A computationally universal phase of quantum matter

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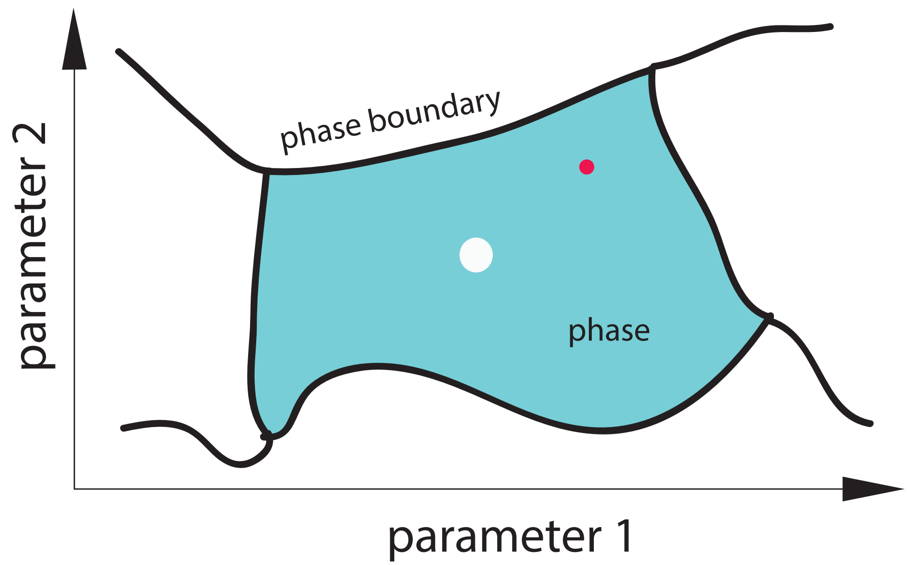
joint work with D.-S. Wang, D.T. Stephen, C. Okay, and H.P. Nautrup

The liquid phase of water

A quantum phase of spins

... which supports universal quantum computation

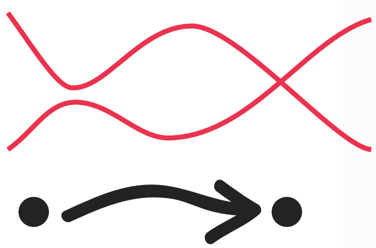


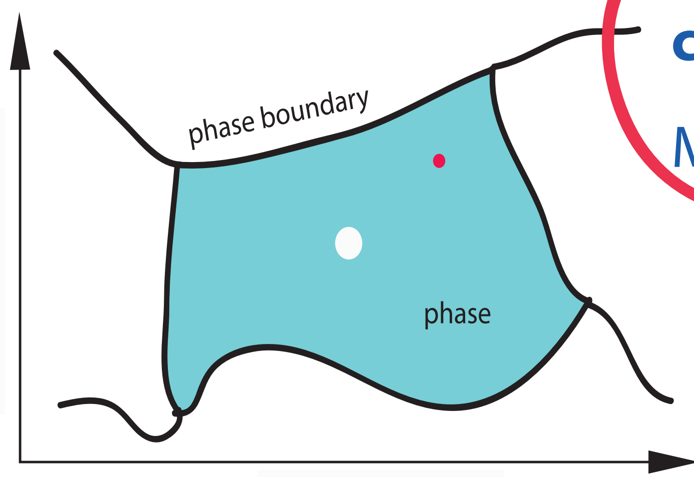
We consider:

- Phases of unique ground states of spin Hamiltonians, at $T = 0$
- In the presence of symmetry

Computational phases of quantum matter

physical characterization





computational characterization

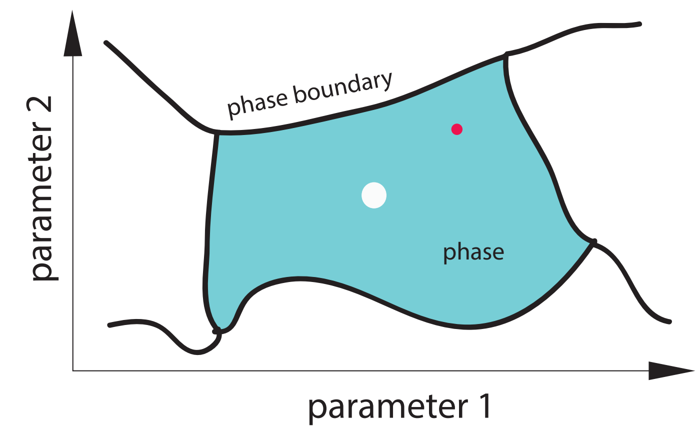
MBQC power

mathematical characterization

group cohomology

A quantum phase of spins in 2D

... which supports universal quantum computation



We show:

- $\boxed{\text{There exists a quantum phase of matter which is universal for quantum computation}}$
- The computational power is *uniform* across the phase.
- Employ measurement-based quantum computation

Outline

1. "Computational phases of quantum matter":

    - Our motivation

    - A short history of the question (1D)

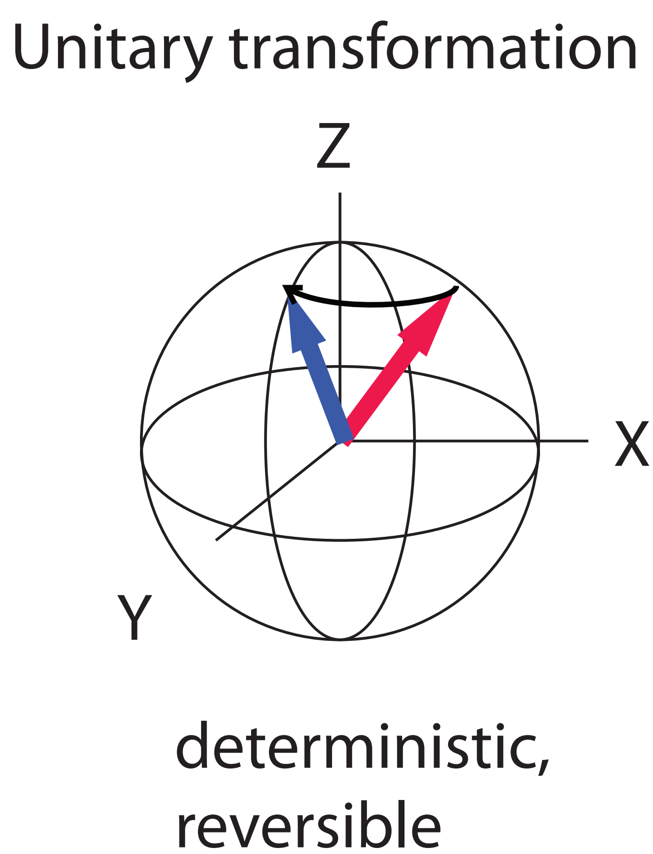
2. A computationally universal phase of matter in 2D

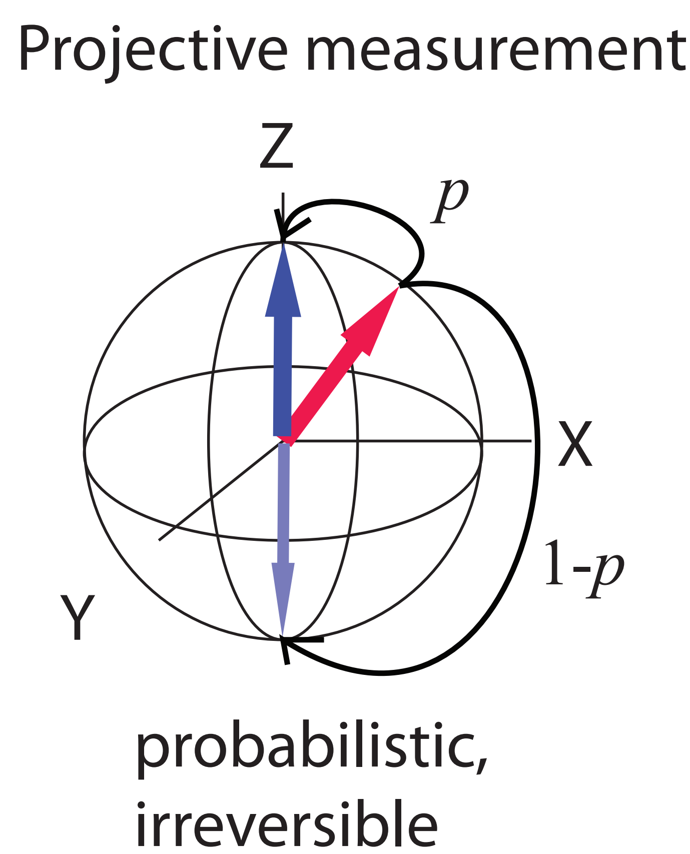
Part I:

