Theory of a Planckian metal

Quantum Information and String Theory 2019
It from Qubit school/workshop,
Yukawa Institute for Theoretical Physics, Kyoto University, June 20, 2019

Aavishkar Patel and Subir Sachdev, arXiv: 1906.03265

Notes on the complex SYK model, Yingfei Gu, Alexei Kitaev, Subir Sachdev, and Grigory Tarnopolsky, to appear

Talk online: sachdev.physics.harvard.edu

PHYSICS
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Ordinary metals and quasiparticles

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ϵ_α

$$E = \sum_\alpha n_\alpha \epsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

Ordinary metals and quasiparticles

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\rm eq} \sim \frac{\hbar E_F^3}{U^2 (k_B T)^2} \quad , \quad \text{as } T \to 0,$$

where U is the strength of interactions, and E_F is the Fermi energy.

Ordinary metals and quasiparticles

We compute the lifetime of a quasiparticle, τ_α, in an eigenstate of the free quasiparticle particle Hamitonian with energy ε_α. By Fermi's Golden rule, for ε_α at the Fermi energy

$$\begin{aligned}\frac{1}{\tau_\alpha} = \pi U^2 \rho_0^2 \int d\varepsilon_\beta d\varepsilon_\gamma d\varepsilon_\delta f(\varepsilon_\beta)(1 - f(\varepsilon_\gamma))(1 - f(\varepsilon_\delta)) \\ \times \delta(\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_\gamma - \varepsilon_\delta) \\ = \frac{\pi^3 U^2 \rho_0^2}{4} T^2\end{aligned}$$

where ρ_0 is the density of states at the Fermi energy, and $f(\epsilon) = 1/(e^{\epsilon/T} + 1)$ is the Fermi function.

Ordinary metals and quasiparticles

- Similarly, a quasiparticle model implies a resistivity

$$\rho = \frac{m^*}{ne^2}\frac{1}{\tau} \sim U^2T^2 \quad \text{with} \quad \tau \sim \tau_{\text{eq}}$$

Ordinary metals and quasiparticles

- These times are much longer than the 'Planckian time' $\hbar/(k_B T)$, which we will find in systems without quasiparticle excitations.

$$\tau \sim \tau_{\rm eq} \gg \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

Remarkable recent observation of 'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ, is

$$\rho = \frac{m^*}{ne^2}\frac{1}{\tau}$$

with a universal scattering rate

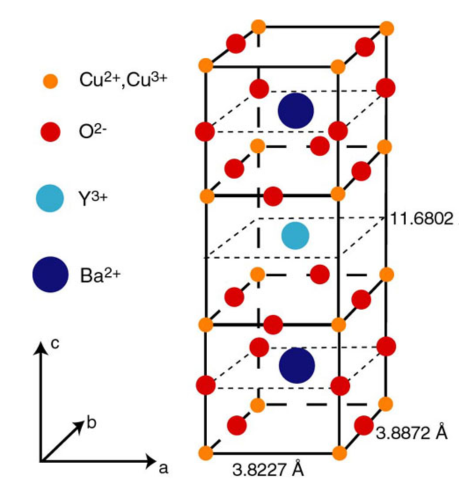
$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar},$$

independent of the strength of interactions!

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of *T*-linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

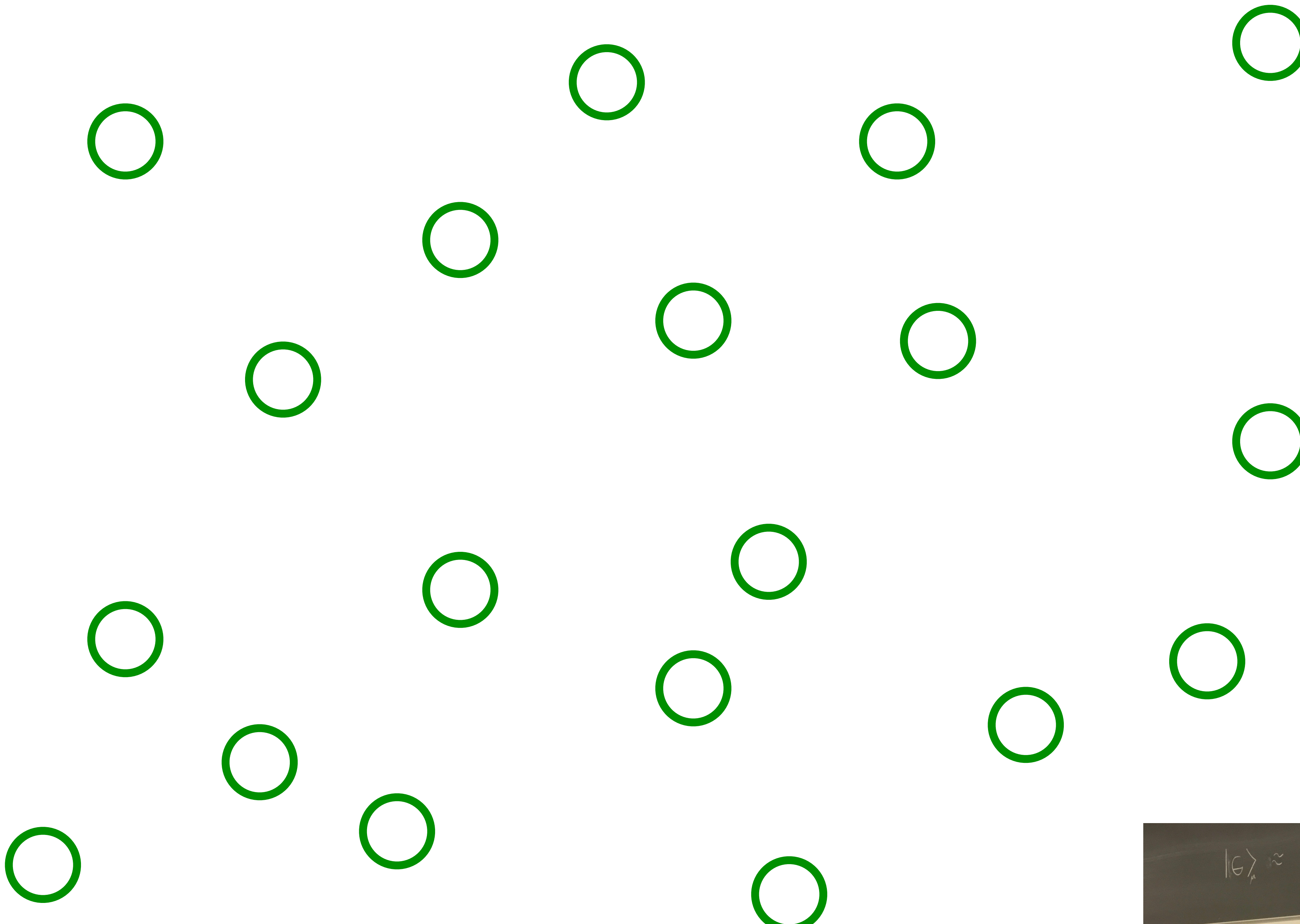


A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

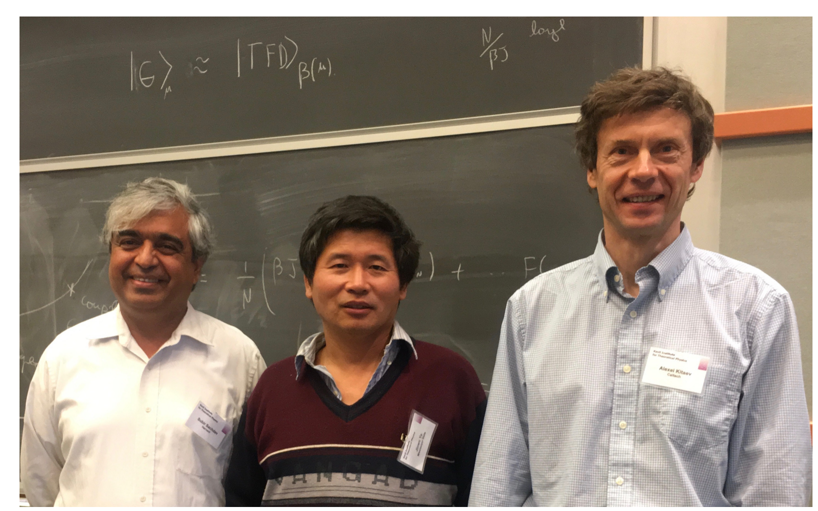
1. The complex SYK model and charged black holes

