# Developments in de Sitter Quantum Gravity

Collaborators: Alishahiha, Dong, Gorbenko, Horn, Karch, Matsuura, Tong, Torroba, + D. Freedman, Coleman, Anderson, Mousatov, Lewkowycz, Liu, Mazenc, Soni,...


[1] V. Gorbenko, E. Silverstein and G. Torroba, "dS/dS and $T\bar{T}$," arXiv:1811.07965 [hep-th].

[2] X. Dong, E. Silverstein and G. Torroba, "De Sitter Holography and Entanglement Entropy," JHEP **1807**, 050 (2018) doi:10.1007/JHEP07(2018)050 [arXiv:1804.08623 [hep-th]].


[3] Further developments; WIP on Subregion dualities and redundant encoding in (AdS) Patches, work in progress with Lewkowycz, Liu, Torroba (cf Sorce); related discussions on 2d gravity formulations w/Mazenc, Soni...

+original background and followups in progress


[4] M. Alishahiha, A. Karch, E. Silverstein, and D. Tong, "The dS/dS correspondence," *AIP Conf. Proc.* **743** (2005) 393–409, arXiv:hep-th/0407125 [hep-th]. [,393(2004)].

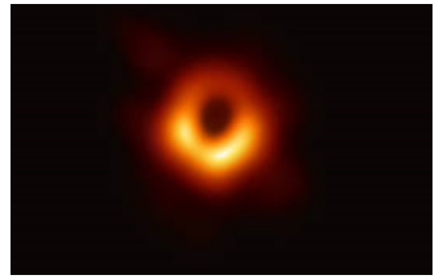
[5] M. Alishahiha, A. Karch, and E. Silverstein, "Hologravity," *JHEP* **06** (2005) 028, arXiv:hep-th/0504056 [hep-th].

[7] X. Dong, B. Horn, E. Silverstein, and G. Torroba, "Micromanaging de Sitter holography," *Class. Quant. Grav.* **27** (2010) 245020, arXiv:1005.5403 [hep-th].

[8] X. Dong, B. Horn, S. Matsuura, E. Silverstein, and G. Torroba, "FRW solutions and holography from uplifted AdS/CFT," *Phys. Rev.* **D85** (2012) 104035, arXiv:1108.5732 [hep-th].

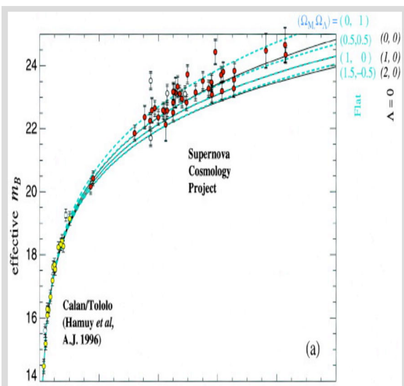
# What is ``It"?

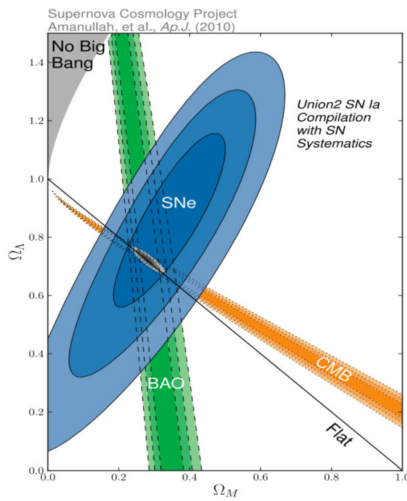
a(t)~exp(Ht)

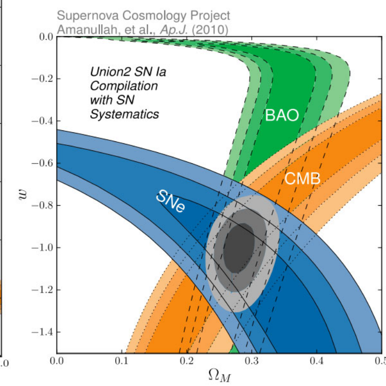


Kerr BHs cf Guica,Apolo/Song,

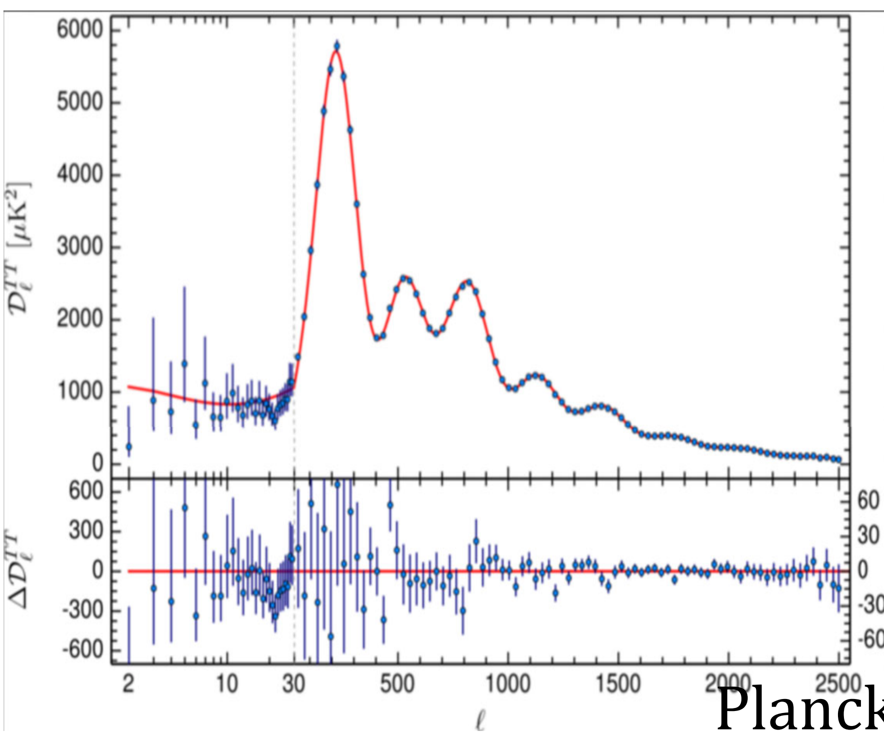
Late Universe



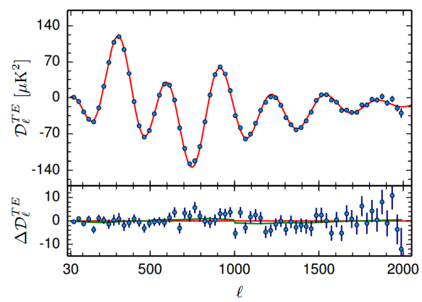



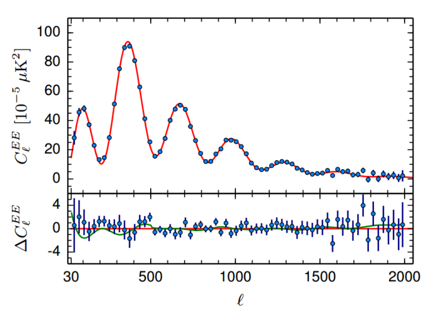




Early Universe: inflation fits well




Planck

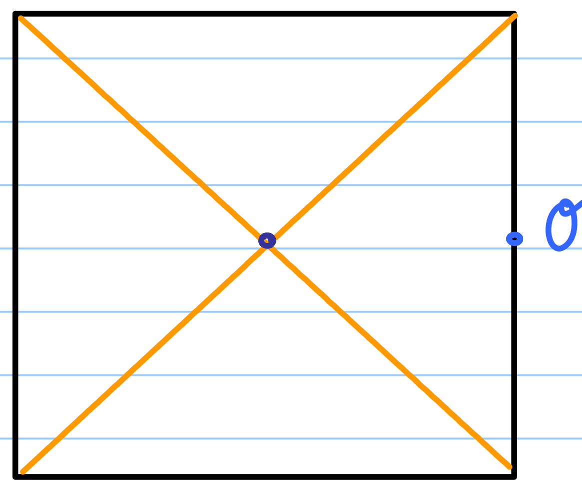


# Effective field theory:

* $\Lambda \sim H^2$ most ``relevant" term in Lagrangian, meaning most important at long times and low energies.

* $\Lambda$ one number, but changes causal structure of spacetime: de Sitter

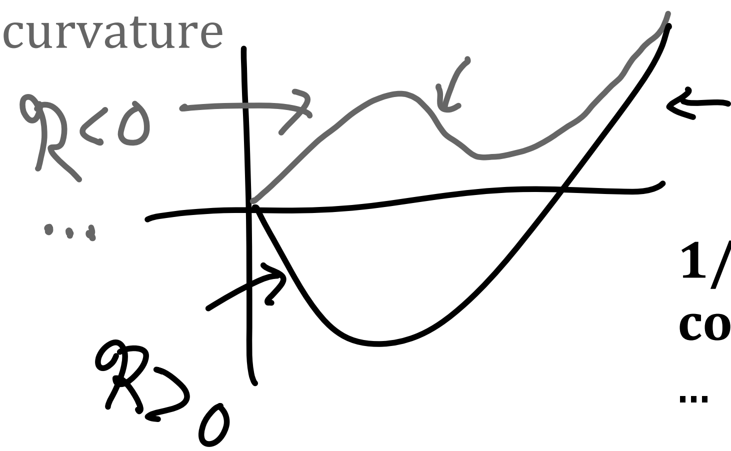



$dS_{d+1}$ Penrose diagram: each point is a (d-1)-sphere; light rays at 45 degrees.
*Observer-dependent horizons whose area behaves like entropy

# Λ and String Theory: Potential energy $V(\phi_i)$ has mostly positive contributions, along with controlled negative sources

D-10
anti D3 brane
negative
curvature

Orientifolds
Quantum




Flux quanta
Bousso-
Polchinski

**1/Volume
coupling**
...

+other
sources to
fully
stabilize

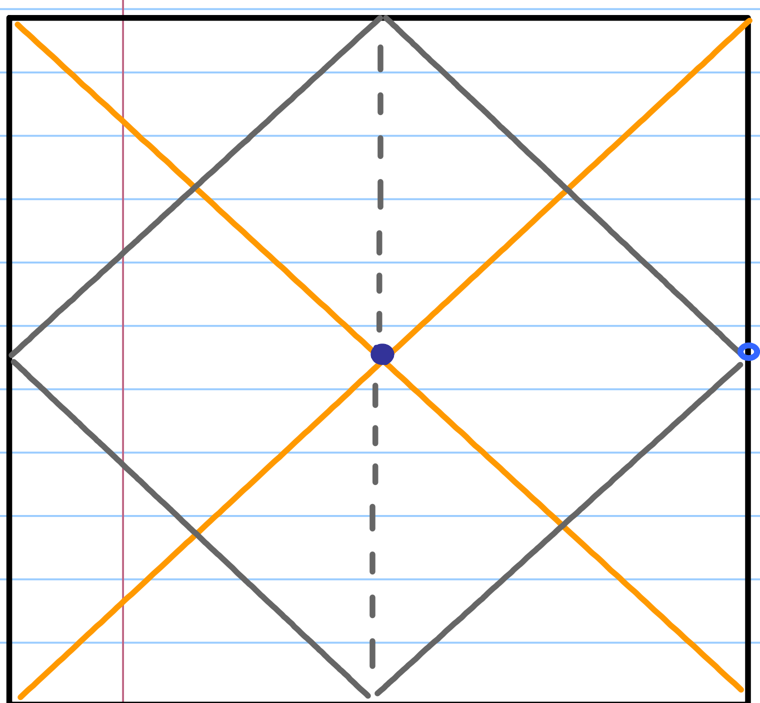
ES, +Maloney Strominger '01 Giddings/Kachru/Polchinski '01/KKLT'03 (studied in great detail), Large Volume, Riemann Surfaces,...Explicit example (2+1 bulk dimensions): Dong, Horn ES, Torroba 2010. Cordova/de Luca/Tomassiello '18.

