Majorana dimers and holographic quantum error-correcting codes

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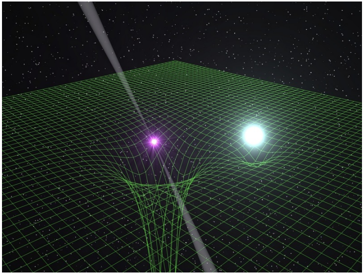
AdS/CFT correspondence

The general idea:

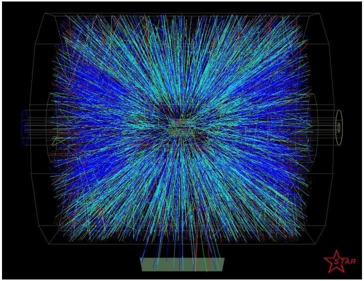
Gravity in $d+1$ dimensions,
weakly coupled



(Antoniadis et al., Science **340** (2013))

$\simeq$

QFT in d dimensions,
strongly coupled



(STAR detector image, Brookhaven RHIC)

The more specific idea:

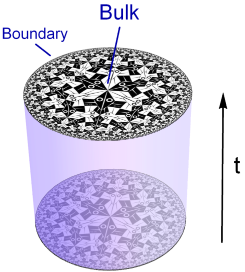
J. M. Maldacena, "The Large N limit of superconformal field theories and supergravity,"
Adv. Theor. Math. Phys. **2** (1998) 231.

E. Witten, "Anti-de Sitter space and holography,"
Adv. Theor. Math. Phys. **2** (1998) 253.

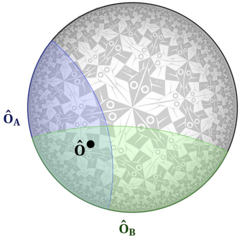
AdS/CFT correspondence

Key features:

- *Einstein gravity* on $\text{AdS}_{d+1} \leftrightarrow$ *conformal field theory* (CFT_d)
- AdS_{d+1} boundary $=$ CFT_d spacetime
- Bulk dynamics $\equiv$ boundary dynamics ($Z_{\text{AdS}} = Z_{\text{CFT}}$)
- Features *quantum error-correction* of bulk information



(full spacetime)



(timeslice)

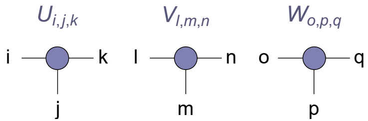
Tensor networks

Quantum state on N physical sites as a tensor T:

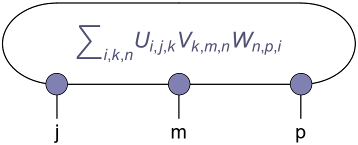
$$|\psi\rangle = \sum_{j_1,\ldots,j_N=0,1} T_{j_1,\ldots,j_N} \, |j_1,\ldots,j_N\rangle$$

Simple ansatz for T: **Contraction** over a product of tensors

Building blocks:



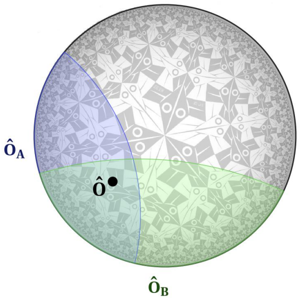
$\Rightarrow T_{j,m,p} =$



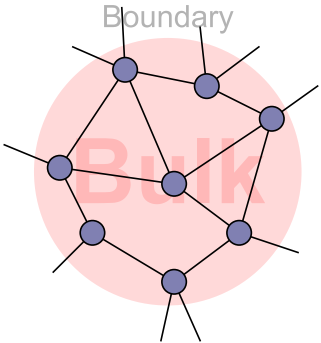
Network structure of contracted tensor indices
$\equiv$ **Entanglement structure** of the quantum state

Tensor networks + AdS/CFT

Tensor network holography:
Mapping between **bulk** tensor content and **boundary** state



$\equiv$

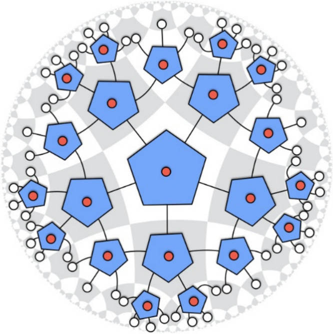


bulk geometry $\equiv$ tensor network structure

boundary regions $\equiv$ open tensor indices

The hyperbolic pentagon code (HyPeC)

Tensor network for **holographic quantum error correction***:



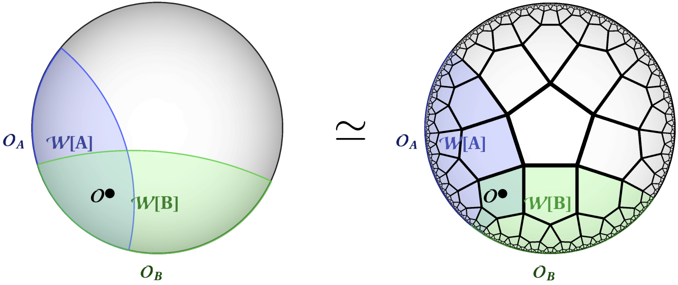
Hyperbolic pentagon tiling of *perfect tensors* corresponding to encoding isometry of $[[5, 1, 3]]$ quantum error-correcting code.

