Independent work by Almheiri, Engelhardt, Marolf and Maxfield was published simultaneously

Entanglement Wedge Reconstruction and the Information Paradox

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arXiv:1905.08255

The Information Paradox in AdS/CFT

- AdS/CFT "solves" the information paradox: the information gets out!
- However we still want to know **how** the information gets out from a bulk perspective. Why was Hawking wrong?
- 2012 (15 years after AdS/CFT): Firewall paradox – everyone still very confused
- Since then, considerable progress in our understanding of AdS/CFT (e.g. ER=EPR, entanglement wedge reconstruction, state-dependence)
- Key tool: thermofield double state (well-understood geometry)
- BUT, an evaporating black hole is never in the thermofield double state

To understand evaporating black holes, we eventually need to study an evaporating black hole!

This talk

Show everything using bulk calculations *(with input from holography via entanglement wedge reconstruction)*

Assumptions:

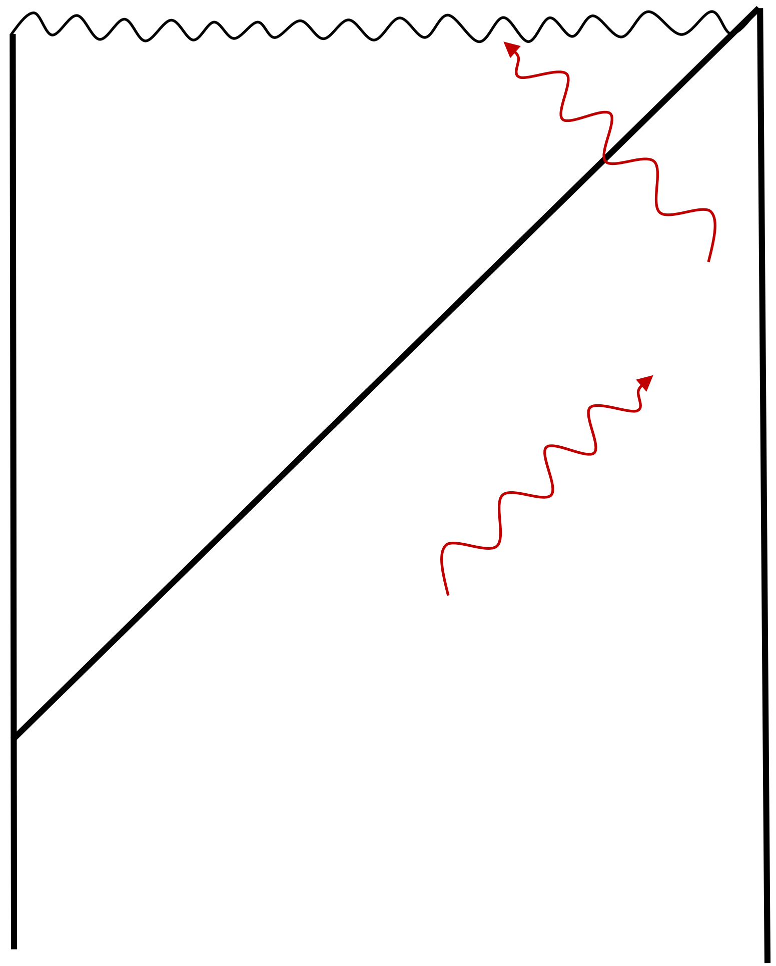
1. GR is valid at small curvature *(even after the Page time)*
2. Entanglement wedge reconstruction

Conclusions:

1. No information escapes **before** the Page time.
2. But, **non-perturbatively small corrections** to thermal Hawking radiation long before the Page time.
3. Entanglement entropy follows **the Page curve**.
4. No AMPS **firewall paradox**.
5. A small diary thrown into a known black hole at an early time can be reconstructed from the Hawking radiation at **the Page time**.
6. If thrown in after the Page time, it can be reconstructed after waiting for **the scrambling time**.
7. Generalisations to large diaries, partially unknown initial black hole states etc.