Fits with interpretation of small Λ as a selection effect.

**In effective field theory, we could simply write any constant Λ. In string theory, it eventually decays.**

# Thought experiments:

Gibbons/Hawking dS entropy demands a microscopic interpretation in quantum gravity



$dS_{d+1}$

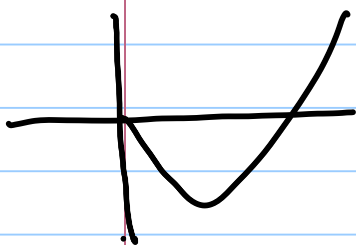
$\mathcal{O}$

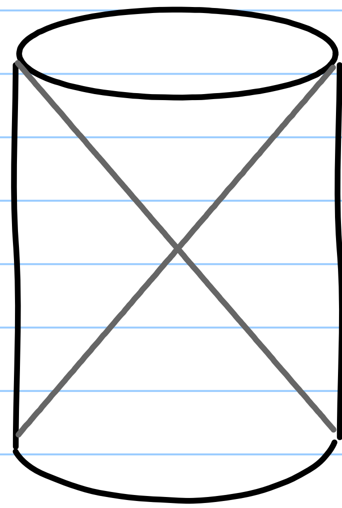
$S_{GH} = V_{d-1}/4G_N$
(cf black holes)

We find that this entropy indeed arises from tracing over $\exp(S_{GH})$ quantum states on one side.

More generally, we need a complete framework for the V>0 landscape.

$\Lambda < 0$



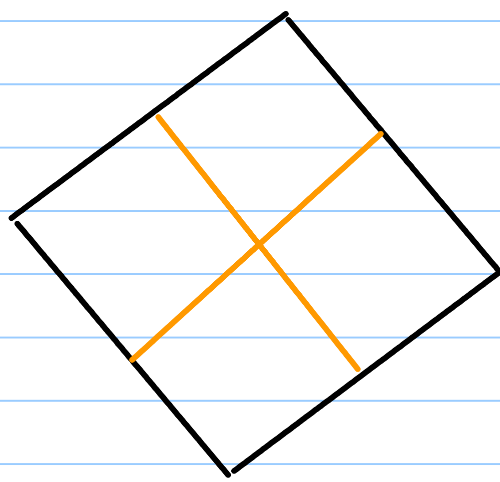


AdS:

observables are conformal field theory (CFT) correlation functions

Timelike boundary at infinity pins fields.

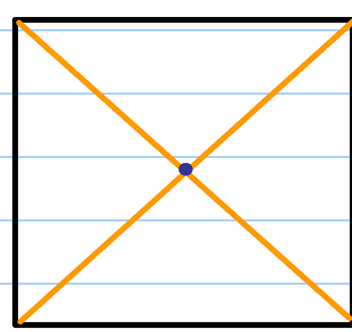
$\Lambda = 0$



Minkowski

Scattering matrix

Asymptotics also very special.



dS

no boundary analogous to the AdS one. (Finite, fluctuating space.)

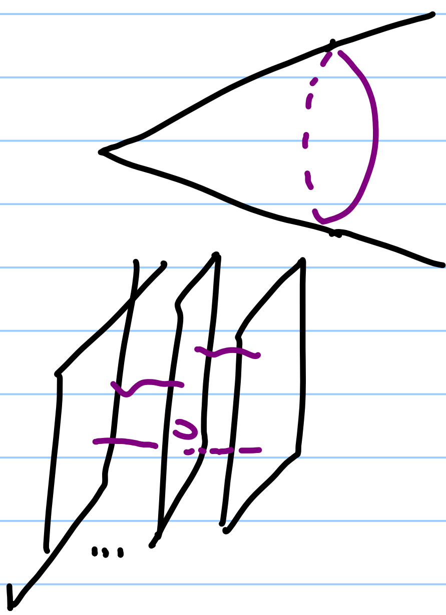
$\Lambda > 0$

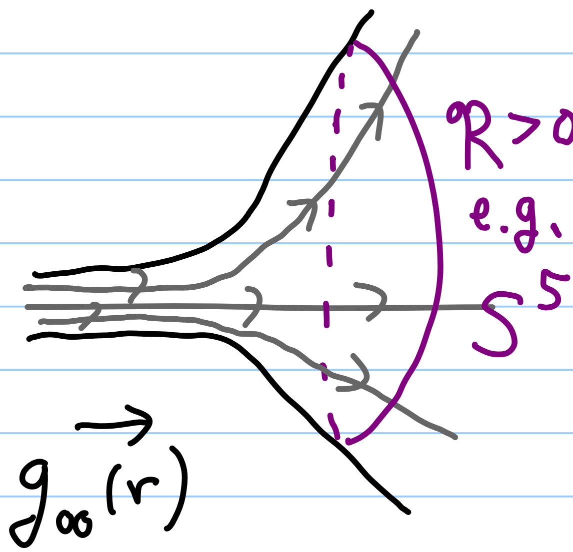
# $AdS_{d+1}/CFT_d$ correspondence formulates $\Lambda < 0$ quantum gravity in terms of non-gravitational dual

Grew out of black hole thermodynamics:

Area/$G_N$ = Entropy

Brane construction in string theory




Nc D3-branes

Polchinski

Nc flux quanta Horowitz/Strominger

Low energy QFT = highly redshifted region

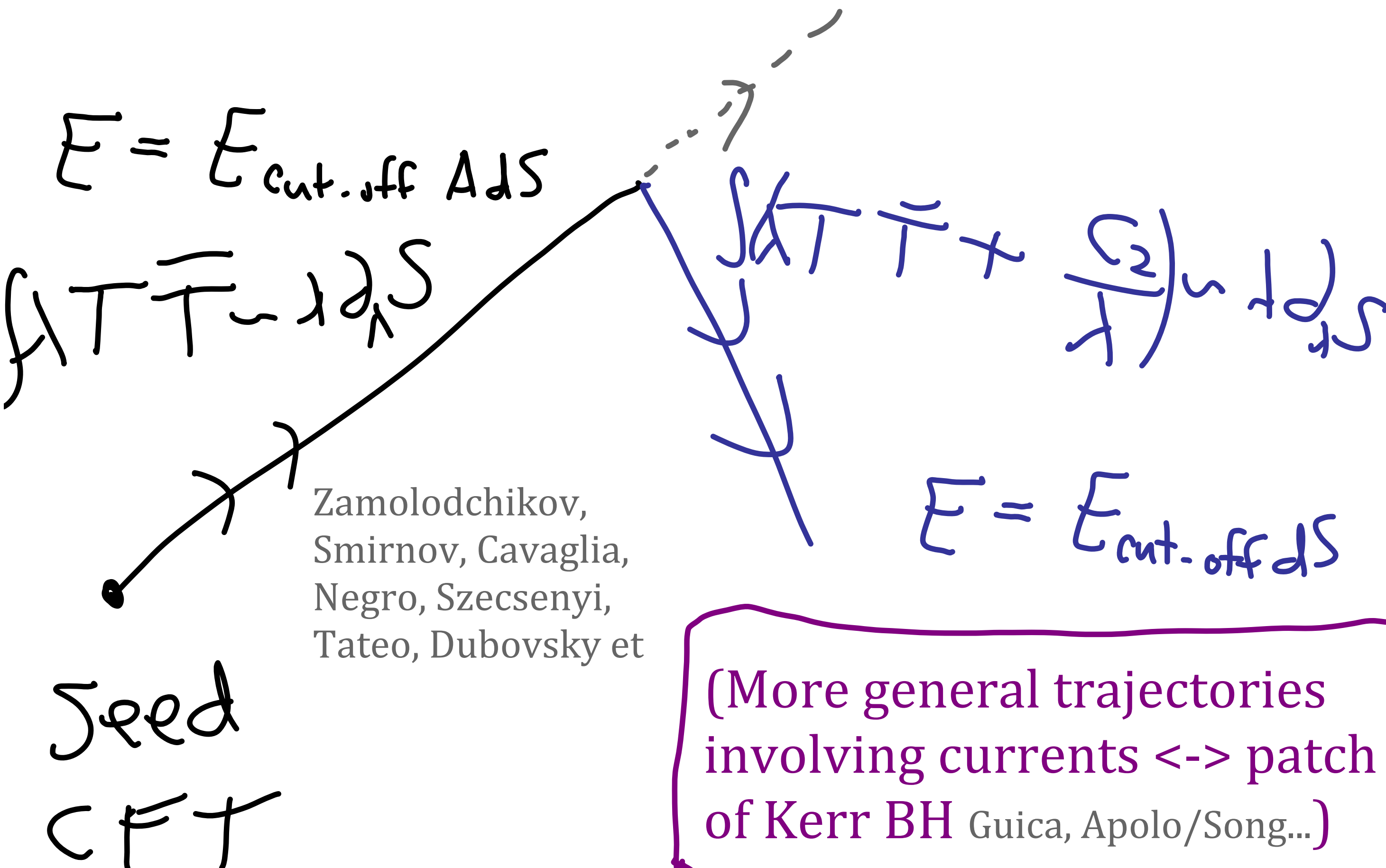
Maldacana; Gubser Polyakov Klebanov Witten

Low energy QFT = highly redshifted region. The near-horizon AdS x S has a timelike boundary at spatial infinity. This, along with the negative cosmological constant, is very special and unrealistic.

