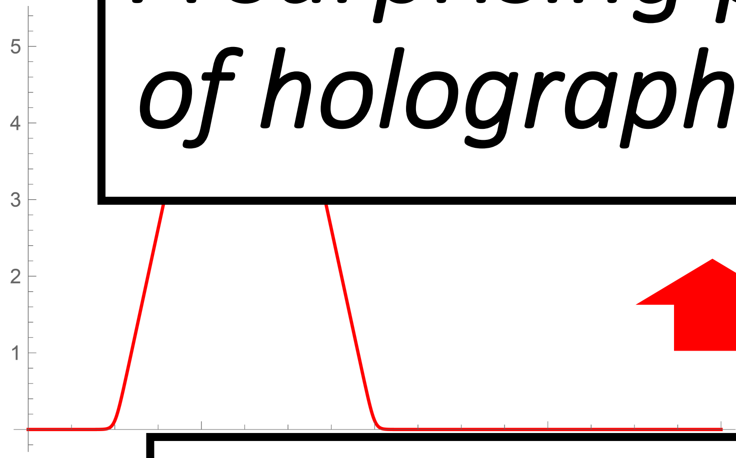
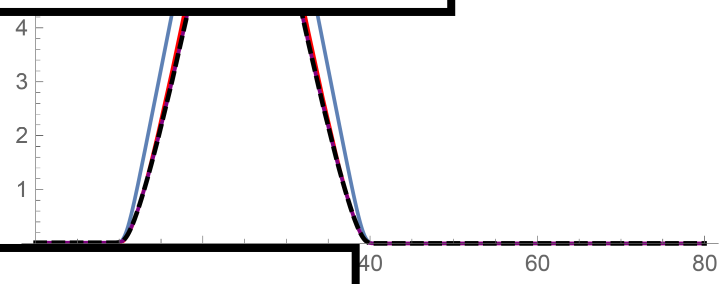
A surprising property of holographic channel.



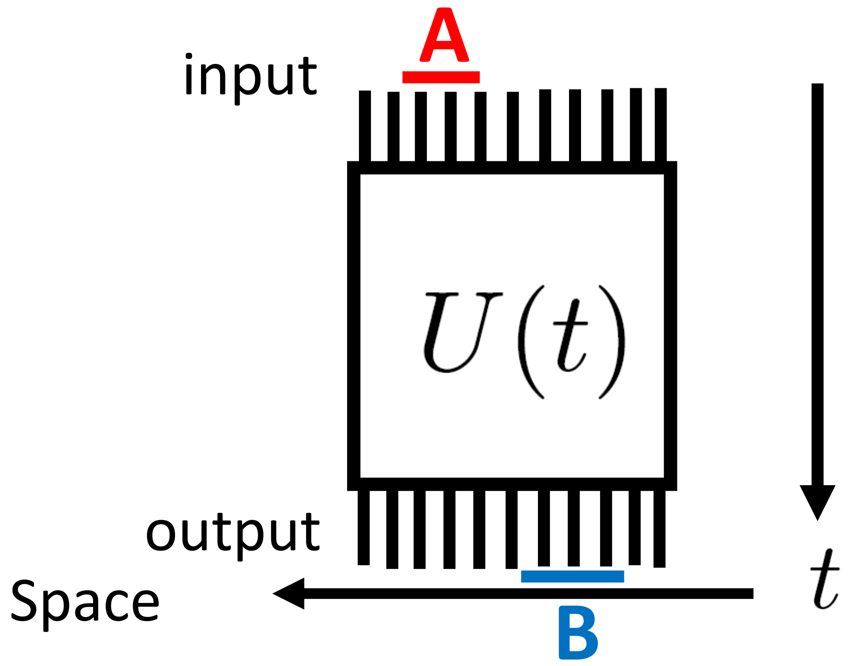


Information Scrambling.

