COVARIANT QEC

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Covariant Quantum Error Correction

Reference frame encoding, symmetries, and time evolution in holography

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

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OUTLINE

- arXiv:1709.04471 and arXiv:1902.07714
- Error correction of quantum reference frame information
- Covariant quantum error correction
- Approximate error correction
- Application to AdS/CFT and symmetries in quantum gravity and fault tolerance

} Equivalent

- With Philippe Faist, Sepehr Nezami, Victor Albert, Fernando Pastawski, Patrick Hayden, Sandu Popescu, John Preskill







REFERENCE FRAME

QUANTUM ERROR CORRECTION

Abstract classical information can be described using a sequence of symbols (e.g., 0,1)

Abstract quantum information can be stored in systems of qubits (or qudits, modes, etc.)

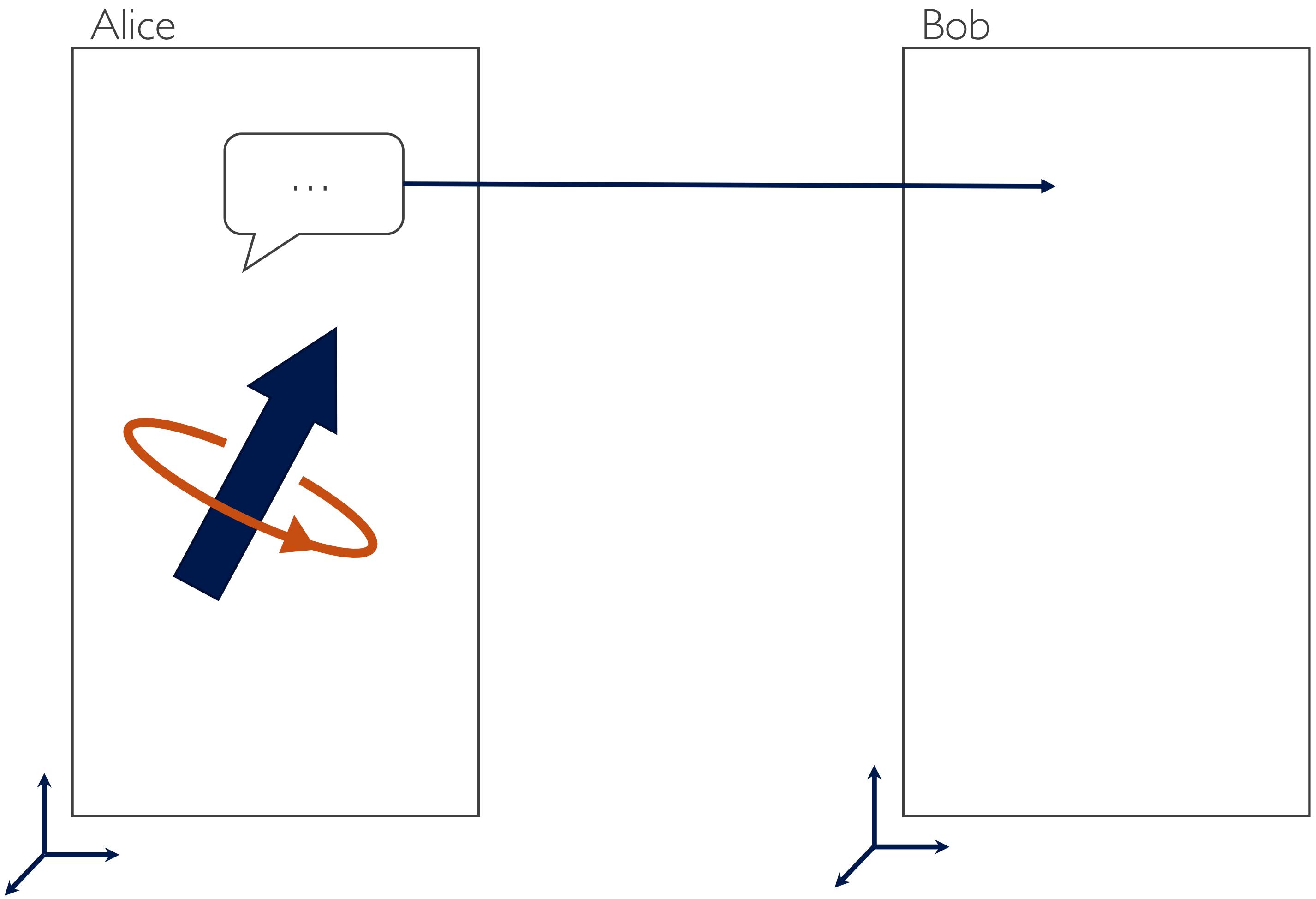
Physical information is any information that cannot be described in this abstract way (e.g., reference frames)

SHARED FRAMES

Alice wants to convey some "directional" information to Bob (e.g., the rotation axis of a gyroscope.)