Within the near-horizon, AdS x S solution, there is a `scale-radius' duality.
To progress toward more realistic QG, want to determine the dual of a finite patch of this and other spacetimes (e.g. a radial cutoff).

Recently, McGough, Mezei, Verlinde and followups proposed a specific answer to this question, isolating a radially cutoff region of AdS as the dual of a tractable `irrelevant' deformation of the dual CFT known as T-Tbar. We then (w/Gorbenko, Torroba) generalize this deformation to capture finite patches of bulk **dS** instead. cf Miyaji Takayanagi Sato, Nomura Rath

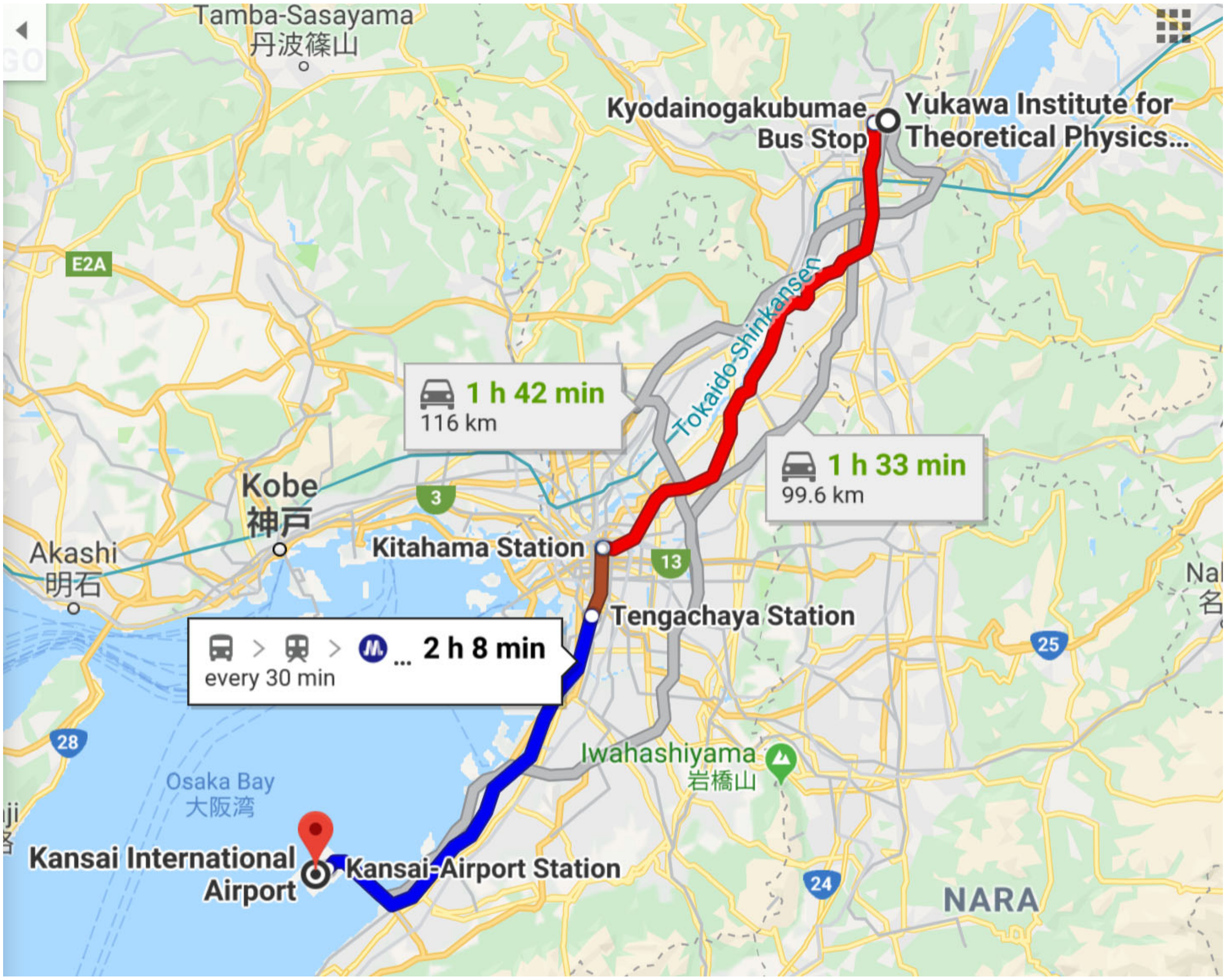
# Pictorial Summary:




(More general trajectories involving currents <-> patch of Kerr BH Guica, Apolo/Song...)

* Both universal, solvable deformations whose dressed energies match quasilocal Brown-York energy of a patch of (A)dS.

*Many interesting questions and directions for research, including QI connections.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Kyodainogakubumae Bus Stop
Yukawa Institute for Theoretical Physics...
E2A
Tokaido Shinkansen
1 h 42 min
116 km
1 h 33 min
99.6 km
Kobe
神戸
Akashi
明石
Kitahama Station
Tengachaya Station
2 h 8 min
every 30 min
Iwahashiyama
岩橋山
Osaka Bay
大阪湾
Kansai International Airport
Kansai-Airport Station
NARA


Baffin Bay
Arctic Ocean
Finland
Kara Sea
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Russia
Bering Sea
Sea of Okhotsk
Mongolia
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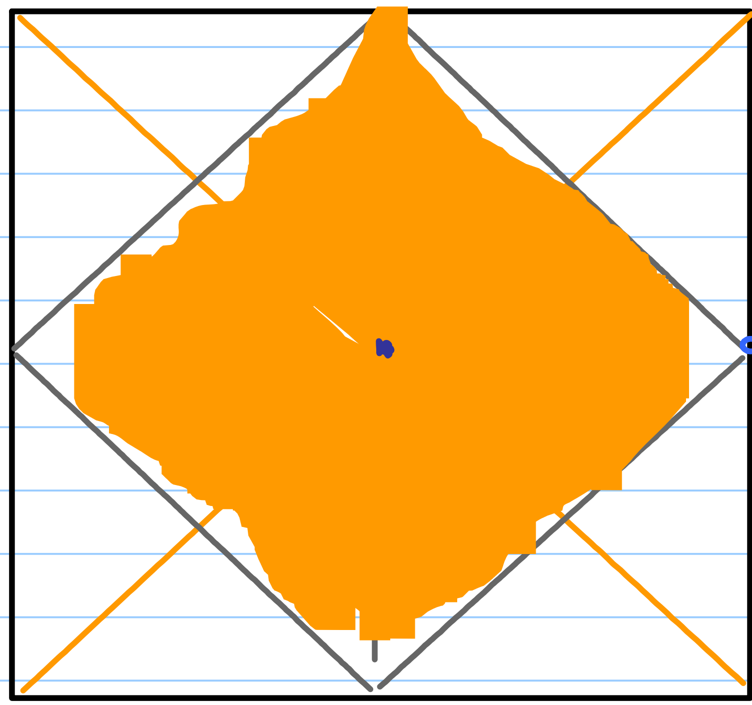
from ¥119,000
China
Sea of Japan
Kansai International Airport
Yukawa Institute for Theoretical Physics...
North Pacific Ocean
HI

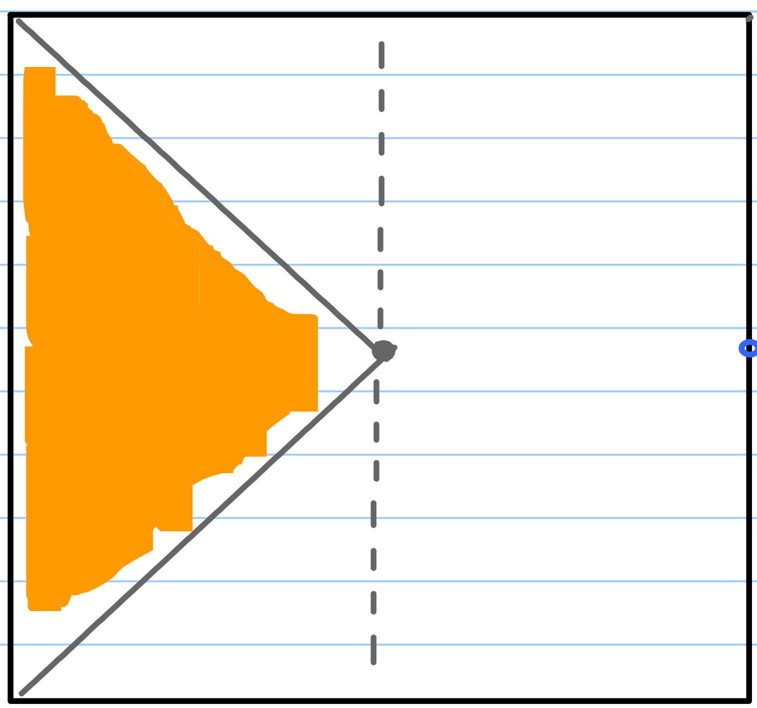
Note: fluctuations of the finite patch, including the ultimate non-perturbative decay of dS, are suppressed at large c but ultimately important.

The large-radius physics of the finite patch is entitled to a dual holographic description (similarly to bulk reconstruction in AdS/CFT, which is necessarily approximate). Residual gravitational fluctuations are those of low dimensional gravity, relatively tractable.

