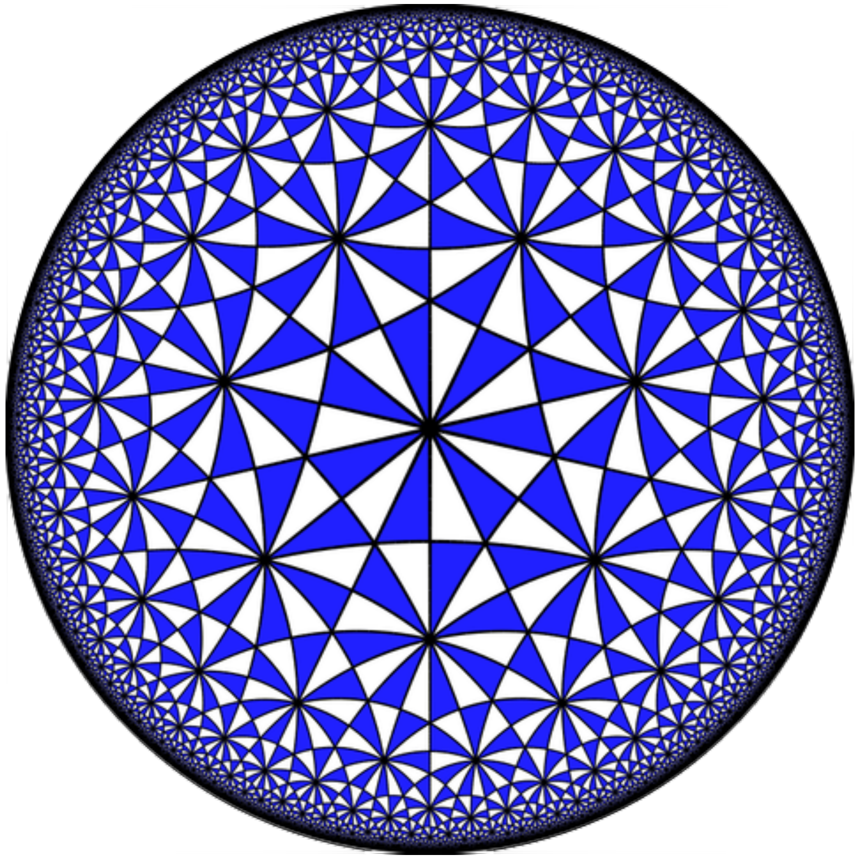
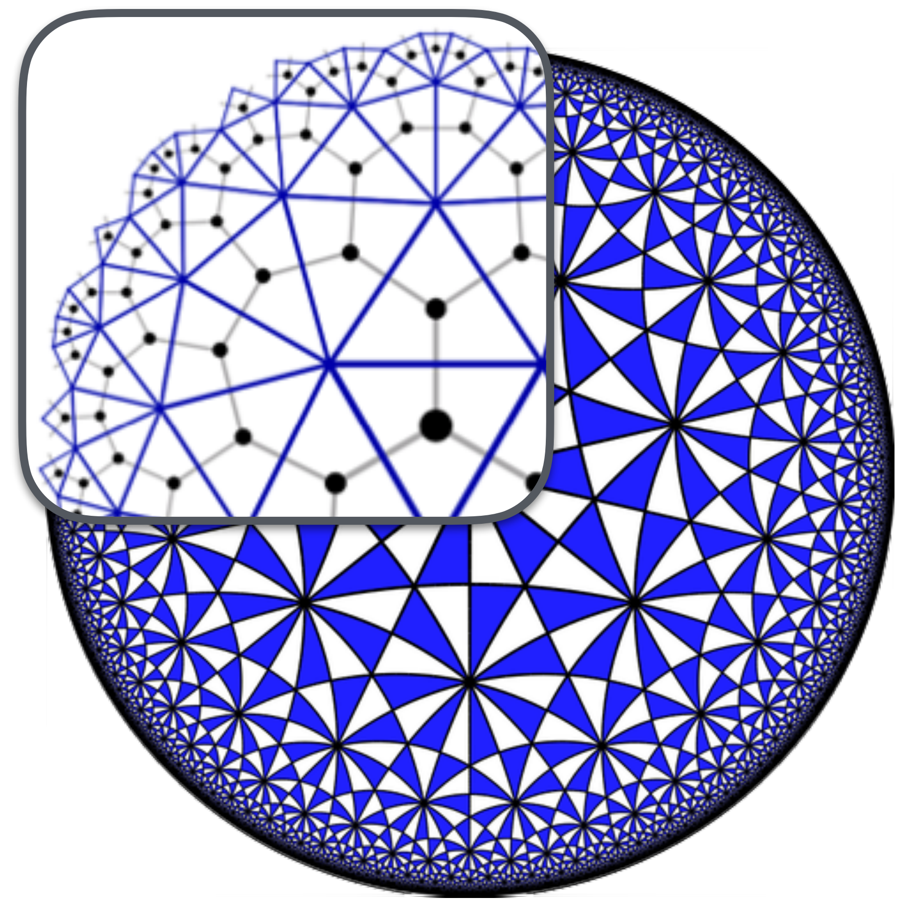
HOLOGRAPHY AND MATCHGATE TENSOR NETWORKS



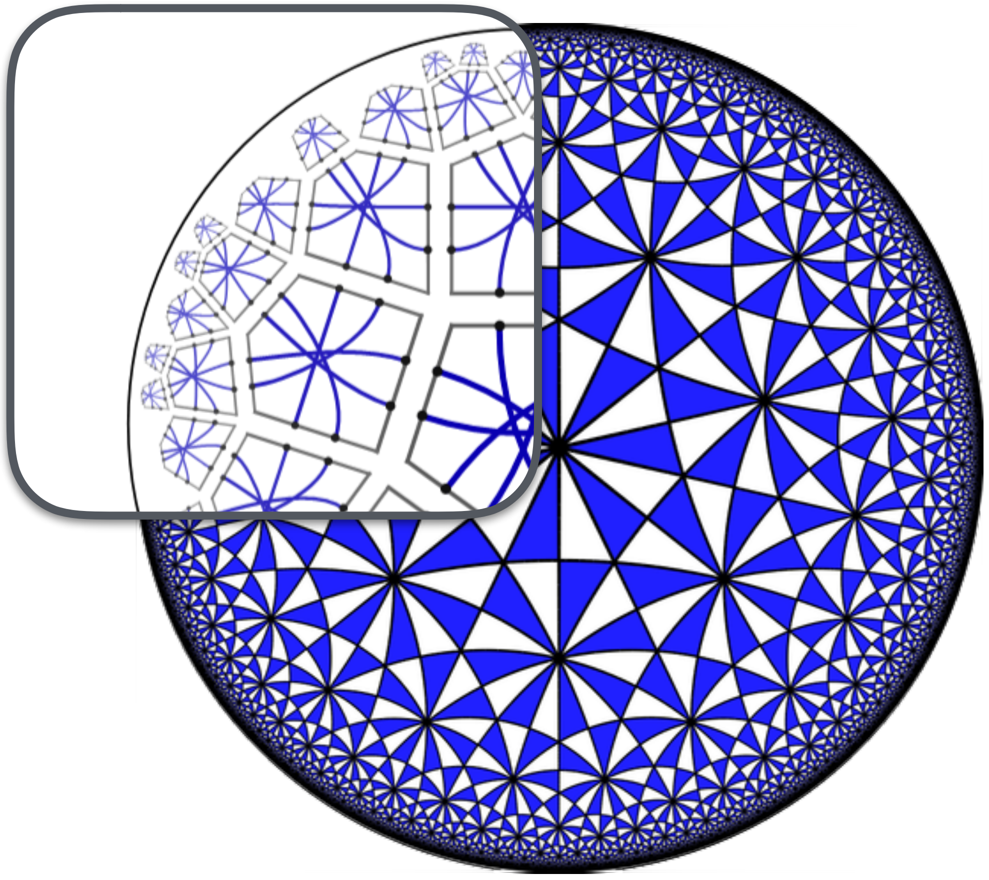
Jens Eisert, Freie Universität Berlin

Joint work with Alexander Jahn, Marek Gluza, Fernando Pastawski

1. MATCHGATE TENSOR NETWORKS

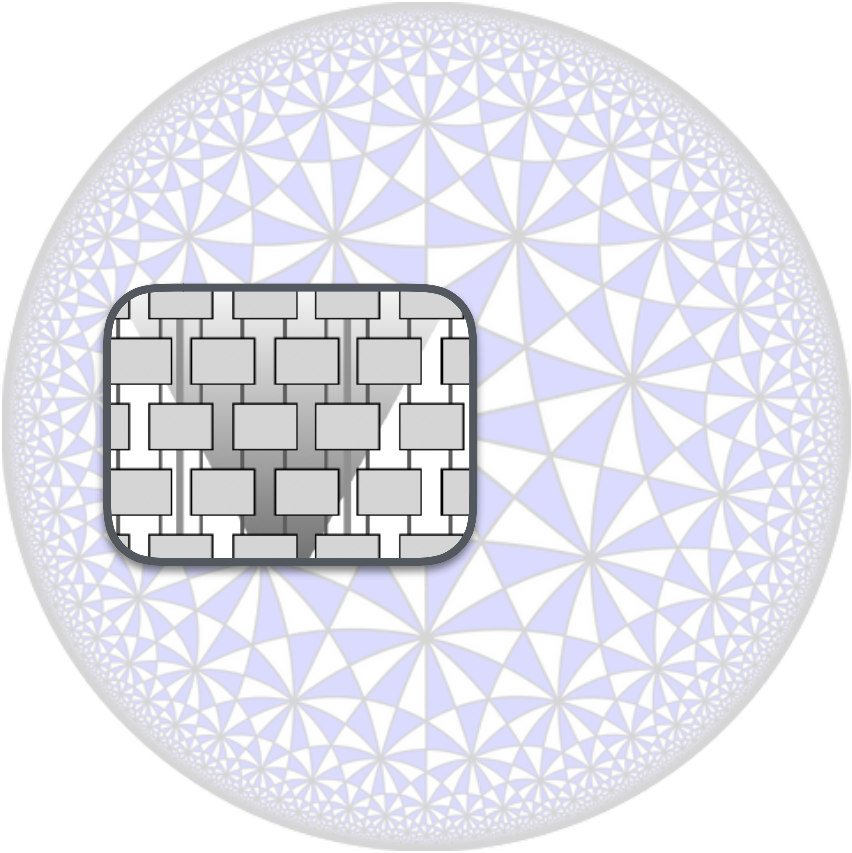


JAHN, GLUZA, PASTAWSKI, EISERT, SCIENCE ADVANCES, in press (2019)



2. MAJORANA DIMERS AND HOLOGRAPHIC QUANTUM ERROR-CORRECTING CODES

JAHN, GLUZA, PASTAWSKI, EISERT, arXiv:1905.03268 (2019)

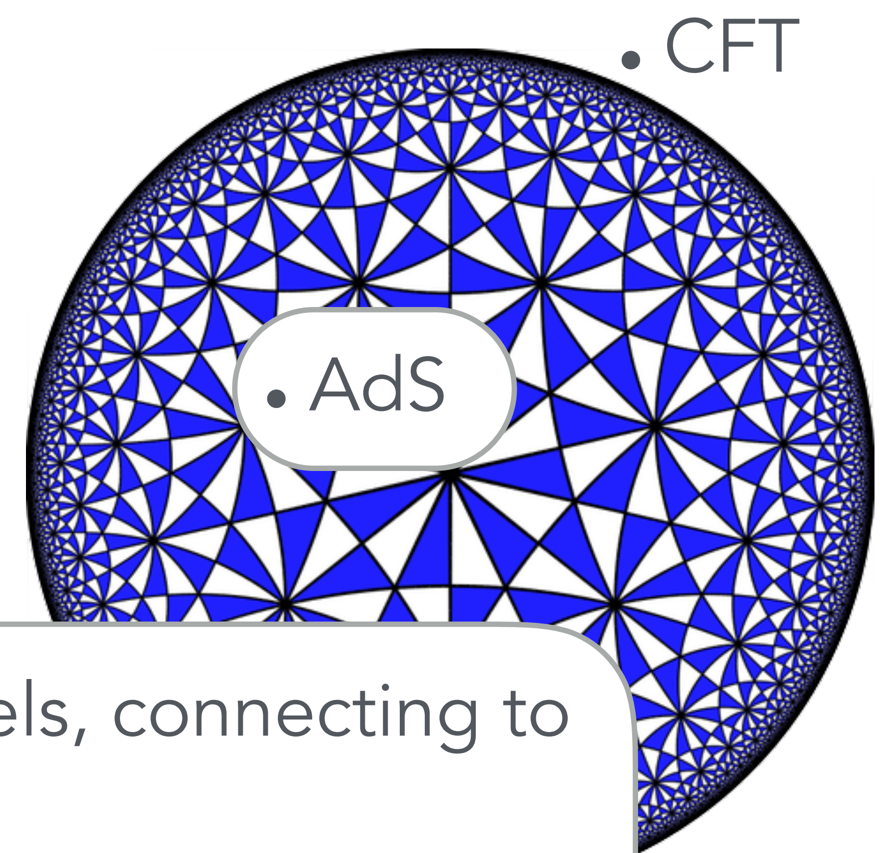


3. COMPLEXITY AND ENTANGLEMENT

EISERT, HELLER, in preparation (2019)