2. Quantum matter without quasiparticles: lattice SYK models and Planckian metals

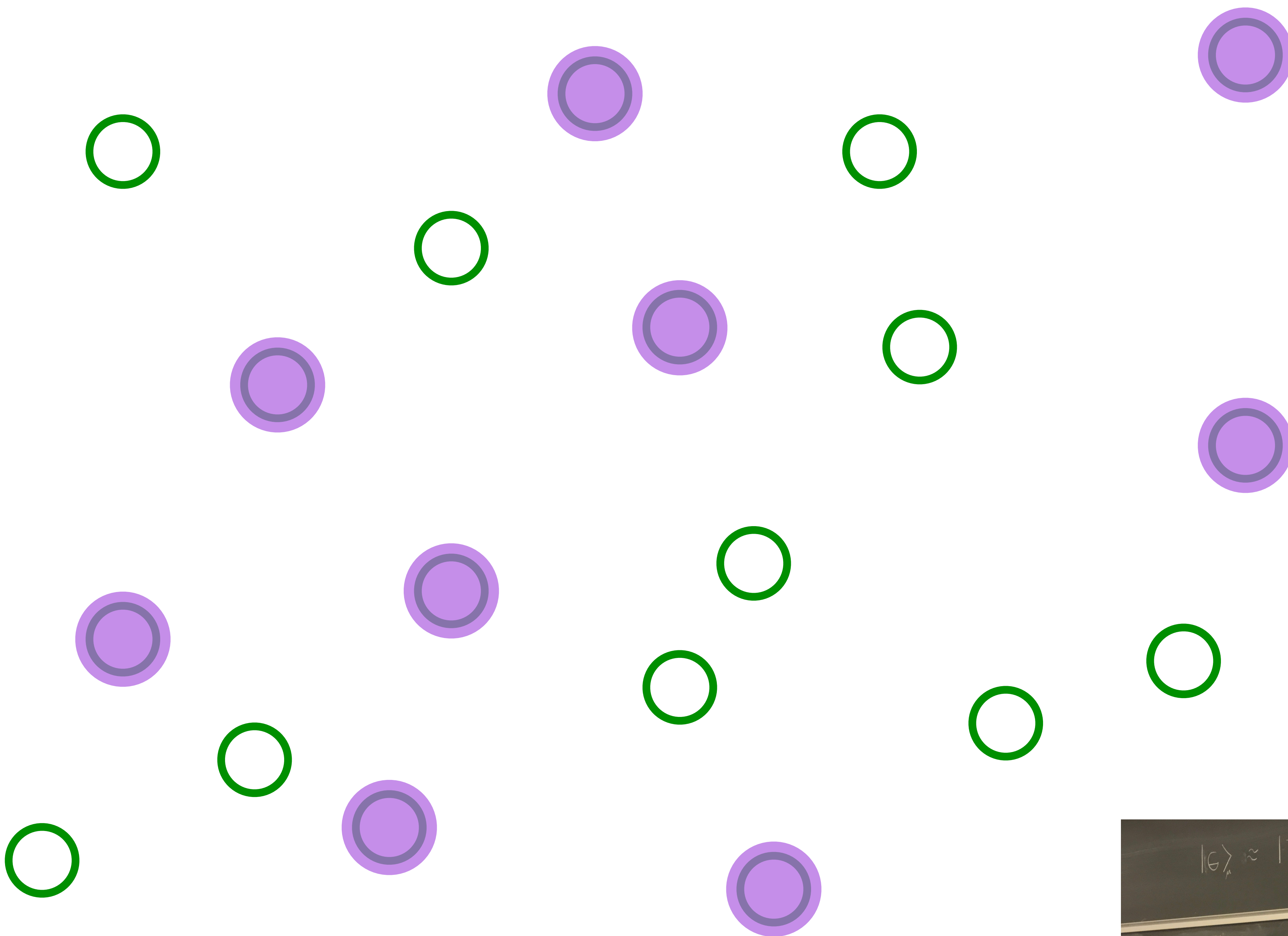
The Sachdev-Ye-Kitaev (SYK) model



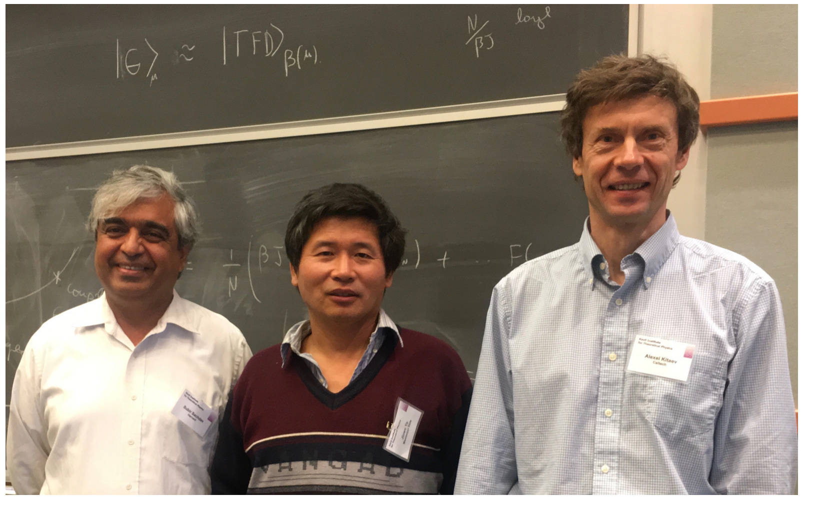
Pick a set of random positions



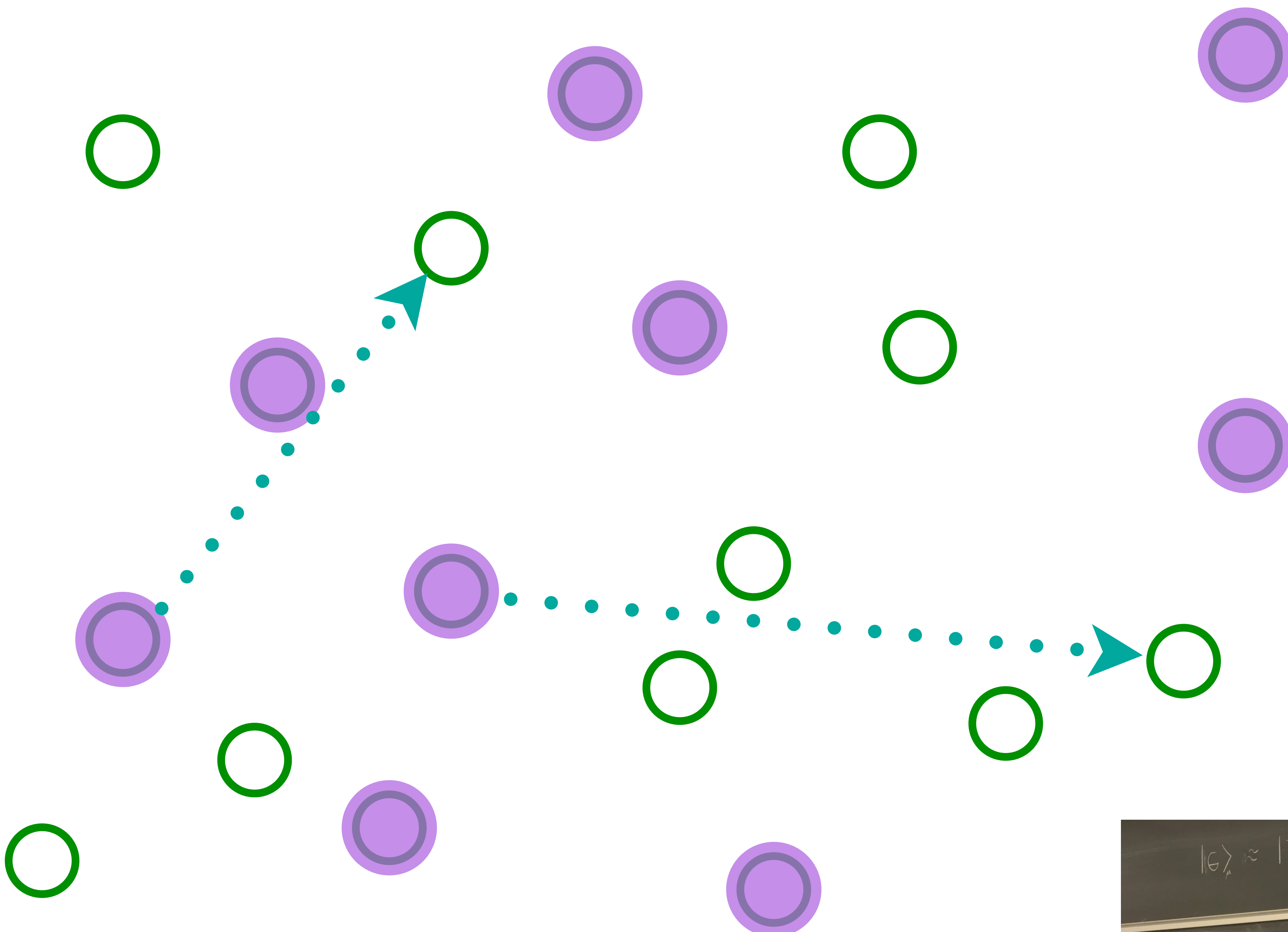
The SYK model



Place electrons randomly on some sites



The SYK model



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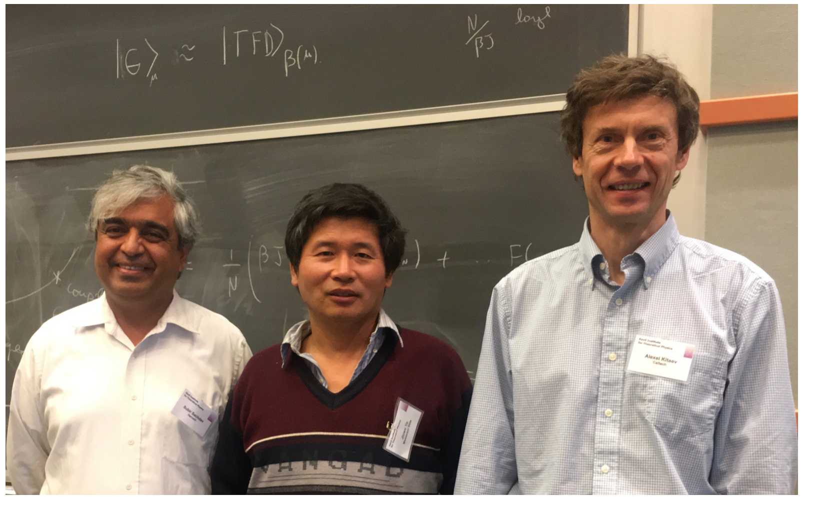
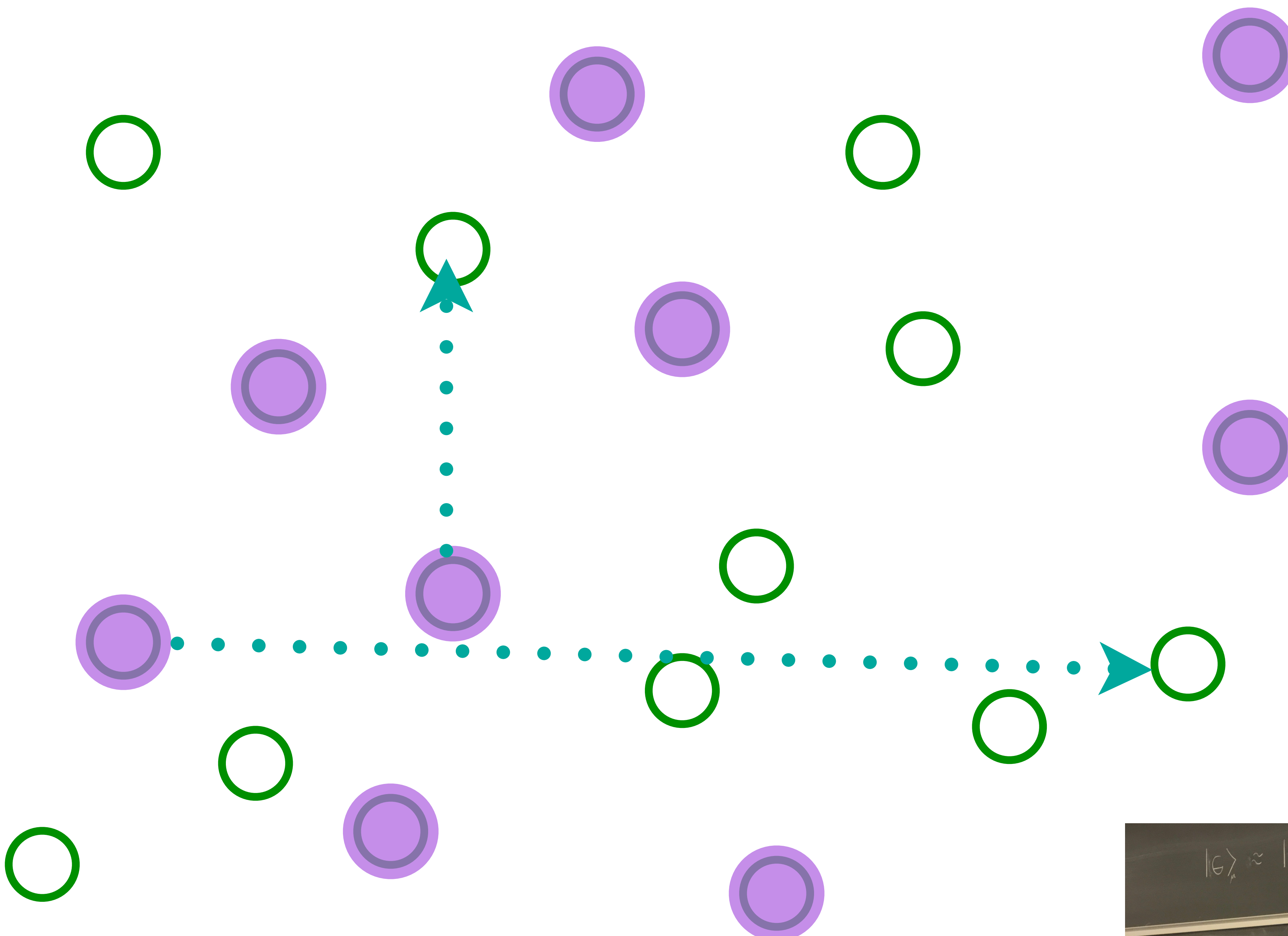


The SYK model



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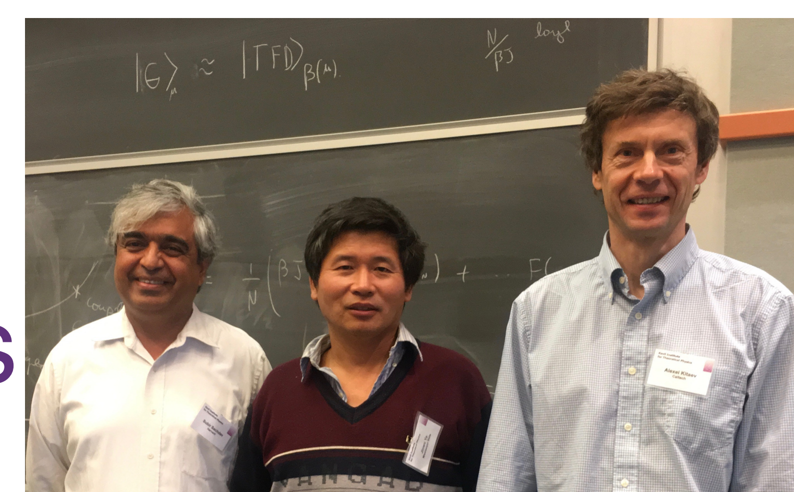
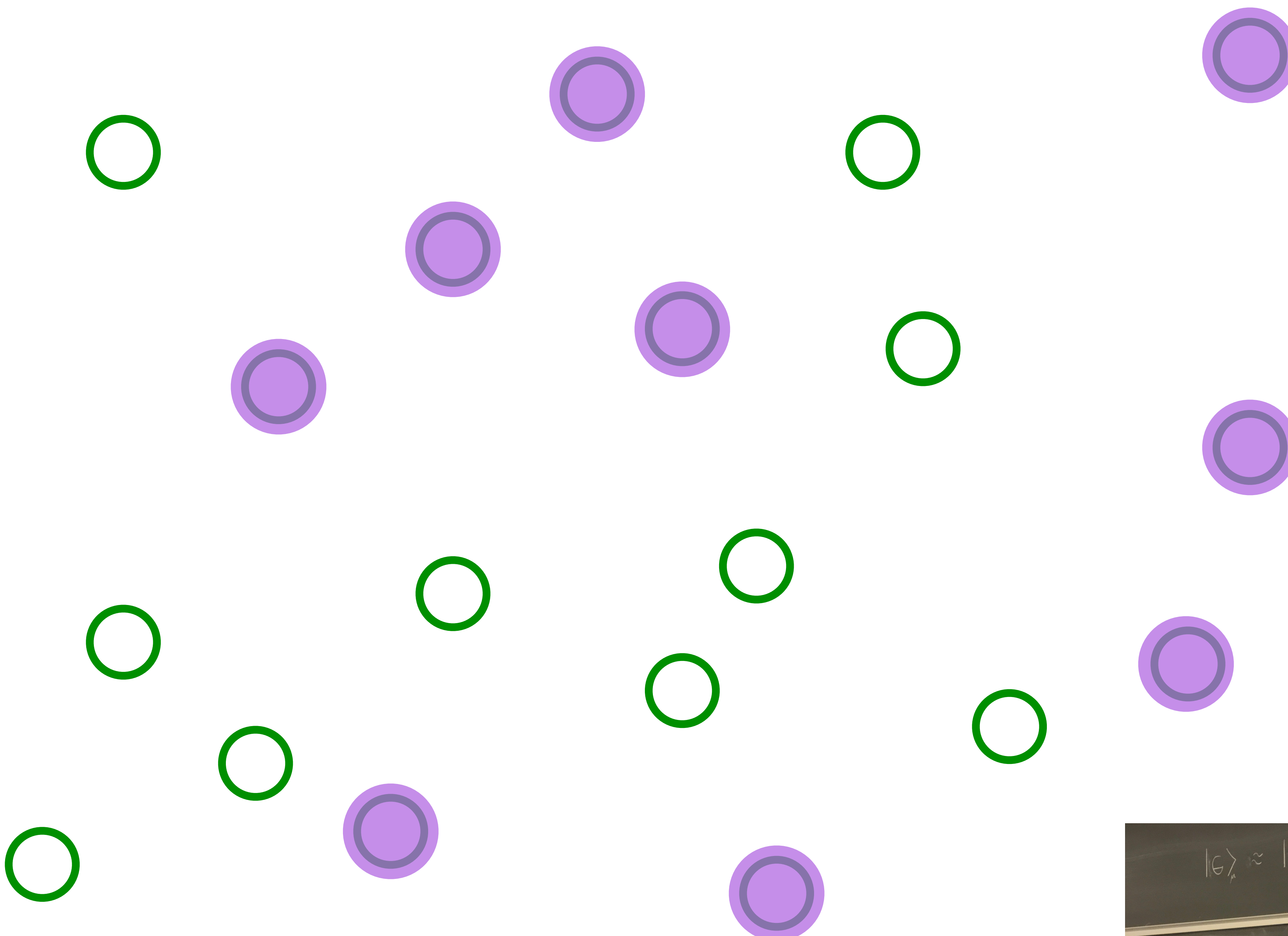
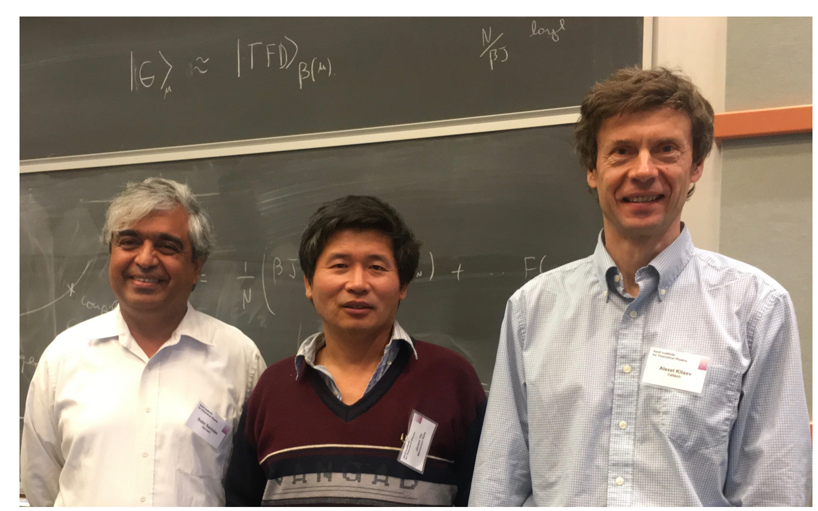


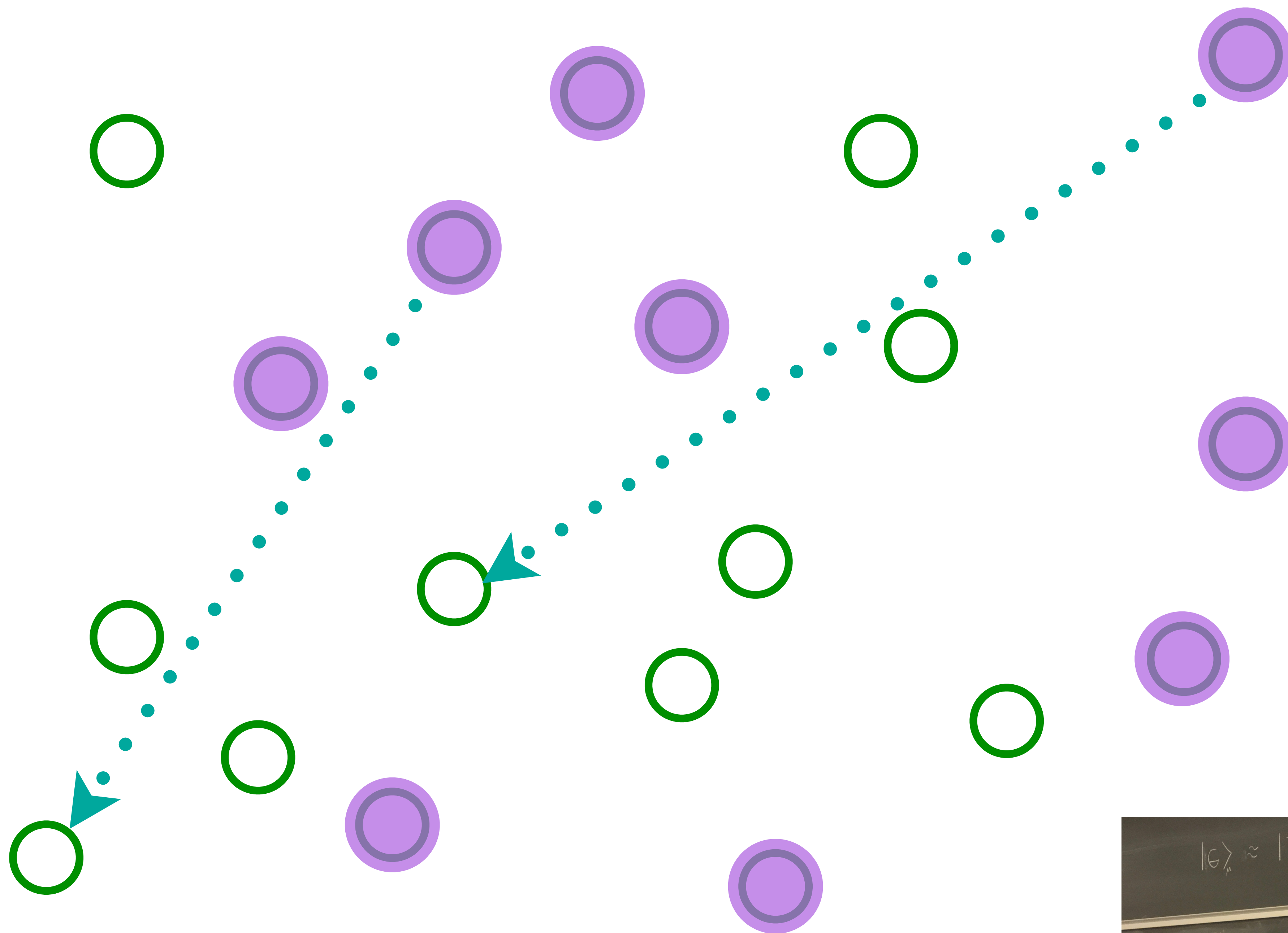
The SYK model



Entangle electrons pairwise randomly



The SYK model



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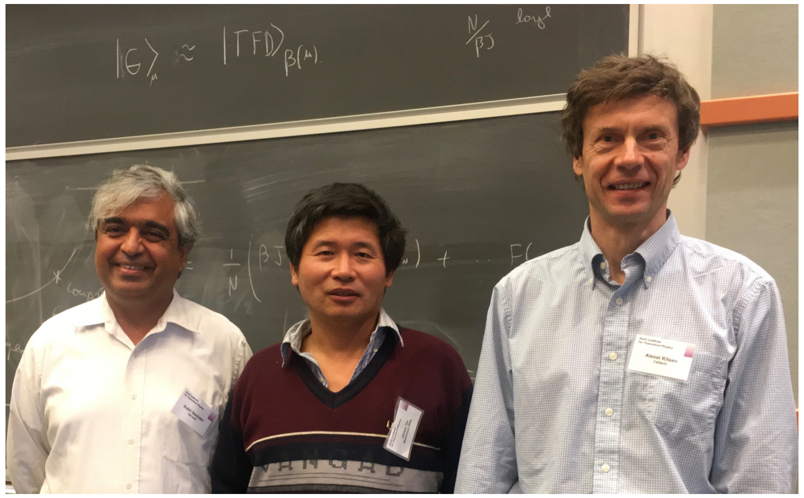
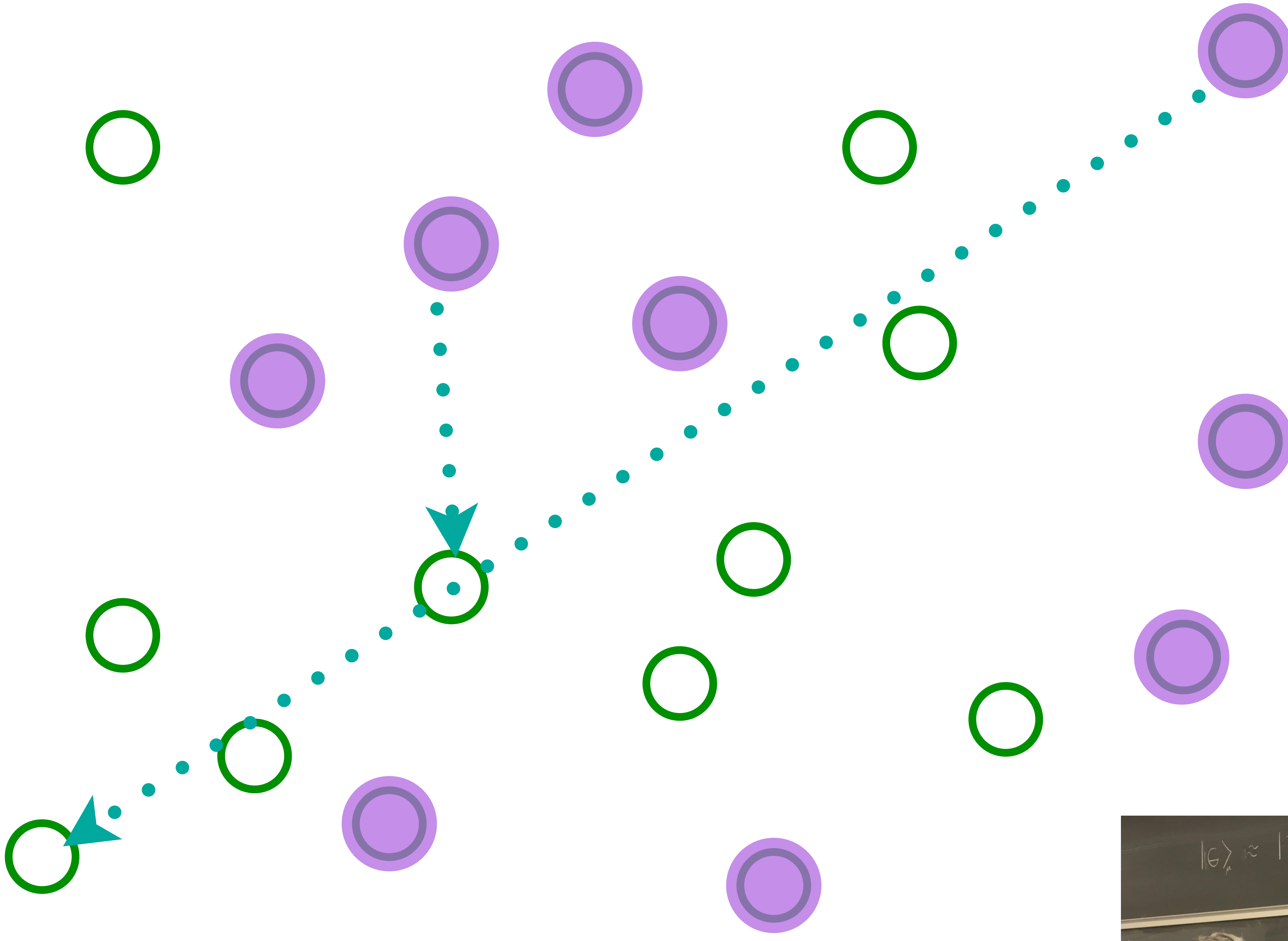