Alice measures the the vector and describes the components to Bob in words

Bob can then create a gyroscope spinning along the same axis



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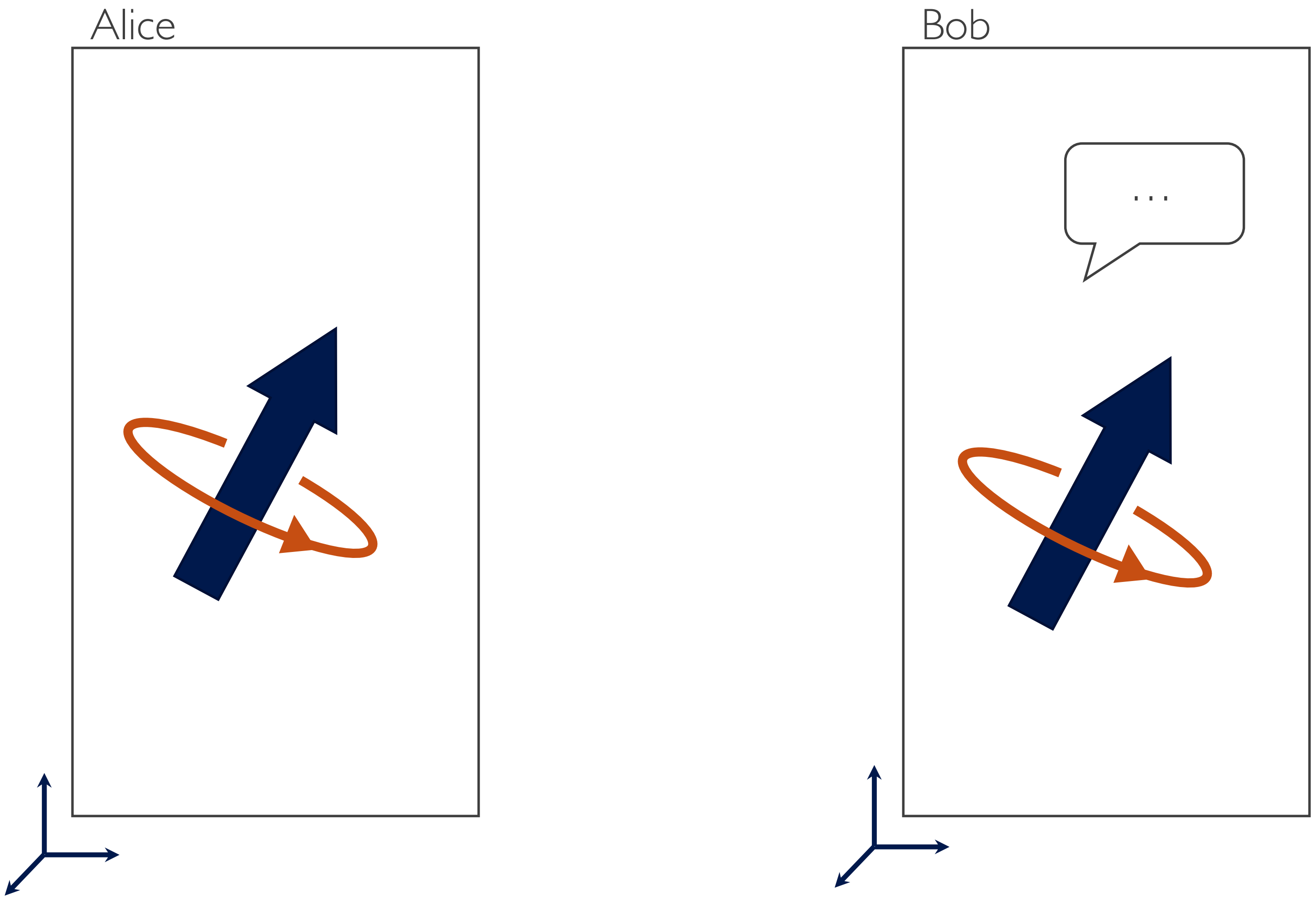


REFERENCE FRAME QEC

NO COMMON REFERENCE FRAME

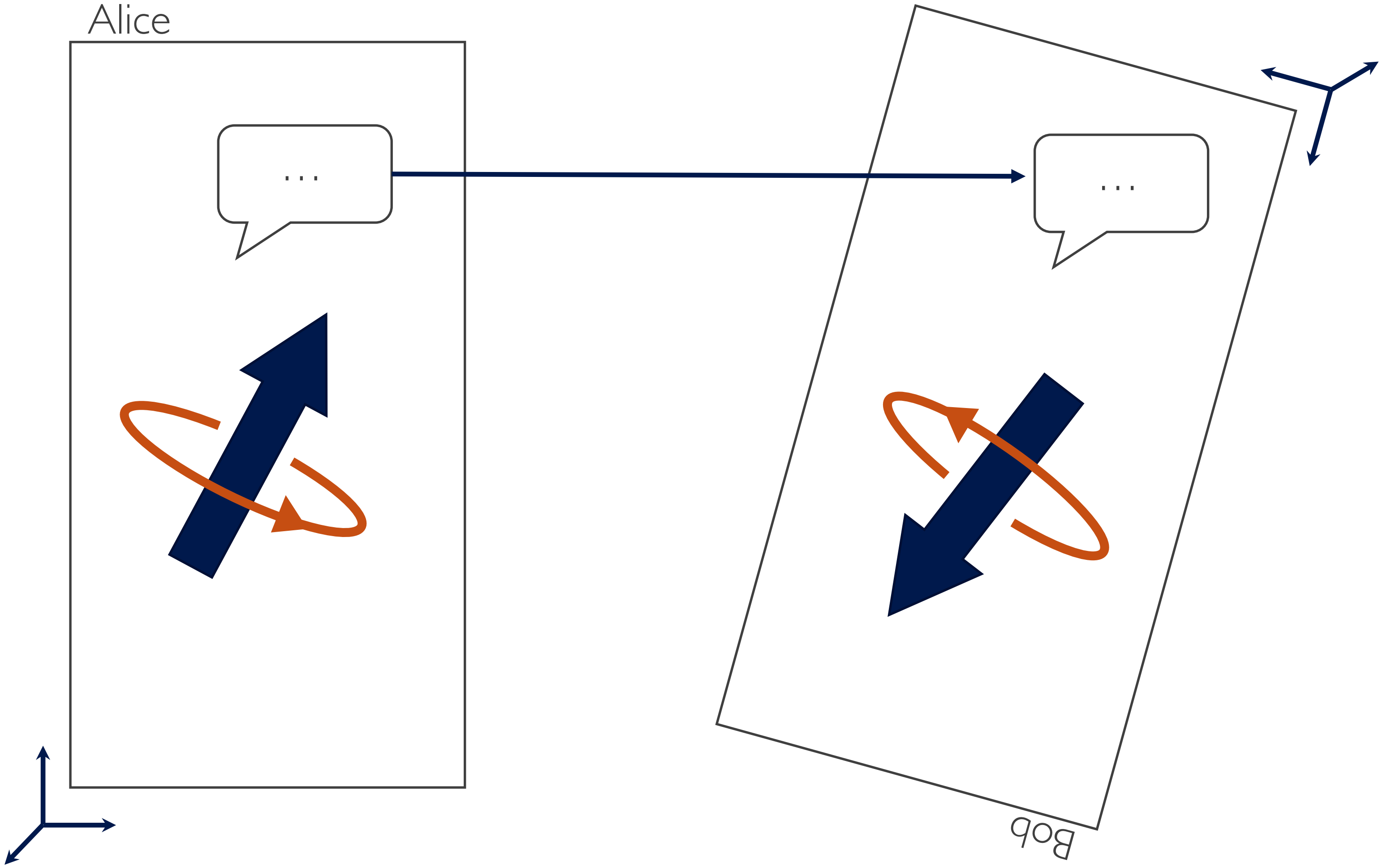
Now suppose Alice and Bob do not share a common reference frame

By sending a string of words to describe the vector, Bob will create the wrong vector

Alice and Bob fail this task.

What can we do in this situation?

What if the channel is noisy?



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Yes! Use a classical repetition code.