A short history of

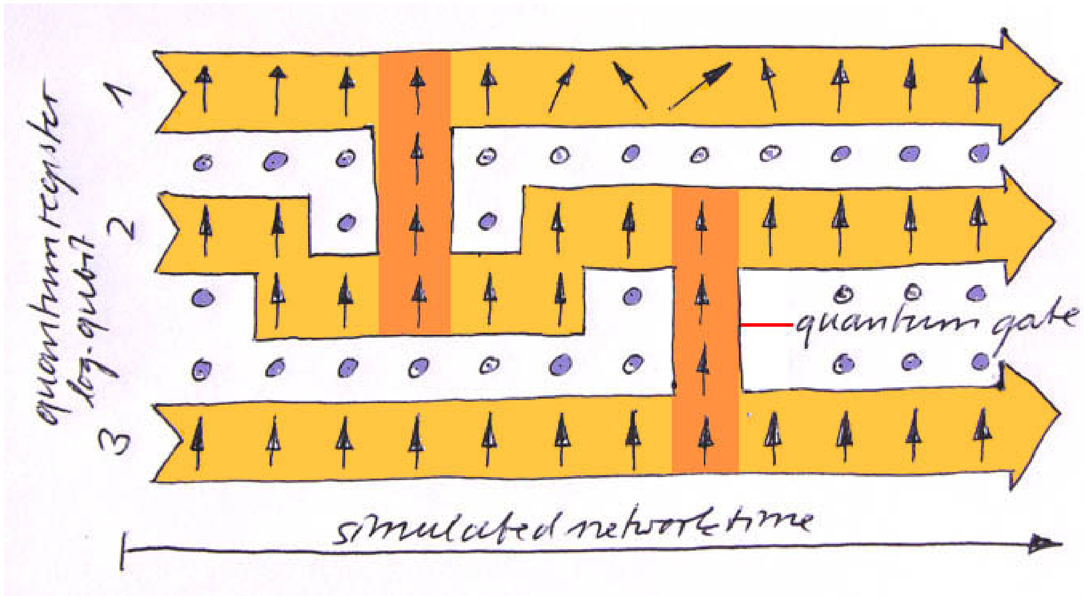
"computational phases of quantum matter"

Measurement-based quantum computation





Measurement-based quantum computation

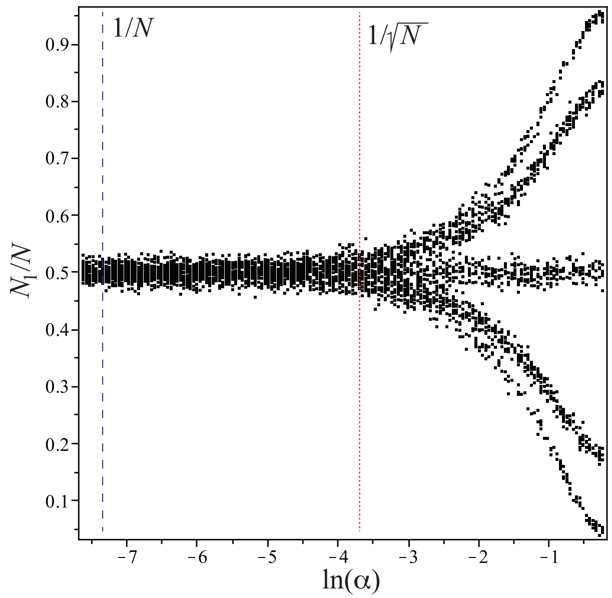


measurement of Z ($\odot$), X ($\uparrow$), $\cos\alpha\, X + \sin\alpha\, Y$ ($\nearrow$)

- Information written onto the resource state, processed and read out by one-qubit measurements only.
- Universal computational resources exist: cluster state, AKLT state.

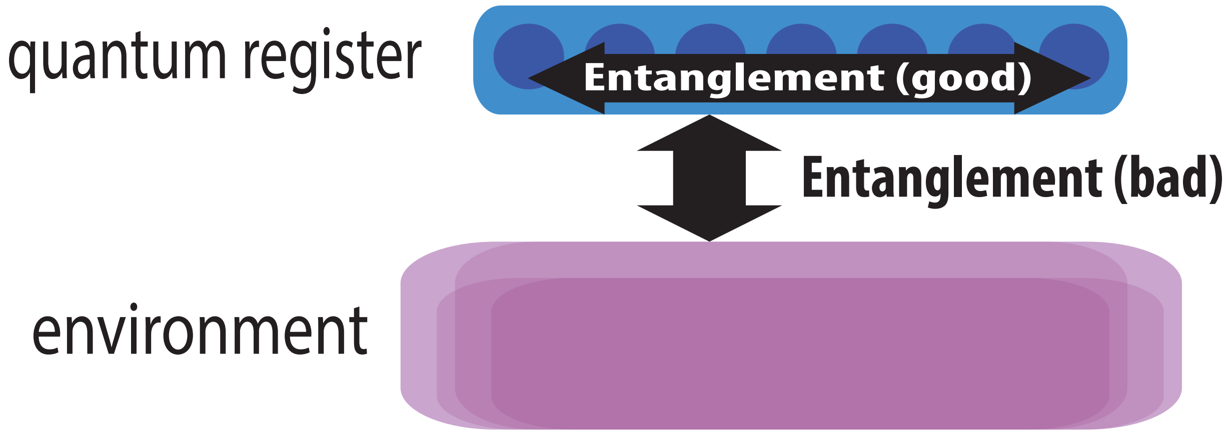
R. Raussendorf, H.-J. Briegel, Physical Review Letters 86, 5188 (2001).

Motivation #1: MBQC and symmetry



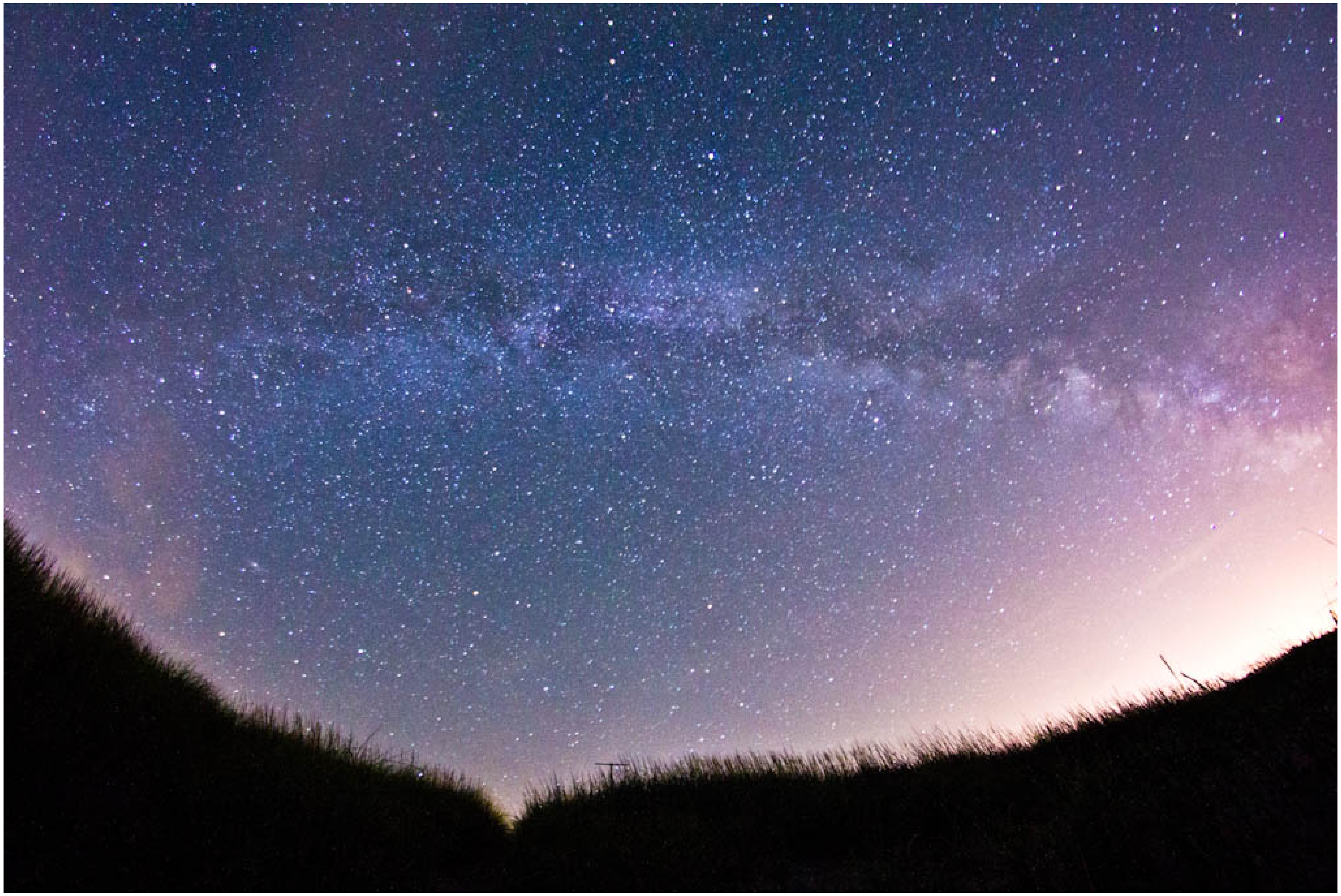
Can MBQC schemes be classified by symmetry, in a similar way as, say, elementary particles can?