Two particular patches of interest in dS:



dS/dS



static patch

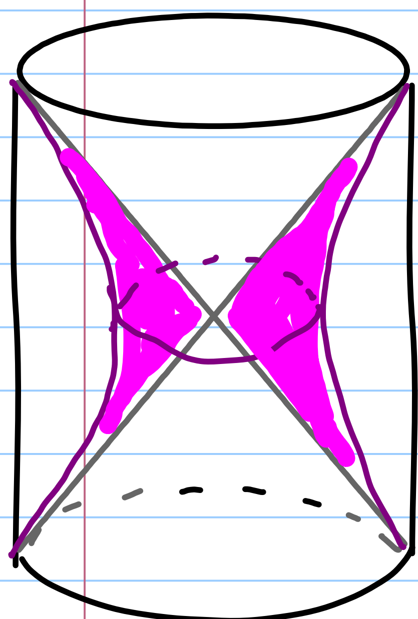
Pre-existing dS/dS conjecture for $\Lambda > 0$:

$dS_{d+1}$ = Two coupled cutoff d-dimensional CFTs, constrained by residual d-dimensional gravity (on approximate $dS_d$ geometry)

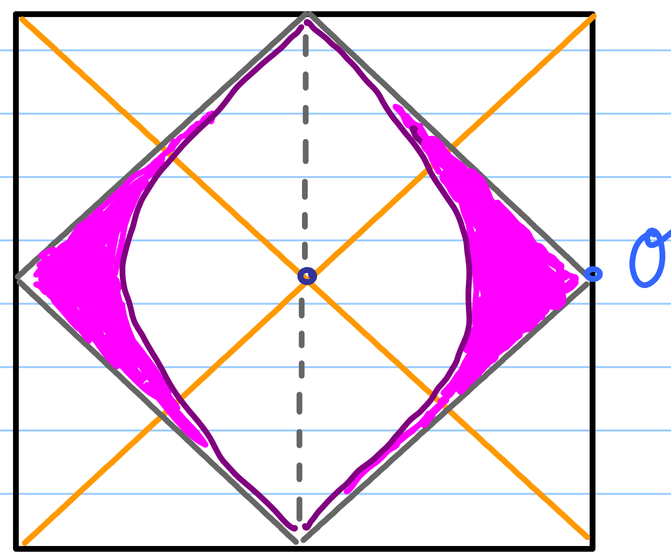
This follows from 2+1 independent arguments, macroscopic and microscopic. They agree <u>because</u> of the meta-stability (no hard cosmological constant in string theory).

# Macroscopic:

$$
\begin{aligned}
ds^2_{(A)dS_{d+1}} &= dw^2 + \sin(h)^2(\frac{w}{\ell_{dS}})ds^2_{dS_d} \\
&= dw^2 + \sin(h)^2(\frac{w}{\ell_{dS}})\left[-d\tau^2 + \ell_{dS}^2 \cosh^2 \frac{\tau}{\ell_{dS}} d\Omega^2_{d-1}\right] .
\end{aligned}
$$



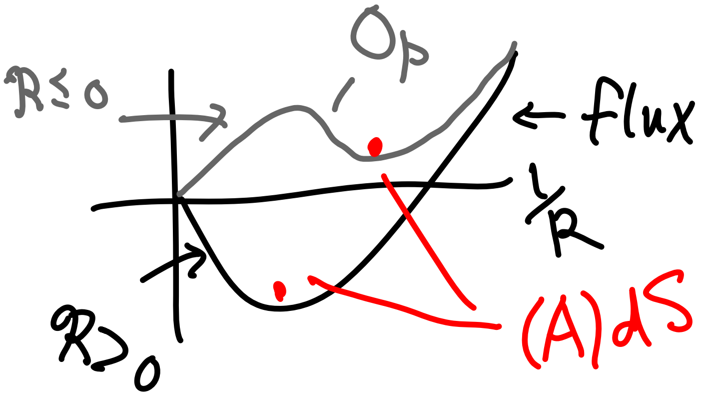
AdS/dS




dS/dS (each point is (d-1)-sphere)

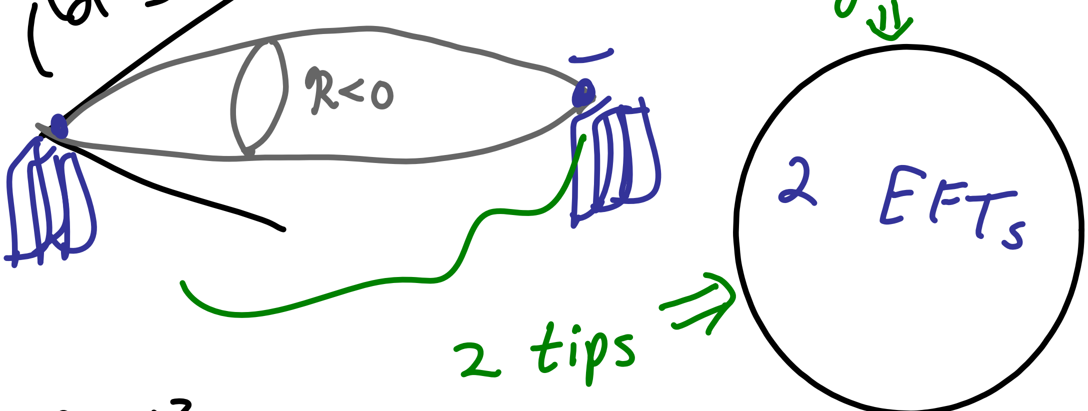
2 highly redshifted (IR) regions, each ~ IR region of AdS/dS

# Microscopic: Uplifting AdS/CFT




$$ds^2_{d+1} = \underbrace{\sin(h)^2 \frac{w}{\ell}\, ds^2_{dS_d}}_{\text{2 redshifted regions}} + dw^2$$

AdS) $\left(\frac{dR}{dr}\right)^2 = +\frac{1}{R^2}$

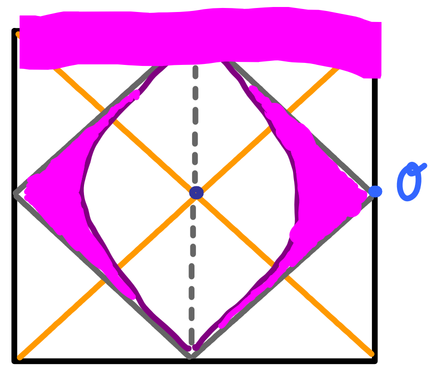



dS) $\left(\frac{dR}{dr}\right)^2 = \frac{-1}{R^2} + \frac{const}{R^{n>2}}$

dS vs AdS brane construction: independent derivation of the two sectors.

# The dS/CFT conjecture gives a third indication of this structure

Strominger; Anninos, Hartman...




$Z_{CFT} = \Psi(g_{\mu\nu})$

$$\langle \mathcal{O} \rangle \sim \int Dg^{(d)}_{\mu\nu} \Psi^\dagger(g^{(d)}_{\mu\nu}) \mathcal{O} \Psi(g^{(d)}_{\mu\nu})$$

Maldacena; Harlow/Stanford,...

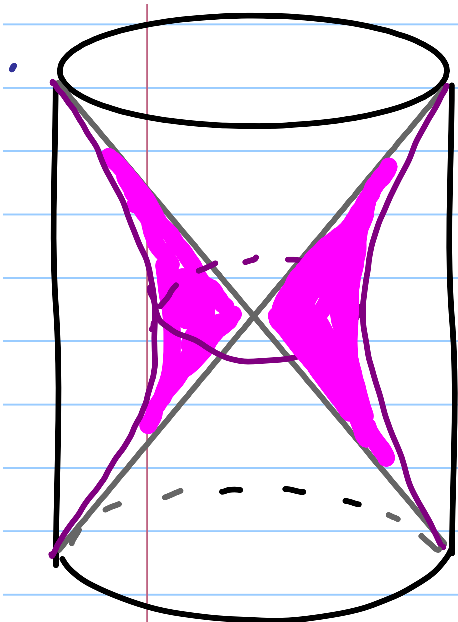
2 CFTs

Symmetries manifest, but bulk unitarity not;

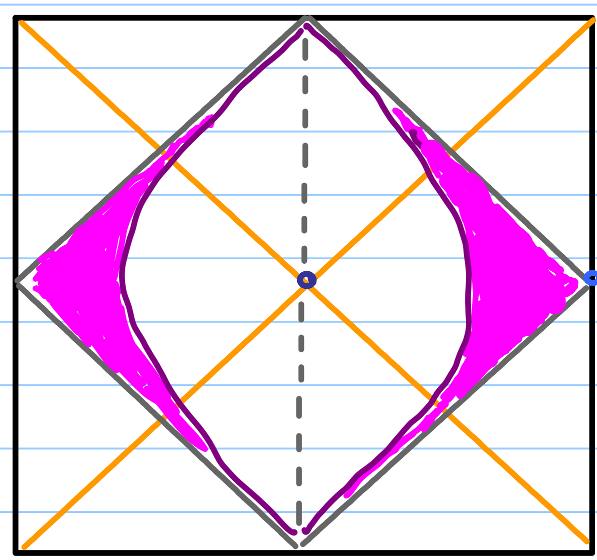
dS not obtained by analytic continuation of AdS in string theory (would yield complex fluxes). Decays infect future infinity.

# dS/dS and $T\bar{T}$

arXiv1811.07965
with Gorbenko
and Torroba



AdS/dS




dS/dS

The finite cutoff scale and the geometry of the (A)dS/dS throats => involves some sort of flow containing an *irrelevant deformation* of a CFT.

BONN-TH-2004-02

# Expectation value of composite field $T\bar{T}$ in two-dimensional quantum field theory

Alexander B. Zamolodchikov*

# On space of integrable quantum field theories



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**Abstract**

We study deformations of 2D Integrable Quantum Field Theories (IQFT) which preserve integrability (the existence of infinitely many local integrals of motion). The IQFT are understood as "effective field theories", with finite ultraviolet cutoff. We show that for any such IQFT there are infinitely many integrable deformations generated by scalar local fields $X_s$, which are in one-to-one correspondence with the local integrals of motion; moreover, the scalars $X_s$ are built from the components of the associated conserved currents in a universal way. The first of these scalars, $X_1$, coincides with the composite field $(T\bar{T})$ built from the components of the energy-momentum tensor. The deformations of quantum field theories generated by $X_1$ are "solvable" in a certain sense, even if the original theory is not integrable. In a massive IQFT the deformations $X_s$ are identified with the deformations of the corresponding factorizable S-matrix via the CDD factor. The situation is illustrated by explicit construction of the form factors of the operators $X_s$ in sine-Gordon theory. We also make some remarks on the problem of UV completeness of such integrable deformations.

This paper is an extended version of the talk given at the Simons Center, 2015-03-04, http://media.scgp.stonybrook.edu/presentations/20150304_Zamolodchikov.pdf

$$\frac{dS}{d\lambda} = 2\pi \int d^2x \sqrt{-g}\, T\bar{T}$$

Apply step by step: generates a universal tractable trajectory in the space of QFTs

Recompute stress energy tensor at each step.

$$8\langle n|T\bar{T}|n\rangle \;=\; \langle n|T^{\alpha\beta}|n\rangle\langle n|T_{\alpha\beta}|n\rangle - \langle n|T^{\alpha}_{\alpha}|n\rangle^2$$

Factorization for 2d theory on flat spacetime for any theory, and any spacetime for large number of degrees of freedom.

