Quantum Gravity and finite cut-off AdS

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Based on:

"Sphere partition functions and cut-off AdS" JHEP 1905 (2019) 112

with Shouvik Datta (UCLA) and Vasudev Shyam (PI)

also earlier work:

"Airy function and 4d quantum gravity" JHEP 1806 (2018) 106

with Shinji Hirano (WITS)



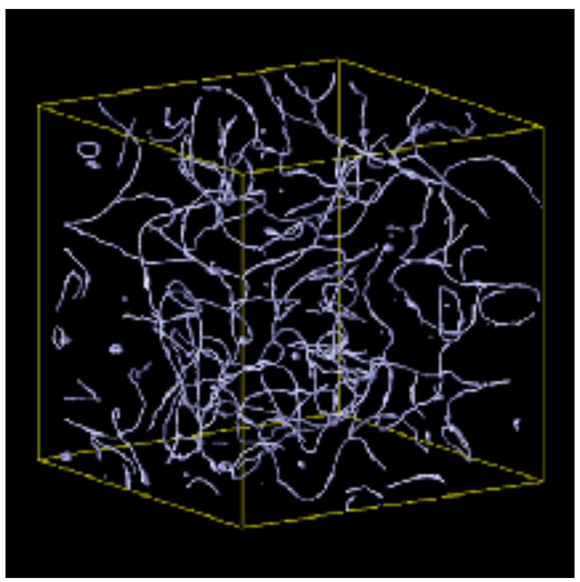
Part of the TT story at this workshop: Shyam, Datta, Rolf, Soni, Silverstein, Verlinde, Song, Nomura, Apolo,… (I will try not to overlap too much)

Plan:

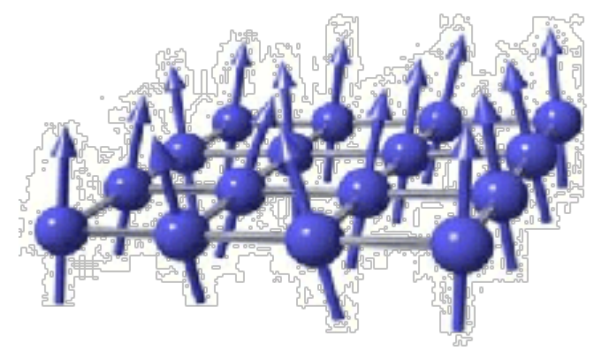
- Summary
- TT flow and sphere partition functions
- QGR: Wheeler-DeWitt equation
- It from Qubit physics and TT
- Open Questions

Take-home (reminder):

AdS/CFT: Tool to understand hol. CFT (QFTs) using Quantum Gravity



QGR in AdS



Holographic CFT

(Quantum) Gravity is the most efficient way of doing (hol.) QFTs!

Summary:

"Radial Hamiltonian" constraint is a powerful tool in AdS/CFT (any dim.)!

Classically: It can be interpreted as a (TT) flow equation in holographic CFTs that reproduces non-trivial features of finite cut-off AdS/CFT.

This talk: Sphere partition functions and RT formula at finite cut-off.

Quantum: Wheeler-DeWitt equation in minisuperspace becomes a differential equation that captures information about finite-N partition functions.

This talk: ABJM sphere partition function (Airy) that contains all orders contributions from perturbative quantum gravity.

“It from GR” (Gauss-Codazzi or quantum WDW)

In the (radial) ADM decomposition $ds^2 = dr^2 + \gamma_{ij}(r,x)dx^i dx^j$

Einstein equations can be written as ([Skenderis,Papadimitriou’04])

$$
\begin{aligned}
K^2 - K_{ij}K^{ij} &= \tilde{R} + 2\kappa^2 T_{d+1d+1}, \\
\nabla_i K^i_j - \nabla_j K &= \kappa^2 T_{jd+1}, \\
\dot{K}^i_j + K\,K^i_j &= \tilde{R}^i_j - \kappa^2\left(T^i_j - \frac{1}{d-1}T^\sigma_\sigma \delta^i_j\right)
\end{aligned}
$$

