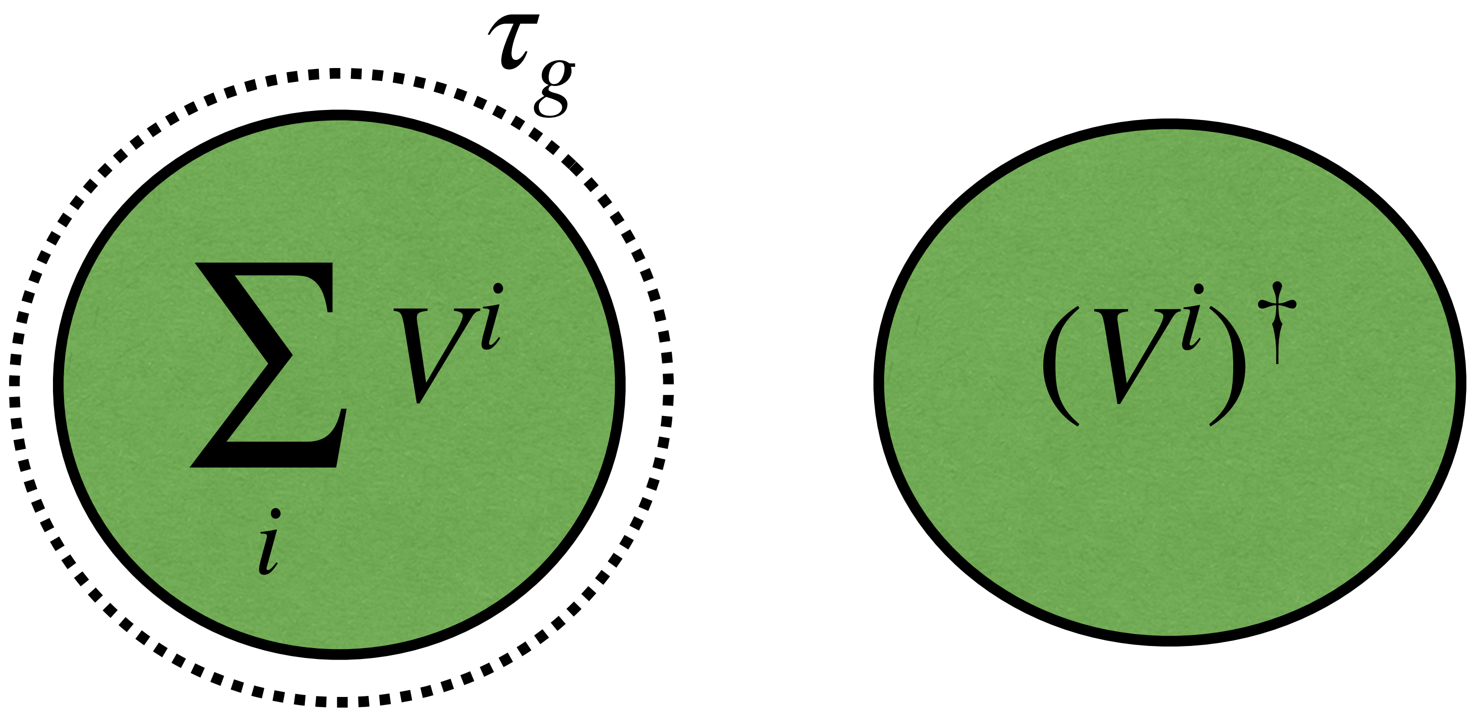
Entanglement, superselection sectors and holography



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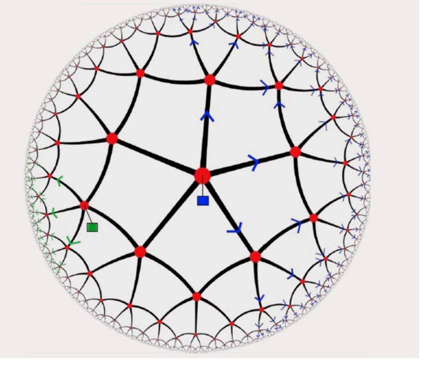
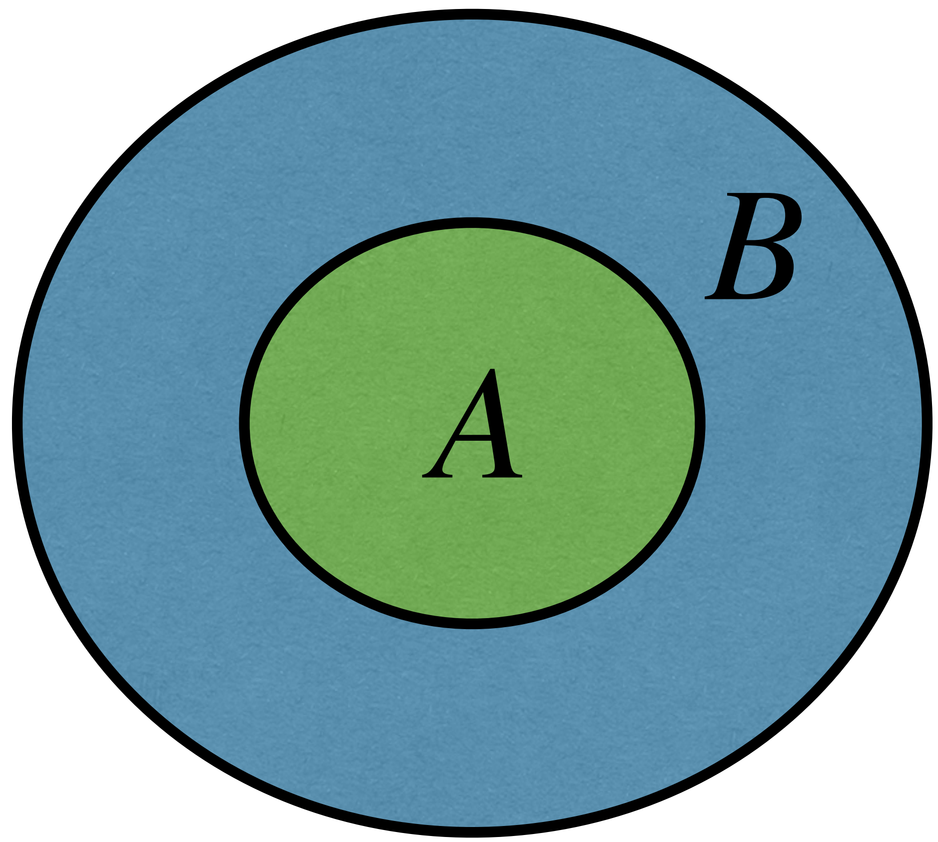


