Dressed states from gauge invariance

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Based on the work with **Hayato Hirai** (Osaka U.)

- arXiv:1901.09935 (to be published in JHEP)

@YITP, June 5, 2019

Introduction

- QFTs generally have UV divergences.

- Some massless theories such as electromagnetism and gravity also suffer from IR divergences.

Dressed state formalism

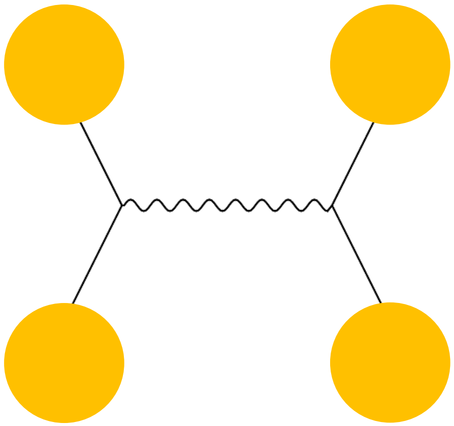
◆ The dressed state formalism in QED

- was developed to define the S-matrix without IR divergences.
  [Chung (1965), Kibble (1968), Kulish & Faddeev (1970)]

- has been reconsidered recently in connection with asymptotic symmetry. [Mirbabayi & Porrati (2016), Gabai & Sever (2016), Kapec, Perry, Raclariu & Strominger (2017), Carney, Chaurette, Neuenfeld & Semenoff (2018),…]

◆ Asymptotic charged particles are dressed.

$$b^\dagger(\vec{p})\,|0\rangle \;\Rightarrow\; e^R b^\dagger(\vec{p})\,|0\rangle$$



Our work

[Hirai & SS (2019)]

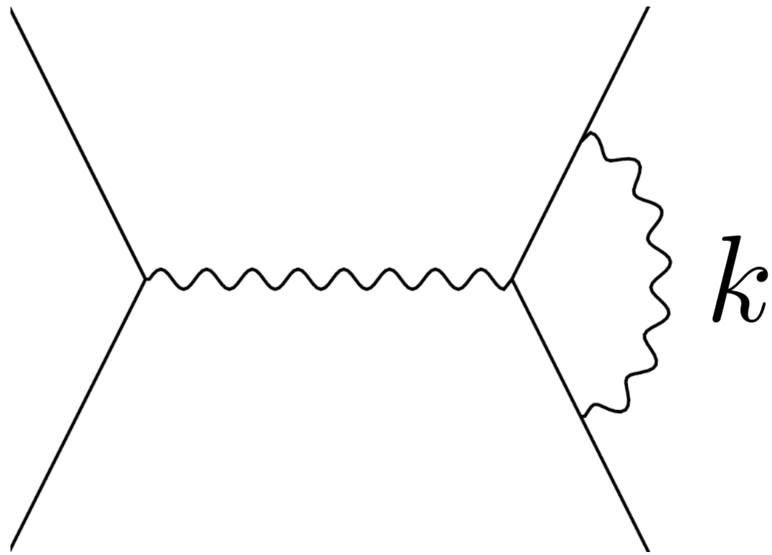
- We show that the dressing is required from the gauge invariance (Gauss's law constraint).

- We give an interpretation of the dressing operator.
  - It corresponds to the Lienard-Wiechert potential.
  - It carries a "hard charge" of asymptotic symmetry.

Outline

1. IR divs in QED and dressed state formalism (Review)
2. Gauge invariance for dressed states
3. Interpretation of the dressing operator
4. Discussions

IR divergences in QED



The loop integral diverges because of the soft virtual photon.

$$k \sim 0$$

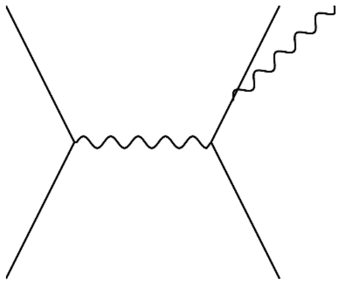
S-matrix elements have IR divergences at loop levels.

Conventional way to avoid IR problem

Stop worrying about the S-matrix. [Bloch & Nordsieck (1937)]

In a realistic setup, any detector has a threshold E_T .

The detector can't see soft photons $\omega < E_T$.



The sum of the cross section is physically observable, and IR finite.

$$d\sigma(X \to X') + d\sigma(X \to X' + \gamma) + \cdots$$

The inclusive formalism

Another way

[Chung (1965)]

No S-matrix formulation?

