Soft modes from black hole microstates

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It from Qubit Workshop
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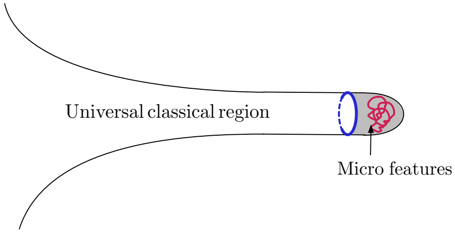
Based on

- V. Balasubramanian, D. Berenstein, A. Lewkowycz, A. Miller, OP & C. Rabideau, arXiv:1810.13440 [hep-th].
- Work in Progress.

Introduction

- It is widely expected that the black hole geometry is a coarse-grained description of a large number of underlying microstates [Strominger, Vafa '96, Lunin, Mathur '01, Balasubramanian, de Boer, Jejjala, Simon '05, Alday, de Boer, Messamah '06...].



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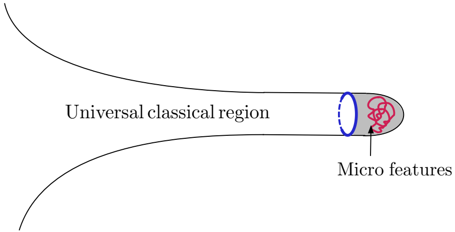

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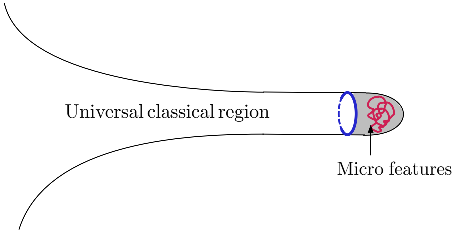

- Our aim in this talk will be to study the effects of this coarse-graining on the classical phase space and symplectic form for excitations around the black hole geometry.
- We will argue that the coarse-graining has a non-trivial effect – it leads to an emergent soft mode on the stretched horizon.

Preliminaries: CFT side

- We will work with an incipient black hole, called the 1/2-BPS superstar, whose microstates are 1/2-BPS states in $\mathcal{N} = 4$ SYM [Myers & Tafjord '01, Balasubramanian, de Boer, Jejjala, Simon '05].

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- The $\frac{1}{2}$-BPS sector of $\mathcal{N} = 4$ SYM theory can be reduced to N free fermions in a harmonic-oscillator potential:

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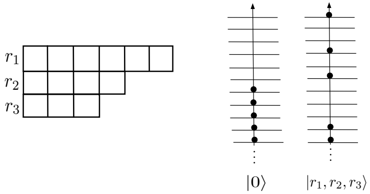
- The ground state is given by filling the first N energy levels of the oscillator, which we refer to as the *Fermi sea*. Excited states can be labelled by *Young diagrams*:



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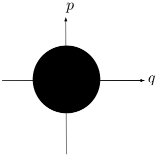
- For example, in the classical limit $N \to \infty$, $\hbar \to 0$ with $N\hbar$ fixed, the Fermi sea is given by

$$u(q,p) = \Theta(2\hbar N - q^2 - p^2).$$

which we can pictorially represent as a black disc:



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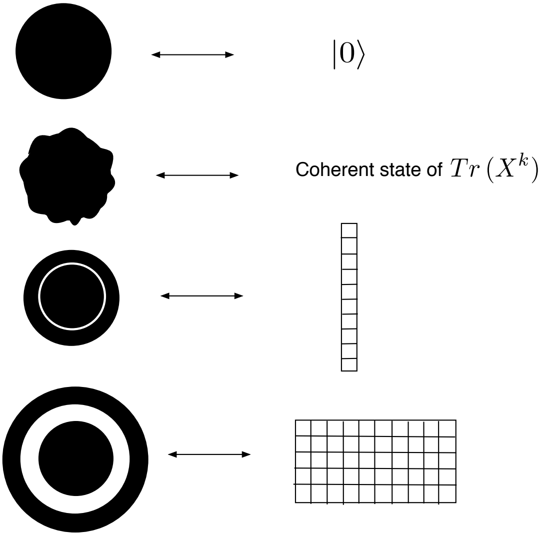
- Here are some further examples:



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- **Remark**: y-evolution has the effect of coarse-graining.

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- **Remark**: The gravity description makes sense for sufficiently "classical" states.

Superstars

- We will be interested in states of energy $O(N^2)$, with gravity duals which look like incipient black holes called *superstars* [Myers & Tafjord '01, Balasubramanian, de Boer, Jejjala, Simon '05].

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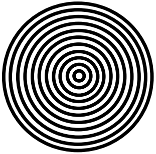

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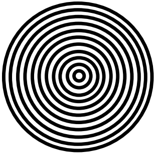

- So, there is no obvious classical limit! On the gravity side,the "geometry" would have Planck-scale topological features close to $y = 0$.

Superstars

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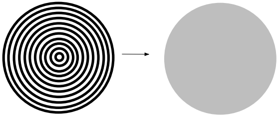

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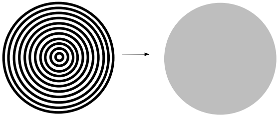
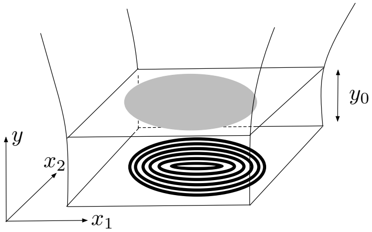


- However, translated into gravity, such a "grey" boundary condition does not correspond to a regular geometry – indeed, if we use such a boundary condition in constructing the LLM metric, the resulting geometry has a singularity at $y = 0$ – the 1/2 BPS-superstar [Myers & Tafjord '01].