# One scale S(λ) ~>

cf McGough Mezei Verlinde, Kraus Liu Marolf

$$T^i_i = -4\pi\lambda T\bar{T}$$

Factorization => for homogeneous states, algebraic equation for pressure in terms of energy density. e.g. zero momentum:

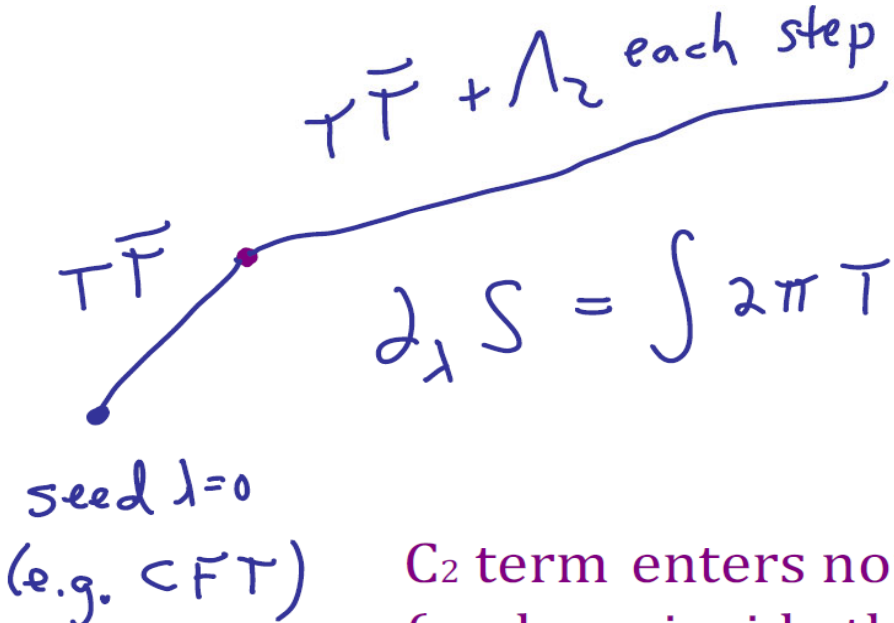
$$\begin{aligned}
\mathrm{Tr}\,T &= T^\tau_\tau + T^\phi_\phi = -4\pi\lambda T\bar{T} \Rightarrow \langle T^\phi_\phi\rangle = \frac{\langle T^\tau_\tau\rangle}{\pi\lambda\langle T^\tau_\tau\rangle - 1} \\
\langle T^\phi_\phi\rangle &= -dE/dL \\
\langle T^\tau_\tau\rangle &= -\frac{E}{L} = \frac{1}{\pi\lambda}\left(1 - \sqrt{1 + 2\pi\lambda\langle {T^\tau_\tau}^{(0)}\rangle}\right)
\end{aligned}$$

Result is exact nonlinear formula for`dressed' energies along trajectory.

Zamolodchikov, Smirnov, Cavaglia, Negro, Szecsenyi, Tateo, Cardy, Flauger, Dubovsky, Gorbenko, Mirbabayi, Hernandez-Chifflet, Aharony, Guica et al, Cotrell, Hashimoto, Giveon, Kutasov, Aharony, Datta, Giveon, Vaknin,...

(These same dressed levels arise if we couple the CFT to 2d Jackiw-Teitelboim', or Polyakov, gravity...)

We'll be led to introduce a second universal deformation, starting from a finite $\lambda$ point on the original T-Tbar trajectory:



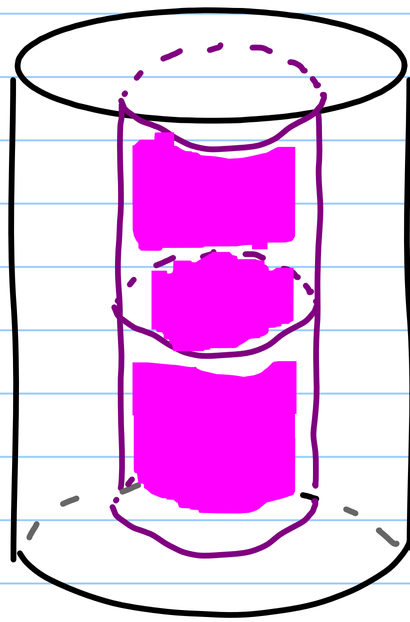

$$\partial_\lambda S = \int 2\pi T\bar{T} + \frac{c_2}{\lambda^2}$$

$C_2$ term enters nonlinearly (ends up inside the square root). Will also find a role for the opposite sign of the square root (far from seed).

This deformation is equally solvable, just not perturbatively connected to $\lambda=0$

This structure (and more) is mirrored precisely on the gravity side of AdS/CFT if we cut it off at a radial scale related to $\lambda$

McGough, Mezei, Verlinde; Kraus, Liu, Marolf; Donnelly, Shyam; Taylor; Hartman Kruthoff Shaghoulian, Tajdini(cf Guica).



plus excited states (particles and black holes).

$T_{ij}$ maps to the Brown York `quasilocal stress-energy tensor' on the gravity side.

This gravity-side calculation readily generalizes to bulk $dS_3$, and to a boundary that is $dS_2$. Meanwhile, on the QFT side the factorization holds at large number of

$$
\begin{aligned}
E &= \frac{r_c}{4G}\left(1-\sqrt{1-\frac{8GM}{r_c^2}+\frac{16G^2J^2}{r_c^4}}\right)\\
r_c &= \frac{L_{proper}}{2\pi\ell_{AdS}}, \quad \lambda = 8G\ell, \quad c=\frac{3\ell}{2G}\\
M\ell &= \Delta+\bar{\Delta}-\frac{c}{12}, \quad J\ell = \Delta-\bar{\Delta}\\
E &= \frac{L}{\pi\lambda}\left(1-\sqrt{1-\frac{4\pi^2\lambda(\Delta+\bar{\Delta}-c/12)}{L^2}+\frac{4\pi^4\lambda^2(\Delta-\bar{\Delta})^2}{L^4}}\right)
\end{aligned}
$$

BH energies go complex above cutoff scale. They are absent in the unitary theory obtained by truncating to real levels. To match GR side, we should have emergent locality down to the bulk string scale.

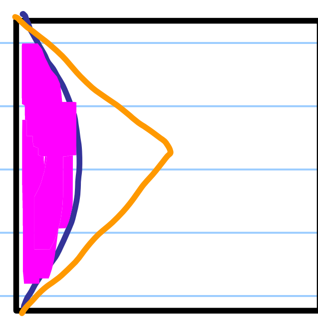
**Various aspects addressed in recent and ongoing work, including bulk matter beyond pure gravity, higher dimensions, classifying well-defined observables, ...** To binge-watch T-Tbar, see talks at Simons Center workshop April 2019...

Before even getting to BTZ black holes, there are particle states which classically introduce a deficit angle in $AdS_3$. These are

$$
\begin{aligned}
ds^2 &= -(r^2+\mu^2)dt^2 + \ell^2\frac{dr^2}{r^2+\mu^2} + r^2\ell^2 d\phi^2 \\
T^t_t &= \frac{1}{8\pi G\ell}\left(b_{CT} - \sqrt{1+\mu^2/r_c^2}\right) \qquad (AdS_3/cylinder_2)
\end{aligned}
$$

$$m = \frac{1}{4G}(1-\mu)$$

The bulk $dS_3$ generalization of these excited levels (e.g. within an observer patch)



$$
\begin{aligned}
ds^2 &= -(\mu^2-r^2)dt^2 + \ell^2\frac{dr^2}{\mu^2-r^2} + r^2\ell^2 d\phi^2 \\
T^t_t &= \frac{1}{8\pi G\ell}\left(b_{CT} - \sqrt{-1+\mu^2/r_c^2}\right) \qquad (dS_3/cylinder_2)
\end{aligned}
$$

Below, we will generalize T-Tbar in a simple way to generate the dS quasilocal stress energy as the dressed energy.

Repeat MMV et al calculation for (A)dS/dS. Solve for stress energy, matching AdS/dS and dS/dS near w=0.

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\,\sqrt{-g}\left(\mathcal{R}^{(3)} + \frac{2\eta}{\ell^2}\right) + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^2x\,\sqrt{-g}\left(K - \frac{b_{CT}}{\ell}\right)$$

Taking radial slices

$$ds_3^2 = dw^2 + g_{ij}\,dx^i dx^j\,,$$

the quasilocal stress-tensor is

$$T_{ij} = \frac{2}{\sqrt{-g}}\frac{\delta S_{\text{on-shell}}}{\delta g^{ij}} = \frac{1}{8\pi G}\left(K_{ij} - K g_{ij} + \frac{b_{CT}}{\ell} g_{ij}\right)\,.$$

Result: 2 new terms in trace flow equation

$$\int d^2x\sqrt{-g}T^i_i \;=\; \int d^2x\sqrt{-g}\left(-4\pi\lambda T\bar{T} - \underline{\frac{c\mathcal{R}^{(2)}}{24\pi} + \frac{1-\eta}{\pi\lambda}}\right)$$

**We next derive a generalization of the step by step trajectory that generates these terms on the QFT side.** First for dS/dS, then returning to dS/cylinder and the static patch.

**First let us consider its implications for the dressed stress-energy: (p=0)**

$$\langle T^{\phi}_{\phi}\rangle = \frac{d(L\,\langle T^{\tau}_{\tau}\rangle)}{dL} = \frac{\langle T^{\tau}_{\tau}\rangle + c\frac{\mathcal{R}^{(2)}}{24\pi} + \frac{\eta-1}{\pi\lambda}}{\pi\lambda\langle T^{\tau}_{\tau}\rangle - 1}\,,$$

$$\langle T^{\tau}_{\tau}\rangle = \frac{1}{\pi\lambda}\left(1 - \sqrt{\eta + c\frac{\mathcal{R}^{(2)}\lambda}{24} - \frac{C_1\lambda}{L^2}}\right)\,.$$

**Similarly on cylinder (static patch case), we get a new term in the differential equation from appropriate Einstein equations:**

$$4\,\frac{\partial}{\partial\lambda}\,E(r,\lambda) = E(r,\lambda)\,\frac{\partial}{\partial r}\,E(r,\lambda) + \frac{4(1-\eta)r}{\lambda^2}\,.$$

A simplification: $dS_3$ masses do not generate full-fledged BHs (just particles sourcing deficit angle Deser-Jackiw) However, Casimir energy can lead to horizons (Arkani-Hamed, Dubovsky et al).

$$ds^2_{3\mu} = dw^2 + \sin^2 w(-d\tau^2 + \ell^2 \cosh^2 \frac{\tau}{\ell} d\phi^2), \qquad \phi = \phi + 2\pi\mu$$

$$m = \frac{1}{4G}\frac{\Delta\phi}{2\pi} = \frac{1}{4G}(1-\mu)$$

For QFT on tall $dS_2$ with period $2\pi\mu$, the quasilocal stress-energy tensor satisfies

$$T^\tau_\tau = \frac{1}{\pi\lambda}\left(1 - \sqrt{\eta + \frac{c\lambda\mathcal{R}^{(2)}}{24}}\right) = \frac{1}{\pi\lambda}\left(1 - \sqrt{\eta + \frac{c\lambda\mu^2\pi^2}{3L^2}}\right)$$

Note that for $\eta$=-1, the neck size L is bounded as it should be for bulk $dS_3$.

