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Quantum Chaos of pure states in Random Matrices and in the SYK model

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Based on work arXiv:1901.02025
+ work in progress with Tomoki Nosaka(KIAS)

Pure State dynamics in RMT and in the SYK

In this talk, we consider the time evolution of pure states

Generically, a time evolved state $|\psi(t)\rangle$ is a complicated superposition of vectors:

$$|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle = \sum_i c_i(t)|\psi_i\rangle$$

What we are considering in this talk is the (square of)amplitude

$$\langle |c_i(t)|^2 \rangle_{\text{ensemble}}$$

in Random Matrix theory (analytically) and in the (mass deformed) SYK model (numerically).

It is related to the spectral form factor, which is diagnostic of quantum chaos [Berry] and brought to BH physics by [CGHPSSSST] [Papadodimas-Raju]

Motivation

- Study black hole microstate dynamics (non-perturbatively)

- To understand state dependence of the interior by studying the state dependent deformation of the theory

- To study the time evolution after projection measurement

[Shiba-TN-Takayanagi-Watanabe, 16] [Maldacena-Stanford-Yang, 17]

Late time is expected to be governed by RMT [CGHPSSSST]

→Can we also see the random matrix behavior in pure states?
Does it give a prediction on non perturbative effect in late time?

- SYK model have a collective field description, which is similar problem to AdS/CFT

Return(Evolution) Amplitude

The overlap between time evolved states and the initial states

$$g_R(t) = |\langle\psi_0|\, e^{-iHt}\, |\psi_0\rangle\,|^2$$

We call this return amplitude according to [Cardy, 14]

sometimes called survival probability

A special case of Loschmidt Echo

We can also consider the amplitude to evolve to orthogonal states:

$$g_{ev}(t) = |\langle\psi_1|e^{-iHt}|\psi_0\rangle\,|^2 \qquad \text{where} \quad \langle\psi_1|\psi_0\rangle = 0$$

We call this evolution amplitude.

Return/Evolution Amplitude in Random Matrices

The ensemble average is defined $\langle \cdot \rangle_{GUE} = \frac{\int dH \ \cdot \ e^{-\mathrm{Tr} H^2}}{\int dH e^{-\mathrm{Tr} H^2}}$

H : Hermitian Matrices

(for GOE/GSE, H is replaced by real/quartanion sym matrices)

A technique to compute

[Cotler-Hunter Jones-Liu-Yoshida, 16]

[Beni Yoshida's Lecture in the 4th week]

Invariance of $\int d(U^\dagger H U) = \int dH$ and $\mathrm{Tr}(U^\dagger H U)^2 = \mathrm{Tr} H^2$

lead to $\int dH f(H) = \int dH \int dU f(U^\dagger H U)$

(for GOE/GSE, U is replaced by orthogonal/symplectic matrices)

Return/Evolution Amplitude in Random Matrices

The results: [TN, 19]

$$\langle g_R(t)\rangle_{\mathrm{GUE}} = \frac{1}{L(L+1)}(\langle g(t)\rangle_{\mathrm{GUE}} + L)$$

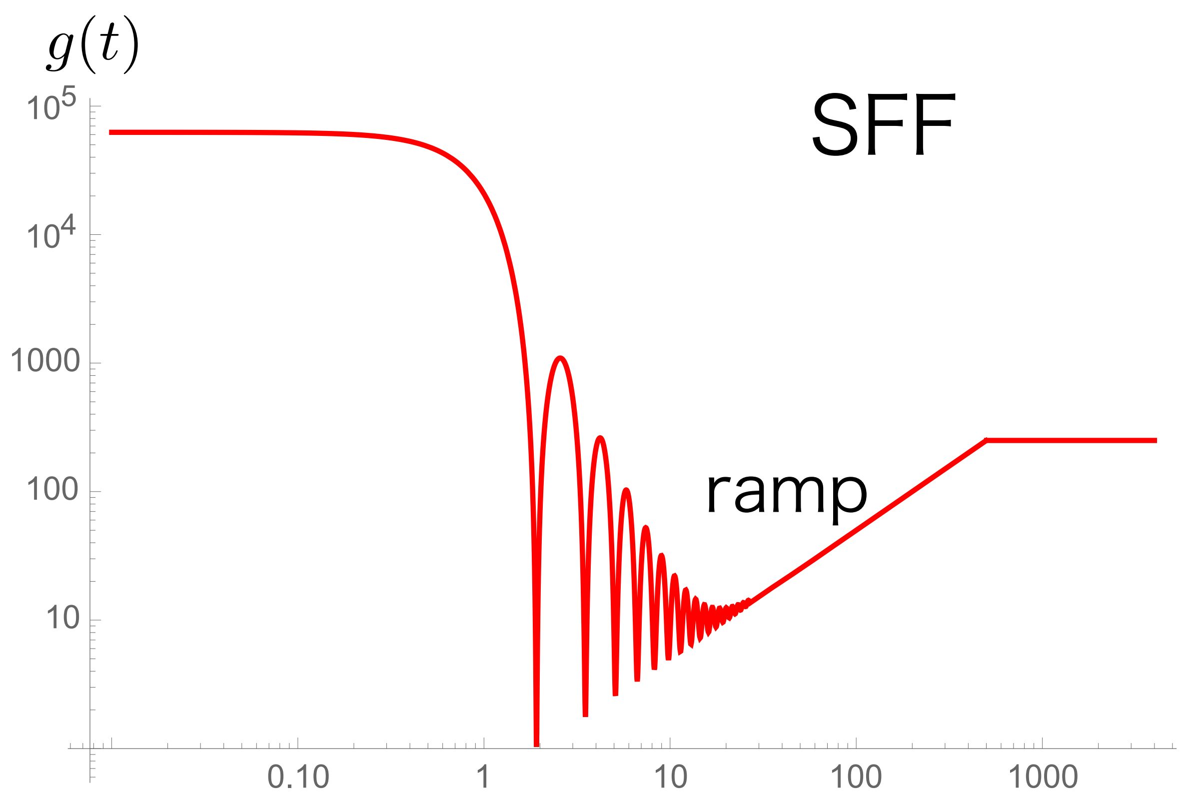
$$\langle g_{ev}(t)\rangle_{\mathrm{GUE}} = \frac{1}{L^2-1}(L - \frac{1}{L}\langle g(t)\rangle_{\mathrm{GUE}})$$

