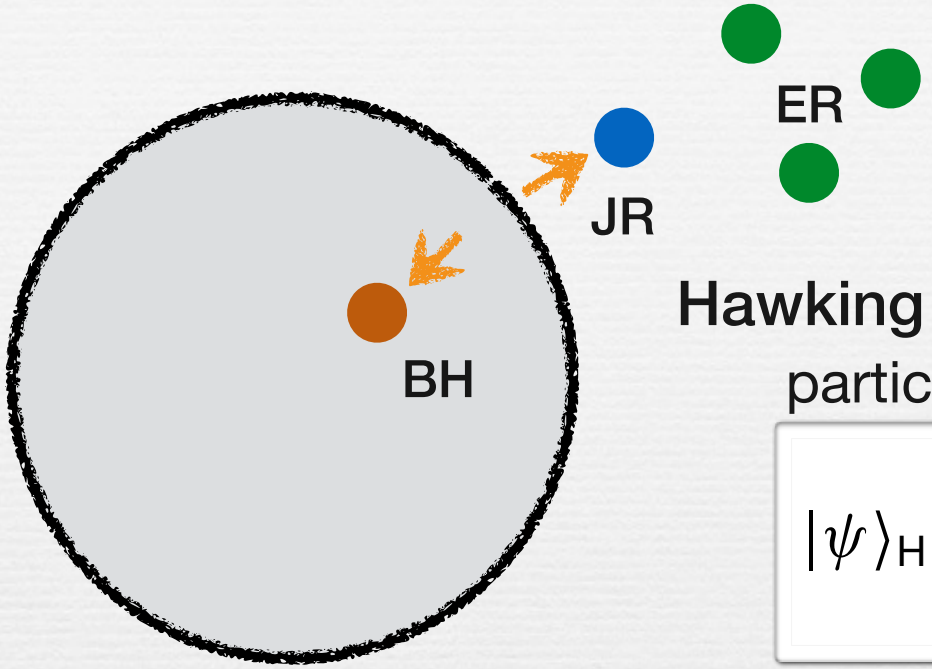
Quantum Circuit Model of Black Hole Evaporation

Yasusada Nambu
Nagoya University, Japan

QIST@YITP, Kyoto
June 12, 2019

T.Tokuzumi, A.Matsumura, Y.Nambu
“Quantum circuit model of black hole evaporation”, CQG 35 (2018) 235013

Black Hole Evaporation and Monogamy



black hole

Early emitted **R**adiation

Just emitted **R**adiation

Hawking radiation

particles are created around BH horizon

$$|\psi\rangle_{\mathrm{H}} = \prod_i \sum_{n_i=0}^{\infty} e^{-4\pi M \omega_i n_i} |n_i\rangle_{\mathrm{BH}} \otimes |n_i\rangle_{\mathrm{JR}}$$

entangled pair of particles

thermal property $T_H = \dfrac{1}{8\pi M}$

- Gravitational energy of BH is extracted by particle creation
- We want to know about
  - Final state of evaporating system ?
  - Possible scenario, remnant ?
  - information issue ?