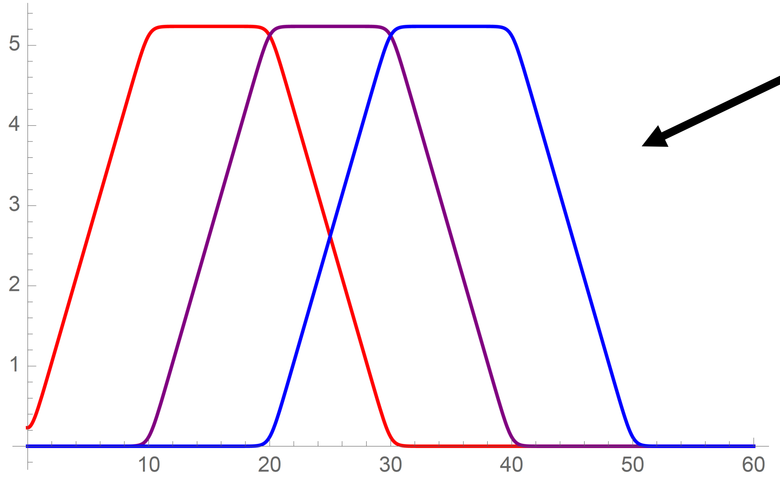
$\eta = 6$:Purple

$\eta = \pi$:Red $\eta = \frac{1}{6}$:Black dash

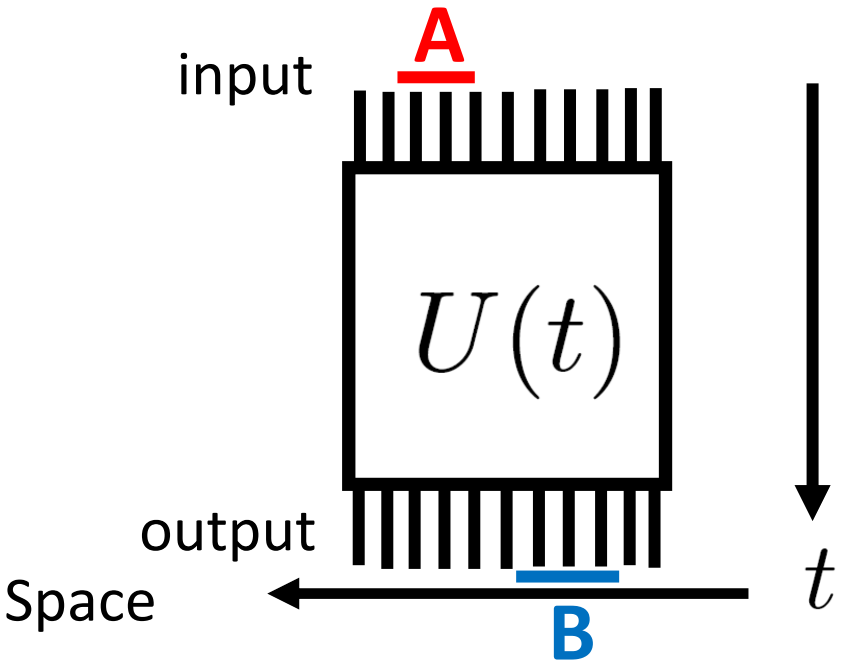
Free fermion channel



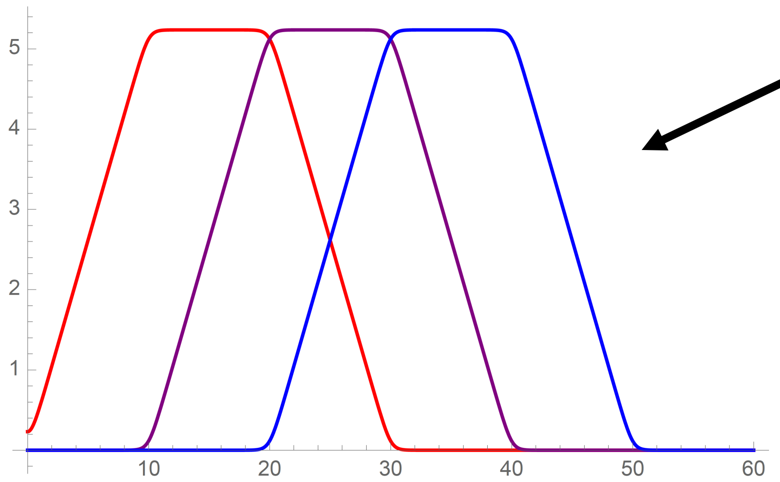
Bumps



Free fermion channel



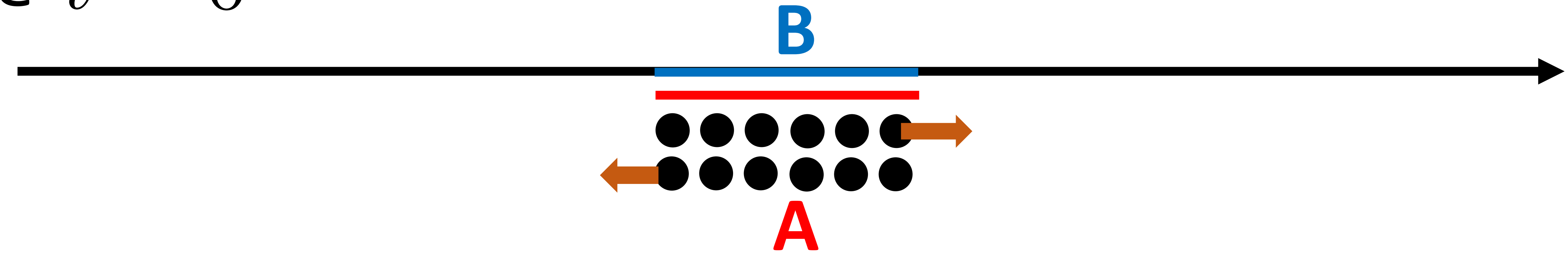
Bumps



Can be interpreted in terms of particles

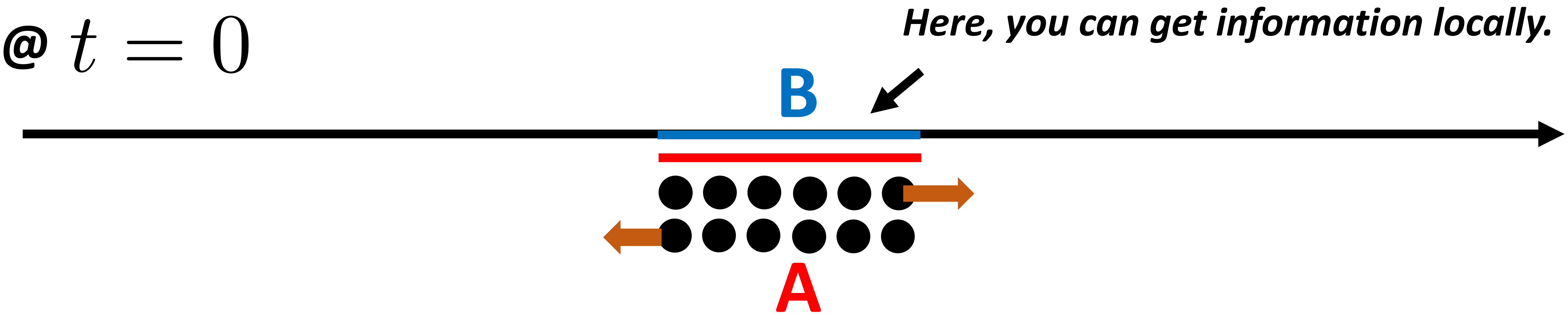
Free fermion channel

@ $t = 0$



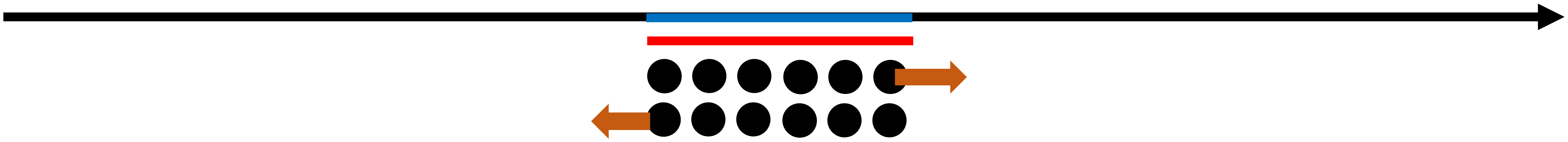
Free fermion channel

$@\ t = 0$



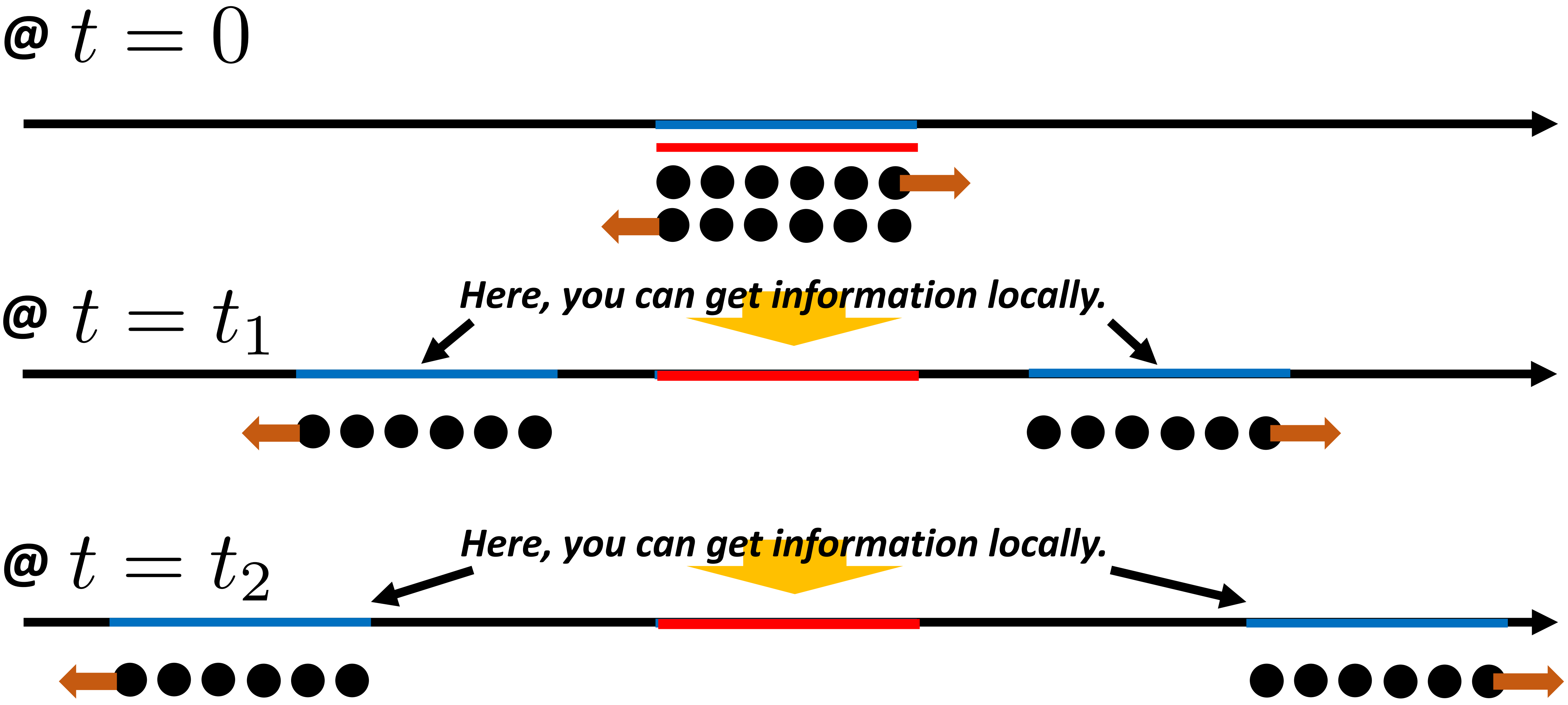
Free fermion channel