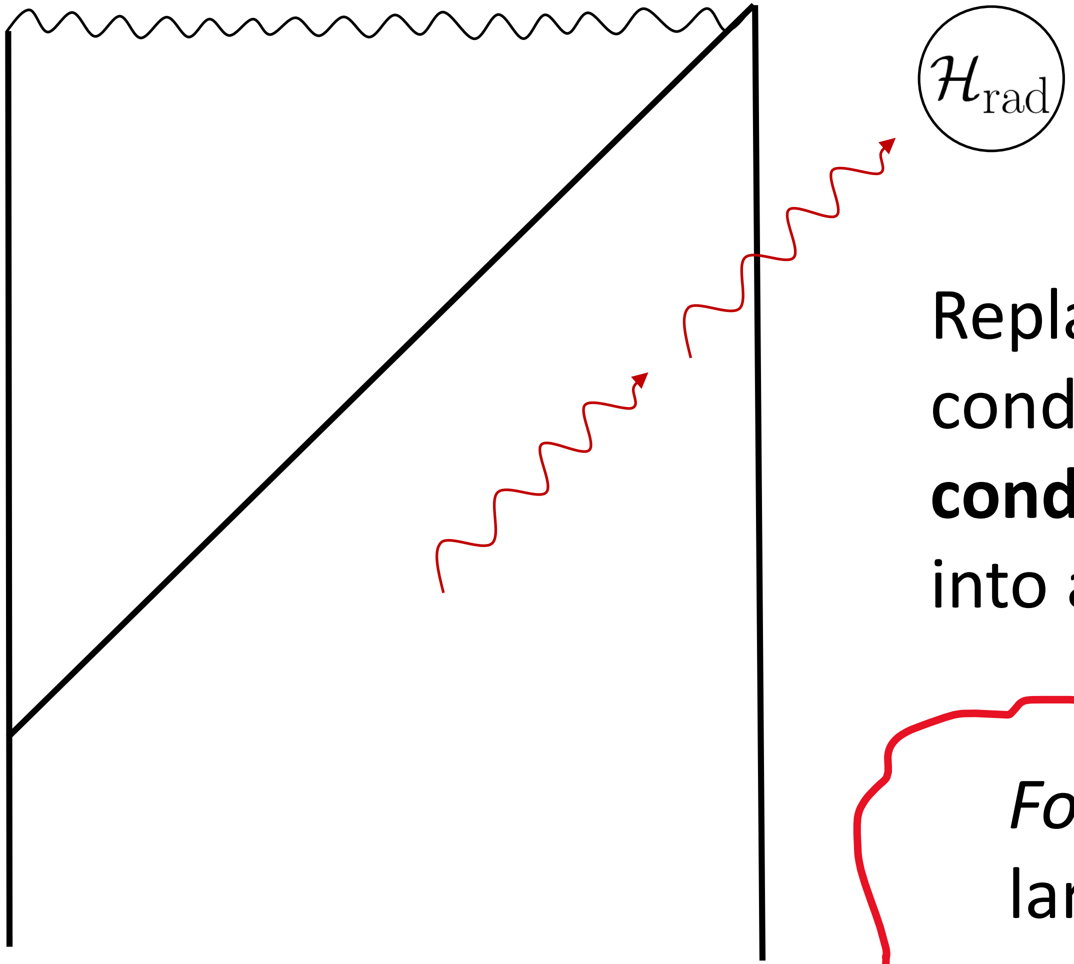
Hayden-Preskill decoding criterion

Evaporating Black Holes



Black holes in AdS do not spontaneously evaporate *(unless very very small)*

Evaporating Black Holes



Boundary perspective: couple CFT to auxiliary system

Replace reflecting boundary conditions with **absorbing boundary conditions**: extract Hawking radiation into an auxiliary system $\mathcal{H}_{\mathrm{rad}}$

For concreteness: assume $\mathcal{H}_{\mathrm{rad}}$ large holographic system (to avoid backreaction)

Entanglement Wedge Reconstruction

Two holographic boundary systems $\mathcal{H}_{\mathrm{CFT}}$ and $\mathcal{H}_{\mathrm{rad}}$. *Key question:* what bulk region is encoded in each boundary system?