ADS-CFT CORRESPONDENCE

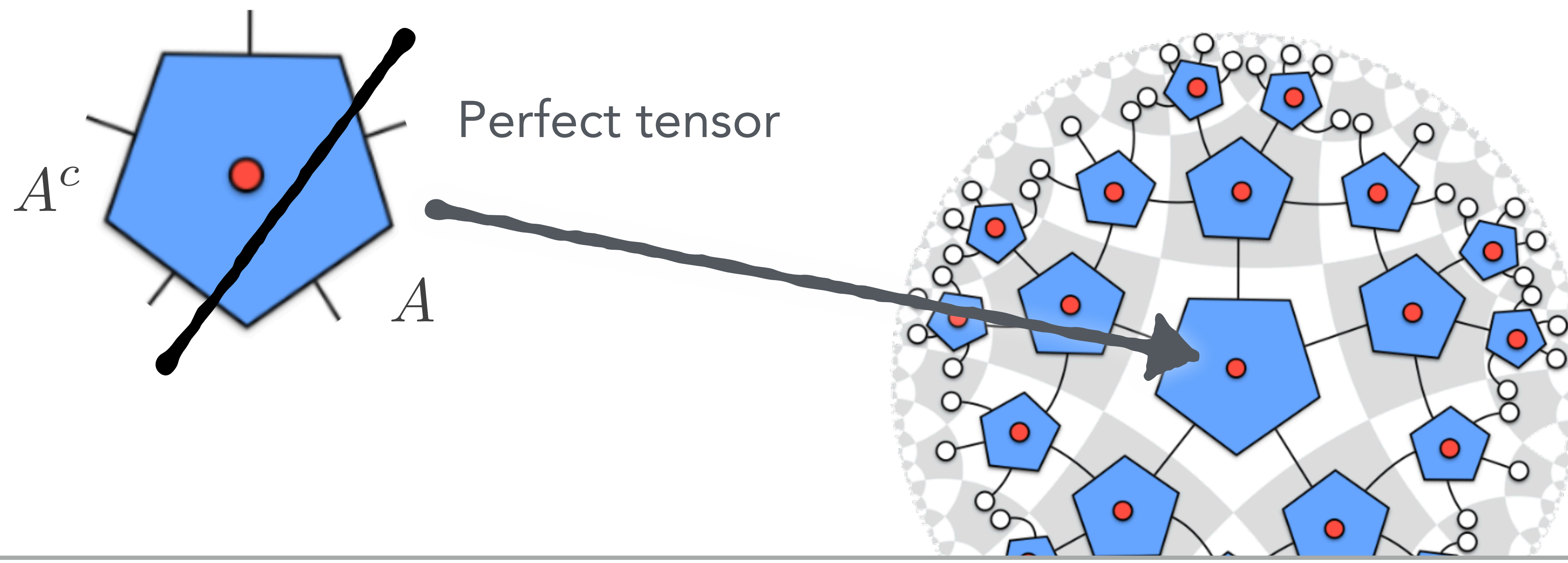
- Duality between **Einstein gravity** in $D+2$ Anti de Sitter spacetime and conformal field theory in $D+1$ dimensions



- **Tensor-network** based toy models, connecting to
  - condensed matter
  - quantum information

Maldacena, Avd Th Math Phys 2, 231 (1998)
Witten, Adv Theor Math Phys 2, 253 (1998)
van Raamsdonk, Gen Rel Grav 42, 2323 (2010)

- **Quantum error correction:** Holographic pentagon code



- Perfect tensor: Any bi-partite cut with $|A| \leq |A^c|$ is proportional to isometry

- $[[2n-1,1,n]]$ quantum error correcting code
- Here, $2n-1=5$, "Pentagon code"

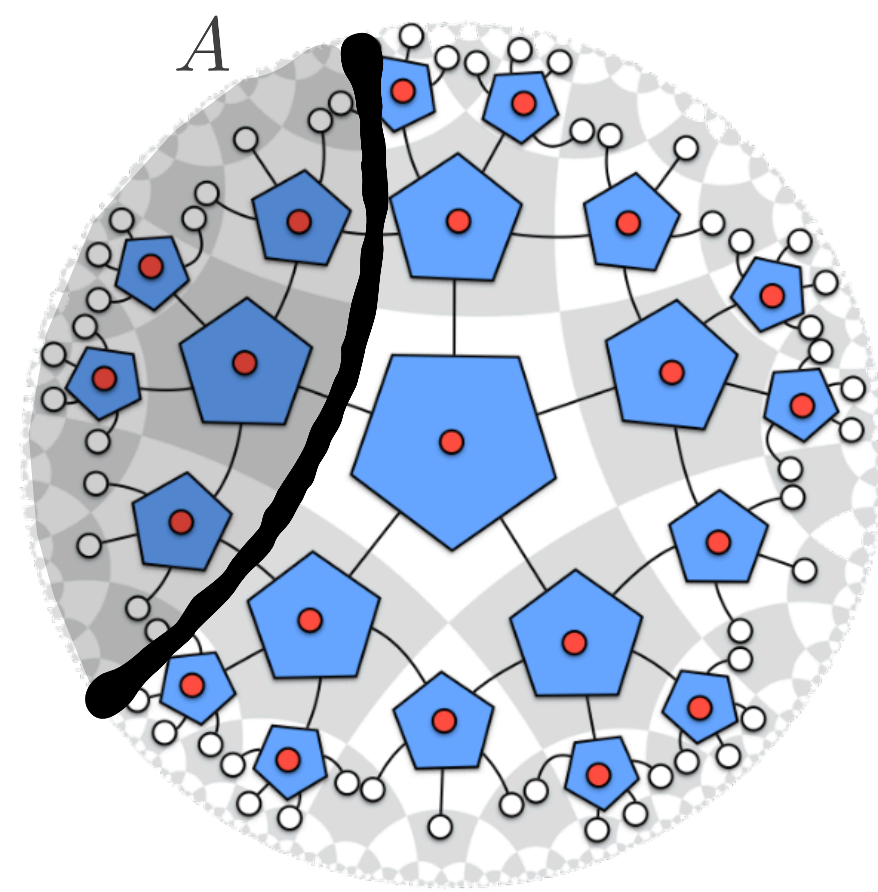
- Holographic state: Product state fed into bulk

Pastawski, Yoshida, Harlow, Preskill, JHEP 2015, 149 (2015)
Helwig, Cui, Latorre, Riera, Lo, Phys Rev A 86, 052335 (2012)

- Entangelement entropy of a connected region of a boundary satisfies

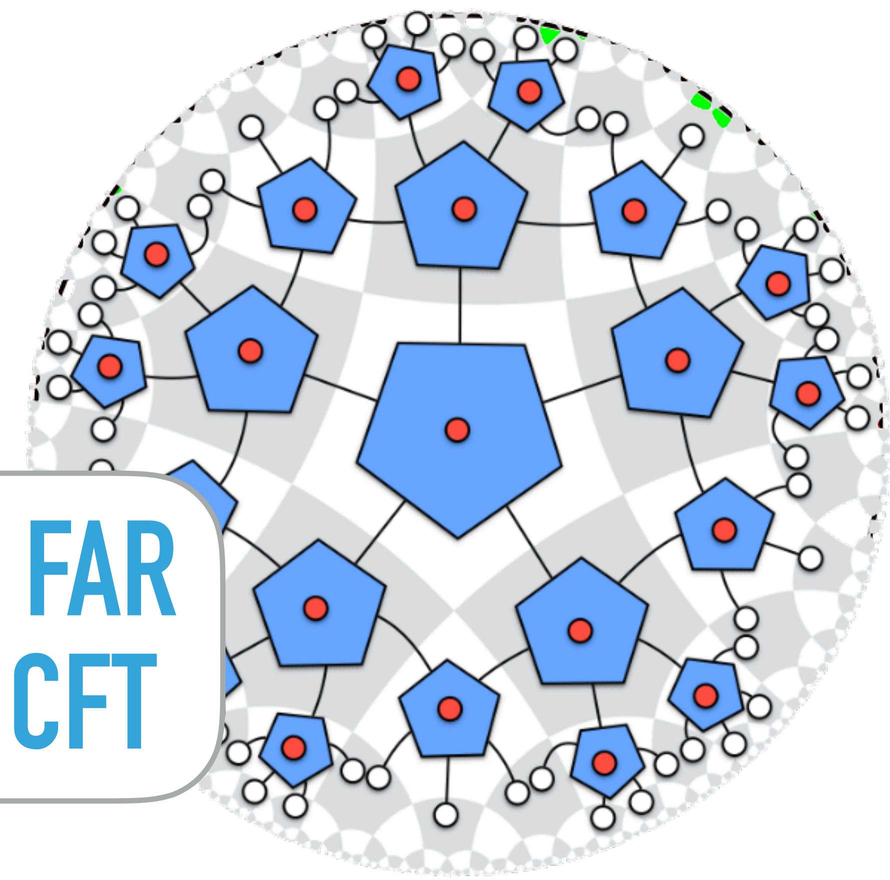
$$S_A = |\gamma_A|$$

- γ_A minimal bulk geodesic
- Lattice version of Ryu-Takayanagi formula



Pastawski, Yoshida, Harlow, Preskill, JHEP 2015, 149 (2015)
Ryu, Takayanagi, Phys Rev Lett 96, 181602 (2006)

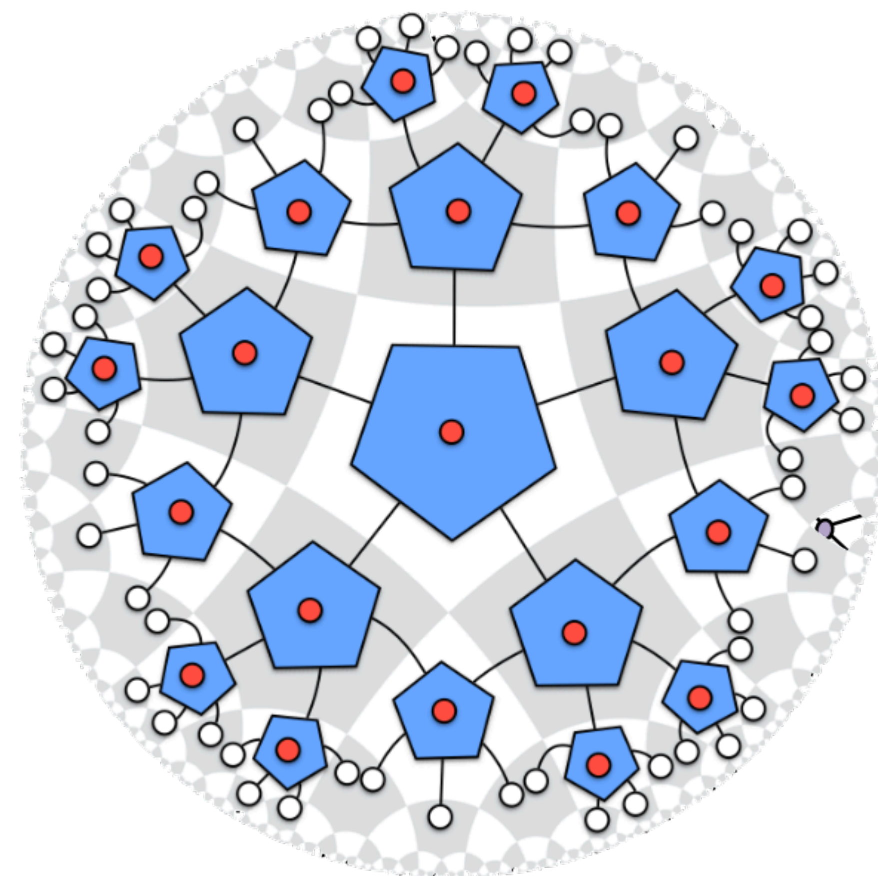
- Connection of **AdS-cft** to **holographic quantum error correction**



BUT, BOUNDARY STATE FAR FROM A REASONABLE CFT

Almheiri, Dong, Harlow, JHEP 1504, 163 (2015)
Harris, McMahon, Brennen, Stace, Phys Rev A 98, 052301 (2018)

- Tensor network consisting of **isometries** and **disentanglers**
- Approximates critical quantum states