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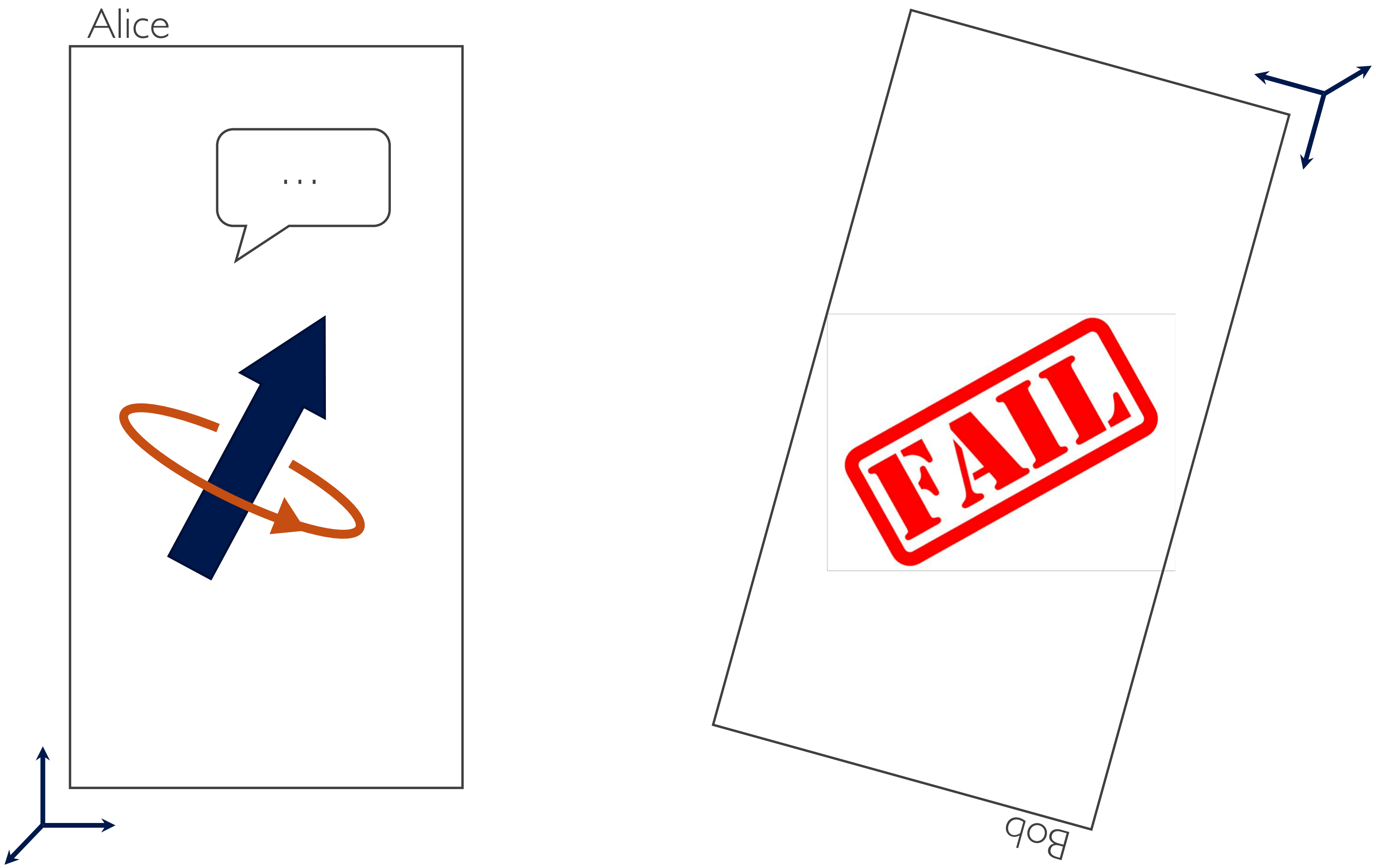

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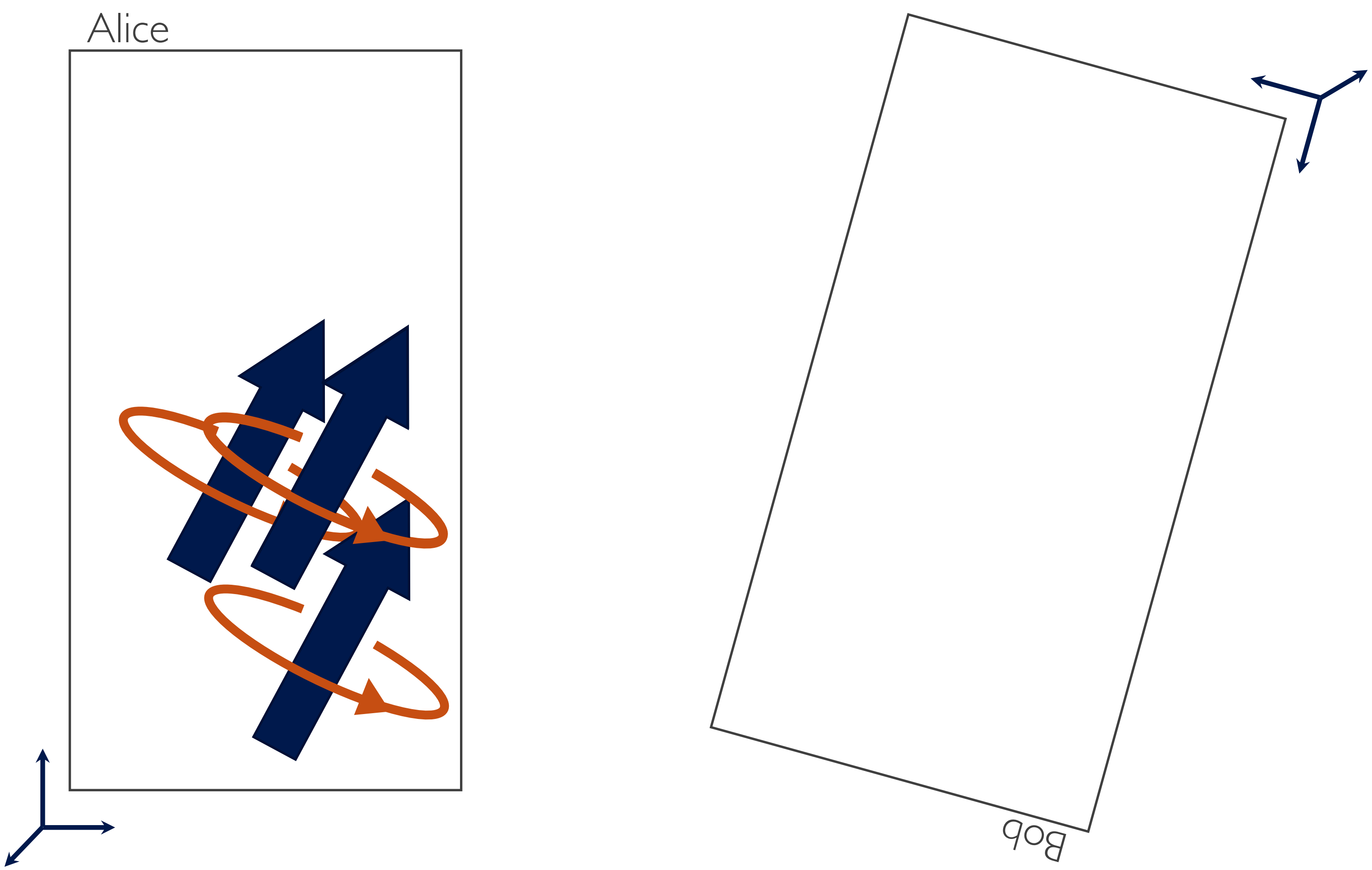