Motivations

- Entanglement Entropy in QFT
- QFT's with superselection sectors **[Doplicher, Haag, Roberts, 1969]**
- Universal terms in the expansion of EE
- Lattice approaches require fine-tuning **[Casini, Huerta, Rosabal, 2014]**
- Mutual Information seems to fail **[Casini, Huerta, 2015]**
- Topological models, gauge theories (Maxwell in particular), spontaneous symmetry breaking, holography…

Algebras and regions in QFT

- Isotonia

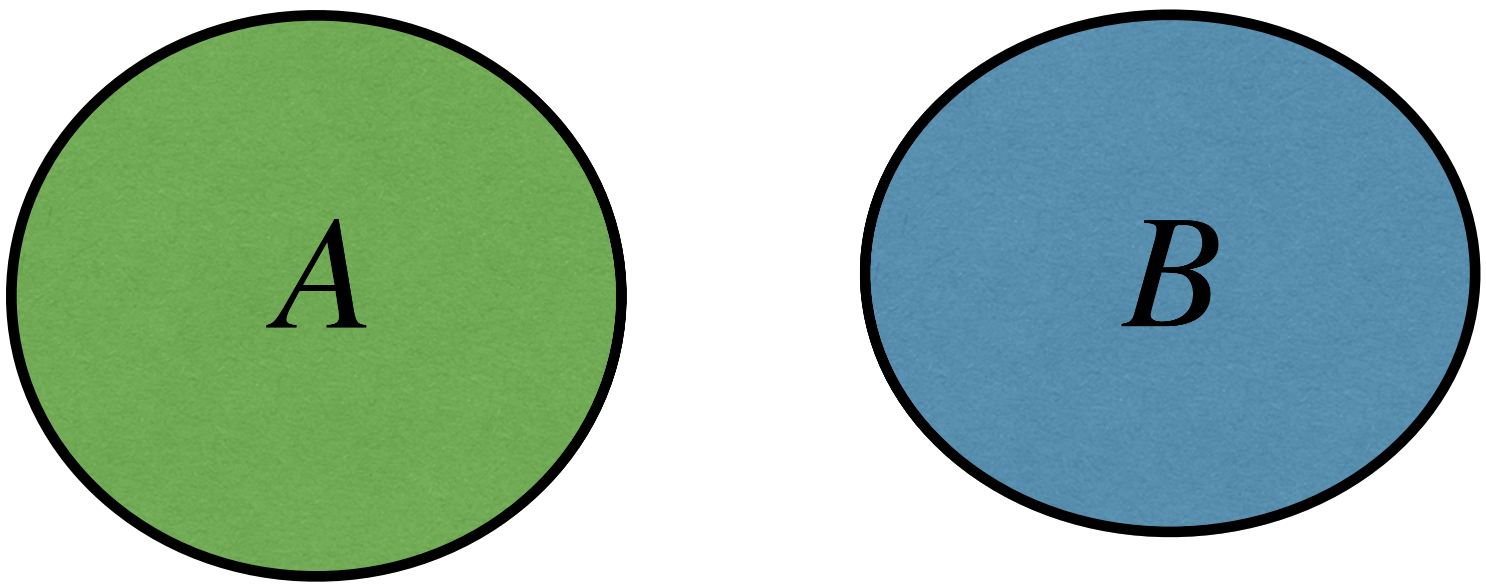


$$A \subseteq B$$

$$\mathcal{O}_A \subseteq \mathcal{O}_B$$

Algebras and regions in QFT

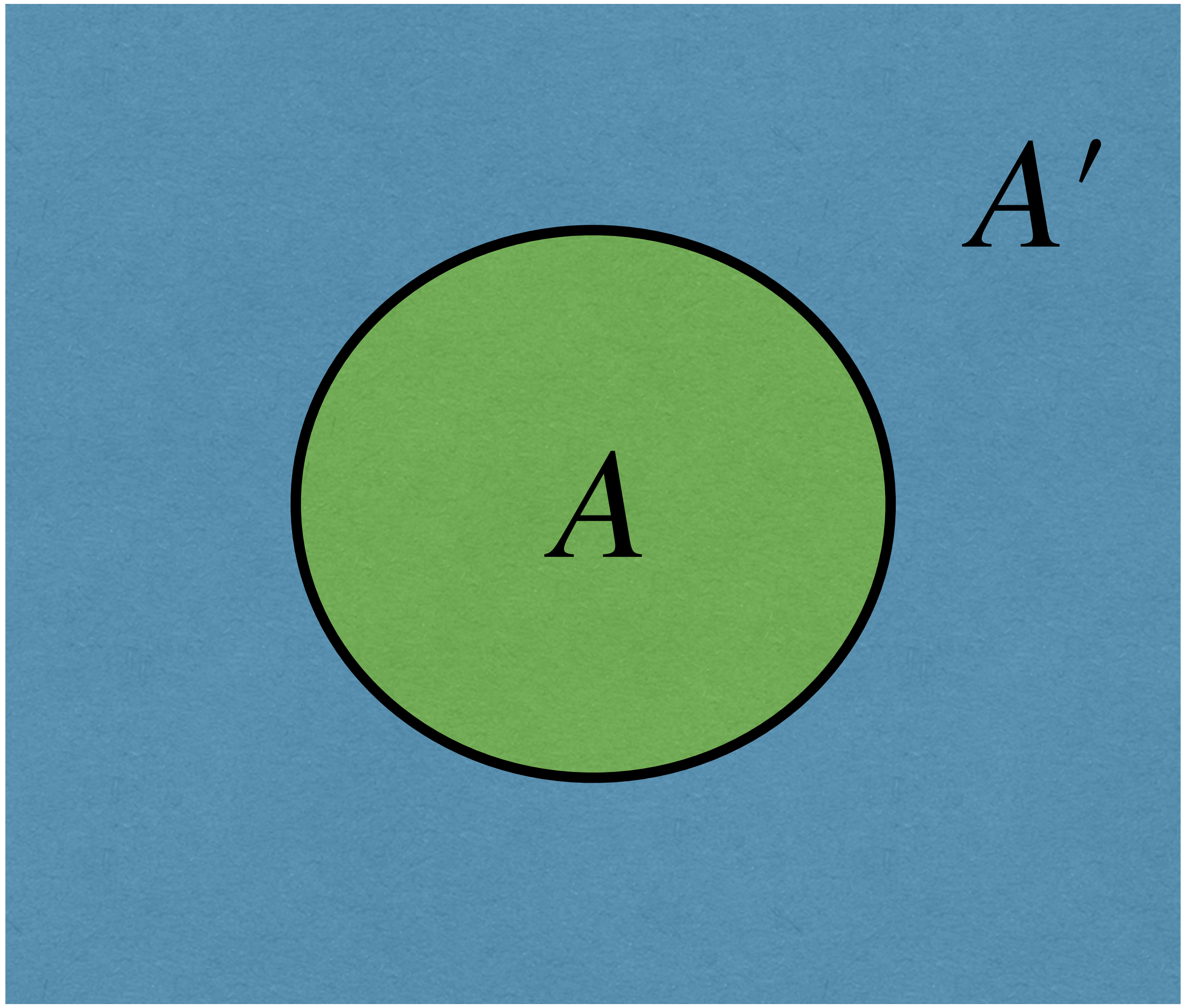
- Isotonia $A \subseteq B \longrightarrow \mathcal{O}_A \subseteq \mathcal{O}_B$
- Additivity