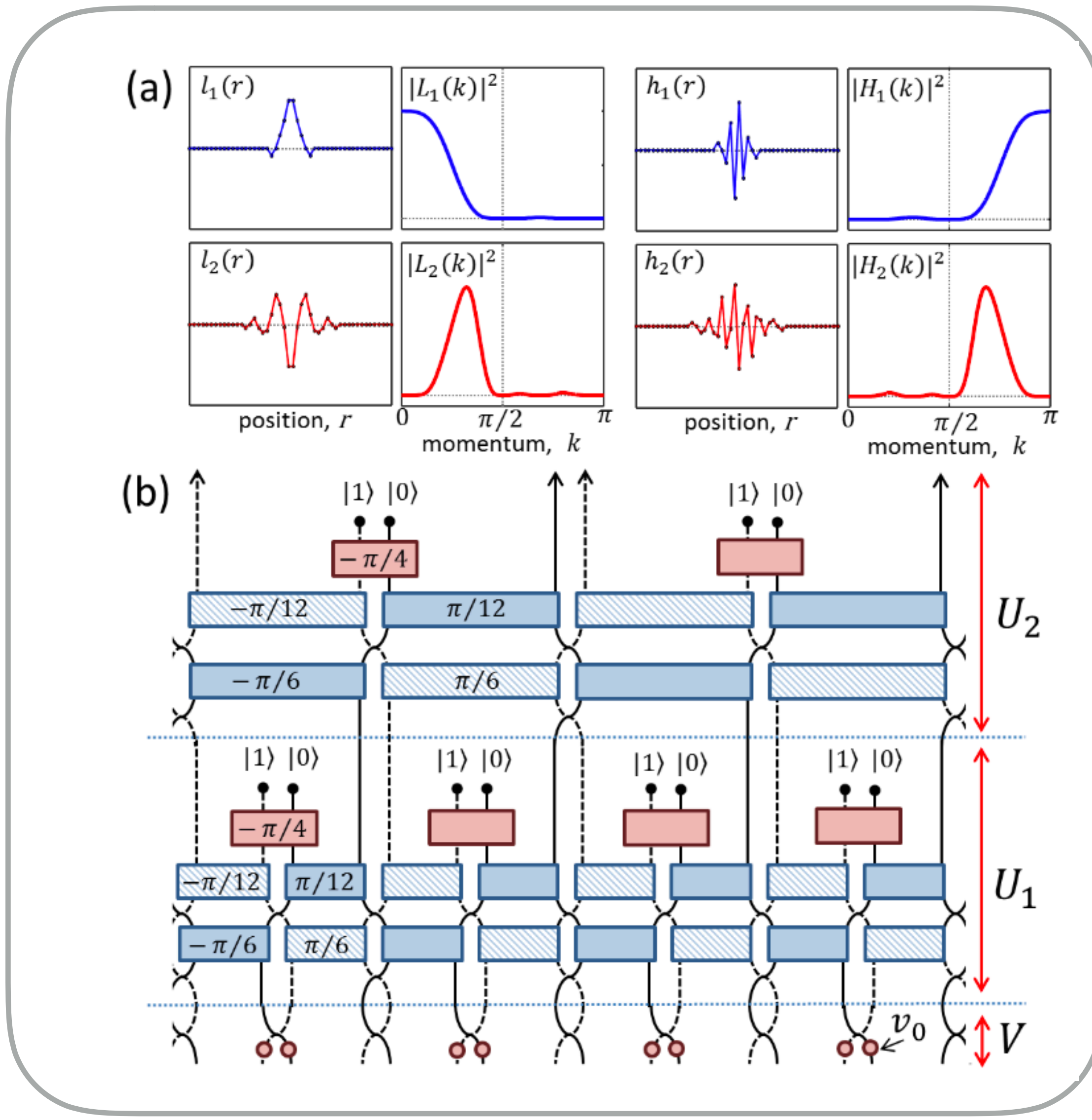


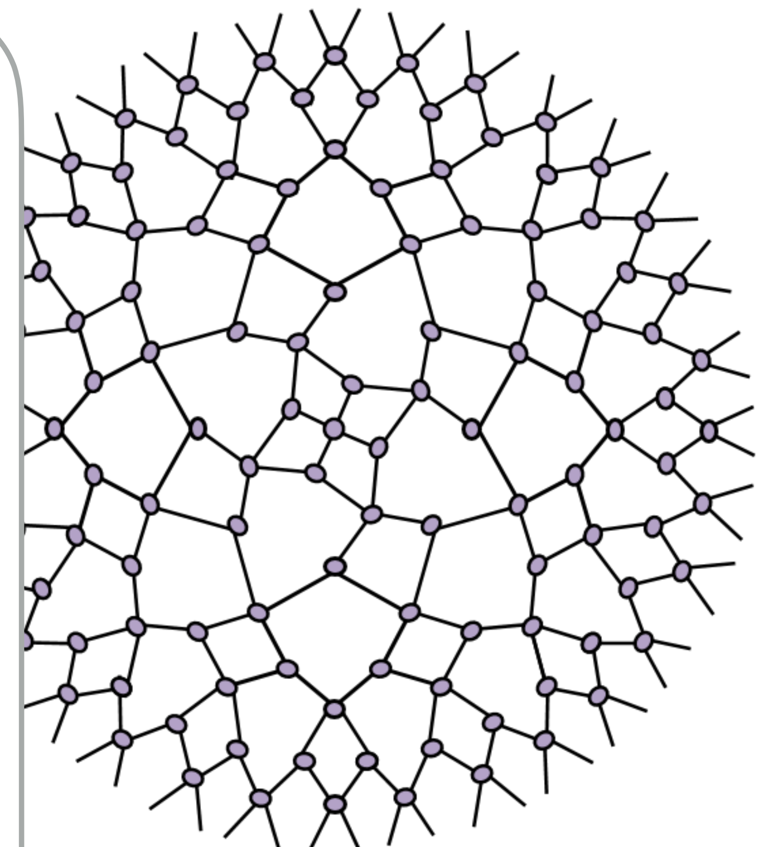
Vidal, Phys Rev Lett 101, 110501 (2008)
Evenbly, Vidal, Phys Rev B, 79, 144108 (2009)
Dawson, Eisert, Osborne, Phys Rev Lett 100, 130501 (2008)
Swingle, Phys Rev D 86, 065007 (2012)

"MODEL 2": MULTISCALE ENTANGLEMENT RENORMALIZATION (MERA)

- Free fermionic MERA based on wavelets approximates Ising critical theory

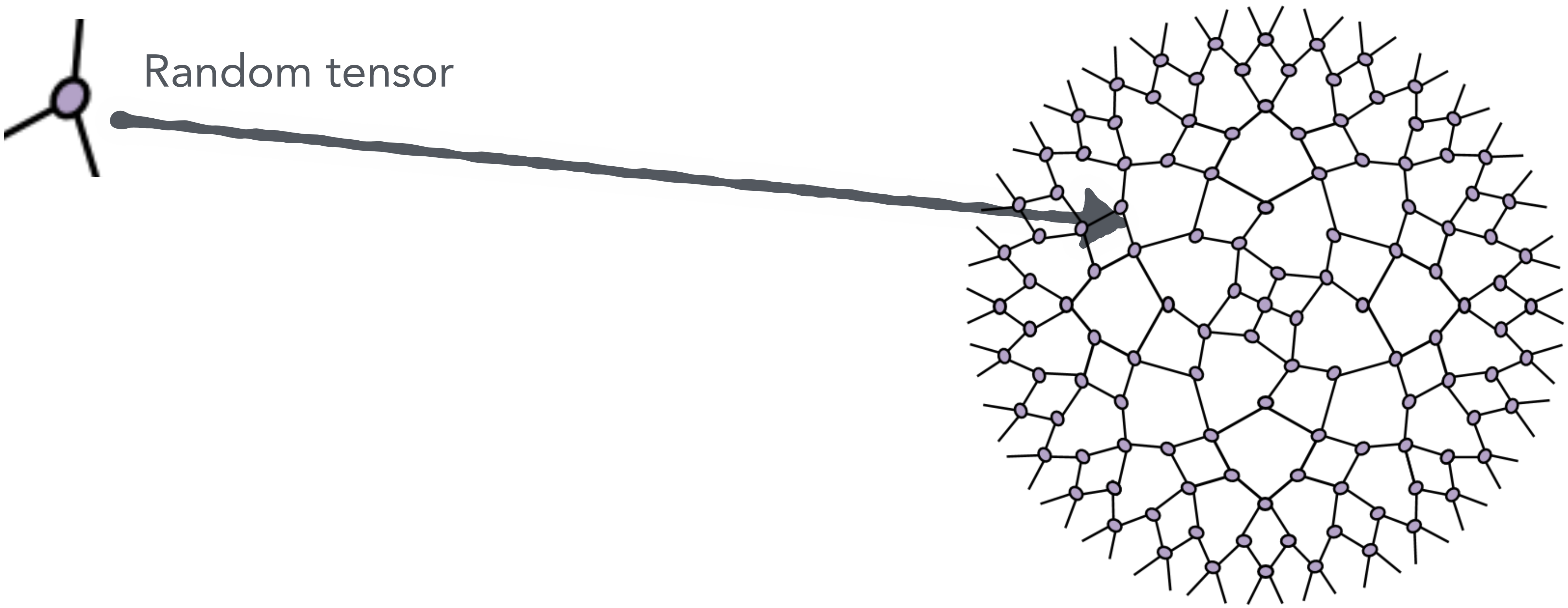
$$\hat{c}_j = \sum_{j,k} A_{j,k} c_k$$



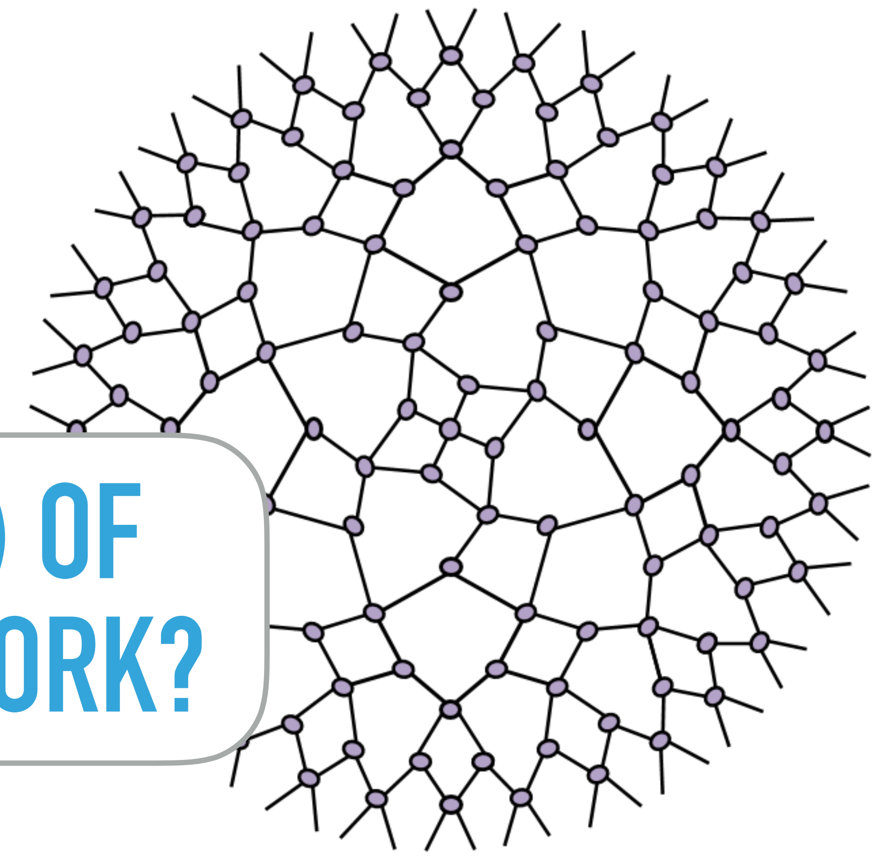


Evenbly, White, Phys Rev Lett 116, 140403 (2016)
Haegeman, Swingle, Walter, Cotler, Evenbly, Scholz, Phys Rev X 8, 011003 (2018)

- Random isometric tensors are with high probability close to being perfect



Hayden, Nezami, Qi, Thomas, Walter, Yang, JHEP 2016, 9 (2016)



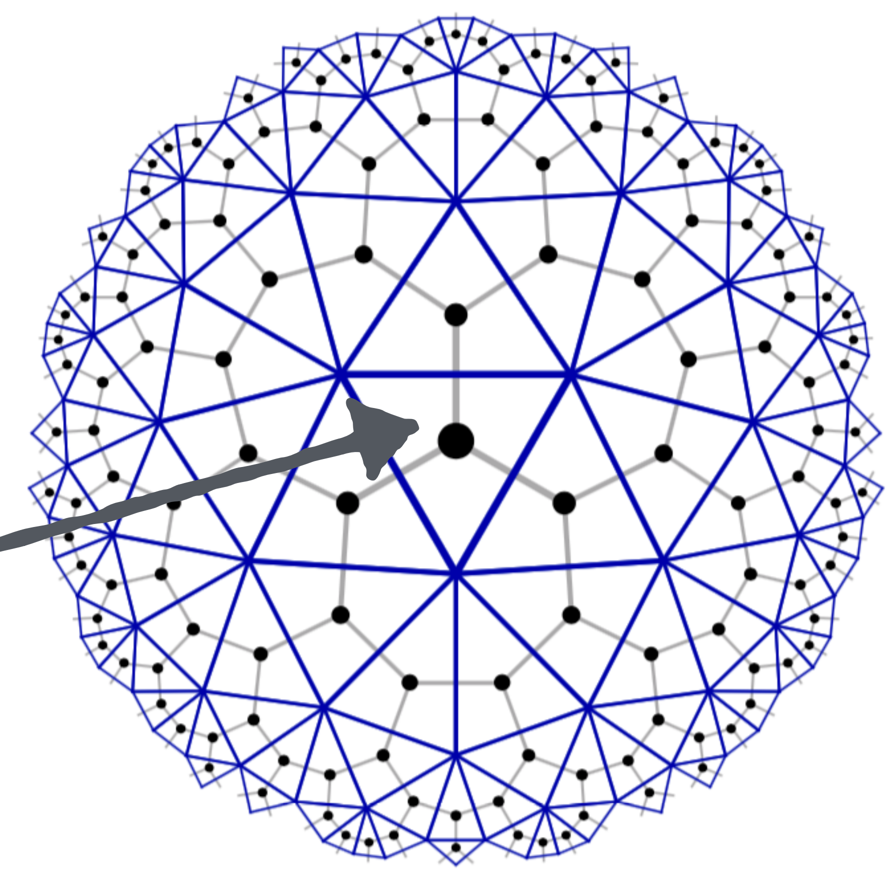
CAN ONE EMBODY (ASPECTS) OF THEM IN A LARGER FRAMEWORK?

Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)
Jahn, Gluza, Pastawski, Eisert, arXiv:1905.03268 (2019)

GETTING TO WORK: MATCHGATE TENSOR NETWORKS

MATCHGATE TENSOR NETWORKS

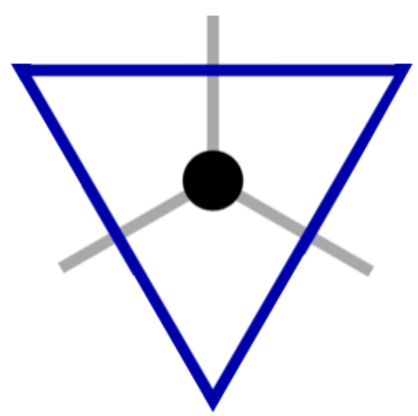
- Choose some some tiling of the plane



- Matchgate tensor

$$T_v : \{0,1\}^{\times r} \to \mathbb{C}$$

per vertex $v \in V$



- Boundary state obtained by tensor contraction $|\psi\rangle = \sum_{j \in \{0,1\}^{\times L}} \mathcal{T}(j)\, |j\rangle$

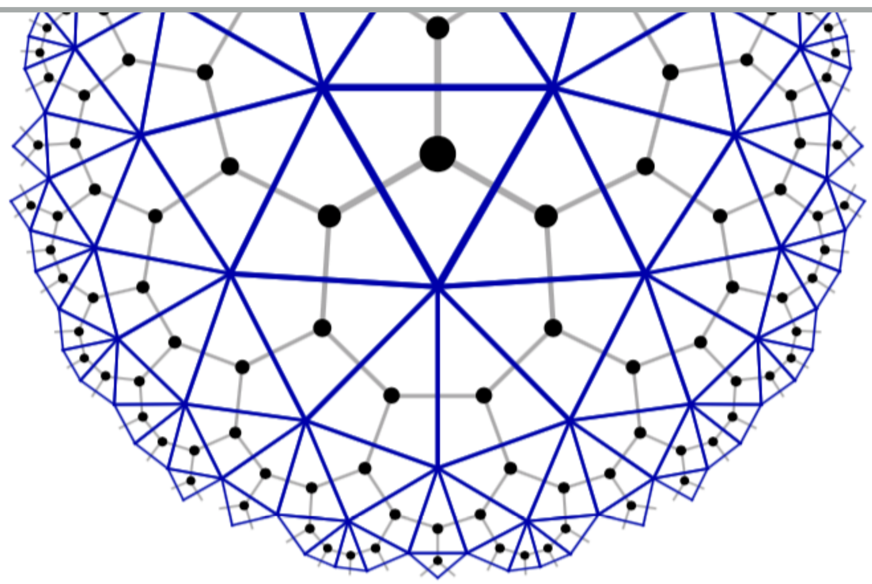
Bravyi, Cont Math 482, 179 (2009)
Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)

- **Matchgate tensors:** Consider a rank-r tensor $T(x)$ with inputs $x \in \{0,1\}^{\times r}$, $T(x)$ is a matchgate if there exists an antisymmetric matrix $A \in \mathbb{C}^{r\times r}$ and a reference index $z \in \{0,1\}^r$ such that

$$T(x) = \mathrm{Pf}(A_{|x\mathrm{XORz}})T(z)$$

where $\mathrm{Pf}(A)$ is the Pfaffian of A and $A_{|x}$ is the submatrix of A acting on the subspace supported by x

Cai, Choudhary, Lu, CCC07, IEEE Conference (2007)