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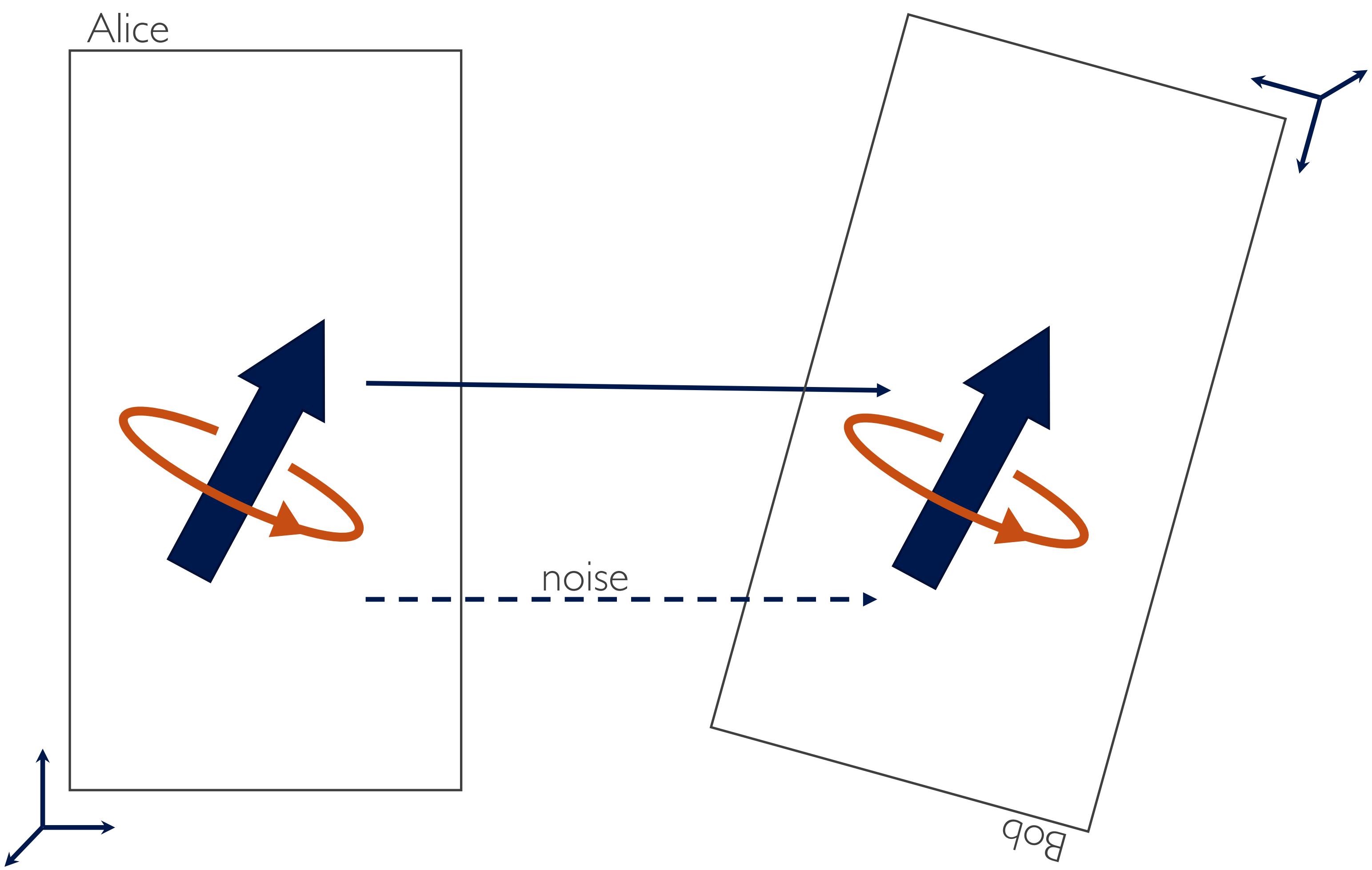

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QUANTUM REFERENCE FRAMES

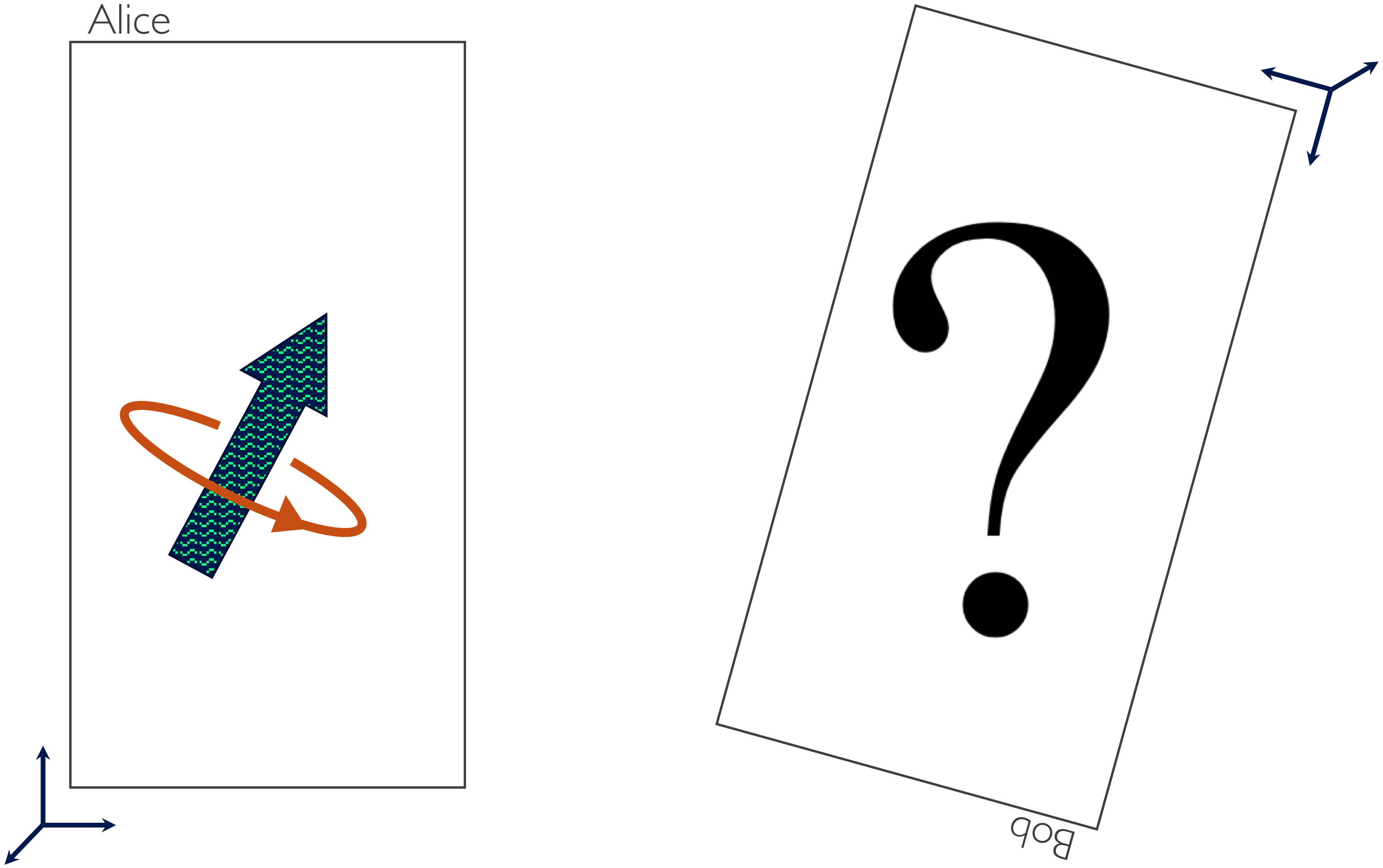
Now the information is *quantum* (e.g., a spin)