Page's Theorem

A+B: pure state

A	B

If $N_A \ll N_B$ for a typical state, entanglement entropy

$$\langle S_A \rangle \approx \log N_A \qquad \rho_A \propto I_A$$

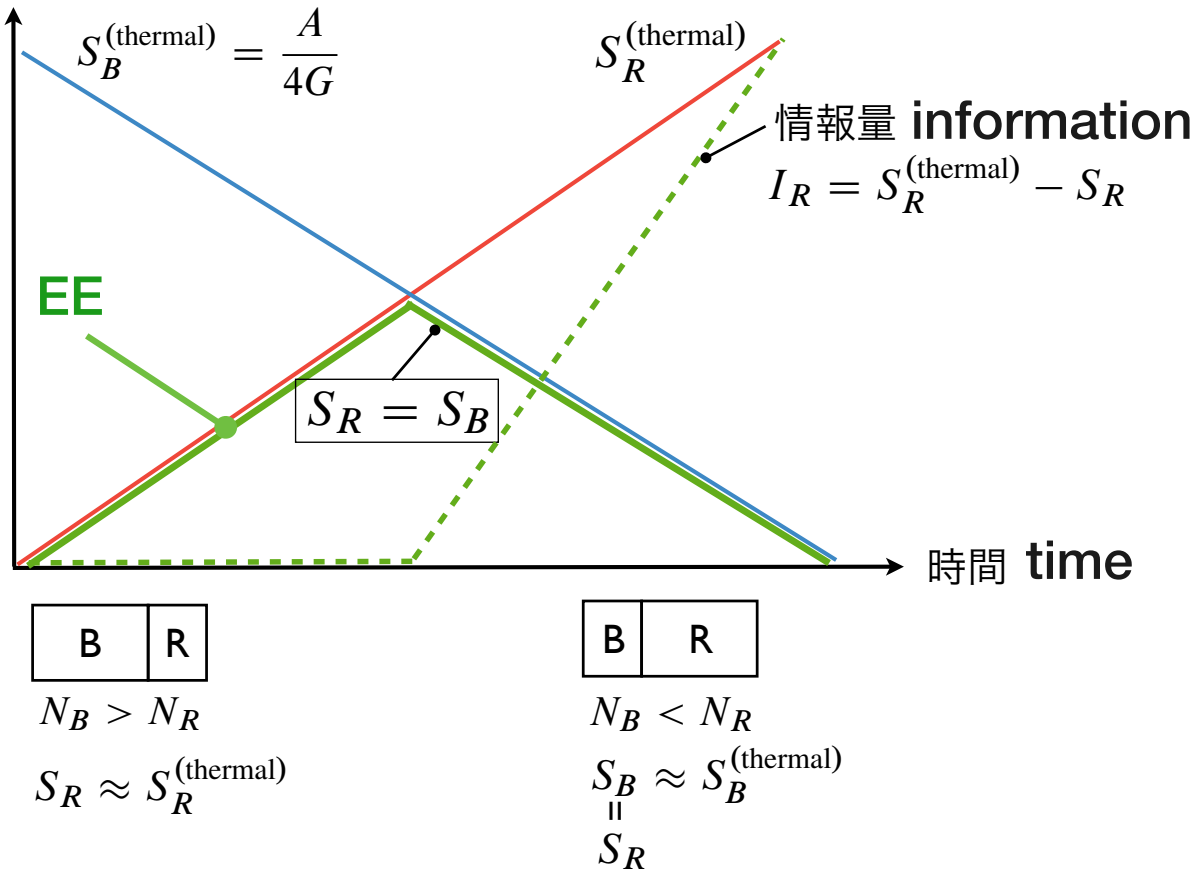
A and B are maximally entangled

Page Curve

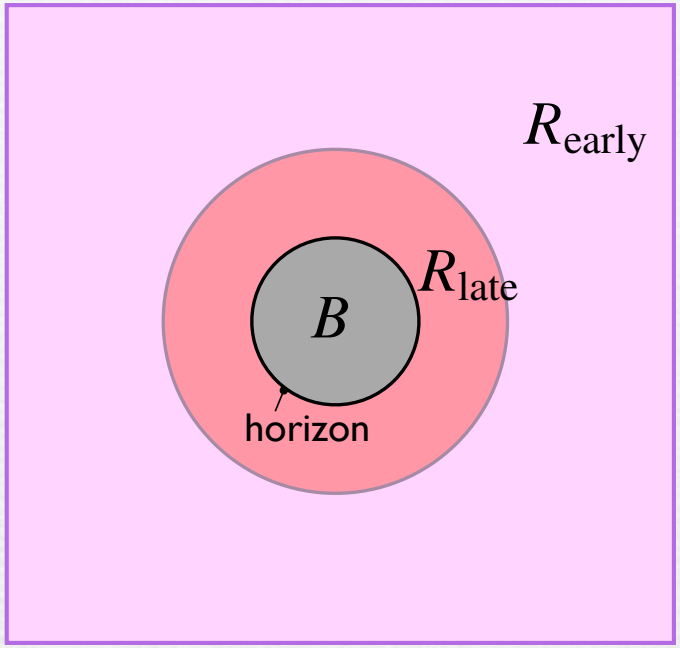
Without detail of models, we can follow evolution of EE during evaporation if we adopt unitarity

Page time

S_{EE}: max



Firewall? AMPS *Almheiri, Marolf, Polchinski and Sully 2013*



After Page time,

$$N_{R_{\text{early}}} \gg N_B N_{R_{\text{late}}}$$

B+R_{late} are maximally entangled with R_{early}

$$\rightarrow \quad \rho_{B+R_{\text{late}}} \propto I_{B+R_{\text{late}}}$$

Correlation of field between B and R_{late} is lost and high energetic curtain emerges around the horizon

- Emergence of field separable (product) state
- This phenomena is related to **entanglement monogamy**

Effect of multipartite entanglement:
entanglement cannot be freely distributed

Entanglement

- state is separable — pure state

$$|A, B\rangle = |A\rangle|B\rangle$$

- state is entangled

$$|A, B\rangle = |a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle + \cdots$$

entangled state:
quantum mechanical non-locality

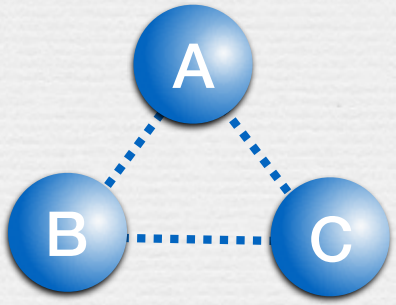
violation of Bell's inequality

2 qubit system

Bell state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

3 qubit system



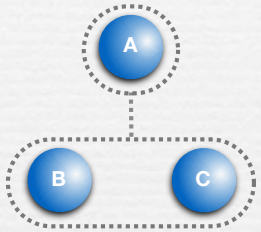
$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |001\rangle)$$

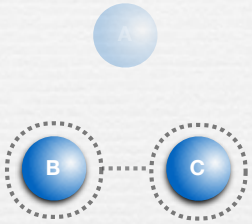
$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|111\rangle + |000\rangle)$$

