Universiteit van Amsterdam

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Holographic Complexity for Systems with Defects

Shira Chapman - University of Amsterdam

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Dongsheng Ge

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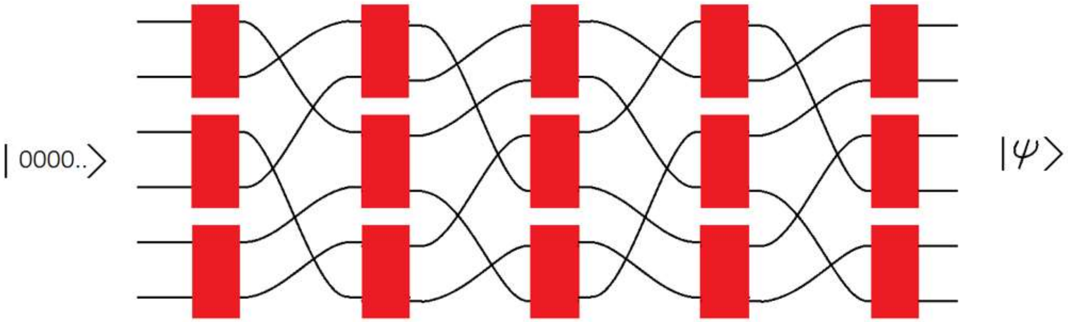
Giuseppe Policastro

Quantum Computational Complexity

Complexity of a quantum state is defined as the minimal number of elementary unitary operations applied to a simple (unentangled) reference state in order to obtain the state of interest.

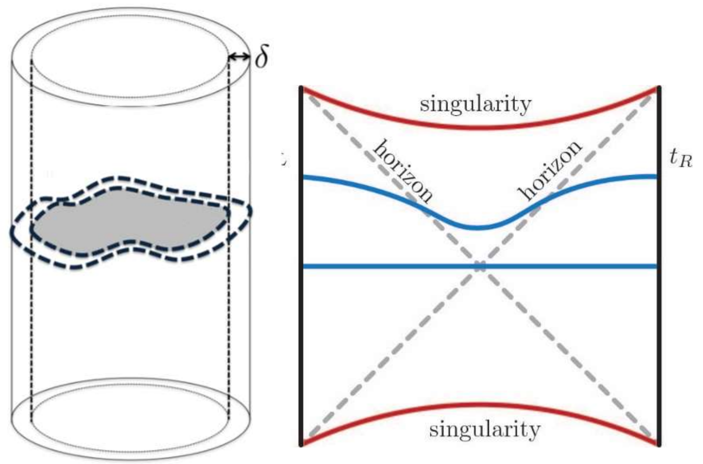


Circuit complexity of $|\psi\rangle$ is the minimum number of gates needed to go from $|0000..\rangle$ to $|\psi\rangle$.

Complexity in Holography - Two Proposals

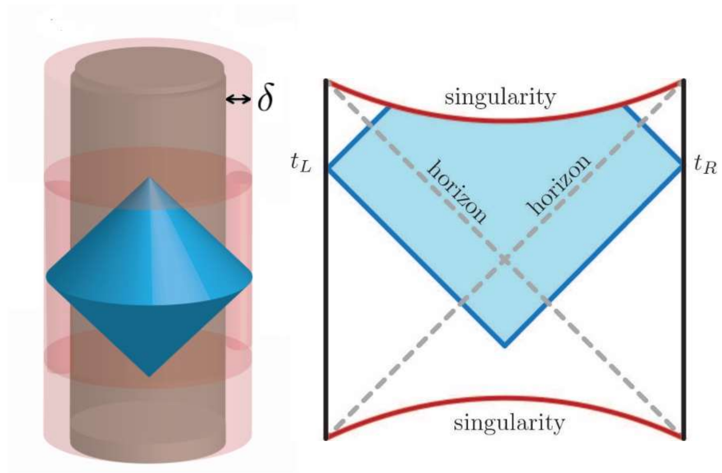
Complexity=Volume of a maximal spacelike slice anchored at the boundary time slice at which the state is defined (Stanford & Susskind)

$$C_V = \max \frac{\mathcal{V}}{G_N \ell}$$



Complexity=Gravitational action of the WDW patch - union of all such spacelike slices (Brown, Roberts, Swingle, Susskind & Zhao)

$$C_A = \frac{I_{WDW}}{\pi \hbar}$$



CA = CV?