@ $t = 0$

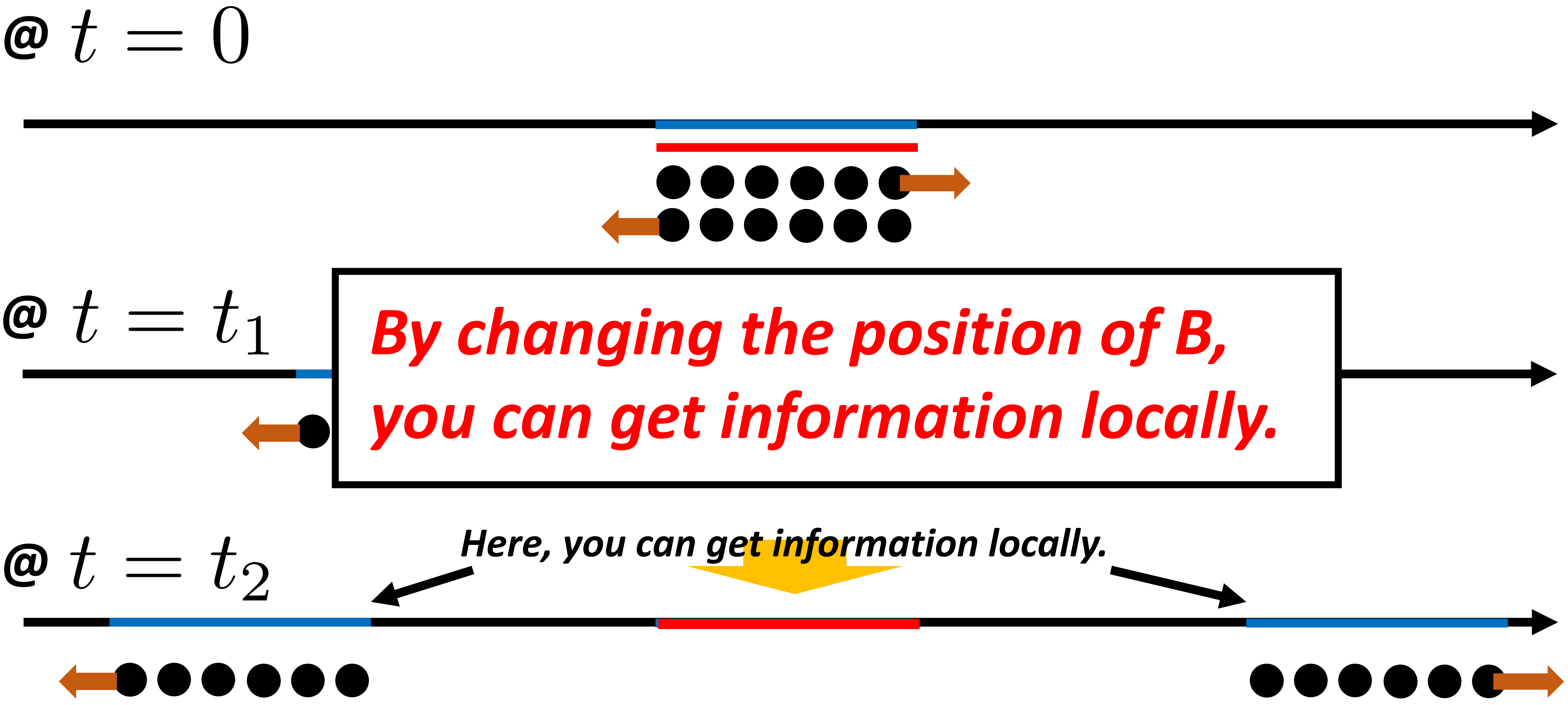


@ $t = t_1$

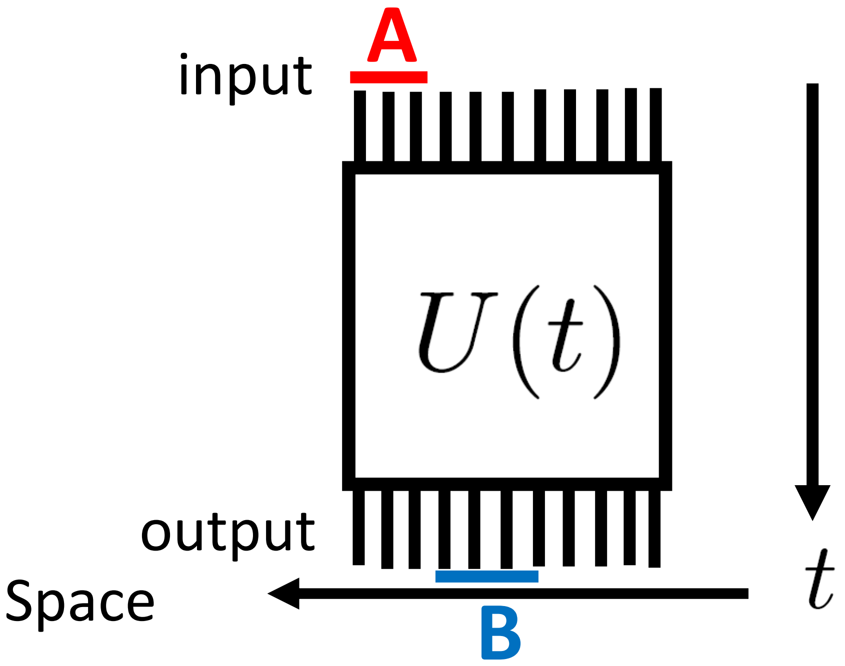
Free fermion channel



Free fermion channel



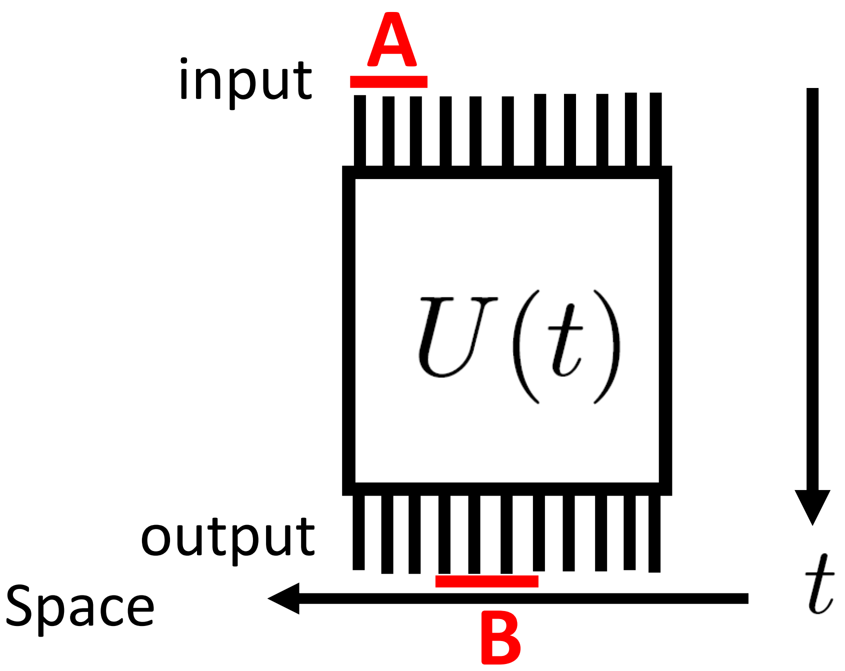
Holographic channel



For ***disjoint*** or ***late-time*** case, ***for any B***

$$I(A, B) = 0$$

Holographic channel



Beyond the quasi-particle model

For ***disjoint*** or ***late-time*** case, ***for any B***

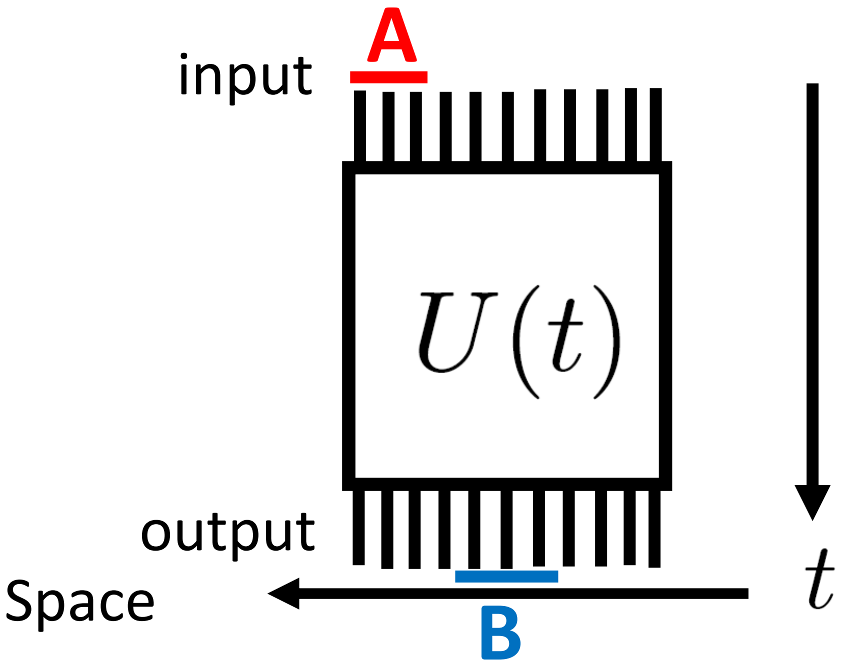
$$I(A, B) = 0$$

Holographic channel

Everywhere , you <u>can't</u> get information locally at late time .