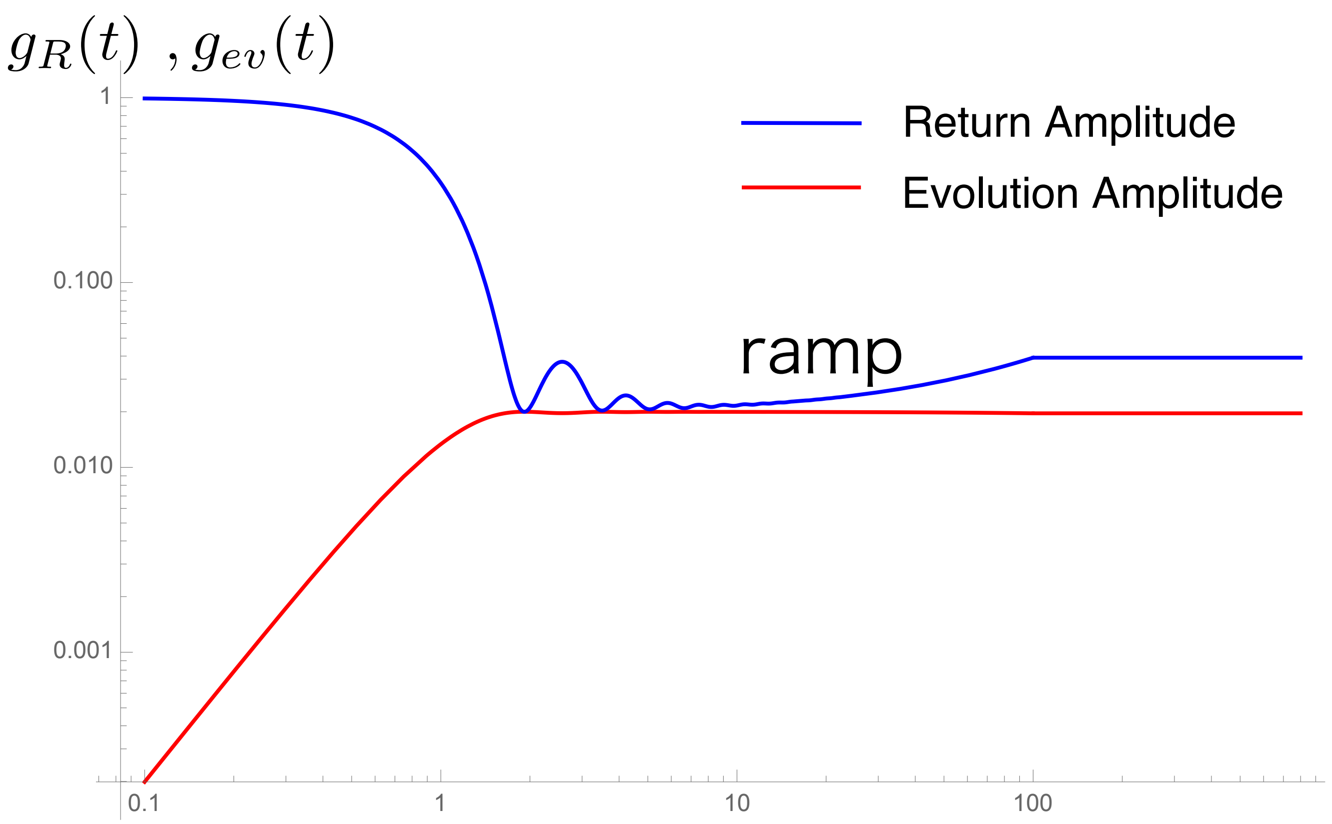
(We can also evaluate in GOE/GSE ensembles)

where $g(t) = \mathrm{Tr}(e^{-iHt})$ is the spectral form factor(SFF), which is

$$\langle g(t)\rangle_{GUE} = L^2\frac{J_1(2t)^2}{t^2} - L(1-\frac{t}{2L})\theta(2L-t) + L$$

