Symmetry Generators of $NAdS_2/NCFT_1$

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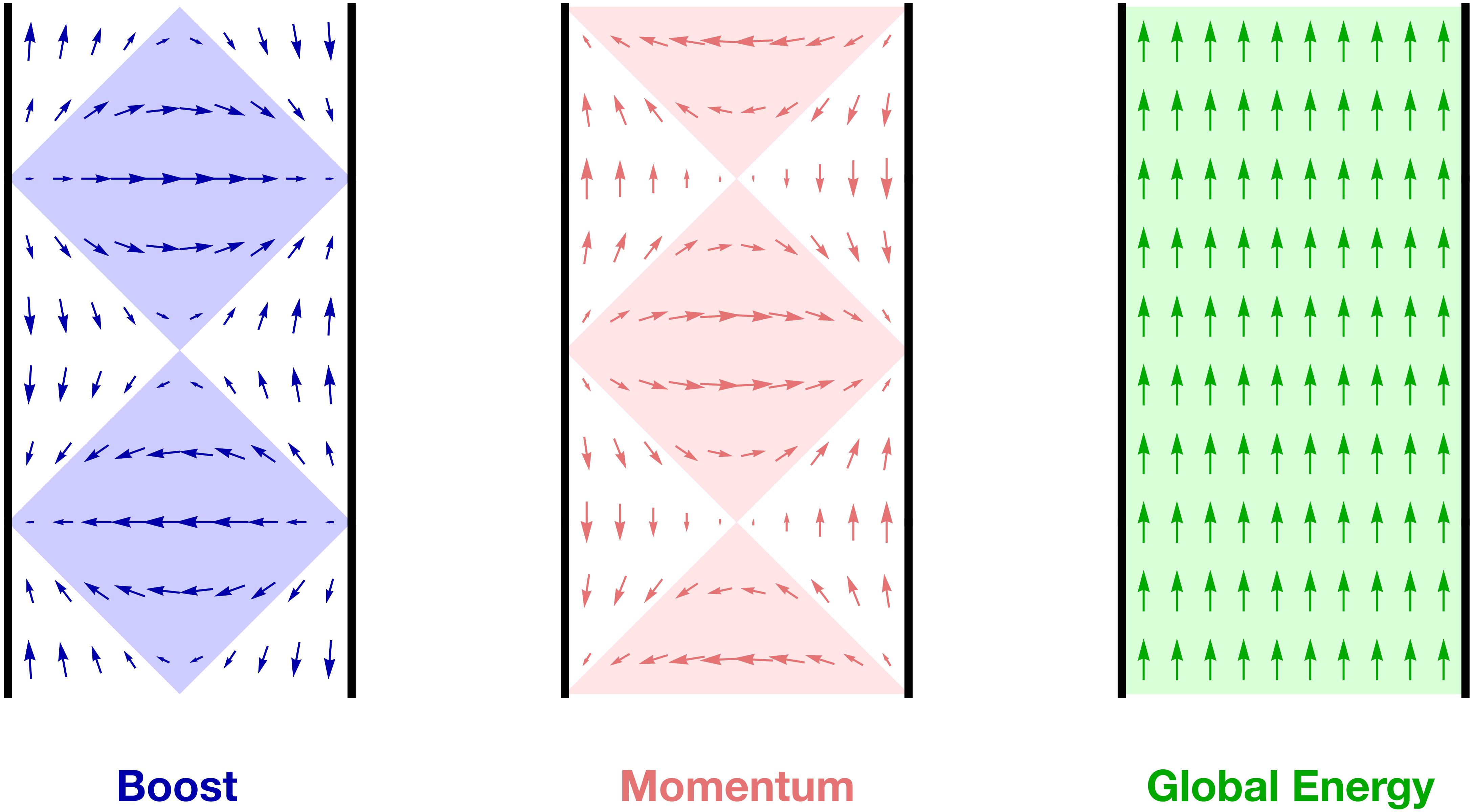
YITP 2019

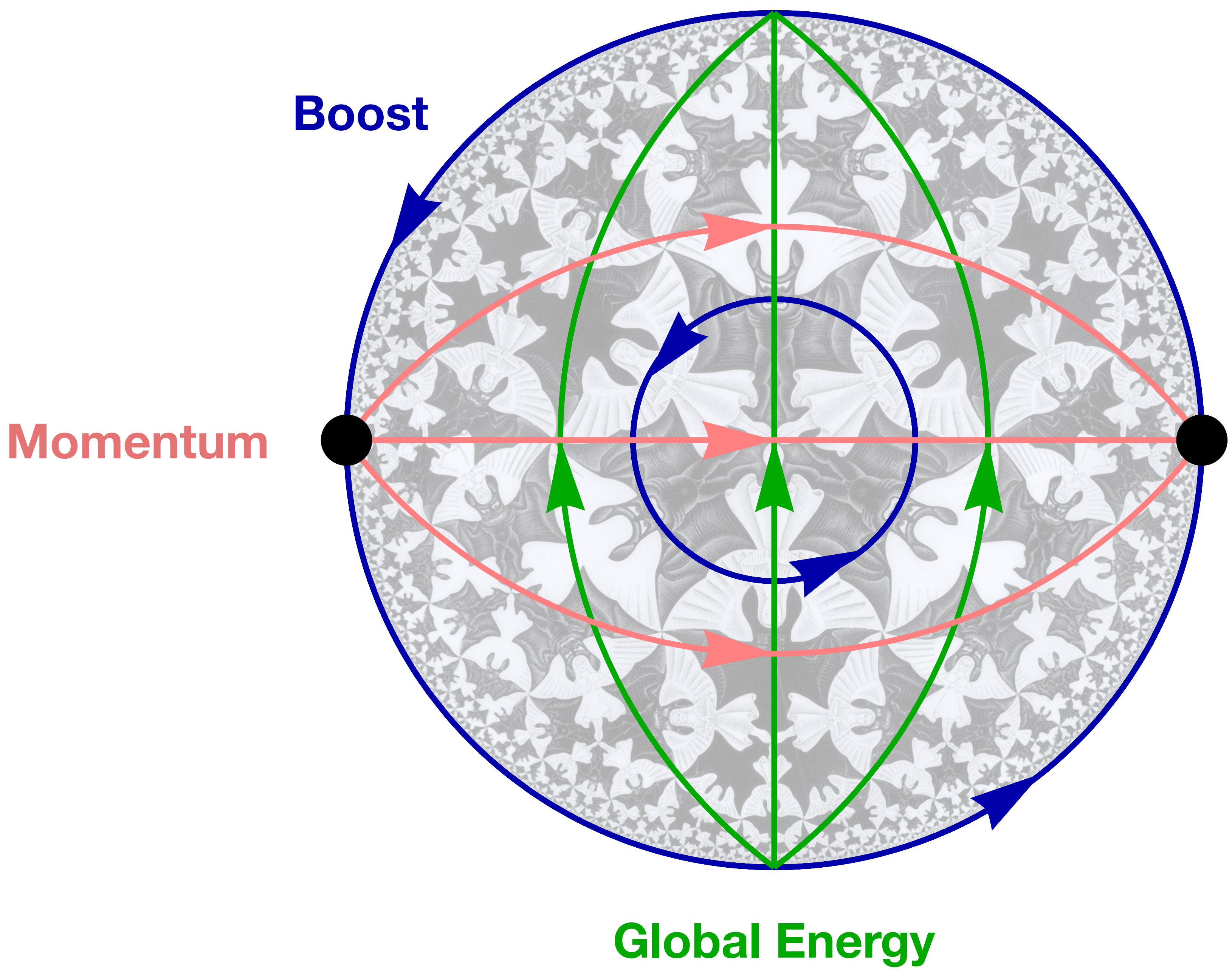
See 1904.1280 w/ J. Maldacena and Y. Zhao