An observation in quantum error-correction



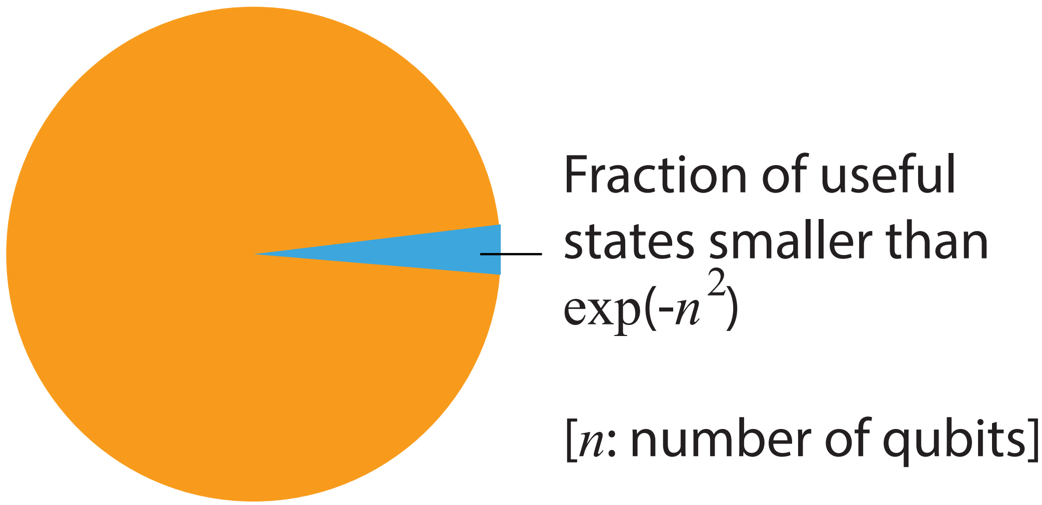
There's good and bad entanglement.
Good entanglement often comes with a symmetry

Motivation #2



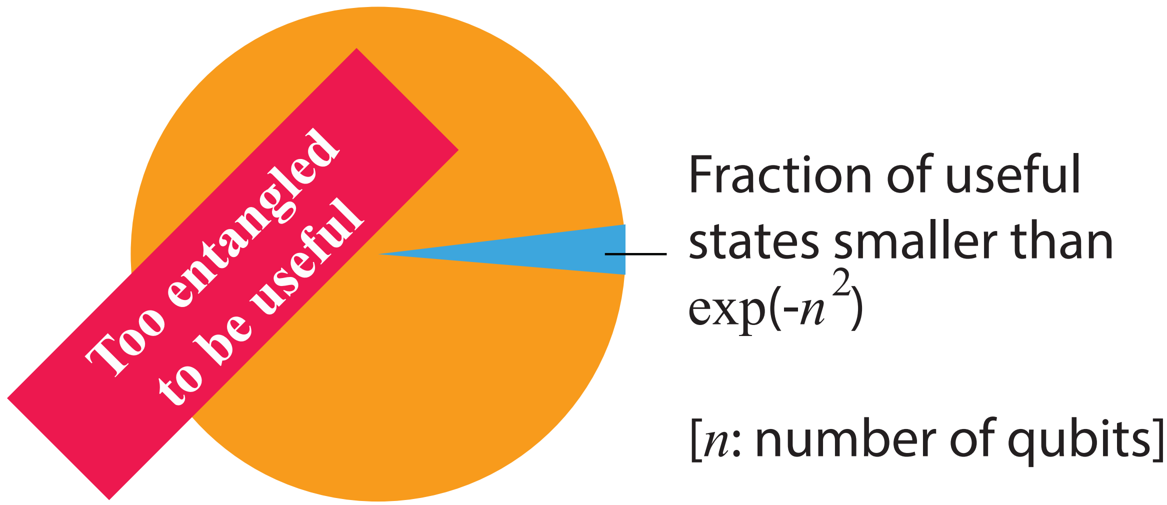
How rare are MBQC resource states?

1. MBQC resource states are rare



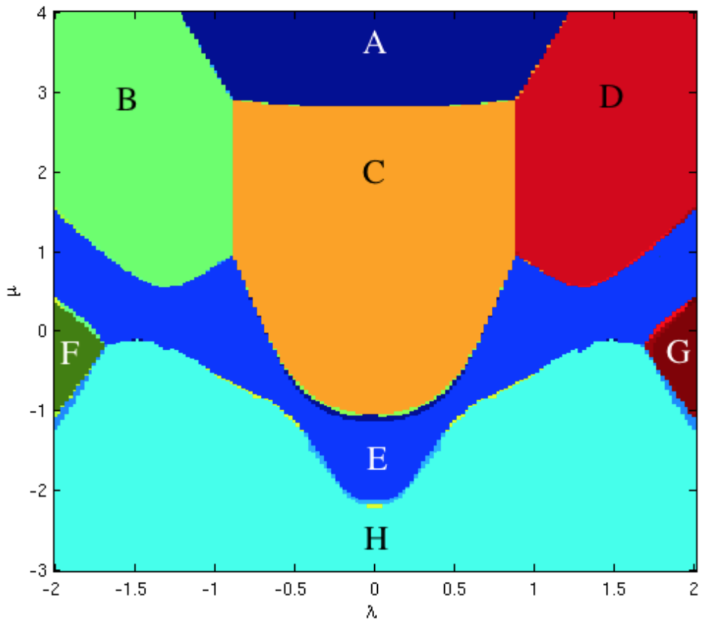
D. Gross, S.T. Flammia, J. Eisert, PRL 2009.

1. MBQC resource states are rare



D. Gross, S.T. Flammia, J. Eisert, PRL 2009.

What about systems with symmetry?

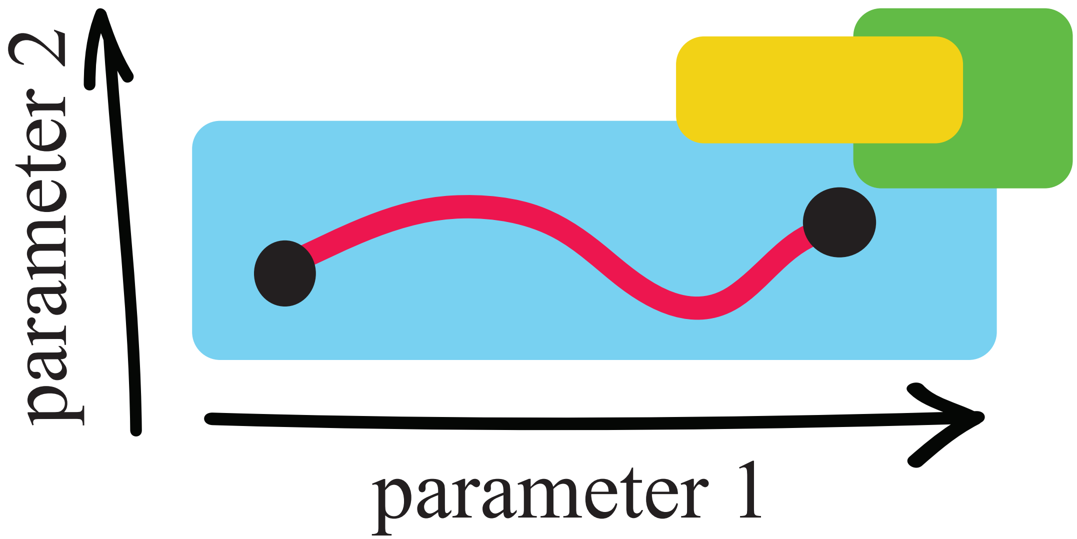


In the presence of symmetry

- Computational power is uniform across physical phases (known in 1D, conjectured beyond).
- Computationally useful quantum states are no longer rare.

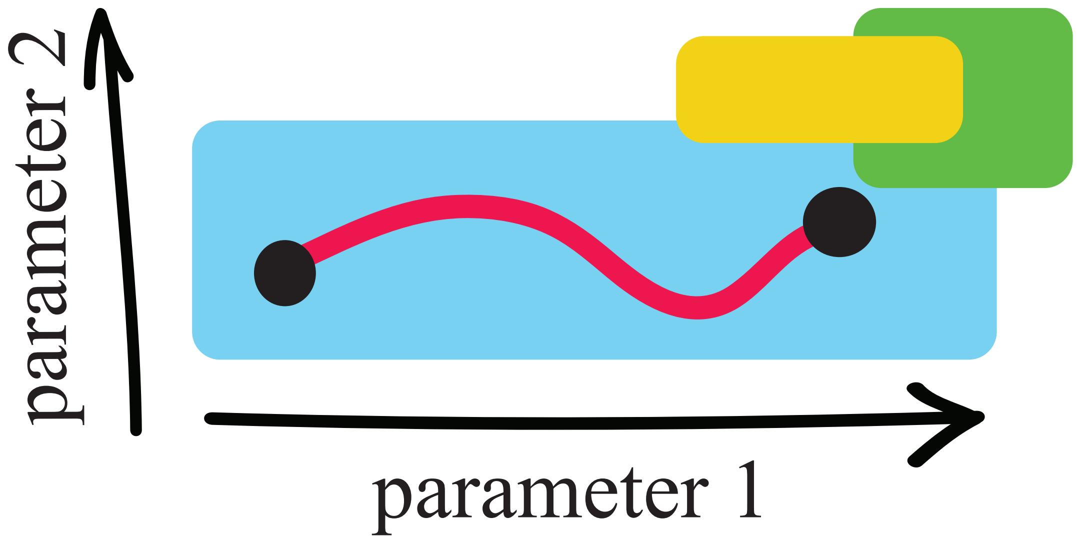
Symmetry-protected topological order

Definition of SPT phases:



We consider ground states of Hamiltonians that are invariant under a symmetry group G.