@ $t = t_1$

Holographic channel



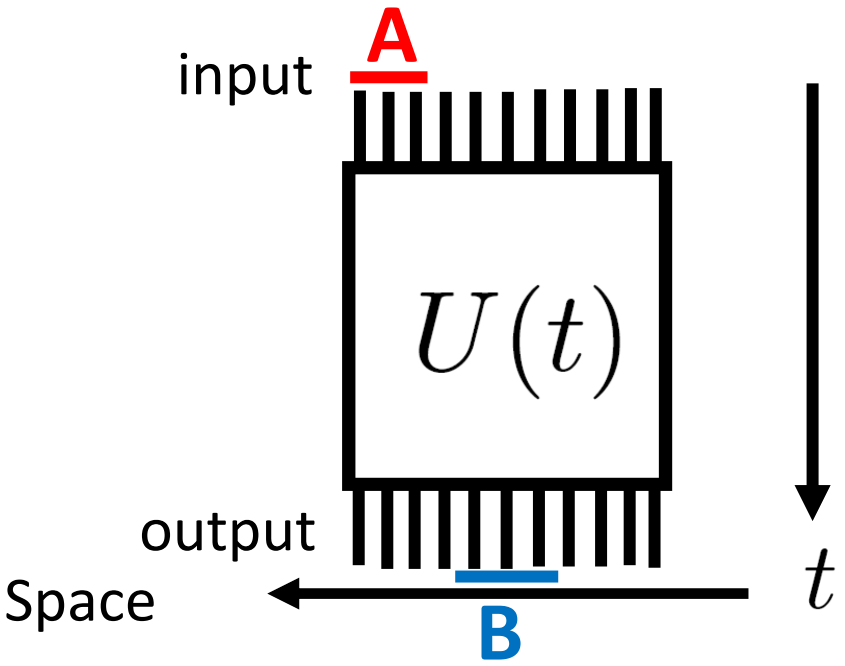
We cannot mine the information in A from B locally.

Beyond the quasi-particle model

For ***disjoint*** or ***late-time*** case, ***for any B***

$$I(A,B)=0$$

Holographic channel



Signature of information scrambling

We cannot mine the information in A from B locally.

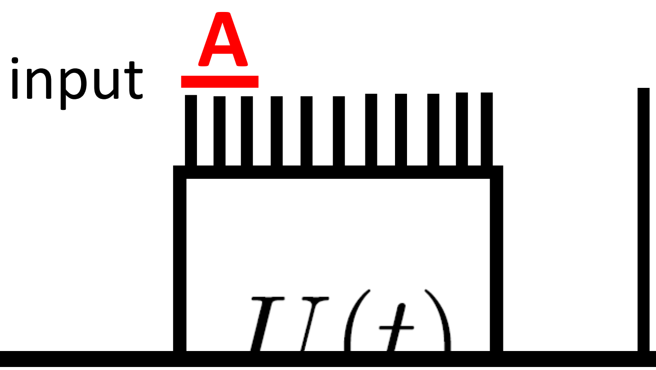
Beyond the quasi-particle model

For ***disjoint*** or ***late-time*** case, ***for any B***

$$I(A,B)=0$$

Holographic channel

Signature of information scrambling



We cannot mine the information in A from B locally.

Can we treat the effect of information scrambling quantitatively?

For ***disjoint*** or ***late-time*** case, ***for any B***

$$I(A, B) = 0$$

A definition of (maximally) Information scrambling

(It might be different from usual one… Sorry.)

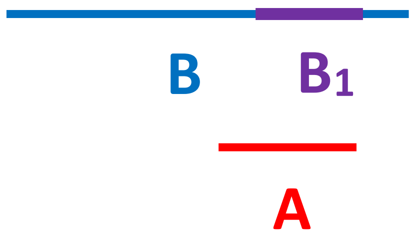
We cannot mine any information about A locally, but we can mine the information from the whole of output system.

A definition of (maximally) Information scrambling

We cannot mine any information about A locally, but we can mine the information from the whole of output system.

$$I(A, B_1) = 0$$

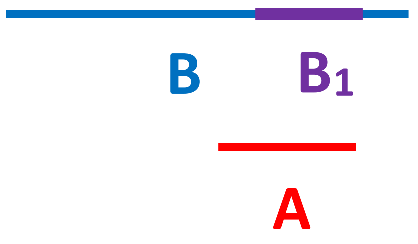
$$\mathcal{E}(A, B_1) = 0$$



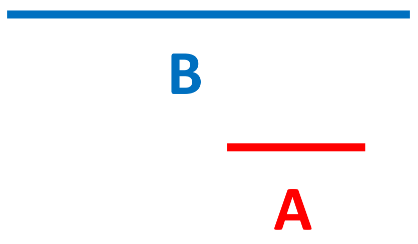
A definition of (maximally) Information scrambling

We cannot mine any information about A locally, but we can mine the information from the whole of output system.

$$I(A, B_1) = 0$$
$$\mathcal{E}(A, B_1) = 0$$



$$I(A, B) \neq 0$$
$$\mathcal{E}(A, B) \neq 0$$



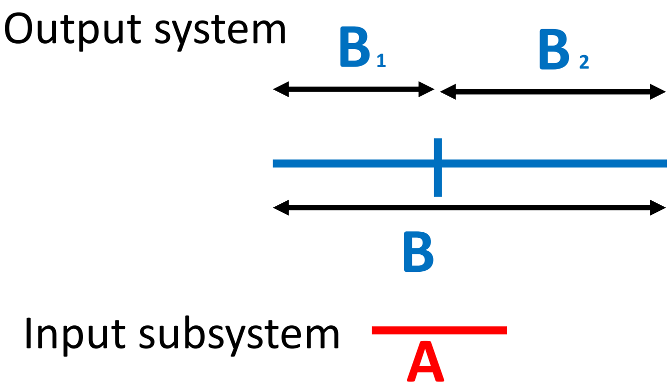
A definition of (maximally) Information scrambling

We cannot mine any information about A locally, but we can mine the information from the whole of output system.

Tripartite information (Tripartite logarithmic negativity) is useful quantity in order to treat this phenomenon, quantitatively. [Hosur-Qi-Roberts-Yoshida'16]

Tripartite operator mutual information

$$I(A, B_1, B_2) = I(A, B_1) + I(A, B_2) - I(A, B)$$



B is the whole of output system.

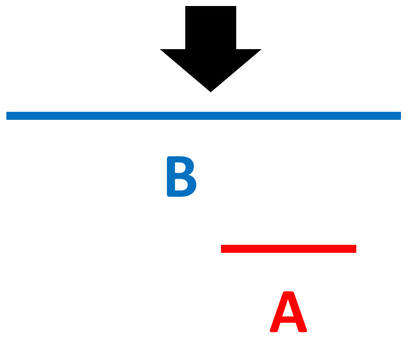
B_1 and B_2 are the halves of output system.

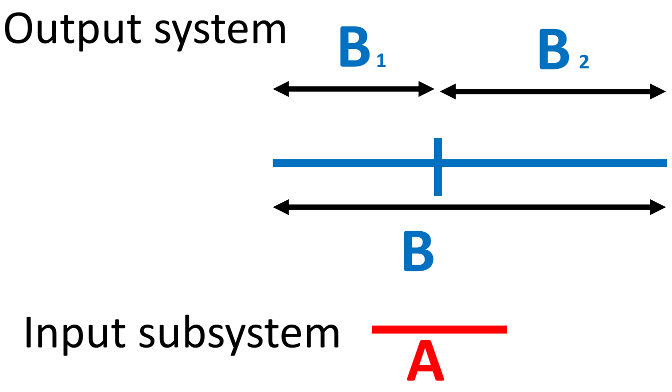
A is a subsystem in input system.