Multipartite Entanglement

3 qubit system

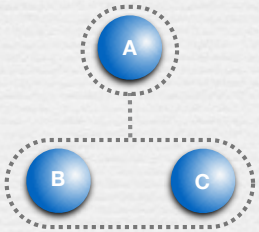
$$|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |001\rangle)$$

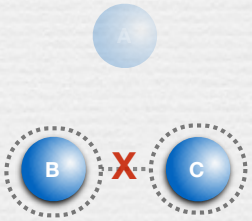




$\mathcal{N}(\mathrm{A}:\mathrm{BC}) = \sqrt{2}/3 \quad \mathcal{N}(\mathrm{B}:\mathrm{C}) = \dfrac{\sqrt{5}-1}{6}$

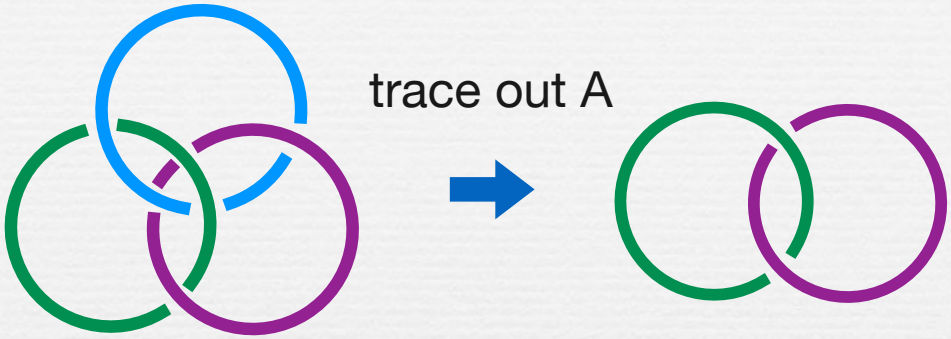
$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|111\rangle + |000\rangle)$$

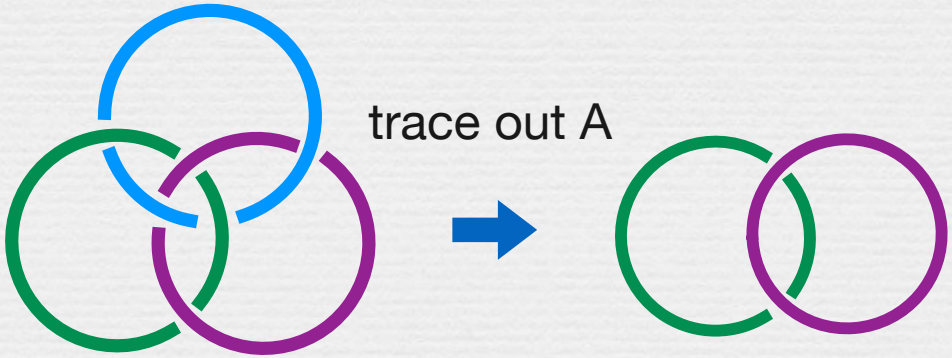




$\mathcal{N}(\mathrm{A}:\mathrm{BC}) = 1/2 \qquad \mathcal{N}(\mathrm{B}:\mathrm{C}) = 0$

analogy of link

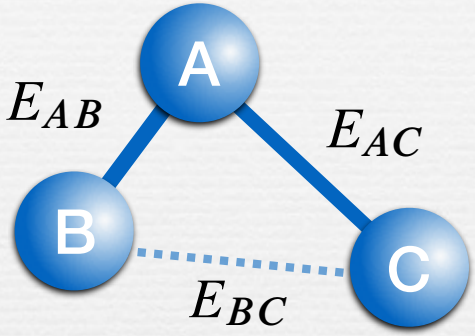




Way of entanglement sharing is restricted depending on structure of states:
entanglement monogamy

Entanglement Monogamy

monogamy relation of entanglement



E :entanglement measure

qubit system
square of concurrence, negativity

Gaussian system
square of negativity

$$E_{\mathrm{A|BC}} \geq E_{\mathrm{AB}} + E_{\mathrm{AC}}$$

V.Coffman, J.Kundu, W.K.Wootters 1998
T.J.Osbone, F.Verstraete 2006
G.Adesso, F.Illuminati 2006

trade off relation between E_{AB} and E_{AC}

- universal relation characterizing multi-partite entanglement
- may provide upper bound of E_{AB} and E_{AC}
- sharing of quantum information, no-cloning theorem

In this talk:

- Emergence of multi-partite entangled state and separable state in Hawking radiation (BH evaporation)
- Does quantum state really evolve such as to develop firewall during evaporation? (related to monogamy of entanglement)
- We investigate this issue using a simple model of evaporation
    - Quantum circuit model of BH evaporation

