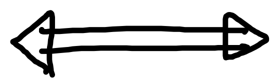
SPACETIME FROM "BITS"

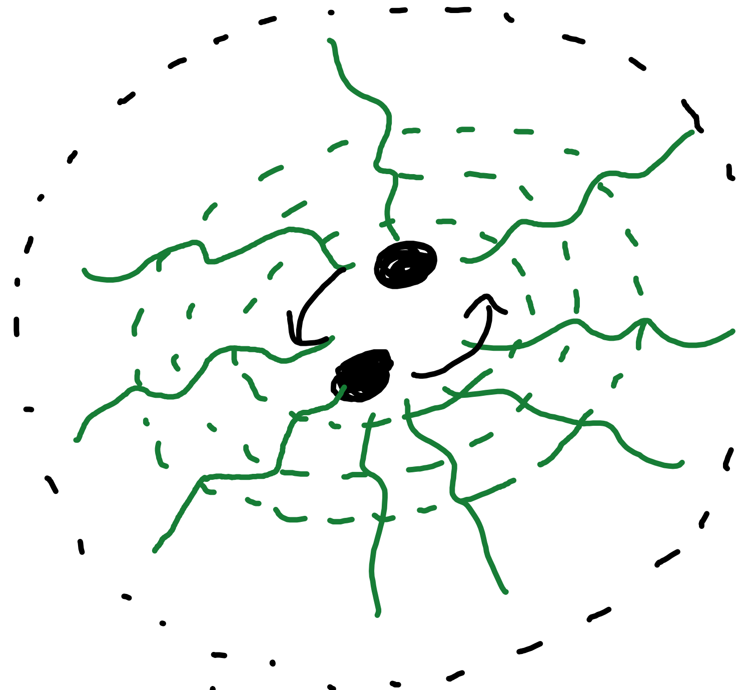
Mark Van Raamsdonk, UBC

YITP workshop, June 2019

based on: 1808.01197

THE AdS/CFT correspondence: spacetime physics + gravitation dynamics encoded in a non-gravitational quantum system



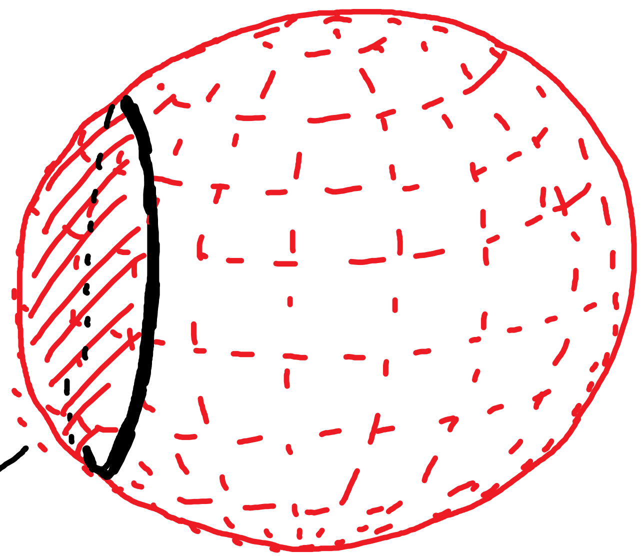


$$i\hbar \frac{d|\Psi\rangle}{dt} = H|\Psi\rangle$$

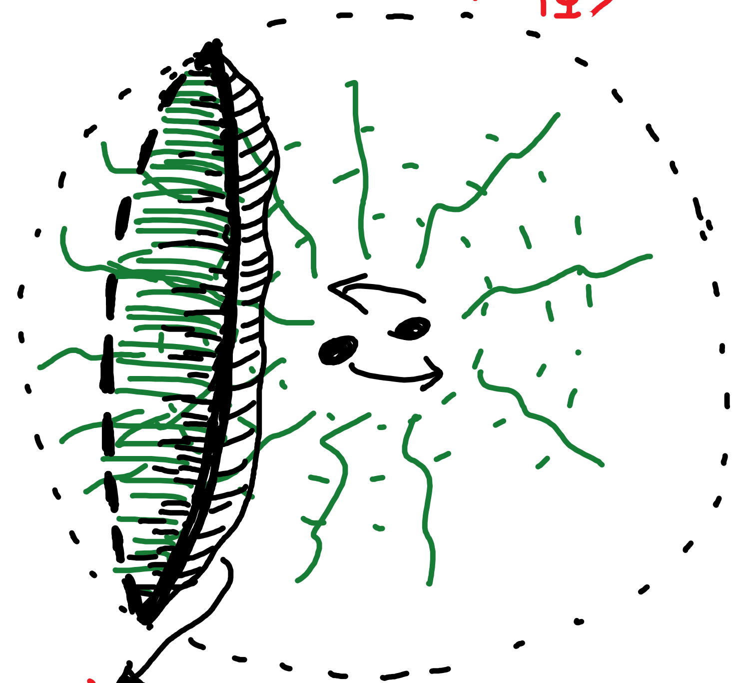
$M_{|\Psi\rangle}$

The Ryu-Takayanagi formula: spacetime geometry is directly related to entanglement structure:

$|\Psi\rangle_{\text{CFT on } S^2}$



$M_{|\Psi\rangle}$



subsystem $A \to \rho_A$

$\tilde{A}$ = extremal area surface in $M_{|\Psi\rangle}$ enclosing boundary region $\sim A$

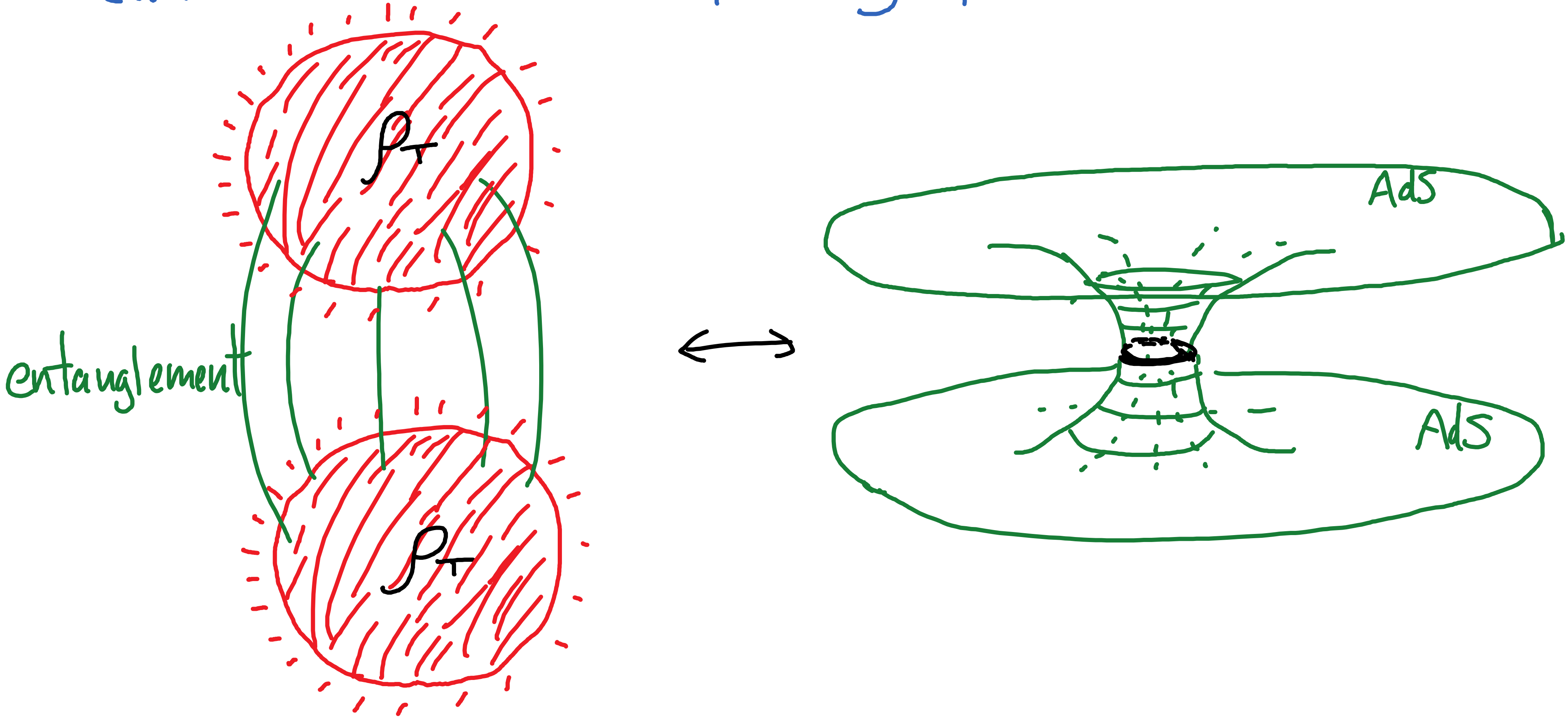
entropy $S_A = -\mathrm{Tr}(\rho_A \log \rho_A)$ $=$ $\frac{1}{4G} \times \mathrm{Area}(\tilde{A})$

Suggests: Spacetime "emerges" from entanglement in underlying degrees of freedom.