The SYK model



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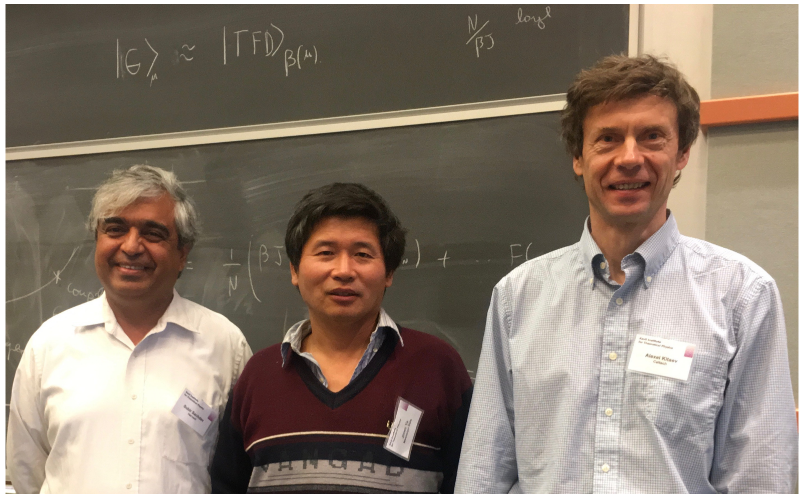
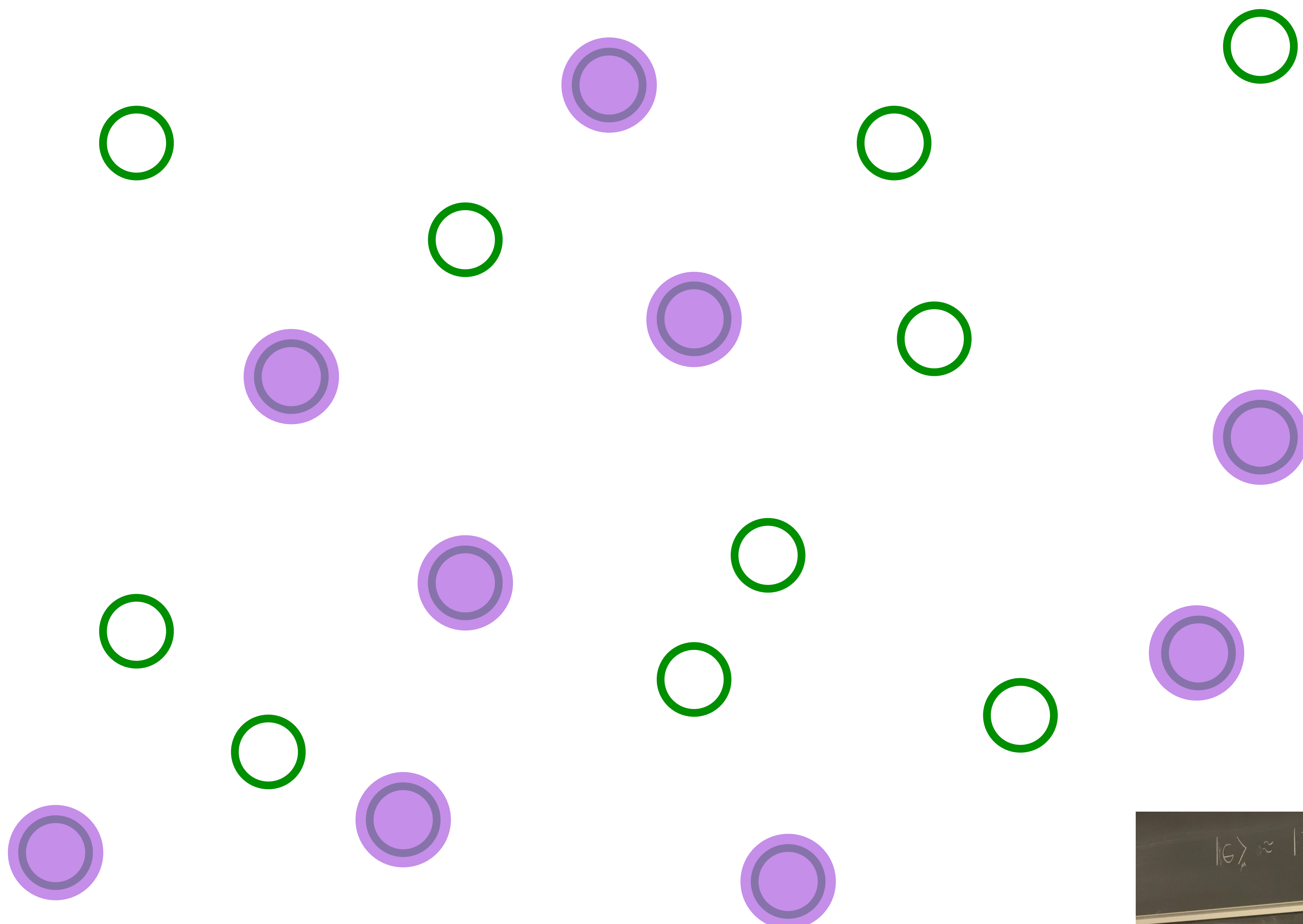


The SYK model



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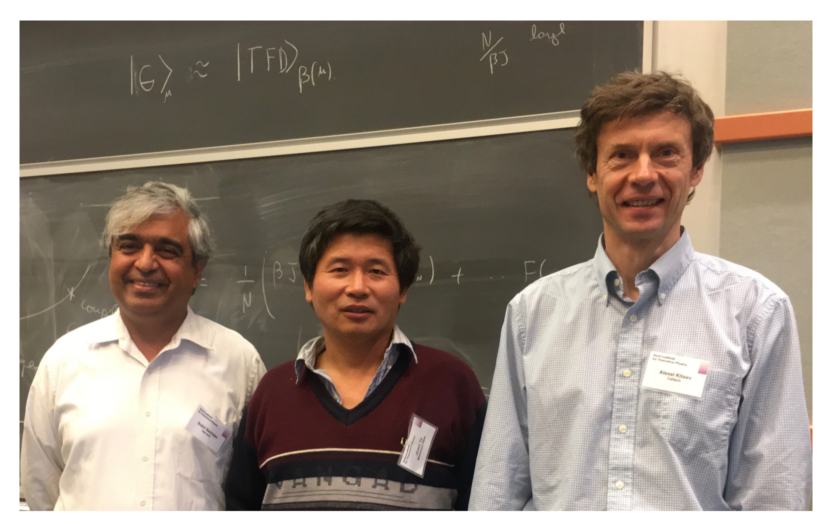

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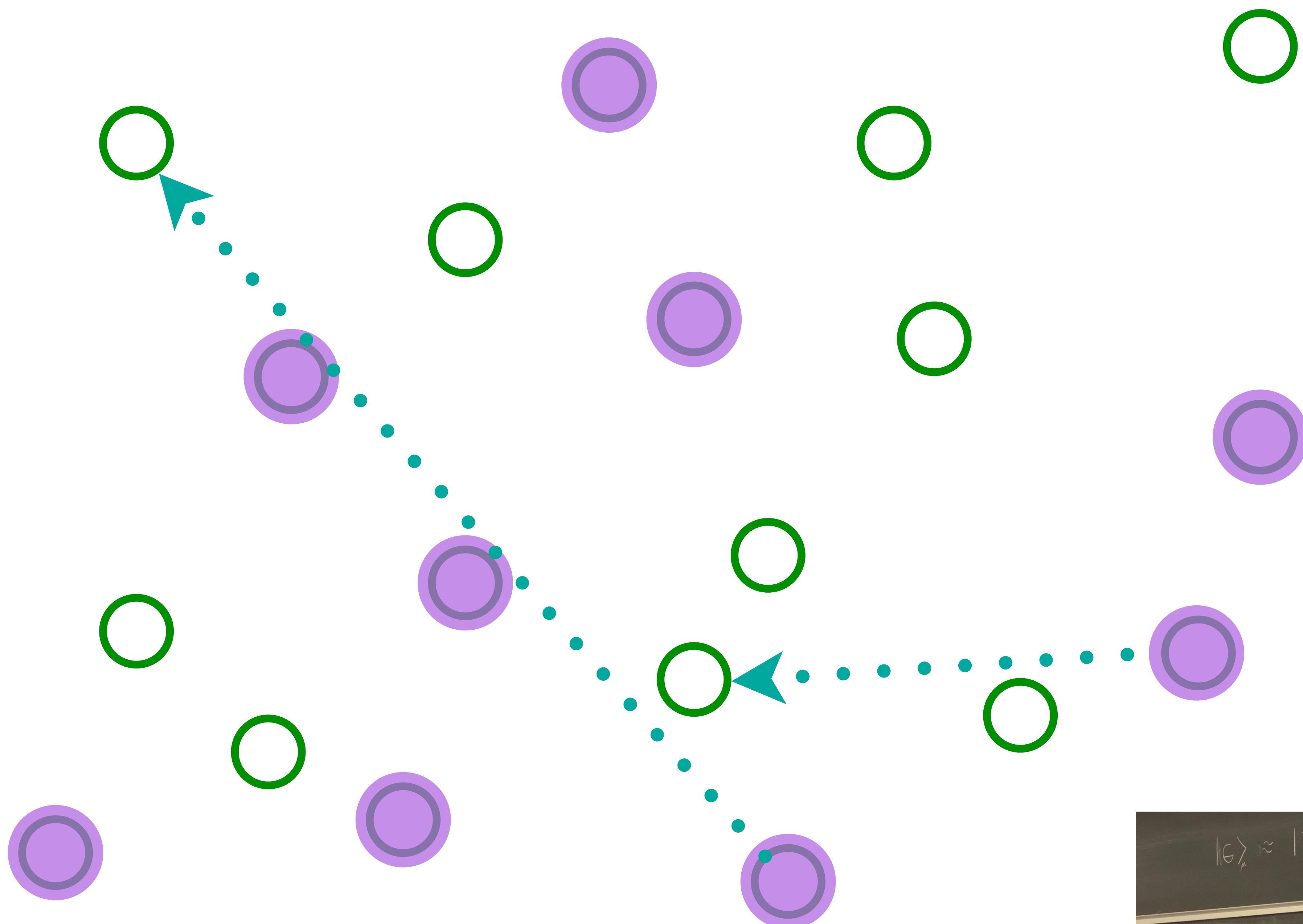

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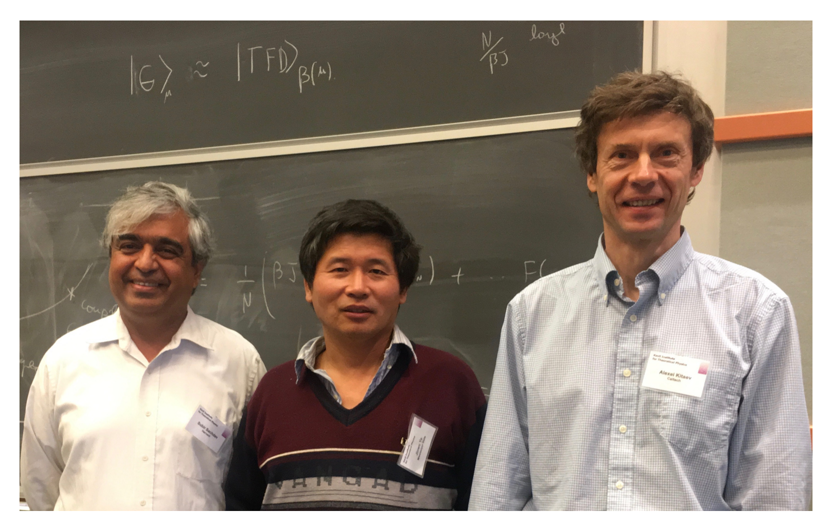

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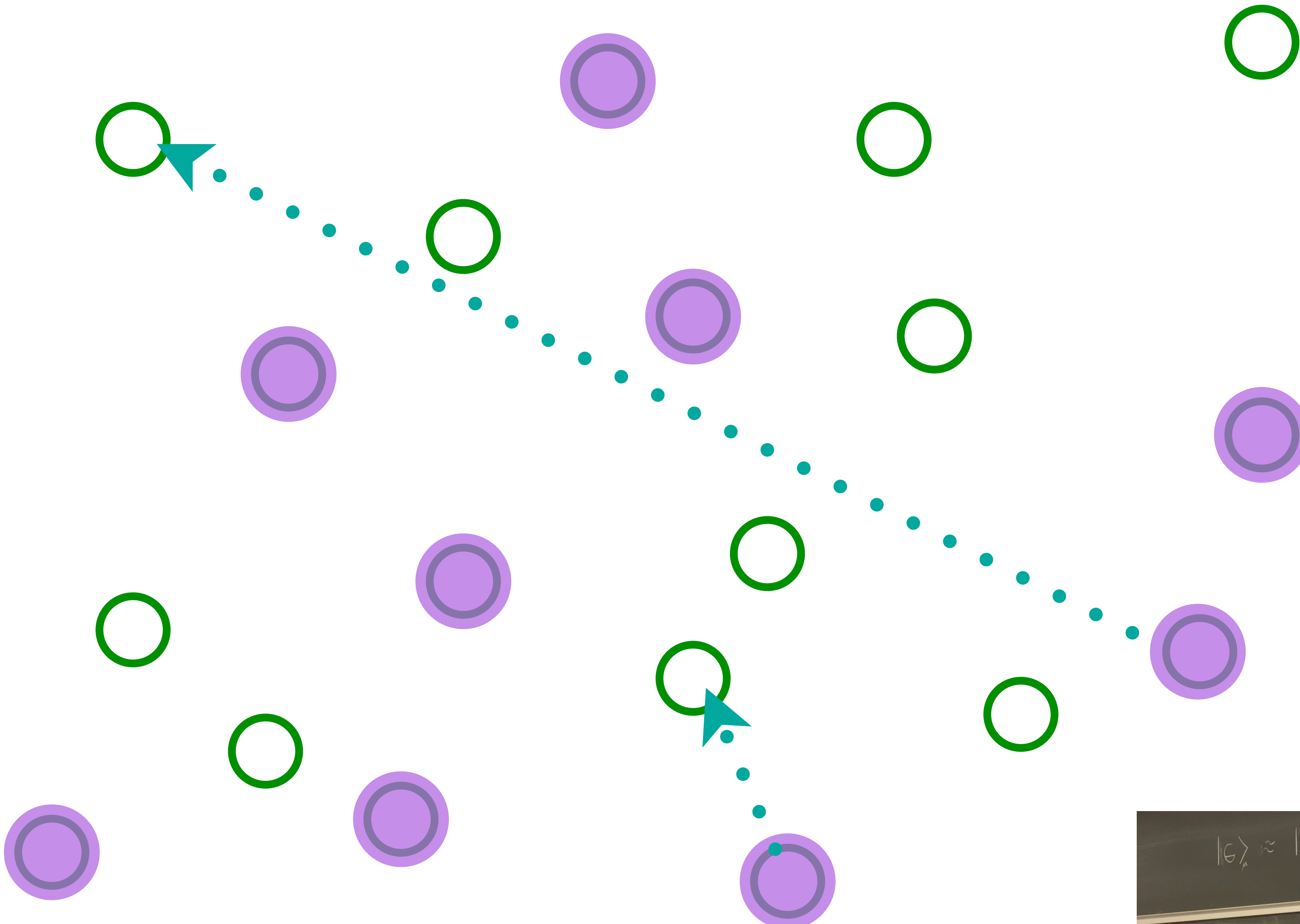

