Maximal volumes and Euclidean variations

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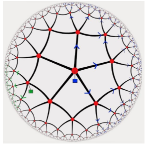
Based on 1811.03097 , with A. Belin and G. Sarosi

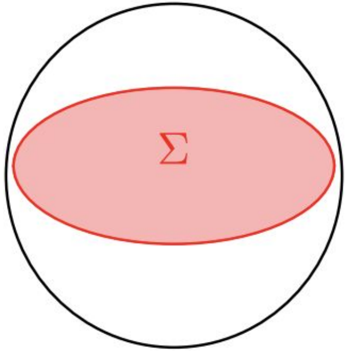


Introduction

In the recent years, we have advanced a lot in our understanding of codim-2 surfaces and their quantum information interpretation: entanglement entropy and subregion - subregion duality.

However, not much has been proven about codimension 1 surfaces : what is their use in QI QG?

Focus on codim 1 surfaces dual to the boundary $t = 0$ slice.



Overlaps!

As discussed by Gabor, a natural way to obtain quantities localized in codimension 1 bulk surfaces is through wave functions overlaps .

In particular the bulk symplectic form is dual to an antisymmetrized boundary overlap:

$$\Omega_{\Sigma_{bulk}}(\delta_1\phi, \delta_2\phi) \sim \langle \delta_1 J | \delta_2 J \rangle - (1 \leftrightarrow 2)$$

where ϕ, J could be the metric and the stress-tensor and the states are naturally defined using the euclidean path integral.

$^*\Omega = \int_{\Sigma_{bulk}} \delta_1\phi\delta_2\Pi - (1 \leftrightarrow 2)$

Volume and EE analogies

For entanglement entropy, we were able to make progress through the replica trick: a boundary deformation creates a conical singularity.

$$\delta\partial_n \log Z = \delta S_{EE}$$

Can we find a similar deformation for the maximal volume slice that ends at t=0?

$$\delta V =_? \delta\partial_\alpha \log Z$$

Volume and EE analogies

Following the analogy with entanglement entropy, it seems natural that the deformation δ_α is smooth in the bulk but becomes singular in the boundary.

Close to the boundary of the entangling region ∂_n creates a conical singularity. We expect that close to the $t = 0$ surface, the metric deformation is discontinuous. As we will see, in the case of the vacuum, the simplest deformation with this property generates the volume:

$$\delta_\alpha \gamma_{ab} = \mathrm{sign}(t)\gamma_{ab}$$

While in principle a similar local description should apply to more complicated cases, that seems hard to make precise.

Use symplectic form to understand volume!

$$\delta V = \Omega_{\Sigma_{bulk}}(\delta_\alpha h, \delta h)$$

Approach

Obtain δV from the symplectic form: $\delta V = \Omega_{\Sigma_{bulk}}(\delta_\alpha h, \delta h)$

Volume generating $\delta_\alpha h$, only around extremal surfaces: constraint equations.

δ_α deforms extremal surface infinitesimally, same canonical fields:

$$\delta_\alpha \phi_{grav} = \delta_\alpha \pi_{grav} = 0, \text{``}\delta_\alpha t\text{''} = 1$$

Not a diffeo: fields don't change (equiv change the fields keep surface fixed). It half a York time translation, so we called it the New York deformation.

Similar to $T\bar{T}$: bulk cutoff surface moves while keeping Dirichlet boundary conditions. In this case the bulk vector field acts trivially on the boundary.

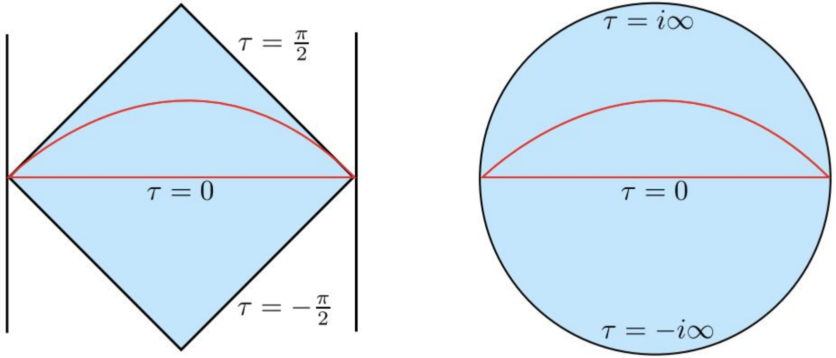
$$^*\delta_\alpha h_{ab} = 0, \delta_\alpha K_{ab} = h_{ab}$$

Vacuum

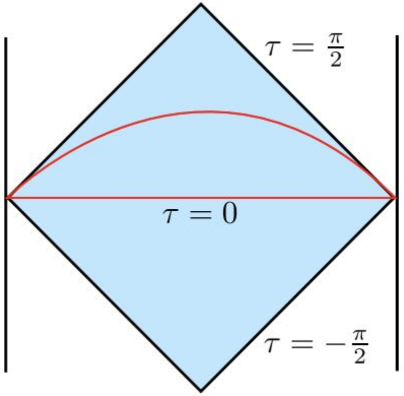
Obtain boundary deformation explicitly for simple geometries.