$$\mathcal{O}_{A \vee B} = \mathcal{O}_A \vee \mathcal{O}_B$$

Algebras and regions in QFT

- Isotonia $A \subseteq B \longrightarrow \mathcal{O}_A \subseteq \mathcal{O}_B$
- Additivity $\mathcal{O}_{A \vee B} = \mathcal{O}_A \vee \mathcal{O}_B$
- Duality for connected region [Haag, 1992]



$$\mathcal{O}_A \subset \mathcal{O}'_{A'}$$

Algebras and regions in QFT

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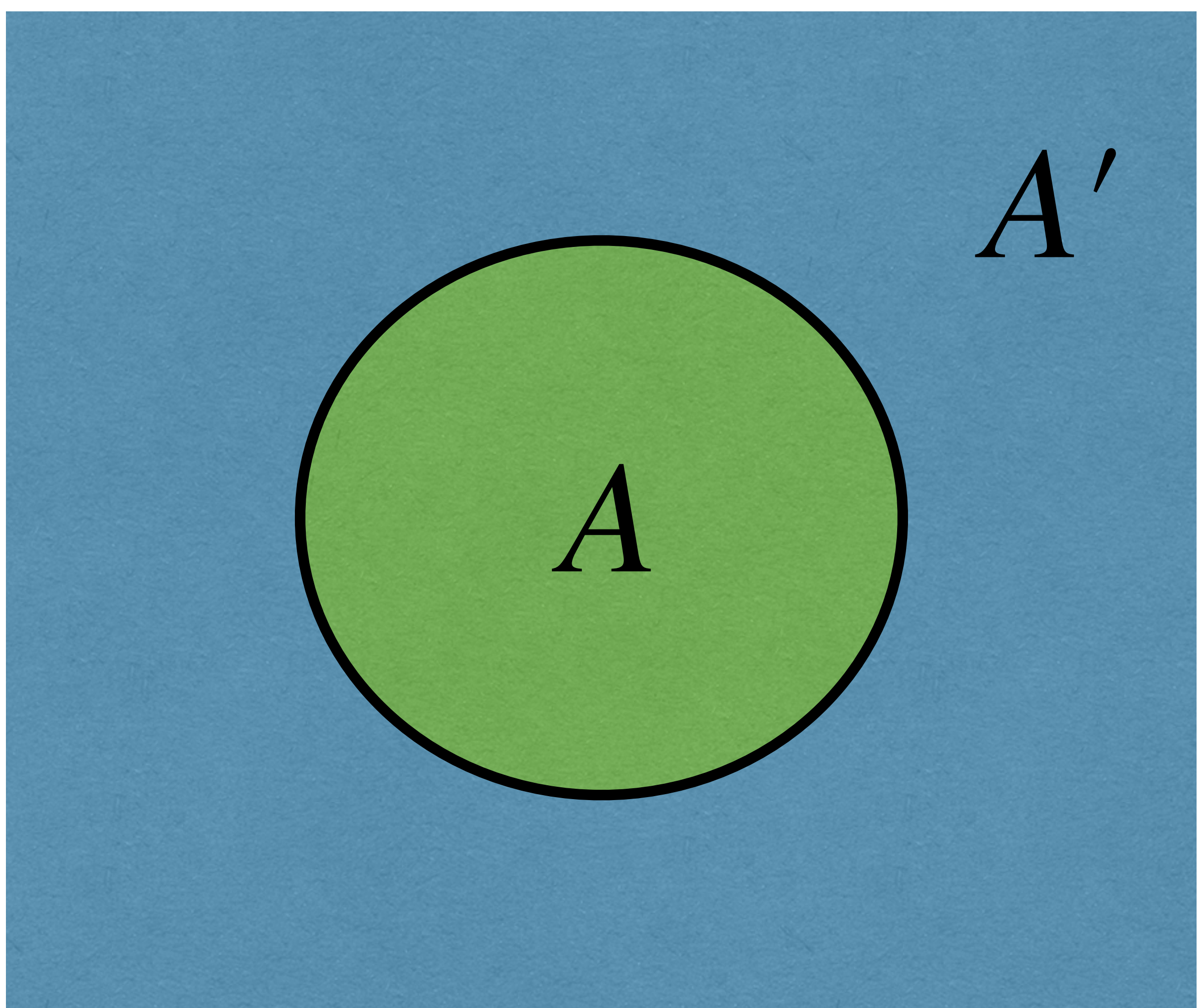

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$$\Downarrow$$

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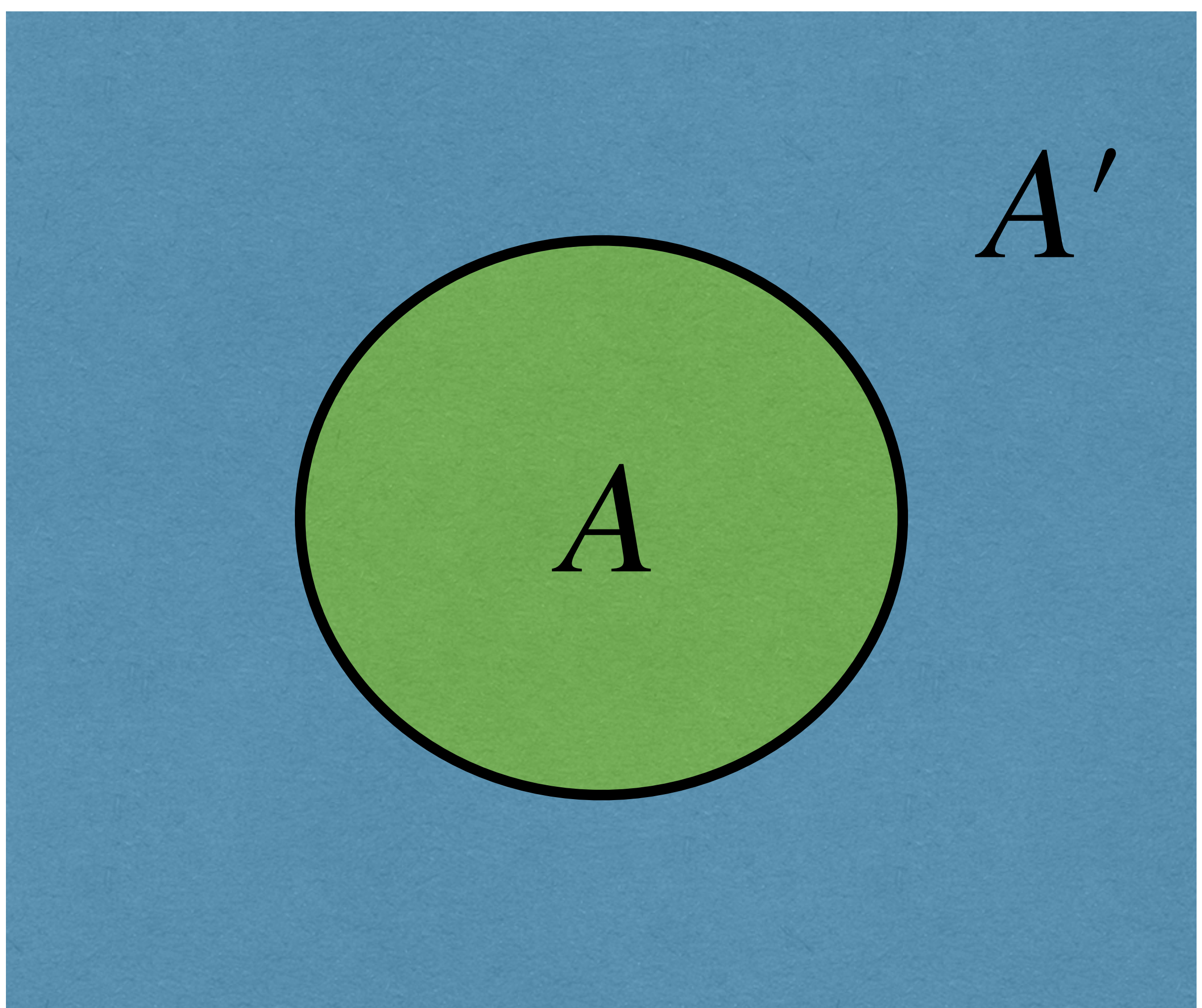