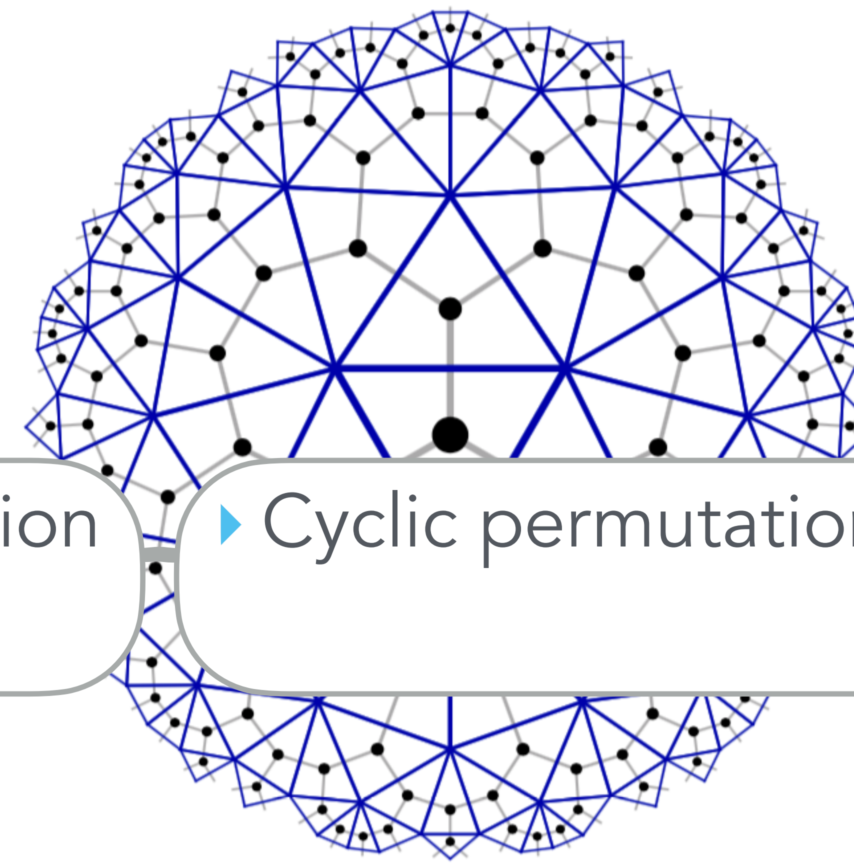
Bravyi, Cont Math 482, 179 (2009)
Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)

- **Observation:** The contraction requires $O(L^2N)$ steps for L boundary sites and N contracted tensors

- Contraction of tensors with generating matrices A and B

- Self-contraction (tedious)

- Cyclic permutations



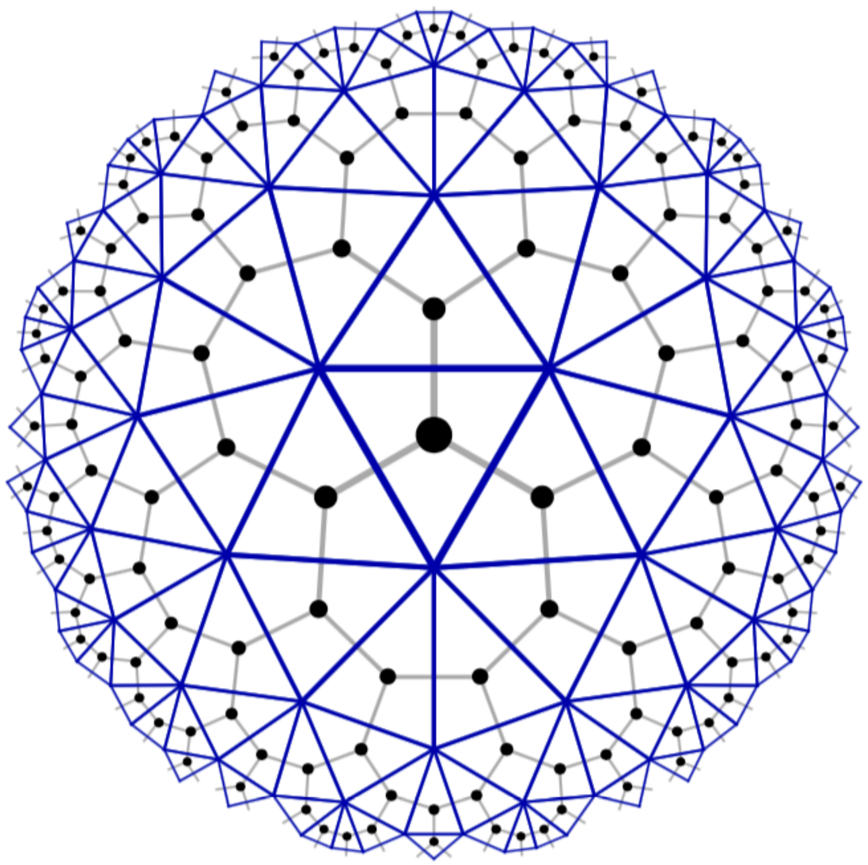
- Generic even matchgate with $\bar{z} = 0$ has

$$\Phi_T(\theta) = T(\bar{0}) \exp\left(\frac{1}{2} \sum_{j,k=1}^{r} A_{j,k} \theta_j \theta_k \right)$$

with generating matrix A

Bravyi, Cont Math 482, 179 (2009)
Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)

Place generating matrices
on some tiling



- Generic even matchgate with $\bar{z} = 0$ has

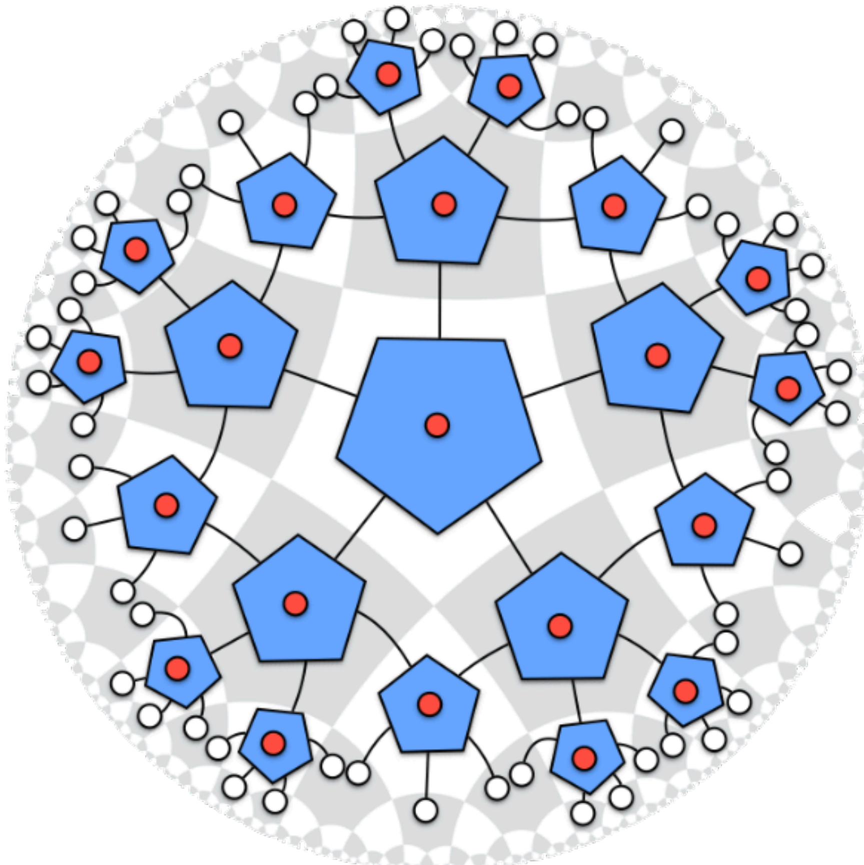
$$\Phi_T(\theta) = T(\overline{0}) \exp\left(\frac{1}{2} \sum_{j,k=1}^{r} A_{j,k} \theta_j \theta_k \right)$$

with generating matrix A

Bravyi, Cont Math 482, 179 (2009)
Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)

LET US PLAY

- **Observation:** The holographic pentagon code with computational basis input in the bulk yields a matchgate tensor network



- Gives rise to stabilizer code $\langle S_j \rangle_{j=1}^{5}$, e.g.,

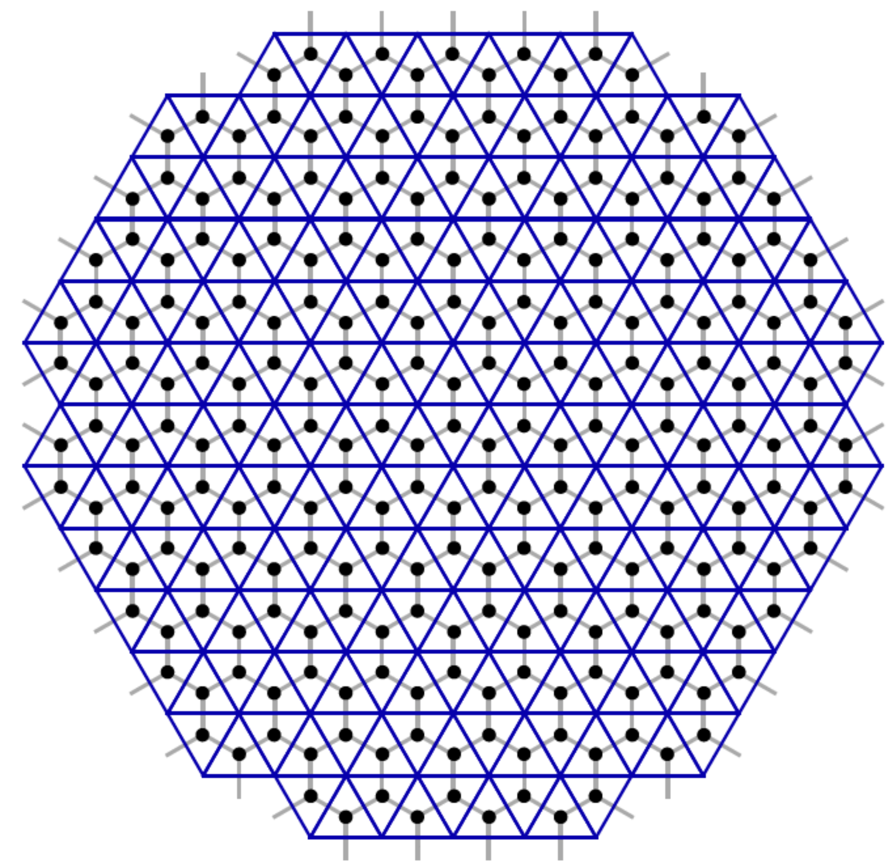
$$S_1 = \sigma^x \otimes \sigma^z \otimes \sigma^z \otimes \sigma^x \otimes \mathbb{1}_2 = \mathrm{i} m_7 m_2$$
$$S_2 = \mathbb{1}_2 \otimes \sigma^x \otimes \sigma^z \otimes \sigma^z \otimes \sigma^x = \mathrm{i} m_9 m_4$$

expressed in Majoranas

Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)