Symmetry-protected topological order

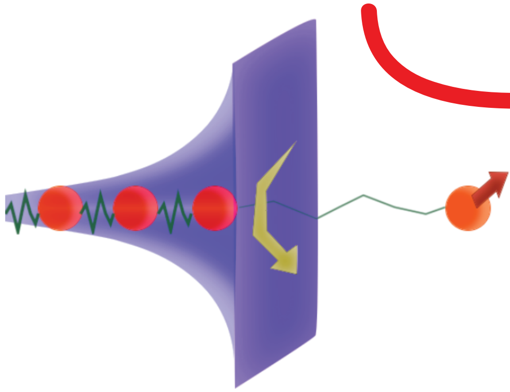


Two points in parameter space lie in the same SPT phase iff they can be connected by a path of Hamiltonians such that

1. At every point on the path, the corresponding Hamiltonian is invariant under G.
2. Along the path the energy gap never closes.

2. Symmetry protects computation

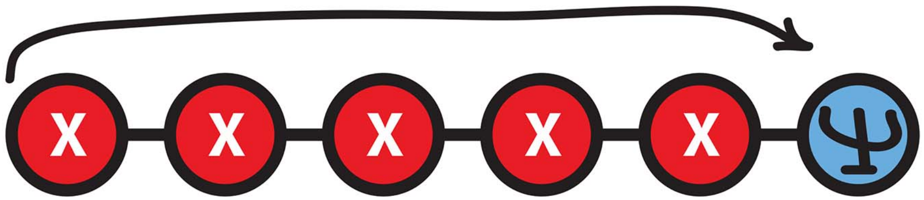
we observe low-maintenance features of the ground-code MQC in that this computation is doable without an exact (classical) description of the resource ground state as well as without an initialization to a pure state. It



It turns out these features are deeply intertwined with the physics of the 1D Haldane phase (cf. Fig. 1), that is well characterized as the symmmetry-protected topological order in a modern perspective [6, 7]. We believe our approach must bring the study of MQC, conventionally based on the analysis of the model entangled states (e.g., [1, 8, 9]), much closer to the condensed matter physics, which is aimed to describe characteristic physics based on the Hamiltonian.

A. Miyake, Phys. Rev. Lett. 105, 040501 (2010).

3. Symmetry-protected wire in MBQC

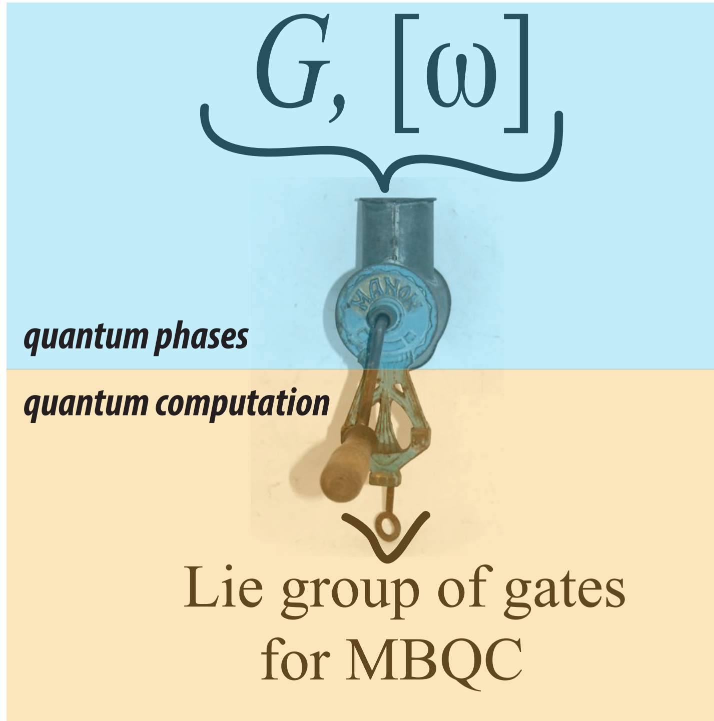


- Computational wire persists throughout symmetry-protected phases in 1D.
- Imports group cohomology from the classification of SPT phases.

D.V. Else, I. Schwartz, S.D. Bartlett and A.C. Doherty, PRL 108 (2012).

F. Pollmann *et al.*, PRB B 81, 064439 (2010); N. Schuch, D. Perez-Garcia, and I. Cirac, PRB 84, 165139 (2011); X. Chen, Z.-C. Gu, and X.-G. Wen, PRB 83, 035107 (2011).

4. The SPT⇒MBQC meat grinder



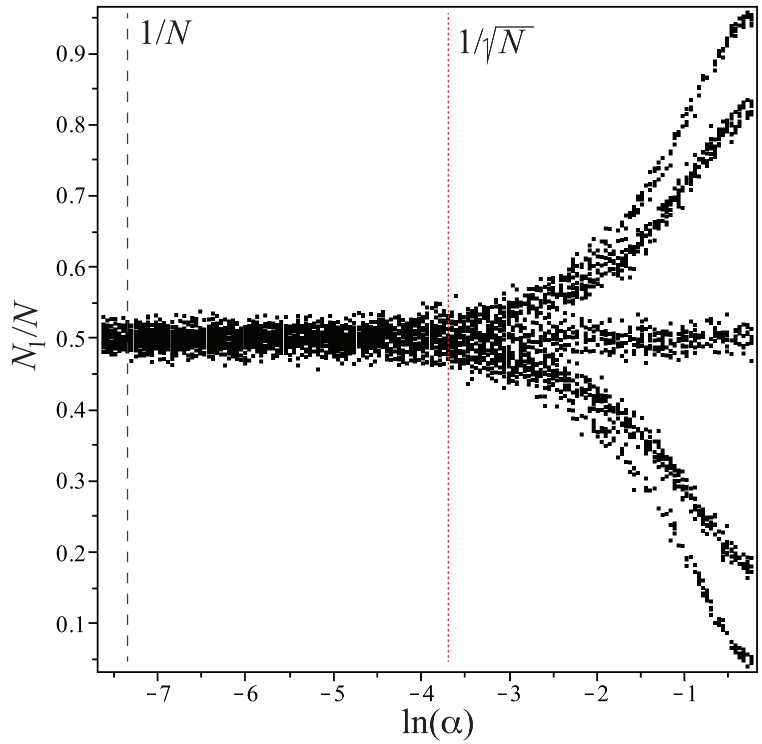
Hints at the classification of MBQC schemes by symmetry.

J. Miller and A. Miyake, Phys. Rev. Lett. 114, 120506 (2015) [first 1D comp. phase].

A. Prakash and T.-C. Wei, Phys. Rev. A (2016) [Wigner Eckart Theorem for MBQC].

RR, A.Prakash, D.-S. Wang, T.-C.Wei, D.T. Stephen, Phys. Rev. A (2017) [meat grinder].

Symmetry's work and asymmetry's contribution



In 1D (at least):

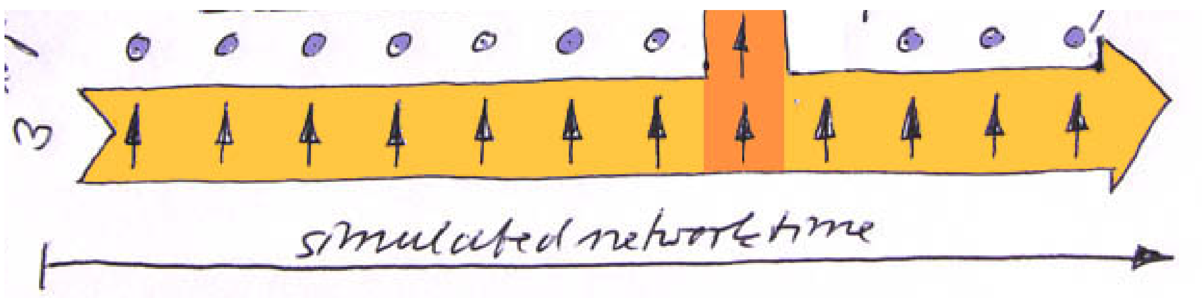
- MBQC schemes classified by symmetry
- MBQC schemes operated using symmetry breaking

Inspection

The above waypoints 2 - 4 are about 1D systems.

$\boxed{\text{1D is not sufficient for universal MBQC}}$

here is why:



- MBQC in spatial dimension D maps to the circuit model in dimension $D-1$

$\Rightarrow$ Require $D \geq 2$ for universality.

Are there computationally universal quantum phases in two dimensions?

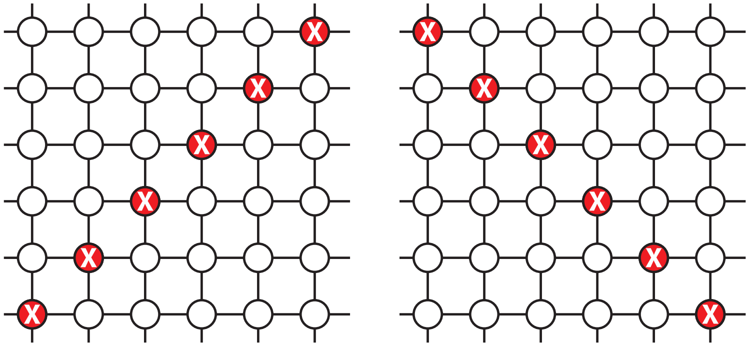
This talk describes one.