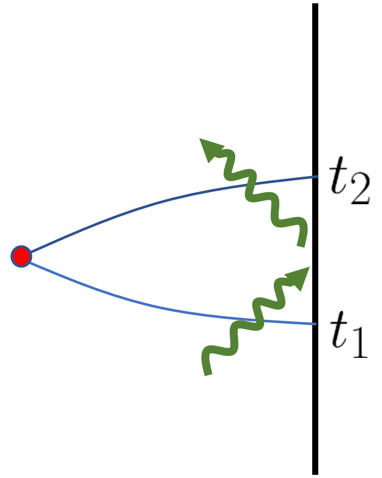
Answer: ***The Entanglement Wedge***

Entanglement Wedge Reconstruction

$$\chi \cup B = \partial C$$

Quantum extremal surface χ

$$\text{ext}\ \left[\frac{A(\chi)}{4G_N} + S_{\text{bulk}}(C)\right]$$



Reflecting boundary conditions $\Rightarrow$ independent of time

Entanglement Wedge Reconstruction

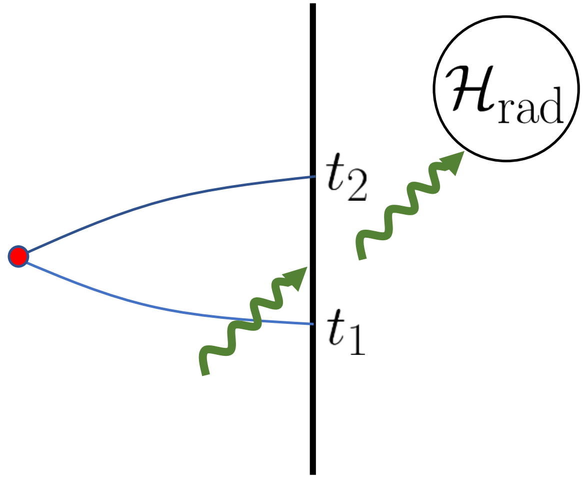
$$\chi \cup B = \partial C$$

Quantum extremal surface

$$\text{ext} \quad \left[\frac{A(\chi)}{4G_N} + S_{\text{bulk}}(C) \right]$$

Quantum RT surface = minimal generalised entropy quantum extremal surface

The entanglement wedge of B is the domain of dependence of C



Absorbing boundary conditions $\Rightarrow$ **time-dependent**

Entanglement Wedge Reconstruction

$$\chi \cup B = \partial C$$

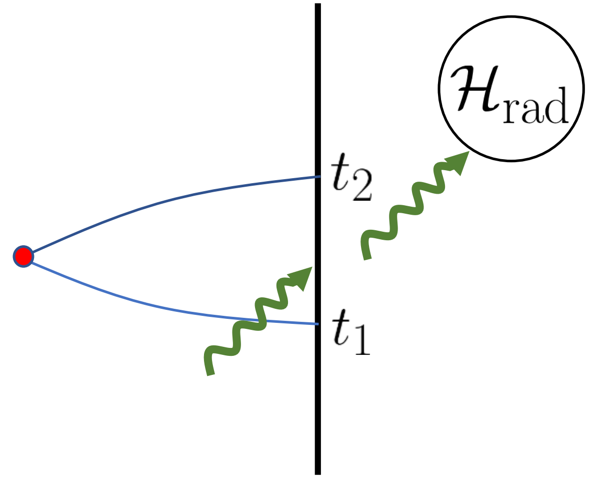
Quantum extremal surface

$$\text{ext} \left[\frac{A(\chi)}{4G_N} + S_{\text{bulk}}(\chi) \right]$$

Quantum RT surface = minimal generalised entropy quantum extremal surface

Very helpful (if unnecessary): assume quantum RT surface can be found by a maximin prescription:

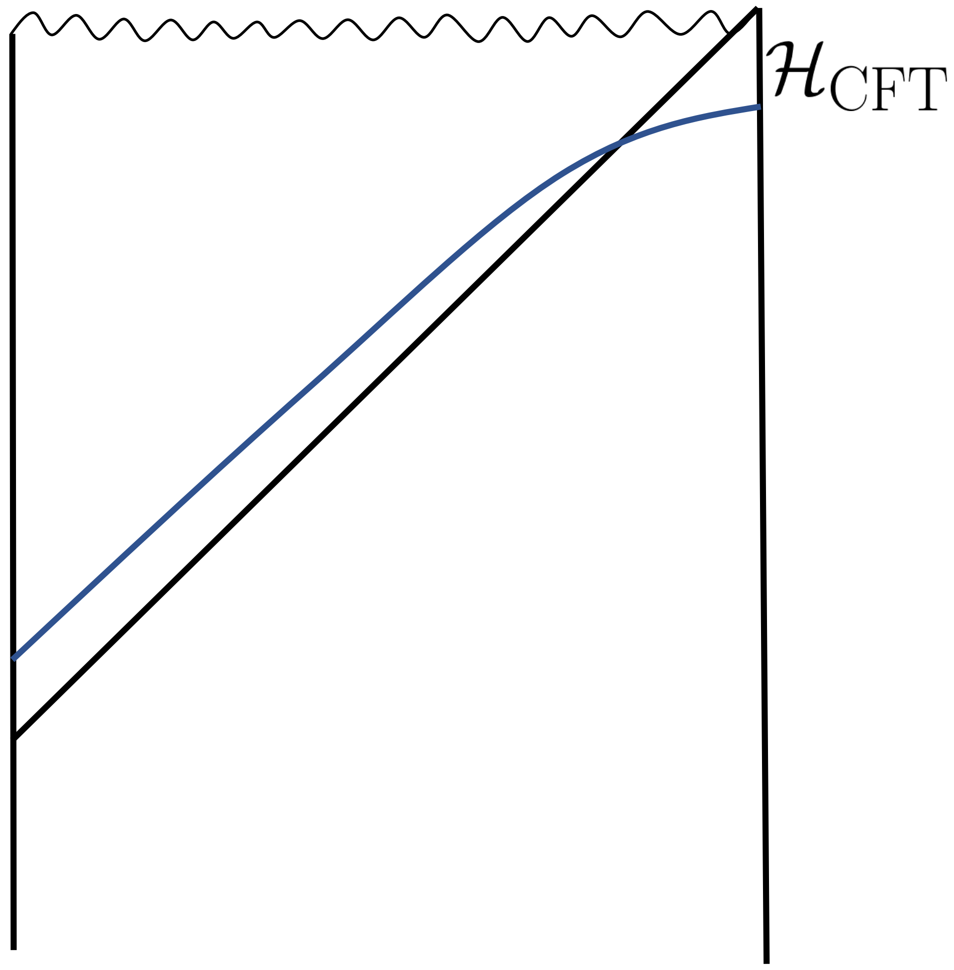
$$\max_{\text{Cauchy}} \min_{\chi} \frac{A(\chi)}{4G_N} + S_{\text{bulk}}(\chi)$$