- $\{3,6\}$ flat tiling

Schläfli symbol

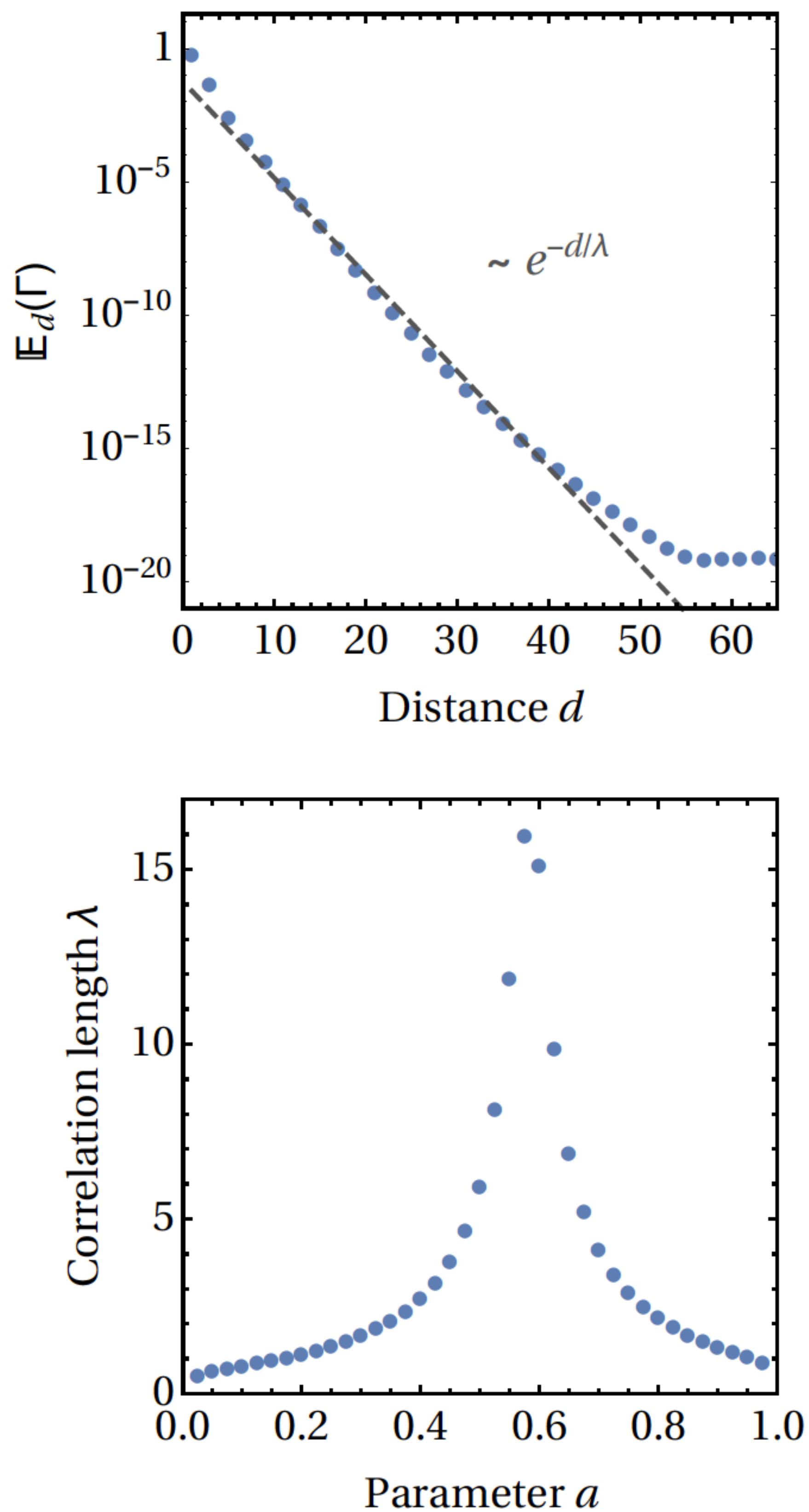


- Anti-symmetric matrix A, single parameter a

$$A = \begin{pmatrix} 0 & a & a \\ -a & 0 & a \\ -a & -a & 0 \end{pmatrix}$$

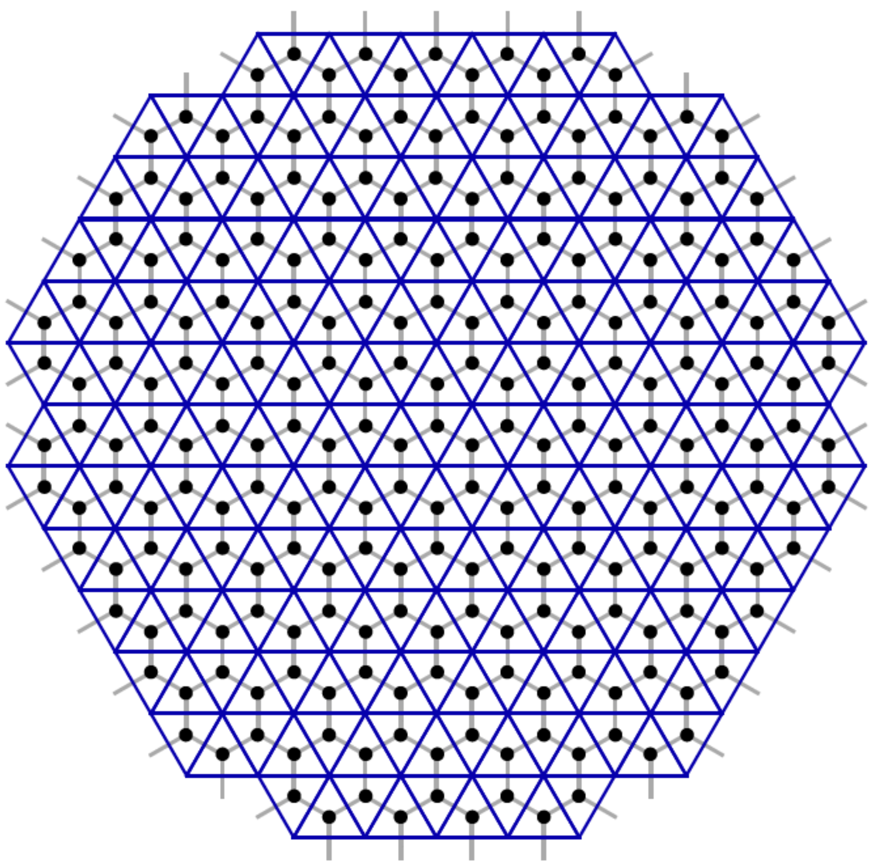
Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)

- Gapped system



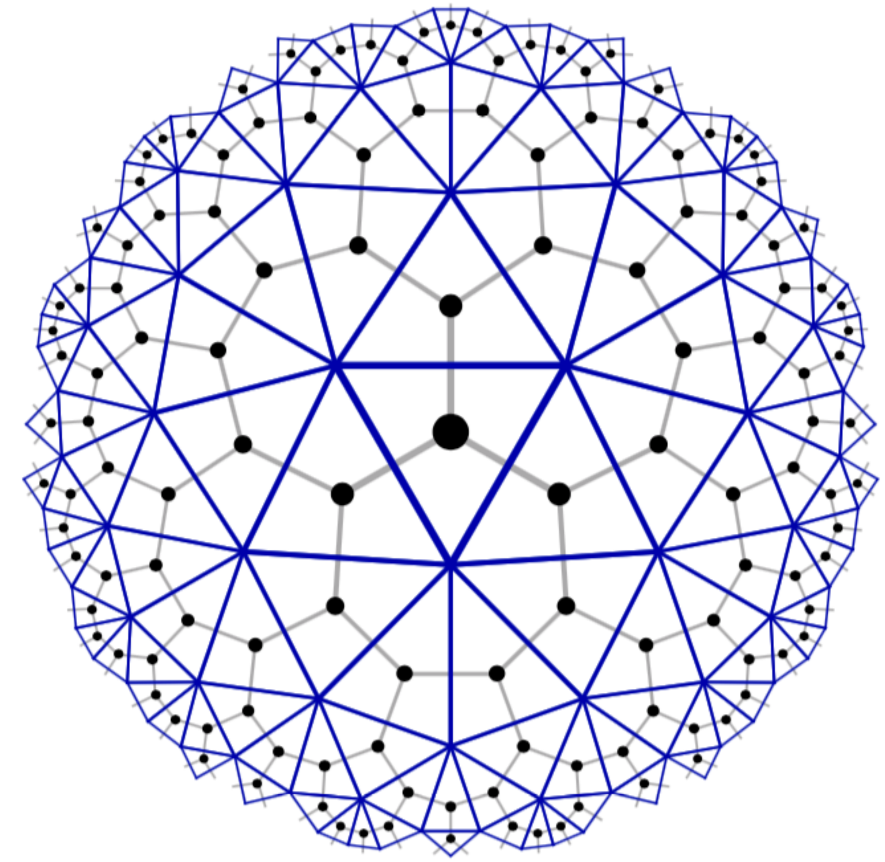
- A... ...ameter a

- $\{3,6\}$ flat tiling



Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)

- $\{3, k\},\ k > 6,$ hyperbolic tiling

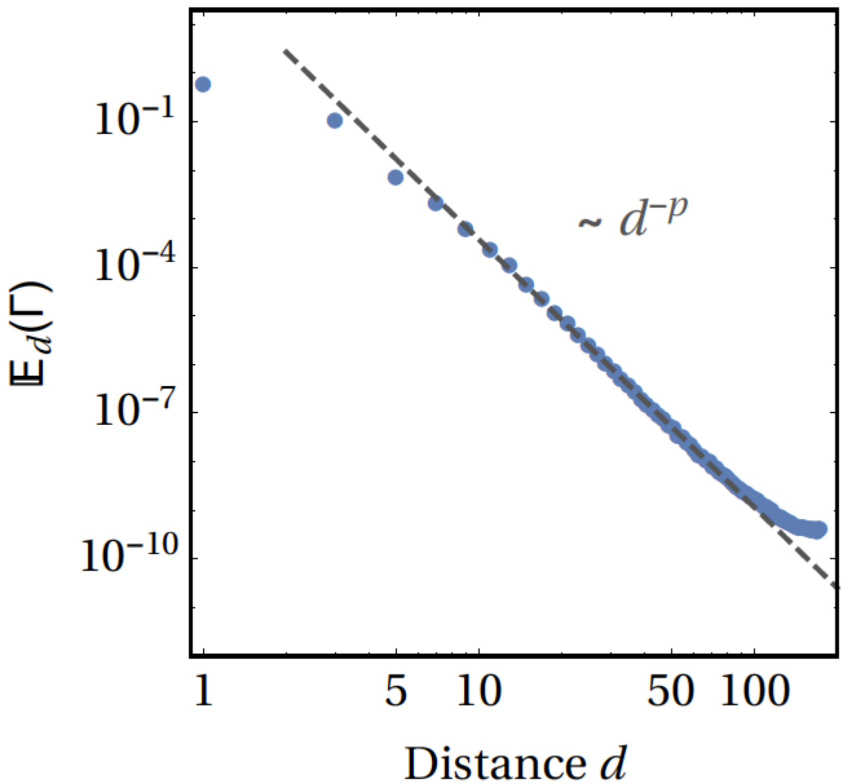


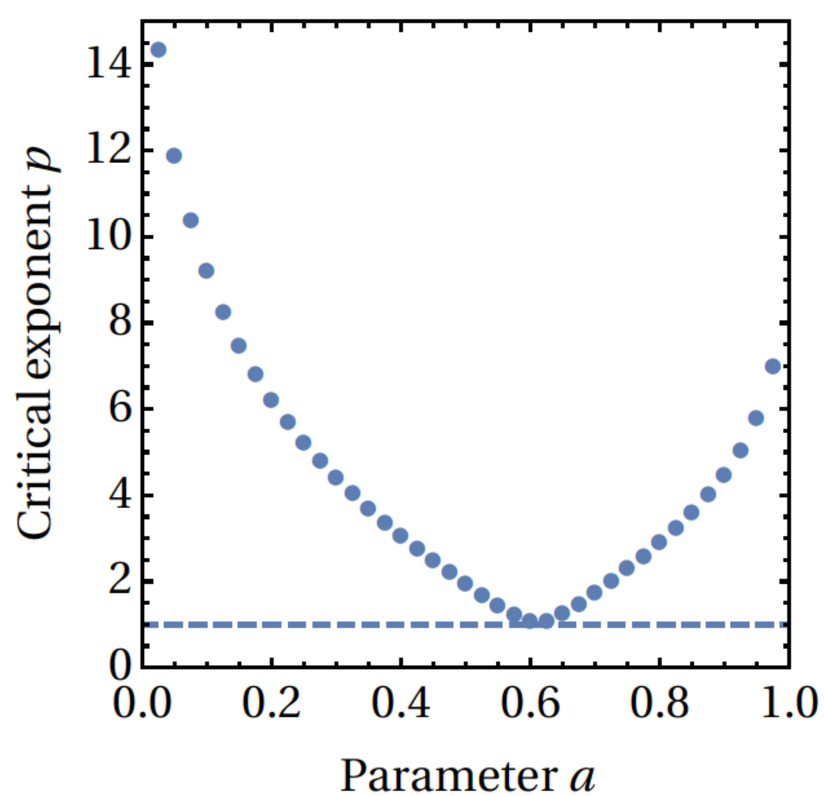
- Anti-symmetric matrix A, single parameter a

$$A = \begin{pmatrix} 0 & a & a \\ -a & 0 & a \\ -a & -a & 0 \end{pmatrix}$$

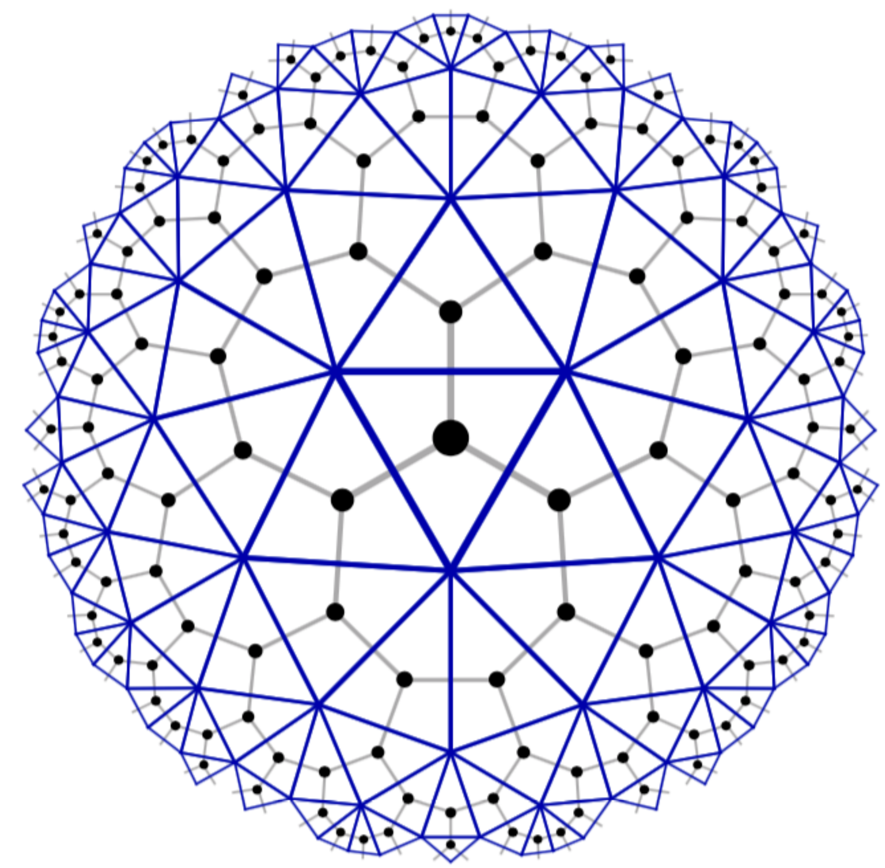
Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)