Qualitative picture: removing the entanglement should destroy spacetime

BUT: all QFT states are highly entangled; would cost infinite energy to remove

Inspirational example (Maldacena): Entangling 2 copies of CFT can connect the corresponding spacetimes



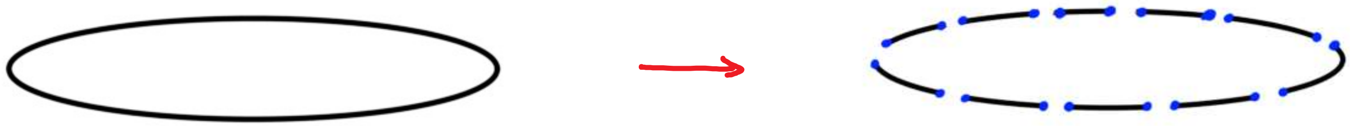
$$|\Psi\rangle = \sum_i e^{-\beta E_i/2} |E_i\rangle \otimes |E_i\rangle$$

"Thermofield Double State"

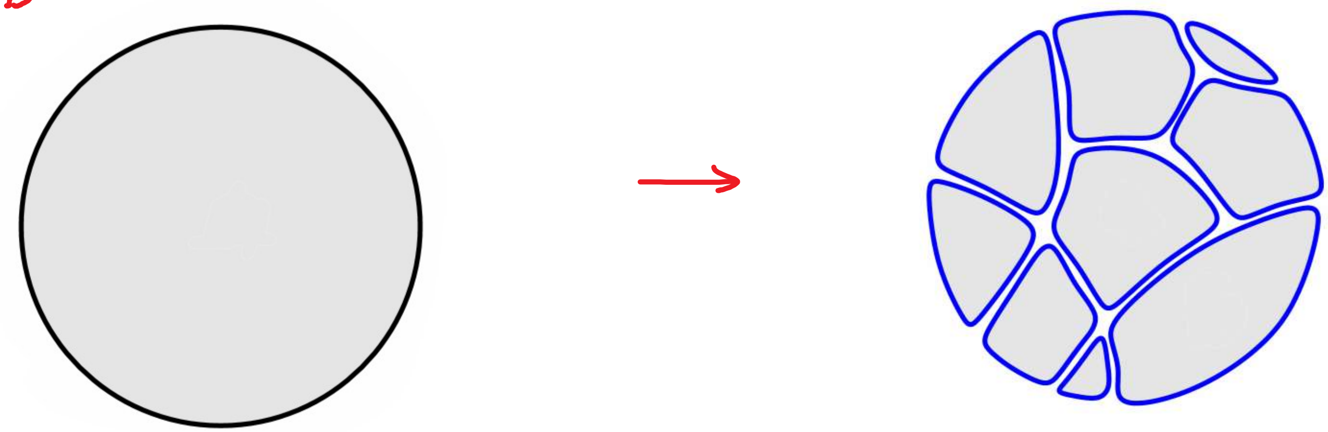
Main question for this talk: can we build up any spacetime by entangling discrete parts?

Basic idea: replace continuous CFT with collection of non-interacting systems (CFTs with boundary)

e.g. 1D



e.g. 2D

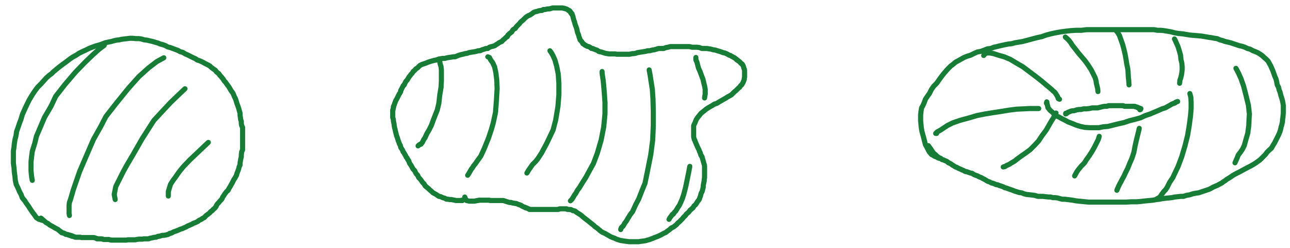


- approximate state $|\Psi\rangle$ of original theory encoding some spacetime M_Ψ with entangled state of "discretized" system.

- argue that new state still encodes connected spacetime closely related to M_Ψ

CFTs with boundary:

- Given a CFT, can define it on various spacetime backgrounds:



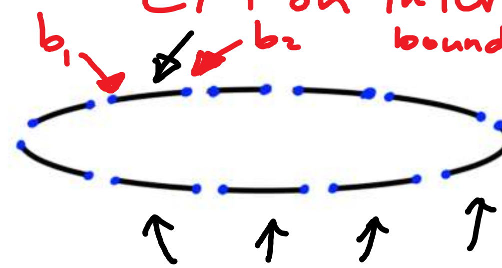
- Can also define it on spaces with boundary if we choose boundary conditions for each boundary component

- example: for free scalar theory, can choose Neumann or Dirichlet boundary conditions.

Examples:

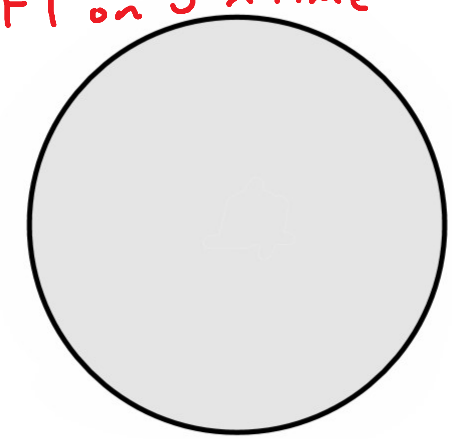
CFT on $S^1 \times$ time

CFT on interval with boundary conditions b_1, b_2

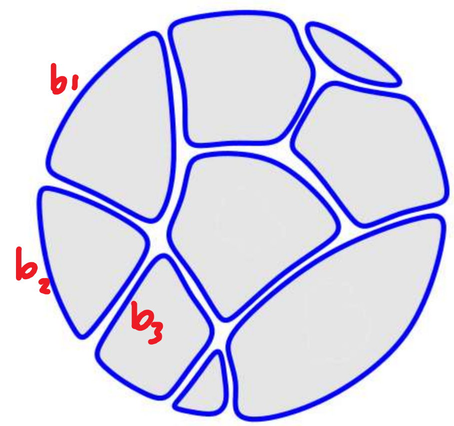


more CFTs on intervals

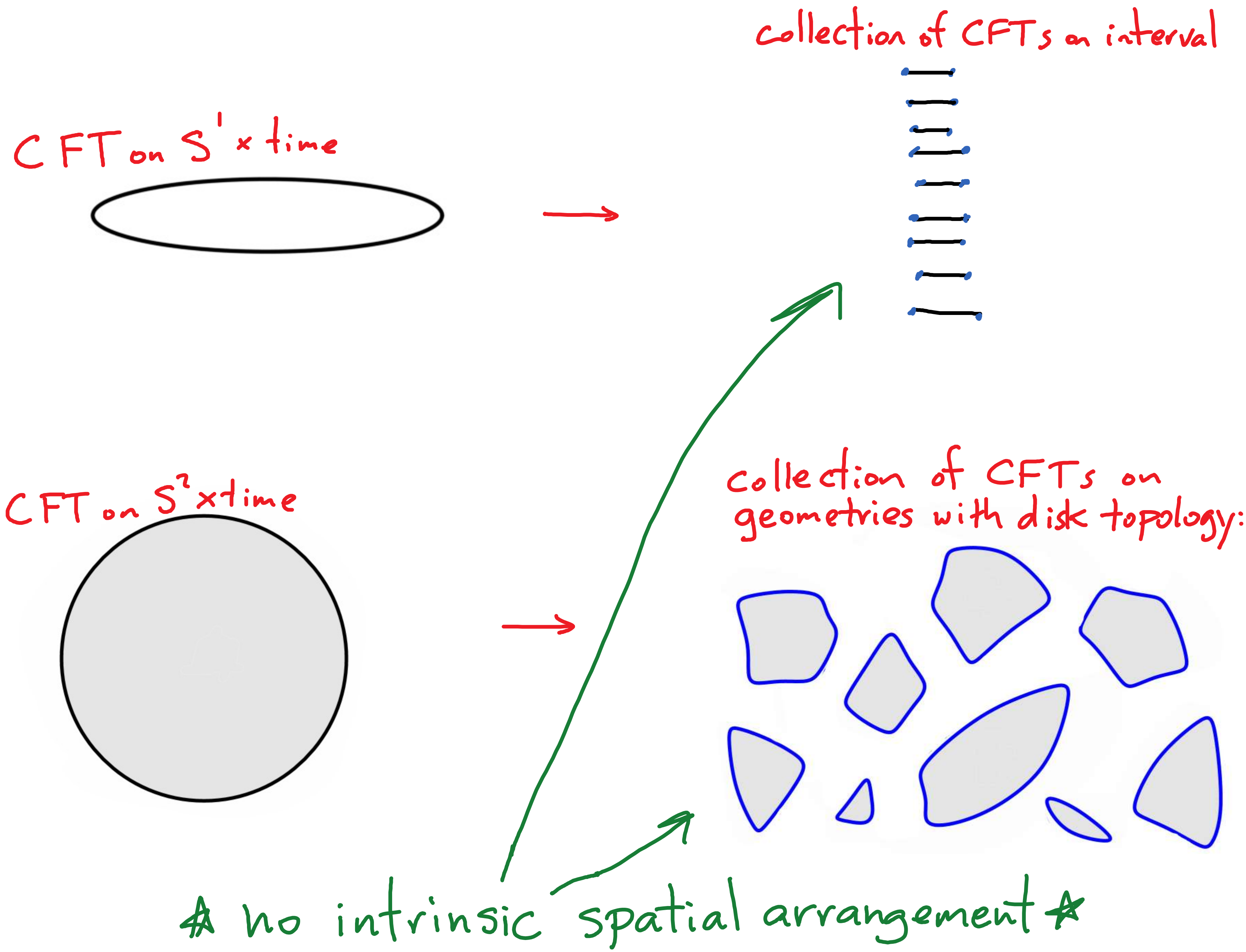
CFT on $S^2 \times$ time



collection of CFTs on geometries w. disk topology



Examples:


CFT on $S^1 \times$ time
collection of CFTs on interval
CFT on $S^2 \times$ time
collection of CFTs on
geometries with disk topology:
☆ no intrinsic spatial arrangement ☆

Next step: how do we approximate CFT states by states of the multipart systems?

Geometrical states via Euclidean path integral

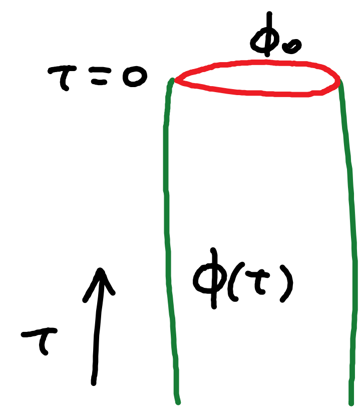
- For "holographic" CFT, only some states encode dual spacetime with nice semiclassical description
- A very large class of these "geometrical" states may be defined via Euclidean path integral.

Standard example: vacuum state

Euclidean action

$$|vac\rangle = \lim_{\beta \to \infty} e^{-\beta H} |\Psi_0\rangle \quad \Longrightarrow \quad \Psi_{vac}[\phi_0] = \int_{\tau<0}^{\phi(0)=\phi_0} [d\phi(\tau)]\, e^{-S_{Euc}}$$

arbitrary state with overlap to vaccuum



average of $e^{-S_{Euc}}$ over all field configurations with:

$$\phi(\tau=0) = \phi_0$$

Harmonic oscillator example:

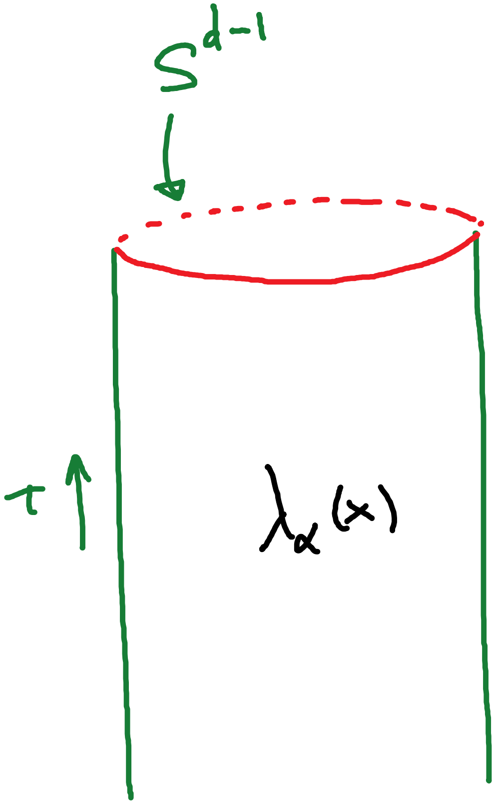
$$\langle x_0 | \text{vac} \rangle = \mathcal{N} \cdot \int_{\tau<0}^{x(0)=x_0} [dx(\tau)]\, e^{-\int d\tau \left\{ \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \omega^2 x^2 \right\}}$$

More general states:

$$\langle x_0 | \Psi_\lambda \rangle = \mathcal{N} \cdot \int_{\tau<0}^{x(0)=x_0} [dx(\tau)]\, e^{-\int d\tau \left\{ \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \omega^2 x^2 + \lambda(\tau) x(\tau) \right\}}$$

source for x operator

AdS/CFT:



wavefunction for CFT on $S^{d-1} \times$ time

low-dimension CFT operators

$$\langle \phi_0 | \Psi_\lambda \rangle = \frac{1}{\sqrt{Z_\lambda}} \int^{\phi(\tau)=\phi_0} [d\phi(\tau)]\, e^{-S_{Euc} - \int \lambda_\alpha(x)\, \mathcal{O}_\alpha(x)}$$

These states have gravity duals with a good classical description.

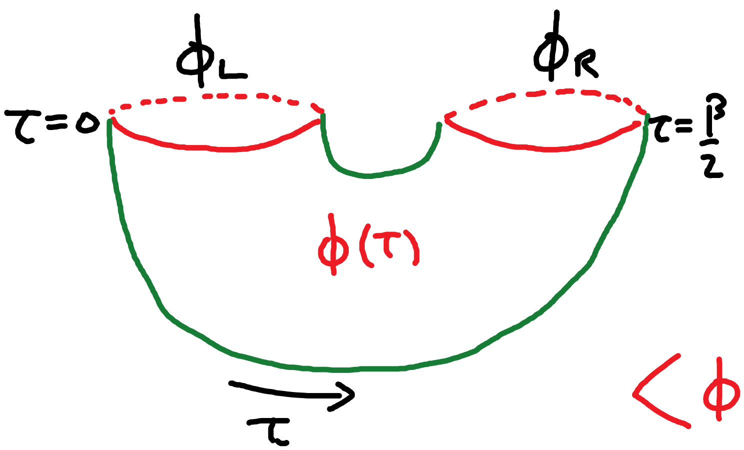
(will desribe dual geometry later)

Entangling via path integrals:

Recall: thermofield double state dual to 2-sided black hole

$$|\Psi_\beta\rangle = \sum_i e^{-\beta E_i/2} |E_i\rangle \otimes |E_i\rangle$$

Can define using a Euclidean path integral:

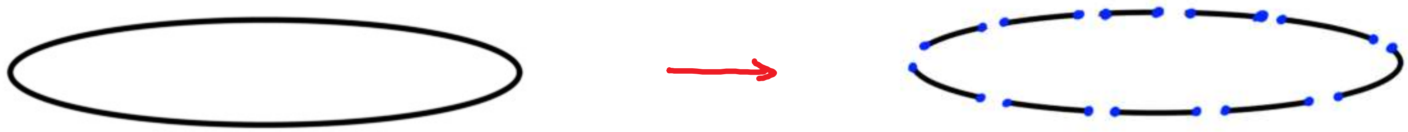


$$\langle \phi_L, \phi_R | \Psi_\beta \rangle \sim \int_{\phi(0)=\phi_L}^{\phi(\beta/2)=\phi_R} [d\phi(\tau)]\, e^{-S_{EUC}}$$

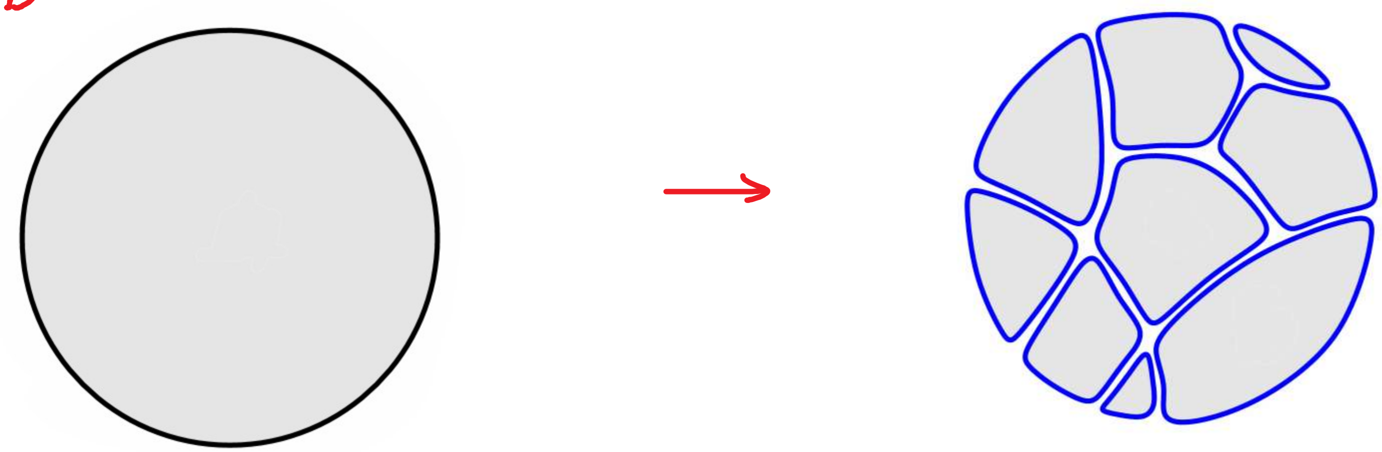
Connected path-integral geometry defines entangled state of non-interacting CFTs.