Tripartite operator mutual information

$$I(A, B_1, B_2) = I(A, B_1) + I(A, B_2) - \underline{I(A, B)}$$

the information of A from the whole of output system B.





B is the whole of output system.

B_1 and B_2 are the halves of output system.

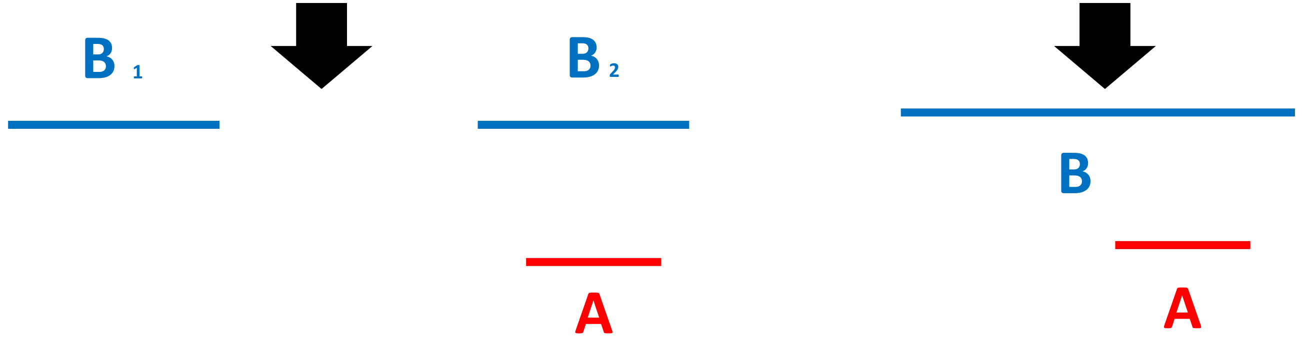
A is a subsystem in input system.

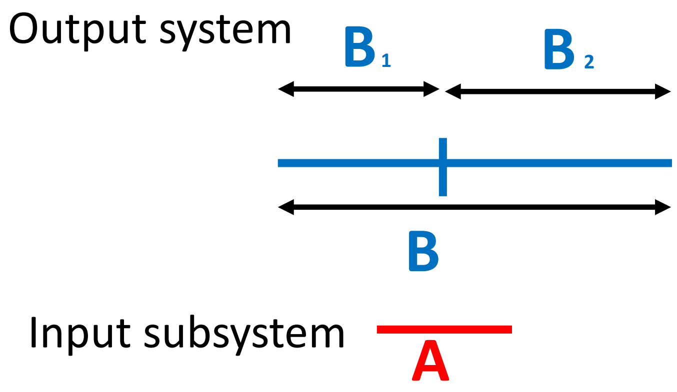
Tripartite operator mutual information

$$I(A,B_1,B_2) = \underline{I(A,B_1)} + \underline{I(A,B_2)} - \underline{I(A,B)}$$

the information of A from the subsystems.

the information of A from the whole of output system B.





B is the whole of output system.

$\mathbf{B_1}$ and $\mathbf{B_2}$ are the halves of output system.

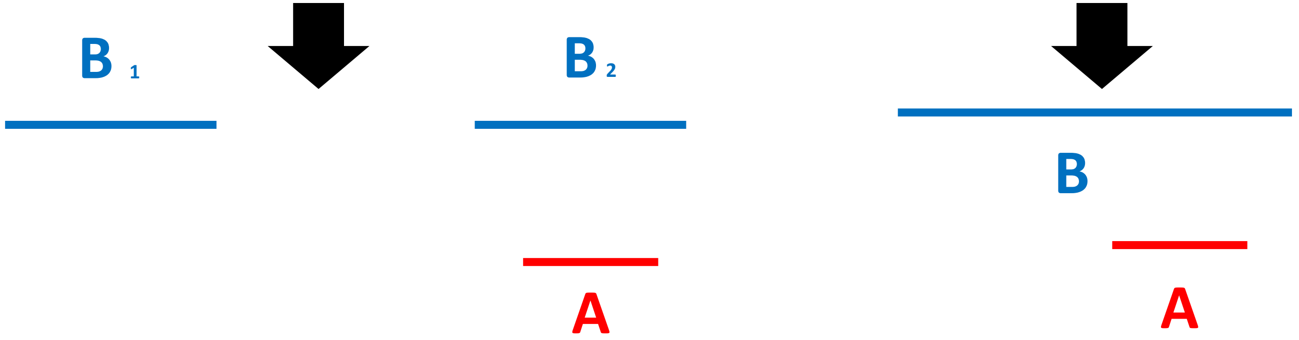
A is a subsystem in input system.

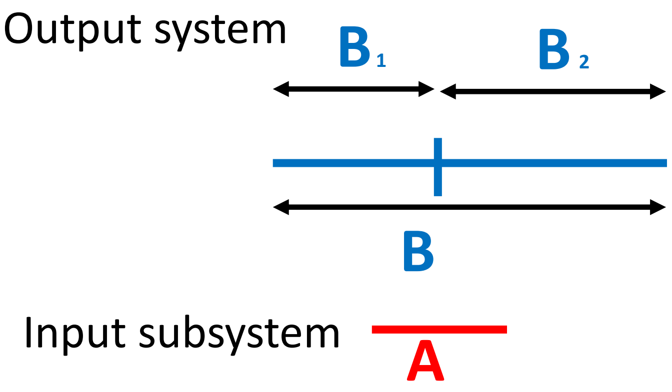
Tripartite operator mutual information

$$I(A,B_1,B_2) = \underline{I(A,B_1)} + \underline{I(A,B_2)} - \underline{I(A,B)}$$

the information of A from the subsystems.

the information of A from the whole of output system B.





B is the whole of output system.

B_1 and B_2 are the halves of output system.

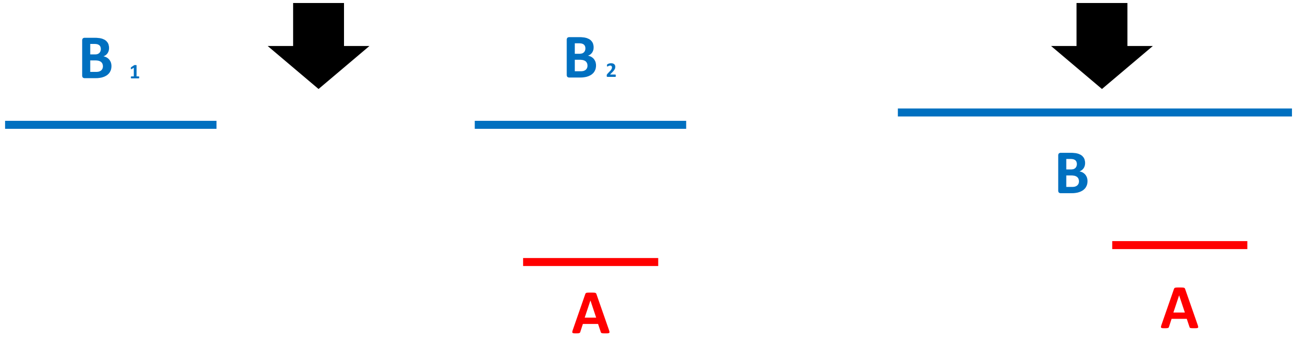
A is a subsystem in input system.

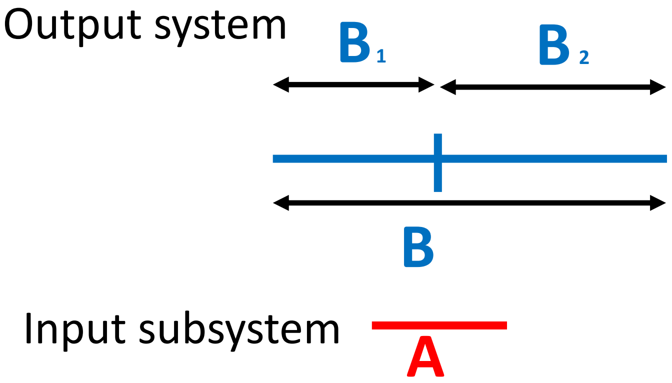
Tripartite operator logarithmic negativity

$$\mathcal{E}_3(A, B_1, B_2) = \underline{\mathcal{E}(A, B_1)} + \underline{\mathcal{E}(A, B_2)} - \underline{\mathcal{E}(A, B)}$$

the information of A from the subsystems.

the information of A from the whole of output system B.