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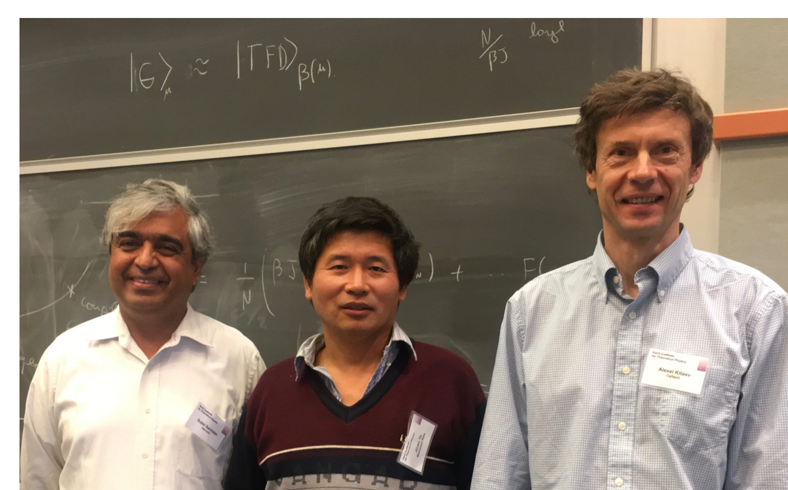

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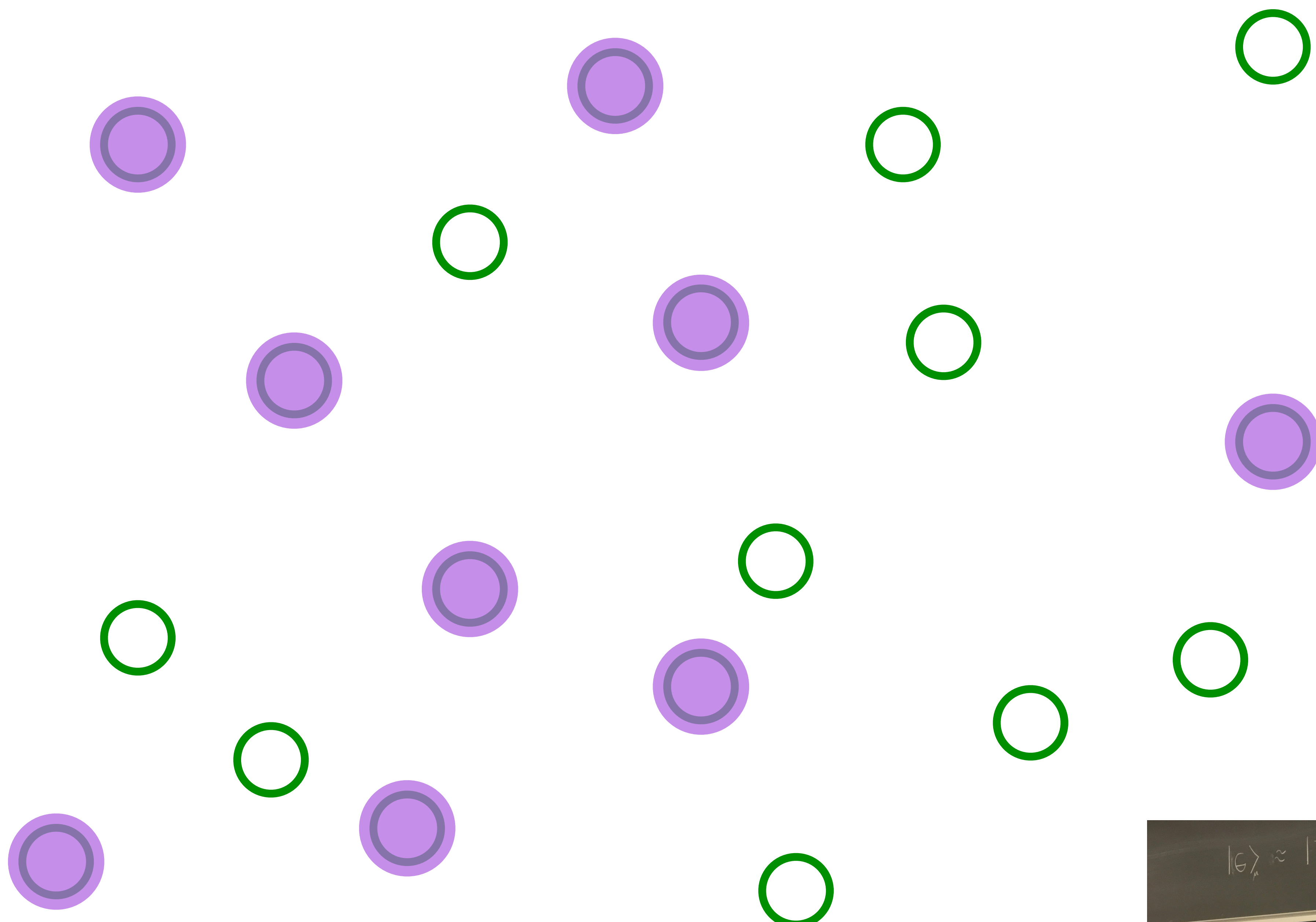

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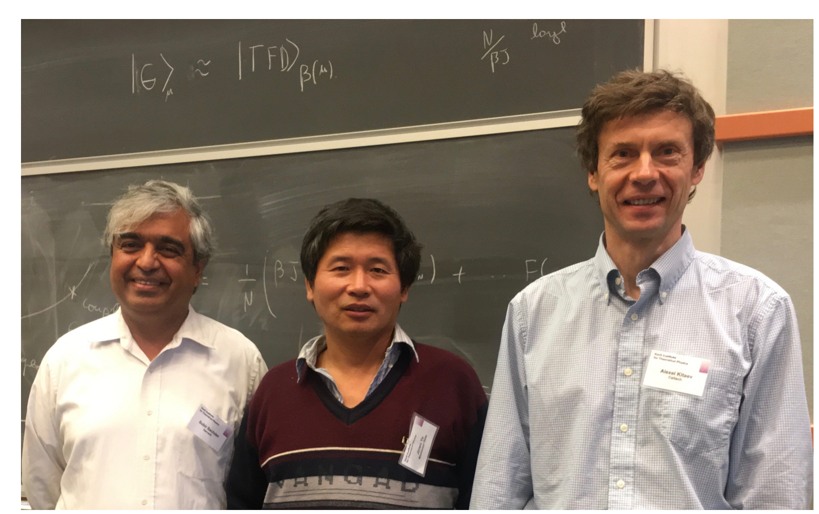
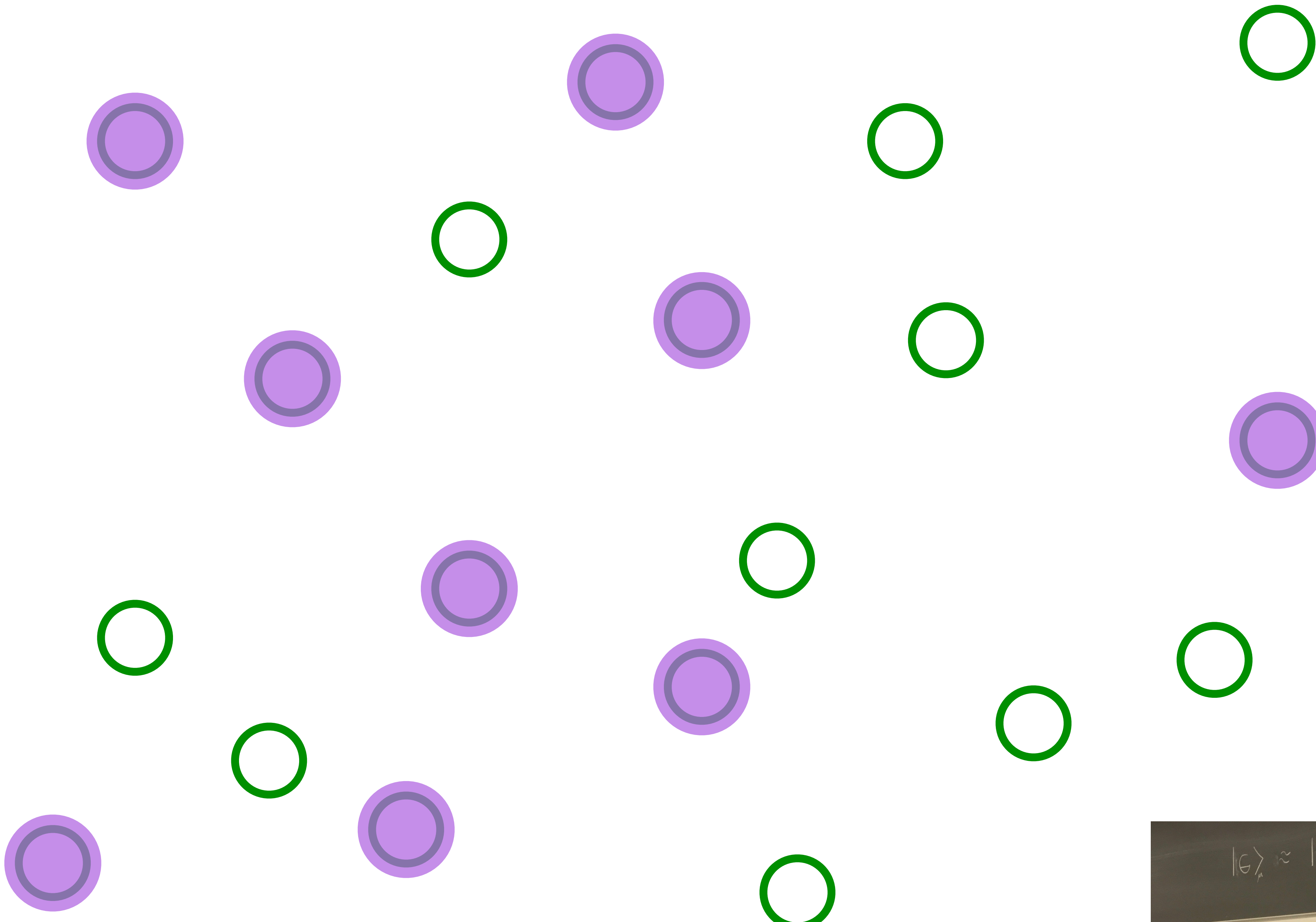
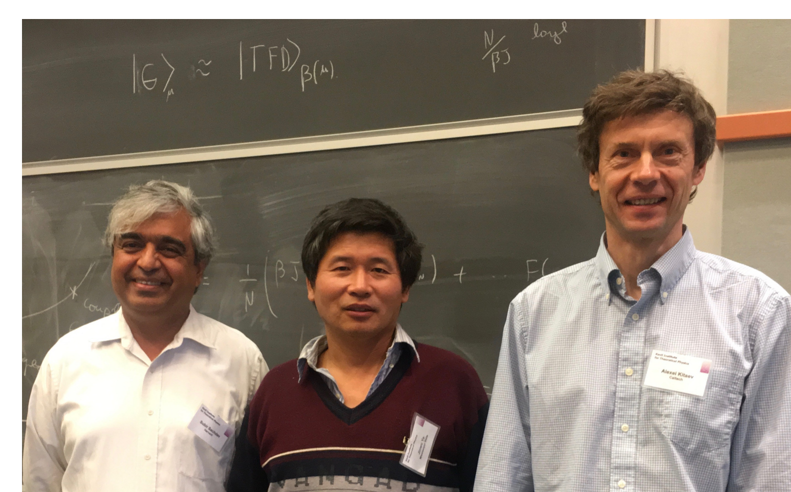


The SYK model



This describes both a strange metal and a black hole!



The complex SYK model

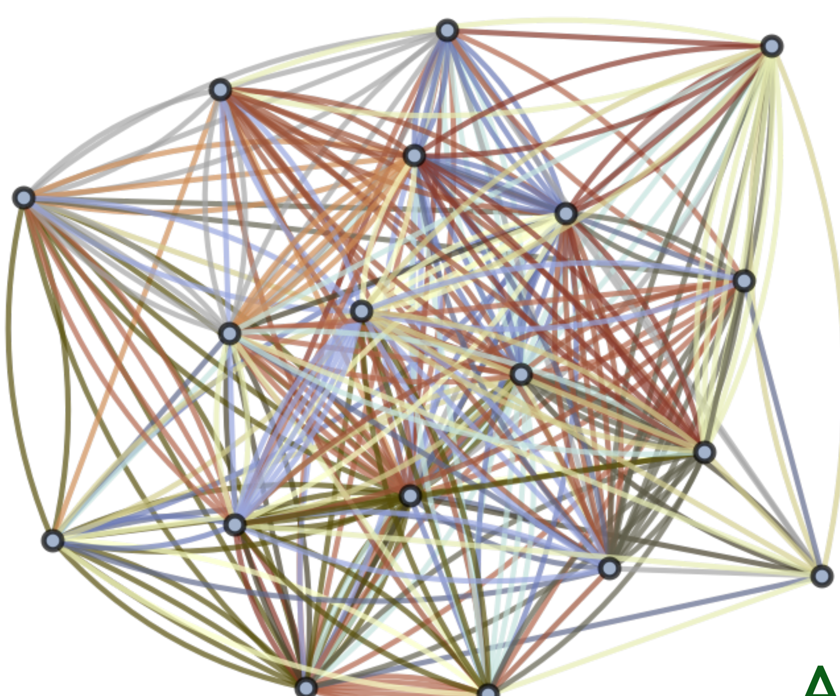
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

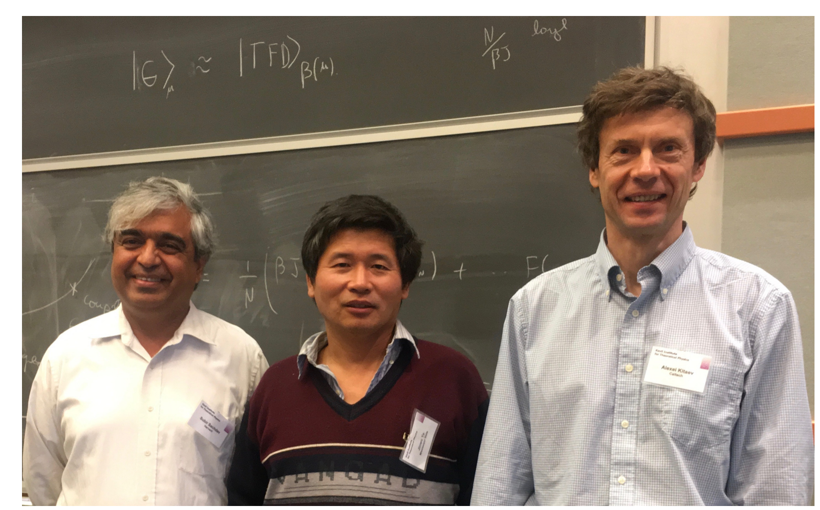
$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta + \epsilon \sum_\alpha c_\alpha^\dagger c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.





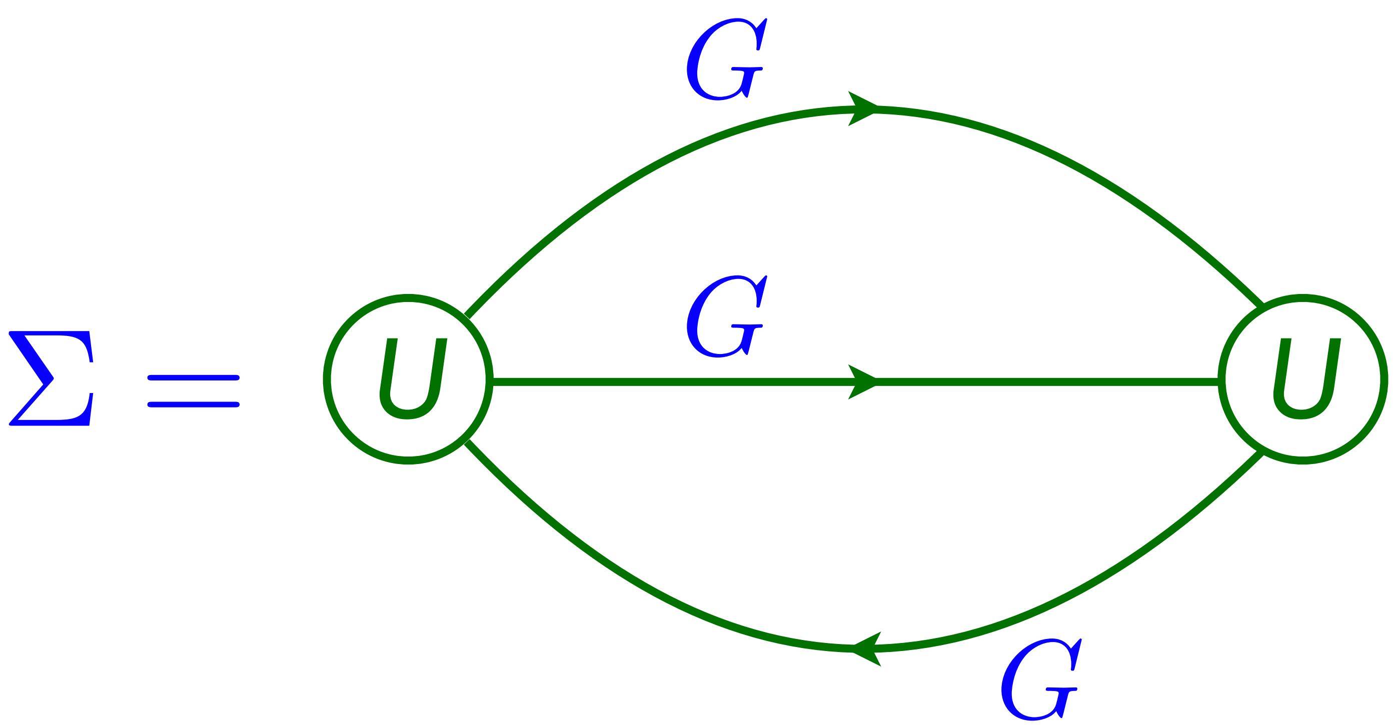
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$\boxed{\begin{array}{c} G(i\omega) = \dfrac{1}{i\omega - \epsilon - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau) \\ G(\tau = 0^-) = \mathcal{Q}. \end{array}}$$



S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

The complex SYK model

There is a one-parameter family of critical solutions with varying $\mathcal{Q}$, characterized by a dimensionless parameter $\mathcal{E}$.