[Cotler-Hunter Jones -Liu-Yoshida, 16]





ramp: Fourier transform of the long range level repulsion

Comparing with Haar Random evolution

$$\langle g_R(t)\rangle_{\mathrm{GUE}} = \frac{1}{L(L+1)}(\langle g(t)\rangle_{\mathrm{GUE}} + L) \;\to\; \frac{2}{L+1}$$

$$\langle g_{ev}(t)\rangle_{\mathrm{GUE}} = \frac{1}{L^2-1}(L - \frac{1}{L}\langle g(t)\rangle_{\mathrm{GUE}}) \;\to\; \frac{1}{L+1}$$

When we replace e^{-iHt} with Haar Random Unitary U

$$\int dU |\langle\psi_1| U |\psi_0\rangle|^2 = \frac{1}{L}$$

This is because generically

$$\lim_{T\to\infty}\int dH\ F(e^{-iHt}) \neq \int dU\ F(U)$$

[cf: Saad-Stanford-Shenker, 18]

Return amplitude acquires the bigger value in RMT because of level correlation

SYK model :

[Sachdev-Ye 93] [Kitaev 14,15]

N Majorana fermions $\{\psi_i, \psi_j\} = \delta_{ij}$ ($\dim\mathcal{H} = 2^{\frac{N}{2}}$)

$$H_{SYK} = \sum_{i<j<k<l} J_{ijkl}\psi_i\psi_j\psi_k\psi_l$$

with $\langle J_{ijkl}\rangle_J = 0$ and $\langle J_{ijkl}^2\rangle_J = \dfrac{3!J^2}{N^3}$

Symmetry of the SYK: Time reversal $\mathcal{T}$

(mod 2) fermion number $(-1)^F$

$N \pmod 8$	$\mathcal{T}^2$	$\mathcal{T}(-1)^F = a(-1)^F\mathcal{T}$	statistics	degeneracy
$N=0$	$+1$	$a=+1$	GOE	1
$N=2$	$+1$	$a=-1$	GUE	2
$N=4$	-1	$a=+1$	GSE	2
$N=6$	-1	$a=-1$	GUE	2

[Fidkowski-Kitaev 11] [You-Ludwig-Xu 16]

Spin operators: $S_k = -2i\psi_{2k-1}\psi_{2k} \quad (k = 1, \cdots, N/2)$

They satisfy $S_k^2 = 1$

Pure states: [Kourkoulou-Maldacena 17]

$|B_s\rangle$: simultaneous eigenstates of $S_k \ (k = 1, \cdots, N/2)$

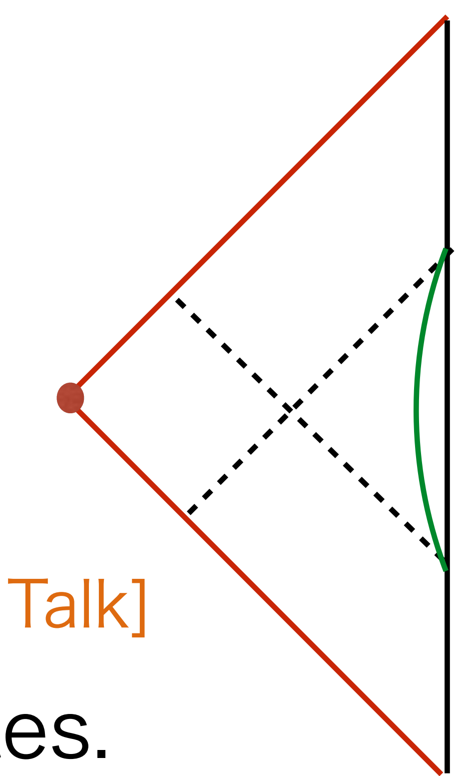
$S_k |B_s\rangle = s_k |B_s\rangle$ ($2^{\frac{N}{2}}$ states, form a basis)

・To produce lower energy states, we can smear as $e^{-\frac{\beta}{2}H_{SYK}} |B_s\rangle$ by the SYK Hamiltonian.

・In large N, there is an emergent $O(N)$ symmetry.

$O(N) \supset N/2$ flip group $\quad \psi_{2k} \to -\psi_{2k}$

$$\langle B_s| e^{-\beta H_{SYK}} |B_s\rangle = 2^{-\frac{N}{2}} \mathrm{Tr}(e^{-\beta H_{SYK}}) + \mathcal{O}(1/N^3)$$



・Have a NAdS2 gravity interpretation [cf: Raamsdonk's Talk]

・Projection measurement of the Left CFT in TFD states.