Where MMV, Rangamani et al found superluminal modes (related to full-fledged BTZ BHs in AdS) our analogous modes do not have this property.

**Generalization of the step by step trajectory that generates these terms on the QFT side:**

$$\delta\mathcal{L} = \delta\lambda \left\{ 2\pi T\bar{T} - \frac{1-\eta}{2\pi\lambda^2} \right\}$$

(with the curvature contribution in the (A)dS/dS case coming from the trace anomaly: **see derivation below**.)

At each step, there is a coordinated flow including the original irrelevant deformation

2d cosmological term $\Lambda_2$ normally drops out of QFT dynamics, but along the trajectory it enters nonlinearly via its contribution to the stress-energy tensor T, recomputed each step.

Again can solve for sequence of dressed energy levels, result matches above GR side formulas.

## Detailed prescription:

$$d\Lambda_2 = \frac{(1-\eta)d\lambda}{2\pi\lambda^2} \;= \frac{d\lambda}{\pi\lambda^2}\,.$$

$$\frac{\partial}{\partial\lambda}\log Z = -2\pi\int d^2x\,\sqrt{g}\,\langle T\bar{T}\rangle + \frac{1-\eta}{2\pi\lambda^2}\int d^2x\,\sqrt{g} = -8\pi^2 r^2\,\langle T\bar{T}\rangle\, + \frac{2(1-\eta)r^2}{\lambda^2}\,.$$

$$\langle T^\theta_\theta\rangle = \langle T^\phi_\phi\rangle = \frac{1}{2}\,\langle \mathrm{Tr}\,T\rangle$$

considering symmetric, homogeneous states on (Euclidean) dS

$$r\frac{\partial}{\partial r}\,\log Z = -\int d^2x\,\sqrt{g}\,\langle \mathrm{Tr}T\rangle = -8\pi r^2\,\langle T^\theta_\theta\rangle\,.$$

**... =>** $$4\,\frac{\partial}{\partial\lambda}\,E(r,\lambda) = E(r,\lambda)\,\frac{\partial}{\partial r}\,E(r,\lambda) + \frac{4(1-\eta)r}{\lambda^2}\,.$$

The solution of this equation is the one we had above starting from the gravity-side trace flow equation:

$$\langle T^\tau_\tau\rangle = \frac{1}{\pi\lambda}\left(1-\sqrt{\eta + c\frac{\mathcal{R}^{(2)}\lambda}{24} - \frac{C_1\lambda}{L^2}}\right)\,.$$

# Starting from a seed CFT, holographically reconstruct a patch of de Sitter spacetime.

$$
\begin{aligned}
c &= \frac{3\ell}{2G}\\
\lambda &= 8G\ell\\
r &= \ell \sin\left(\frac{w_c}{\ell}\right)\\
L &= 2\pi\mu\ell \sin\left(\frac{w_c}{\ell}\right)\\
\Rightarrow\quad \Lambda_2 = -\frac{1}{\pi\lambda} &= -\frac{c}{12\pi}\frac{1}{r^2} \qquad (w_c = \ell\pi/2)
\end{aligned}
$$

$c\lambda/r^2 >> 1$

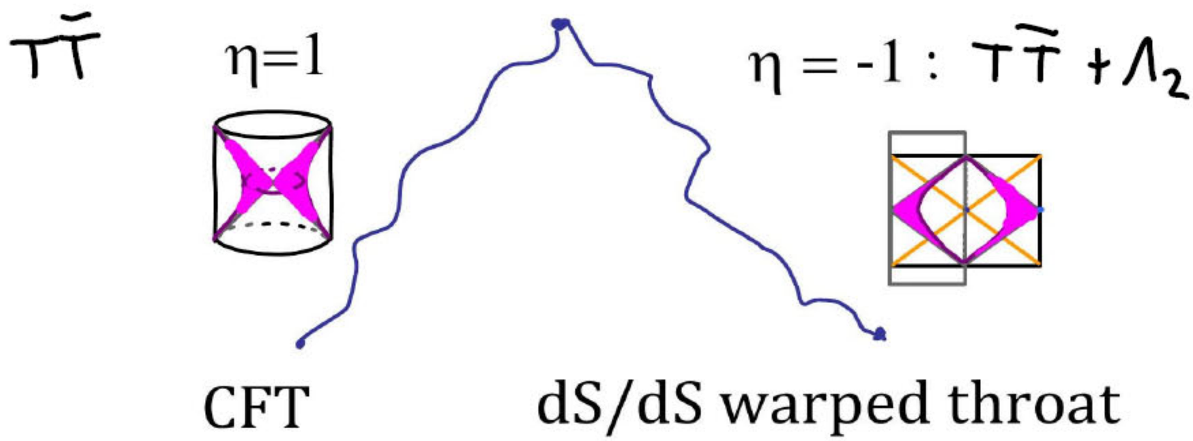



**Figure 1:** The reconstruction of the $dS/dS$ throat from a seed CFT proceeds via two joined trajectories as described in the text. The first trajectory (on the left) evolves the system from a pure CFT, via a sequence of cutoff AdS/dS systems, to the limit where this cutoff scale goes to zero, indicated by the point at the top of the figure. That point is the start of a new trajectory incorporating $\Lambda_2 \propto \eta - 1$, with increasing cutoff scale, culminating in the full $dS/dS$ warped throat.

We can go further, beyond the middle slice, by connecting to the opposite branch of the square root:

$T^r_r = \frac{1}{\pi\lambda}\left(1+\sqrt{\dots}\right)$

$\sqrt{(\dots)} = 0$

There was a concern that bulk matter is a problem or complication for the MMV holographic interpretation. KLM At the boundary:

$$T^i_i = -\frac{\ell}{16\pi G}\mathcal{R}^{(2)} - 4\pi G\ell\left(T^{ij}T_{ij} - (T^i_i)^2\right) - \frac{\eta - 1}{8\pi G\ell} + T_{matter}{}^w_w$$

1) If we want to impose a Dirichlet condition for matter, e.g. bulk scalar, we do get new term: Hartman Kruthoff Shaghoulian, Tajdini; D. Freedman et al (in progress)

$\Pi_{\phi,\,radial} \sim \mathcal{O} \rightarrow$

$$T^i_i = -4\pi\lambda T\bar{T} - \frac{1}{\pi\lambda}\left(\frac{c\lambda}{L^2}\right)^{\Delta}\tilde{\mathcal{O}}^2 - \frac{cR^{(2)}}{24\pi}$$

2) Interesting open question: what exactly is *pure* $\lambda$T-Tbar+c/$\lambda$ holographically? Unitarity ensured by truncation to real energy levels, but nonlocal.

3) Can also add c2/$\lambda$ to single-trace TTbar Giveon et al

# Beyond pure gravity, the matching the two parts of the trajectory for local bulk examples involves interpolation between the AdS and dS minima e.g. in the connected scalar sector of string theory.