For long times $\tau > 0$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = e^{\pi \mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

$$\left\langle c_\alpha^\dagger(\tau) c_\alpha(0) \right\rangle = e^{-\pi \mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

$\mathcal{E}$ determines the particle-hole asymmetry, and $A(\mathcal{E})$ is a known function. $\mathcal{E}$ is determined by ϵ/U.

In a Fermi liquid,

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = \left\langle c_\alpha^\dagger(\tau) c_\alpha(0) \right\rangle = \widetilde{A}/\tau$$

The complex SYK model

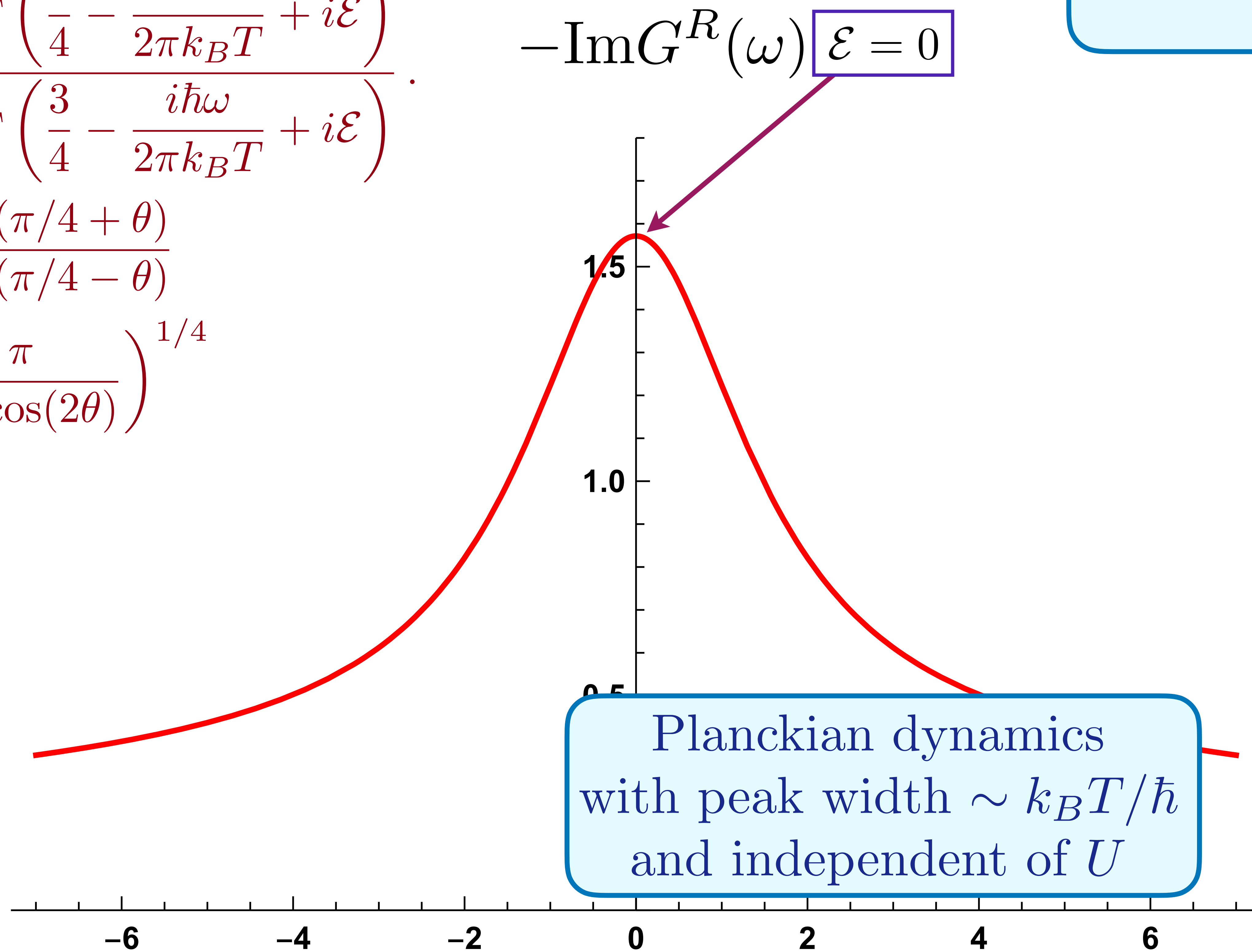
$$G^R_{\rm SYK}(\epsilon, \hbar\omega/(k_B T)) =$$

$$\frac{-iCe^{-i\theta}}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}.$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

$$C = \left(\frac{\pi}{U^2 \cos(2\theta)}\right)^{1/4}$$

$$\mathcal{E} = \mathbb{C}\,\frac{\epsilon}{U}$$



Planckian dynamics
with peak width $\sim k_B T/\hbar$
and independent of U

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

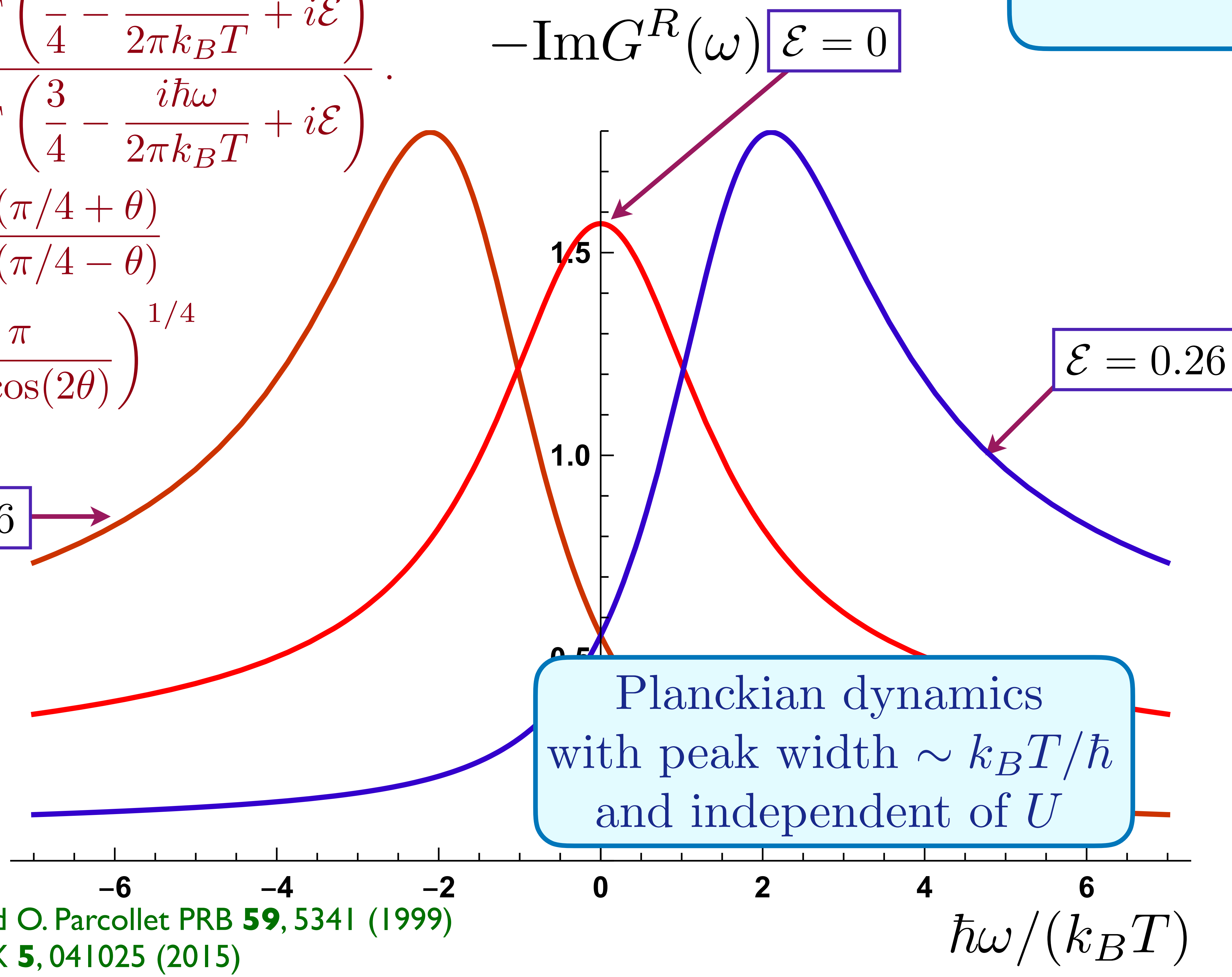
The complex SYK model

$$\mathcal{E} = \mathbb{C}\,\frac{\epsilon}{U}$$

$$G^R_{\rm SYK}(\epsilon, \hbar\omega/(k_B T)) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}.$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

$$C = \left(\frac{\pi}{U^2 \cos(2\theta)}\right)^{1/4}$$



Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
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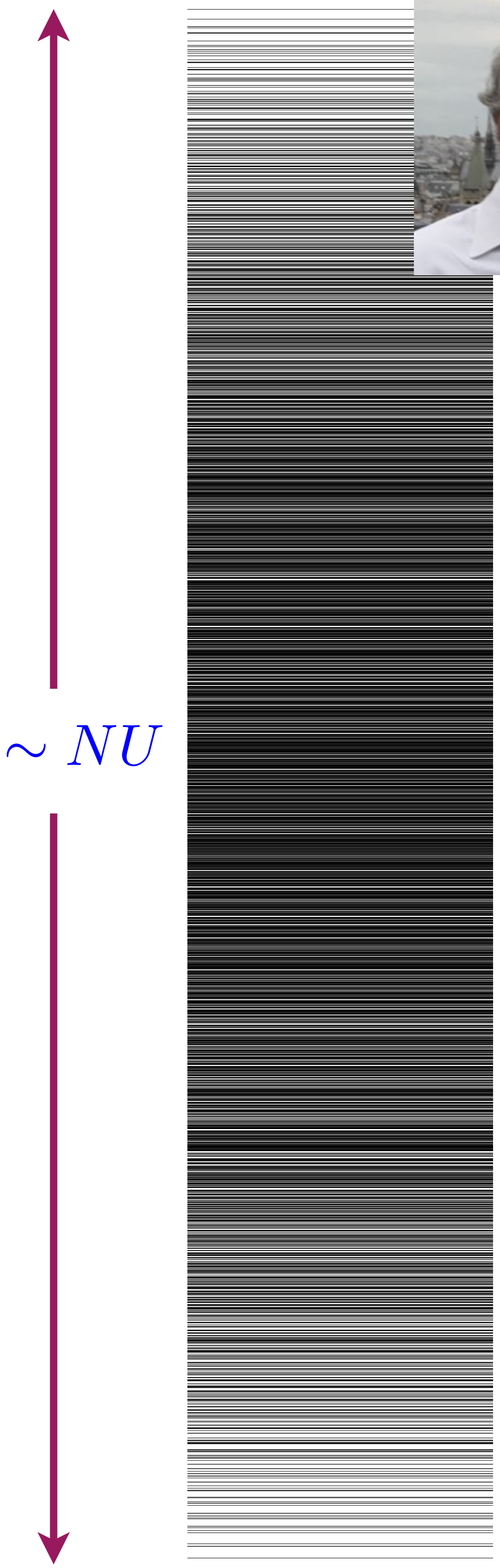
The SYK model



GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

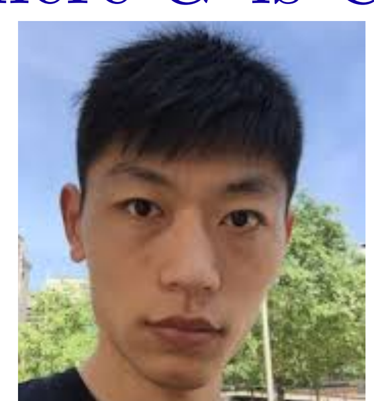
There are 2^N many body levels with energy E. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$, where $s_0 < \ln 2$ is determined by integrating

$$\boxed{\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}\,.}$$

At $\mathcal{Q} = 1/2$,

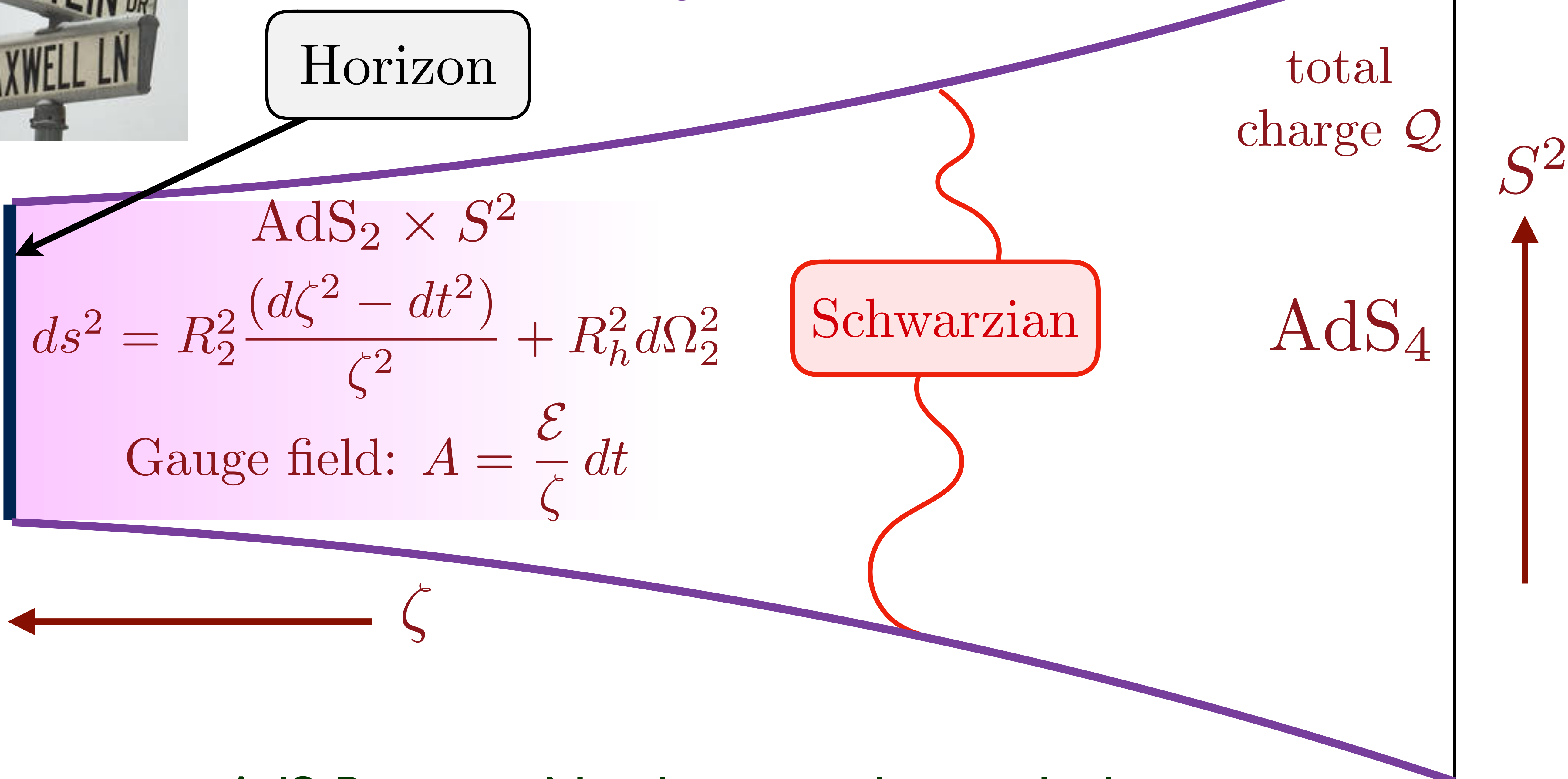
$$s_0 \quad = \quad \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where G is Catalan's constant.