Just pure gravity (cc) the “radial Hamiltonian” constraint

$$
\boxed{K^2 - K_{ij}K^{ij} = \tilde{R} + \frac{d(d-1)}{l^2}}
$$

With canonical momenta $\pi_{ij} = \frac{\sqrt{\gamma}}{\kappa^2}\left(K\gamma_{ij} - K_{ij}\right), \qquad \pi^i_i = \frac{\sqrt{\gamma}}{\kappa^2}(d-1)K,$

$$
\frac{\kappa^2}{\sqrt{\gamma}}\left[\pi^{ij}\pi_{ij} - \frac{1}{d-1}(\pi^i_i)^2\right] + \frac{\sqrt{\gamma}}{\kappa^2}\left[\tilde{R} + \frac{d(d-1)}{l^2}\right] = 0
$$

(QGR: WDW eq. see later)

AdS/CFT flow equation from Gauss-Codazzi

[McGough'18] [Kraus,Liu,Marolf'18]

[Taylor'18] [Hartman et al.'18]

Pure gravity

$$K^2 - K_{ij}K^{ij} = \tilde{R} + \frac{d(d-1)}{l^2}$$

Given a general holographic energy-momentum tensor [Balasubramanian,Kraus'99]

$$\langle T_{ij} \rangle = -\frac{1}{\kappa^2}\left[K_{ij} - K\gamma_{ij} - \frac{d-1}{l}\gamma_{ij}\right] - a_d C_{ij} \qquad a_d = \frac{l}{(d-2)\kappa^2}$$

GC can be rewritten as a "flow equation" $\hat{T}_{ij} = T_{ij} + a_d C_{ij},$

$$\boxed{\hat{T}^i_i = -\frac{l\kappa^2}{2}\left[\hat{T}_{ij}\hat{T}^{ij} - \frac{1}{d-1}\left(\hat{T}^i_i\right)^2\right] - \frac{l}{2\kappa^2}\tilde{R}.}$$

[Taylor'18]

And translated/interpreted as a large N QFT flow ($\gamma_{ij} \to r_c^2 \gamma^b_{ij}$ etc.)

$$\langle T^i_i \rangle = -d\lambda \langle X_d \rangle$$

Large N Flow equation

[Hartman,Kruthoff,Shaghoulian,Tajdini'18]
[PC,Datta,Shyam'19]

In terms of the "renormalized" EM tensor $\hat{T}_{ij} = T_{ij} + a_d C_{ij}$,

$$
\begin{aligned}
T_i^i &= -d\lambda \Bigg[T_{ij}T^{ij} - \frac{1}{d-1}(T_i^i)^2 + \frac{2\alpha_d}{\lambda^{\frac{d-2}{d}}}\left(T_{ij}C^{ij} - \frac{1}{d-1}T_i^i C_i^i \right) \\
&+ \left(\frac{\alpha_d}{\lambda^{\frac{d-2}{d}}} \right)^2 \left(C_{ij}C^{ij} - \frac{1}{d-1}(C_i^i)^2 \right) + \frac{1}{d\lambda}\frac{\alpha_d}{\lambda^{\frac{d-2}{d}}}\left(\frac{(d-2)}{2}\tilde{R} + C_i^i \right) \Bigg]
\end{aligned}
$$

with "dictionary" for λ, α_d

$$
\lambda = \frac{l\kappa^2}{2dr_c^d}, \qquad \frac{\alpha_d}{\lambda^{\frac{d-2}{d}}} \equiv \frac{l r_c^{d-2}}{(d-2)\kappa^2}
$$

Matching with anomalies and known holographic setups we have e.g.

$$
\alpha_3 = \frac{N_{\mathrm{ABJM}}}{6\,2^{1/3}\pi^{2/3}}, \qquad \alpha_4 = \frac{N_{\mathrm{SYM}}}{2^{7/2}\pi}, \qquad \alpha_6 = \frac{N_{(2,0)}}{24\pi}
$$

IDEA: We can define (effective) "TT-deformed" hol. CFTs dual to AdS with finite cut-off by the above flow equation.