Is QEC possible for this type of information?

In which frame do we encode and decode?

In full generality, Alice and Bob are related by an *unknown* element of a symmetry group G

e.g., the rotation group SO(3) for reference frames, and U(1) for a shared time standard



COVARIANT QEC

- An encoding $\mathcal{E}$ from a logical input to $\boldsymbol{n}$ physical systems is *covariant* if

$$\mathcal{E}(U_{\text{in}}\rho_{\text{in}}U_{\text{in}}^{\dagger}) = U^{\otimes n}\mathcal{E}(\rho_{\text{in}})U^{\dagger\otimes n}$$

- The encoding operation *commutes* with the action of the symmetry group G.

- This requirement places severe constraints on $\mathcal{E}$, and in principle such a map need not exist

- If a covariant encoding *does* exist, it means we can correct errors:

$$\mathcal{R}_j[\text{Tr}_j(\mathcal{E}(\rho_{\text{in}}))] = \rho_{\text{in}}$$

- It turns out that the existence of a covariant code for a group G implies that one can error correct physical information that transforms under G.

RESULTS

EXISTENCE OF COVARIANT QUANTUM CODES

Symmetry group: / Code dimension:	Lie group (continuous symmetry)	Finite group
Finite dimensional code	NO-GO THEOREM	Explicit construction
Infinite dimensional code	Explicit construction	Explicit construction

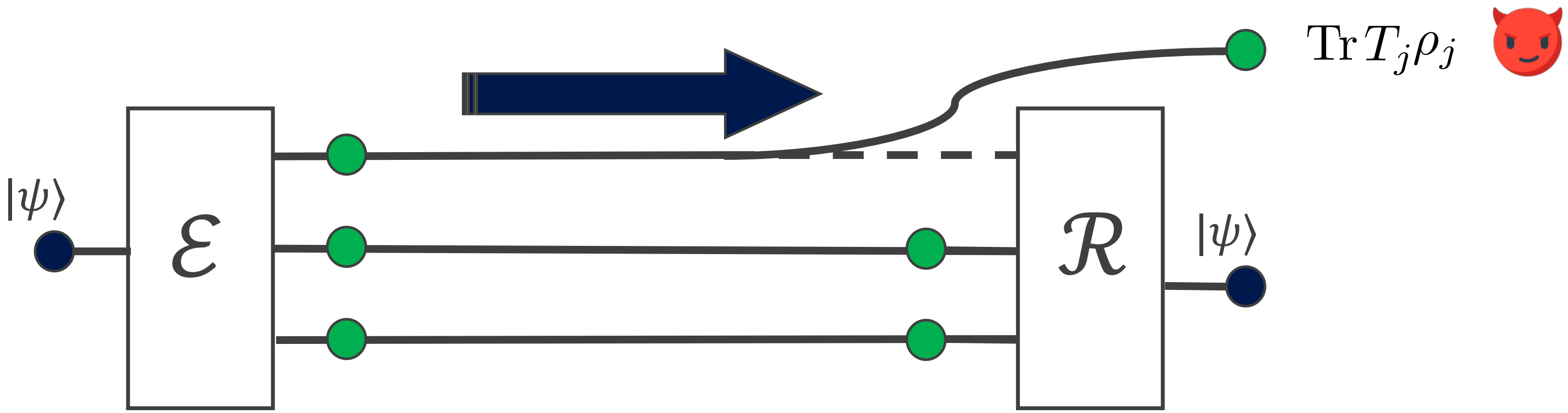
$$\mathcal{E}(U_{\rm in}\rho_{\rm in}U_{\rm in}^{\dagger}) = U^{\otimes n}\mathcal{E}(\rho_{\rm in})U^{\dagger\otimes n}$$

Possible
Impossible

SKETCH OF NO-GO THEOREM

- Continuous symmetry (e.g., Lie Group), with at least one infinitesimal generator and conserved charge
- Logical generator T_L and physical generator T_A, assumed to be nontrivial
- On the physical space, the generator is a *sum of local terms* $T_A = \sum_i T_i$

- Physical subsystem j is erased and given to the environment
- The environment can measure T_j and *learn* some information about the charge
- But for an erasure correcting code, each reduced state ρ_j should be *independent* of the input



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$$\text{const}_i = \text{Tr}(T_i \rho_i) = \text{Tr}(T_i \mathcal{E}(\rho_{\text{in}})) = \text{Tr}(\mathcal{E}^\dagger(T_i)\rho_{\text{in}})\ \forall \rho_{\text{in}}$$

$$\implies \mathcal{E}^\dagger(T_i) \propto I \implies \mathcal{E}^\dagger(T_A) \propto I$$

To avoid a contradiction, the generators must be trivial

$$T_L \propto I$$

APPROXIMATE

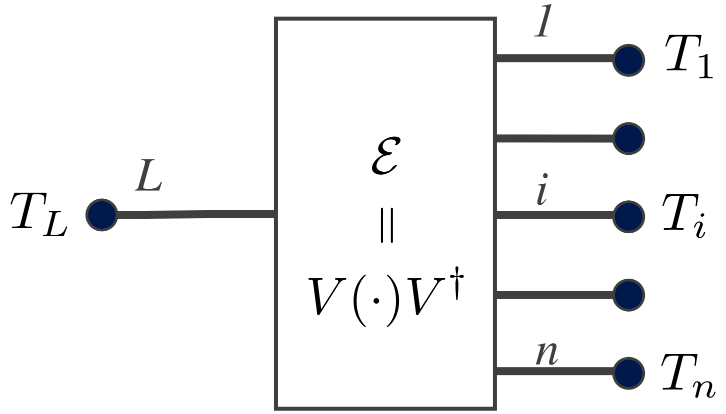
QUANTUM ERROR CORRECTION

What if we only need to recover *approximately*?