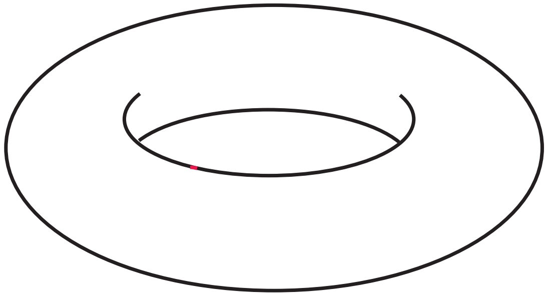
Part II:

A computationally universal SPT phase in 2D

Description of the 2D phase & result

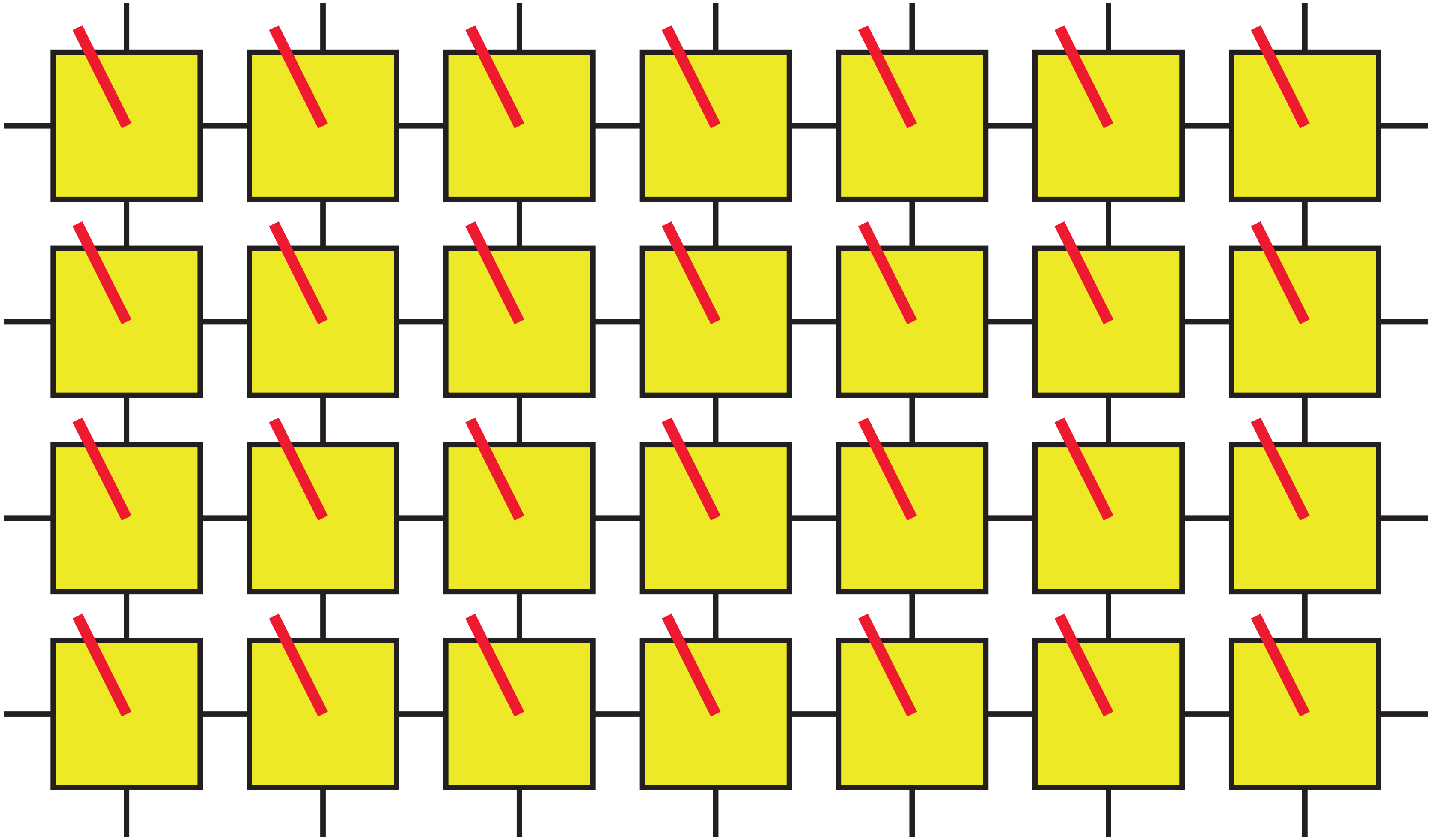
- The symmetries of the phase are





- The 2D cluster state is inside the phase

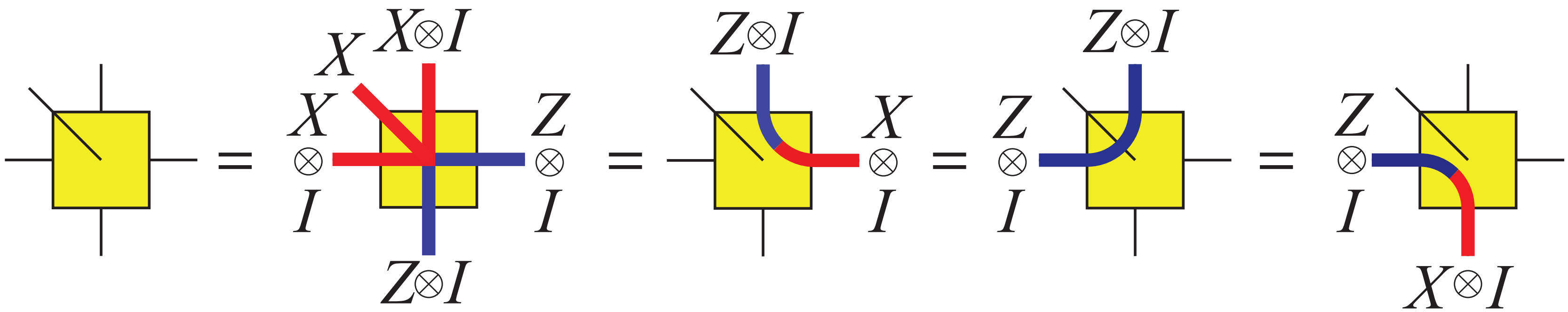
Result. For a spin-1/2 lattice on a torus with circumferences n and Nn, with n even, all ground states in the 2D cluster phase, except a possible set of measure zero, are universal resources for measurement-based quantum computation on $n/2$ logical qubits.



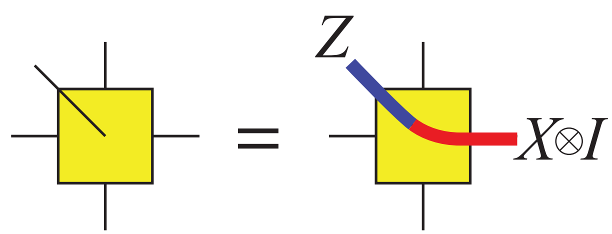
Consider MBQC resource states as tensor networks

Cluster-like states

... have PEPS tensors with the following symmetries



The cluster states have the additional symmetry



(We do not require the latter symmetry for cluster-like states)

Splitting the problem into halves

Part A:

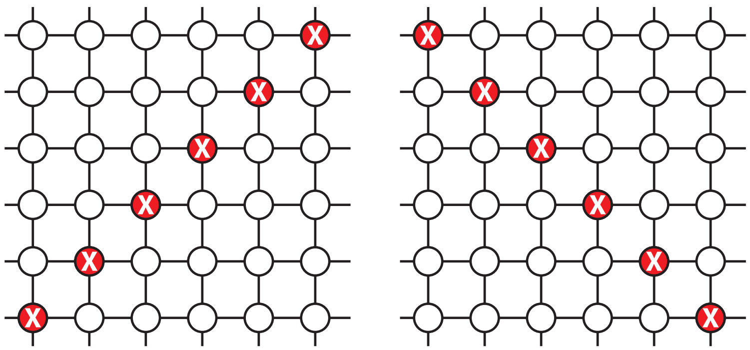
Lemma 1. All states in the 2D cluster phase are cluster-like.

Part B:

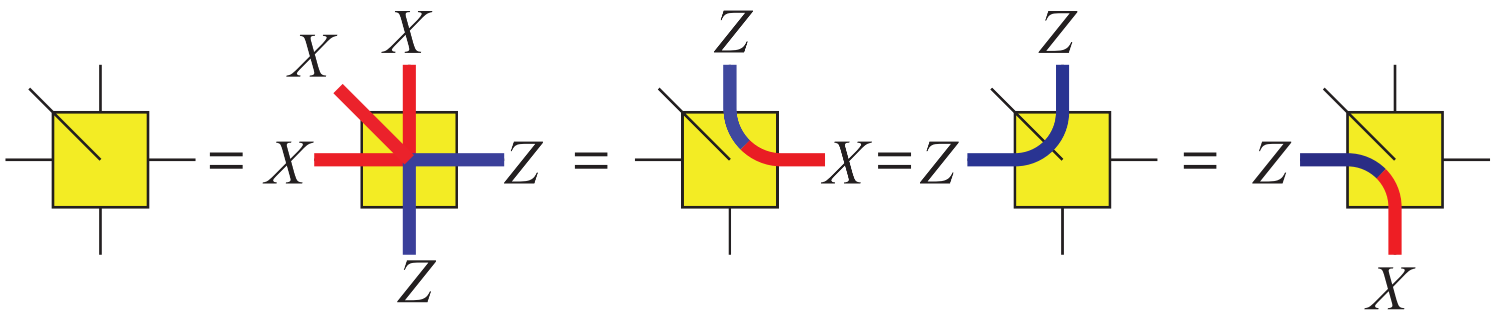
Lemma 2. All cluster-like states, except a set of measure zero, are universal for MBQC.