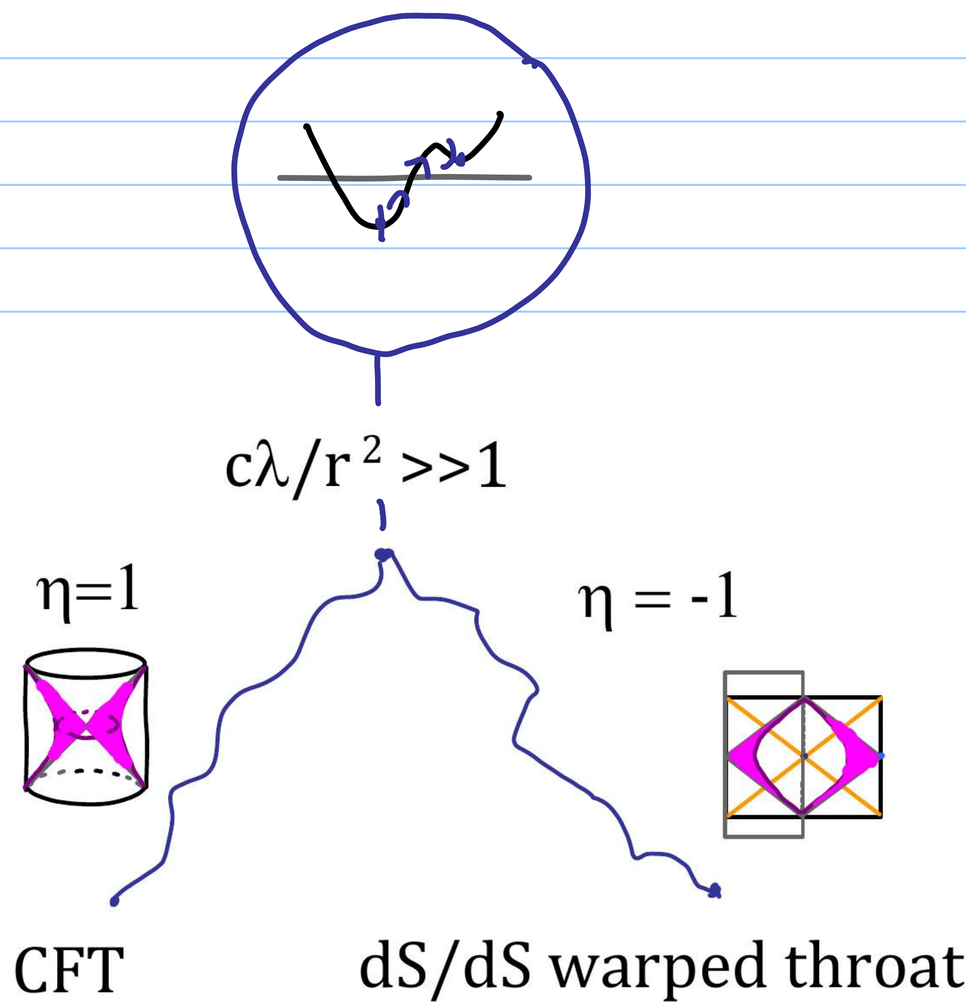
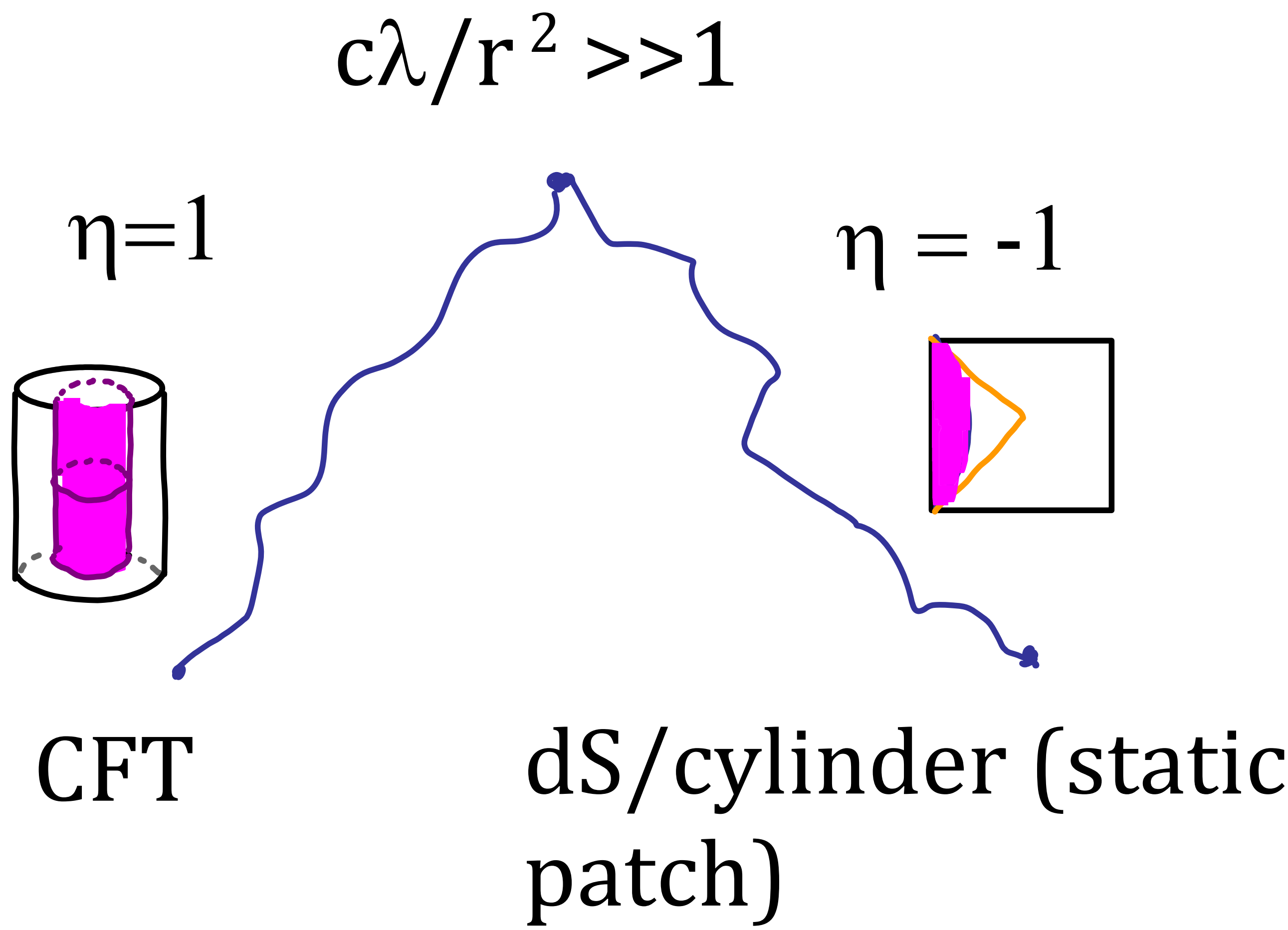



**Figure 1:** The reconstruction of the $dS/dS$ throat from a seed CFT proceeds via two joined trajectories as described in the text. The first trajectory (on the left) evolves the system from a pure CFT, via a sequence of cutoff AdS/dS systems, to the limit where this cutoff scale goes to zero, indicated by the point at the top of the figure. That point is the start of a new trajectory incorporating $\Lambda_2 \propto \eta - 1$, with increasing cutoff scale, culminating in the full $dS/dS$ warped throat.

# Static patch version




$$\delta\mathcal{L} = \delta\lambda \left\{ 2\pi T\bar{T} - \frac{1-\eta}{2\pi\lambda^2} \right\}$$

# Static patch version

$$
\begin{aligned}
ds^2 &= -(r^2+\mu^2)dt^2 + \ell^2 \frac{dr^2}{r^2+\mu^2} + r^2\ell^2 d\phi^2 \\
T^t_t &= \frac{1}{8\pi G\ell}\left(b_{CT} - \sqrt{1+\mu^2/r_c^2}\right) \qquad (AdS_3/cylinder_2)
\end{aligned}
$$

$$
\begin{aligned}
ds^2 &= -(\mu^2-r^2)dt^2 + \ell^2 \frac{dr^2}{\mu^2-r^2} + r^2\ell^2 d\phi^2 \\
T^t_t &= \frac{1}{8\pi G\ell}\left(b_{CT} - \sqrt{-1+\mu^2/r_c^2}\right) \qquad (dS_3/cylinder_2)
\end{aligned}
$$

$$
\delta\mathcal{L} = \delta\lambda\left\{2\pi T\bar{T} - \frac{1-\eta}{2\pi\lambda^2}\right\}
$$

**2d gravity formulations** **Dubovsky Flauger Gorbenko Mirbabayi Hernandez-Chifflet...:**

TTbar theory analogous to worldsheet string theory wrapped on a fixed torus target space. Dressed energy is the spacetime energy of the string states:

$$L_0 - a = 0 \Rightarrow p^0 = \sqrt{(\cdots)}$$

$$Z_{dressed} = \int \frac{\mathcal{D}g_{\alpha\beta}}{Diff \times Weyl} \mathcal{D}X^\mu e^{-\int \sqrt{g}\frac{1}{2\ell^2}(\partial_\alpha X^\mu)^2} Z_{CFT}(g_{\alpha\beta})$$

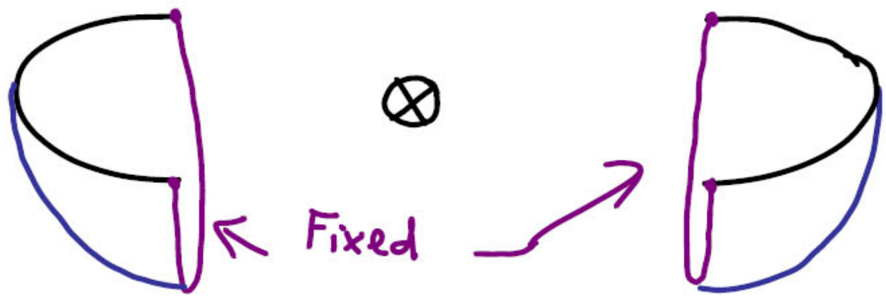
$$Z_\lambda = \int \frac{DeDX}{Vol(\text{diff})} e^{\frac{1}{2\lambda}\int d^2x \varepsilon^{\mu\nu}\varepsilon_{ab}(\partial X - e)^a_\mu (\partial X - e)^b_\nu} Z_0[e^a_\mu].$$

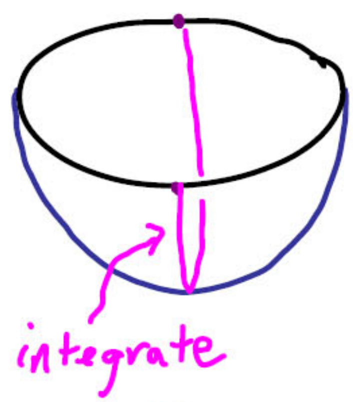
**(How) Do these generalize to curvature and $\Lambda_2$ ?** Different treatment of Weyl factor, e.g. for $\Lambda_2$ keep residual `worldsheet' c.c.

**cf Mazenc,Shyam,Soni...WIP on curvature.**

**Joining throats:** Given the duality for each throat, generalized to any fixed boundary metric and fields, we can construct the ground state of the joined system and compute the Renyi entropies.

$S_n = 0$

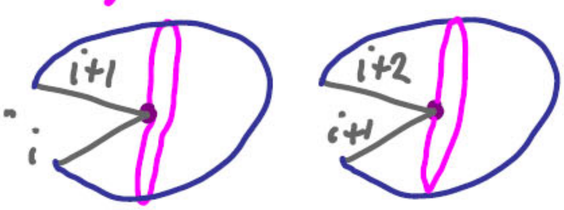



cf Dong; Dong-ES-Torroba; Soni et al, Akers-Rath/ Dong-Harlow-Marolf, Murdia Nomura Rath Salzetta, ...