- $\{3,7\}$ critical system





- $\{3,k\}$, $k>6$, hyperbolic tiling



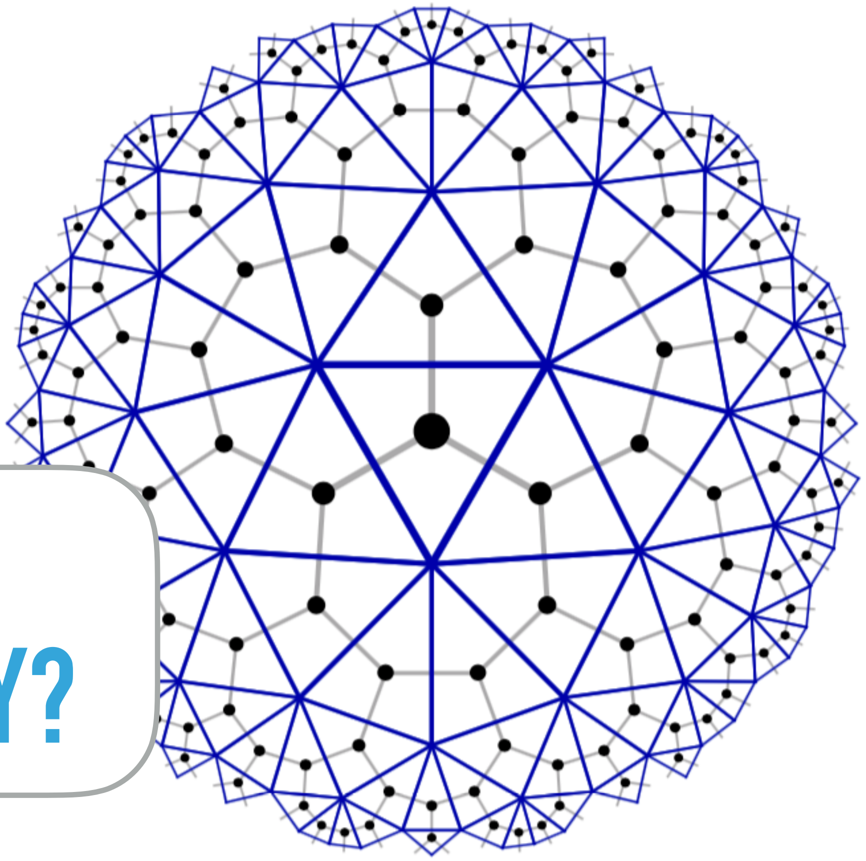
- A ameter a

- For $a \approx 0.61$, critical Ising theory

$$H = \mathrm{i}\left(\sum_{i=1}^{N-1} m_i m_{i+1} + m_1 m_N\right)$$

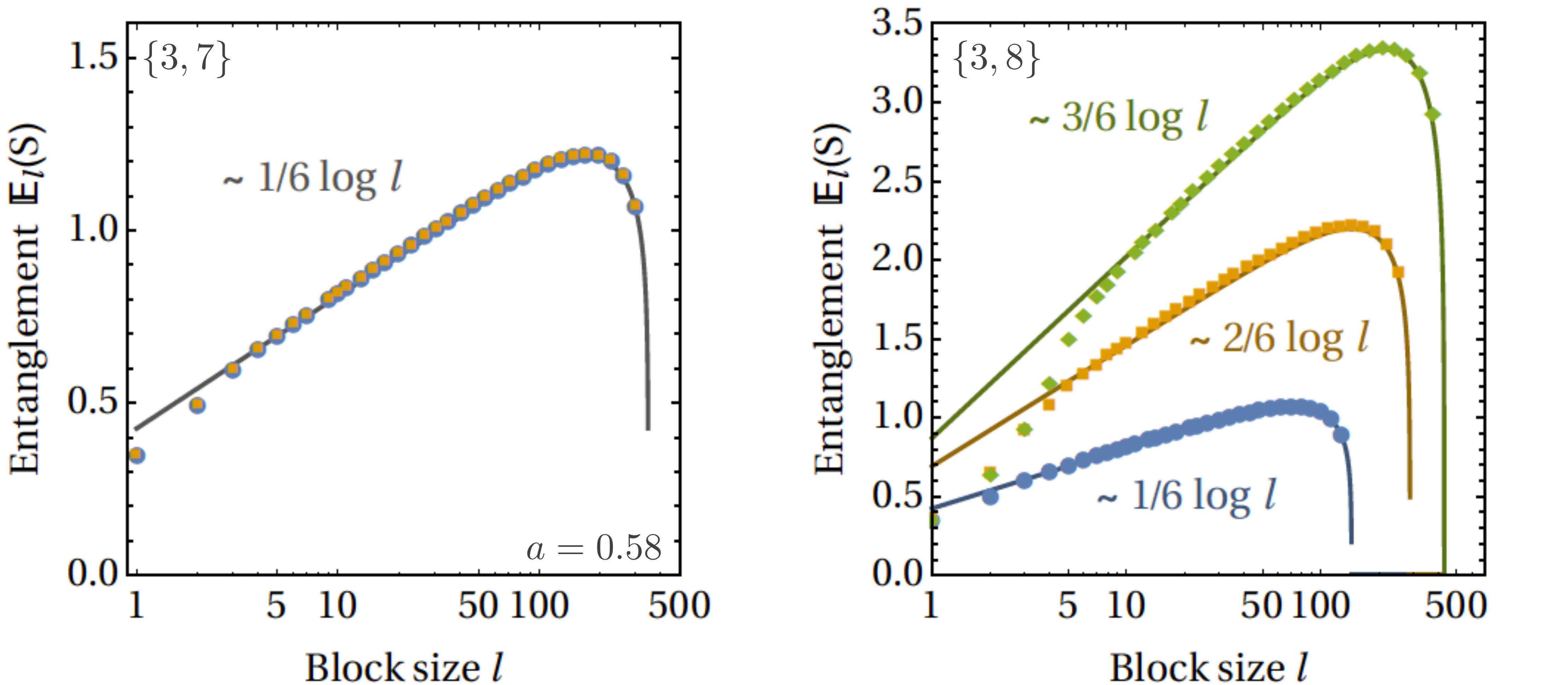
Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)

- $\{3,k\}$, $k>6$, hyperbolic tiling

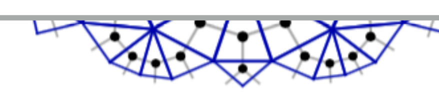


HOW ABOUT THE ENTANGLEMENT ENTROPY?

Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)



Tiling with higher bond dimensions χ (144, 288, 432 Majorana fermions for χ = 2, 4, 8, respectively)

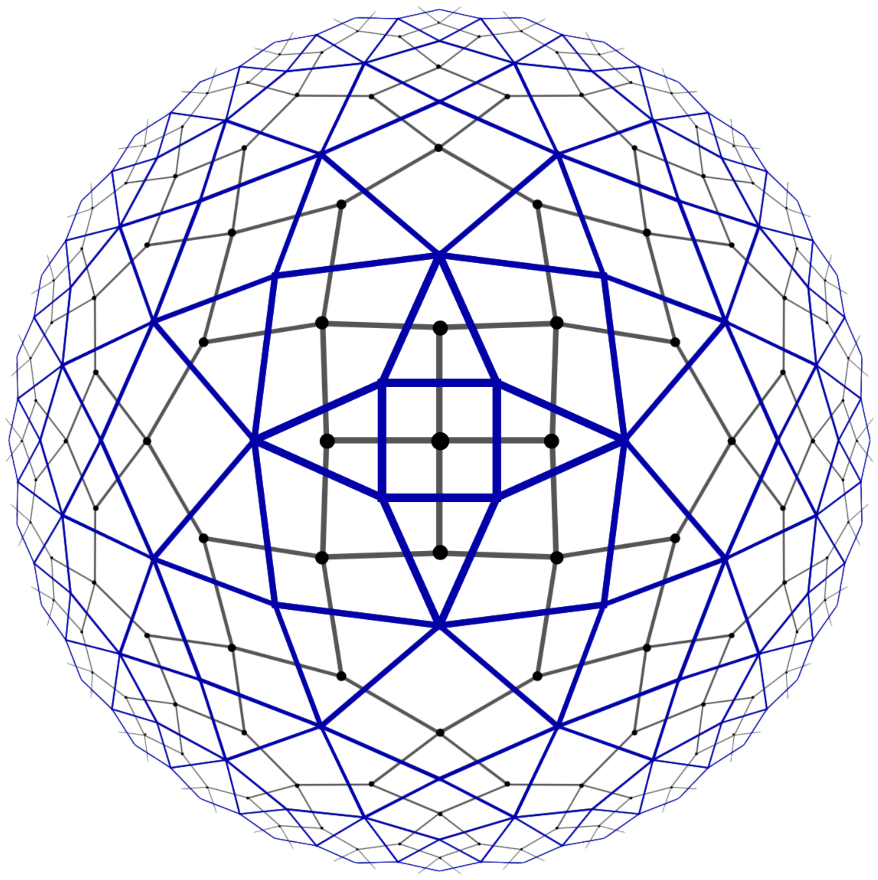


- CFT entanglement entropy of a block

$$S_\ell = \frac{c}{3} \ln \left(\frac{L}{\pi \epsilon} \sin \frac{\pi \ell}{L} \right) \simeq \frac{c}{3} \ln \frac{\ell}{\epsilon} + O\left((\ell / L)^2 \right)$$

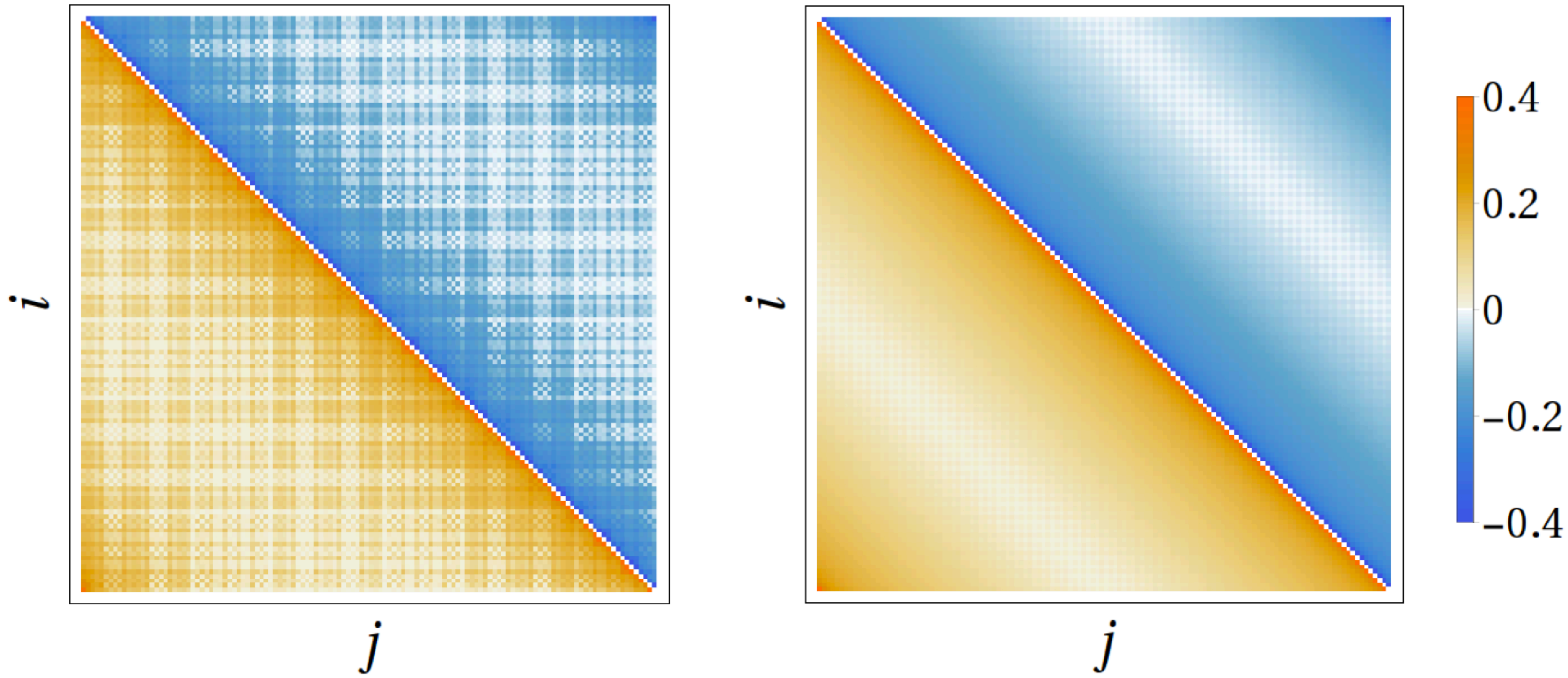
Holzhey, Larsen, Wilczek, Nucl Phys B 424, 443 (1994)
Calabrese, Cardy, J Stat Mech 0406, 06002 (2004)
Eisert, Cramer, Plenio, Rev Mod Phys 82, 277 (2010)

MERA?

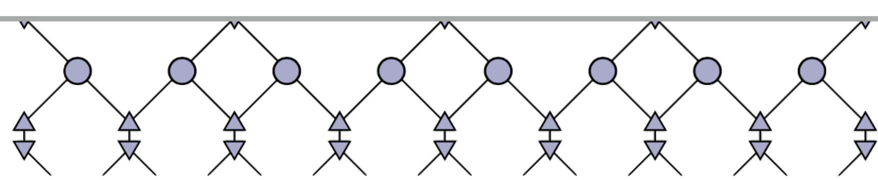


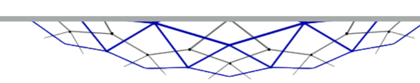
Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)

- Tiling with 3-and 4-leg **MERA tensor network**



Entries of the fermionic correlation matrix as a function of distance, left for $\{3, 7\}$ bulk tiling with 252 Majorana fermions, right for MERA with 256





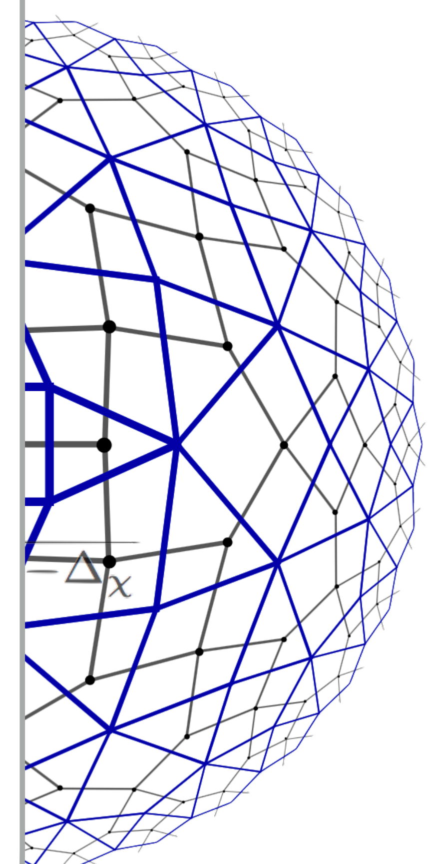
Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)

EXTRACTING CRITICAL DATA

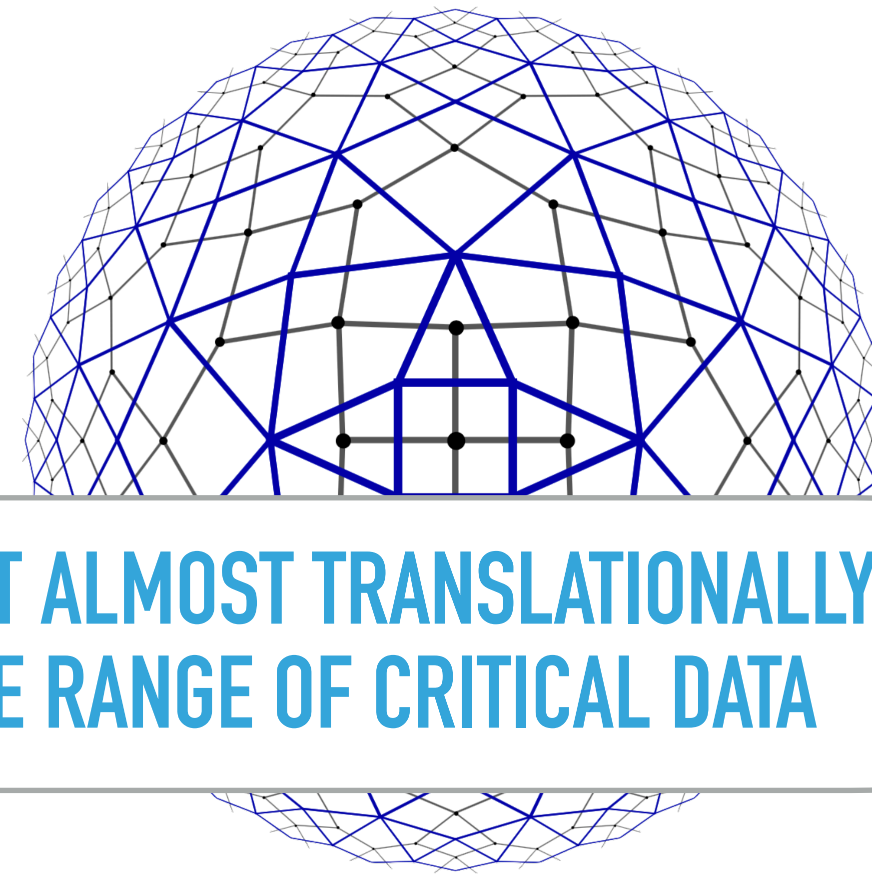
- Ising theory at criticality described by a 1+1-dimensional CFT