Chung thought that IR divs are caused by inappropriate choices of the initial and the final states, and considered dressed states:

$$||\vec{p}\rangle\rangle = e^{R(\vec{p})}\,|\vec{p}\rangle\,, \qquad R(\vec{p}) \sim \int \frac{d^3k}{(2\pi)^3(2\omega)}\,\frac{e\,p^\mu}{p\cdot k}\left(a_\mu(\vec{k}) - a^\dagger_\mu(\vec{k})\right)$$

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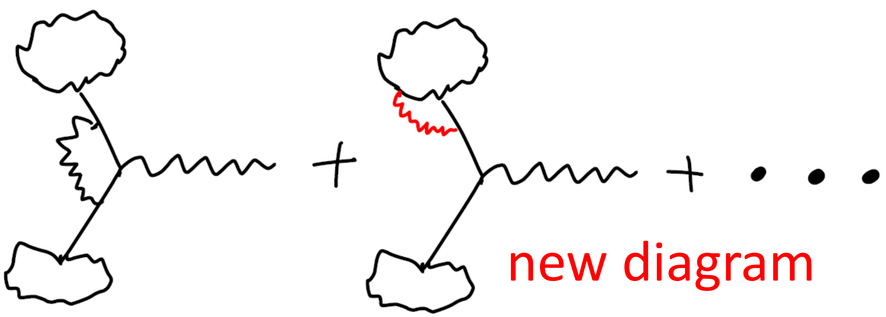
Each charge is surrounded by a cloud of photons.



Chung proved that the S-matrix elements btw dressed states are IR finite.

$$\langle\langle\vec{p}'||S||\vec{p}\rangle\rangle$$

S-matrix

The dressed states can be derived from the asymptotic dynamics. [Kulish & Faddeev (1970)]

Let's recall the usual interaction picture.

$$H_{QED} = H_0 + V, \quad V = -\int d^3x \, j^\mu A_\mu$$

S-matrix

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Scattering state $|\alpha\rangle_{in}$ behaves as a **free** particle at $t \to -\infty$:

$$U(t)\int d\alpha g(\alpha)\,|\alpha\rangle_{in} \xrightarrow[t \to -\infty]{} U_0(t)\int d\alpha g(\alpha)\,|\alpha\rangle_0$$

wave packet

$$\Longleftrightarrow \qquad |\alpha\rangle_{in} = \lim_{t\to-\infty} U^{-1}(t)U_0(t)\,|\alpha\rangle_0$$

S-matrix element $S_{\beta,\alpha} = {}_{out}\langle\beta|\alpha\rangle_{in} = {}_0\langle\beta|\, S_0\,|\alpha\rangle_0\,,$

$$S_0 = \mathrm{T}\exp\left(-i\int_{-\infty}^{+\infty} dt V^I(t)\right)$$

Asymptotic interaction 1

Faddeev and Kulish proposed that the asymptotic Hamiltonian is given by $H_{as} = H_0 + V_{as}$ rather than H_0 .

$$V_{as}(t) = -\int d^3x\, j^\mu_{cl}(t,\vec{x}) A_\mu(\vec{x})$$

classical trajectory

$$j^\mu_{cl}(t,\vec{x}) = e\int \frac{d^3p}{(2\pi)^3(2E_p)} \frac{p^\mu}{E_p}\, \delta^3(\vec{x}-\vec{p}t/E_p)\rho(\vec{p}),$$

$$\rho(\vec{p}) = b^\dagger(\vec{p})b(\vec{p}) - d^\dagger(\vec{p})d(\vec{p})$$

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commutes with H_0.

In their formalism, scattering states are given by

$$|\alpha\rangle_{in} = \lim_{t\to-\infty} U^{-1}(t)\underline{U_{as}(t)}\,|\alpha\rangle_0$$

Not the free time-evolution U_0

$$U_{as} = \mathrm{T}\exp\left(-i\int_0^t dt' H_{as}(t')\right)$$

Asymptotic interaction 2

$$|\alpha\rangle_{in} = \lim_{t\to-\infty} U^{-1}(t)\underline{U_0(t)}\,|\alpha\rangle_0 \quad \Rightarrow \quad |\alpha\rangle_{in} = \lim_{t\to-\infty} U^{-1}(t)\underline{U_{as}(t)}\,|\alpha\rangle_0$$

S-matrix element changes as

$$S_{\beta,\alpha} = {}_{out}\langle\beta|\alpha\rangle_{in} = {}_0\langle\beta|\,S_0\,|\alpha\rangle_0 \quad \Rightarrow \quad {}_0\langle\beta|\,e^{R^\dagger_{out}}S_0 e^{R_{in}}\,|\alpha\rangle_0$$