B is the whole of output system.

B 1 and **B** 2 are the halves of output system.

A is a subsystem in input system.

Tripartite operator mutual information (logarithmic negativity)

If information mined from subsystems B_1 ***and*** B_2 ***is smaller than the information from whole of output system*** B***,***

Tripartite operator mutual information (logarithmic negativity)

If information mined from subsystems $B_1$and B_2 is smaller than the information from whole of output system B,

$$I(A,B_1)+I(A,B_2)$$

$$\mathcal{E}(A,B_1)+\mathcal{E}(A,B_2)$$

Tripartite operator mutual information (logarithmic negativity)

If information mined from subsystems $B_1$and B_2 is smaller than the information from whole of output system B,

$$I(A,B_1)+I(A,B_2)-\ I(A,B)<0$$

$$\mathcal{E}(A,B_1)+\mathcal{E}(A,B_2)-\ \mathcal{E}(A,B)<0$$

Tripartite operator mutual information (logarithmic negativity)

If information mined from subsystems $B_1$and B_2 is smaller than the information from whole of output system B,

$$I(A,B_1)+I(A,B_2)-\ I(A,B)<0$$

$$\mathcal{E}(A,B_1)+\mathcal{E}(A,B_2)-\ \mathcal{E}(A,B)<0$$

$$\boxed{I(A,B_1,B_2)<0\ ,\mathcal{E}_3(A,B_1,B_2)<0}$$

Tripartite operator mutual information (logarithmic negativity)

If information mined from subsystems B_1 and B_2 is smaller than the information from whole of output system B,

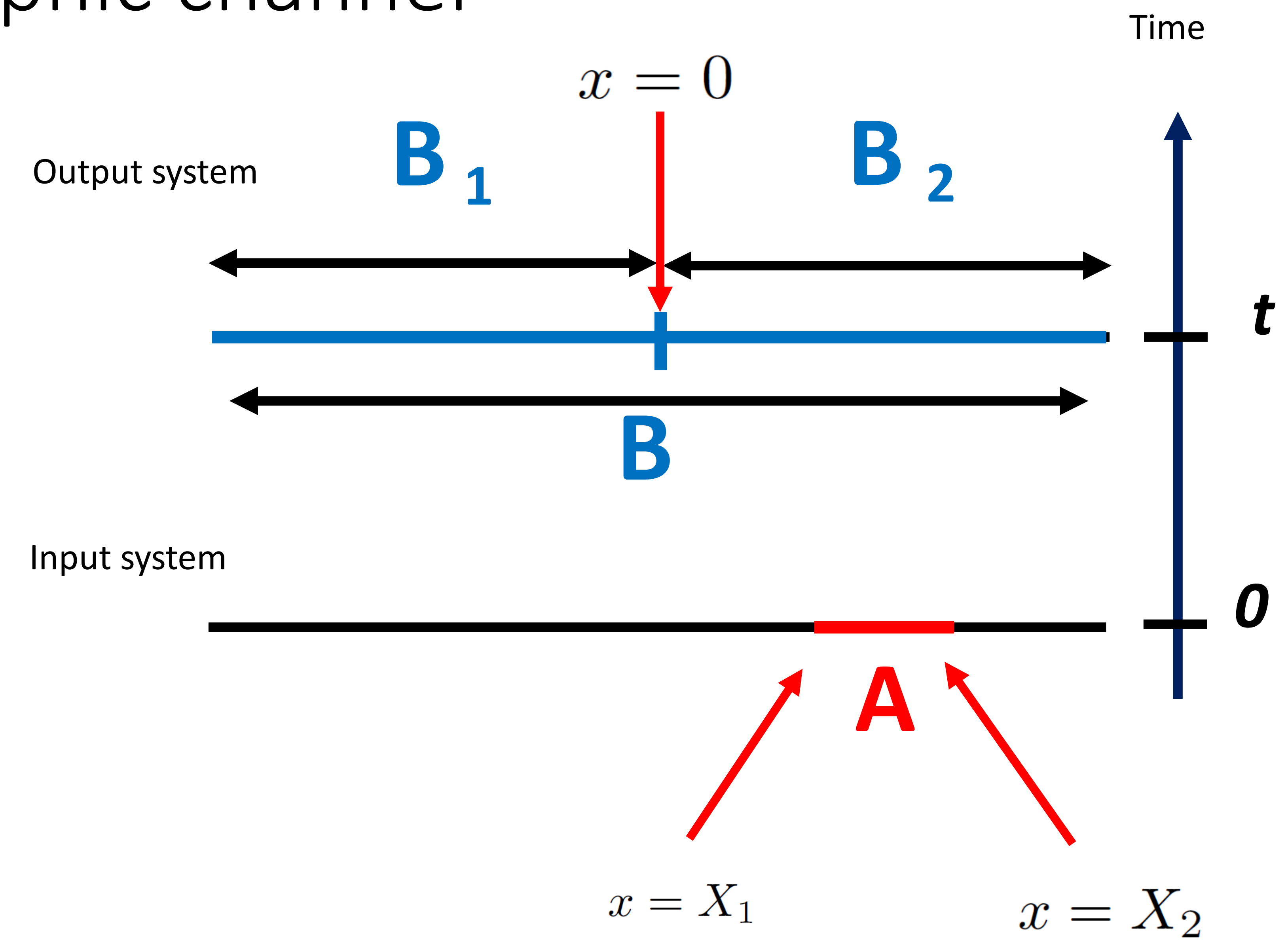
$$\boxed{I(A, B_1, B_2) < 0 \ , \mathcal{E}_3(A, B_1, B_2) < 0}$$

Some information is hidden in whole of output system due to information scrambling effect.

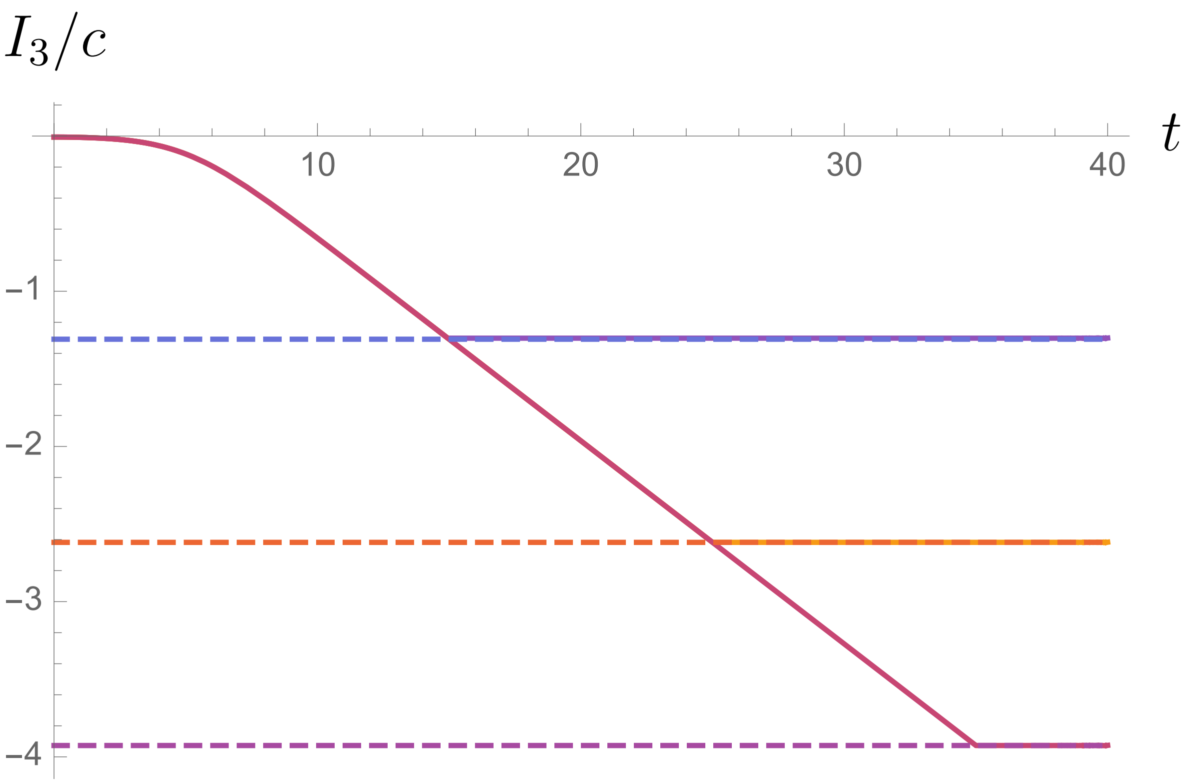
This quantity can quantify the effect of information scrambling.