In many cases the two proposals yield very similar results

Structure of divergences (Carmi, Myers, Rath; Reynolds, Ross)

$$C_V = \frac{L^{d-1}}{(d-1)G_N}\frac{Vol}{\delta^{d-1}} \qquad C_A = \frac{L^{d-1}}{4\,\pi^2(d-1)G_N}\frac{Vol}{\delta^{d-1}}\log\left(\frac{\ell_{ct}(d-1)}{L}\right)$$

Linear growth at late times

(Stanford, Susskind; Brown, Roberts, Swingle, Susskind, Zhao)

$$\frac{dC_V}{dt}\Big|_{t\to\infty} = \frac{8\,\pi\,M}{d-1} \qquad \frac{dC_A}{dt}\Big|_{t\to\infty} = \frac{2\,M}{\pi}$$

The switchback effect (Stanford, Susskind; Zhao)

Chaotic evolution with Lyapunov exponent $\lambda_L = \frac{2\pi}{\beta}$, followed by delayed linear evolution after $t^*_{scr} = \frac{1}{2\pi T}\ln\frac{T}{\Delta T}$ following the injection of a perturbation.

We want to understand cases where the proposals are not equivalent.

A Conformal Defect in AdS_3

Conformal defect in 1+1 dimensions

Preserves one copy of the Virasoro algebra

Simple Holographic Model

AdS_2 Brane in AdS_3

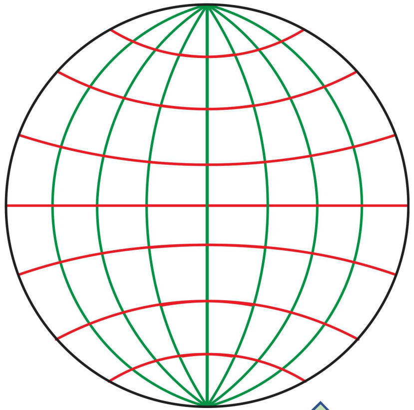
$$S = \frac{1}{16\,\pi\,G_N} \int d^3\,x\,\sqrt{-g}\left(R + \frac{2}{L^2}\right) - \lambda \int d^2\,x\sqrt{-h}$$

Exact Solution Including Backreaction

$$ds^2 \;=\; L^2\left(dy^2 + \cosh^2(\,|y| - y^*)\;\left(-\cosh^2 r\,dt^2 + dr^2\right)\right)$$