Part A: PEPS tensor symmetries

The physical symmetries



in the 2D cluster phase imply the local PEPS tensor symmetries,



A: In cluster phase $\Rightarrow$ cluster-like

Lemma 3. [*] Symmetric gapped ground states in the same SPT phase are connected by symmetric local quantum circuits of constant depth.

For any state $|\Phi\rangle$ in the phase,

$$|\Phi\rangle = U_k U_{k-1} .. U_1 \, |\text{2D cluster}\rangle.$$

Look at an individual symmetry-respecting gate in the circuit,

$$U = \sum_j c_j T_j, \text{ with } T_j \in \mathcal{P}.$$

Which Pauli observables T_j can be admitted in the expansion?

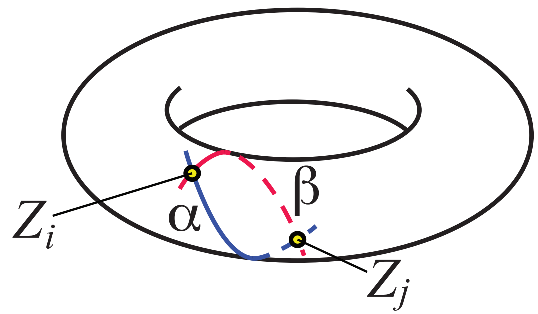
[*] X. Chen, Z.C. Gu, and X.G. Wen, Phys. Rev. B **82**, 155138 (2010).

A: In cluster phase $\Rightarrow$ cluster-like

Which Paulis T_j can be admitted in the expansion $U = \sum_j c_j T_j$?

$$X$$

$$\begin{array}{ccc} & Z & \\ Z & I & Z \\ & Z & \end{array}$$



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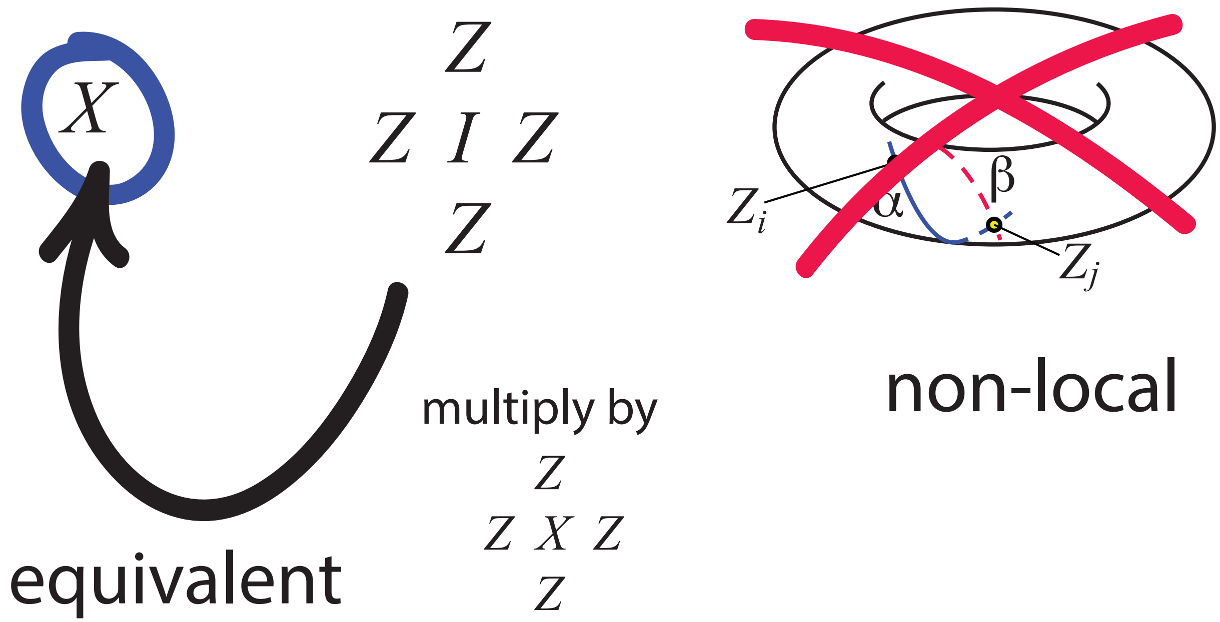
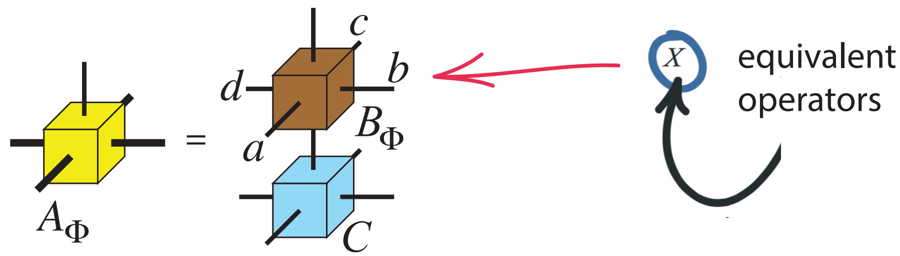
Which Paulis T_j can be admitted in the expansion $U = \sum_j c_j T_j$?

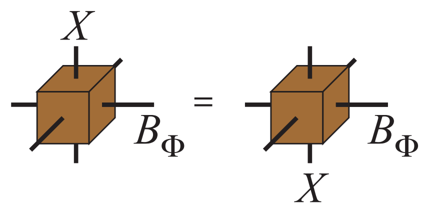


Only X-type Pauli operators survive in the expansion.

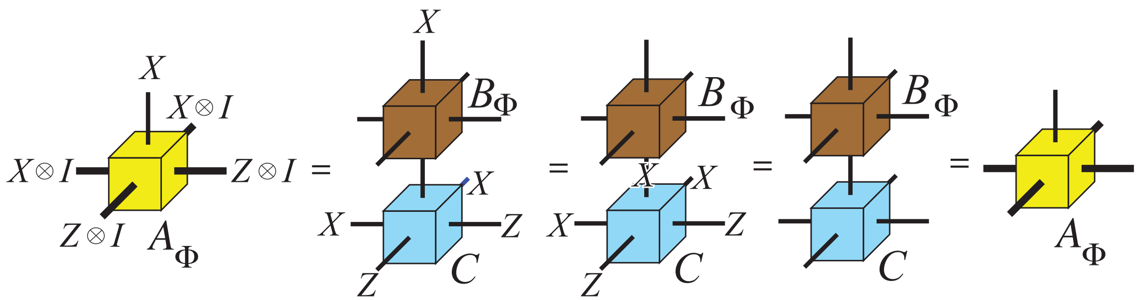
Description of the local tensors:



With



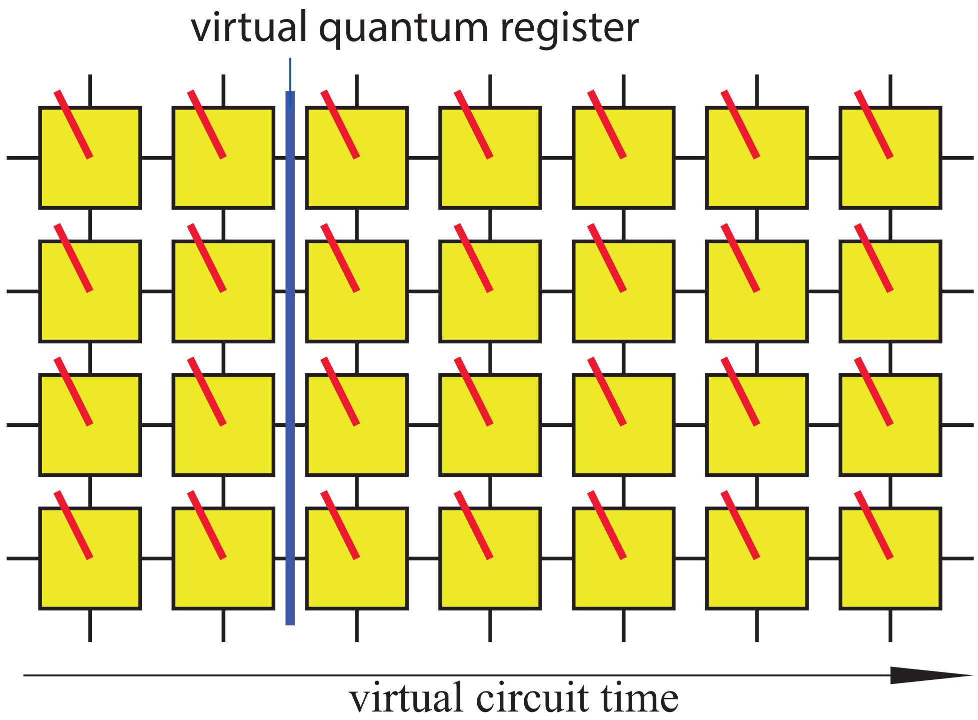
Hence



- Local tensors A_Φ describing $|\Phi\rangle$ are invariant under the cluster-like symmetries.