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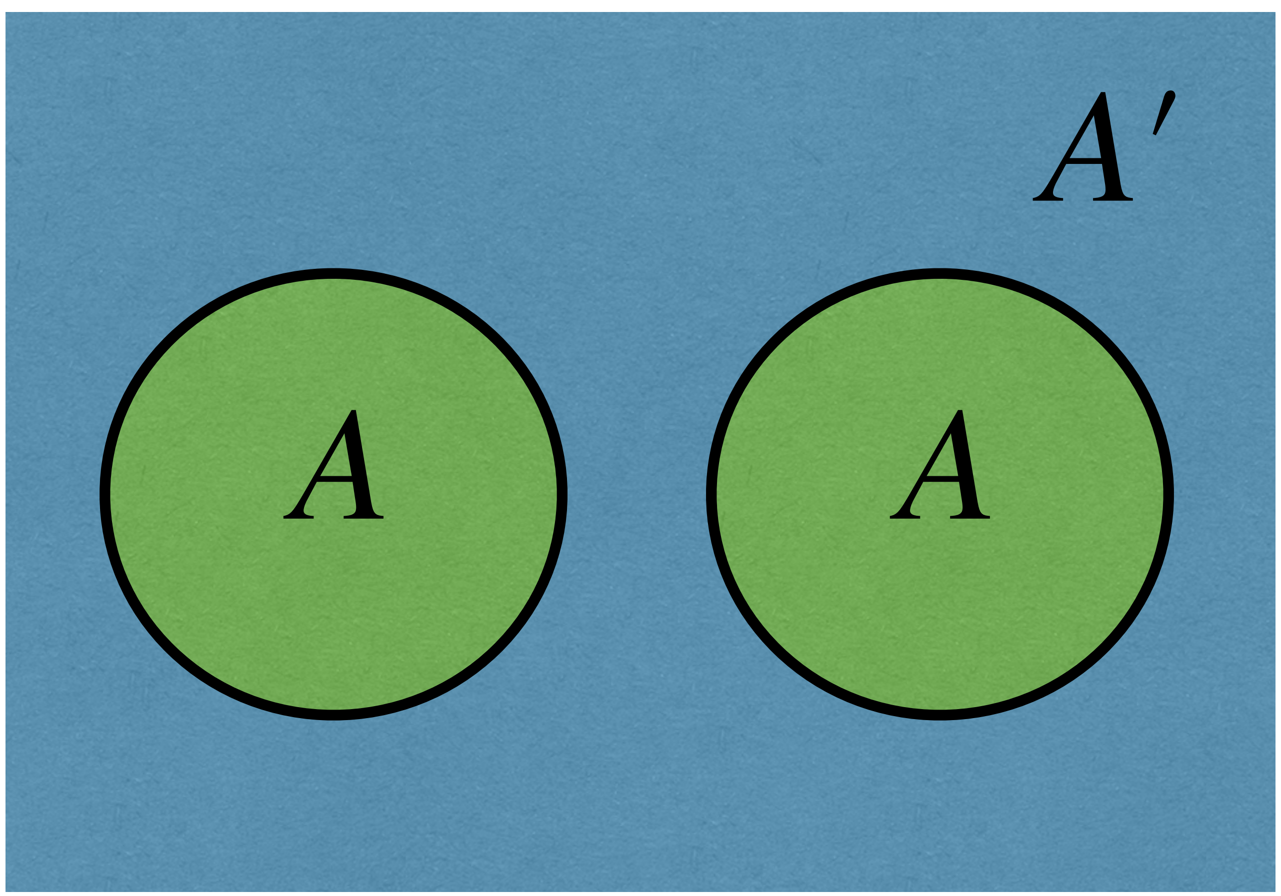
- Duality for connected region $\mathcal{O}_A = \mathcal{O}'_{A'}$
- Duality for disconnected region…



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QFT with global symmetries (DHR sectors)

[Doplicher, Haag, Roberts, 1969] [Doplicher, Longo 1984] [Doplicher, Roberts, 1990]

- Charge creating operators are local $V^i = \int \alpha(x) V^i(x)$
- Transforms naturally under symmetry as $\tau_g \, V^i \, \tau_g^{-1} = \sum_j R_{ij}(g) \, V^j$
- The algebra of neutral operators $\mathcal{O}$ arises from the field algebra $\mathcal{F}$ as

$$\mathcal{O} = \frac{1}{G} \sum_g \tau_g \, \mathcal{F} \, \tau_g^{-1} = E(\mathcal{F})$$