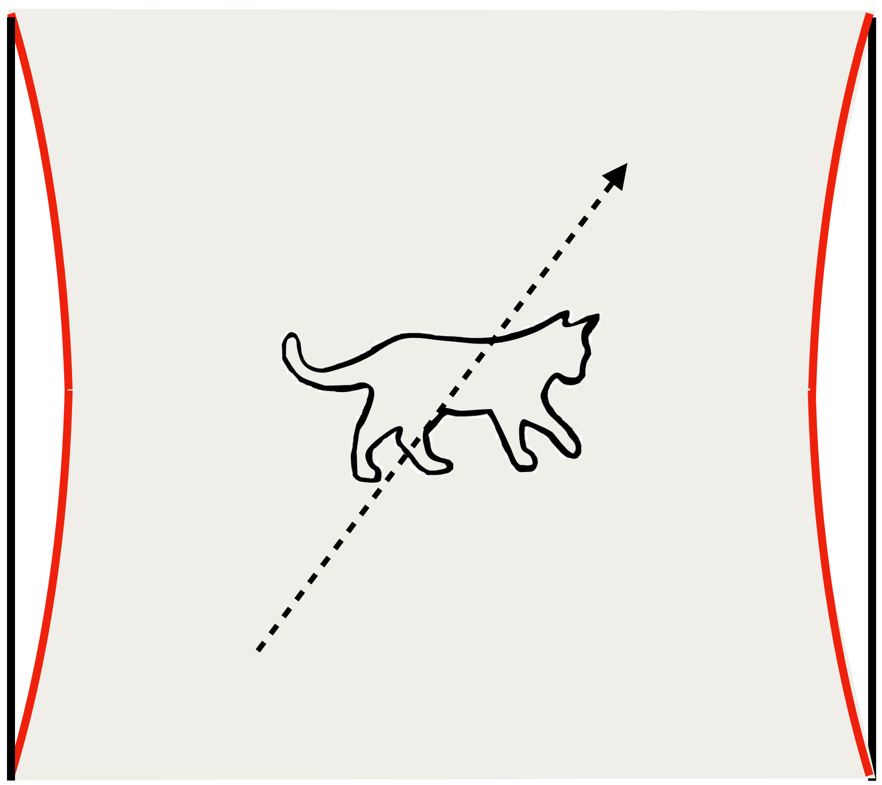
also 1804.00491, 1810.11958, 1904.12819

AdS_2



Euclidean AdS_2





$NAdS_2$

- *Previous work*: SL(2) symmetry gives rise to a **gauge** symmetry; since only relative positions of bulk objects w.r.t. the boundaries are physical, an overall $SL(2)_g$ transformation to the entire system (boundary + matter) leaves physical states invariant.

- *Here*, this symmetry also gives rise to a separate **physical** symmetry SL(2) that ~ translates matter relative to the boundary. Moves the worldline of the cat into the interior!

Outline

Construction → Algebra; State Operator Correspondence

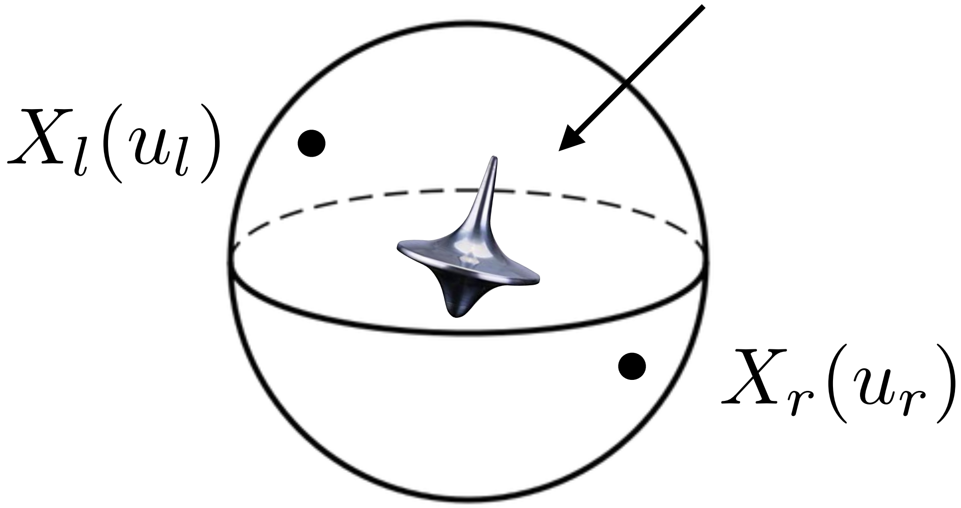
Applications

- Chaos
- Bulk Reconstruction/Exploring the bulk
- **Traversable Wormholes**
- Connection to “Size” (see talk by YZ)

Ask me about this!

aside: SU(2) analogy

"matter" = irreps of su(2)



"embedding space" constraint

$$X^2 = 1$$

$$J_l^a + J_r^a + J_m^a = 0$$

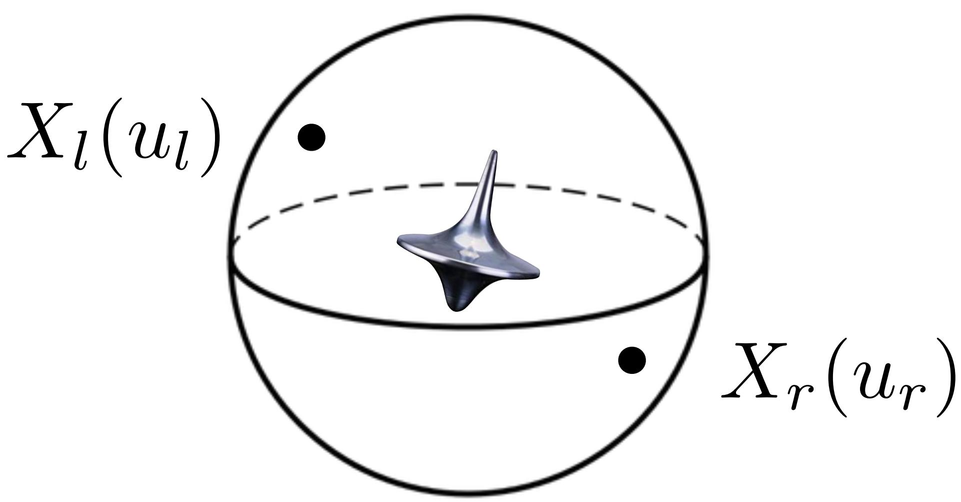
Gauge constraint

$$\mathcal{H}_{\text{Physical}} = (\mathcal{H}_l \times \mathcal{H}_{\text{matter}} \times \mathcal{H}_r)/SU(2)_g$$