$$S_n = \frac{\hat{A}}{4G_N}$$

$$\mathrm{Tr}\rho_1^{\,n} = \cdots$$

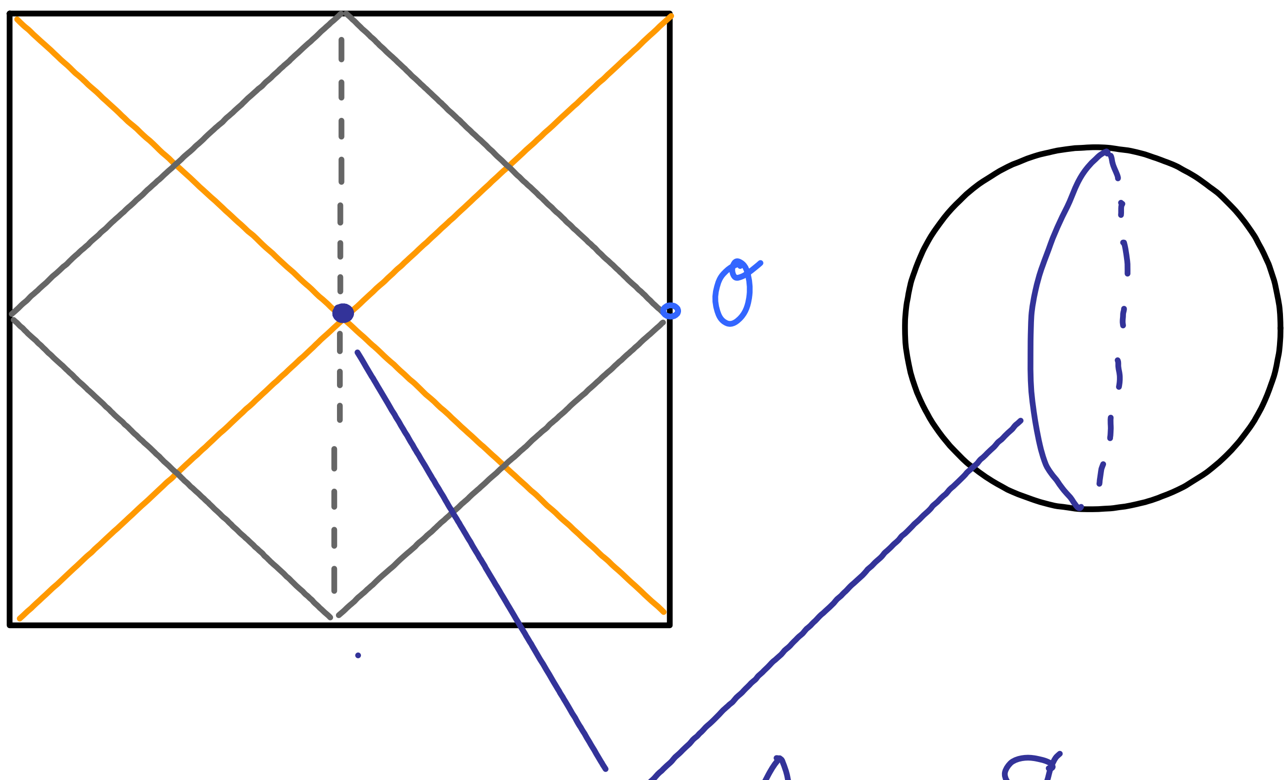



$+ \cdots$

In our case, the area of the shared locus has a saddle point independent of n, flat entanglement spectrum

Interactions -> highly mixed grnd state

# Interpretation of $S_{\text{Gibbons-Hawking}}$:




Ryu-Takayanagi Surface

$$\frac{A}{4G_N} = S_{RT}$$

$$\approx S_n = S_{vN} = \log \dim H = S^{(O)}_{G-H}$$

Observer O cannot interact with 2nd matter sector, traces over it.

Can generalize to more divisions, matter field profiles, etc. cf Geng Grieninger Karch

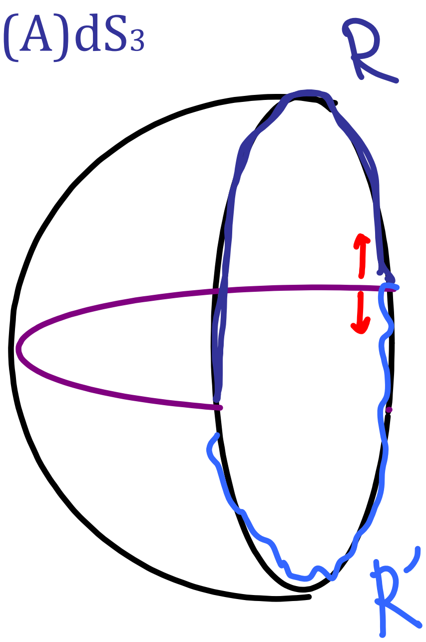
# Subregion dualities and redundant encoding <u>work in progress</u>

w/Aitor Lewcowycz, Junyu Liu, Gonzalo Torroba...

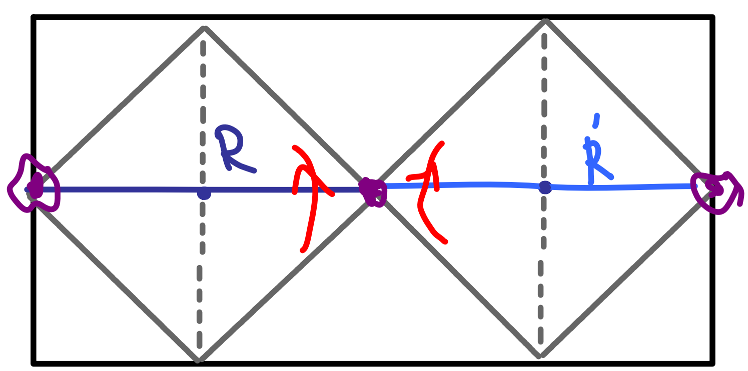
## Finite patches of (A)dS: Causal Wedge can exceed Entanglement Wedge. cf Headrick Hubeny Lawrence Rangamani Wall...

Start from half sphere case: here CW=EW and VN+Renyi entropies have been calculated on both sides. (Donelly, Shyam, Caputa, Datta, GST)

spatial slice of bulk (A)$dS_3$

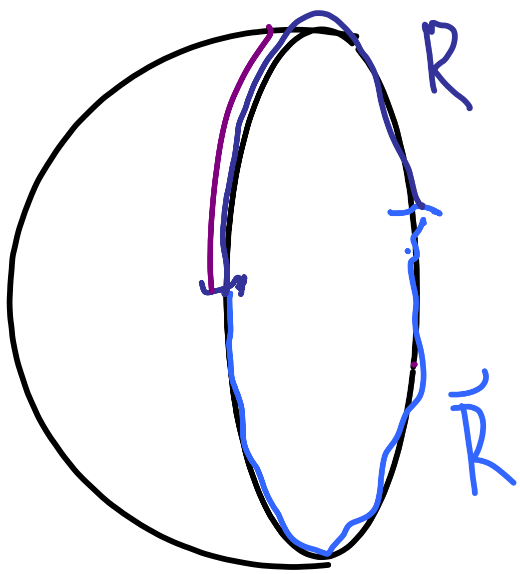



Boundary $dS_2$




Modular Hamiltonian $K_R$ is local in this case (just $T_{tt}$) and modular evolution takes Ops(R)->Ops(D(R)).

Now reduce the region R: The extremal surface moves to the boundary for a full dS/dS throat; similar inequalities in other cut off (A)dS patches. But CW(R) extends further into bulk.




Now CW(R)>EW(R)

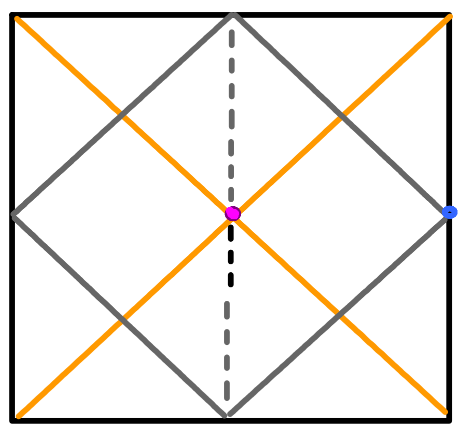
Also, EW(R') overlaps with CW(R) so the corresponding bulk operators cannot all commute.

This is not a contradiction because the TTbar theory is nonlocal (<=>2d gravity), Hilbert space will not factorize.

cf Mazenc/Soni,...Pastawski/Preskill

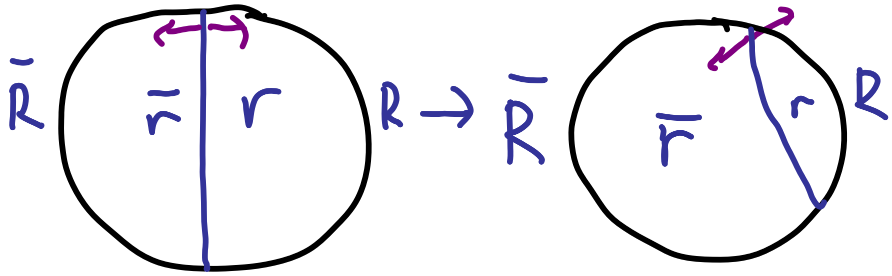
Now modular Hamiltonian flow should not generate all of Ops(D(R)) from Ops(R). Conversely it should generate more than Ops(D(R')) from Ops(R').

In fact, we already saw this above: the flat entanglement spectrum in dS/dS for $\rho_1$ means the corresponding modular Hamiltonian $K_1$ is a c-number (so no evolution). The EW is trivial.



Expect to see the novel modular flow in more general cases by perturbing S and K from half space, following earlier works e.g. JLMS, Balakrishnan Dong Harlow Dutta Faulkner Leigh Parrikar Lewkowycz Wall Wang...

May be amenable to 2d path integral analysis, e.g. if 2d gravity formulations of TTbar+$\Lambda_2$ generalizes



In all these cases, the CW extends into the bulk and admits an HKLL analysis.  This leads to similar picture of redundant encoding of bulk points as in Almheiri-Dong-Harlow

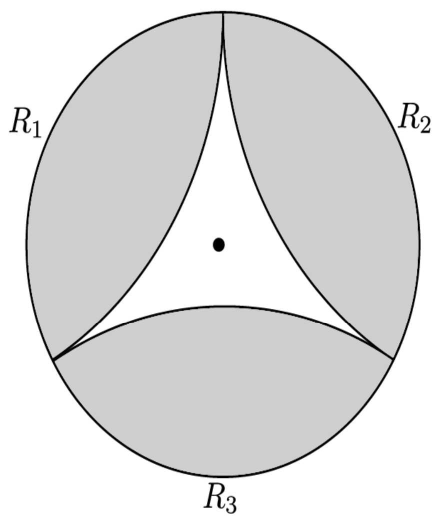



**Figure 11:** Redundant encoding of a bulk field $\phi(x)$, which can be represented on any two of the boundary regions.

Note that the λT-Tbar+C/λ  theory is not a lattice model:  the tensor network toy models do not directly apply.  It is a smooth UV softening (analogous to string theory).

# Many other ongoing/Future directions, e.g.:

Generalize 2d gravity path integral formulations of T-Tbar and the single-trace version Giveon et al to include the $\Lambda_2$ deformation. 2d side solvable, must have gravity-side realization. Analogy: double-trace deformations were novel on GR side since nonlocal on the internal sphere, but standard on QFT side.

In the single-trace version, the sign of $\lambda$ corresponding to the cutoff spacetime introduces singularities. Resolved in string theory, e.g. via enhancon mechanism? wip w/Anderson, Coleman, Mousatov