$$y^* = 4\,\pi\,G_N L\,\lambda$$ brane tension parameter

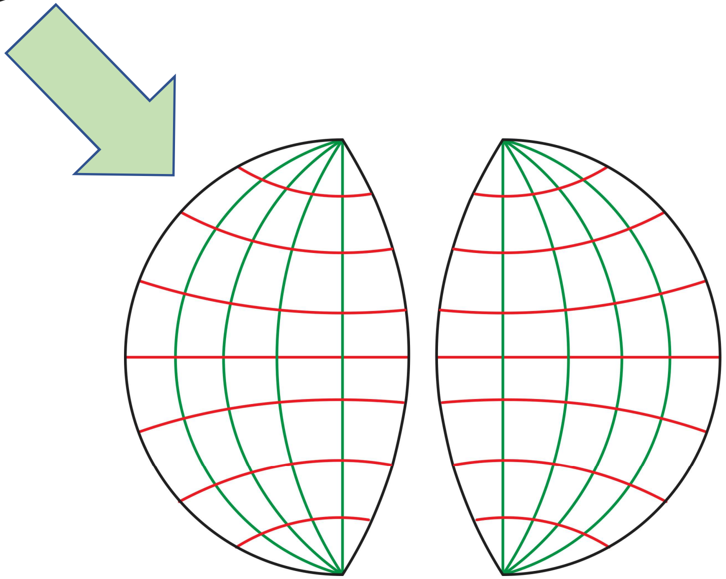
A Conformal Defect in AdS_3

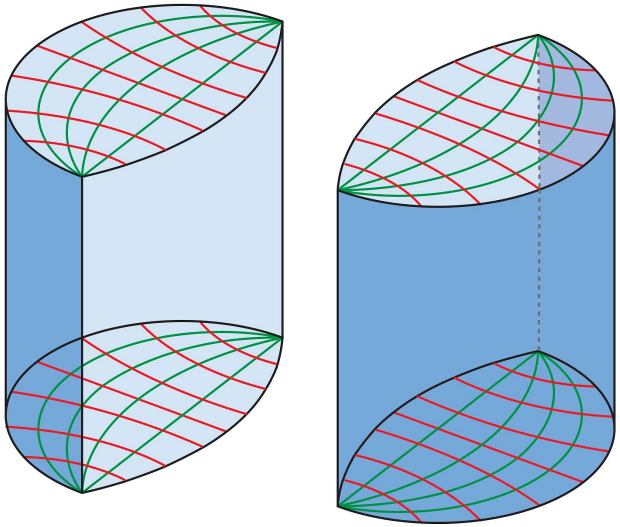


Exact Solution Including Backreaction

$$ds^2 = L^2\left(dy^2 + \cosh^2(\,|y| - y^*)\,\left(-\cosh^2 r\, dt^2 + dr^2\right)\right)$$

$\tanh y^* = 4\,\pi\, G_N L\, \lambda$ *brane tension parameter*



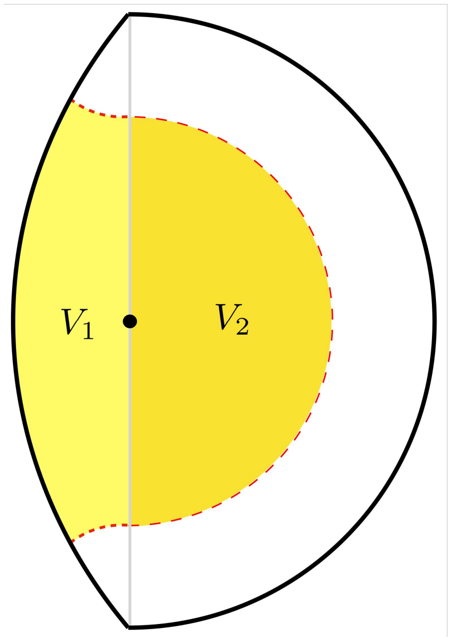


Complexity=Volume

Fixing the Cutoff in the defect region along a line of constant r - connects smoothly across the defect

More volume - larger complexity

$$C_V = \frac{4\,c_T}{3}\left(\frac{\pi L_B}{\delta} + 2\sinh y^* \ln\frac{2L_B}{\delta} - \pi\right)$$



Volume law
Boundary size $2\pi L_B$

Topological Contribution
(Abt, Erdmenger, Hinrichsen, Melby-Thompson, Meyer, Northe, Reyes)

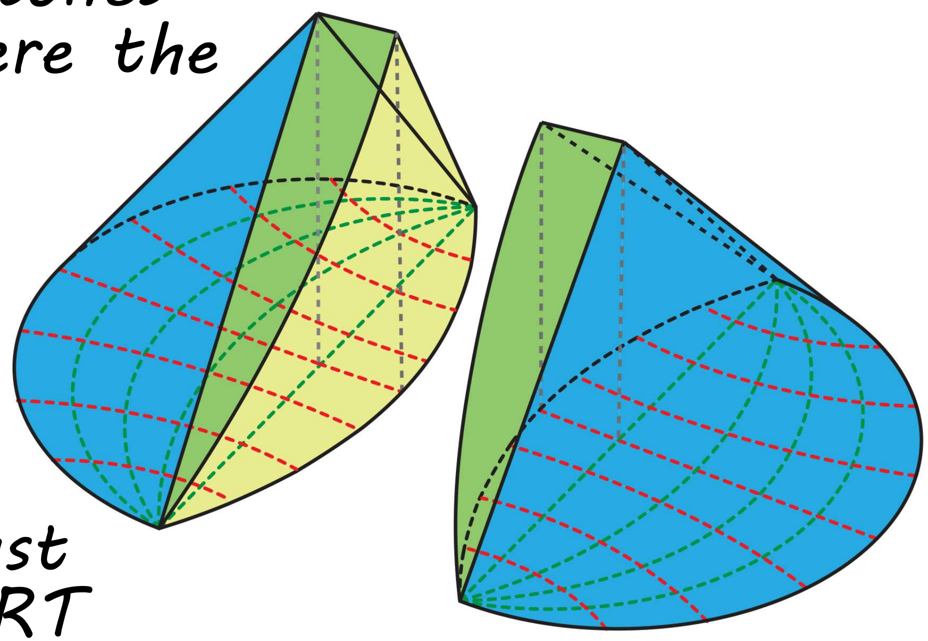
Logarithmic Defect Contribution Related to the Affleck-Ludwig boundary Entropy