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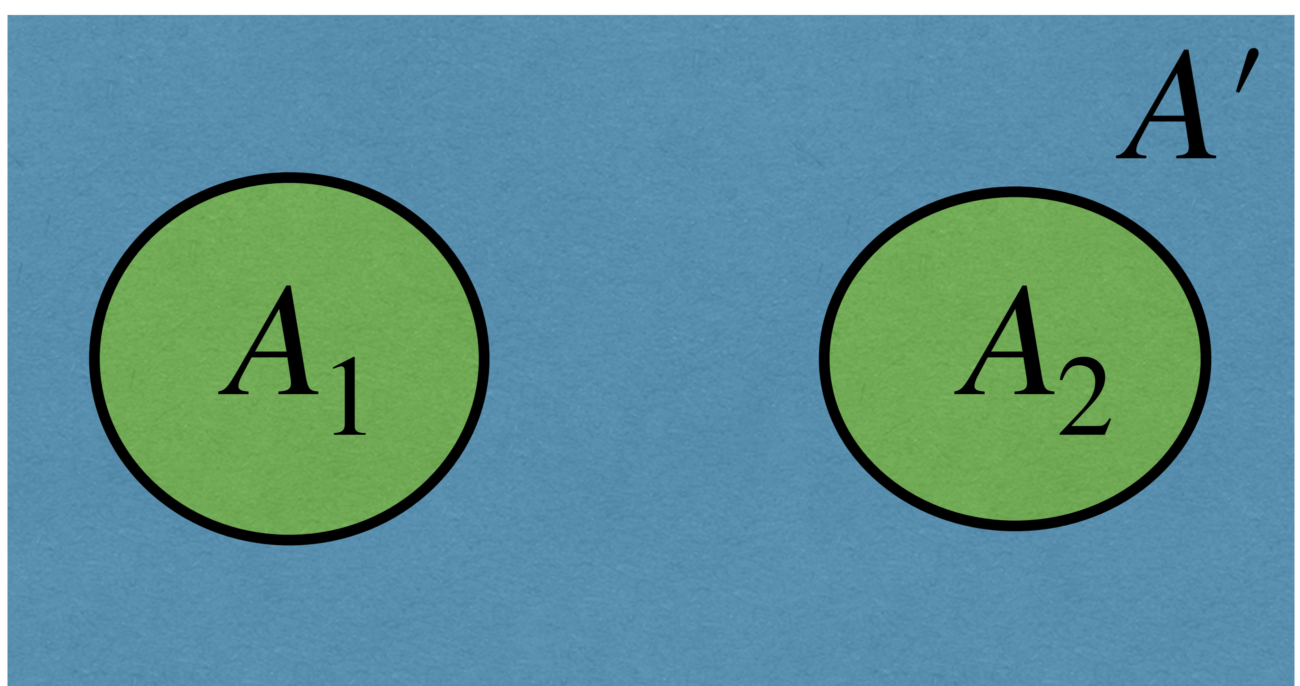
On one hand we have the additive algebra

$$A = A_1 \vee A_2$$

$$\mathcal{O}_A = \mathcal{O}_{A_1} \vee \mathcal{O}_{A_2}$$



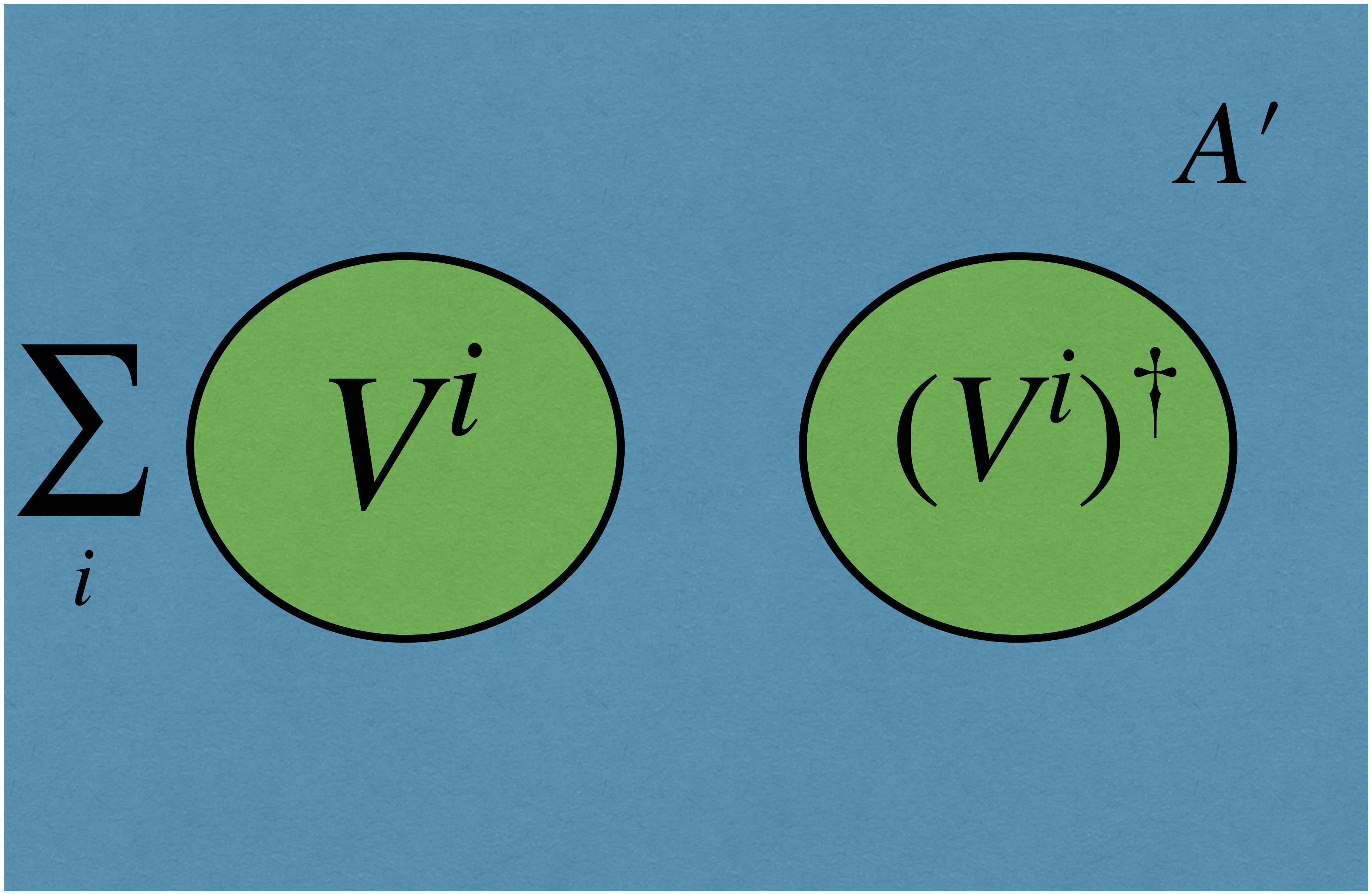
But on the other

$$\mathcal{O}'_{A'} = \mathcal{O}_{A_1} \vee \mathcal{O}_{A_2} \vee \cdots ?$$

QFT with global symmetries (DHR sectors)