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$$S_{\beta,\alpha} = \;_{out}\langle\beta|\alpha\rangle_{in} = \;_0\langle\beta|\,S_0\,|\alpha\rangle_0 \Rightarrow \;_0\langle\beta|\,e^{R^\dagger_{out}}S_0 e^{R_{in}}\,|\alpha\rangle_0$$

Dressing operator

$$R_{in} = \lim_{t\to-\infty}\int\frac{d^3p}{(2\pi)^3(2E_p)}\rho(\vec{p})\underline{\int\frac{d^3k}{(2\pi)^3(2\omega)}\frac{ep^\mu}{p\cdot k}\left[a_\mu(\vec{k})e^{i\frac{p\cdot k}{E_p}t} - a^\dagger_\mu(\vec{k})e^{-i\frac{p\cdot k}{E_p}t}\right]}$$

$+$ phase op.

Asymptotic states are dressed states

$$||\vec{p}\rangle\rangle = e^{R_{in}}\,|\vec{p}\rangle_0 \sim e^{R(\vec{p})}\,|\vec{p}\rangle_0$$

$$\boxed{S_{\beta,\alpha} = \langle\langle\beta||S_0||\alpha\rangle\rangle}$$

Physical condition?

QED is a gauge theory.
A physical cond should be imposed on the dressed states.

Faddeev and Kulish required the Gupta-Bleuler condition:

$$\boxed{k^\mu a_\mu(\vec{k})||\alpha\rangle\rangle = 0}$$

Physical condition?

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Faddeev and Kulish required the Gupta-Bleuler condition:

$$\boxed{k^\mu a_\mu(\vec{k})||\alpha\rangle\rangle = 0}$$

However, they concluded that their dressing op cannot accord with this condition $\Rightarrow$ $k^\mu a_\mu(\vec{k})||\alpha\rangle\rangle \neq 0,$
and they modified the dressing op:

$$R_{in} \sim e \int \frac{d^3k}{(2\pi)^3(2\omega)} \frac{p^\mu}{p\cdot k} \left[a_\mu(\vec{k}) e^{i\frac{p\cdot k}{E_p}t} - a_\mu^\dagger(\vec{k}) e^{-i\frac{p\cdot k}{E_p}t} \right]$$

$$\Rightarrow R_{in}^{new} \sim e \int \frac{d^3k}{(2\pi)^3(2\omega)} \left(\frac{p^\mu}{p\cdot k} - \underline{c^\mu(\vec{k})} \right) \left[a_\mu(\vec{k}) e^{i\frac{p\cdot k}{E_p}t} - a_\mu^\dagger(\vec{k}) e^{-i\frac{p\cdot k}{E_p}t} \right]$$

$$c^\mu k_\mu = 1$$

Our question

Unlike the original dressing op, the modified one cannot be derived from the asymptotic dynamics.

Question: Why do you use the Gupta-Bleuler condition? That is not a gauge invariant condition for the interacting case.

I will present the correct physical condition for asymptotic states. This condition naturally requires dressed states.

(Bonus) No need to introduce the asymptotic Hamiltonian.

$$H_{as} = H_0 + V_{as}$$

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Exact physical condition

BRST condition: $$Q_B \left|\psi\right\rangle = 0$$

Restrict states to the no-ghost sector

$$\boxed{\left[k^\mu a_\mu(\vec{k}) + e^{i\omega t_s}\tilde{j}^{0s}(\vec{k})\right]\left|\psi\right\rangle = 0}$$

Operators in the Schrödinger picture at $t = t_s$

$$A^s_\mu(\vec{x}) = \int \frac{d^3k}{(2\pi)^3(2\omega)}\left[a_\mu(\vec{k})e^{-i\omega t_s + i\vec{k}\cdot\vec{x}} + a^\dagger_\mu(\vec{k})e^{i\omega t_s - i\vec{k}\cdot\vec{x}}\right]$$

$$\tilde{j}^s_\mu(\vec{k}) = \int d^3x\, e^{-i\vec{k}\cdot\vec{x}} j^s_\mu(\vec{x})$$

This is Gauss's law. It is nonperturbative.

Gauss's law in the interaction picture

$$\begin{cases} Q_B^s \left|\alpha\right\rangle_{in} = 0 \\ \left|\alpha\right\rangle_{in} = \lim_{t \to -\infty} U^{-1}(t) U_0(t) || \alpha \rangle\rangle \end{cases} \Rightarrow \lim_{t \to -\infty} Q_B^I(t) || \alpha \rangle\rangle = 0$$

$$\Rightarrow \boxed{\lim_{t \to -\infty} \left[k^\mu a_\mu(\vec{k}) + e^{i\omega t} \tilde{j}^{0I}(t, \vec{k}) \right] || \alpha \rangle\rangle = 0}$$

In the decoupling limit $e \to 0$, it reduces to the usual Gupta-Bleuler condition $k^\mu a_\mu(\vec{k}) || \alpha \rangle\rangle = 0$.