Holographic sphere partition functions in cut-off AdS

One of the oldest and very interesting holographic probes

Precision tests of AdS/CFT (localization)

Even-d: sensitive to a-type holographic anomalies

“Good” holographic measure of degrees of freedom

Closely related to entanglement entropy

See also applications in dS/dS [Gorbenko,Silverstein,Torroba’18]

Holographic sphere partition functions and EM tensor

Gravity Action

$$I^{(d+1)}_{on-shell} = -\frac{1}{2\kappa^2}\int_{\mathcal{M}} d^{d+1}x\sqrt{g}\,(R-2\Lambda) + \frac{1}{\kappa^2}\int_{\partial\mathcal{M}} d^d x\sqrt{\gamma}K + S_{ct},$$

Holographic counter-terms (up to d=6) [Emparan,Johnson,Myers'99]

$$S_{ct} = \frac{1}{\kappa^2}\int_{\partial\mathcal{M}} d^d x\sqrt{\gamma}\left[\frac{d-1}{l}c_d^{(1)} + \frac{c_d^{(2)}\,l}{2(d-2)}\tilde{R} + \frac{c_d^{(3)} l^3}{2(d-4)(d-2)^2}\left(\tilde{R}^{ij}\tilde{R}_{ij} - \frac{d}{4(d-1)}\tilde{R}^2\right)\right]$$

On-shell solution

$$ds^2 = \frac{l^2\,dr^2}{l^2+r^2} + r^2 d\Omega_d^2 \;\equiv \frac{l^2\,dr^2}{l^2+r^2} + \gamma_{ij}(r,x)dx^i dx^j$$

Holographic Partition functions

$$\log Z_{\mathbb{S}^d}[r] = -I^{(d+1)}_{on-shell}[r]$$

E-M tensor (Brown-York)

$$T^d_{ij}[r] \equiv -\frac{2}{\sqrt{\gamma}}\frac{\delta I^{(d+1)}_{on-shell}[r]}{\delta\gamma^{ij}}$$

Sphere partition function in cut-off AdS

[PC,Datta,Shyam'19]

The large N sphere partition function in finite cut-off holography

$$\log Z_{\mathbb{S}^d}[r_c] = -\frac{dS_d r_c^d}{\kappa^2 l}\Bigg[-\frac{l}{r_c}\ {}_2F_1\left(-\frac{1}{2},\frac{d-1}{2},\frac{d+1}{2},-\frac{r_c^2}{l^2}\right) + c_d^{(1)}\frac{(d-1)}{d} + \frac{c_d^{(2)}(d-1)}{2(d-2)}\frac{l^2}{r_c^2} - \frac{c_d^{(3)}(d-1)}{8(d-4)}\frac{l^4}{r_c^4}\Bigg]$$

Holographic energy-momentum tensor

$$T_{ij}^d[r_c] = \frac{(d-1)}{\kappa^2 l}\left[c_d^{(1)} + \frac{c_d^{(2)} l^2}{2r_c^2} - \frac{c_d^{(3)} l^4}{8r_c^4} - \sqrt{1+\frac{l^2}{r_c^2}}\right]\gamma_{ij} \;\equiv \omega[r_c]\gamma_{ij}$$

They satisfy

$$r_c\partial_{r_c}\log Z_{\mathbb{S}^d}[r_c] = -\int d^d x\sqrt{\gamma}\langle T_i^i\rangle = -r_c^d S_d\, d\,\omega[r_c]$$

Holographic Field Theory

[PC,Datta,Shyam'19]

For the spheres the symmetry fixes $\langle T_{ij}\rangle = \omega_d \gamma_{ij}$

This way the flow equation becomes quadratic equation for ω_d

$$d\,\omega_d \;=\; d\lambda\left[\frac{d}{d-1}\omega_d^2 + \frac{2\alpha_d}{\lambda^{\frac{d-2}{d}}}\frac{1}{d-1}\,C^i_i\omega_d - \frac{1}{d\lambda}\frac{\alpha_d}{\lambda^{\frac{d-2}{d}}}f_d(R)\right]$$

The two solutions are (- to match anomalies)

$$\omega_d^{(\pm)} = \frac{d-1}{2d\lambda}\left(1 - \frac{2\alpha_d\lambda^{\frac{2}{d}}}{d-1}C^i_i \pm \sqrt{\left(1 - \frac{2\alpha_d\lambda^{\frac{2}{d}}}{d-1}C^i_i\right)^2 + \frac{4\alpha_d\lambda^{\frac{2}{d}}}{d-1}f_d(R)}\right)$$

with explicit form of C_{ij} and holographic dictionary we reproduce the gravity bulk computation from the TT flow equation!