References

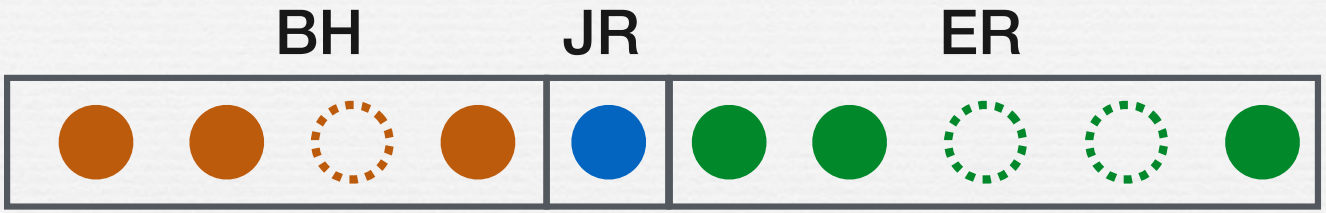
T.Tokuzumi, A.Matsumura, Y.Nambu
“Quantum circuit model of black hole evaporation”, CQG 35 (2018) 235013

Quantum circuit model of BH evaporation

T.Tokuzumi, A. Matsumura & YN, 2018

Rule 1: the system is composed of n qubits

$$\mathcal{H}_{\rm tot} = \mathcal{H}_{\rm BH} \otimes \mathcal{H}_{\rm JR} \otimes \mathcal{H}_{\rm ER}.$$

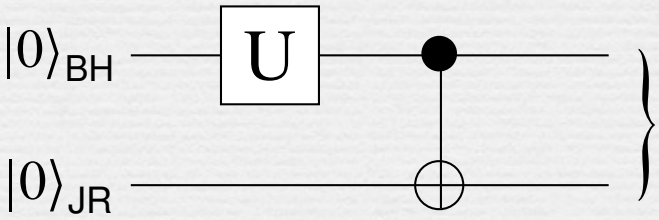


$$|\psi\rangle = |{\rm BH}\rangle \otimes |{\rm JR}\rangle \otimes |{\rm ER}\rangle,$$

$|0\rangle$ no particle

$|1\rangle$ particle

Rule 2: entanglement between Hawking particles is generated by CNOT-U gate



$$\cos\gamma\,|0\rangle_{\rm BH}|0\rangle_{\rm JR} + \sin\gamma\,|1\rangle_{\rm BH}|1\rangle_{\rm JR}$$

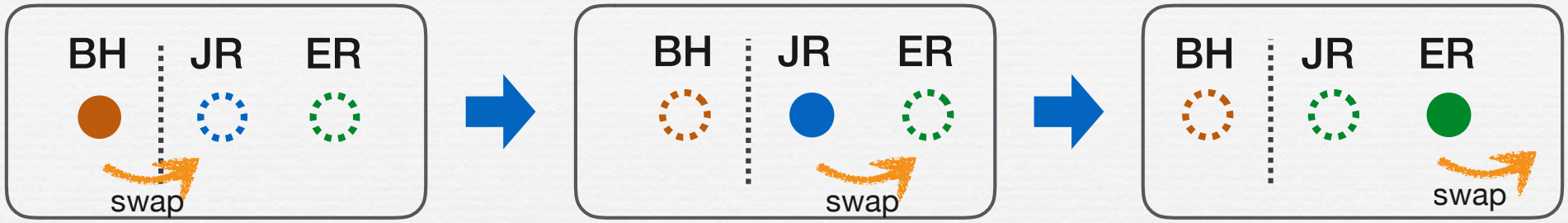
$$\tan\gamma = \exp\left(-4\pi M\omega\right)$$

$$\longleftrightarrow\ |0\rangle_{\rm BH}|0\rangle_{\rm JR} + \exp(-4\pi M\omega)\,|1\rangle_{\rm BH}|1\rangle_{\rm JR}$$