$$[J_i, J_j] = i\varepsilon_{ijk}J_k, \qquad [J_i, X_j] = i\varepsilon_{ijk}X_k, \qquad [X_i, X_j] = 0$$

not gauge invariant

aside: SU(2) analogy



$$X^2 = 1$$

$$J_l^a + J_r^a + J_m^a = 0$$

$$\mathcal{H}_{\text{Physical}} = (\mathcal{H}_l \times \mathcal{H}_{\text{matter}} \times \mathcal{H}_r)/SU(2)_g$$

$$G^1 = -X_l \cdot J_m, \quad G^2 \sim -X_r \cdot J_m, \quad G^3 \sim -(X_l \times X_r) \cdot J_m.$$

Gauge invariant & satisfy exact su(2) algebra!

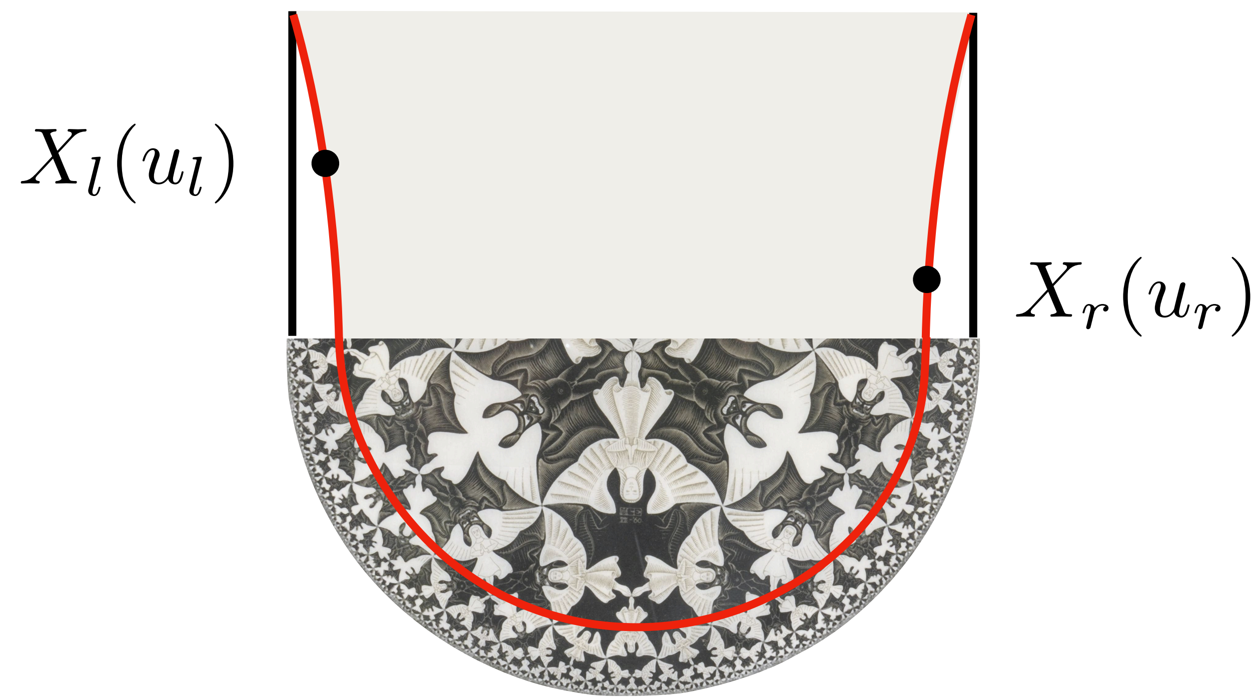
Embedding space

$$Y \cdot Y = \eta_{ab} Y^a Y^b = -1$$

- View AdS_2 as a hyperboloid in 3D spacetime
- Matter charges are vectors (carry an SL(2,R) index)
- “Gravitationally dressing” bulk charges

$$[Q^a, Q^b] = i\epsilon^{abc} Q_c$$

Schwarzian Theory $^{+matter}$



Gauge constraint

$$\mathcal{H}_{\text{Physical}} = (\mathcal{H}_l \times \mathcal{H}_{matter} \times \mathcal{H}_r)/SL(2)_g \ ,$$

$$X_r = (X^{-1}, X^{+}, X^{-}) = \left(\frac{1}{t_r{}'}, \frac{e^{t_r}}{t_r{}'}, -\frac{e^{-t_r}}{t_r{}'} \right)$$

$$X_l = \left(\frac{1}{t_l{}'}, -\frac{e^{-t_l}}{t_l{}'}, \frac{e^{t_l}}{t_l{}'} \right) .$$