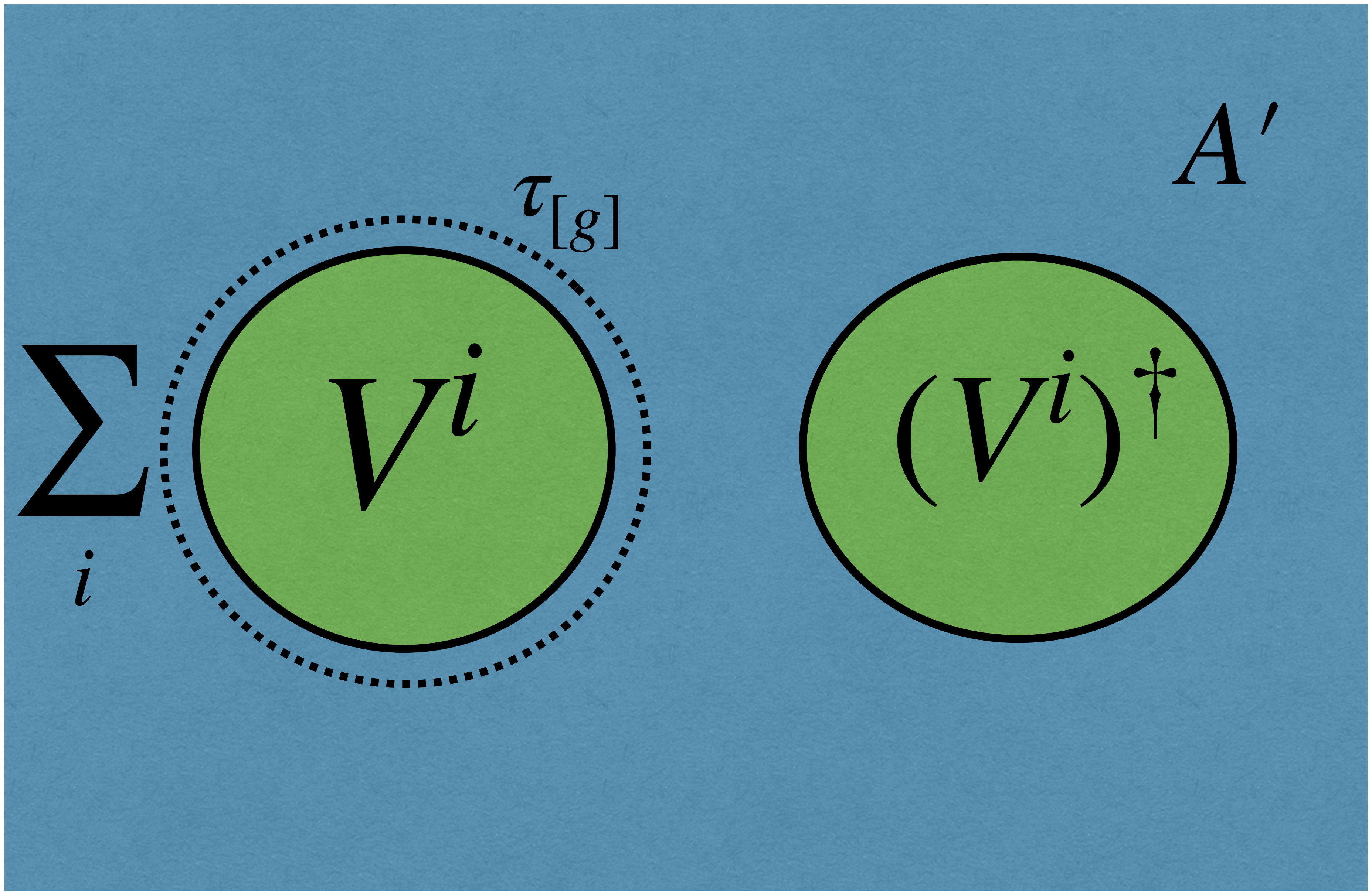
$$\sum_i V^i \ (V^i)^\dagger$$

QFT with global symmetries (DHR sectors)



$$\mathcal{O}_A \not\subset \sum_i V^i (V^i)^\dagger \subset \mathcal{O}'_{A'}$$

QFT with global symmetries (DHR sectors)



$$\mathcal{O}_{A'} \not\subset \tau_{[g]} \subset \mathcal{O}_A'$$

Entropy for DHR sectors

- Mutual Information of the neutral additive algebra (without intertwiners)

$$S_{\mathcal{O}_{12}}(\omega_{12}, \omega_1 \otimes \omega_2) = I_{\mathcal{O}}(1,2)$$

- Relative entropy of the full neutral algebra (with intertwiners)

$$S_{(\mathcal{O}_{(12)'})'}(\omega_{12} \,|\, (\omega_1 \otimes \omega_2) \circ E_{12}) = I_{\mathcal{F}}(1,2)$$

- Most importantly, we have the master formulas

$$I_{\mathcal{F}}(1,2) = S_{\mathcal{F}}(\omega_{12} \,|\, \omega_{12} \circ E_{12}) + I_{\mathcal{O}}(1,2)$$

$$S_{\mathcal{F}}(\omega_{12} \,|\, \omega_{12} \circ E_{12}) = S_{(\mathcal{O}_{(12)'})'}(\omega_{12} \,|\, \omega_{12} \circ E_{12})$$

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$$S_{\mathcal{F}}(\omega_{12} | \omega_{12} \circ E_{12}) = S_{(\mathcal{O}_{(12)'})'}(\omega_{12} | \omega_{12} \circ E_{12})$$

Entropy for DHR sectors

- Finite groups

$$\Delta I = \log G = \log D^2$$

[Kitaev, Preskill 2006] [Levin, Wen, 2006] [Longo, Xu, 2017]

- Lie groups

$$\Delta I \simeq \frac{1}{2}(d-2)\,\mathcal{G}\,\log\frac{R}{\epsilon}$$

- Muticomponent regions

$$S_{\mathcal{F}}(\omega_{AB}\,|\,\omega_{AB}\circ\otimes_i E_{A_i}\otimes_j E_{B_j}) = n_\partial \log|G|$$