W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

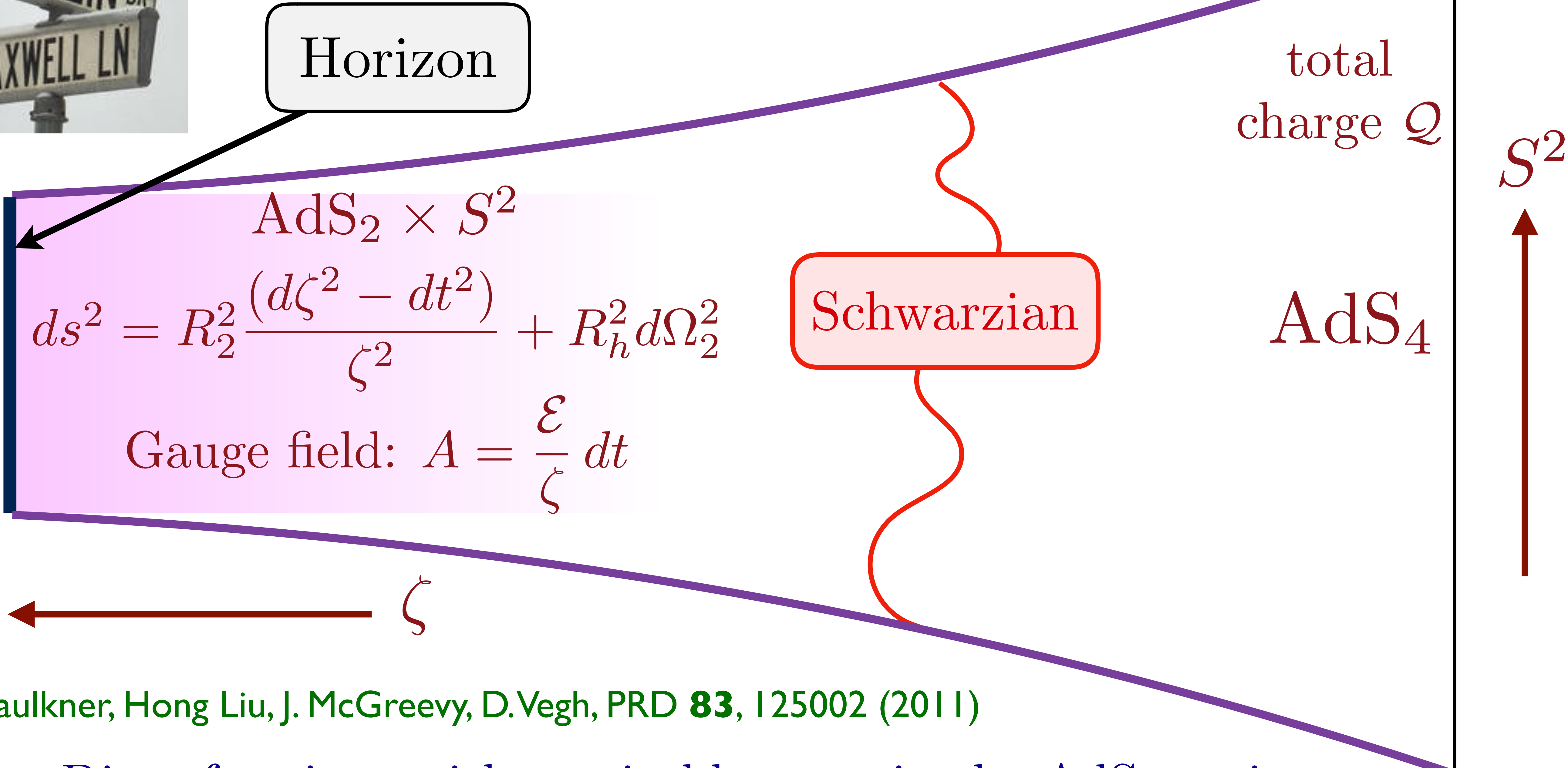
Charged black holes



AdS-Reissner-Nordstrom solution dual
via AdS/CFT correspondence
to large-*N* SYM_3 in a chemical potential

A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD **60**, 064018 (1999)

Charged black holes



T. Faulkner, Hong Liu, J. McGreevy, D. Vegh, PRD **83**, 125002 (2011)

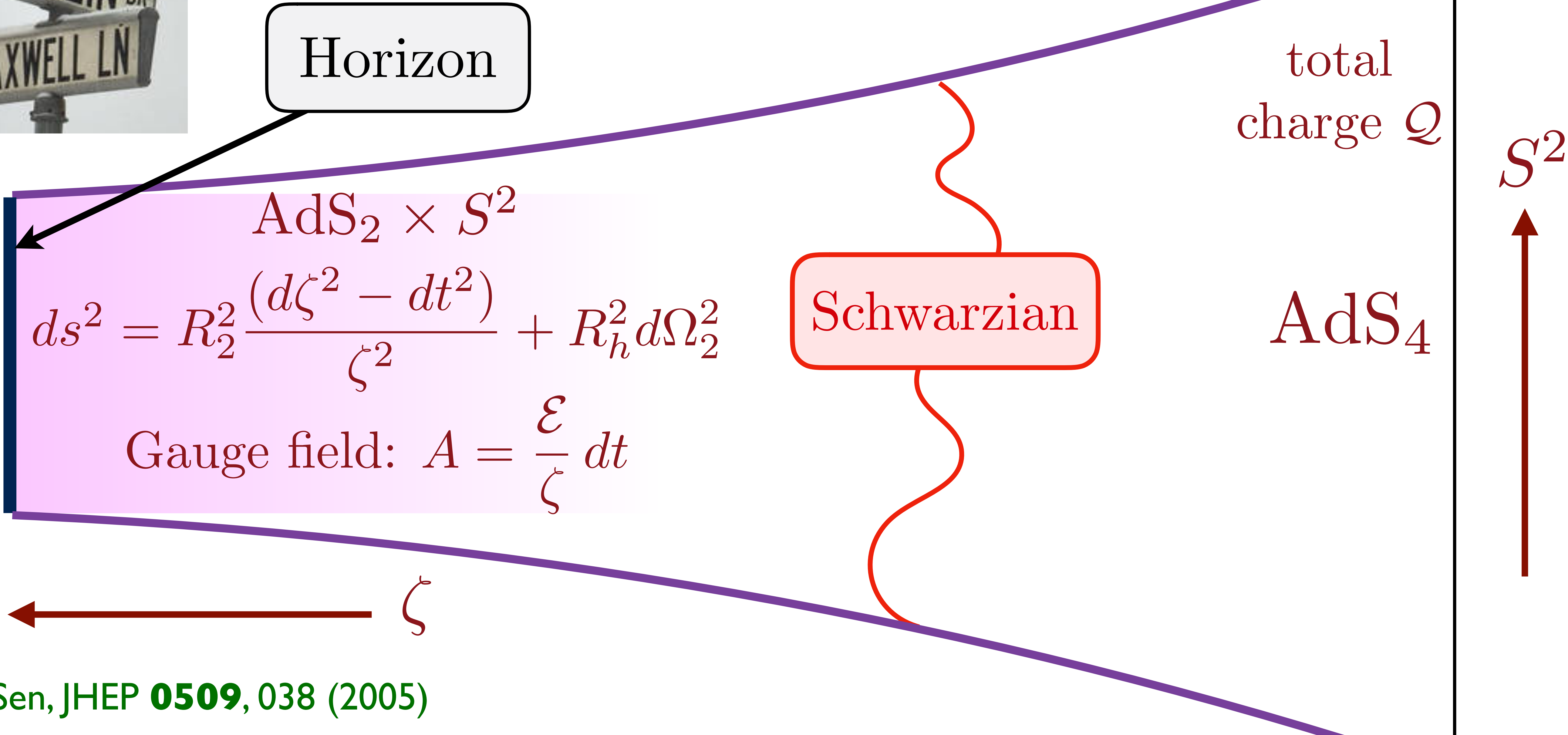
For Dirac fermions with a suitable mass in the AdS_2 region,

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = e^{\pi \mathcal{E}} \frac{A}{\sqrt{\tau}} \quad , \quad \left\langle c_\alpha^\dagger(\tau) c_\alpha(0) \right\rangle = e^{-\pi \mathcal{E}} \frac{A}{\sqrt{\tau}}$$

At $T > 0$, there is a non-zero entropy s_0, and the fermion correlators also precisely match those of the SYK model.

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

Charged black holes



A. Sen, JHEP **0509**, 038 (2005)

- The entropy s_0, the charge $\mathcal{Q}$, and the dimensionless electric field $\mathcal{E}$ obey

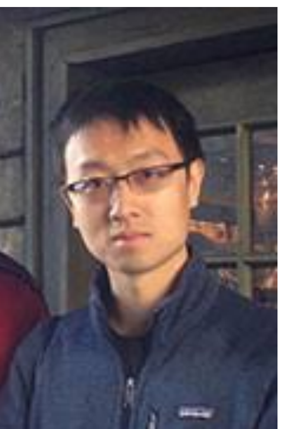
$$\boxed{\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}}$$

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010); PRX **5,** 041025 (2015)

The complex SYK model has a universal Luttinger-like relation between the charge $\mathcal{Q}$ and the spectral asymmetry $\mathcal{E}$, which can be integrated to obtain s_0 as a function $\mathcal{E}$.

$$\mathcal{Q} = \frac{1}{4}(3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi}\tan^{-1}\left(e^{2\pi\mathcal{E}}\right)$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)



Precisely the same s_0 as a function of $\mathcal{E}$ is obtained by the path integral of Dirac fermions on AdS$_2$.

Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, to appear

In Einstein-Maxwell theory with action

$$I_{EM} = \int d^{d+2}x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(\mathcal{R}_{d+2} + \frac{d(d+1)}{L^2}\right) + \frac{1}{4g_F^2}F^2\right],$$

the black-hole 'equation of state' is $s_0 = \left(2\pi s_d/\kappa^2\right) R_h^d$,

$$\mathcal{Q} = \frac{s_d R_h^{d-1}\sqrt{d\left[(d+1)R_h^2 + (d-1)L^2\right]}}{L\kappa g_F}, \quad \mathcal{E} = \frac{g_F R_h L\sqrt{d\left[(d+1)R_h^2 + (d-1)L^2\right]}}{\kappa\left[d(d+1)R_h^2 + (d-1)^2L^2\right]}$$

A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD **60**, 064018 (1999)

1. The complex SYK model and
charged black holes

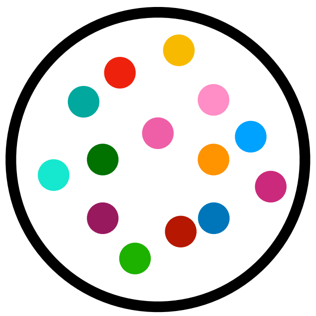
2. Quantum matter without quasiparticles:
lattice SYK models
and Planckian metals

The complex SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta + \epsilon \sum_\alpha c_\alpha^\dagger c_\alpha$$

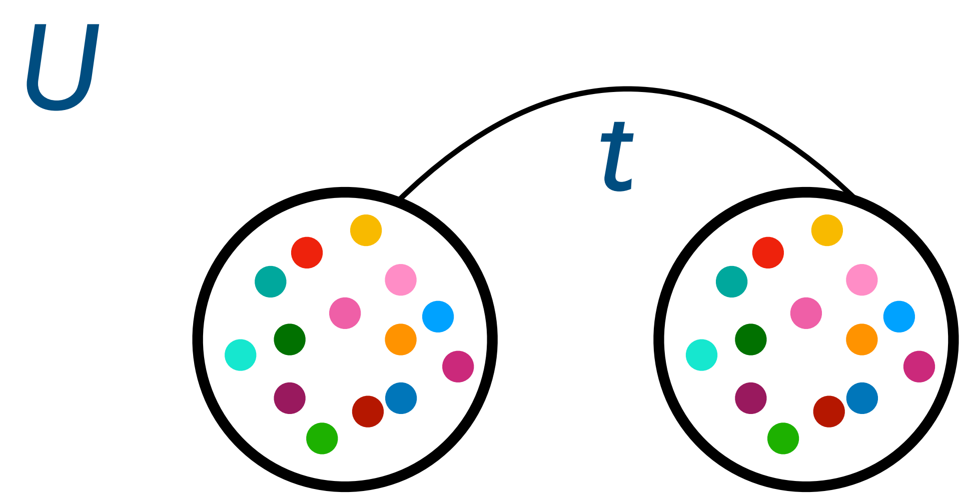
$U_{\alpha\beta;\gamma\delta}$ are independent random variables

with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$



$$H = \frac{1}{(2N)^{3/2}} \sum_{i} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_{i\alpha} c^\dagger_{i\beta} c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c^\dagger_{i\alpha} c_{j\alpha}$$

Equivalent to an
"eternal traversable wormhole"
between two black holes with
AdS_2 horizons



J. Maldacena and Xiao-Liang Qi, arXiv:1804.00491

A lattice SYK model

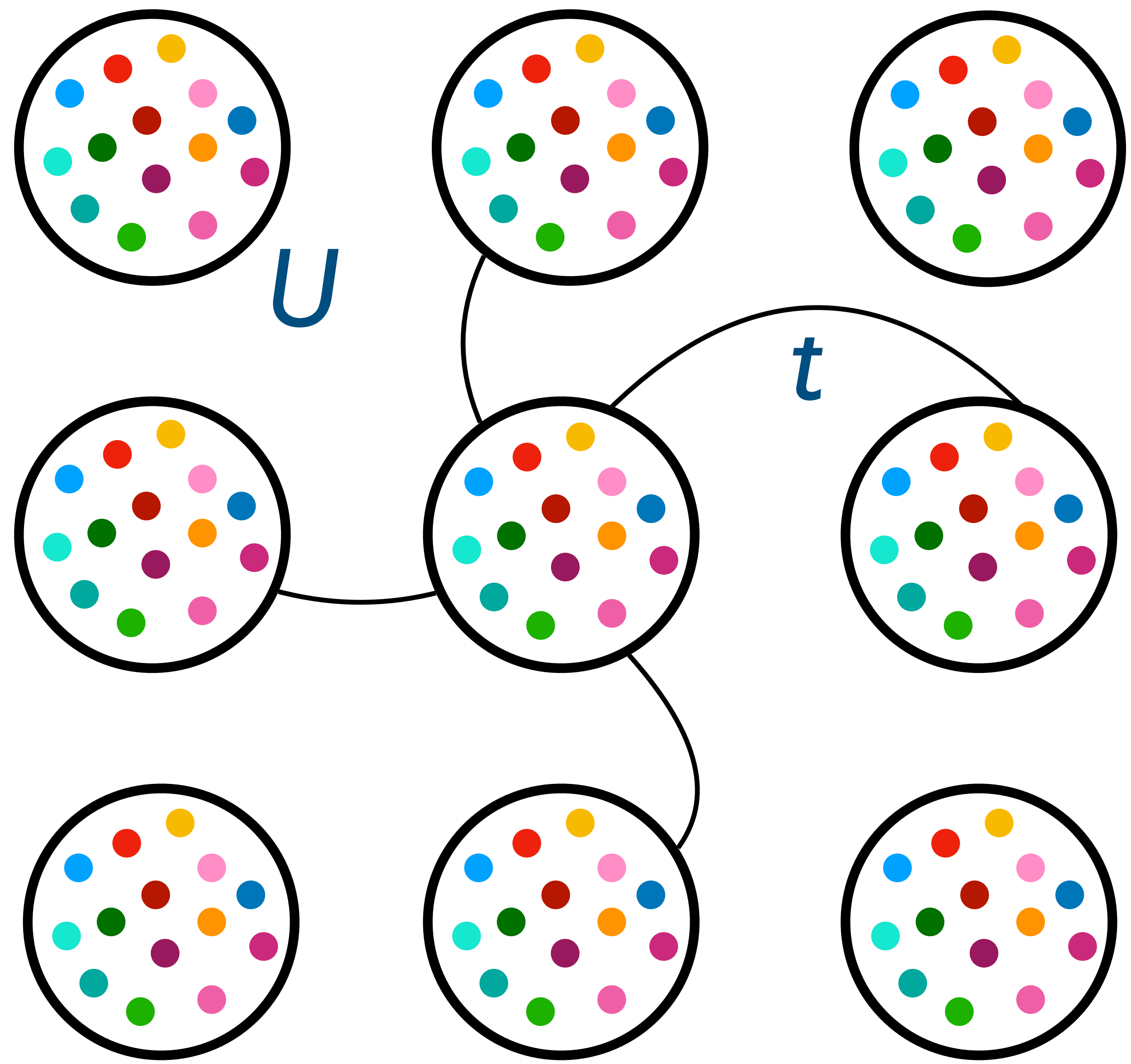
$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c_{i\alpha}^\dagger c_{j\alpha}$$

Choose U on-site, and the same on all sites; yields 'incoherent metal' with no Fermi surface for $t^2/U \ll k_B T \ll U$ with

$$G(\mathbf{k},\omega) = G_{\rm SYK}(\epsilon, \hbar\omega/(k_B T))$$

independent of $\mathbf{k}$. There is linear-in-T resistivity but only with bad metal behavior with $\rho > h/e^2$, and co-efficient dependent upon U:

$$\rho \sim \frac{h}{e^2} \frac{k_B T}{t^2/U}$$



Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017); Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

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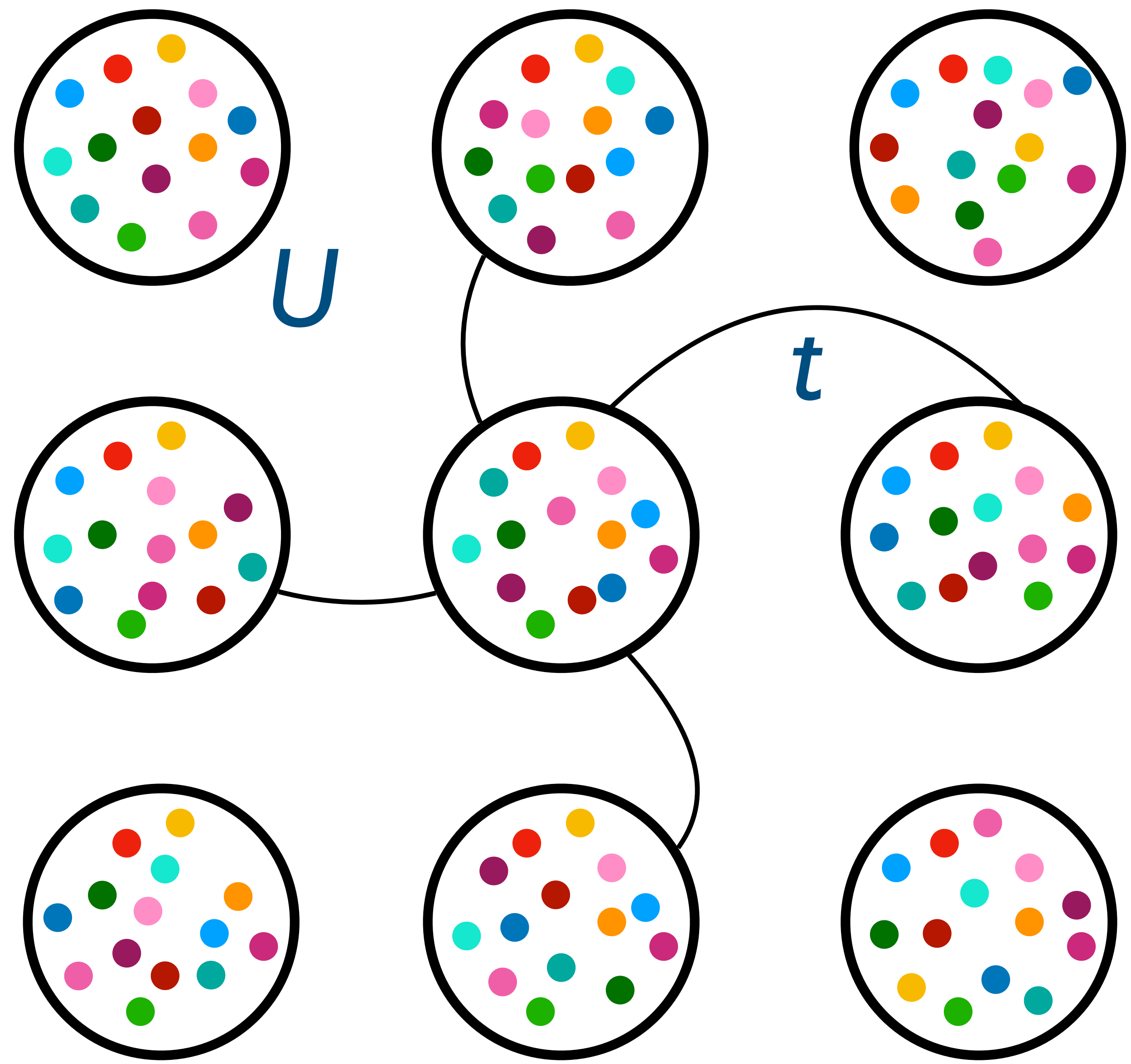