Mass is evolved following formula of Bekenstein-Hawking entropy

$$M_n = M_0\sqrt{1-\frac{n-1}{N_{\rm BH}}}$$

Rule 3: created particles are moved outward via SWAP operation



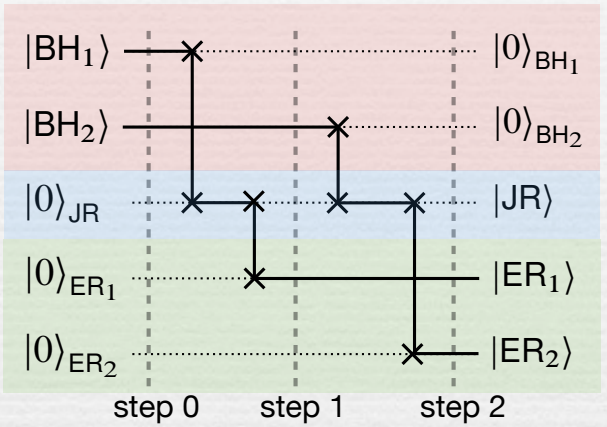
For initial state

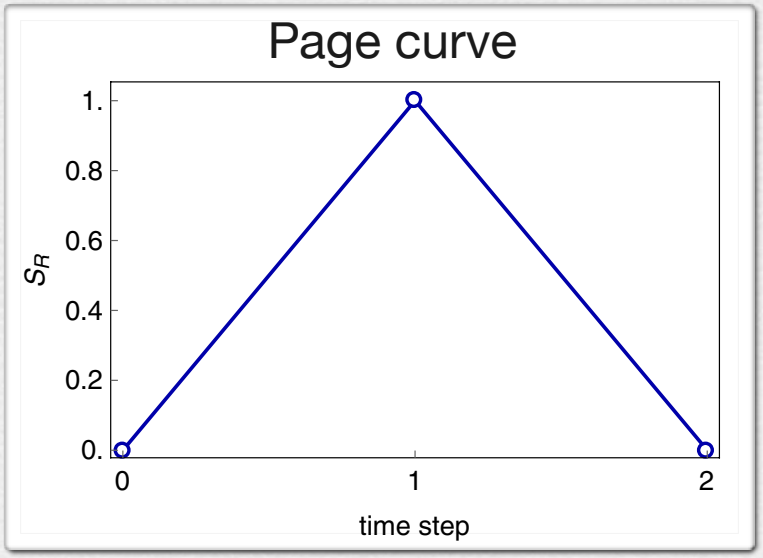
$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\Big[|10\rangle + |01\rangle\Big]_{\mathrm{BH}_{12}} |0\rangle_{\mathrm{JR}} |00\rangle_{\mathrm{ER}_{12}}$$

$$\Rightarrow |\psi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle_{\mathrm{BH}_1}|0\rangle_{\mathrm{JR}}\Big[|01\rangle + |10\rangle\Big]_{\mathrm{BH}_2,\mathrm{ER}_1}|0\rangle_{\mathrm{ER}_2}$$

$$\Rightarrow |\psi_2\rangle = \frac{1}{\sqrt{2}}|00\rangle_{\mathrm{BH}_{12}}|0\rangle_{\mathrm{JR}}\Big[|10\rangle + |01\rangle\Big]_{\mathrm{ER}_{12}}$$

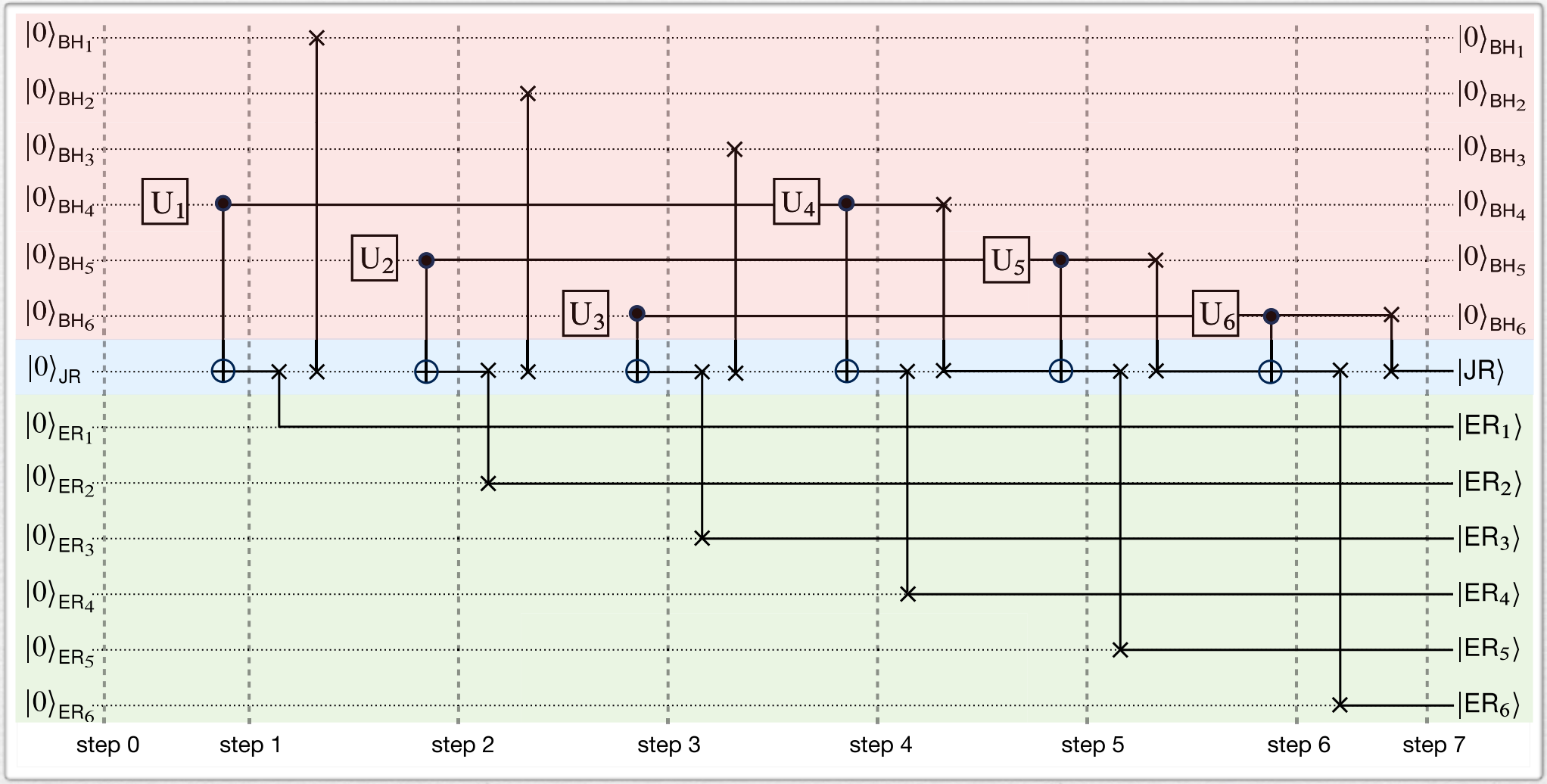
Entanglement of BH is transferred to Radiation by SWAP operations