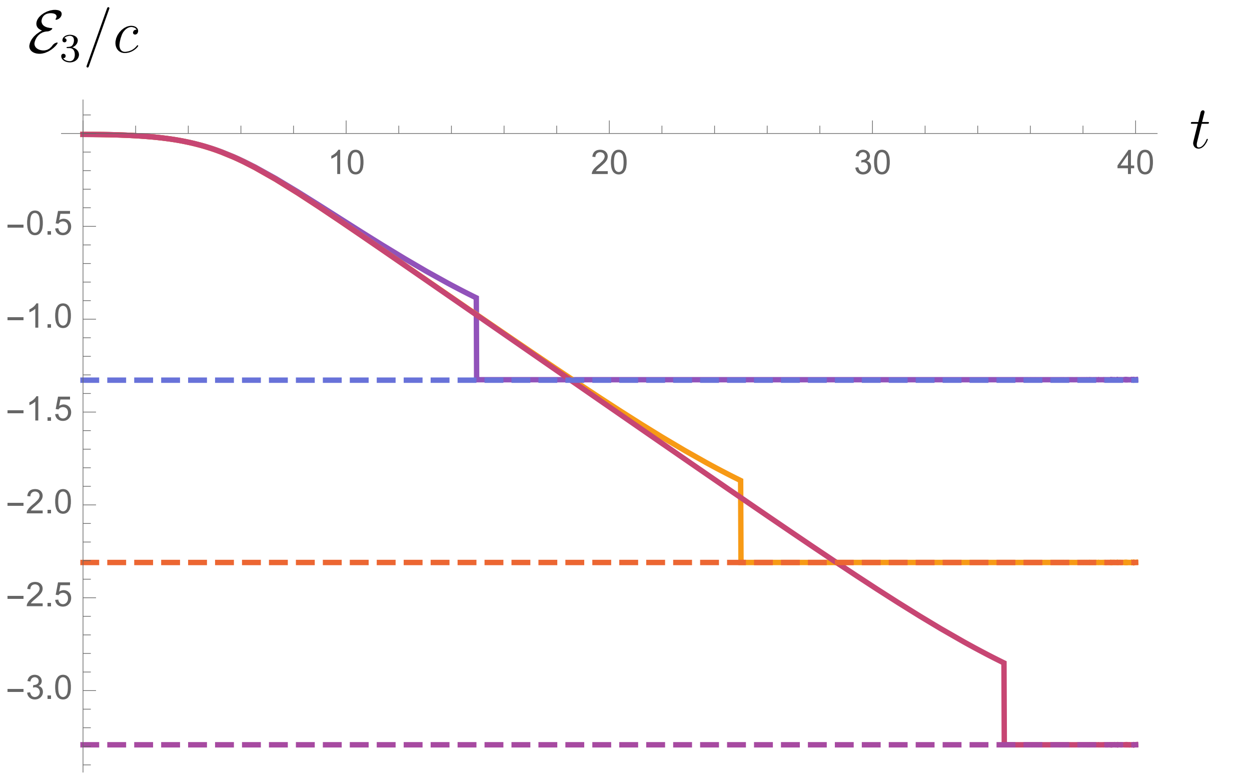
Holographic channel

Setup:



Holographic channel





@ late time,

$$\underline{I(A,B_1,B_2) \to -2S_A} \quad \underline{\mathcal{E}_3(A,B_1,B_2) \to -2\mathcal{E}(A,\bar{A})}$$

Holographic channel

At late time,

$$I(A,B_1,B_2) = I(A,B_1) + I(A,B_2) - I(A,B)$$

$$\mathcal{E}_3(A,B_1,B_2) = \mathcal{E}(A,B_1) + \mathcal{E}(A,B_2) - \mathcal{E}(A,B)$$

0 0 ***Constant***

We cannot mine information locally, but we can mine the information about A from the whole of output system B.

Holographic channel

At late time,

$$I(A,B_1,B_2)=\underset{\overset{\downarrow}{0}}{I(A,B_1)}+\underset{\overset{\downarrow}{0}}{I(A,B_2)}-\underset{\overset{\downarrow}{\text{Constant}}}{I(A,B)}$$

$$\mathcal{E}_3(A,B_1,B_2)=\mathcal{E}(A,B_1)+\mathcal{E}(A,B_2)-\mathcal{E}(A,B)$$

We cannot mine information locally, but we can mine the information about A from the whole of output system B.

Holographic channel

At late time,

$$I(A, B_1, B_2) = \underset{\substack{\downarrow \\ 0}}{I(A, B_1)} + \underset{\substack{\downarrow \\ 0}}{I(A, B_2)} - \underset{\substack{\downarrow \\ \text{Constant}}}{I(A, B)}$$

$$\mathcal{E}_3(A, B_1, B_2) = \overset{\substack{0 \\ \uparrow}}{\mathcal{E}(A, B_1)} + \overset{\substack{0 \\ \uparrow}}{\mathcal{E}(A, B_2)} - \overset{\substack{\text{Constant} \\ \uparrow}}{\mathcal{E}(A, B)}$$

We cannot mine information locally, but we can mine the information about A from the whole of output system B.

Holographic channel

At late time,

$$I(A,B_1,B_2) = \underbrace{I(A,B_1)}_{0} + \underbrace{I(A,B_2)}_{0} - \underbrace{I(A,B)}_{\textit{Constant}}$$

$$\mathcal{E}_3(A,B_1,B_2) = \overbrace{\mathcal{E}(A,B_1)}^{0} + \overbrace{\mathcal{E}(A,B_2)}^{0} - \overbrace{\mathcal{E}(A,B)}^{\textit{Constant}}$$

We cannot mine information locally, but we can mine the information about A from the whole of output system B.

→ All information sent from A is scrambled.

Holographic channel

At late time,

$$I(A,B_1,B_2) = \underset{0}{\underbrace{I(A,B_1)}} + \underset{0}{\underbrace{I(A,B_2)}} - \underset{\textit{Constant}}{\underbrace{I(A,B)}}$$