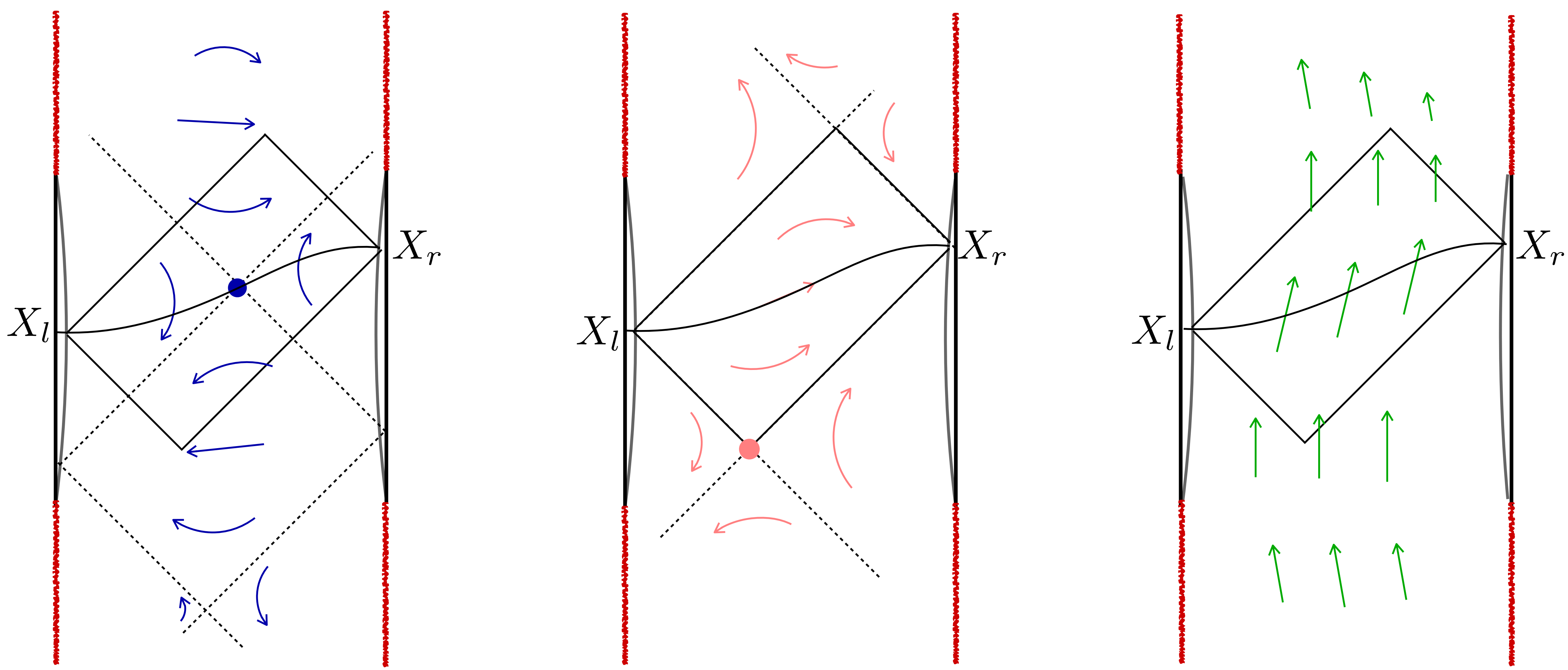
Noether

$$Q_l^a + Q_m^a + Q_r^a = 0$$

$$I = \frac{N}{\beta J} \int \mathrm{d}u \, \{e^t, u\}$$

see also 1606.01857

Geometric construction



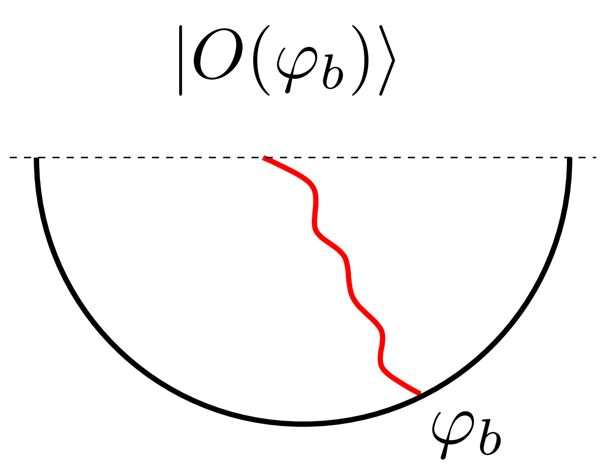
$\tilde{B}$ $\tilde{P}$ $\tilde{E}$

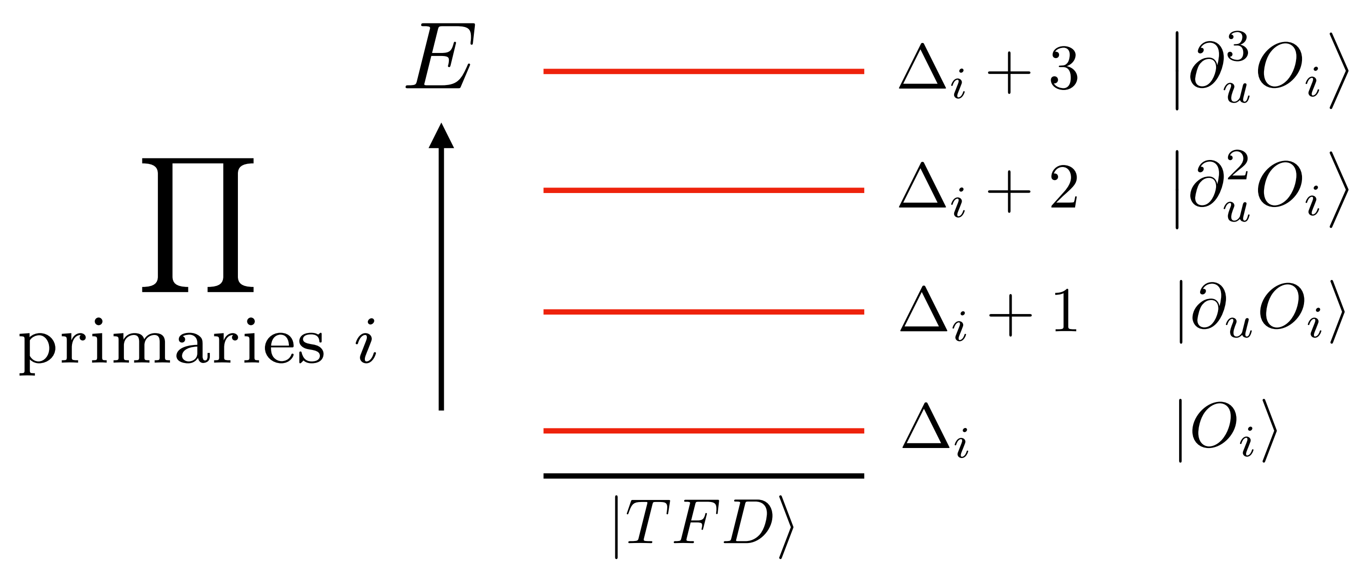
● $\frac{X_r + X_l}{\sqrt{-2X_r \cdot X_l}}$