$\square$

Between Parts A and B

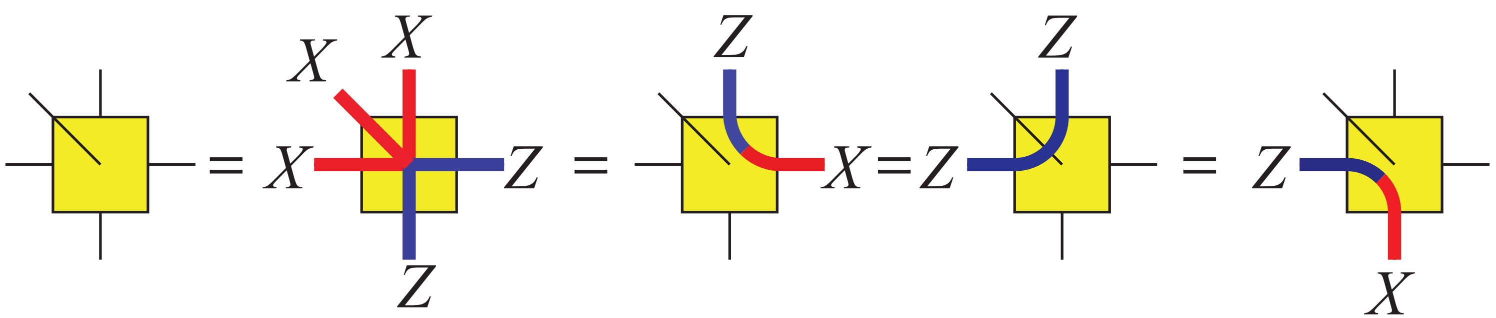


- The "virtual" quantum register is located on the horizontal tensor legs

D. Gross and J. Eisert, Phys. Rev. Lett. 98, 220503 (2007).

Part B: Symmetry Lego

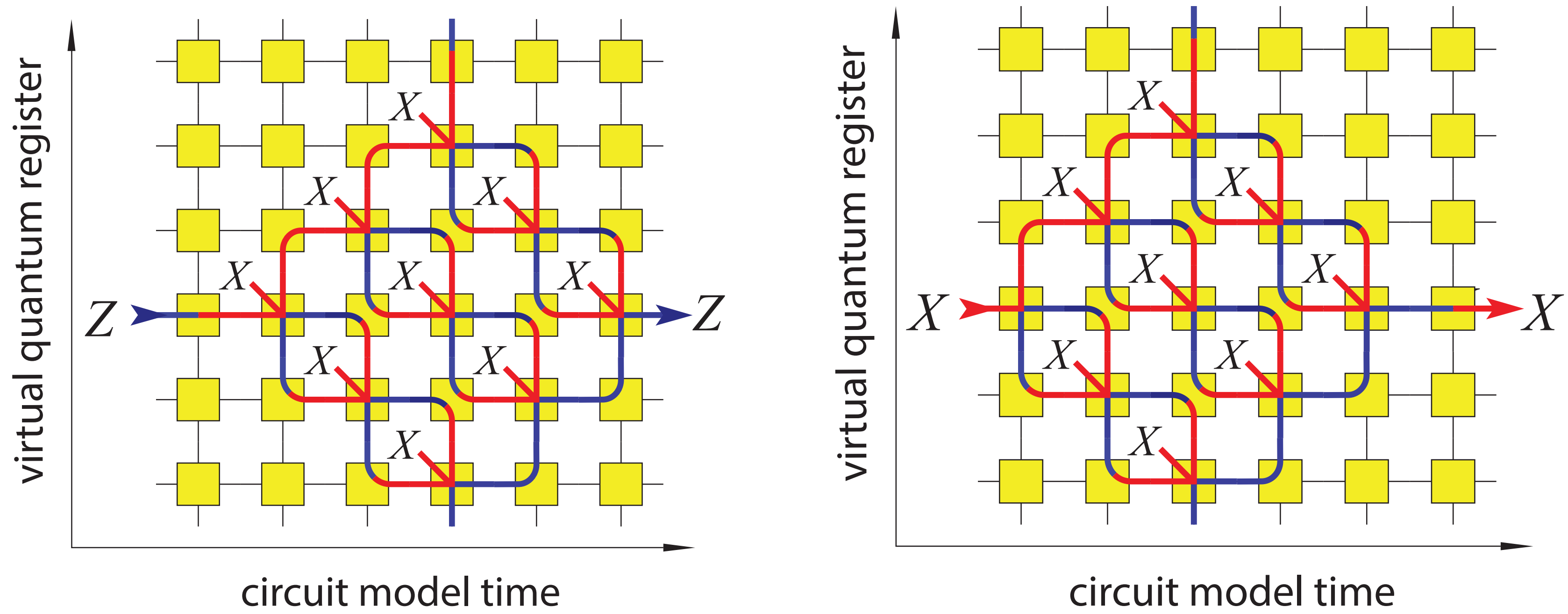
Just shown:
PEPS tensor symmetries hold throughout the 2D cluster phase



- Now weave them into larger patterns.

B: Cluster-like $\Rightarrow$ universal

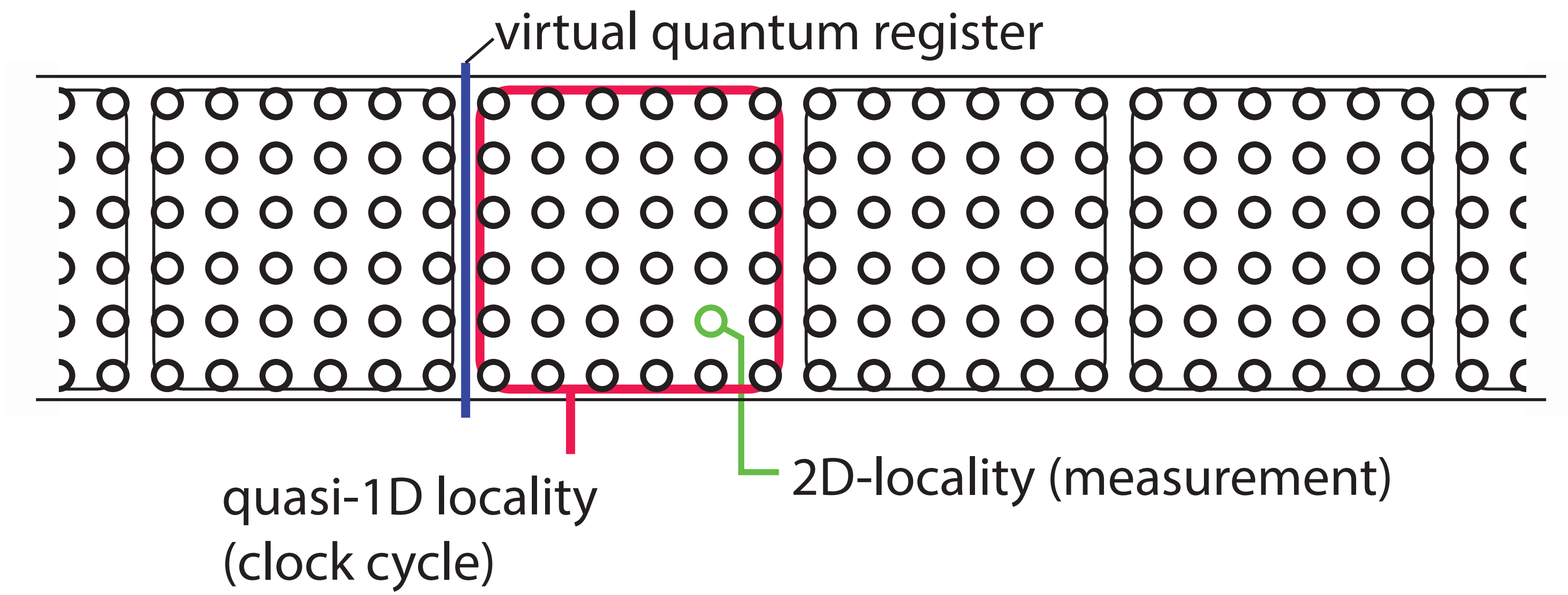
The clock cycle:



- Every logical operator is mapped back to itself after n columns (n = circumference).