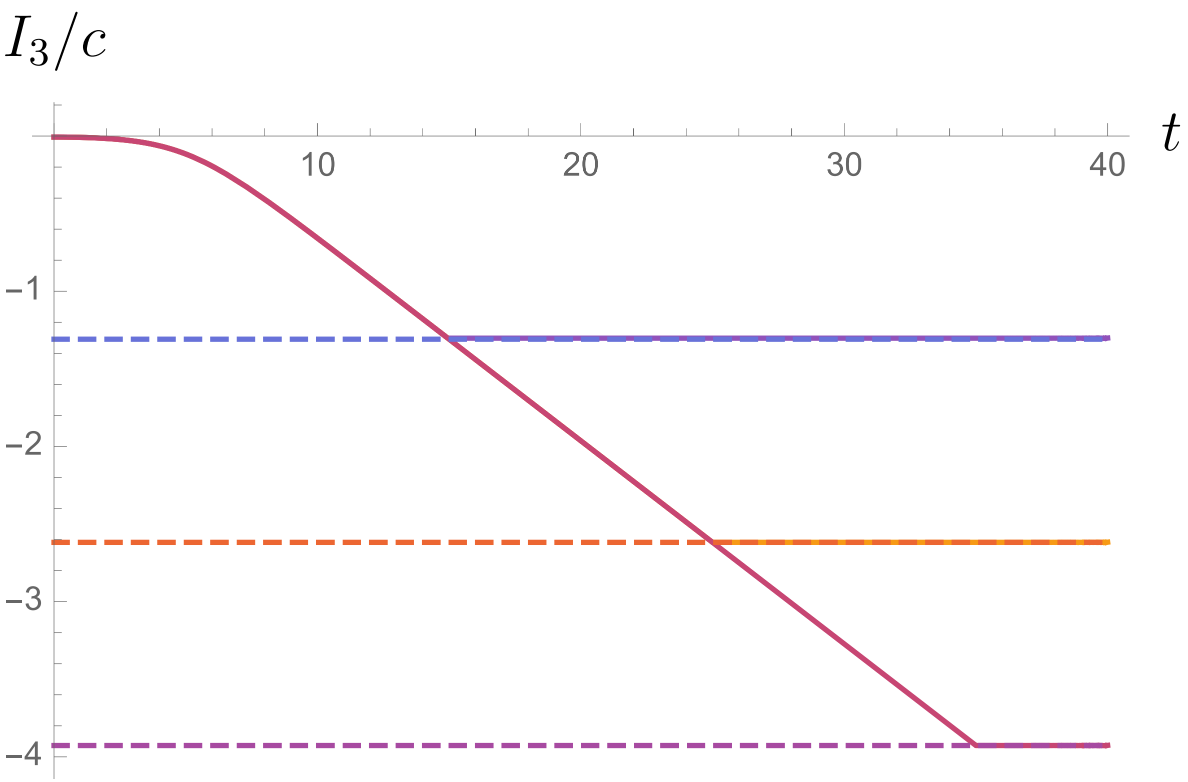
$$\mathcal{E}_3(A,B_1,B_2) = \mathcal{E}(A,B_1) + \mathcal{E}(A,B_2) - \mathcal{E}(A,B)$$

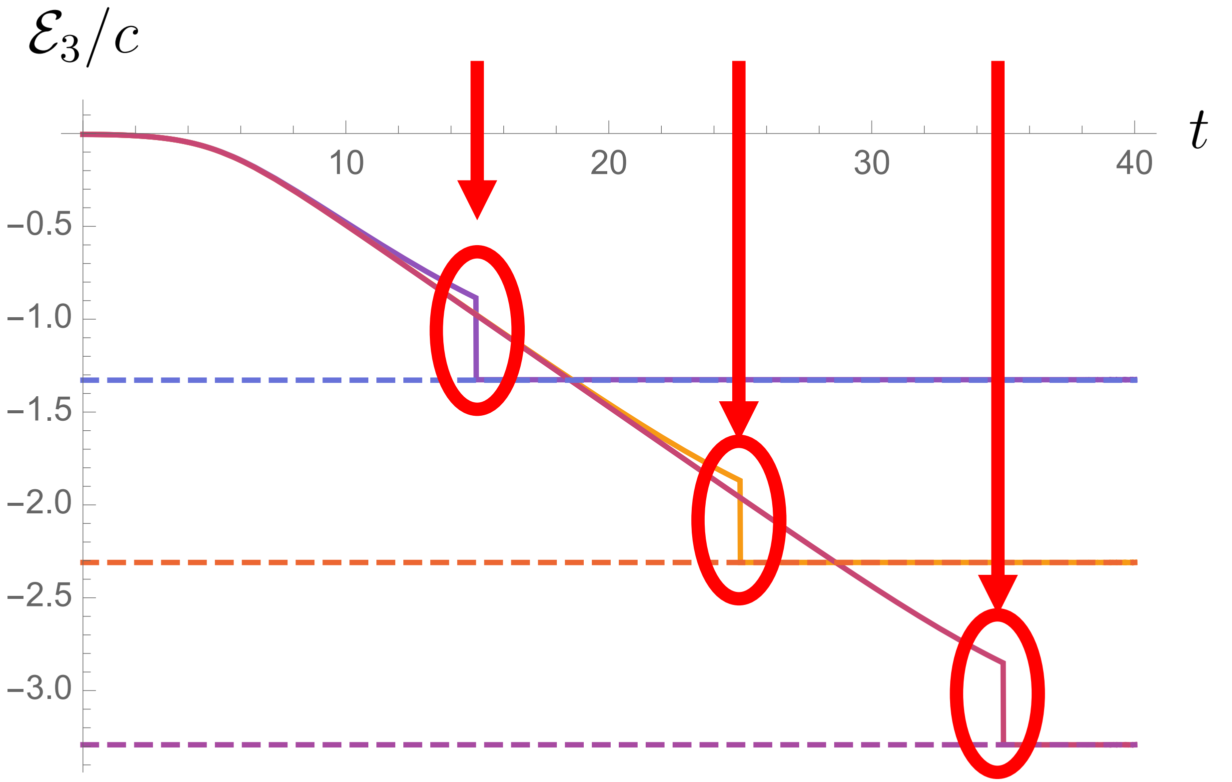
We cannot mine information locally, but we can mine the information about A from the whole of output system B.

They measure how much information is scrambled.

Holographic channel

In the low energy limit,
these kinks can be negligible.





@ late time,

$$I(A, B_1, B_2) \to -2S_A \quad \mathcal{E}_3(A, B_1, B_2) \to -2\mathcal{E}(A, \bar{A})$$

Summary

Bipartite operator mutual information (logarithmic negativity)

Free fermion and Compact boson channels:

Here, you can get information locally.

Holographic channel:

Everywhere , you can't get information locally.

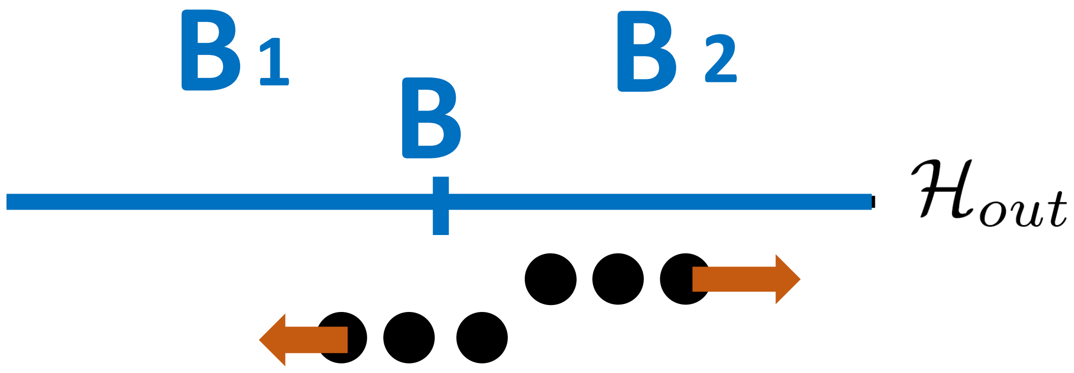
Summary

Tripartite operator mutual information (Tripartite operator logarithmic negativity)

Free fermion channels:

$$I(A, B_1, B_2) = 0$$

$$\mathcal{E}_3(A, B_1, B_2) = 0$$

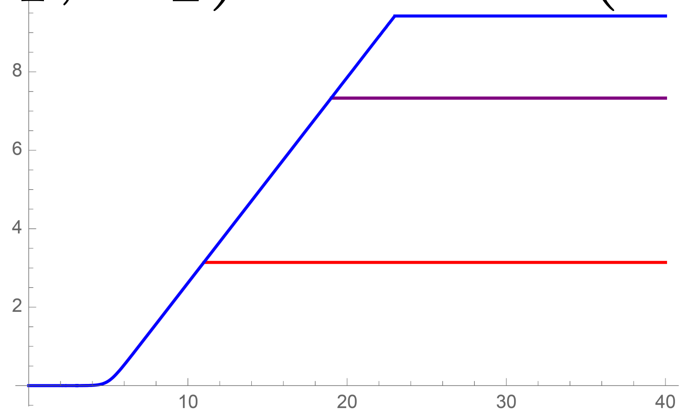


Quasi-particles

Holographic channel:

$$I(A, B_1, B_2) \to -2S_A$$

$$\mathcal{E}_3(A, B_1, B_2) \to -2\mathcal{E}(A, \bar{A})$$



All initial information is scrambled.

Future directions

1. Operator entanglement of local operator

2. Complexity

3. Operator entanglement of CMERA

4. Many-body localization

5. Quantum Chaos and thermalization

6. Wormhole (double trace deformation)