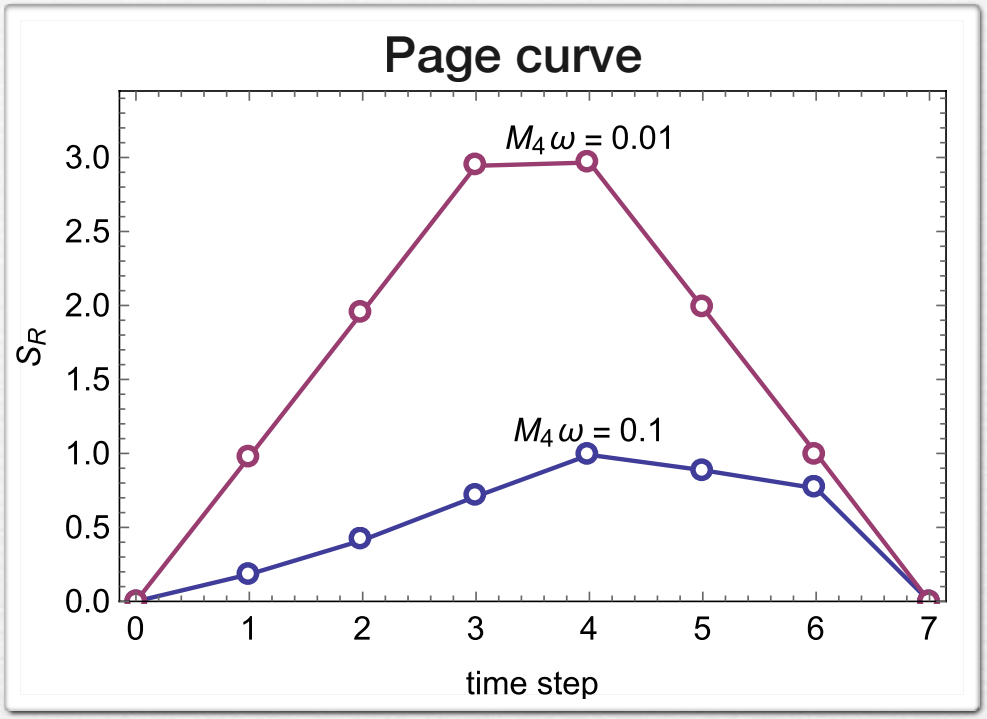
Our circuit model

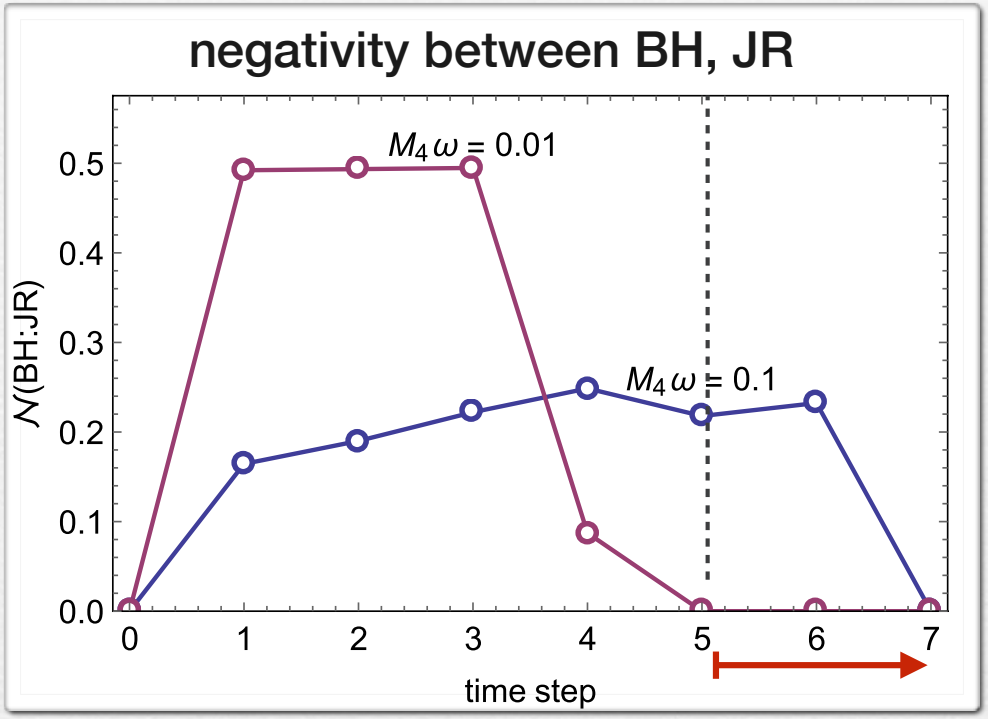
13 qubits $|\mathrm{BH}| = 2^6$ $|\mathrm{JR}| = 2^1$ $|\mathrm{ER}| = 2^6$