- Excited states

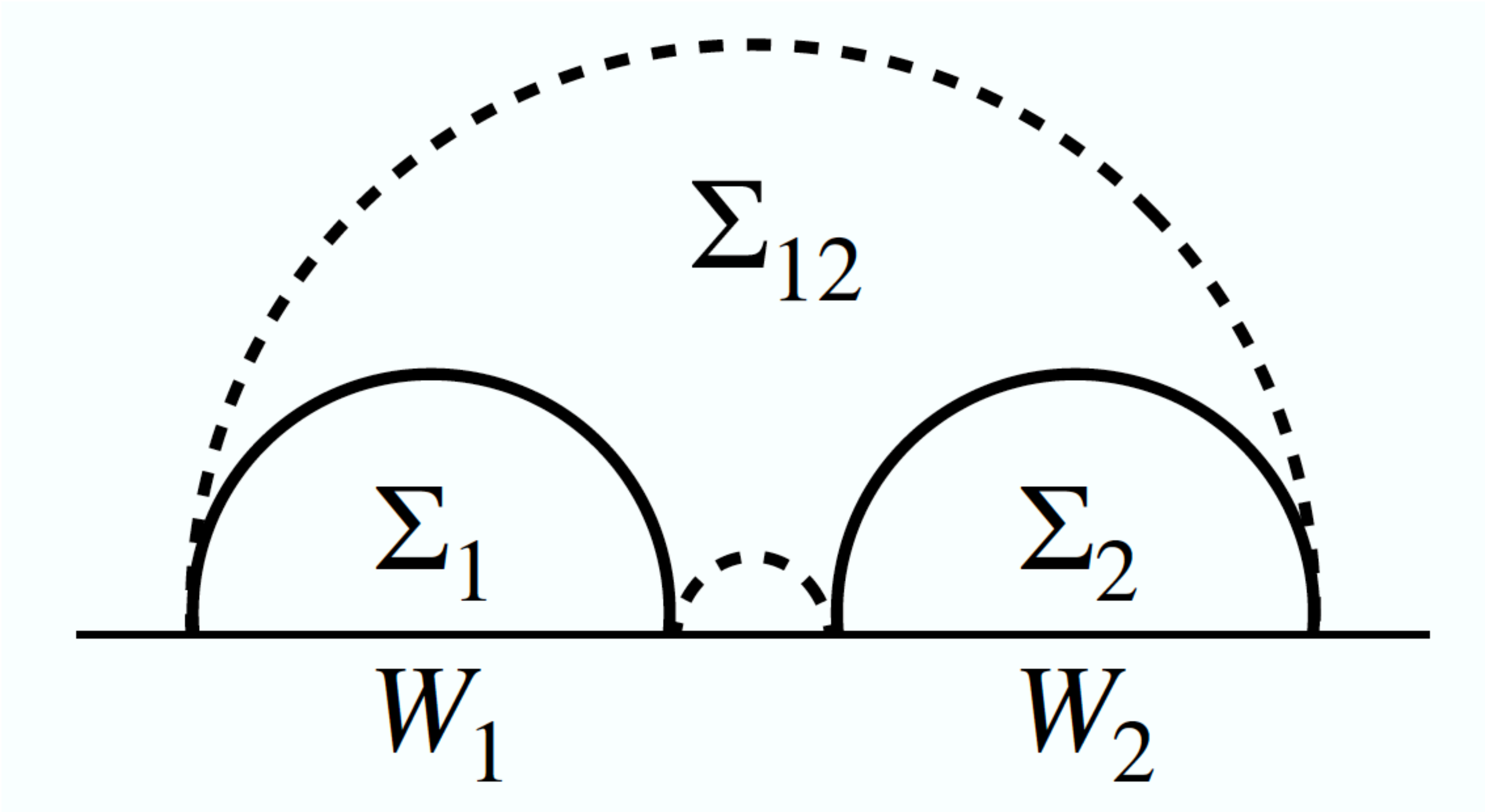
$$I^{\varphi}_{\mathcal{O}}(1,2) - I^{0}_{\mathcal{O}}(1,2) = I^{\varphi}_{\mathcal{F}}(1,2) - I^{0}_{\mathcal{F}}(1,2) = 2\log d_r$$

[Dong et all, 2008] [Alcaraz et all, 2011] [Nozaki et all, 2014] [Lewkowycz, Maldacena, 2014]