Recall: we wanted to replace CFT with collection of non-interacting systems

e.g. 1D



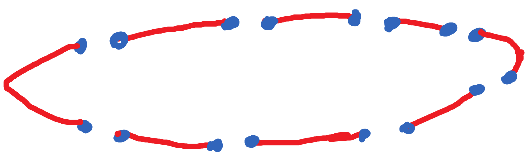
e.g. 2D



We are now ready to describe how to approximate the CFT states with entangled states of the new sytem

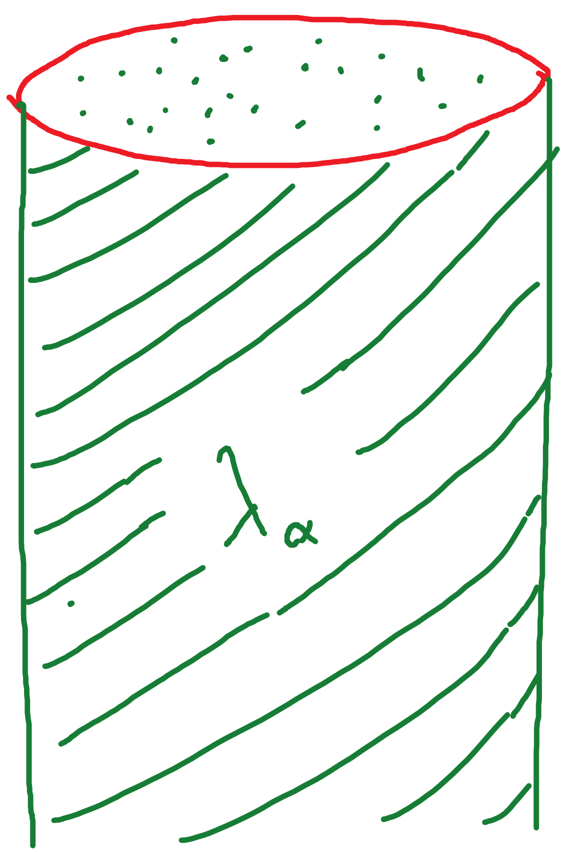
Old system:

New system:

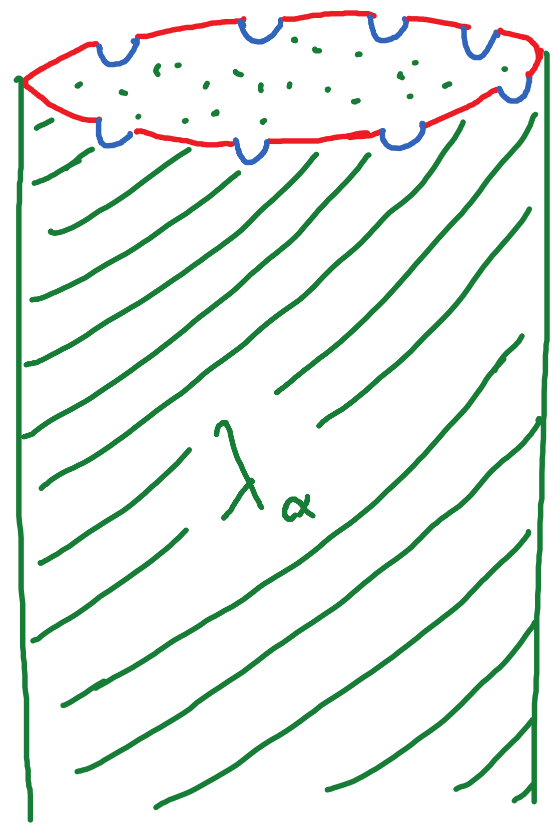


Main idea:

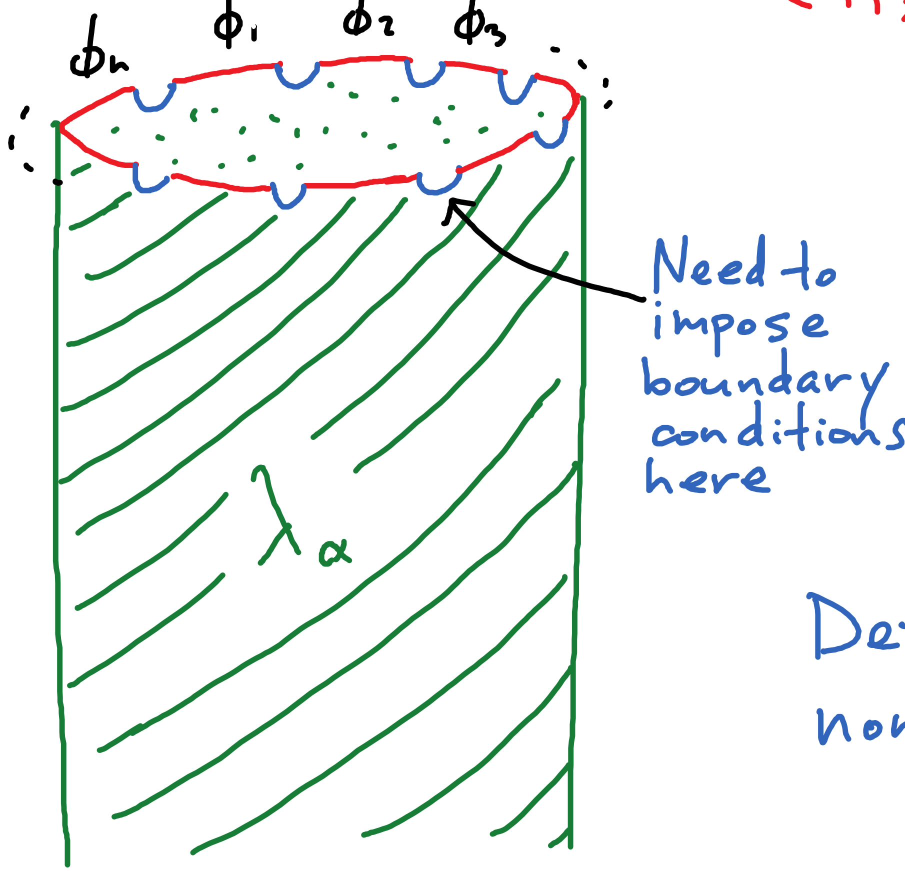
Given CFT state defined via path integral:



Define a new path integral by introducing boundary components:



Explicit expression:

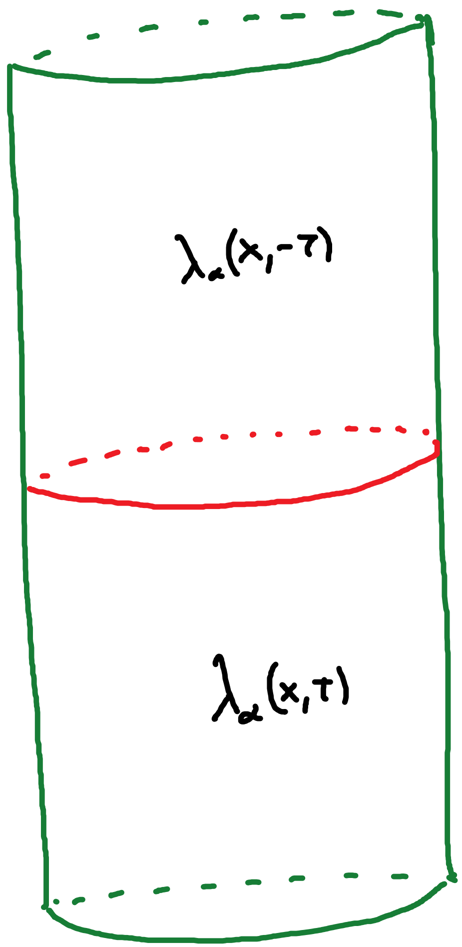


$$\langle \phi_1, \ldots, \phi_n | \Psi \rangle \sim \int [d\phi] \, e^{-S_{Euc} - \int \lambda_\alpha \mathcal{O}_\alpha}$$

Defines state of n non-interacting CFTs w. boundary.