$$S_{EE} = \frac{c_T}{3}\ln\left[\frac{2L_B}{\delta}\sin\left(\frac{\ell}{2L_B}\right)\right] + \log g; \qquad \ln g = \frac{c_T y^*}{3}$$

(Azeyanagi, Karch, Takayanagi and Thompson) folding trick

Complexity=Action: The WDW Patch

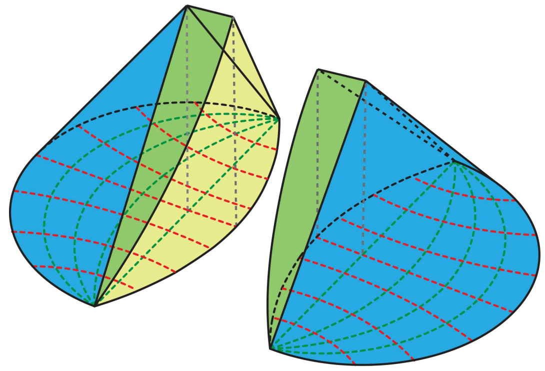
The usual cone is extended by light cones starting at the antipodal points where the brane meets the boundary



The two light cones meet at a ridge to form a tent like shape

They are actually portions of the past entangling wedge of a region whose RT surface includes this ridge (therefore the expansion Θ vanishes)

Complexity=Action



$$I = \frac{1}{16\pi G_N}\int_M d^3x\,\sqrt{-g}(R-2\,\Lambda) + \frac{\epsilon_K}{8\pi G_N}\int_{B_{t/s}} d^2x\,\sqrt{|h|}K$$

$$+\frac{\epsilon_\kappa}{8\pi G_N}\int_{B_n} d\lambda d\theta\sqrt{\gamma}\kappa - \frac{1}{8\pi G_N}\int_{B_n} d\lambda d\theta\sqrt{\gamma}\Theta\ln(\ell_{ct}|\Theta|) + \frac{\epsilon_a}{8\pi G_N}\int_\Sigma dx\,\sqrt{\gamma}a$$

$$-\lambda\int_{D\cap WDW} d^2x\,\sqrt{-h}$$

Defect Contribution

$$I_d = I_\lambda + I_{EH} = -I_\lambda$$

Includes the discontinuity of the Einstein Hilbert term

Null surface contributions

1. $k^\mu \nabla_\mu k_\nu = \kappa\, k_\nu$
   (replaces $K = h^{ab}K_{ab}$ on the null surface)
2. Expansion parameter $\Theta = \partial_\lambda \ln\sqrt{\gamma}$ (ℓ_{ct} counter-term length scale).
3. Joint contribution $a = \ln(k_1\cdot k_2/2)$

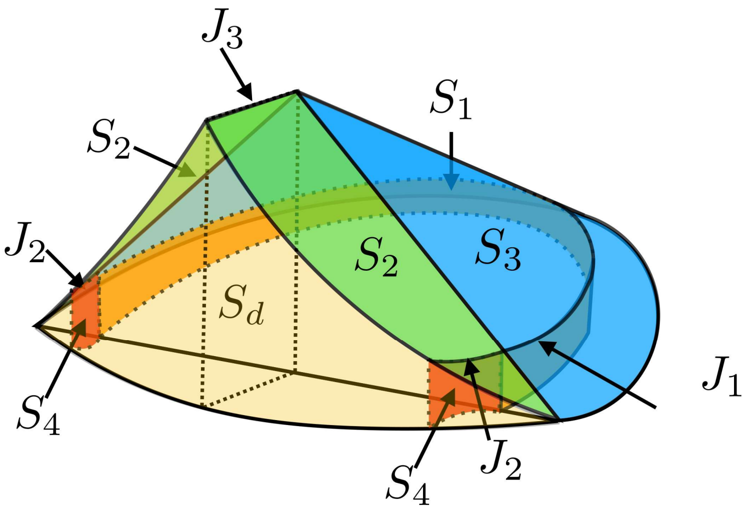
(L. Lehner, R. C. Myers, E. Poisson and R. D. Sorkin)