What if we only need approximate covariance?

Can we find an approximate Eastin-Knill Theorem?

MAIN THEOREM (one of several)



$T_A = \sum_i T_i$

noise channel $\mathcal{N}$

The error of the isometric encoding $\mathcal{E}(\cdot) = V(\cdot)V^\dagger$ is bounded below by

$$\epsilon_{\text{worst}}(\mathcal{N} \circ \mathcal{E}) \geq \frac{1}{2n} \frac{\Delta T_L}{\max_i \Delta T_i}$$

$$\epsilon_{\text{e}}(\mathcal{N} \circ \mathcal{E}) \geq \frac{1}{n} \frac{\|T_L - \text{Tr}(T_L)\mathbb{1}_L/d_L\|_1/(2d_L)}{\max_i \Delta T_i}$$

$$\epsilon(\mathcal{N} \circ \mathcal{E}) := \sqrt{1 - F(\mathcal{R} \circ \mathcal{N} \circ \mathcal{E})^2}$$

Good codes: $\epsilon \approx 0$

$$\Delta T_i = \max(\text{eig}(T_i)) - \min(\text{eig}(T_i))$$

≈

SYMMETRIES IN ADS/CFT

BULK GLOBAL SYMMETRIES AND TIME

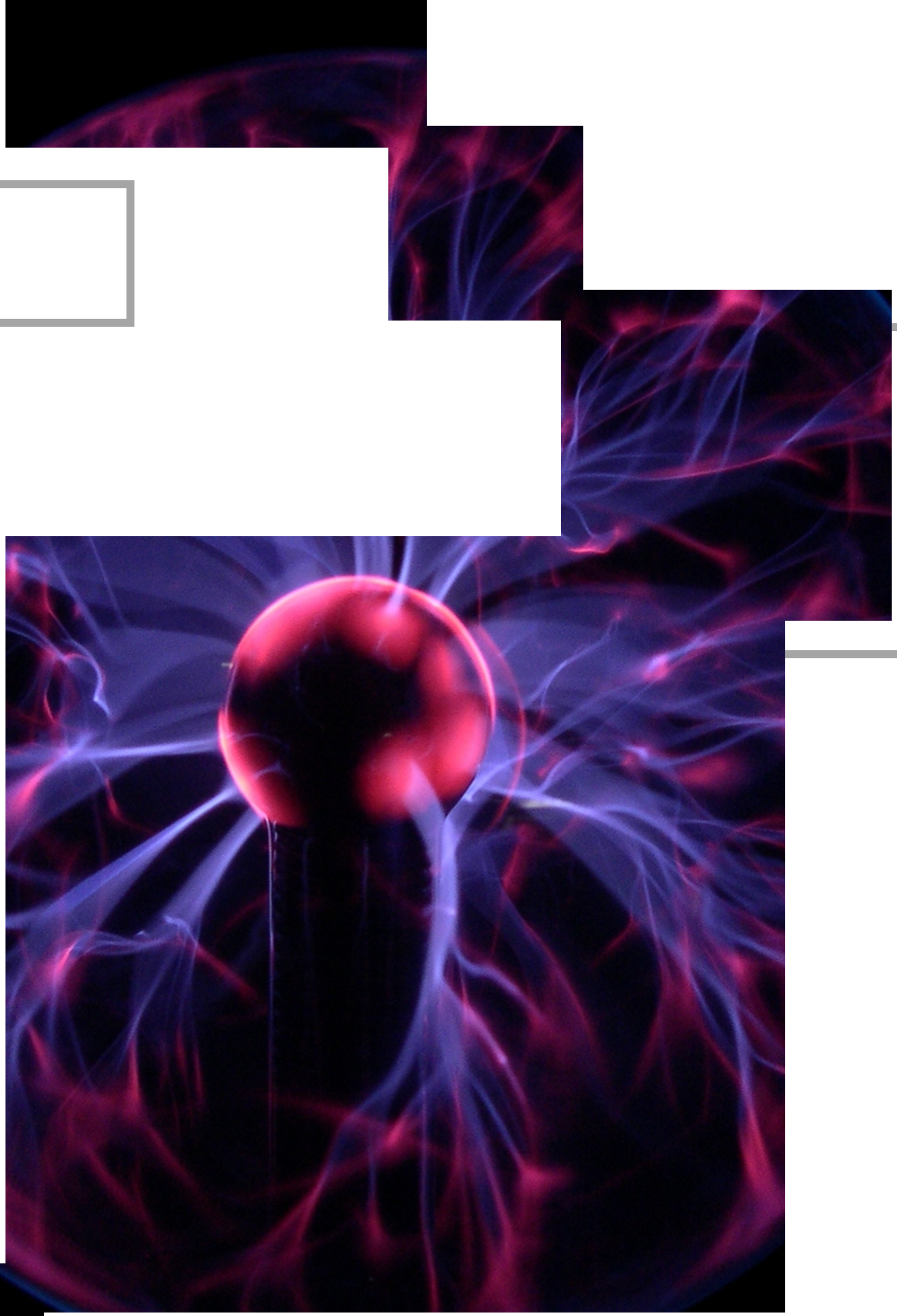
Standard lore in AdS/CFT:

"No bulk global symmetries"

Bulk-to-boundary map defines a QECC

Bulk time evolution = boundary time evolution

Apparent violation of Eastin-Knill?