Next step: understand dual geometries

AdS/CFT recipe:

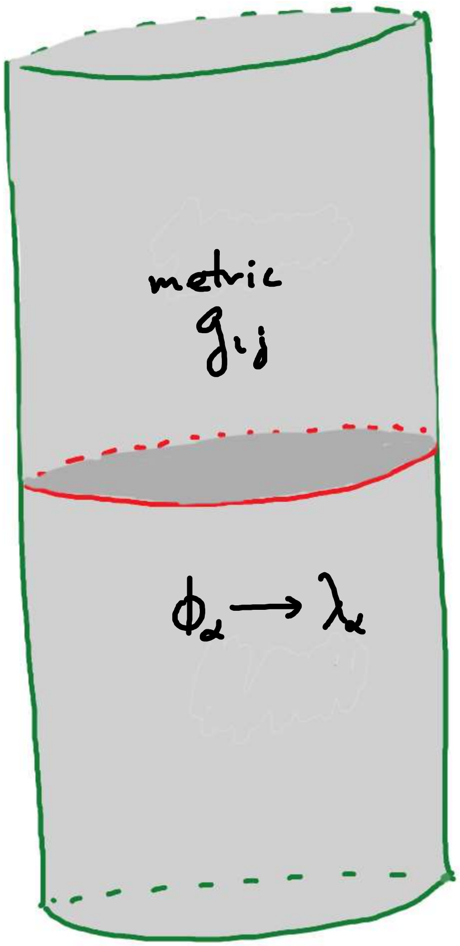


Step 1: Find Euclidean asymptotically AdS space with:

boundary geometry = geometry of path integral for Z_λ

boundary conditions for fields determined by sources λ_α

AdS/CFT recipe:



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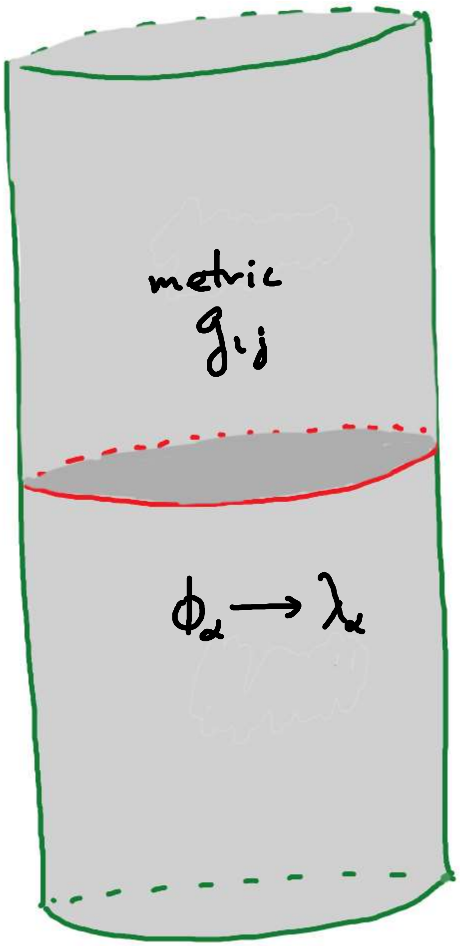

Step 1: Find Euclidean asymptotically AdS space with:

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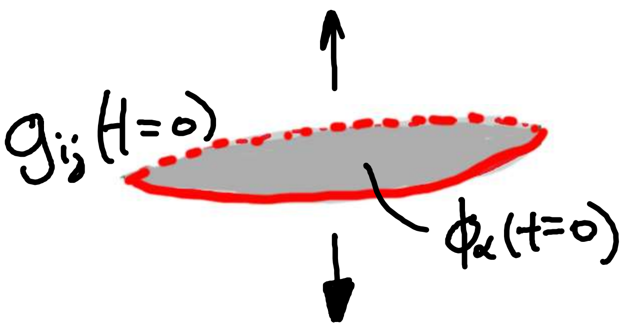
Step 2: Take slice at $\tau = 0$ surface (shaded). This provides initial data for Lorentzian evolution.

i.e: evolve this forward & backward in time to find spacetime dual to CFT state.

AdS/CFT recipe:



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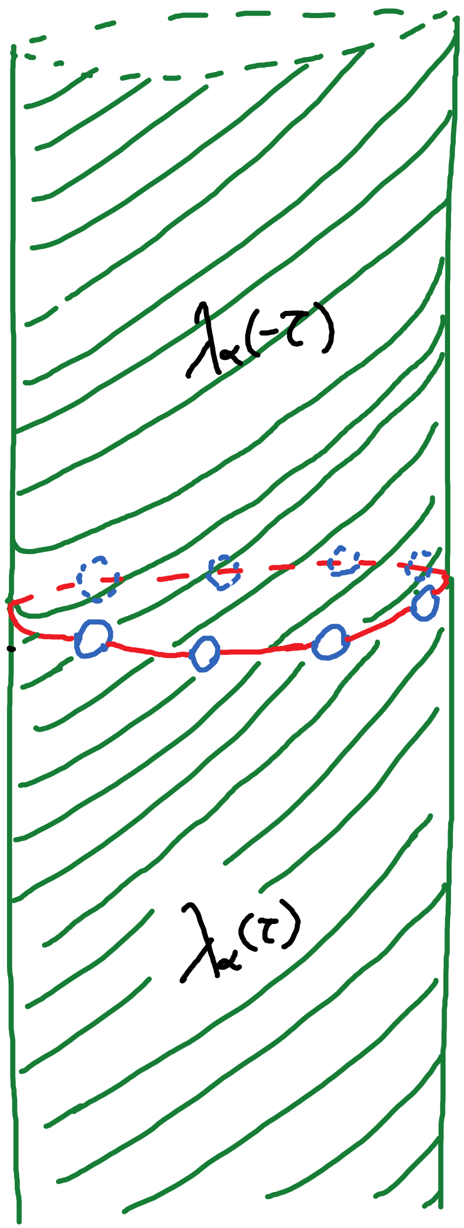
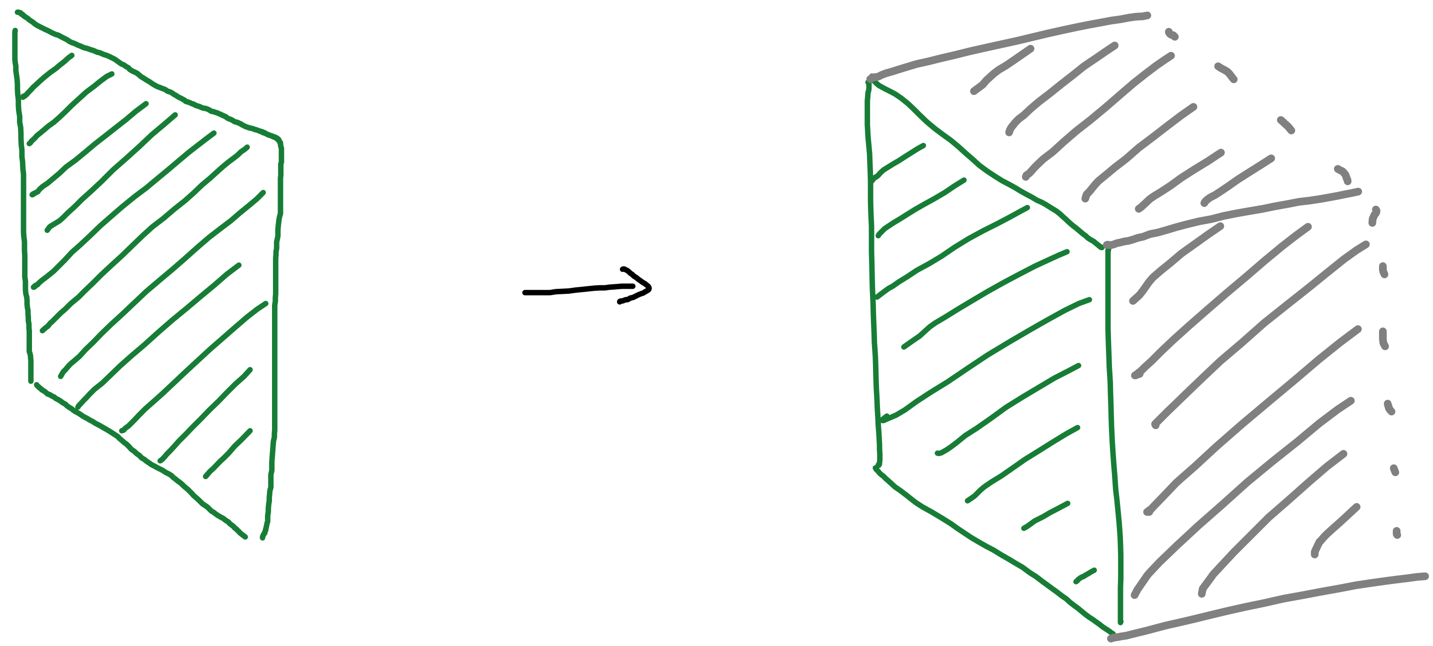
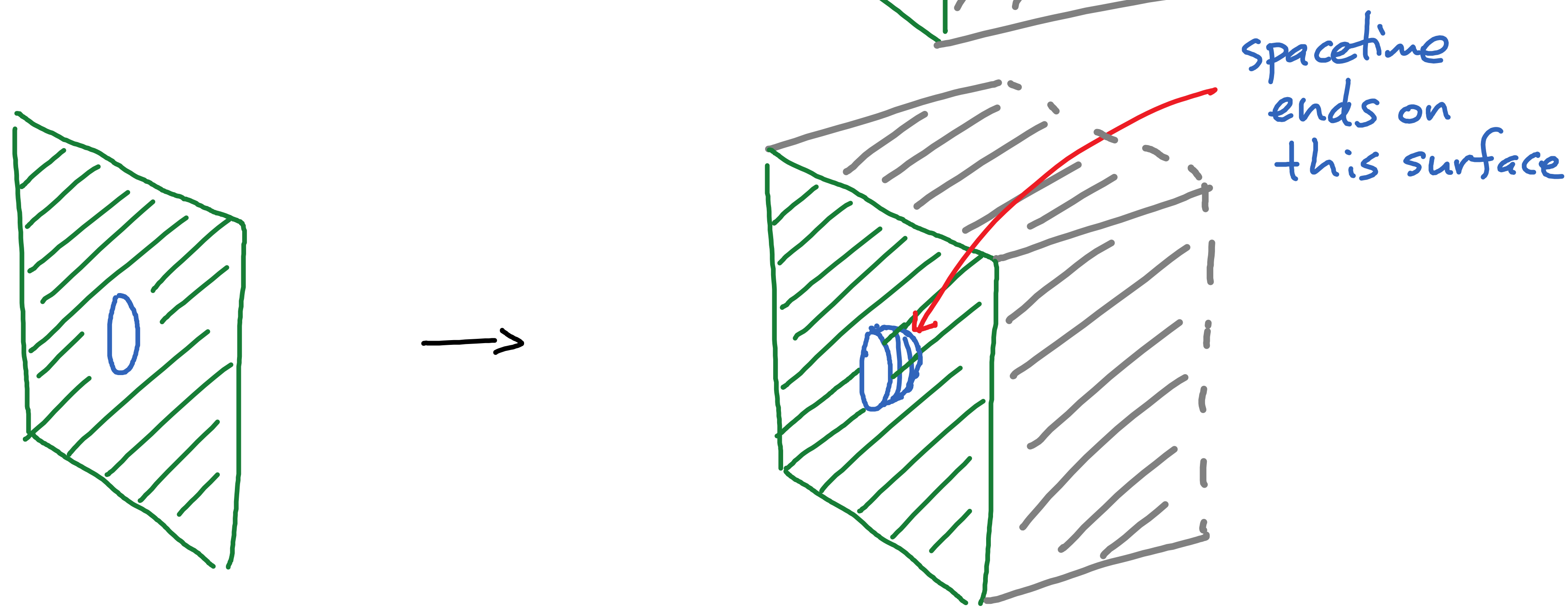
Repeat this for modified path integral