● $\frac{X_l \times X_r}{X_l \cdot X_r}$

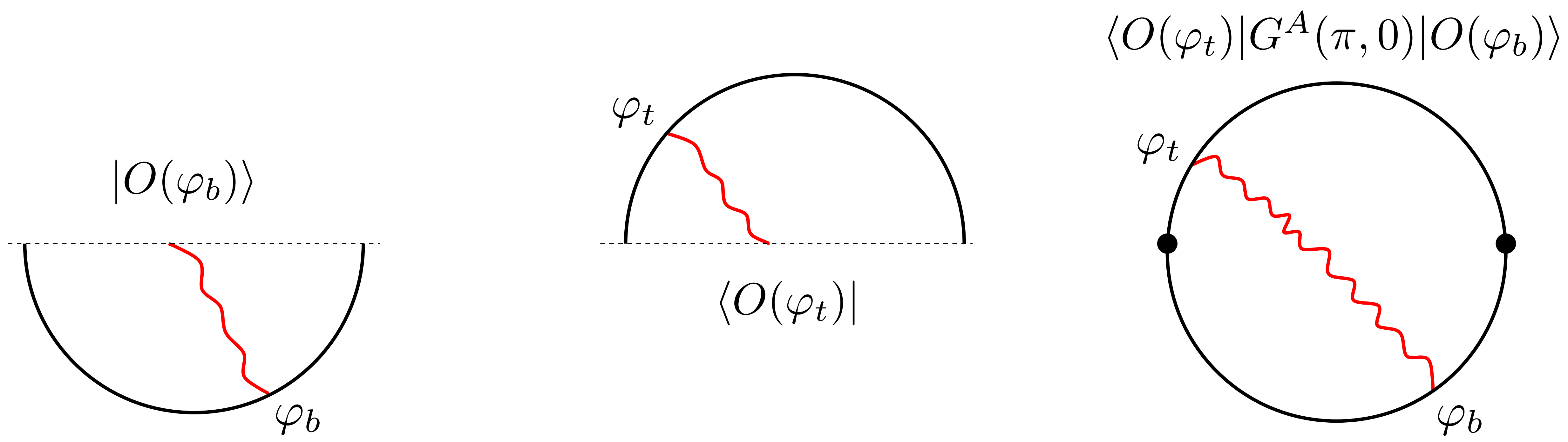
$\frac{1}{\sqrt{-2X_l.X_r}}(X_{ra} - X_{la})$