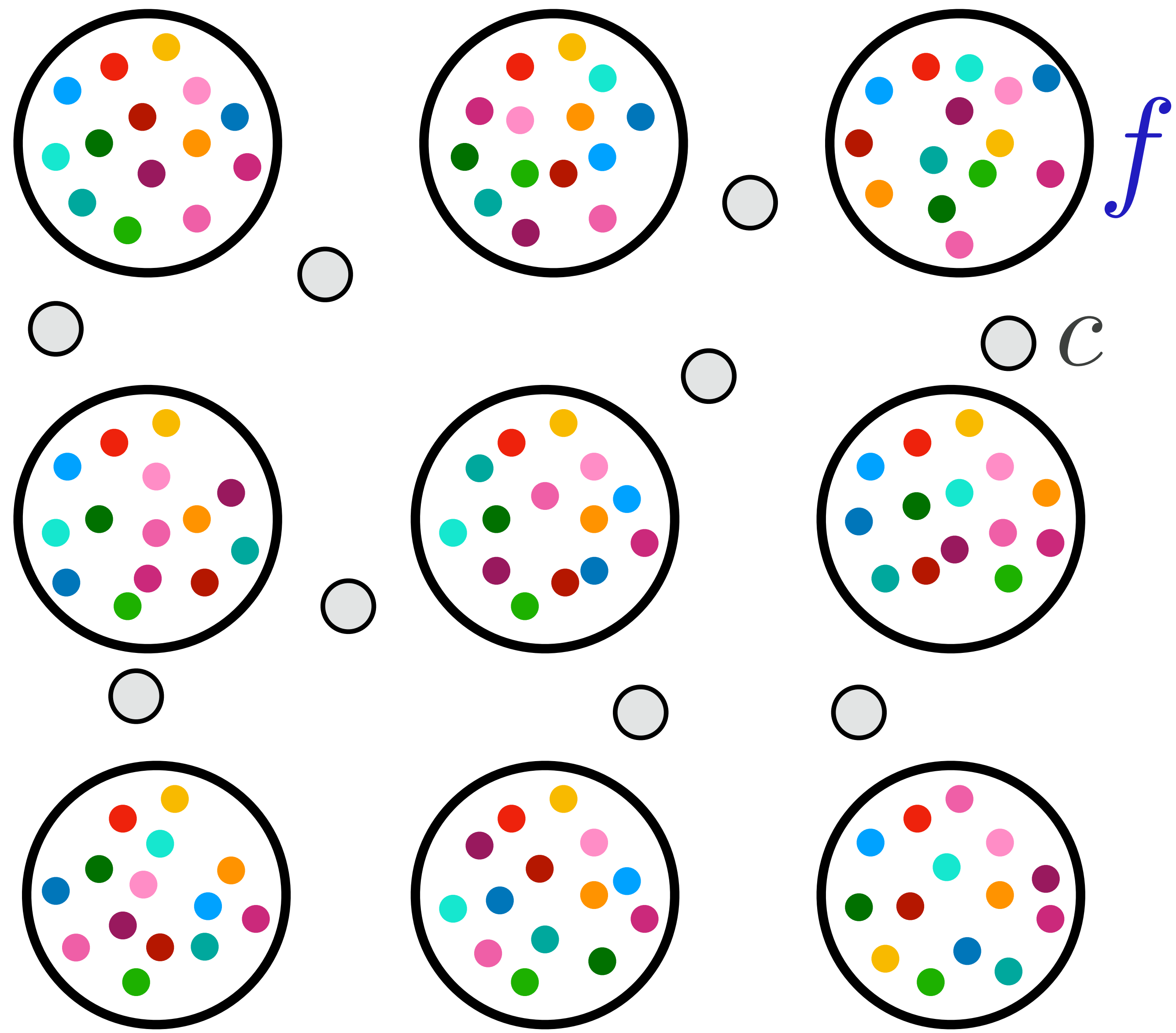
Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017); Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

A Kondo-SYK model

Mobile electrons (c) coupled to SYK quantum islands (f) with exchange interactions.

Has a regime where the c electrons form a marginal Fermi liquid with a linear-in-T resistivity dependent upon interaction strength, and a small Fermi surface which does not count the f electrons.



Similar results for many earlier `marginal Fermi liquid' and holographic models

Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018)
Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

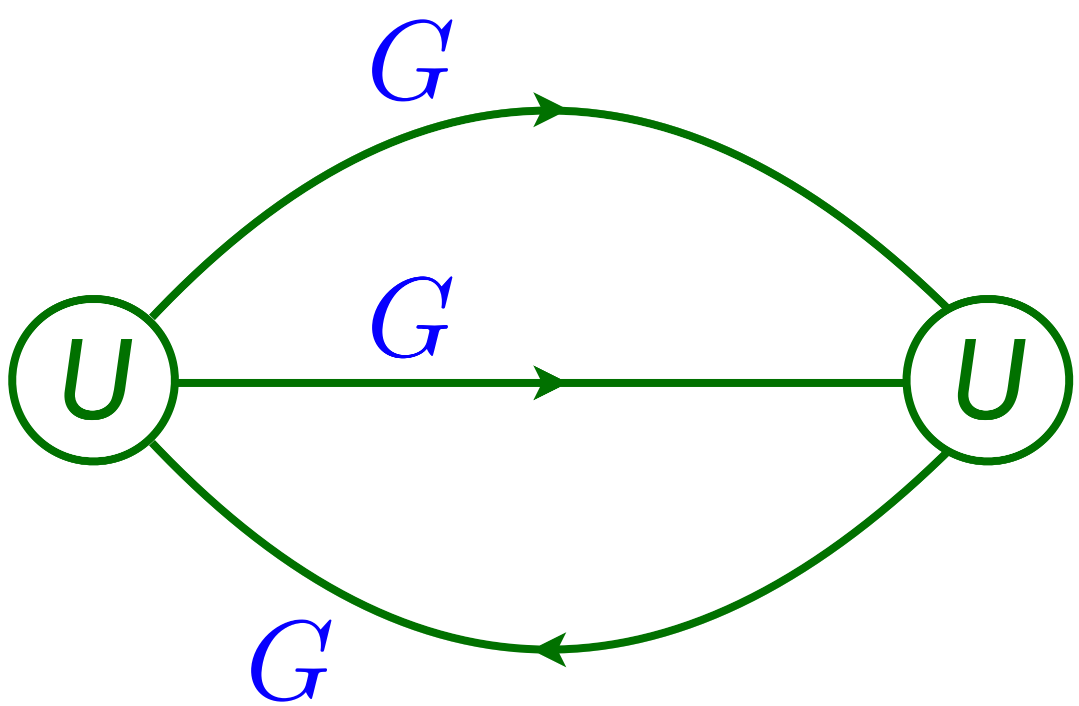
Generalized SYK models

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c^\dagger_{k_1\alpha} c^\dagger_{k_2\beta} c_{k_3\gamma} c_{k_4\delta}$$
$$+ \sum_{k\alpha} \epsilon_k c^\dagger_{k\alpha} c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$ (as before)
ϵ_k has a range of values of width W.

The large N limit is still given by the sum of “melon” diagrams.

$$G(k, i\omega) = \frac{1}{i\omega - \epsilon_k - \Sigma(k, i\omega)}$$

$\Sigma =$ 

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ϵ_k has a range of values of width W.

The large N limit is still given by the sum of "melon" diagrams.

For many generic models in this class, $\hbar\omega/(k_B T)$ scaling of SYK holds for $W^2/U \ll k_B T \ll U$, and Fermi liquid theory is recovered for $k_B T \ll W^2/U$.

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)
Pengfei Zhang, PRB **96**, 205138 (2017)
Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018)
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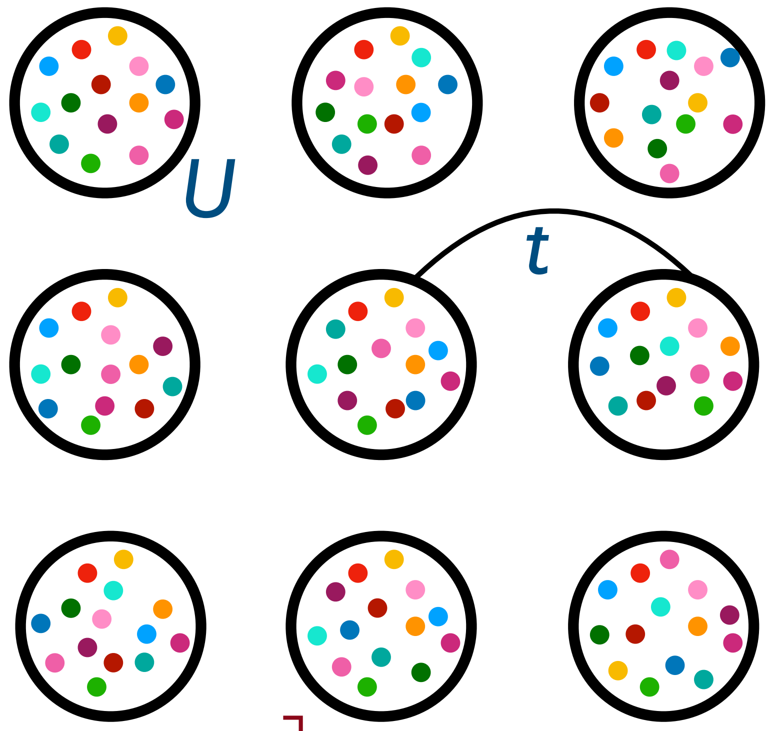
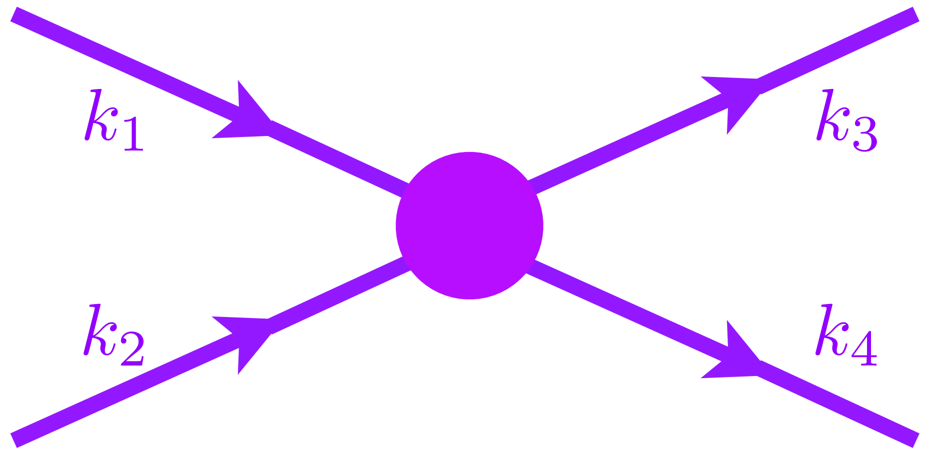


Rewriting of lattice model of incoherent and bad metal in momentum space

$$\overline{U(k_1,k_2,k_3,k_4)U^*(k_5,k_6,k_7,k_8)} = U^2\left[\delta(k_1+k_2-k_3-k_4-k_5-k_6+k_7+k_8)\right]$$

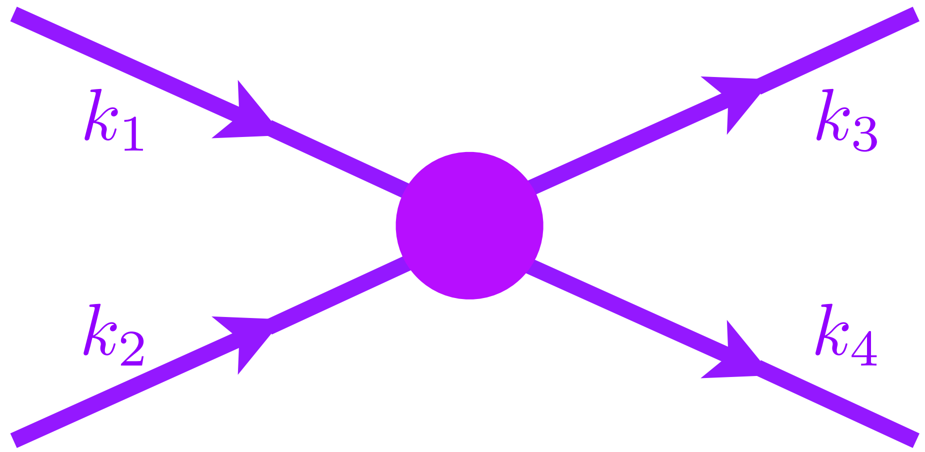
Resonant SYK model



Interactions with $\epsilon_{k_1} + \epsilon_{k_2} \neq \epsilon_{k_3} + \epsilon_{k_4}$ are non-resonant: we "integrate these out" in a RG procedure, and assume that their main effect is a renormalization of the quasiparticle dispersion ϵ_k, which we have already accounted for.

A. Patel and S. Sachdev, arXiv:1906.03265

Resonant SYK model



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Keep only the **interactions resonant in the bare quasiparticle energy** with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$ and account for them with a self-consistent SYK-like analysis.

A. Patel and S. Sachdev, arXiv:1906.03265

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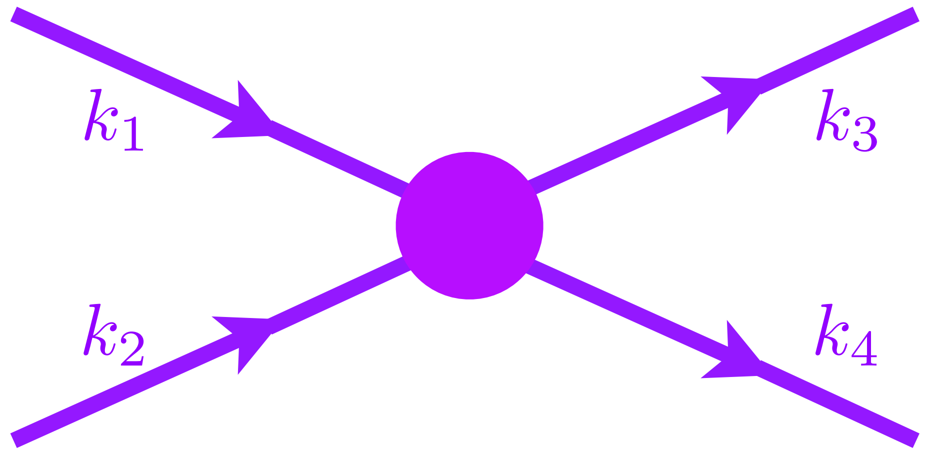

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Keep only the

interactions resonant in the bare quasiparticle energy

with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$ and account for them with a self-consistent SYK-like analysis.

This is precisely the effective Hamiltonian method, when low energy states are separated from high energy states by a gap; we are assuming it can also apply in a gapless system.

A. Patel and S. Sachdev, arXiv:1906.03265

Resonant SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c^\dagger_{k_1\alpha} c^\dagger_{k_2\beta} c_{k_3\gamma} c_{k_4\delta}$$

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This implies off-site interactions with correlations which decay with a power-law in space.