What is the effect of the extra boundaries?

Takayanagi: model by "end of the world" brane





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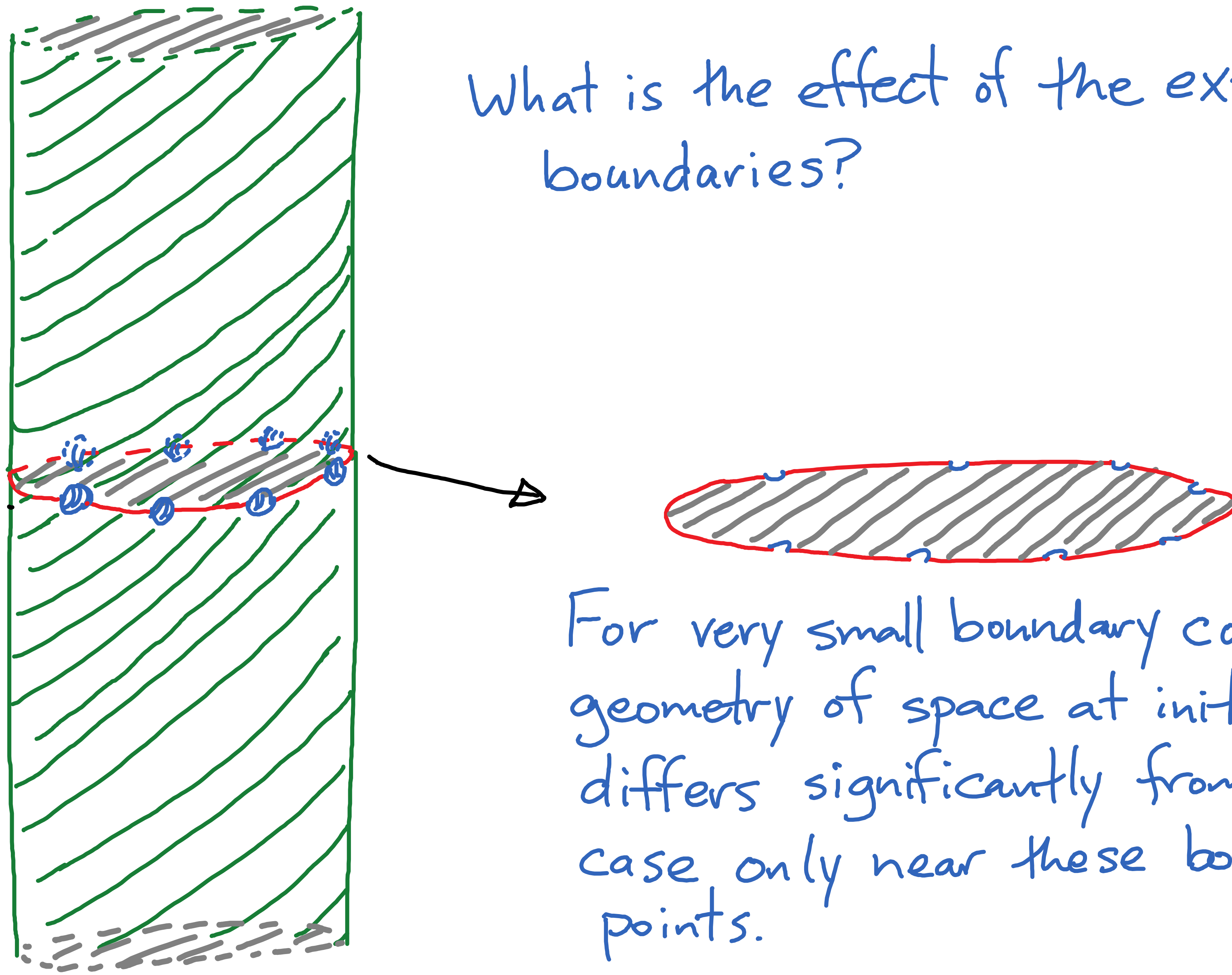
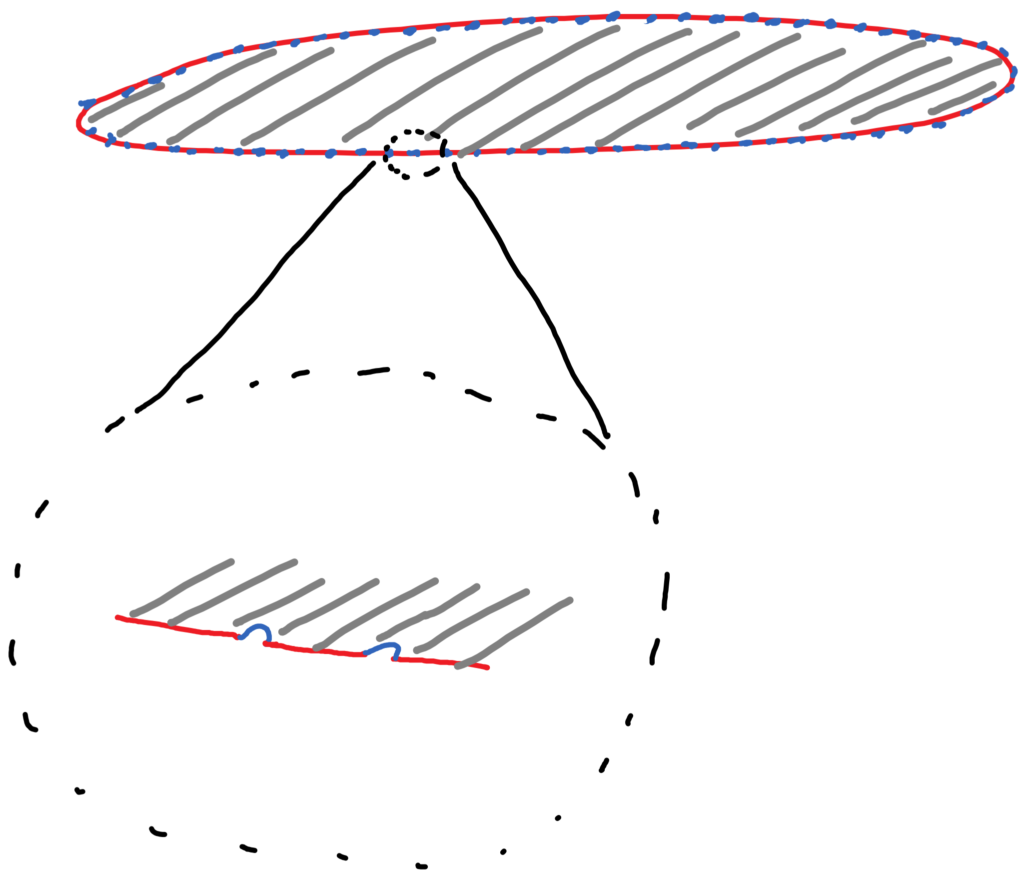
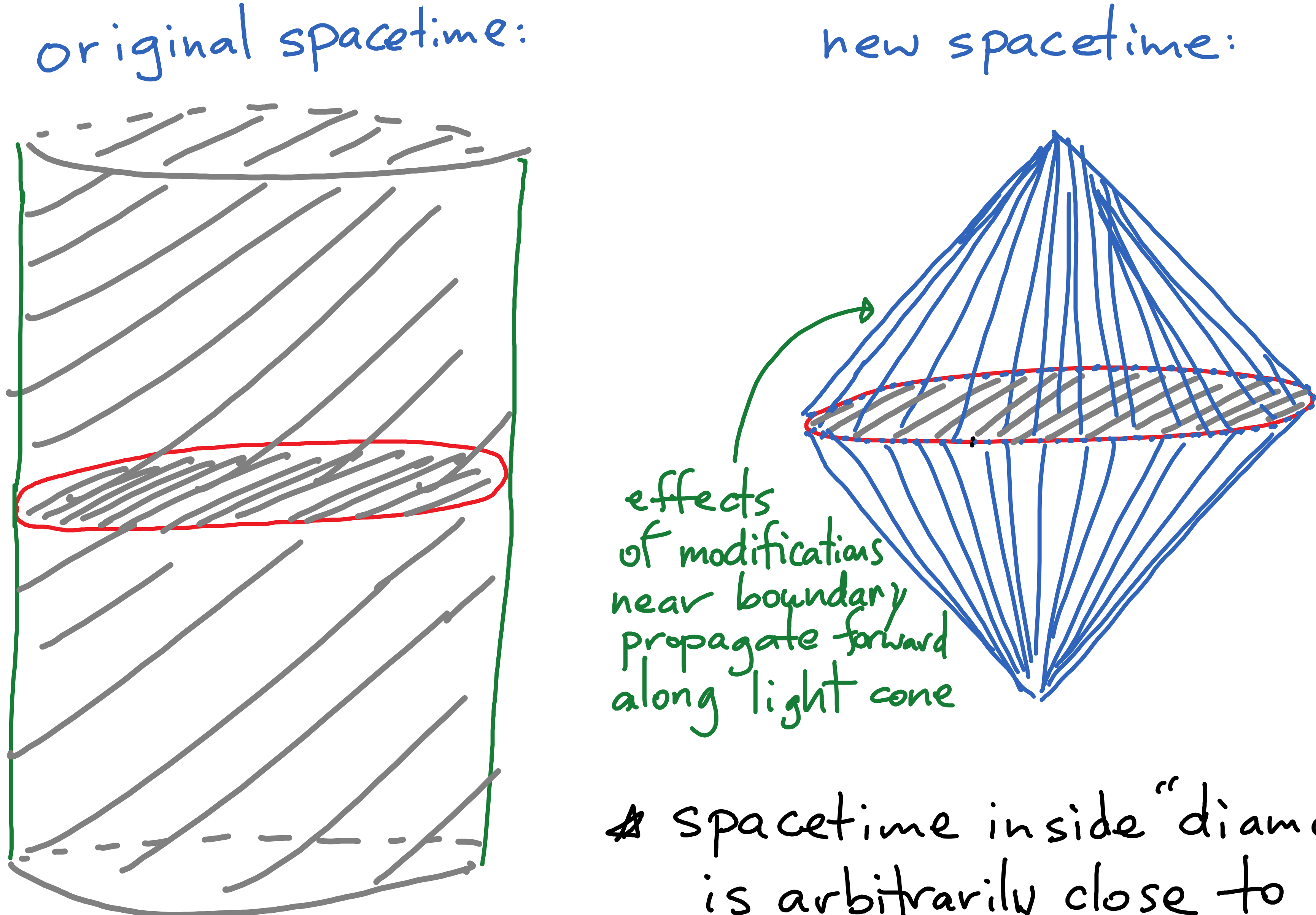