[Shiba-TN-Takayanagi-Watanabe, 16] [Maldacena-Stanford-Yang, 17]

[Goel-Lam-Turiaci-Verlinde, 18]

Return Amplitudes in the SYK

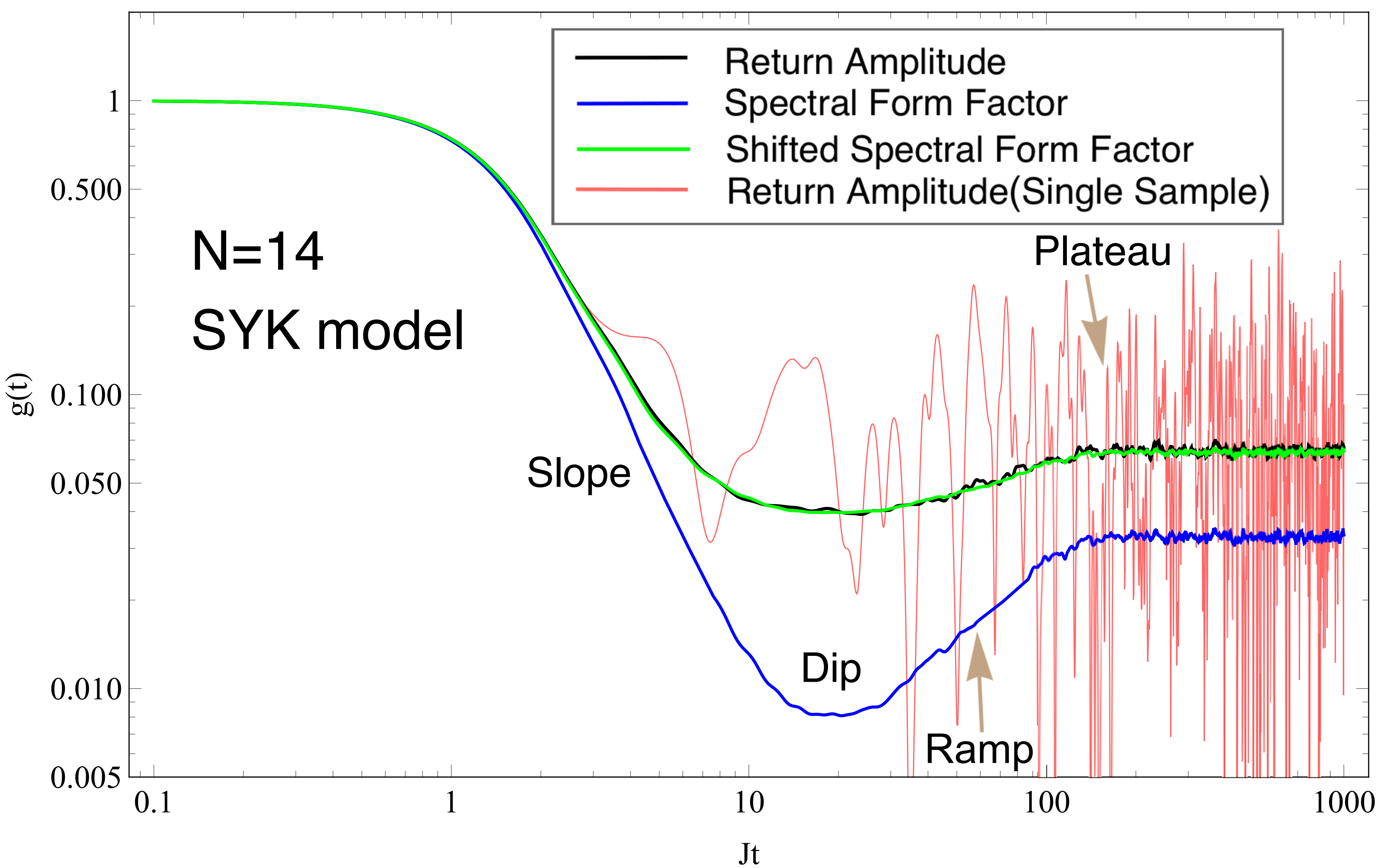
[TN, 19]

- finite N, Exactly diagonalize. $\beta = 1.5$, average over 1500 samples

$$g_p(t;\beta) = |\langle B_{\uparrow\uparrow\uparrow\cdots}|e^{-iH_{SYK}t}e^{-\beta H_{SYK}}|B_{\uparrow\uparrow\uparrow\cdots}\rangle|^2$$

return amplitude: $\langle g_p(t;\beta)\rangle_J$

shifted SFF: $\dfrac{\langle g(t;\beta)\rangle_J + \mathrm{Tr}(e^{-2\beta H_{SYK}})}{\langle g(0;\beta)\rangle_J + \mathrm{Tr}(e^{-2\beta H_{SYK}})}$