Operator-State Correspondence

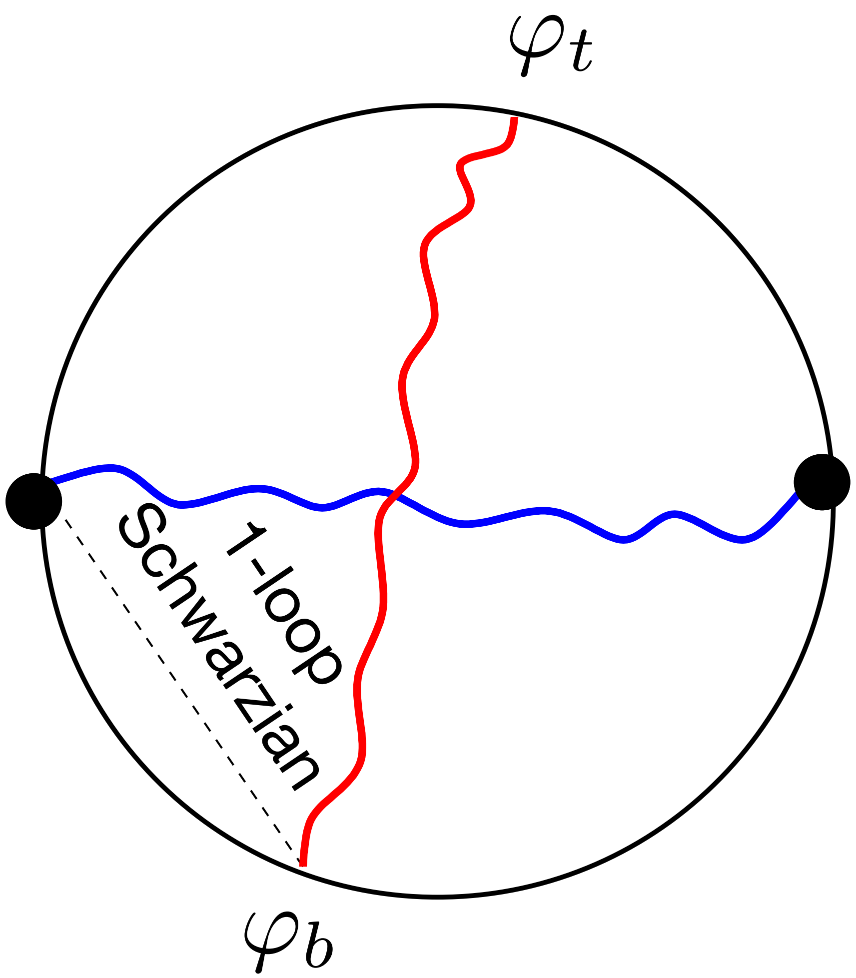


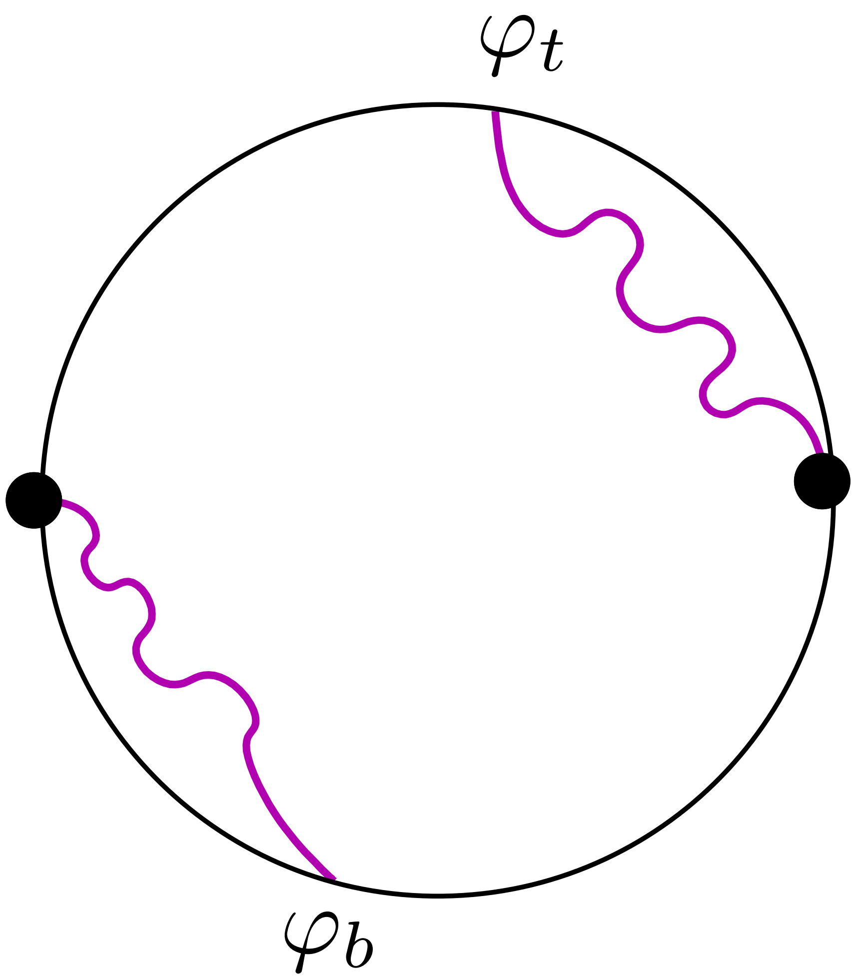


Operator-State Correspondence



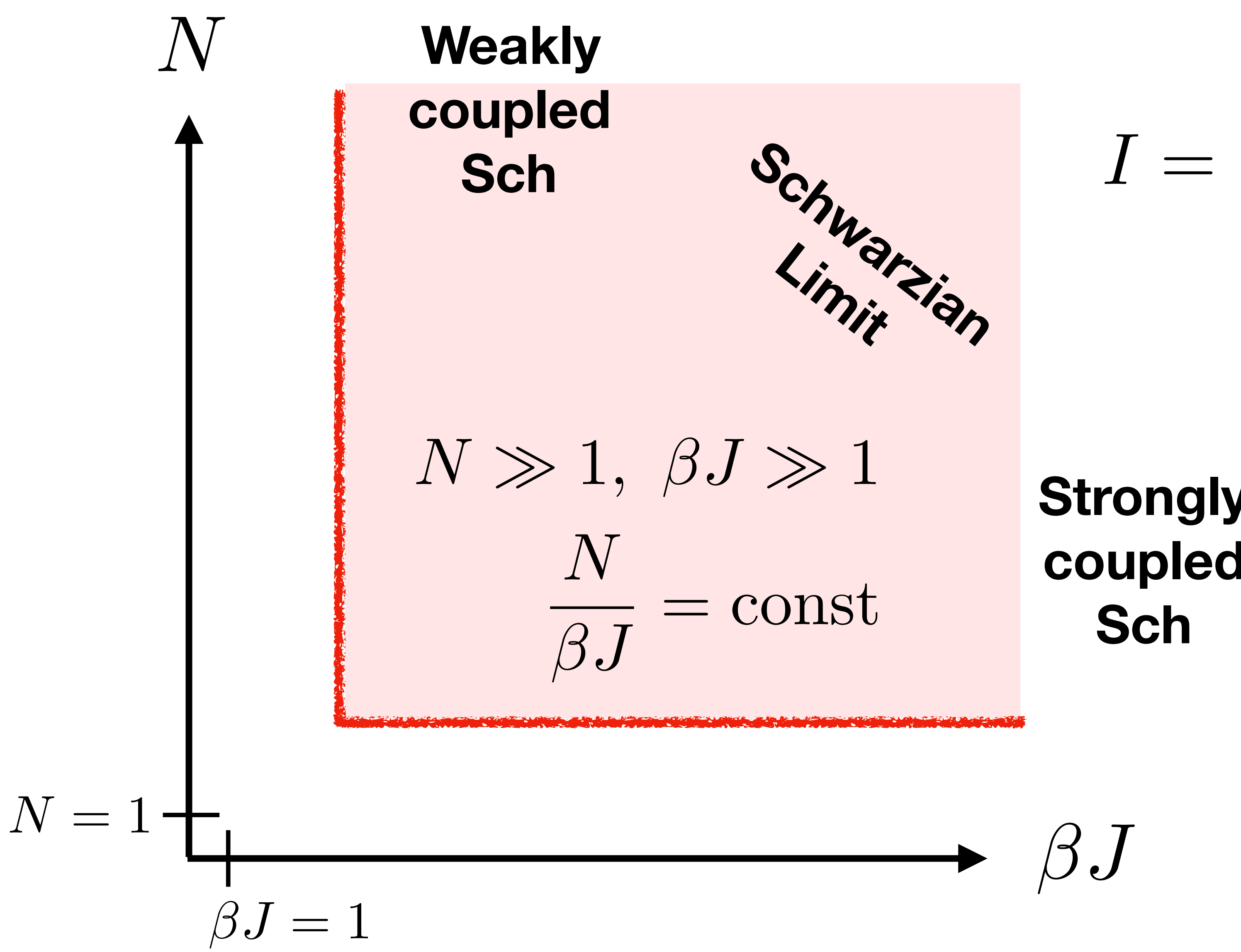
Operator-State Correspondence





bad diagrams
(suppressed by 1/N in a model like SYK)

SYK Model

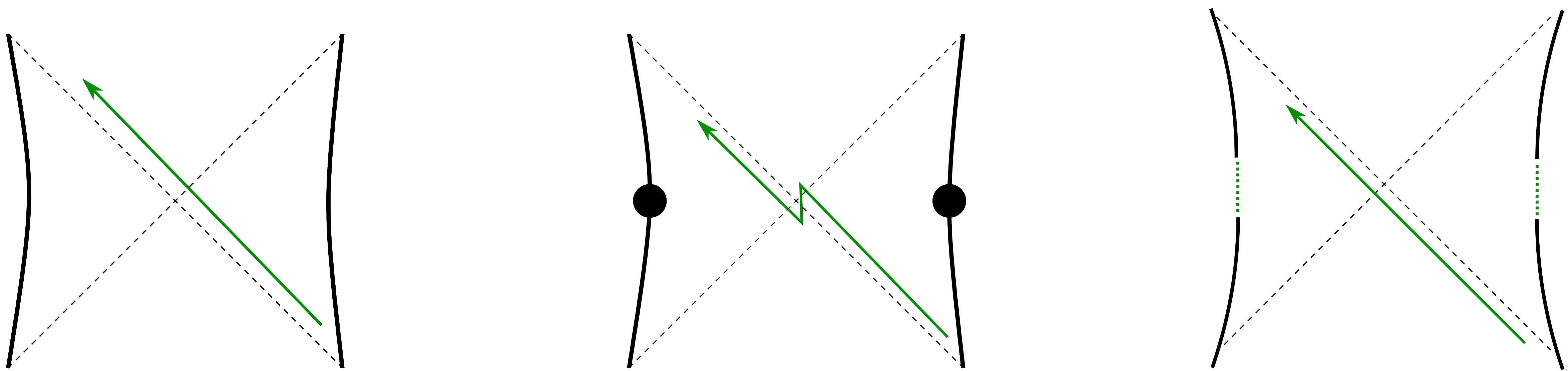


$$I = \frac{N}{\beta J} \int \{e^t, u\}$$

Kitaev talks, Maldacena Stanford 1604.07818, Kiteav Suh 1711.08467

Applications

Traversable wormhole



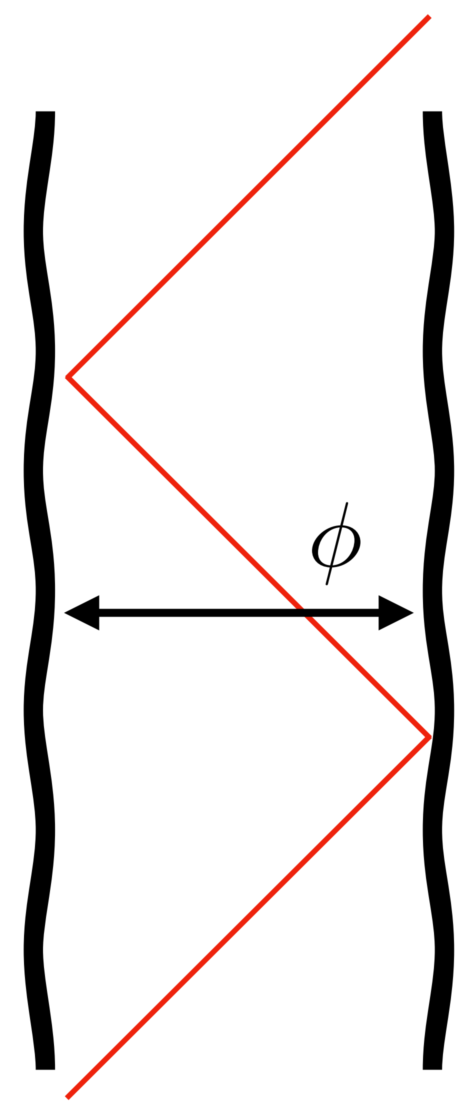
See also Maldacena Stanford Yang 1704.05333