Wheeler-DeWitt equation in mini-superspace

[PC,S.Hirano'18]

Quantum Gravity Path-Integral in the "minisuperspace" approximation

$$ds^2 = \mathbf{N}^2(r)dr^2 + a^2(r)d\Omega_d^2,$$

Quantum Gravity action (q=a^2 for a canonical kinetic term)

$$S_{EH} + S_{GH} = -\frac{d(d-1)S_d}{2\kappa^2}\int dr\left[\frac{q'^2}{4\mathbf{N}} + \mathbf{N}\left(q^{d-3} + l^{-2}q^{d-2}\right)\right]$$

Hamiltonian

$$H = \mathbf{N}\hat{H} = -\frac{2\kappa^2}{S_d d(d-1)}\mathbf{N}\left[p^2 - \left(\frac{d(d-1)S_d}{2\kappa^2 l}\right)^2\left(l^2 q^{d-3} + q^{d-2}\right)\right]$$

Quantum Wheeler-DeWitt equation

$$\boxed{\hat{H}\Psi[q] = \left[\hbar^2\frac{d^2}{dq^2} - \left(\frac{d(d-1)S_d}{2\kappa^2 l}\right)^2\left(l^2 q^{d-3} + q^{d-2}\right)\right]\Psi[q] = 0}$$

Wheeler-DeWitt equation: (HJ) WKB solution

[deBoer,Verlinde,Verlinde'99]

[PC,S.Datta,V.Shyam'19]

Quantum Gravity "radial Hamiltonian" yields the WDW equation (q=a^2)

$$\hat{H}\Psi[q] = \left[\hbar^2 \frac{d^2}{dq^2} - \left(\frac{d(d-1)S_d}{2\kappa^2 l}\right)^2 \left(l^2 q^{d-3} + q^{d-2}\right)\right]\Psi[q] = 0$$

Using the WKB expansion we have the leading order solution (HJ)

$$\Psi_{\mathrm{WKB}}(q) \approx \exp\left[\pm\left(\frac{d(d-1)S_d}{2\kappa^2 l\hbar}\right)\int_0^q \sqrt{l^2 q^{d-3} + q^{d-2}}\, dq\right] \qquad q = r_c^2$$

Which is precisely the "bare" gravity on-shell action with finite cut-off

$$\Psi_{\mathrm{WKB}}[r_c] = e^{-\left(I_{GR}^{\mathrm{on-shell}}[r_c] - S_{ct}[r_c]\right)}$$

Extra canonical transformation to include the counter-terms needed

[Freidel'08]

ABJM and the Airy function

[Fuji,Hirano,Moriyama’11]
[Marino,Putrov’12]

[Hatsuda,Moriyama,Okuyama’12]

$$Z_{ABJM}(S^3) \sim \mathrm{Ai}\left[\left(\frac{\pi N^2}{\sqrt{2\lambda}}\right)^{2/3}\left(1-\frac{1}{24\lambda}-\frac{\lambda}{3N^2}\right)\right]$$ + ~instant.

All orders pert. QGR.!

Non-perturbative [Bergman,Hirano’09]

Classical GR [Klebanov,Tseytlin’96]

$$\mathrm{Ai}\,[x] \sim \frac{1}{x^{1/4}} e^{-\frac{2}{3}x^{3/2}}\left(1-\frac{5}{48x^{3/2}}+\frac{385}{4608x^3}+\ldots\right)$$

1 loop QGR

[Bhattacharyya,Grassi,Marino,Sen’12]

2 loop QGR

3 loop QGR

$x^{-3/2} \sim N^{-3/2} \sim G_N$

Localization in SUGRA [Dabholkar,Drukker,Gomes’14]

WDW and Airy

[PC,S.Hirano'18]

WdW equation in AdS in 4 dim. => Airy equation!

$$\left[\frac{d^2}{dq^2} - \frac{9\pi^2}{16G_N^2\ell^2}\left(q+\ell^2\right)\right]\Psi(q) = 0$$