Complexity=Action

$$I_{\text{bulk}} = \frac{L}{\pi G_N}\left(-\frac{\pi L_B}{\delta} + \left(y^* + \frac{\sinh 2y^*}{2}\right)\left(\ln\frac{\delta}{L_B} - 1\right) + \frac{\pi^2}{4}\right)$$

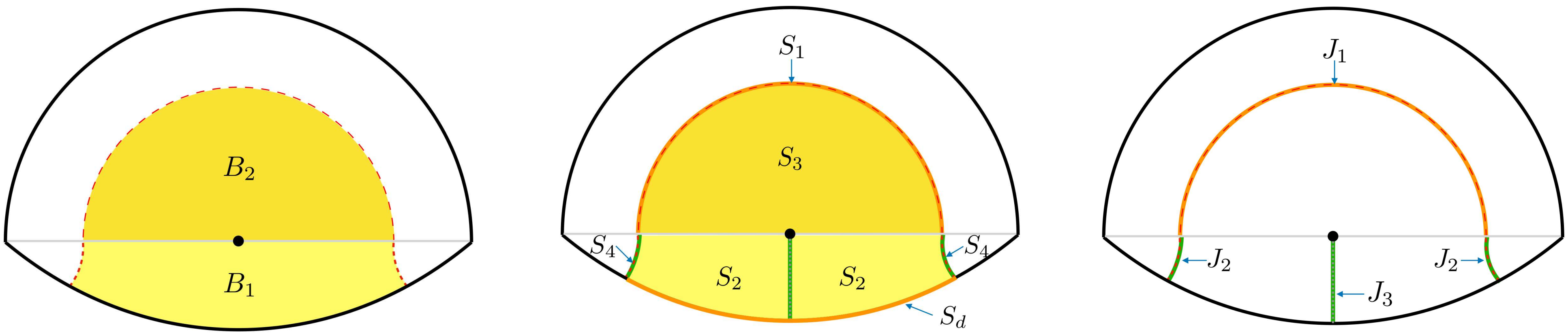
$$I_{\text{surf,joint}} = -\frac{L}{\pi G_N} y^* \left(\ln\frac{\delta}{L_B} - 1\right) + \frac{L\, L_B}{2G_N\delta}\left(\ln\left(\frac{\ell_{ct}}{L}\right) + 3\right)$$

$$I_{\text{d}} = -\frac{L \sinh 2y^*}{2\pi G_N}\left(\ln\frac{\delta}{L_B} - 1\right)$$



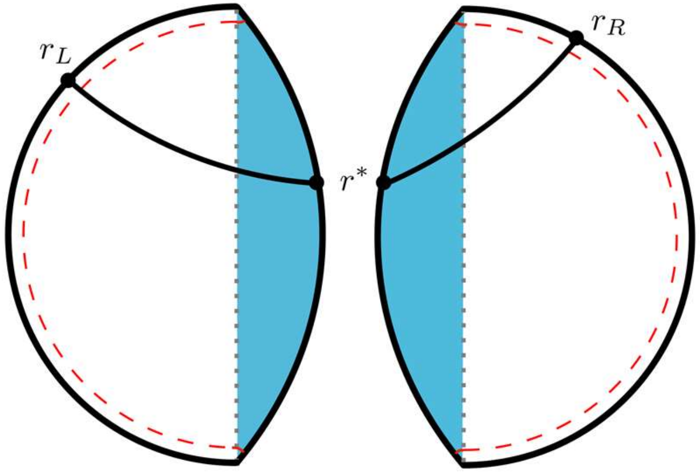
$$C_A = \frac{I_{WDW}}{\pi} = \frac{c_T}{3\pi}\left(\frac{L_B}{\delta}\left[\ln\left(\frac{\ell_{ct}}{L}\right) + 1\right] + \frac{\pi}{2}\right)$$ **Defect influence cancels**

$$c_T = \frac{3L}{2G_N}$$



Subregion complexity=Volume

The complexity is proportional to the maximal volume enclosed between the boundary subregion and its Ryu-Takayanagi surface (Alishahiha)