Parameter	Exact	$\{3,6\}$ bulk	$\{3,7\}$ bulk	mMERA	Wavelets
ϵ_0	-0.6366	-0.6139	-0.5617	-0.6365	-0.6211
c	0.5000	0.5006	0.5018	0.4958	0.4957
$\Delta_\psi, \Delta_{\bar{\psi}}$	0.5000	0.4948	0.4951	0.5023	0.5000
Δ_ϵ	1.0000	0.9856	1.0121	1.0027	1.0000
Δ_σ	0.1250	0.1403	0.1368	0.1417	0.1402
$C_{\sigma,\sigma,\epsilon}$	0.5000	0.5470	0.5336	0.5156	0.4584

TABLE I. Table of *conformal data* for the regular $\{3,6\}$ and $\{3,7\}$ bulk tilings as well as the mMERA, compared to the exact results and the wavelet MERA [16]. Listed are the ground-state energy density ϵ_0, central charge c, scaling dimensions Δ_ϕ of the fields $\phi = \psi, \bar{\psi}, \epsilon, \sigma$, and the structure constant $C_{\sigma,\sigma,\epsilon}$



Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)

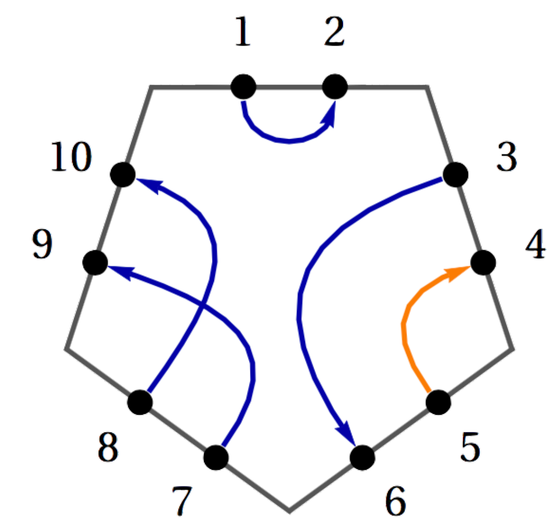


SO WITH FEW PARAMETERS, ONE ARRIVES AT ALMOST TRANSLATIONALLY INVARIANT STATES AND CAN EXTRACT A WIDE RANGE OF CRITICAL DATA

Jahn, Gluza, Pastawski, Eisert, Science Advances, in press (2019)

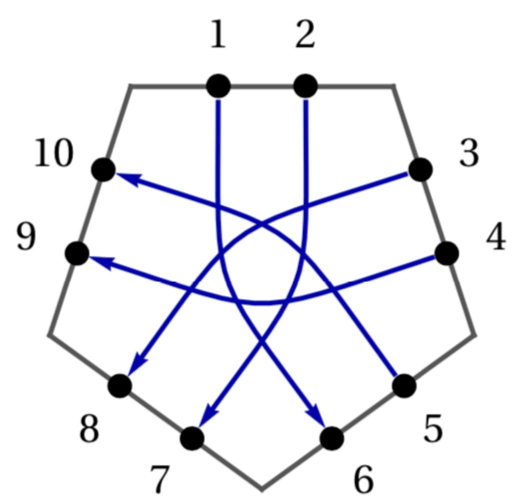
GETTING MORE SERIOUS ON QUANTUM ERROR CORRECTION

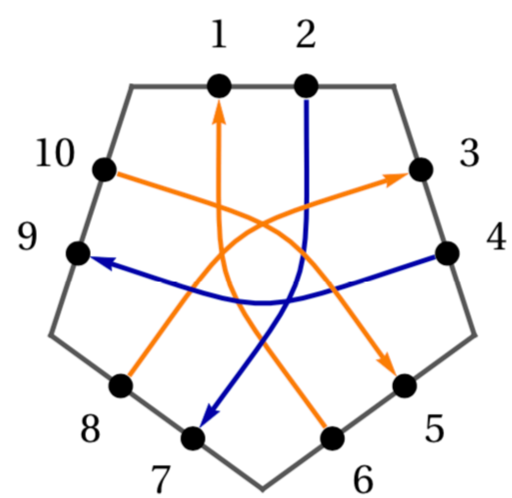
- For holographic stabilizer codes such as pentagon code develop picture of **paired Majorana dimers**



$$\Gamma_{i,j} = \begin{cases} -1 & \text{for an arrow } i \to j \\ 1 & \text{for an arrow } j \to i \\ 0 & \text{if no arrow connects } i \text{ and } j \end{cases}$$

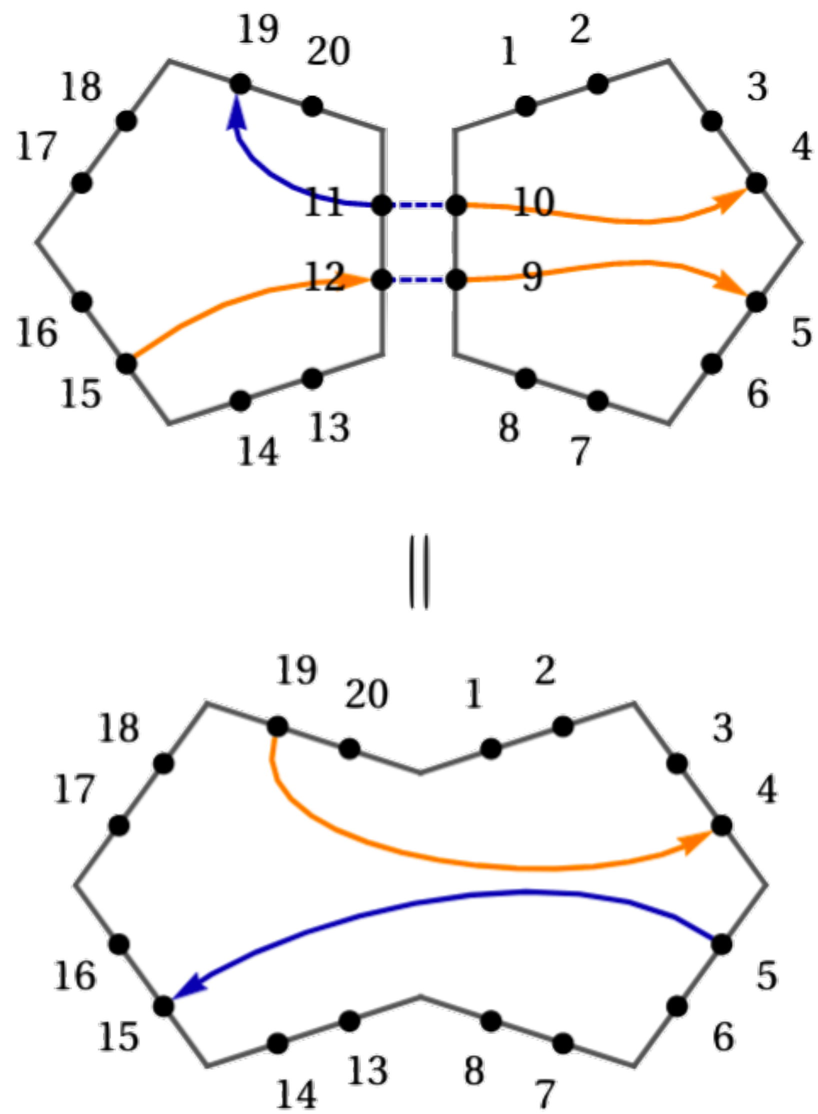
- Pentagon code logical states spanned by basis states $\bar{0}$ and $\bar{1}$

$|\bar{0}\rangle_5 =$ 

$|\bar{1}\rangle_5 =$ 

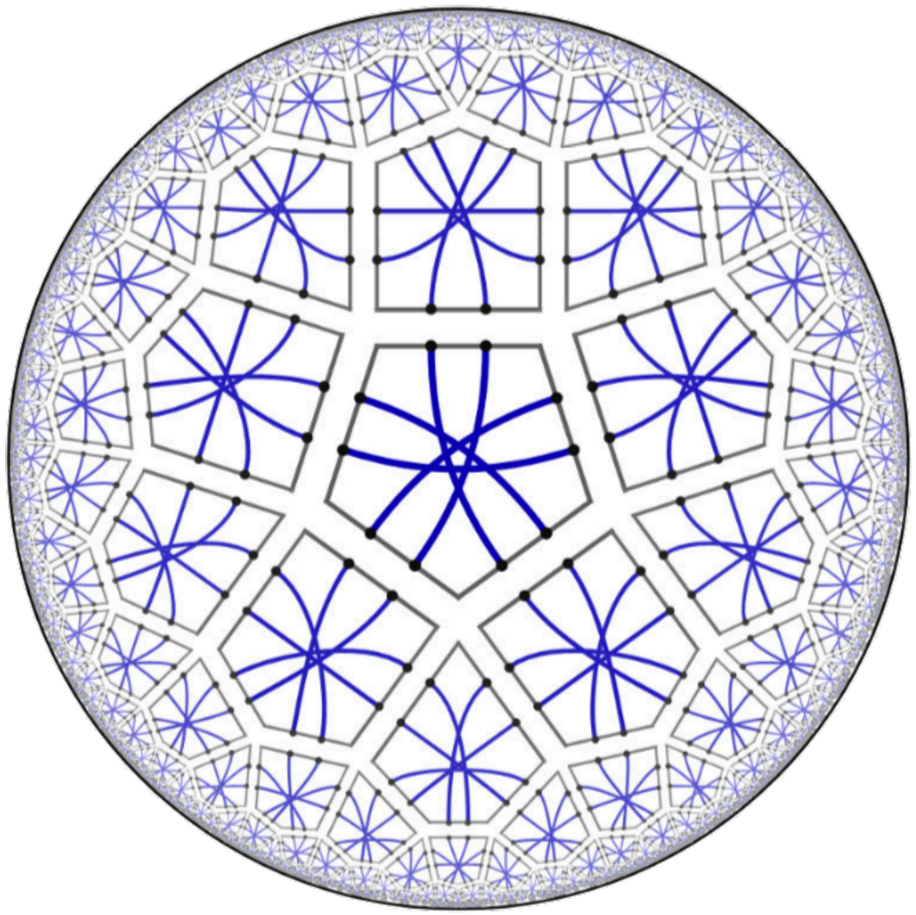
Jahn, Gluza, Pastawski, Eisert, arXiv:1905.03268 (2019)

- Diagrammatic contraction rules amenable to analytical analysis
- "Fusing" of dimers along edges
-



Jahn, Gluza, Pastawski, Eisert, arXiv:1905.03268 (2019)

- New picture of **holographic QEC**: Geodesic structure of dimers



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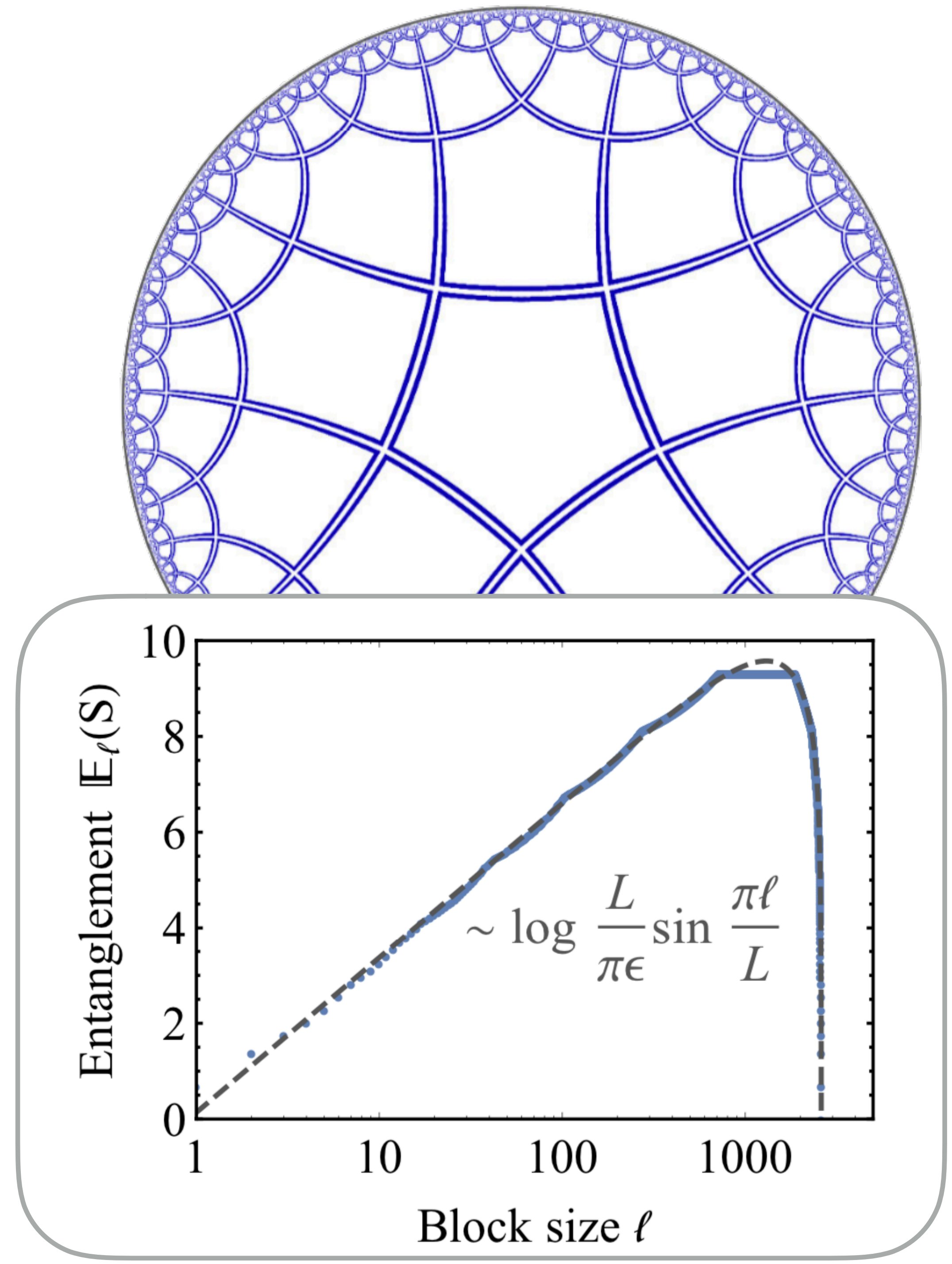
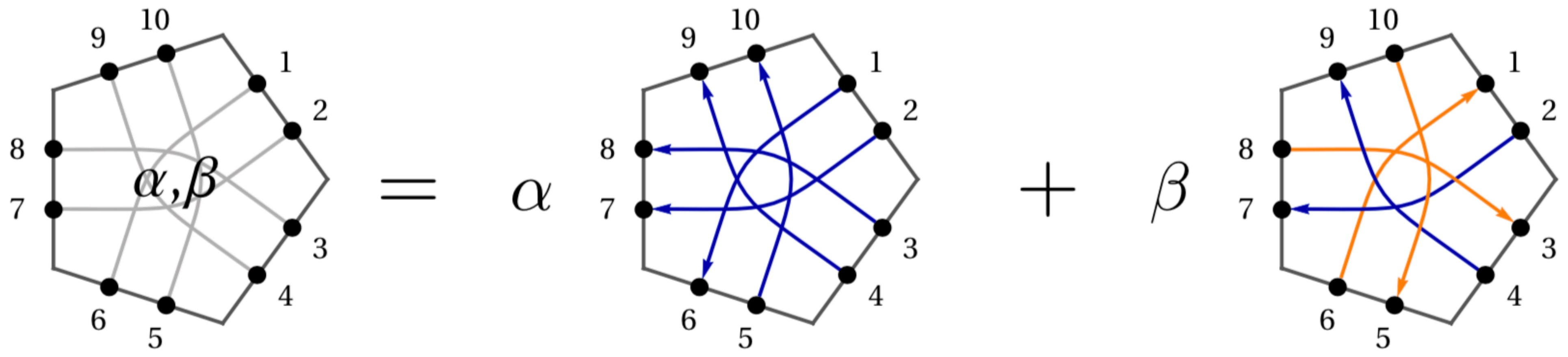