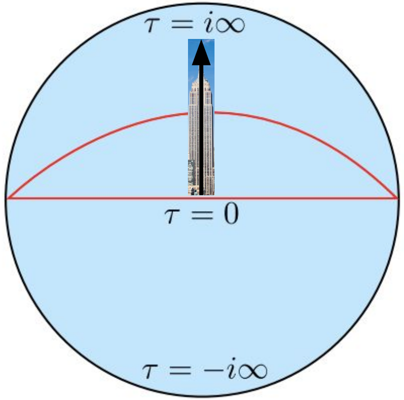
Consider the vacuum in WdW coordinates (Euclidean bdy is at $\tau = \pm i\infty$):

$$ds^2 = -d\tau^2 + \cos^2 \tau dH_d$$



Vacuum



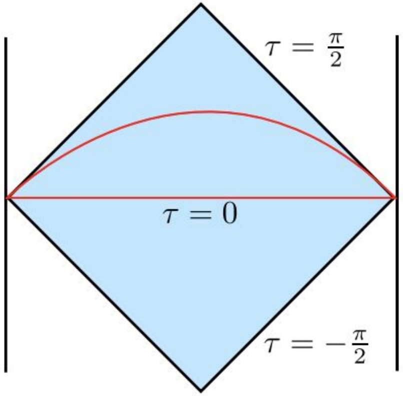


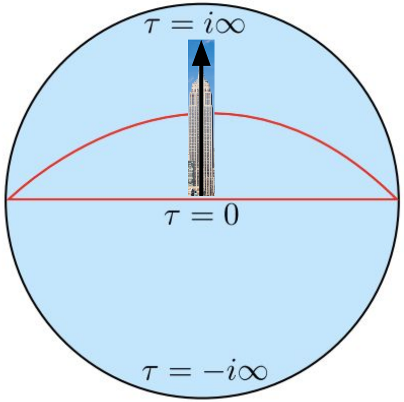
It is easy to show that $\delta_\alpha \sim \partial_\tau$. When mapped to Poincare coordinates, we have that $\delta_\alpha \gamma_{ab} = 2i \text{ sign} t \, \delta_{ab}$

The deformation is a diffeo only in the case when there is a time reflection symmetry and thus $K_{ij} = 0$. Equivalent of Casini-Huerta-Myers for volumes.

All these cases have $\delta V = 0$ for normalizable deformations.

Vacuum + matter fields





In the presence of matter fields, $\delta V = \int_\Sigma T_{\tau\tau}$

Can recover this from the boundary point of view. One needs to look for the deformation of the scalar fields that satisfies $\delta_\alpha \phi = \delta_\alpha \pi = 0$

This is a combination of the previous diffeomorphism plus a transformation of the sources, can be solved explicitly at large mass.

Thermofield double

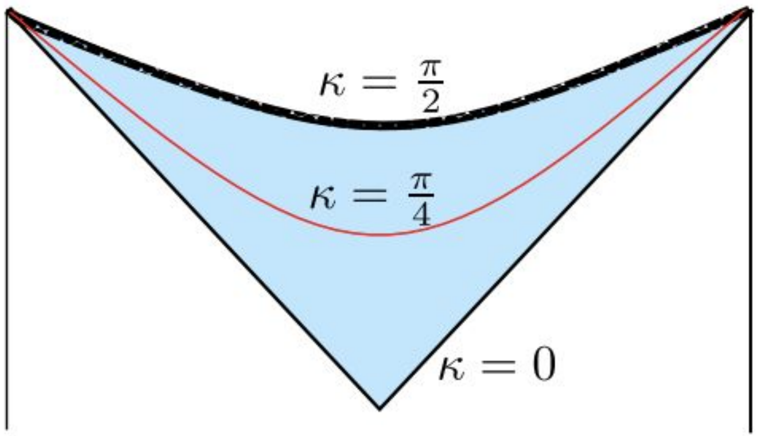
Need a $K_{ij} \neq 0$ extremal surface, simplest case $|TFD(T \to \infty)\rangle$

$$ds^2 = -d\kappa^2 + f(\kappa)dt^2 + g(\kappa)dx^2_{d-1}$$

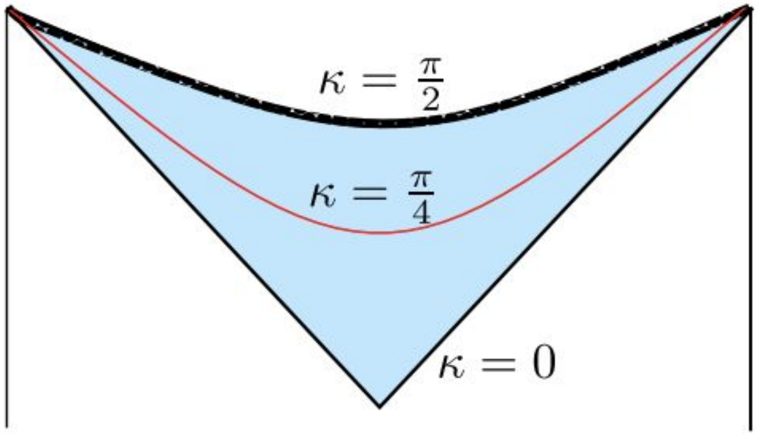
Very symmetric case, extremal surface

at $\kappa = \frac{\pi}{4}$.

t is spacelike in this WdW patch.



Thermofield double



$$V = \frac{MT}{(d-1)} + \dots$$

Can solve for the α deformation in this background.

We can plug the deformation into the boundary overlaps to obtain

$\partial_T V, \partial_\beta V$ from a purely boundary calculation.

The linear in T behaviour arises from the fact that in the path integral preparation of the state, one has to integrate over long times .

$^* 16\pi G_N = 1$

Conclusion

I have presented some motivation to approach the maximal volume in a similar way to entanglement.

Then have pursued this using the dual of the symplectic form.

Main limitation: general picture of the boundary deformation? Should be related with boundary states / singular deformations at $t=0$. Local picture should be good to understand the divergences, but they are subtle.

Alternatively, use $T\bar{T}$ in hyperbolic coordinates to generate these New York transformations **[WIP]**. Bulk arguments allow us to write the volume in terms of the trace of the stress tensor at large deformations.

THANKS
FOR LISTENING!

THANKS
IT FROM QUBIT!

THANKS
HEP-TH!