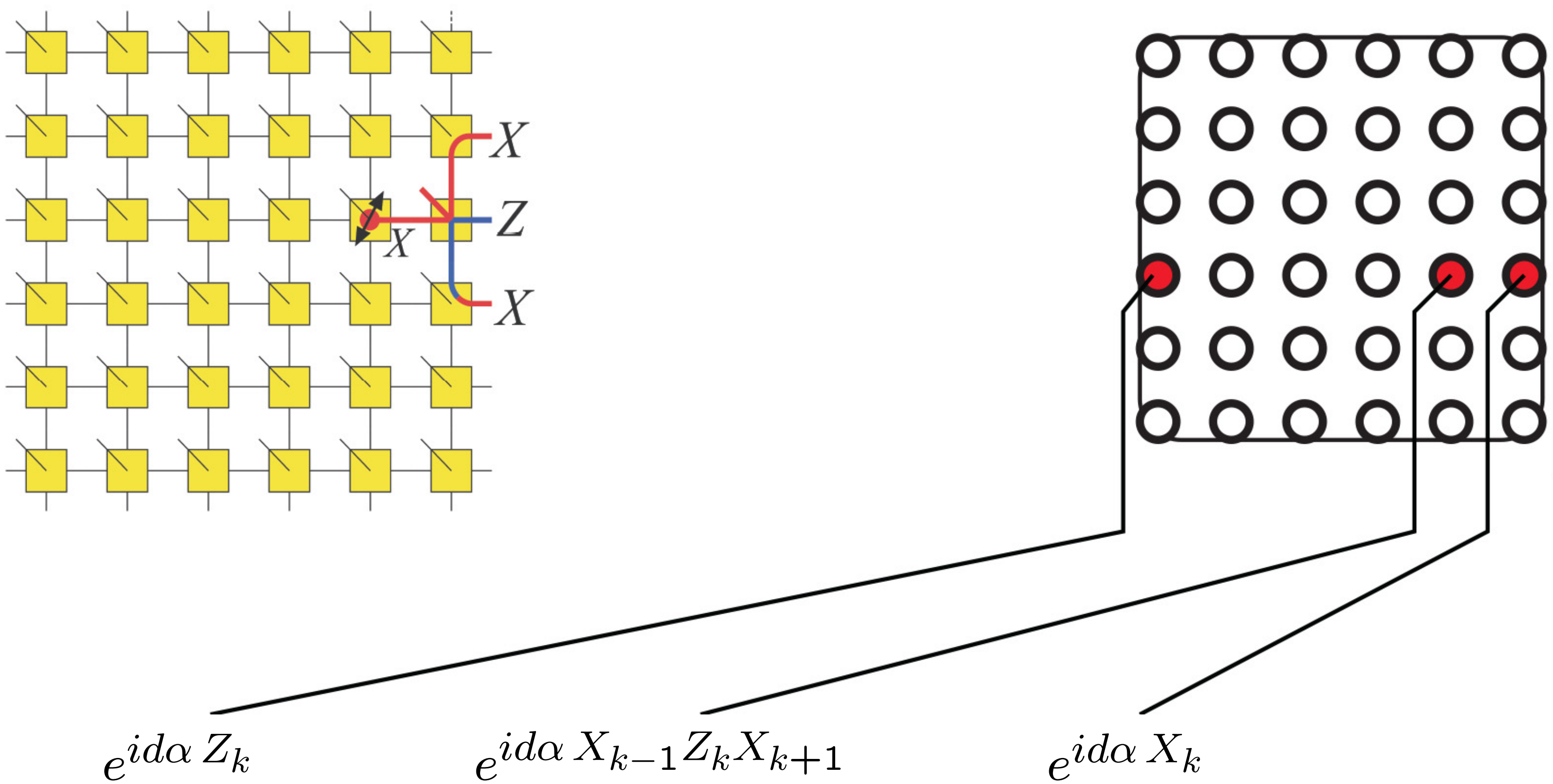
$\Rightarrow$ This defines the *clock cycle* for gate operation.

B: Cluster-like $\Rightarrow$ universal



- Map 2D system to effective 1D system

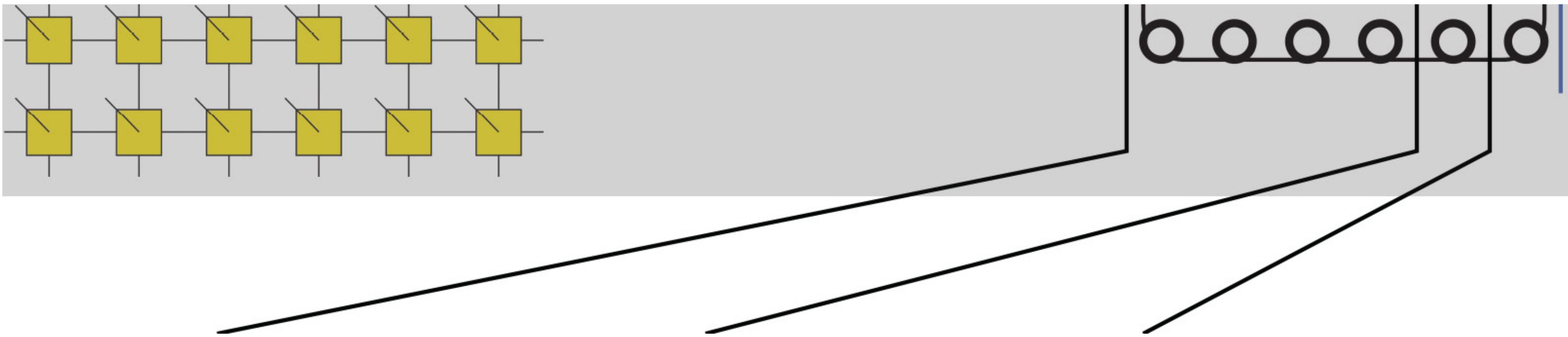
B: Cluster-like $\Rightarrow$ universal



$\boxed{\text{Universal gate set on } n/2 \text{ qubits}}$

B: Cluster-like $\Rightarrow$ universal

2D cluster state:



$e^{id\alpha\, Z_k}$ $\quad$ $e^{id\alpha\, X_{k-1}Z_kX_{k+1}}$ $\quad$ $e^{id\alpha\, X_k}$

Throughout the phase:

$e^{i|\nu|d\alpha\, Z_k}$ $\quad$ $e^{i|\nu|d\alpha\, X_{k-1}Z_kX_{k+1}}$ $\quad$ $e^{i|\nu|d\alpha\, X_k}$

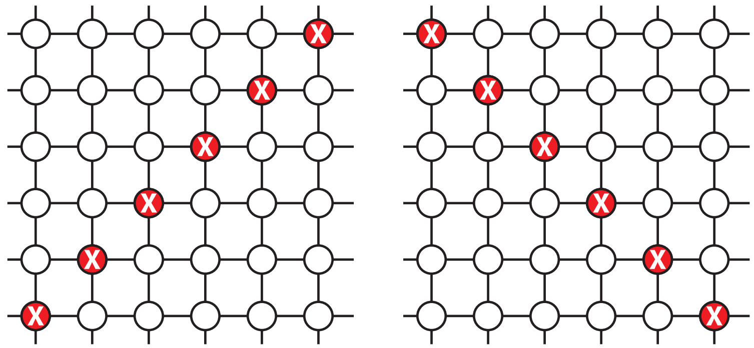
$$|\nu| \leq 1$$

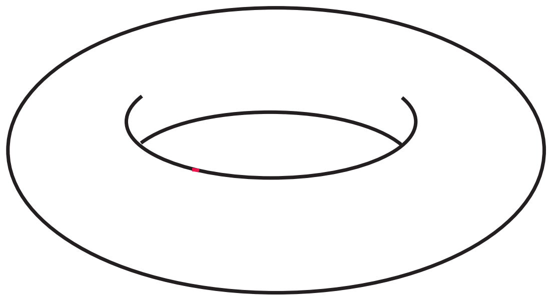
(ν depends on the location in the phase)

About ν: RR, A.Prakash, D.-S. Wang, T.-C.Wei, D.T. Stephen, Phys. Rev. A (2017).

Result

- The symmetries of the phase are





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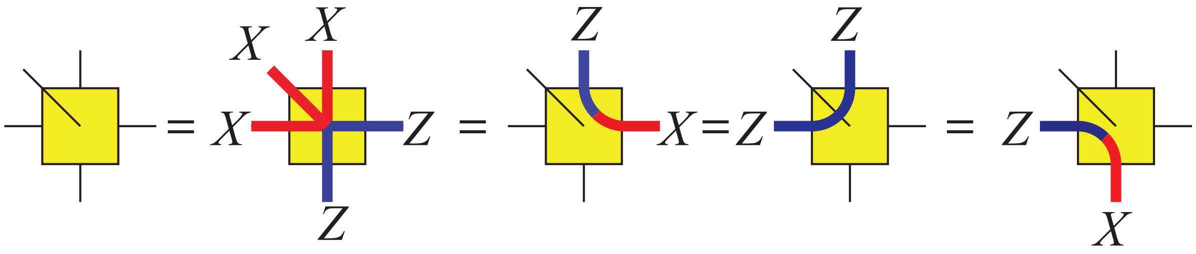
Summary and outlook

- There exists a symmetry-protected phase in 2D with uniform universal computational power for MBQC.
- *Goal:* Classification of MBQC schemes by symmetry.
- Symmetry Lego is fun—Try it!

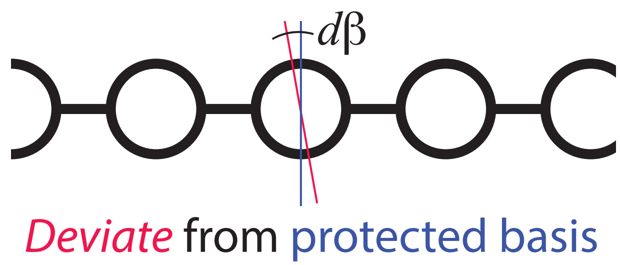
Phys. Rev. Lett. 122, 090501 (2019)

Also see: **Quantum 3, 142 (2019)**



The parameter ν

There is a complex-valued parameter ν, $|\nu| \leq 1$, that needs to be known about the location of the resource state within the phase.



For infinitesimal angles $d\beta$, this results in a logical rotation [*]

$$e^{i d\beta |\nu|\, T},$$

for some Pauli operator T. (E.g., $T = Z_k, X_k, X_{k-1} Z_k X_{k+1}$).

We require that $\nu \neq 0$.

[*] RR, D.-S. Wang, A. Prakash, T.-C. Wei, D.T. Stephen, PRA 96 (2017).