- On average, the pentagon code has a CFT-like log entanglement scaling with

$$c \approx 4.2$$

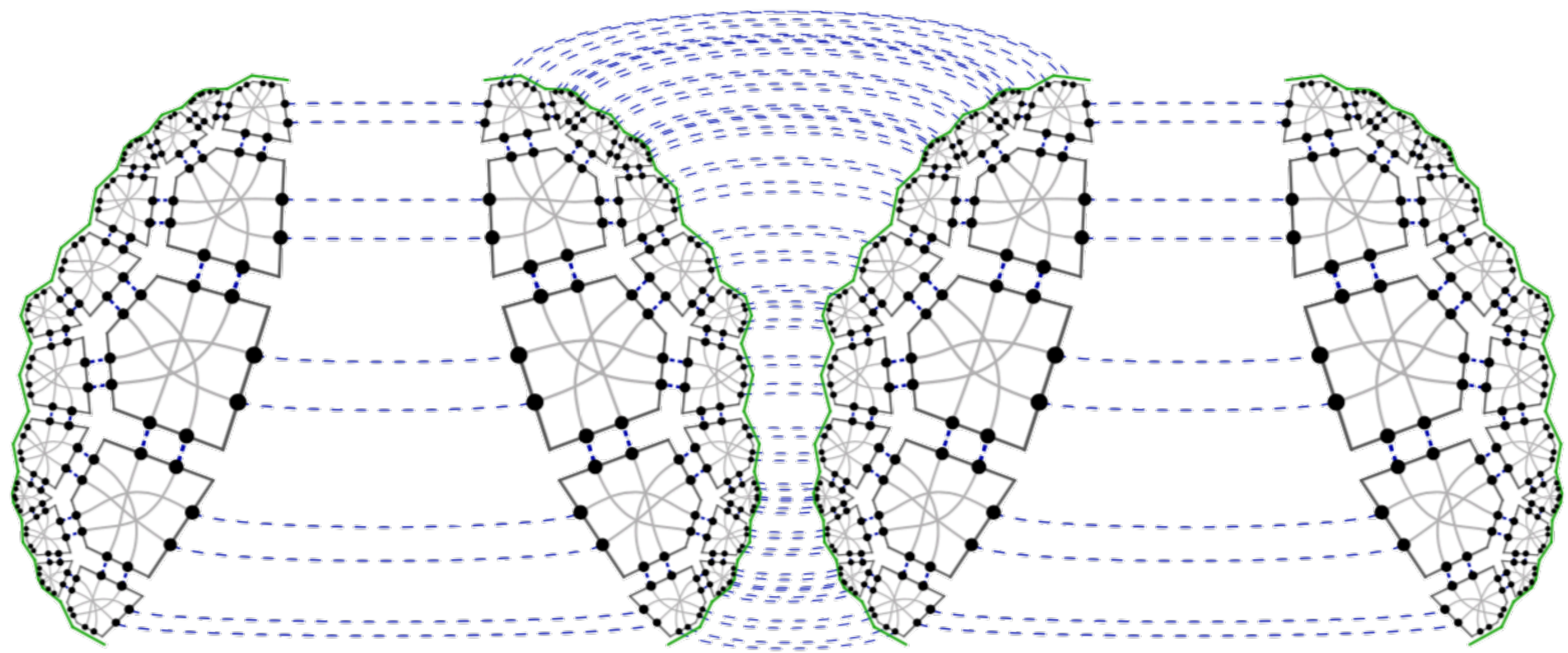
Jahn, Gluza, Pastawski, Eisert, arXiv:1905.03268 (2019)



- **Theorem:** Computational basis state vectors of the bulk are dual to Majorana dimer states on the boundary
- Can compute second moments of **non-Gaussian states** arising in **quantum error correcting codes** etc

Jahn, Gluza, Pastawski, Eisert, arXiv:1905.03268 (2019)

$$\rho_A^2 = 2^{2N_{C,w}}$$



- **Theorem:** Can compute Renyi entropies

Jahn, Gluza, Pastawski, Eisert, arXiv:1905.03268 (2019)

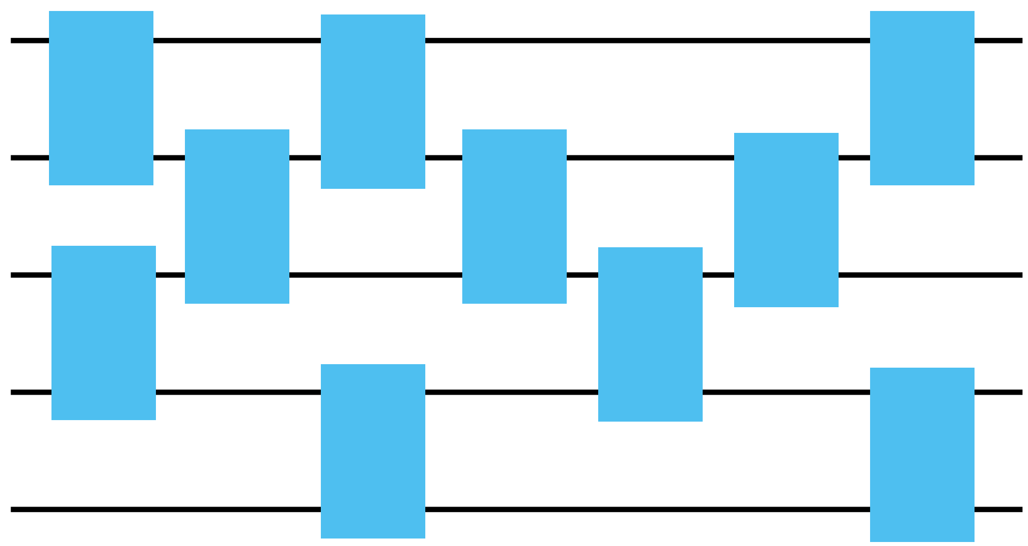
MATCHGATE TENSOR NETWORKS PROVIDE VERSATILE FRAMEWORK, ALLOWING FOR NEW INSIGHTS INTO HOLOGRAPHIC CODES

Jahn, Gluza, Pastawski, Eisert, arXiv:1905.03268 (2019)

OUTLOOK: COMPLEXITY AND ENTANGLEMENT

- **"Definition"**: Complexity of a quantum state vector is defined as the minimum number of gates needed to prepare it from a product

$|T\rangle$



$|R\rangle$

$$|T\rangle = U|R\rangle$$

Nielsen, Dowling, Gu, Doherty, Science 311, 1133 (2006)

- **Nielsen's geometric approach:** Unitary as time-dependent Hamiltonian

$$U = \mathcal{P} \exp\left(-i \int_0^1 dt \sum_I y_I(t) M_I\right)$$

in terms of two-local Pauli matrices M_I

- Complexity (common l_1 choice):

$$C := \inf \int_0^1 \sum_I |y_I(t)| ds$$

Nielsen, Dowling, Gu, Doherty, Science 311, 1133 (2006)

- Much studied in the holographic context
  - "Complexity equals volume"
    Stanford, Susskind, Phys Rev D 90, 126007 (2014)
  - "Complexity equals action"
    Brown, Roberts, Susskind, Swingle, Zhao, Phys Rev Lett 116, 191301 (2016)

Chapman, Marrochio, Myers, JHEP 1701, 062 (2017)
Jefferson, Myers, arXiv:1707.08570
Chapman, Heller, Marrochio, Pastawski, Phys Rev Lett 120, 121602 (2018)
Chapman, Eisert, Hackl, Heller, Jefferson, Marrochio, Myers, SciPost Phys 6, 034 (2019)
Goto, Marrochio, Myers, Quimada, Yoshida, arXiv:1901:00014 (2019)

- But **notoriously hard** to compute - for good reasons

- Complexity (common l_1 choice):

$$C := \inf \int_0^1 \sum_I |y_I(t)| ds$$

Nielsen, Dowling, Gu, Doherty, Science 311, 1133 (2006)

CAN WE OBTAIN COMPUTABLE TIGHT LOWER BOUNDS?

- **Theorem:** The complexity is lower bounded by

$$C \geq \frac{1}{c\log(d)} \sum_{s=1}^{S} E(|\psi\rangle\langle\psi|, \{1,\ldots,A(s)\} : \{A(s)+1,\ldots,n\})$$

the sum entanglements over cuts, where c is a universal constant

- Upper bounds to entanglement generation rates with Hamiltonians using auxiliary systems

Marien, Audenaert, Vam Acoleyen, Verstraete, Commun Math Phys 346, 35 (2016)
Eisert, Heller, in preparation (2019)

- **Definition:** A unitary $U \in U(d^2)$ has **potential entangling power**

$$e(U) = \min_{\delta}\{U = e^{-i\delta H}, \|H\| = 1\}$$

Eisert, Heller, in preparation (2019)
Compare Balasubramanian, DeCross, Kar, Parrikar, arXiv:1811.04085

- **Theorem:** The complexity is lower bounded by

$$C \geq \frac{1}{c\log(d)} \sum_{s=1}^{S} E(|\psi\rangle\langle\psi|, \{1,\ldots,A(s)\} : \{A(s)+1,\ldots,n\})$$

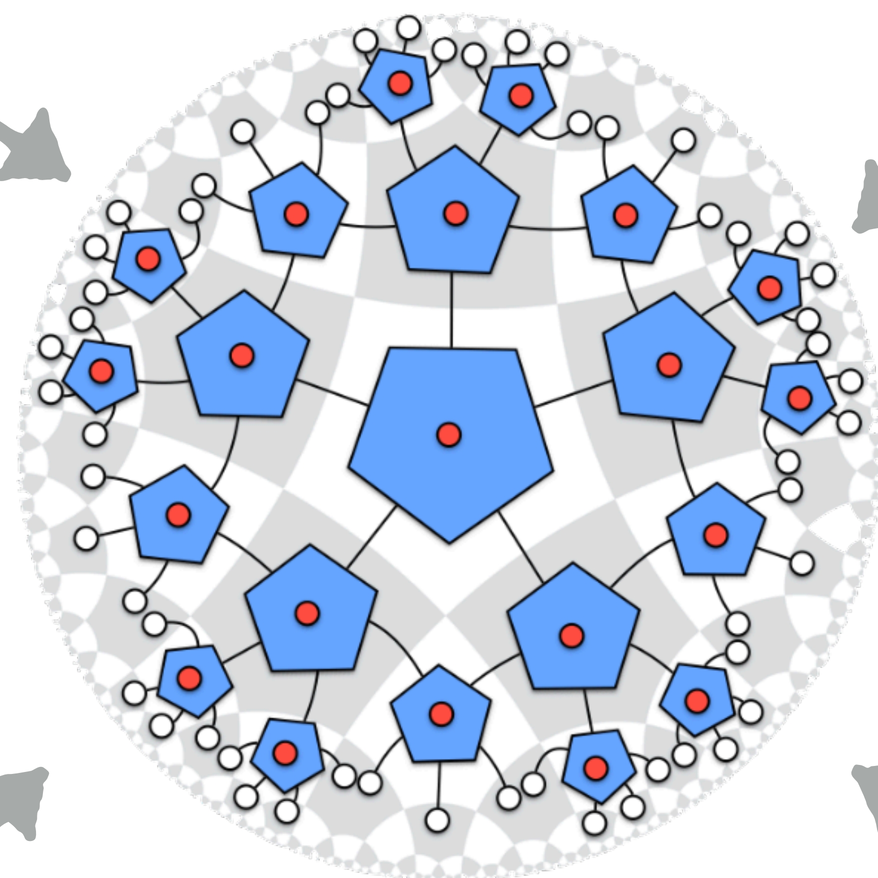
the sum entanglements over cuts, where c is a universal constant

EASILY COMPUTABLE, GROWS LINEARLY FOLLOWING QUENCHES, ETC

Eisert, Heller, in preparation (2019)

Interesting endeavor to think of tensor network models capturing holographic aspects

Fun tool provided by matchgate tensor networks



Tunable correlations, curvature, entanglement, MERA

Quantum error correction and Majorana dimers

Complexity

SUMMARY

- Random matchgate tensors?
  Hayden, Nezami, Qi, Thomas, Walter, Yang, JHEP 2016, 9 (2016)
- Interacting theories, connection to string nets?
  Wille, Buerschaper, Eisert, Phys Rev B 95, 245127 (2017)
  Bultinck, Williamson, Haegeman, Verstraete, Phys Rev B 95, 075108 (2017)
- Steps towards parametrizing physical CFTs?
- Non-unitary MERA, further perspectives?
- Quasi-periodic tilings?
  Boyle, Dickens, Flicker, arXiv:1805.02665

THANKS FOR YOUR ATTENTION!

Seminar on quantum advantages tomorrow 12:30