For very small boundary components, geometry of space at initial time differs significantly from original case only near these boundary points.

Same is true even in limit of a very large number of boundary components if size of "gaps" small relative to size of subsystems



Effects on Lorentzian geometries



★ spacetime inside "diamond" is arbitrarily close to previous spacetime ★

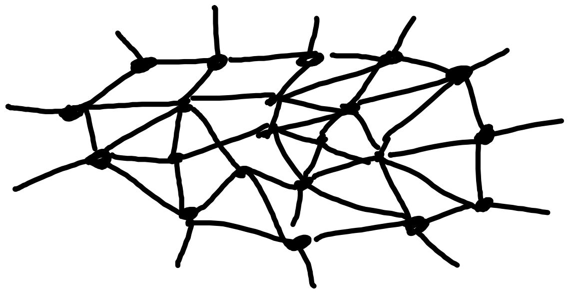
Key message: we can describe very general connected spacetimes via multipart entanged states of many non-interacting systems.

* Almost no information about spacetime geometry in state of individual "bits"

* Removing entanglement manifestly destroys spacetime *

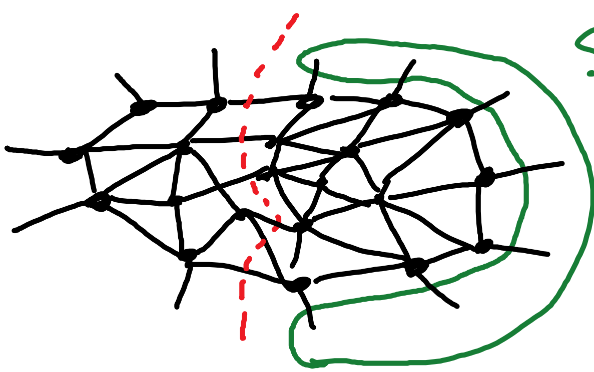
Interesting question: does it matter what the bits are?

EXTRA: Connection to tensor network models of holography.



Tensor network states based on perfect tensors or random tensors for large bond dimension have been shown to obey a Ryu-Takayanagi formula.

(Pastawski, Yoshida, Harlow, Preskill; Hayden et. al.)



Subsystem A

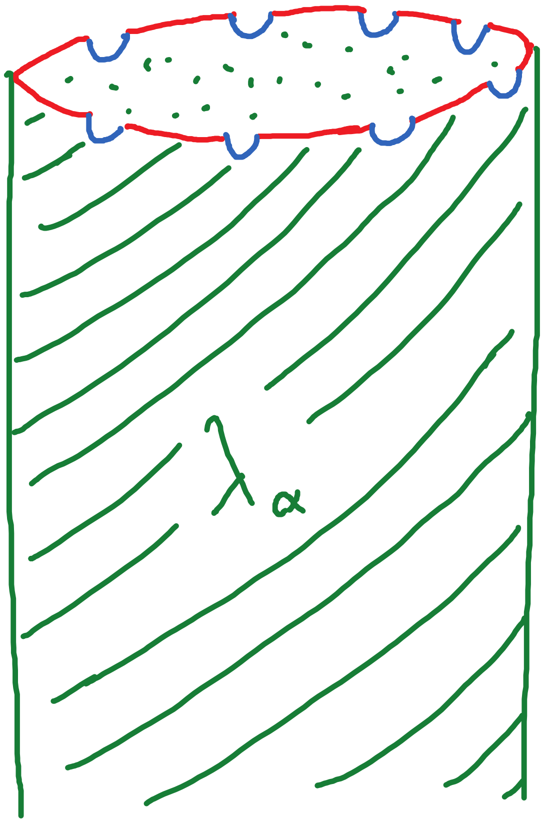
$$S(A) \sim \min\left(\begin{array}{c}\#\text{ of bonds cut}\\ \text{to separate } A\end{array}\right)$$

Suggests that tensor network states provide good toy model for holographic states, with network geometry playing role of dual spatial geometry (Swingle)

- Hard to understand if CFT states have this structure since CFT is continuous system.

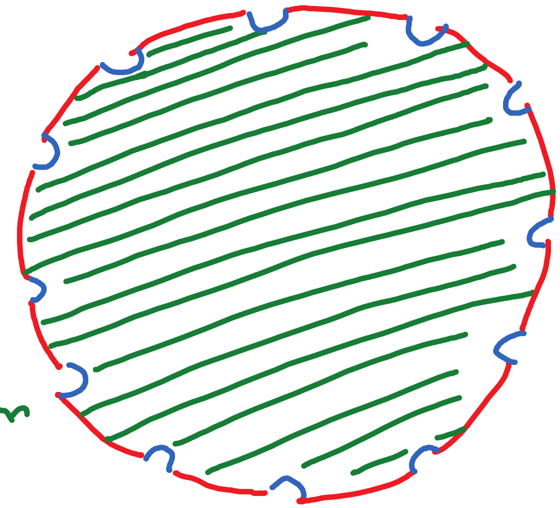
- With our discrete version, can make a more direct connection to tensor networks:

Start with:



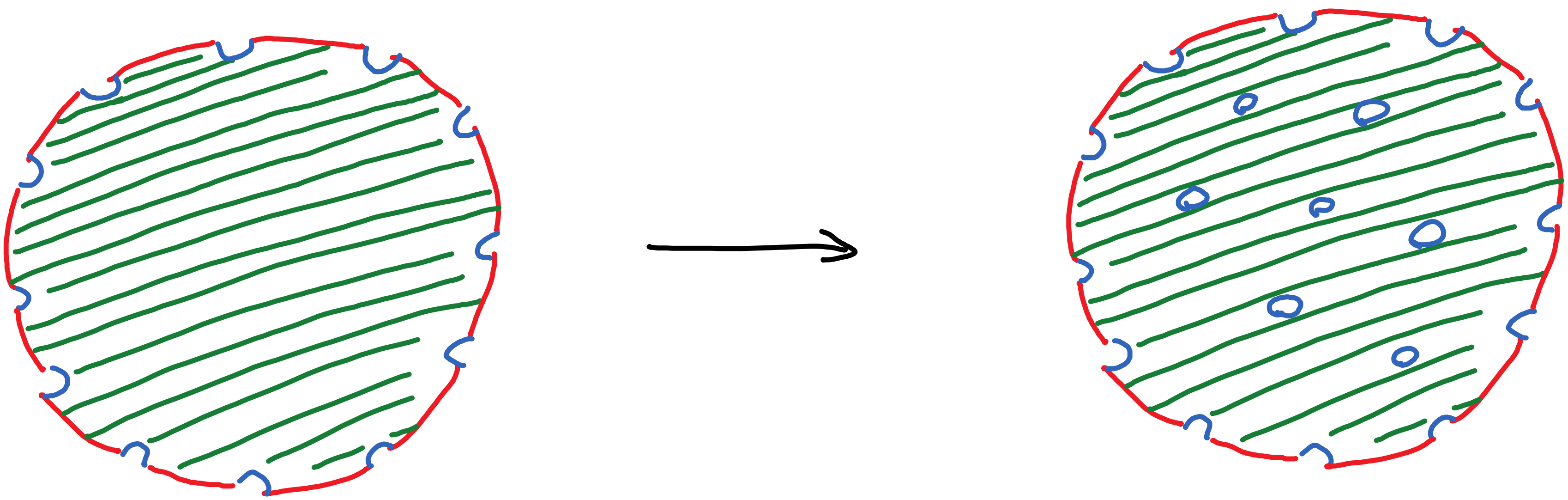
conformal transform

Equivalent to:



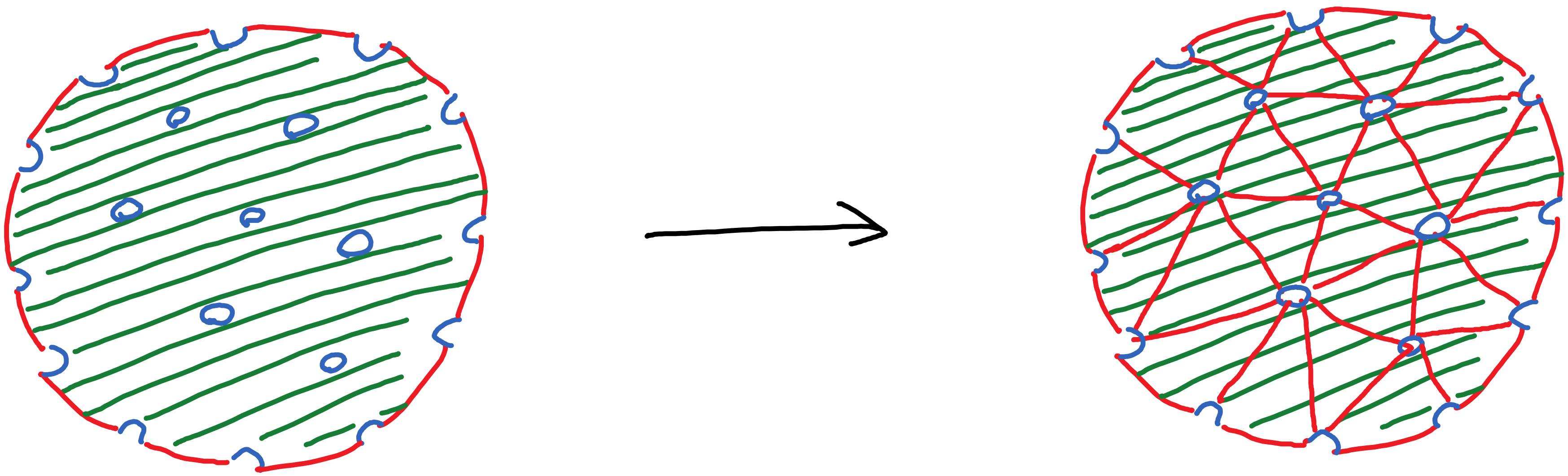
(this is the geometry of our path integral)

Add some extra boundary components:

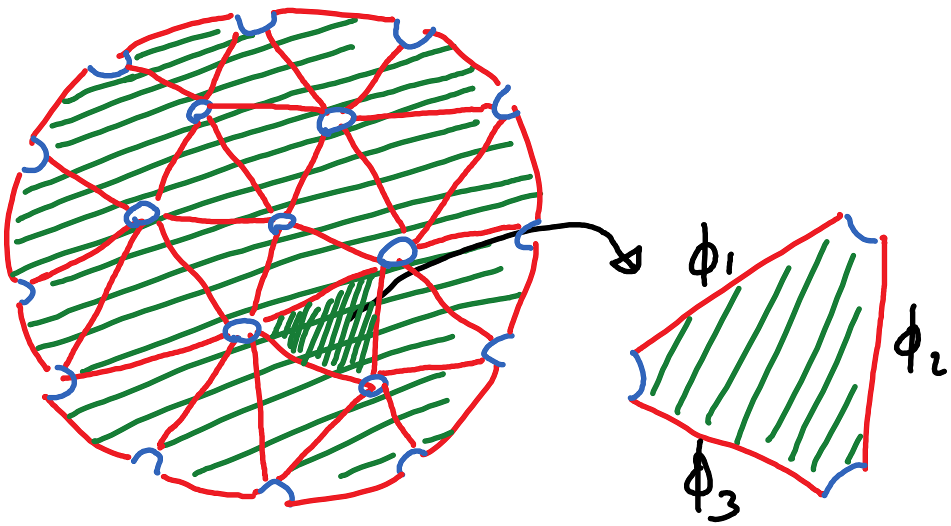


Negligible effect on state in limit of small size

Divide up path integral into pieces:

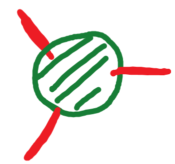


Each piece can be understood as path integral for a state of a small number of CFTs with boundary.



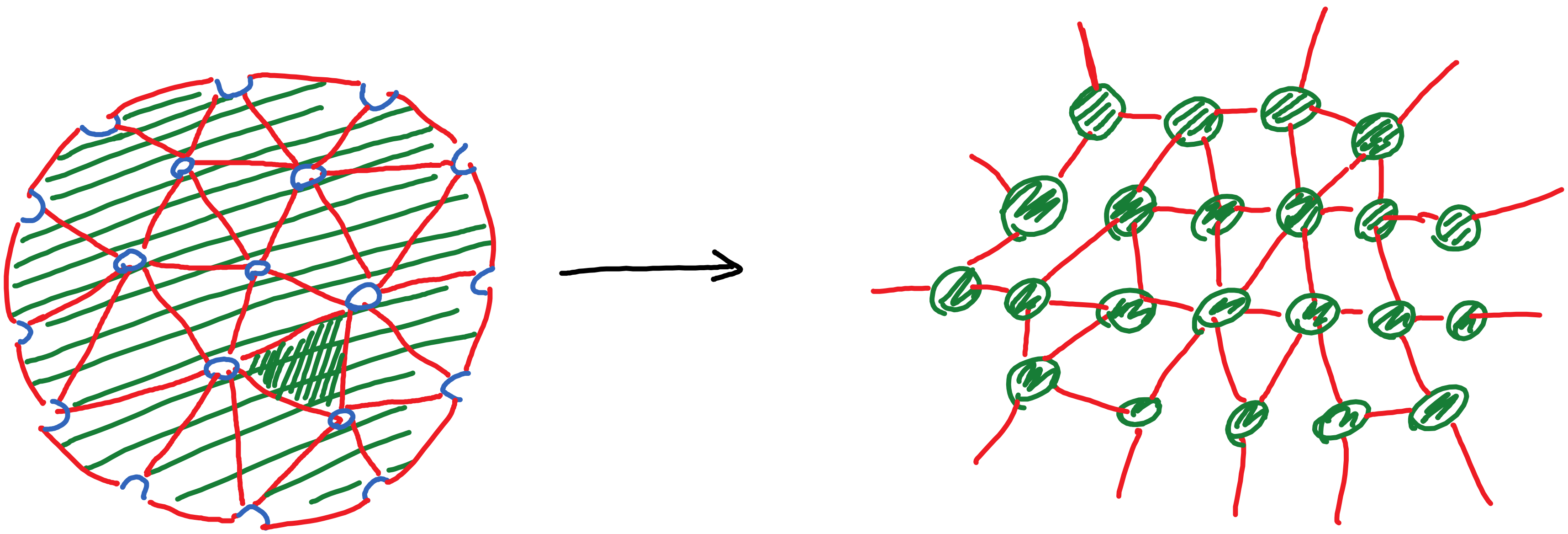
$$\langle \phi_1, \phi_2, \phi_3 | \Psi \rangle = \int [d\phi]\, e^{-S}$$

"



tensor

Full path integral corresponds to contracting up these states into tensor network (as in PEPS)



- Many possible networks since many choices for where to place extra boundaries + divisions

- Is there a rule that gives a network whose geometry matches geometry of the dual space in gravity theory?

(c.f. Kaputa, Kundu, Miyagi
Takayanagi, Watanabe: Path integral optimization)

Thank you!