* F. Pastawski, B. Yoshida, D. Harlow and J. Preskill, JHEP **1506**, 149 (2015).

The hyperbolic pentagon code (HyPeC)

HyPeC properties:

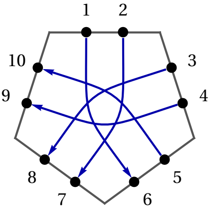
- Each pentagon tile encodes a logical qubit
- Entire network encodes bulk qubits on boundary
- Logical qubit reconstructable from *different* boundary regions



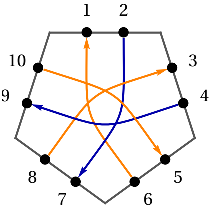
HyPeC $\rightarrow$ Majorana dimers

Pentagon logical state is spanned by basis states $\bar{0}$ and $\bar{1}$. These states are characterized by *Majorana dimers**:

$|\bar{0}\rangle_5 =$



$|\bar{1}\rangle_5 =$



Each **arrow** between Majorana modes $j \rightarrow k$ defines an **operator** $\gamma_j + \mathrm{i}\, \gamma_k$ that annihilates the total state.

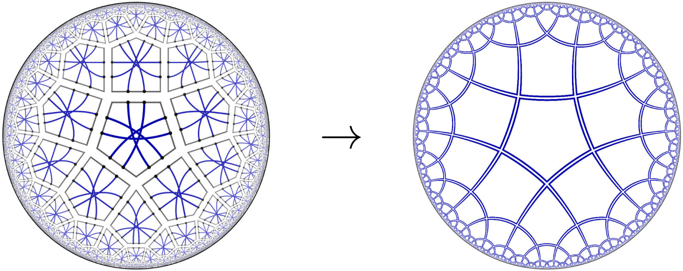
What happens during **tensor contraction** of dimer states?

* B. Ware et al., Phys. Rev. B **94**, 115127 (2016).

HyPeC → Majorana dimers

Dimer state contraction $\equiv$ “Fusing” of dimers along edges!

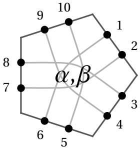
Applied to basis-state HyPeC: **Geodesic** structure of dimers



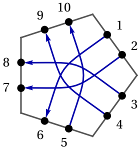
On boundary: Average **polynomial** $1/d$ decay of correlations!

HyPeC $\rightarrow$ Majorana dimers

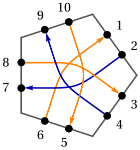
General HyPeC: Local **superpositions** of $\bar{0}$ and $\bar{1}$ inputs:



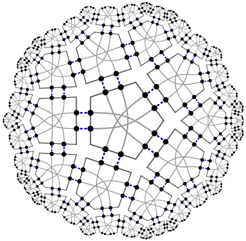
$=\ \alpha$



$+\ \beta$



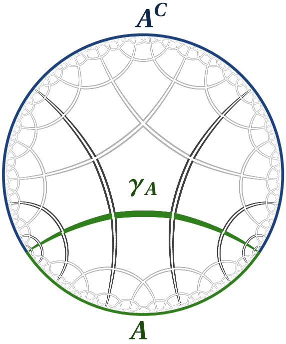
$$\Rightarrow\ |\psi\rangle\ =$$



Contraction of n tiles $\equiv$ sum of 2^n Majorana dimer states $|\psi_k\rangle$.
But: $\langle\psi_j|\,\gamma_a\,\gamma_b\,|\psi_k\rangle \propto \delta_{j,k}$. 2-point functions become easy!

HyPeC → Majorana dimers

Entanglement between regions mediated by dimers:



Realizes holographic *Ryu-Takayanagi*[1] formula

$$S_A = \frac{|\gamma_A|}{4G_N}$$

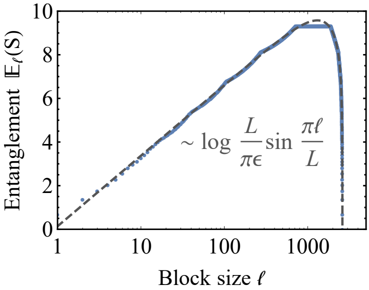
with the *minimal bulk geodesic* γ_A.