Matching the RT surface across the defect

- minimize total length
- Locally dy/dr is continuous

$$\boxed{r^* = \frac{r_L + r_R}{2}}$$

$$C_V(r_L, r_R) = \frac{2c_T}{3}\left(\frac{\ell}{\delta} + \sinh y^* \left(2\ln\left(\frac{2L_B}{\delta}\right) - r_L - r_R\right) - \pi\right)$$

$$\mathrm{S_{EE}(r_L, r_R)} = \frac{\mathrm{c_T}}{3}\ln\left(\frac{\mathrm{2L_B}}{\delta}\ \sin\left(\frac{\ell}{\mathrm{2L_B}}\right)\right) + \frac{\mathrm{c_T}}{3}\ln\left(\cosh y^* + \frac{\sinh y^*}{\cosh\frac{r_R - r_L}{2}}\right)$$

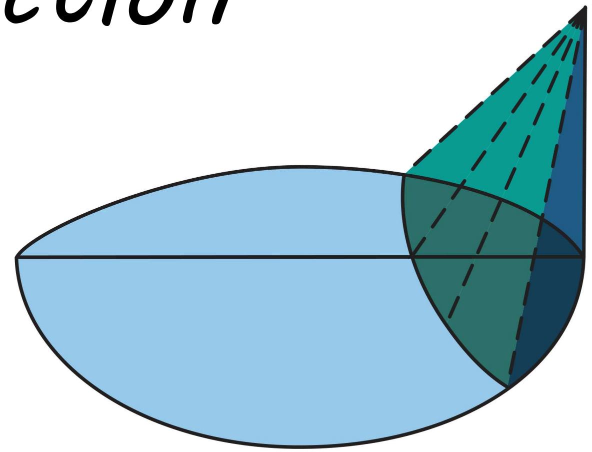
$r_L = r_R$ related to the boundary entropy by folding trick (Azeyanagi, Karch, Takayanagi and Thompson)

$$\theta_R = \cos^{-1}\tanh r_R$$
$$\theta_L = -\cos^{-1}\tanh r_L$$

Subregion Complexity=Action

The complexity is given by the action of
WDW Patch ∩ Entanglement wedge

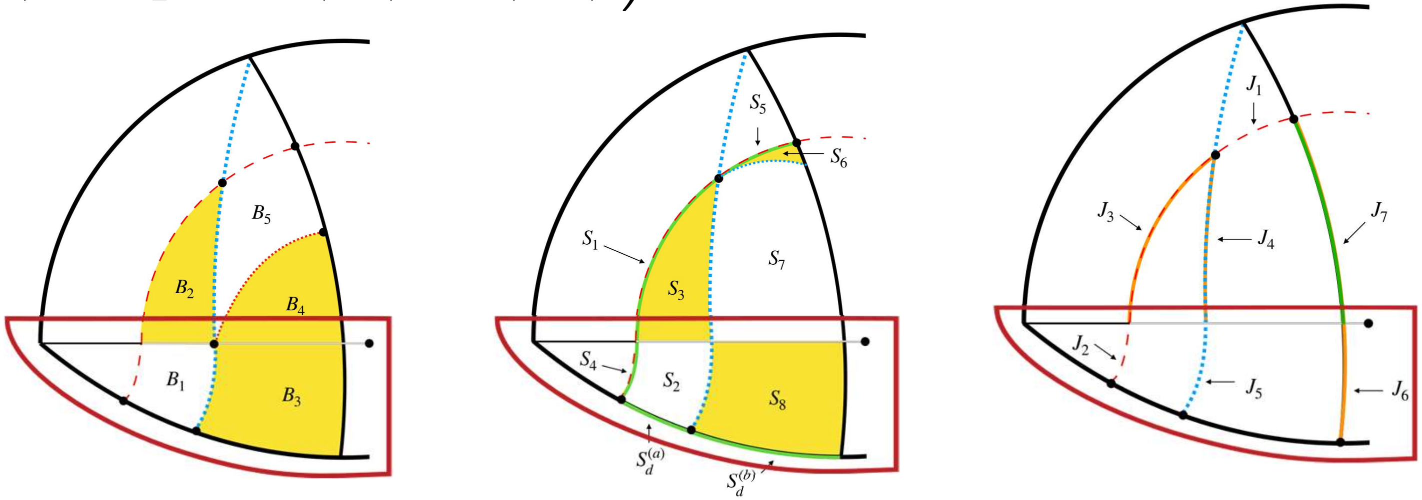
(D. Carmi, R. C. Myers and P. Rath)