If there is a charge, there are longitudinal photons.
Gauss's law implies charged particles are dressed by photons.

Dressed states from Gauss's law

As Faddeev and Kulsih did, the current can be replaced by the classical one

$$\lim_{t\to\pm\infty} \tilde{j}^{0I}(t,\vec{k}) = \lim_{t\to\pm\infty} j^0_{cl}(t,\vec{k})$$

(saddle pt approximation which is exact in this limit) [Campiglia & Laddha (2015)]

- The physical state condition $\lim_{t\to-\infty} G(t,\vec{k})||\alpha\rangle\rangle = 0$

$$G(t,\vec{k}) = k^\mu a_\mu(\vec{k}) + e^{i\omega t} j^0_{cl}(t,\vec{k})$$

- The condition is solved by the original FK dressed states.

$$\boxed{||\alpha\rangle\rangle = \lim_{t\to-\infty} e^{R(t)} |\alpha\rangle_0 \ , \quad |\alpha\rangle_0 \in \mathcal{H}_{GB}}$$

because $G(t,k)e^{R(t)} = e^{R(t)} k^\mu a_\mu(\vec{k})$

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Dress is EM potential

Dressed charged particle state $||p\rangle\rangle = e^{R(t)}\,|p\rangle_0$

The expectation value of EM field

$$\boxed{\langle\langle p\|A^I_\mu(x)\|p\rangle\rangle = A^{LW}_\mu(x;\vec{p})\langle\langle p\|p\rangle\rangle}$$

$A^{LW}_\mu(x;\vec{p})$ is the classical Lienard-Wiechert potential for uniformly moving charge with momentum $\vec{p}$.

In our paper, we also clarified the iε-prescription to obtain the retarded or advanced potential.

Asymptotic symmetry

QED in the Minkowski space has the asymptotic symmetry.
It is given by the large gauge transformations.
The generator in the interaction picture is

$$Q^I_{as}[\epsilon] = -\int d^3x\, \partial_i (F^I_{0i}\epsilon) \quad \text{up to the BRST exact term.}$$

(ϵ is a large gauge parameter $\Box\epsilon = 0$)

ϵ is parametrized by a function on two-sphere. We have an infinite number of charges.

If $\epsilon = 1$, Q_{as} is just a global electric charge.

For the classical system, the conservation of asymptotic charges leads to the electromagnetic memory effect. [Hirai & SS (2018)]

Asymptotic charge of dressing op.

- Commutator with the dressing operator

$$\boxed{[Q^I_{as}[\epsilon], e^R] = e^R \int d^3x\,[F^{0i}_{cl}\partial_i\epsilon + j^0_{cl}\epsilon]}$$

$$F^{cl}_{0i}(x) = \int \frac{d^3p}{(2\pi)^3(2E_p)}\rho(\vec{p})\underline{[\partial_0 A^{LW}_i(x;\vec{p}) - \partial_i A^{LW}_0(x;\vec{p})]}$$

Classical electric field for a pt charge

- Dressed creation op. $e^R b^\dagger(\vec{p})$ carries a hard charge

$$\boxed{Q^H_{as} = \int d^3x\,[F^{0i}_{cl}\partial_i\epsilon + j^0_{cl}\epsilon]}$$

- contribution from a charged particle + its Coulomb field

  not from radiation

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Other dressings

The original Faddeev-Kulish dressing factor carries only the hard part of asymptotic charges.

The conservation law holds only when we include the soft part.

$$Q^H_{as} + Q^S_{as}$$

We need other dressing operators to realize the eigenstates of the full asymptotic charges.

Unlike the derivation by Faddeev and Kulish, the dressing op is not uniquely fixed by the gauge invariant condition.
It’s possible to get different types of dressed states. More investigation is needed to classify the dressing op, and asymptotic sym is helpful.

Summary

- IR finite S-matrix can be computed in the dressed state formalism.
- We revisited the gauge invariant condition of dressed states.
- The correct physical condition requires dressed states.
- The original Faddeev-Kulish dressing corresponds to the Lienard-Wiechert potential, and carries a hard part of asymptotic charges.

Future works

- Other dressing ops. The relation to IR divergences.
- Prediction different from the inclusive formalism
- Extension to gravitational theory [Ware, Saotome & Akhoury (2013)]