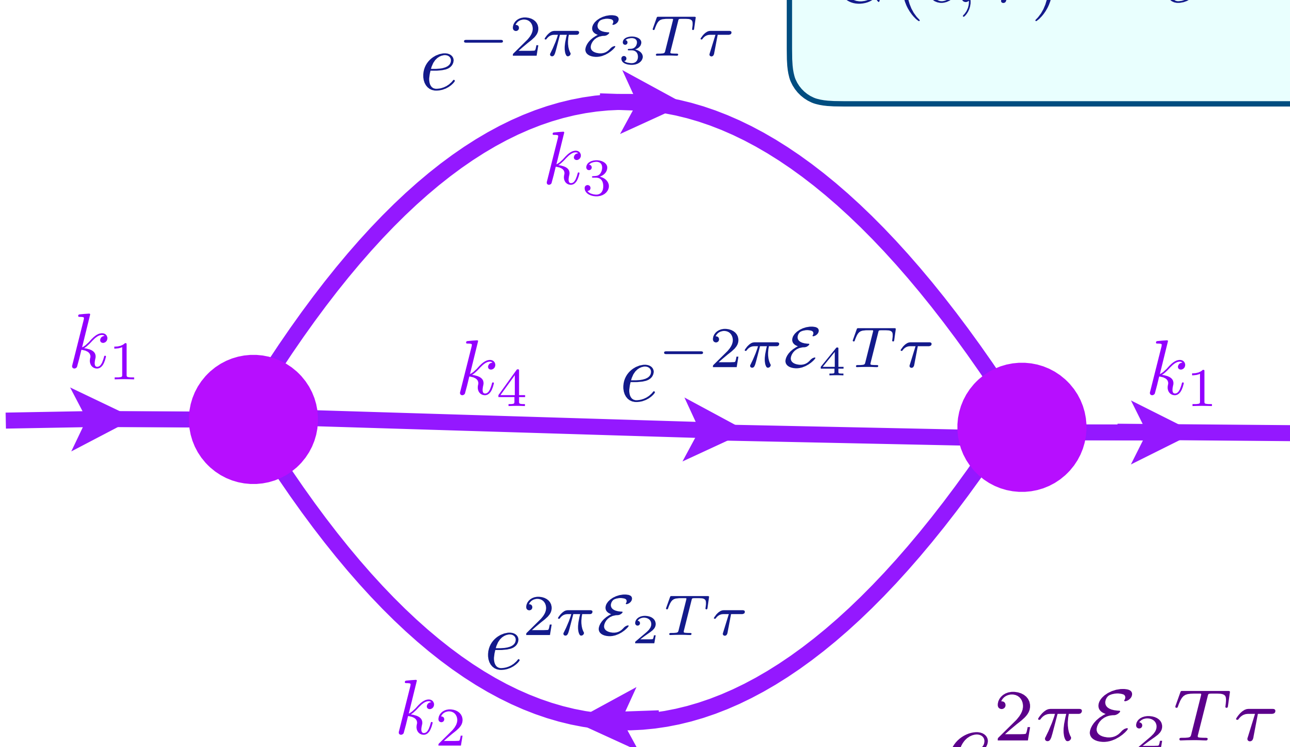
Aavishkar Patel

A. Patel and S. Sachdev, arXiv:1906.03265

Resonant SYK model

Conformal Green's function at $T > 0$ must have the form

$$\boxed{G(\epsilon,\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2}} \quad , \quad 0 < \tau < 1/T\,.$$



$$e^{2\pi\mathcal{E}_2 T\tau}\, e^{-2\pi\mathcal{E}_3 T\tau}\, e^{-2\pi\mathcal{E}_4 T\tau} = e^{-2\pi\mathcal{E}_1 T\tau}$$

if

$$\mathcal{E}_a = \mathbb{C}\epsilon_a/U$$

and

$$\epsilon_1 + \epsilon_2 = \epsilon_3 + \epsilon_4$$

SYK behavior in a
Planckian metal as $T \to 0$
with a remnant Fermi surface:
$G(k,\omega) = G_{\rm SYK}(\epsilon_k, \hbar\omega/(k_B T))$,
with $\mathcal{E}_k = \mathbb{C}\,\epsilon_k/U$

A. Patel and S. Sachdev, arXiv:1906.03265

Incoherent metal

For long times $\tau > 0$

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle = e^{\pi \mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

$$\left\langle c_k^\dagger(\tau) c_k(0) \right\rangle = e^{-\pi \mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

The parameter $\mathcal{E}$ is independent of k, and determined by the total density

Planckian metal with remnant Fermi surface

For long times $\tau > 0$

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle = e^{\pi \mathbb{C} \epsilon_k / U} \frac{A(\epsilon_k/U)}{\sqrt{U\tau}}$$

$$\left\langle c_k^\dagger(\tau) c_k(0) \right\rangle = e^{-\pi \mathbb{C} \epsilon_k / U} \frac{A(\epsilon_k/U)}{\sqrt{U\tau}}$$

The particle-hole asymmetry changes as we cross the Fermi surface

Aavishkar Patel

A. Patel and S. Sachdev, arXiv:1906.03265

The complex SYK model

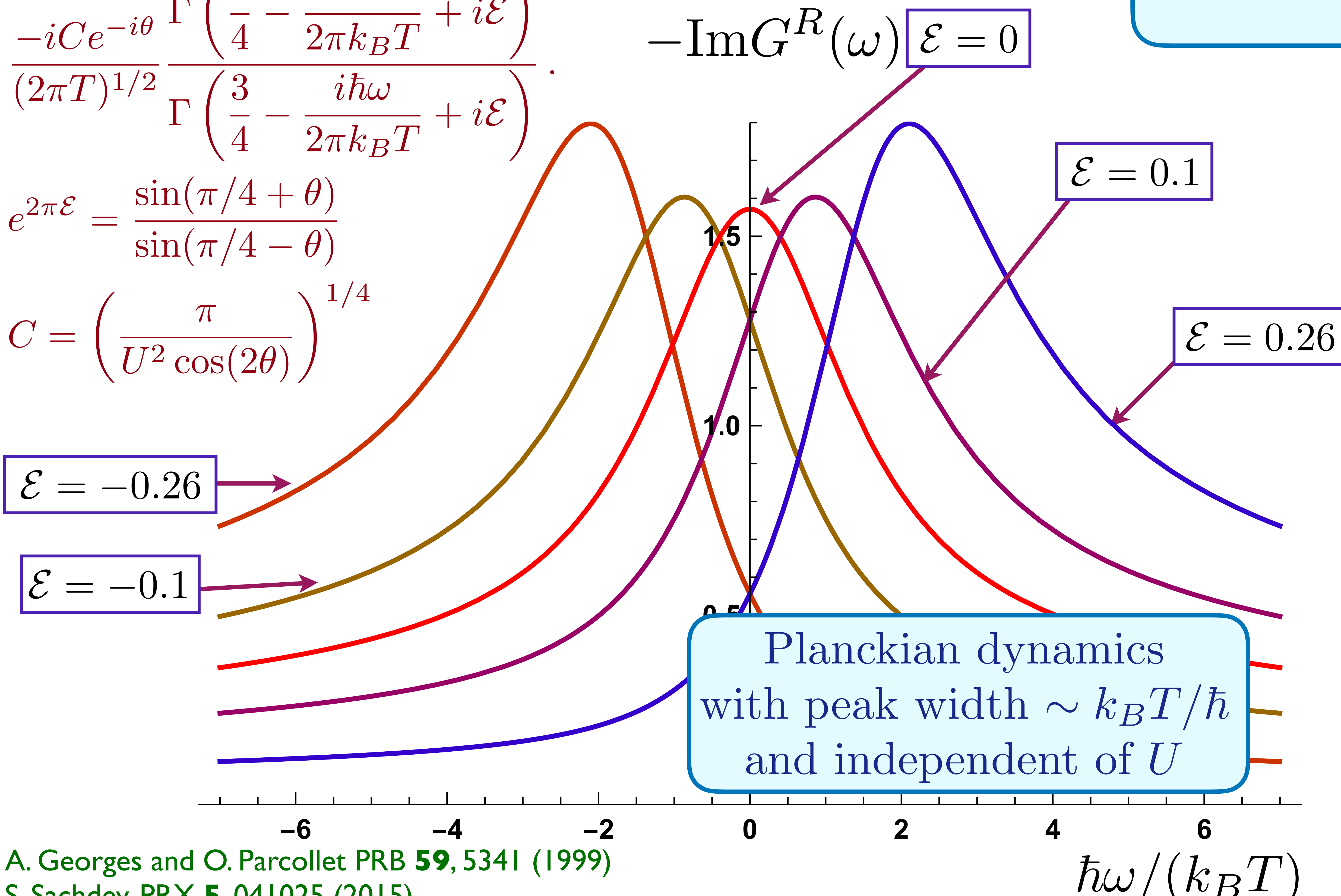
$$\mathcal{E} = \mathbb{C}\,\frac{\epsilon}{U}$$

$$G^R_{\rm SYK}(\epsilon, \hbar\omega/(k_B T)) =$$

$$\frac{-iCe^{-i\theta}}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)} \,.$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

$$C = \left(\frac{\pi}{U^2 \cos(2\theta)}\right)^{1/4}$$



Planckian dynamics
with peak width $\sim k_B T/\hbar$
and independent of U

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

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$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4+\theta)}{\sin(\pi/4-\theta)}$$

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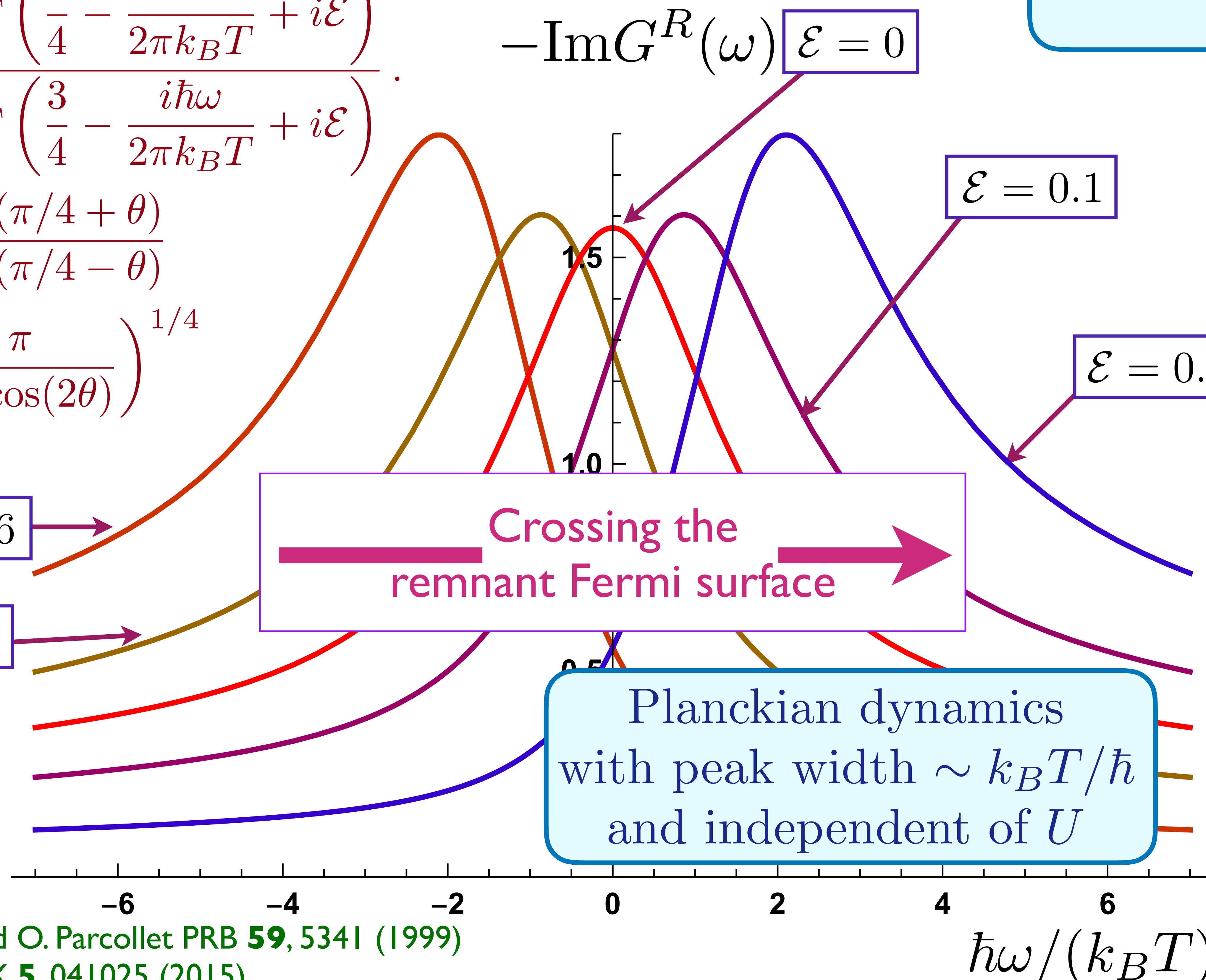
$$\boxed{\mathcal{E} = \mathbb{C}\frac{\epsilon}{U}}$$



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with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$.

Resistivity of a Planckian metal as $T \to 0$

From the Kubo formula, in the large N limit

$$\sigma = \frac{Ne^2 m^* v_F^2}{2T} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{4\pi} \left[\operatorname{Im} G^R_{\rm SYK}\left(\epsilon, \frac{\omega}{T}\right)\right]^2 \operatorname{sech}^2\left(\frac{\omega}{2T}\right)$$

A. Patel and S. Sachdev, arXiv:1906.03265

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$$\boxed{\rho = \frac{m^*}{ne^2}\, 2.71\mathbb{C}\, \frac{k_B T}{\hbar}} \quad , \quad \text{using } \mathcal{E} = \mathbb{C}\epsilon/U\,,$$

where

$$m^* = \frac{d\, V_{FS}}{\oint_{FS} |\boldsymbol{v}_F|}\,,$$

where d is spatial dimensionality and V_{FS} is the volume enclosed by the Fermi surface. For a circular Fermi surface, this is the usual m^*.

Resonant SYK model

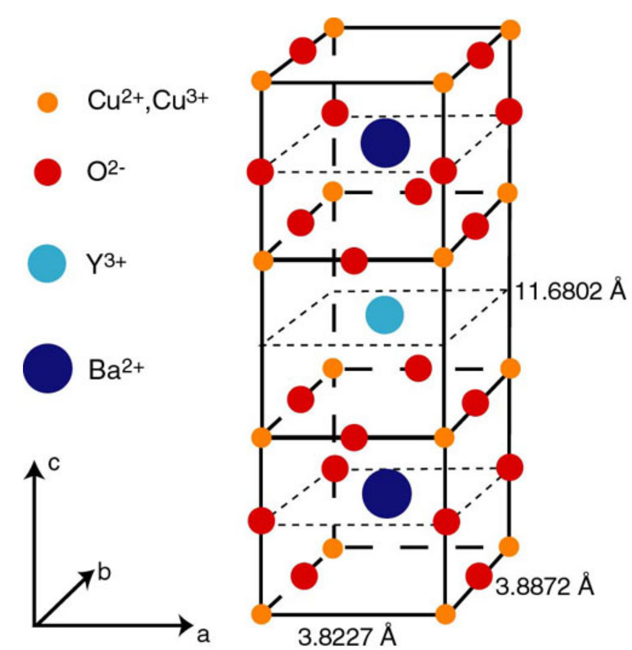
Resistivity of a Planckian metal as $T \to 0$

$$\boxed{\rho = \frac{m^*}{ne^2}\, 2.71\mathbb{C}\, \frac{k_B T}{\hbar}}$$

Note that all explicit dependence on U has cancelled out!

The number $\mathbb{C}$ is defined by $\mathcal{E}_k = \mathbb{C}\, \epsilon_k / U$ as $|\epsilon_k| \to 0$. This is determined by UV physics, and is very weakly dependent upon the ratio of the energy width of the interactions, W_U, to U.

Aavishkar Patel



A. Patel and S. Sachdev, arXiv:1906.03265

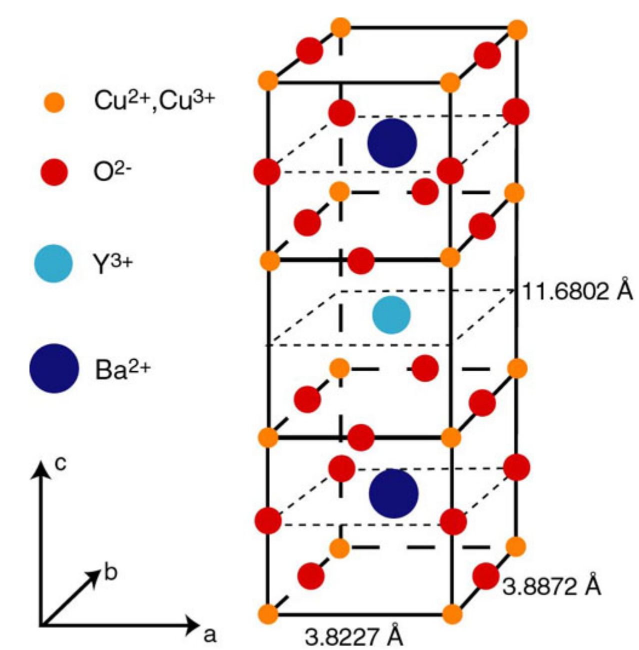
Resonant SYK model

Take the independent momentum shell limit, $W_U/U \to 0$,

$$\overline{U(k_1,k_2,k_3,k_4)U^*(k_5,k_6,k_7,k_8)} = U^2\left[\delta(k_1+k_2-k_3-k_4-k_5-k_6+k_7+k_8)\right] \times \left[\delta(\epsilon_{k_1}-\epsilon_{k_2})\delta(\epsilon_{k_2}-\epsilon_{k_3})\delta(\epsilon_{k_3}-\epsilon_{k_4})+\delta(\epsilon_{k_5}-\epsilon_{k_6})\delta(\epsilon_{k_6}-\epsilon_{k_7})\delta(\epsilon_{k_7}-\epsilon_{k_8})\right]$$

$\mathbb{C} = 0.41$ as in a single SYK model,
and we obtain a Planckian metal with

$$\boxed{\rho = \frac{m^*}{ne^2}\, 1.11\, \frac{k_B T}{\hbar}}$$



Aavishkar Patel

A. Patel and S. Sachdev, arXiv:1906.03265

Planckian metals with a remnant Fermi surface

- Resonant SYK models are compressible and dispersive quantum systems with $\hbar\omega/(k_B T)$ scaling as $T \to 0$.

- The resonance condition is supported by a RG argument: non-resonant interactions mainly renormalize the underlying quasi-particle dispersion ϵ_k, while resonant interactions have to be treated self-consistently.

- The resonance is a single 'fine-tuning' condition designed to obtain $\hbar\omega/(k_B T)$ scaling as $T \to 0$. However, then many other nice features follow: we obtain a Planckian metal with remnant large Fermi surface at $\epsilon_k = 0$, and an effective mass m^* defined by the dispersion of ϵ_k, with a resistivity $\rho \sim (m^*/(ne^2))k_B T/\hbar$ independent of the strength of interactions.

Aavishkar Patel (Harvard $\longrightarrow$ Miller Fellow at Berkeley)

A. Patel and S. Sachdev, arXiv:1906.03265