Eternal Traversable wormhole

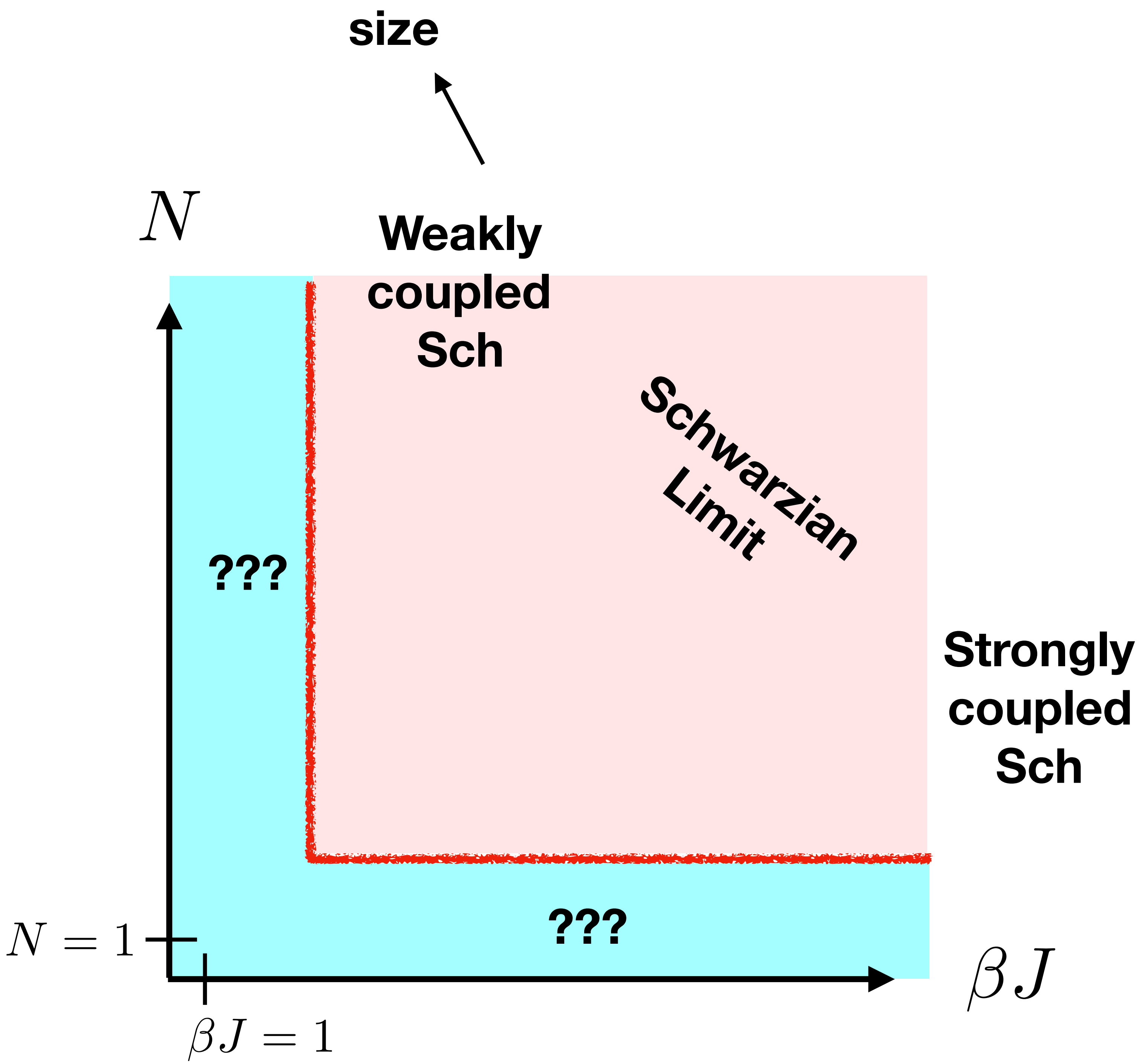
- The Maldacena-Qi traversable wormhole has non-trivial “breathing modes”, which breaks the SL(2) spectrum

$$L = \dot{\phi}^2 - V(\phi)$$

- Our *E* generator “subtracts off” both potential and kinetic energy. The result is generator w/out a temperature dependence



Maldacena Qi 1804.00491


size
N
Weakly coupled Sch
Schwarzian Limit
???
Strongly coupled Sch
???
$N=1$
βJ
$\beta J = 1$