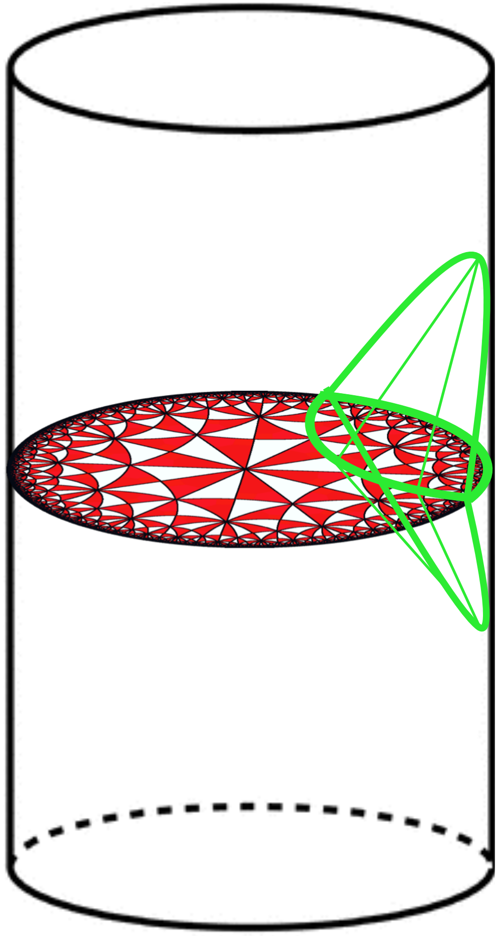


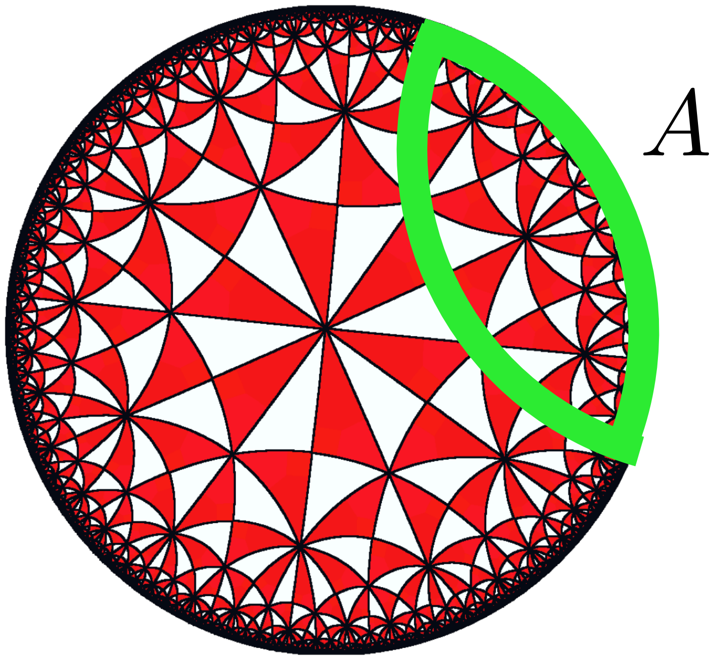
ADS/CFT IN ONE SLIDE

Gravity in AdS in d+1 dimensions

dual to

Conformal field theory in d dimensions





Consider a boundary subregion A, constructed by tracing over $\bar{A}$

We can try to find all bulk operators expressible with support only on A

There is a procedure for mapping bulk operators to boundary operators

This mapping from bulk to boundary defines a QECC!

Harlow, Daniel, and Hirosi Ooguri. "Symmetries in quantum field theory and quantum gravity." *arXiv preprint arXiv:1810.05338* (2018).

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NO GLOBAL BULK SYMMETRIES

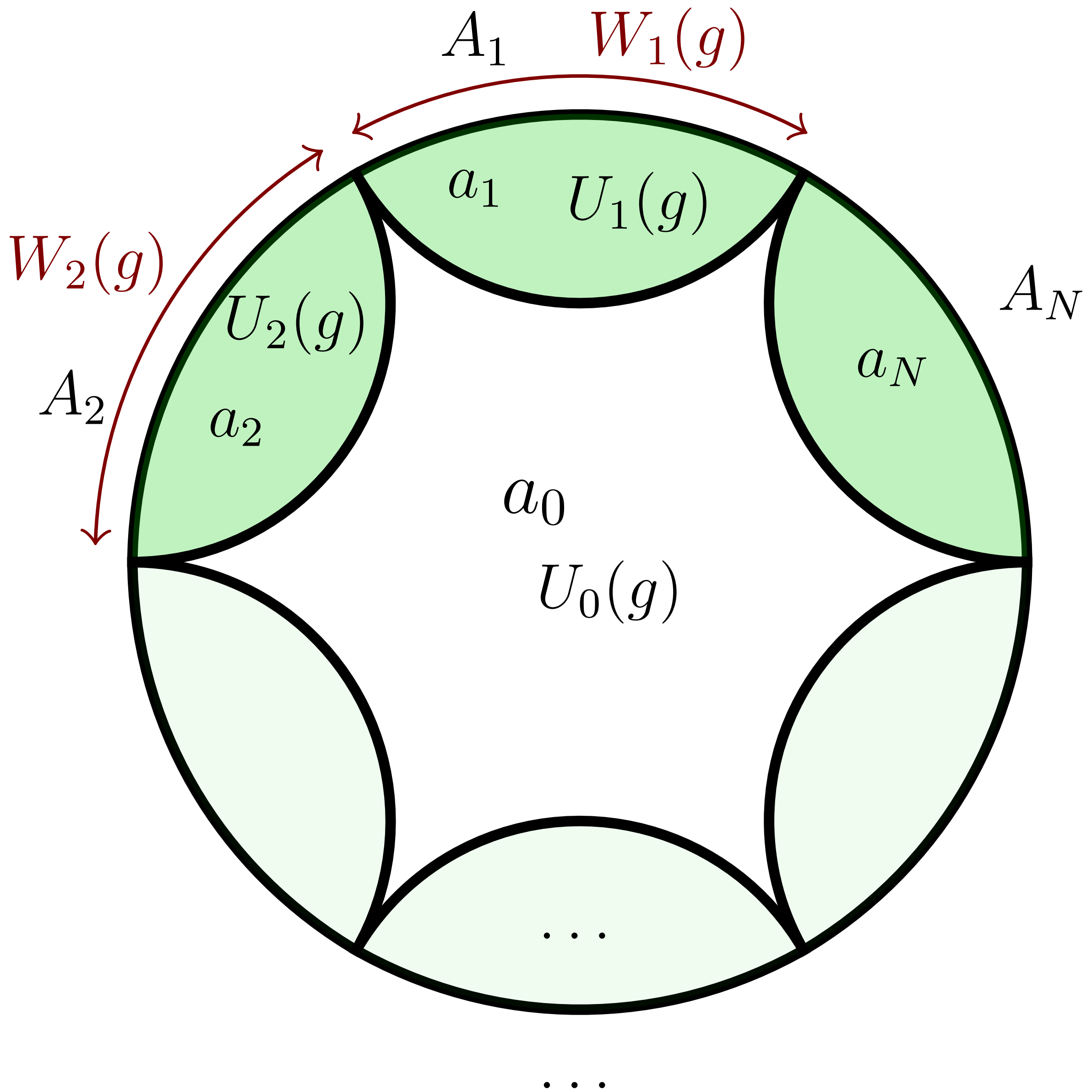
IN ADS/CFT

- Boundary CFT decomposed into N regions $\{A_i\}_{i=1}^{N}$
- Bulk has corresponding decomposition into entanglement wedges $\{a_i\}_{i=0}^{N}$
- Bulk global symmetry: product of local unitaries

$$U_L(g) = \bigotimes_{i=0}^{N} U_i(g)$$

- Mapped to boundary. Splittable CFT $\Rightarrow$ local unitaries

$$U_{\text{CFT}}(g) = \bigotimes_{i=1}^{N} W_i(g)$$



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MORE DETAIL

- The boundary CFT is **splittable**: $U_{\mathrm{CFT}}(g) = \bigotimes_{i=1}^{N} W_i(g)$
  - Boundary operators can be decomposed