This circuit is just one example realizing basic features of Hawking radiation and black hole evaporation

Result and interpretation





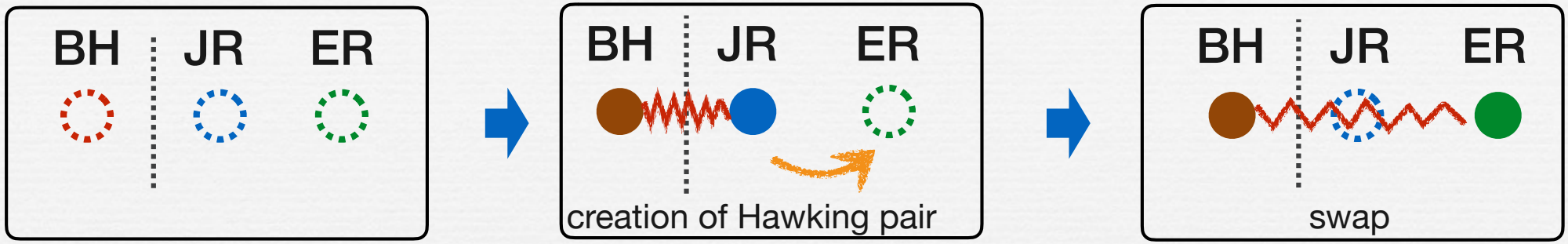
- Information contained in BH is transferred to Radiation
  Shape of Page curve depends on frequency of mode

- After Page time, BH:JR becomes separable for low frequency modes

$$M_4\omega \leq 0.041$$

"firewall" structure but classical correlation remains (separable state)