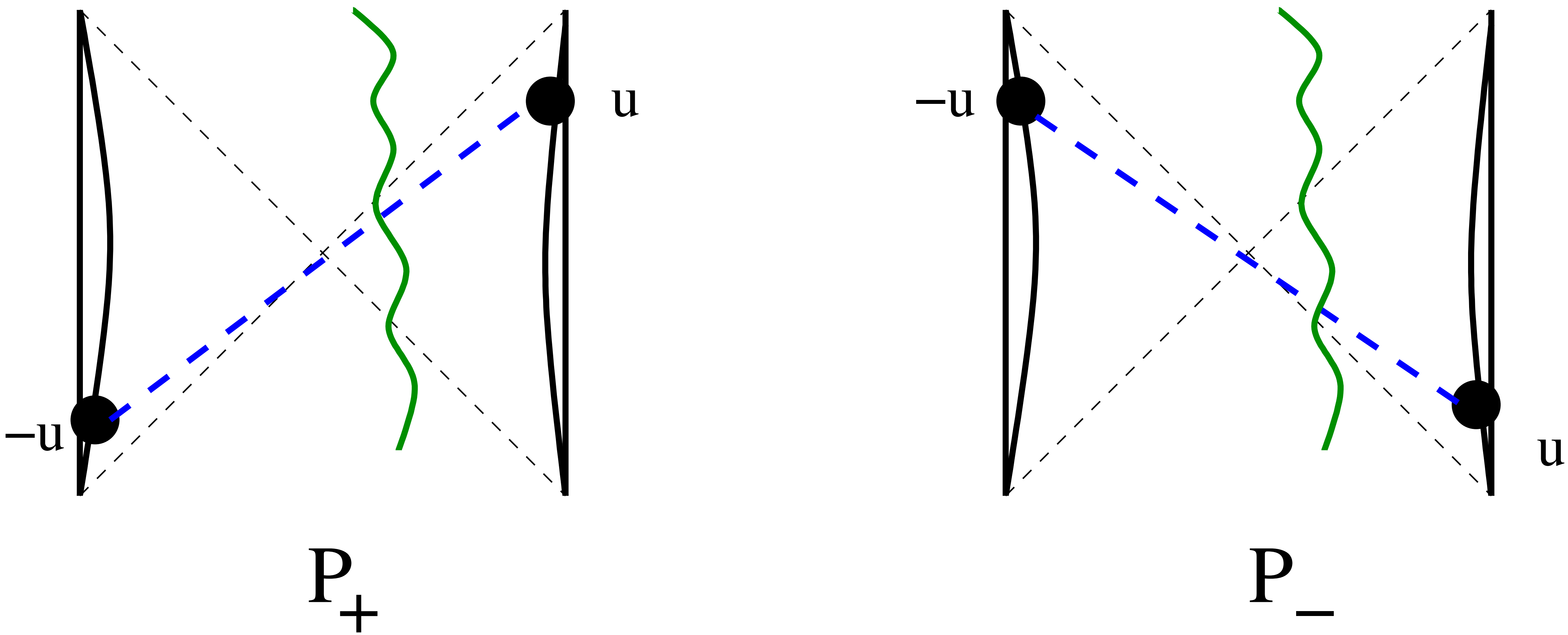
Questions

Order from Chaos

$$
\begin{aligned}
e^{i\tilde{u}\hat{B}}\hat{E}e^{-i\tilde{u}\hat{B}} &= \frac{\beta}{2\pi}\left[H_r + H_l - \tilde{\eta}\sum_j O_l^j(-u)O_r^j(u) - \langle\cdots\rangle_{\text{TFD}}\right] \simeq \cosh\tilde{u}\hat{E} - \sinh\tilde{u}\hat{P} \\
e^{i\tilde{u}\hat{B}}\hat{P}e^{-i\tilde{u}\hat{B}} &= \partial_{\tilde{u}}\left(\frac{\beta\tilde{\eta}}{2\pi}\sum_j O_l^j(-u)O_r^j(u)\right) \simeq -\sinh\tilde{u}\hat{E} + \cosh\tilde{u}\hat{P} \qquad (4.73)
\end{aligned}
$$

Exponential growth determined by the symmetries

Order from Chaos

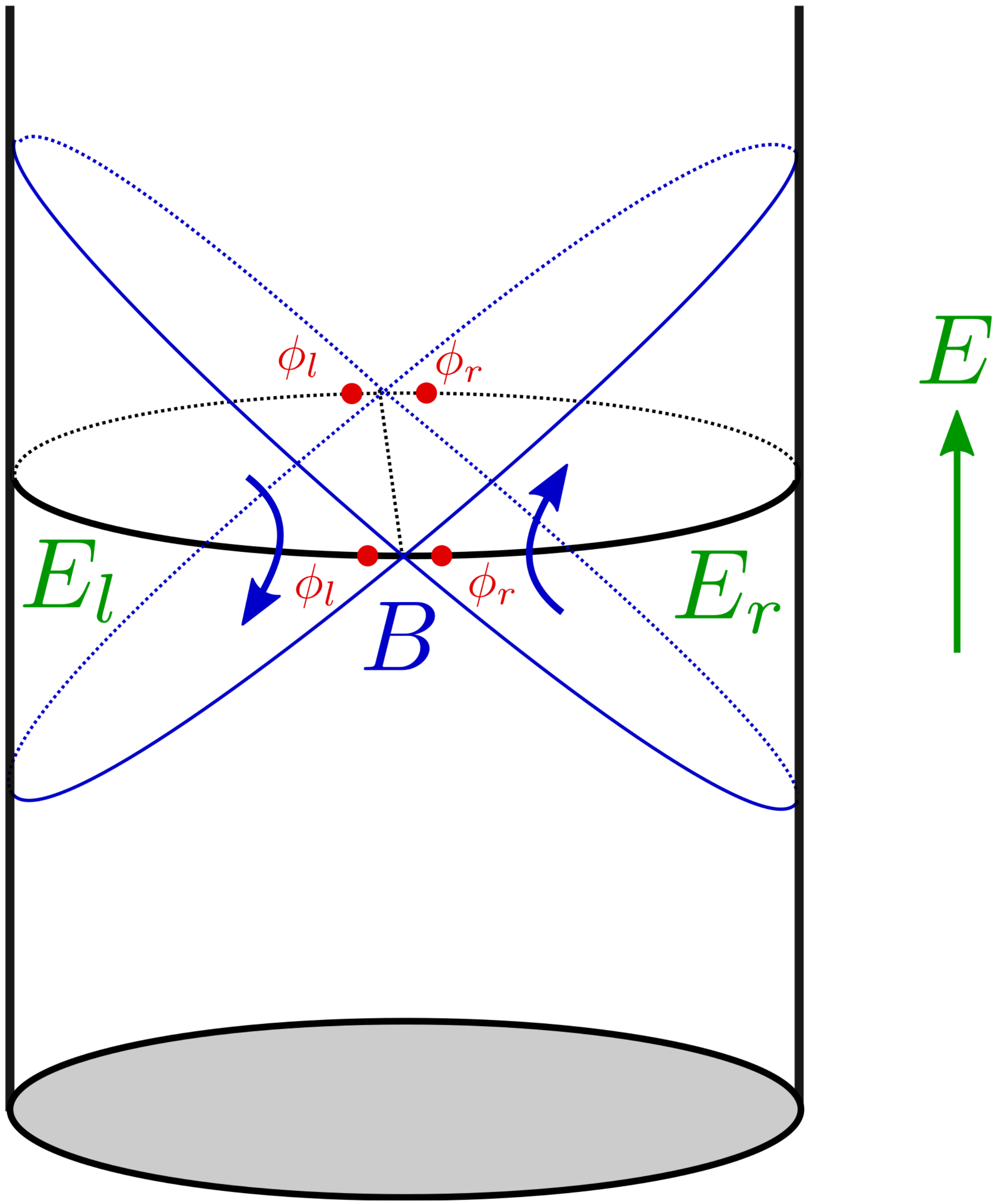


How are these symmetries?

- The Casimir commutes with the Hamiltonian.
- However, generators do **not** commute with the usual Hamiltonian $H_l + H_r$. Nevertheless, their time evolution is simple and geometric. We can undo the time evolution.
- For any QFT on background AdS_2 the generators do *not* commute with the global energy. This is related to the fact that the Killing vectors are time dependent.

$$Q_\zeta = \int_\Sigma n^\mu T_{\mu\nu} \zeta^\nu$$

Higher dimensional analog



Coleman de Luccia Instanton