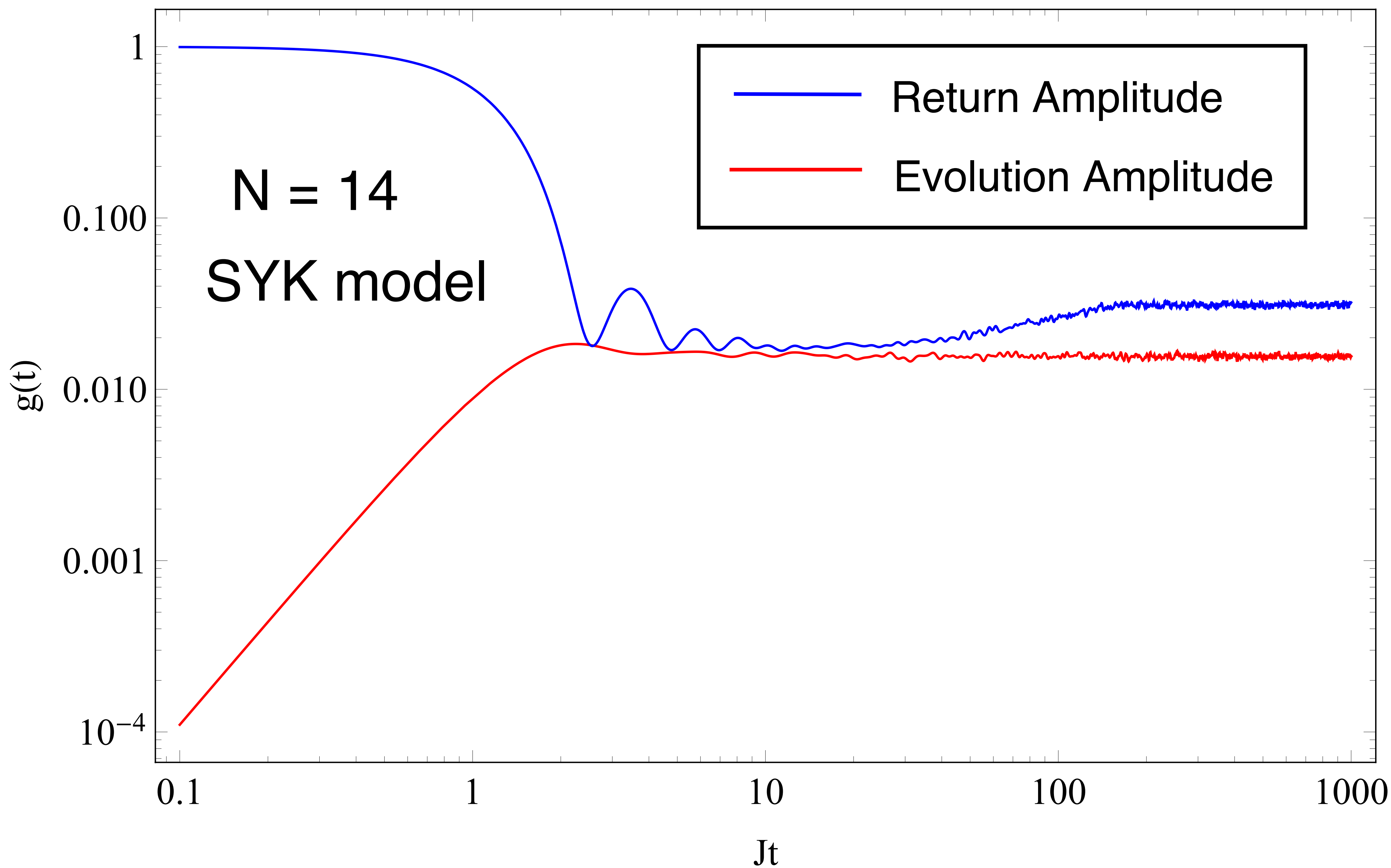


Plot of Evolution amplitude in the SYK

$\beta = 0$, average over 1500 samples

$|\psi_0\rangle = |B_{\uparrow\uparrow\uparrow\ldots}\rangle$ $\qquad$ $|\psi_1\rangle = |B_{\downarrow\downarrow\uparrow\ldots}\rangle$