Superstars

We may think of this as a toy model for black hole singularities in general relativity [Balasubramanian, de Boer, Jejjala, Simon '05].



The geometry for $y \gg y_0$ is well-approximated by the superstar, but for $y < y_0$ it is perfectly regular. The purported singularity is replaced by a topologically complex, but regular LLM geometry.

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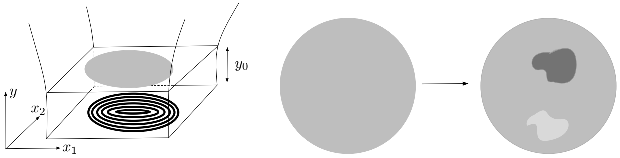
- For deformations, we can consider greyscale deformations at $y = y_0$: $0 \leq \delta u(p,q) \leq 1$:



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- But this leads to a puzzle – what are we to make of this edge mode from the CFT side?

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- But first – how do we extract the microscopic phase space and symplectic form from the CFT?
- Answer: We can get the microscopic symplectic form by using coherent states and the method of coadjoint orbits [Kirillov, Yaffe '82..., Belin, Lewkowycz, Sarosi '18, Verlinde].

$$\mathbf{\Omega}_{CFT}(u_0; \delta_1 u_0, \delta_2 u_0) = \int \frac{dpdq}{2\pi} u_0 \{\delta_1\pi, \delta_2\pi\}_{PB}.$$

where $\delta\pi$ is defined in terms of δu_0 by the equation

$$\delta u_0 = \{\delta\pi, u_0\}_{PB} = (\partial_p \delta\pi \partial_q u_0 - \partial_p u_0 \partial_q \delta\pi).$$

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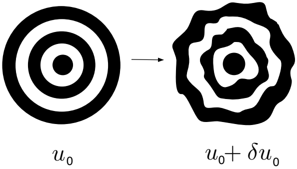
- Let us now apply this to the case of interest:



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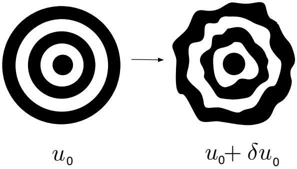

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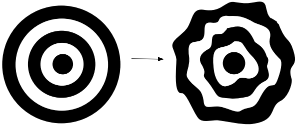

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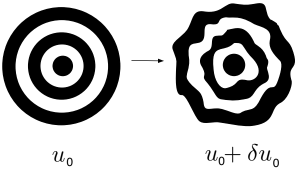

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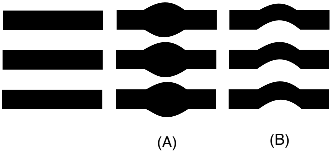
- Using $\delta u_0 = \{\delta\pi, u_0\}_{PB}$, we can solve for $\delta\pi$ and explicitly obtain the microscopic symplectic form in terms of $\delta u_n^{(0)}$.
- Crucially, we must now coarse grain these at some scale $y_0 \gg \epsilon$ to obtain the effective phase space variables.

Phase space: CFT side

- We can do this by rewriting $\delta u_n^{(0)}$ in terms of two slowly-varying modes:

$$\delta u_n^{(0)}(\theta) = \delta A(r_n, \theta) + \frac{e^{i\pi r/\epsilon}}{\epsilon}\delta B(r_n, \theta).$$



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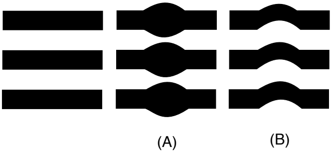

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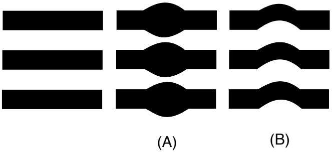

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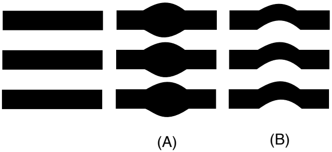

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- On the other hand, δB is a slowly varying mode which is not visible in the coarse-grained density because of the osclllatory factor.
- But if we compute the symplectic form, we find

$$\boldsymbol{\Omega}_{CFT} = \int \frac{dpdq}{2\pi}\,\mathrm{Sign}(\theta - \theta')\boldsymbol{\delta}A(r,\theta)\,\boldsymbol{\delta}B(r,\theta').$$

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- So in summary, upon coarse-graining the full UV density δu_0, we obtain two effectively independent slowly-varying modes: the greyscale fluctuation δu_{coarse} and its canonical momentum

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- This leads us to identify the gravitational soft mode at the horizon as an emergent slowly varying mode in the CFT corresponding to *microstate deformations*.

Summary

- We studied the phase space of fluctuations around an incipient 1/2 BPS black hole.
- On the gravity side, we found a new physical soft mode at the stretched horizon.
- We explicitly constructed the coarse-grained phase space from the CFT side, and found that the soft mode is associated with microstate deformations.
- Note that this mode is still an effective IR mode – it is merely hidden in the UV part of phase space, and needs to be delicately extracted.
- In recent discussions of the information paradox, analogous "soft hair" have been discussed [Hawking, Perry, Strominger '16, Donnay et al '16...].
- Our discussion may provide hints about how aspects of microstates might get imprinted on such soft modes.