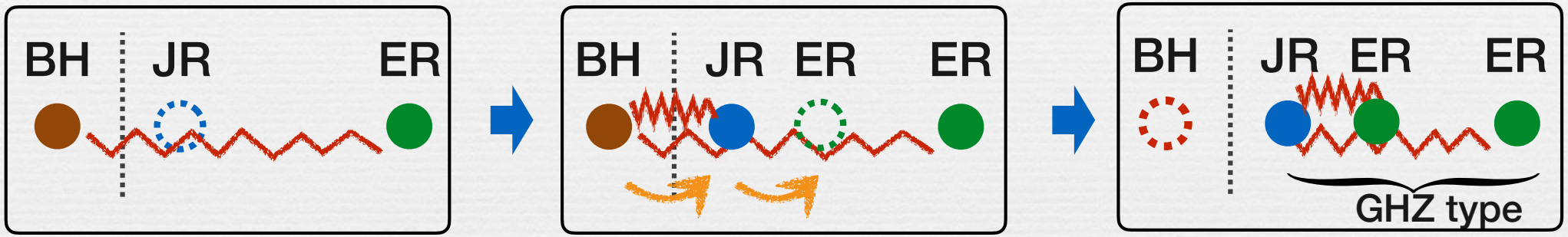
Before Page time



BH:ER pair is created as a entangled pair

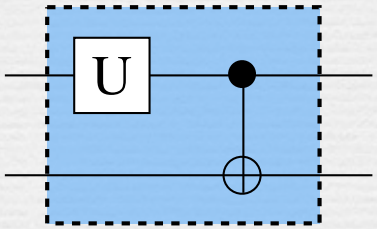
After Page time

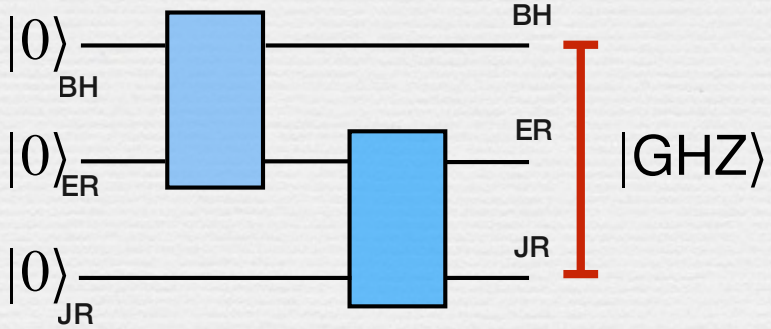
step 4



After successive application of CNOT-U gate, GHZ type state is created

CNOT-U gates create GHZ type state



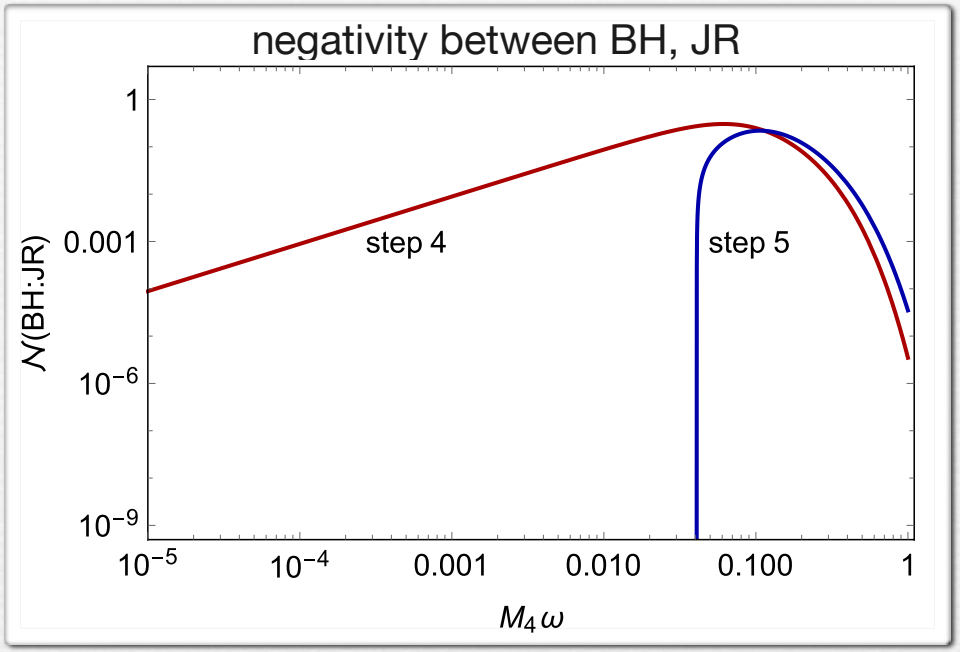


$T_H \to \infty$ U=H

exact GHZ state

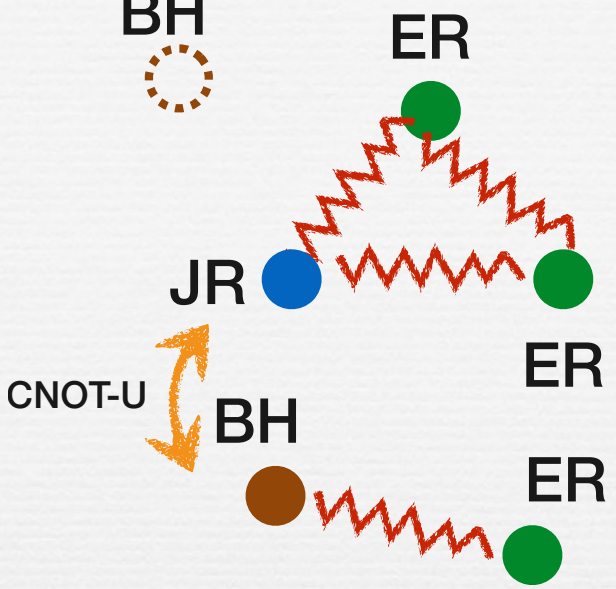
(maximally entangled)

After Page time

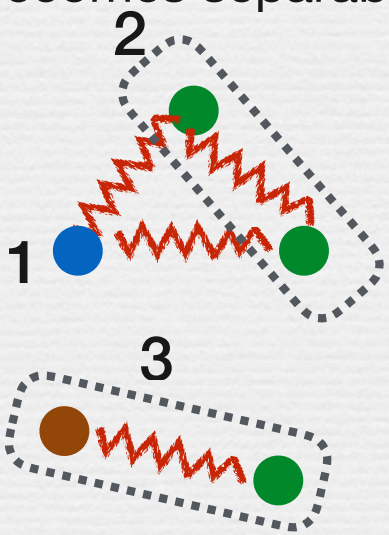


BH: JR becomes separable for $M_4\omega \leq 0.041$

step 5



CNOT-U gate tries to establish a new correlation between GHZ type state and previously generated Hawking pair



$$E_{1|23} \geq E_{12} + E_{13}$$

As 1 and 2 are maximally entangled,
1 and 3 cannot establish new correlation due to monogamy of entanglement

Monogamous property of multipartite state is related to emergence of separable state (firewall)

Summary

- Circuit model of BH evaporation
  - Formation of GHZ type state after Page time

    emergence of separable state
    between BH & R for low frequency modes
  - We observed effect of multipartite entanglement (monogamy) is a reason why the separable state (firewall-like structure) emerges in our evaporation model.

- Different evolution scenario of evaporation

  initial condition, structure of circuit

- Entanglement harvesting in de Sitter space

  A pair of qubit detectors cannot reveal entanglement beyond super horizon scale

  Local noise of de Sitter space kills quantum correlation (but this is related to monogamy of entanglement)

  This behavior can be understood from monogamy of qubit-qubit-environment (quantum field)