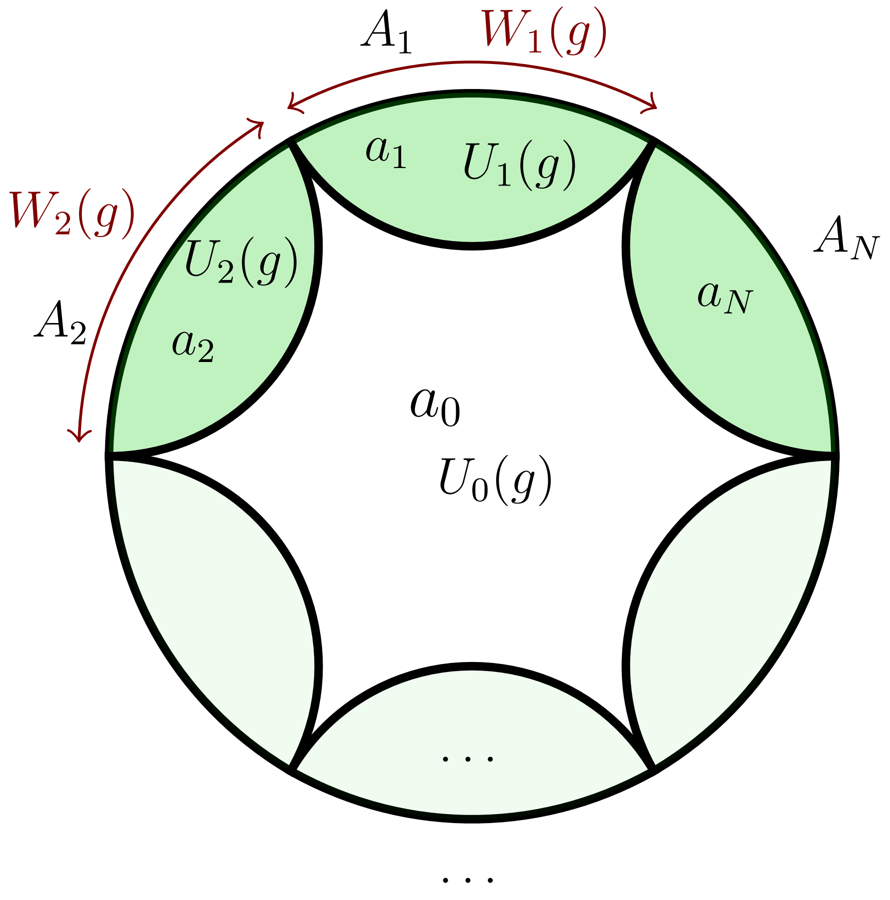
- **Erasure** of A_i is a **correctable error** for a_0.

- The code space for AdS/CFT is **low energy** excitations above the vacuum:

$$\mathcal{H}_{\mathrm{code}} = \mathrm{span}\{|\Omega\rangle, \hat{\phi}(x)|\Omega\rangle, \hat{\phi}(x)\hat{\phi}(y)|\Omega\rangle, \ldots\}$$

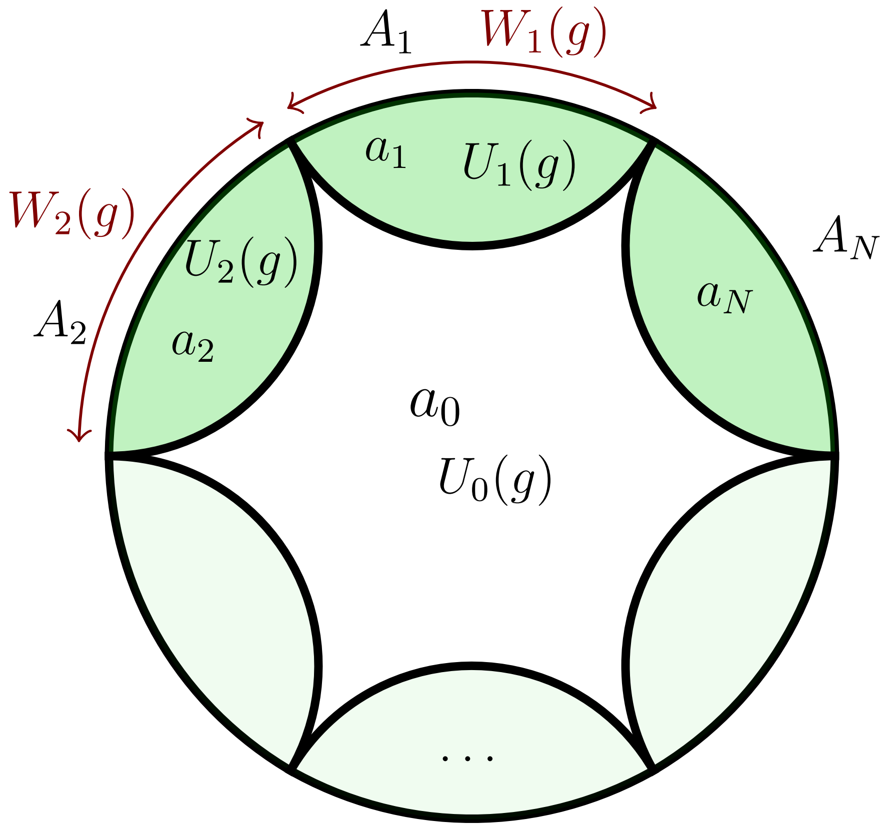
- The W_i are argued to be **low energy**. Therefore they preserve the code space and are **logical operators**

- **Logical operators** that are also **correctable errors** must be **trivial**



TIME EVOLUTION COVARIANCE

- Bulk time evolution = boundary time evolution
- The mapping is covariant w.r.t. this symmetry
- Why does this not violate the previous slide?



- **Gravitational dressing**, previously ignored, is crucial
- This dressing transforms non-trivially under the group action
  - This transformation implements bulk time evolution
- The dressing can be detected locally. Hence, the error correction must be **approximate** and **covariant**