evolution amplitude: $|\langle\psi_1|e^{-iH_{SYK}t}|\psi_0\rangle|^2$

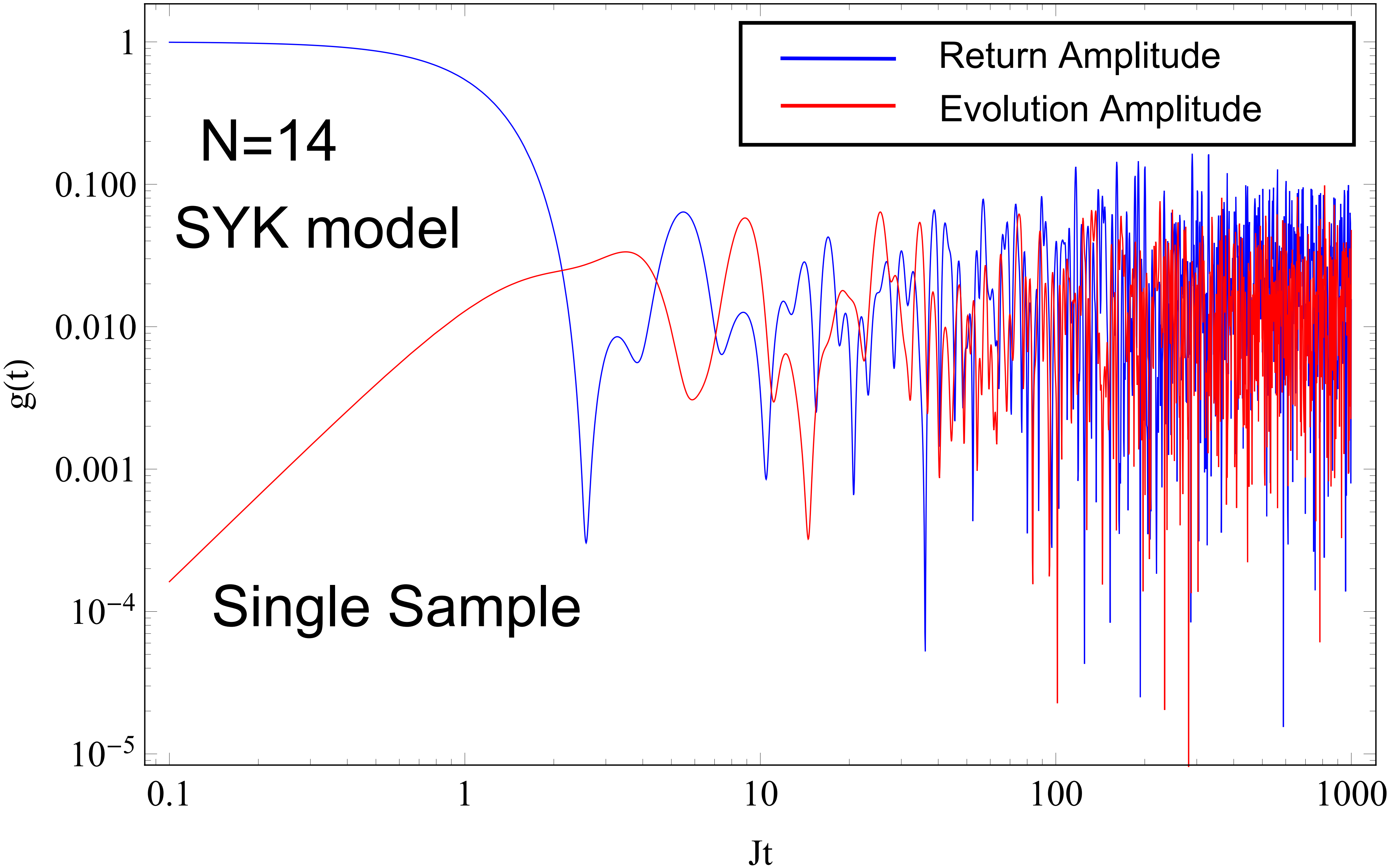


Plot of single sample SYK results

$\beta = 0$, single samples

$|\psi_0\rangle = |B_{\uparrow\uparrow\uparrow\cdots}\rangle \qquad |\psi_1\rangle = |B_{\downarrow\downarrow\uparrow\cdots}\rangle$

$$|\langle\psi_1|e^{-iH_{SYK}t}|\psi_0\rangle|^2$$



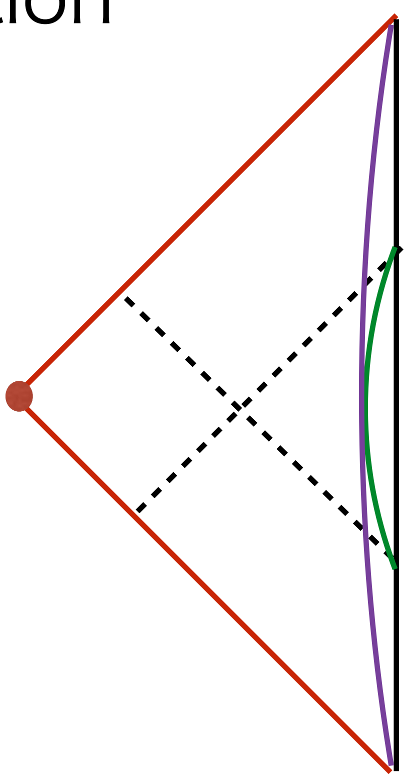
Mass deformation: [Kourkoulou-Maldacena 17]

$$H_{def} = H_{SYK} + \mu H_M \qquad H_M = -\frac{1}{2}\sum_{k=1}^{N/2} s_k S_k = \sum_{k=1}^{N/2} i s_k \psi_{2k-1}\psi_{2k}$$