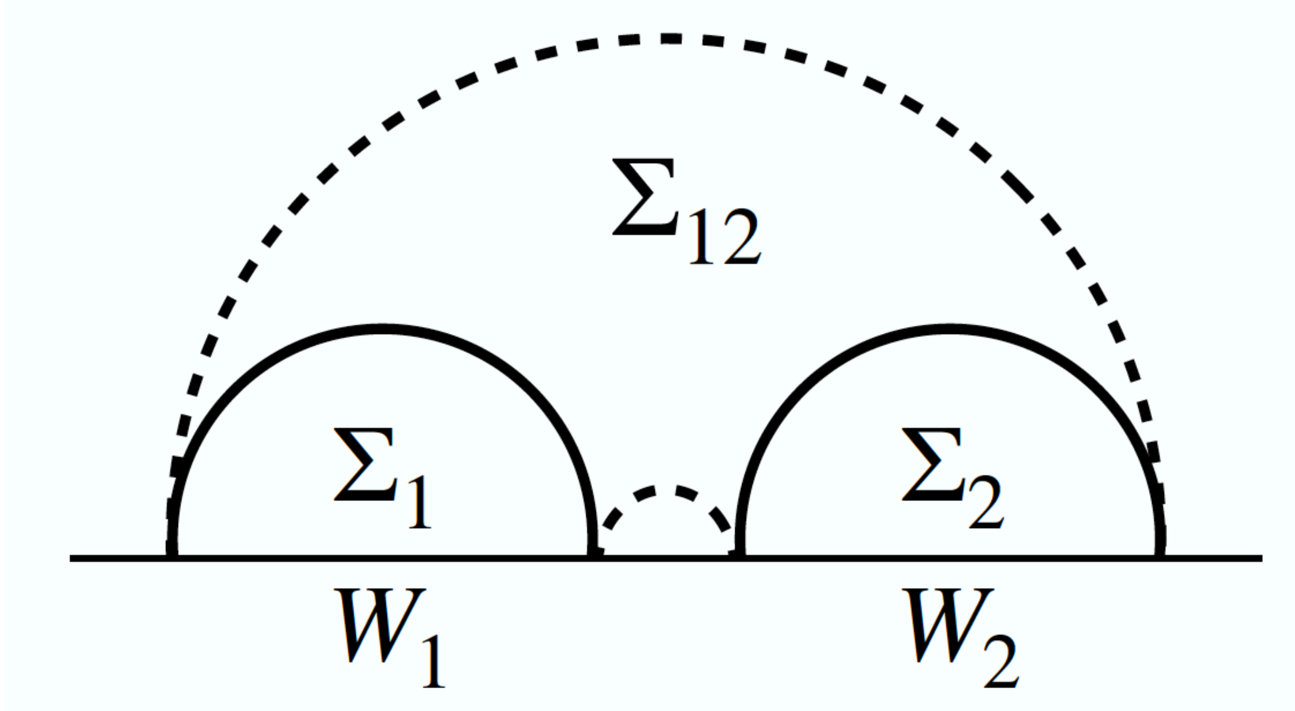
- Entropic certainty relation

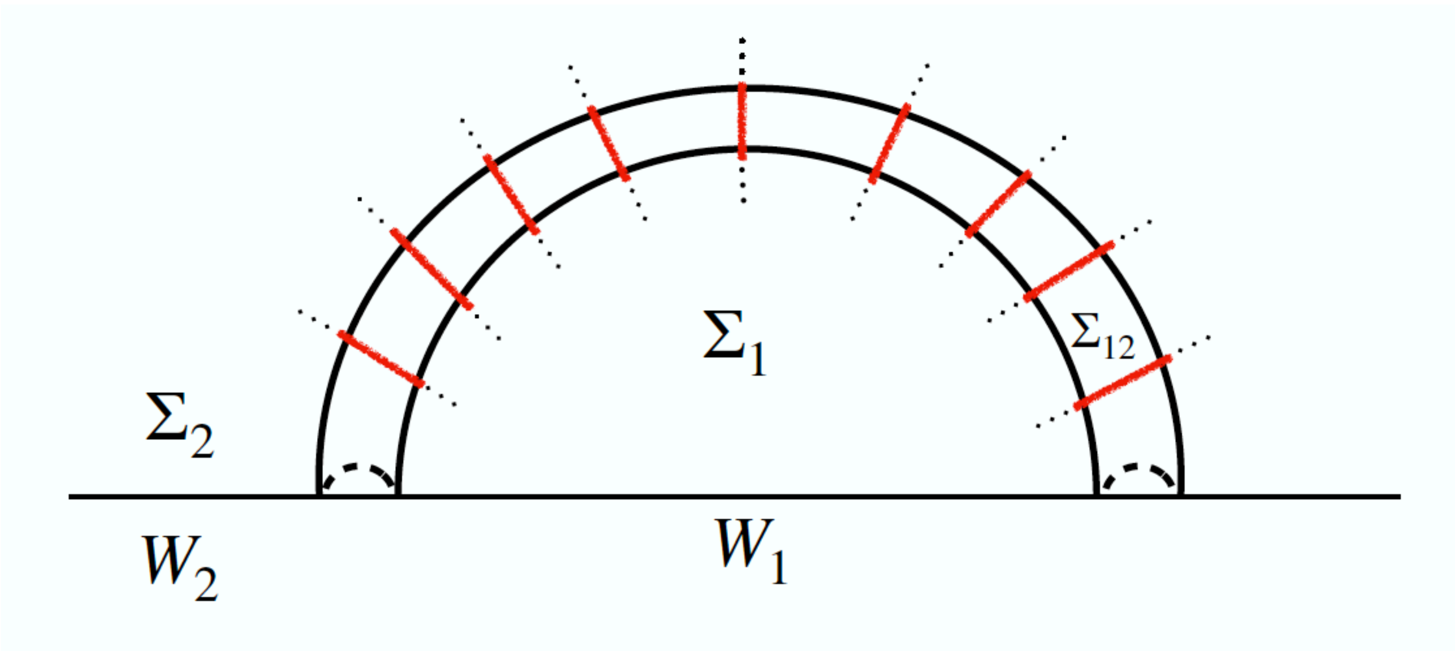
$$S_{\mathcal{F}_{W_1W_2}}(\omega\,|\,\omega\circ E_1\otimes E_2) + S_{\mathcal{F}_{S\vee G_\tau}}(\omega\,|\,\omega\circ E_\tau) = \log|G|$$

Holographic entanglement entropy



Holographic entanglement entropy



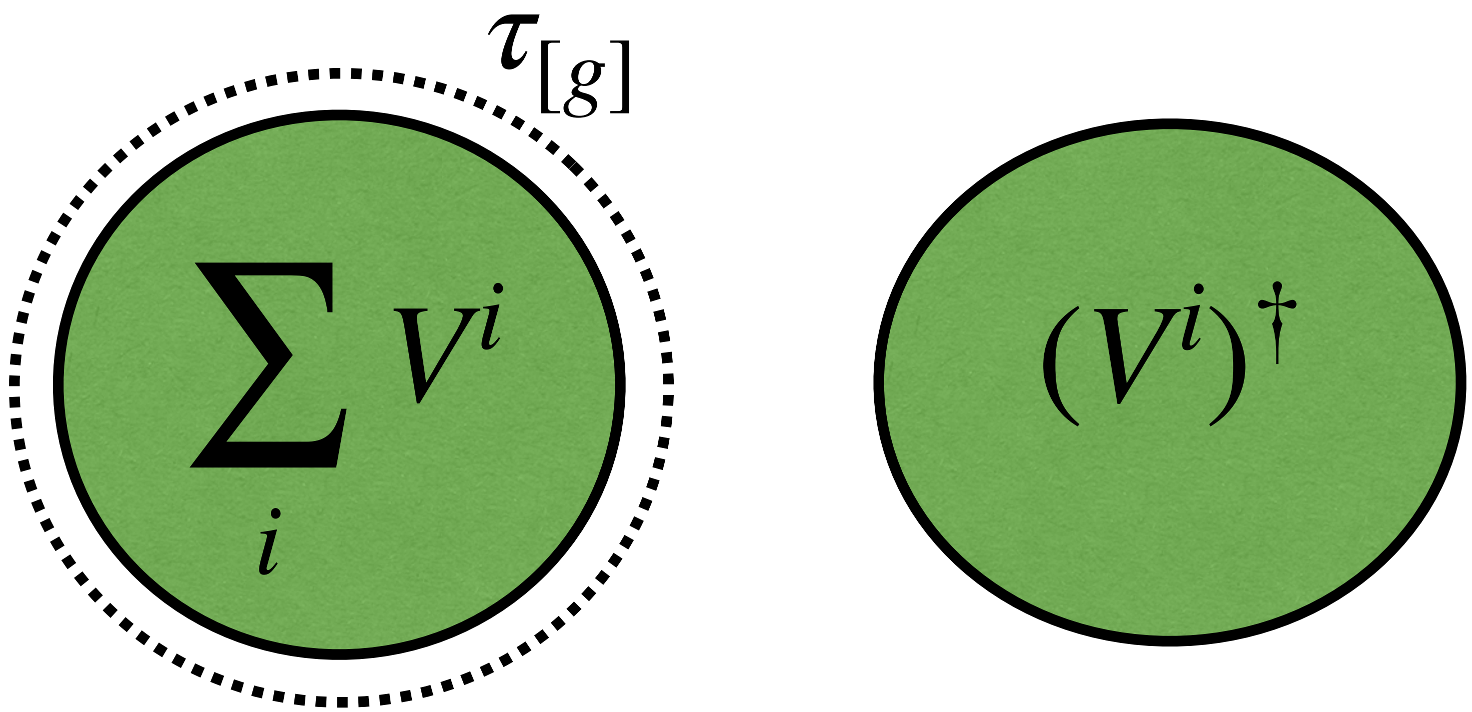


Minimal areas measure the size of Haag duality violation, the 'number' of superselection sectors.

Take away messages!

- Superselection sectors lead to Haag duality violation
- Reason: Existence of localized twist operators and non local intertwiner operators
- Contribution to EE unambiguously accounted by a relative entropy. Provides right universal terms
- Right structure for applications to HEE

Thanks!



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