General (quantum) solution

$$\Psi(q) = C_1 \mathrm{Ai}\left[\left(\frac{3\pi\ell^2}{4G_N}\right)^{\frac{2}{3}}\left(\ell^{-2}q+1\right)\right] + C_2 \mathrm{Bi}\left[\left(\frac{3\pi\ell^2}{4G_N}\right)^{\frac{2}{3}}\left(\ell^{-2}q+1\right)\right]$$

Holographic dictionary (ABJM) and the decaying part for larger N (Ai, C2=0)

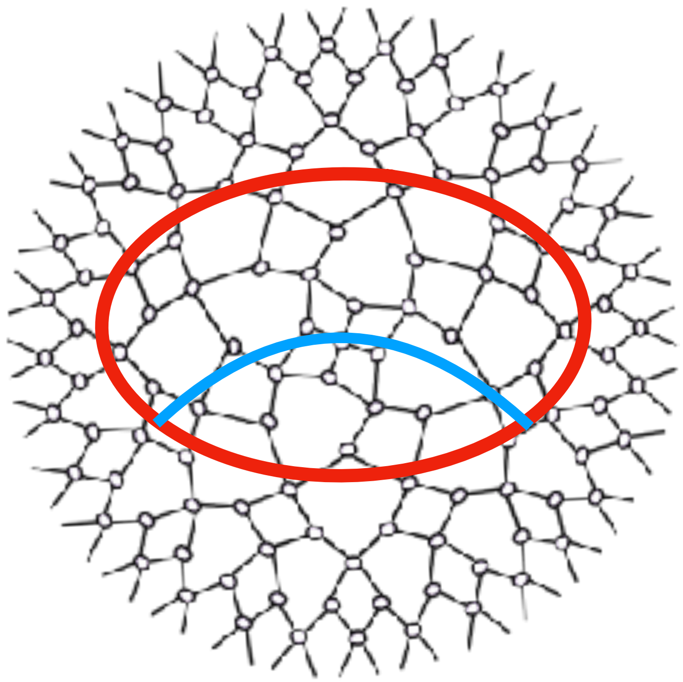
$$\frac{3S_3 l^2}{\kappa^2} = \frac{\pi N^2}{\sqrt{2\lambda}}$$

We find the relation (Pure Gravity!)

$$Z_{ABJM}(\mathbb{S}^3) \sim \mathrm{Ai}(x) = \Psi_{WDW}(q \to 0)$$! ?

Works for 1/2 BPS Wilson Loops!

Is bulk really a Tensor Network?



?

Surface/state-correspondence?

[Miyaji&Takayanagi'15]

Is TT-bar one realization of this correspondence?

Finite cut-of RT from Sphere Partition Functions

[Donnelly,Shyam'18]
[PC,Datta,Shyam'19]
[Banerjee,Bhattacharyya,Chakraborty'19]

Given the partition function for a CFT on a replicated geometry

$$d\Omega_d^2 = n^2 \sin^2\theta d\tau^2 + d\theta^2 + \cos^2\theta d\Omega_{d-2}^2$$

Trick:

$$S_1 = (1 - n\partial_n)\log Z_n[r]|_{n\to 1} = \left(1 - \frac{r}{d}\partial_r\right)\log Z_{\mathbb{S}^d}[r]$$

So using "bare" partition functions at scale r

RT no c.t.!

[Murdia,Nomura,Rath,Salzeta'19]

$$S_1 = \frac{S_d}{8\pi G_N} r^{d-1} {}_2F_1\left(\frac{1}{2}, \frac{d-1}{2}, \frac{d+1}{2}, -\frac{r^2}{l^2}\right)$$

This matches RT for (finite cut-off) surface with induced metric

$$ds^2 = l^2\left(d\rho^2 + \sinh^2\rho d\Omega_{d-2}^2\right)$$

$$S_A = \frac{\mathcal{A}}{4G_N} = \frac{l^{d-1}S_{d-2}}{4G_N}\int_0^\rho d\rho \sinh^{d-2}\rho$$

Is it related to EE in TT deformed theory? Meaning on EE in TT?

Some open questions:

Better understanding of the flow equation from QFT side?

Meaning of entanglement in TT deformed CFTs?

1/N and quantum gravity? Lessons from WDW?

Deforming ABJM or N=4 SYM ?

What are the “bulk” (universal GR) constraints on Holographic TN?

Thank You!

Stay Tuned!