$e^{-iH_{def}t}e^{-\frac{\beta}{2}H_{SYK}}|B_s\rangle$ has a NAdS2 gravity interpretation

whole the space can be seen after deformation

[cf:Maldacena-Qi 18, Cottrell-Freivogel-Hofman-Lokhande 18]



H_M is diagonalized in $|B_s\rangle$ basis.

spectrum: $E_m^{(0)} = -\frac{N}{4} + m$

with degeneracy $d_m = \binom{N/2}{m}$

large μ : spectrum localize near $E_m^{(0)}$

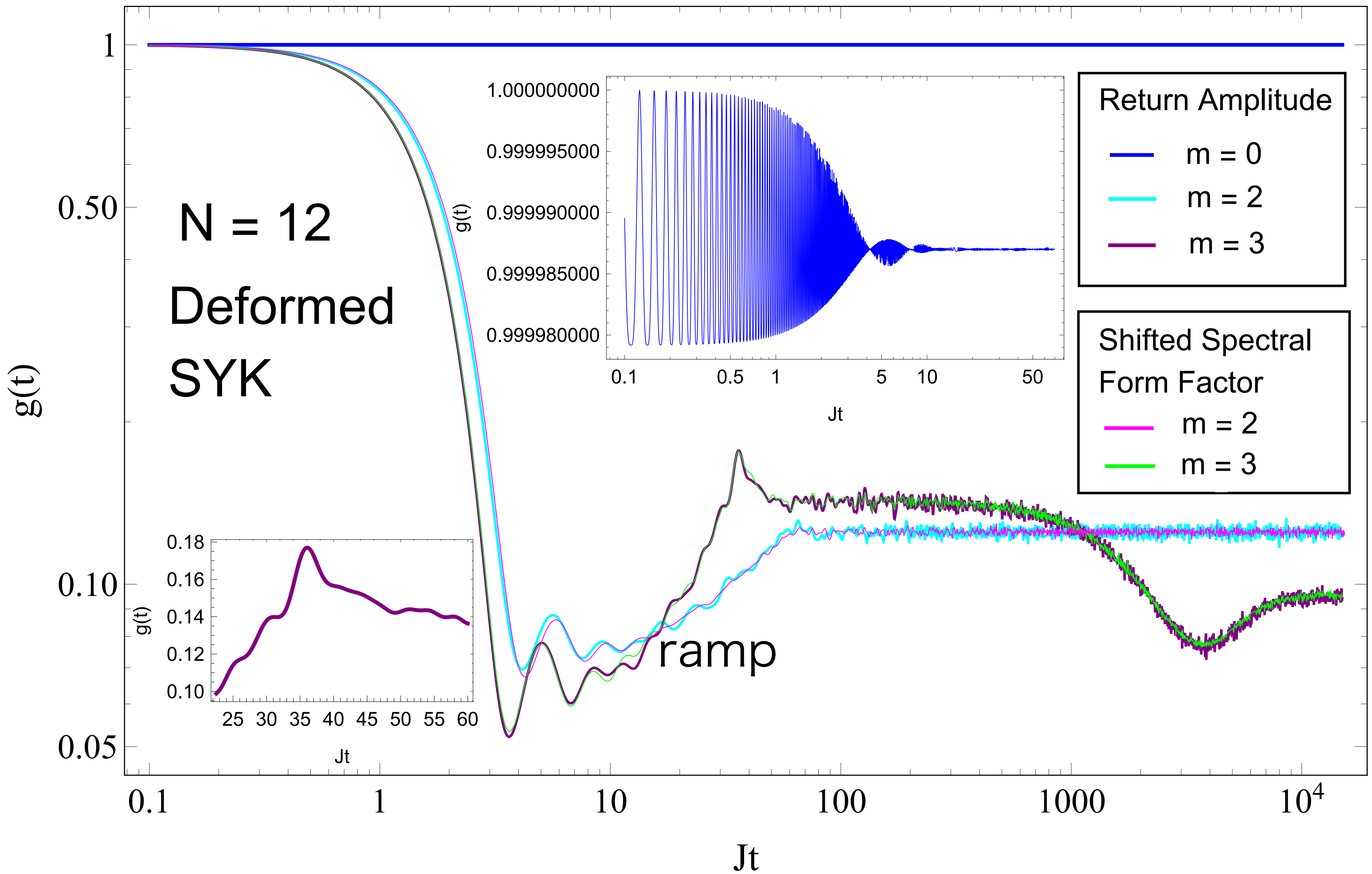
chaotic perturbation of H_M by H_{SYK}

Plot of Return amplitude in deformed SYK

[TN, 19]