Resemblance to the holographic *bit thread*[2] proposal!

[1] S. Ryu and T. Takayanagi, Phys. Rev. Lett. **96**, 181602 (2006).
[2] M. Freedman and M. Headrick, Commun. Math. Phys. **352** (2017) no.1, 407.

HyPeC $\to$ Majorana dimers

Entanglement entropy scaling of the HyPeC:



(with a boundary of $L = 2605$ sites)

$\Rightarrow$ CFT-like logarithmic scaling with central charge $c \approx 4.2$

Quasiregular symmetries suggest an underlying *aperiodic* system!

Summary and outlook

Our work

- Diagrammatic notation of Majorana dimers + contractions
- Application to the hyperbolic pentagon code; computation of two-point correlators, entanglement entropies
- Explicit bulk/boundary mapping of Majorana modes

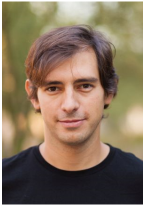
Future directions

- Other models of dimer-based tensor networks
- Entanglement scaling for disjoint regions
- Generalization to other holographic models
- Connection to translation-invariant CFTs (MERA)

Thank you for your attention!

Acknowledgements:

Jens Eisert



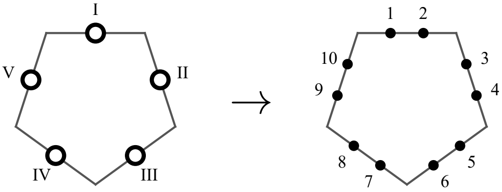
Fernando Pastawski

Marek Gluza

Paper available on arXiv:1905.03268

From spins to Majorana modes

Mapping chain of N spins to $2N$ Majorana modes:



Jordan-Wigner transformation from spin to fermionic operators:

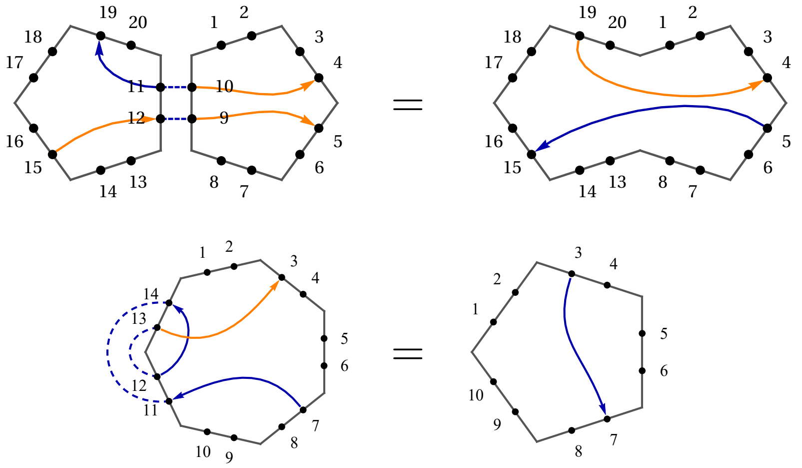
$$\gamma_{2k-1} = (\sigma^z)^{\otimes(k-1)} \otimes \sigma^x \otimes \mathbb{1}^{\otimes(N-k)},$$
$$\gamma_{2k} = (\sigma^z)^{\otimes(k-1)} \otimes \sigma^y \otimes \mathbb{1}^{\otimes(N-k)},$$

In terms of standard fermionic operators f_k and $\mathrm{f}_k^\dagger$:

$$\mathrm{f}_k = (\gamma_{2k-1} + \mathrm{i}\,\gamma_{2k})/2 \;, \quad \mathrm{f}_k^\dagger = (\gamma_{2k-1} - \mathrm{i}\,\gamma_{2k})/2 \;.$$

Contraction rules for Majorana dimers

Using a lot of Grassmann algebra, we found that Majorana dimer states are closed under contraction. Examples:

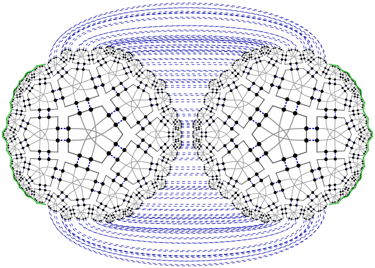


Dimers “fuse” under contraction!

Greedy algorithm in dimer language

Reduced density matrix for HyPeC of local dimer superpositions:

$\rho_A \ = \ 2^{N_C}$



$= \ 2^{N_{C,\mathcal{W}}}$