Focus on the symmetric case

$$C_A = \frac{c_T}{3\pi^2}\left(\frac{\ell}{2\delta}\left[\ln\left(\frac{\ell_{ct}}{L}\right)+1\right]+\ln\left(\frac{\delta}{L_B}\right)\ln\left(\frac{\ell_{ct}}{L}\right)\right)+\text{finite}$$

Defect contribution cancels, additional log divergence



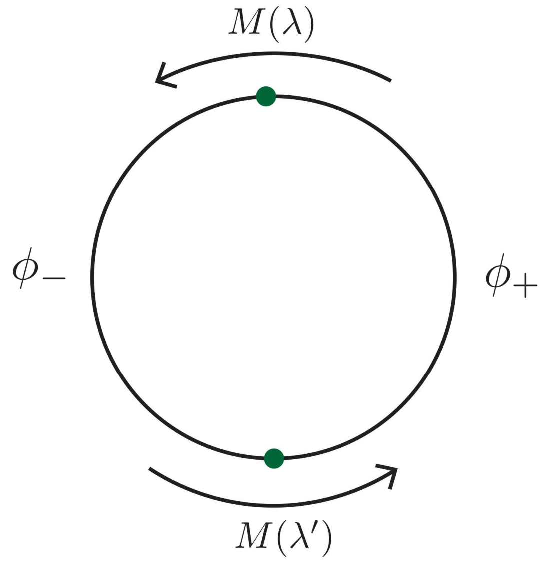
A conformal defect in free QFT

Matching conditions (Bachas, de Boer, Dijkgraaf, Ooguri)

$$\begin{pmatrix}\partial_x\phi_- \\ \partial_t\phi_-\end{pmatrix} = M(\lambda)\begin{pmatrix}\partial_x\phi_+ \\ \partial_t\phi_+\end{pmatrix}, \qquad M(\lambda) = \begin{pmatrix}\lambda & 0 \\ 0 & \lambda^{-1}\end{pmatrix}$$

$$\begin{pmatrix}\partial_x\phi_- \\ \partial_t\phi_-\end{pmatrix} = M'(\lambda)\begin{pmatrix}\partial_x\phi_+ \\ \partial_t\phi_+\end{pmatrix}, \qquad M'(\lambda) = \begin{pmatrix}0 & \lambda^{-1} \\ \lambda & 0\end{pmatrix}$$

$$\Delta \equiv \frac{1}{\pi}\tan^{-1}\left(\frac{\lambda'\lambda - 1}{\lambda + \lambda'}\right)$$



Complexity for Gaussian states in free QFT (Jefferson, Myers; SC, Heller, Marrochio, Pastawski). Use the spectrum assuming that the QFT formula naively generalizes (the boundary size is $2\pi L_B$)

$$C = 4L_B\Lambda\left[\ln\left(\frac{\omega_0}{\Lambda}\right) + 1\right] - \ln(2L_B\Lambda) - \ln\left(\frac{2\sin(\pi\Delta)}{\Delta}\right)$$

$\omega_0 = \Lambda\, e^{\sigma}$

reference state frequency

When the scalar is compact $\lambda = \frac{R_+}{R_-}$ and $\lambda' = \lambda^{-1}$ and we recover the vacuum result.

Log contribution does not depend on the defect parameter - favors CA

Zero modes for compact boson?

Future Directions

Higher Dimensions?

Other holographic defects? Janus Solution

Different codimension defects?

Asymmetric defects?

Lots to explore!

Zero modes?

Complexity in CFT?

Thank you!

ご清聴
ありがとう
ございました