$\beta = 0 \quad \mu = 50$, average over 2000 samples

$m = 0 : |B_{\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow}\rangle$ $\quad m = 2 : |B_{\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow}\rangle$ $\quad m = 3 : |B_{\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow}\rangle$



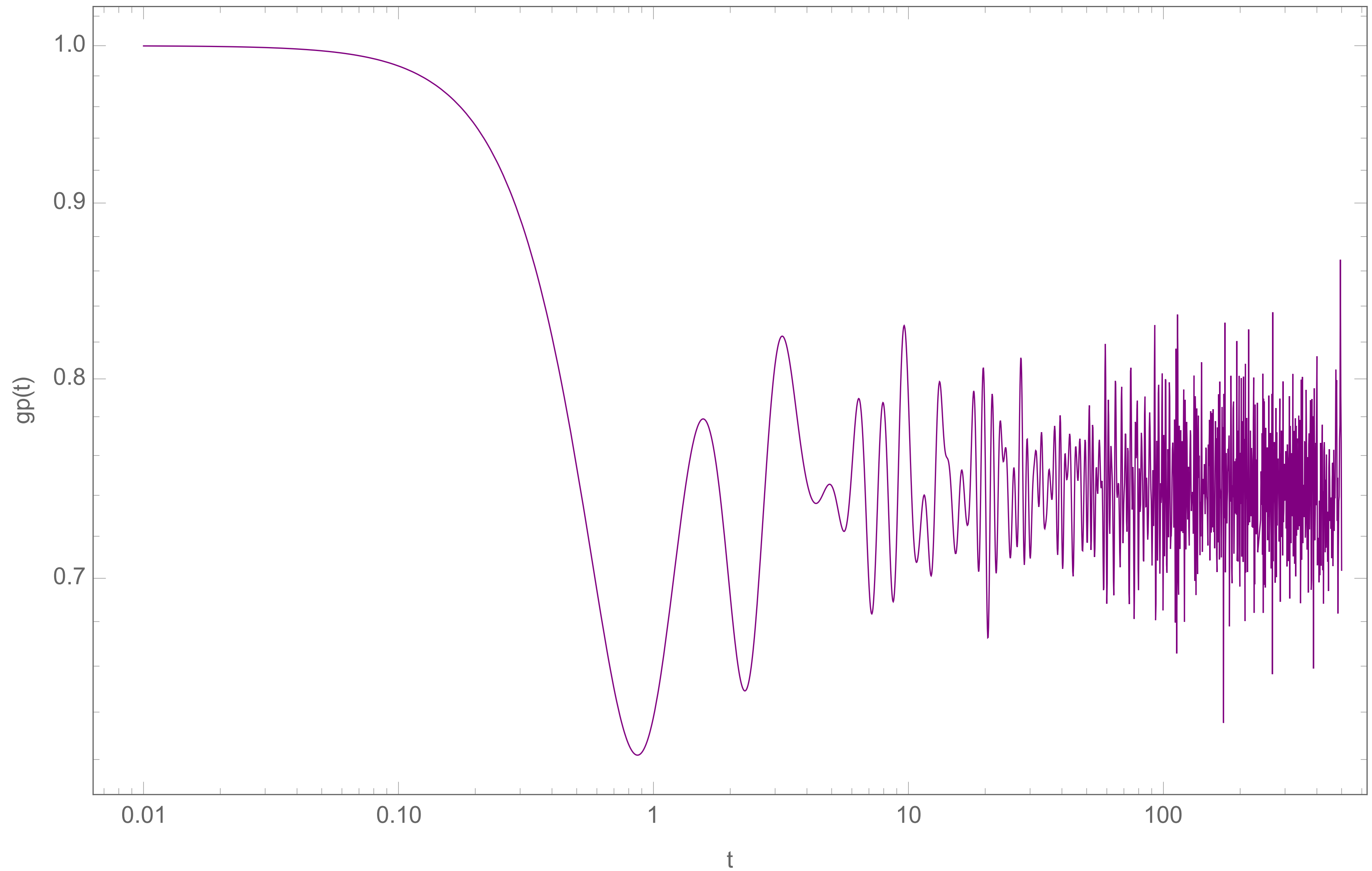
Kink: sign of the Symplectic ensemble [Mehta]

Return amplitude in Finite β pure states

[WIP with T. Nosaka]

- Choosing the parameter β so that we maximize $|\langle G(\mu)|B_s(\beta)\rangle|$
- Keep big value, but smaller than 1.

$\mu = 0.1$ single sample



- In Two coupled SYK [Maldacena-Qi], the return amplitude keep 1at late time in large q limit.

Conclusion

- We derive the analytic expression that relates Return Amplitude to the Spectral Form Factor

- A simple relation between the spectral form factor and the return amplitude

- The SYK model also obeys the same relation.

- State dependent deformation keeps the value of RA, but if there is an error they show random matrix behaviors.