Absorbing boundary conditions $\Rightarrow$ **time-dependent**

Before the Page Time

Consider a traditional ‘nice’ Cauchy slice



Before the Page Time

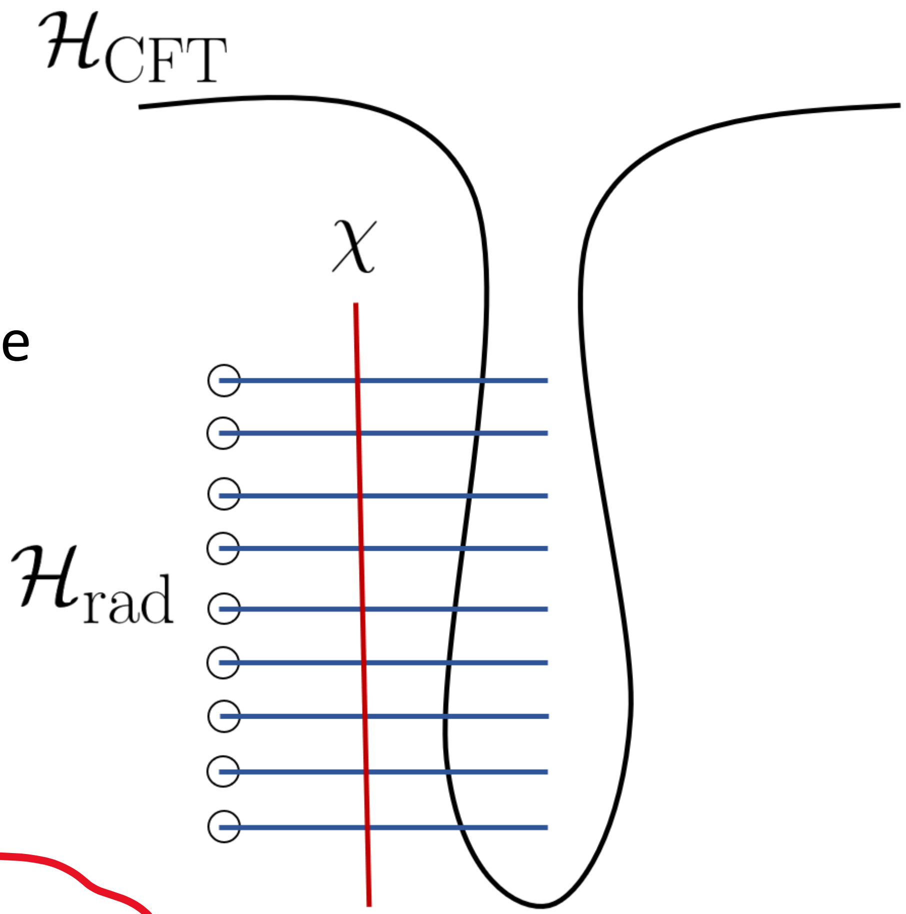
Minimal generalised entropy surface (in this Cauchy slice) is the **empty surface**

in **every** Cauchy slice

Quantum maximin $\Rightarrow$ **quantum RT surface is empty**

The interior is in the entanglement wedge of the CFT

No information has escaped!



$$\frac{A_{\rm hor}}{4G_N} < S_{\rm rad}$$

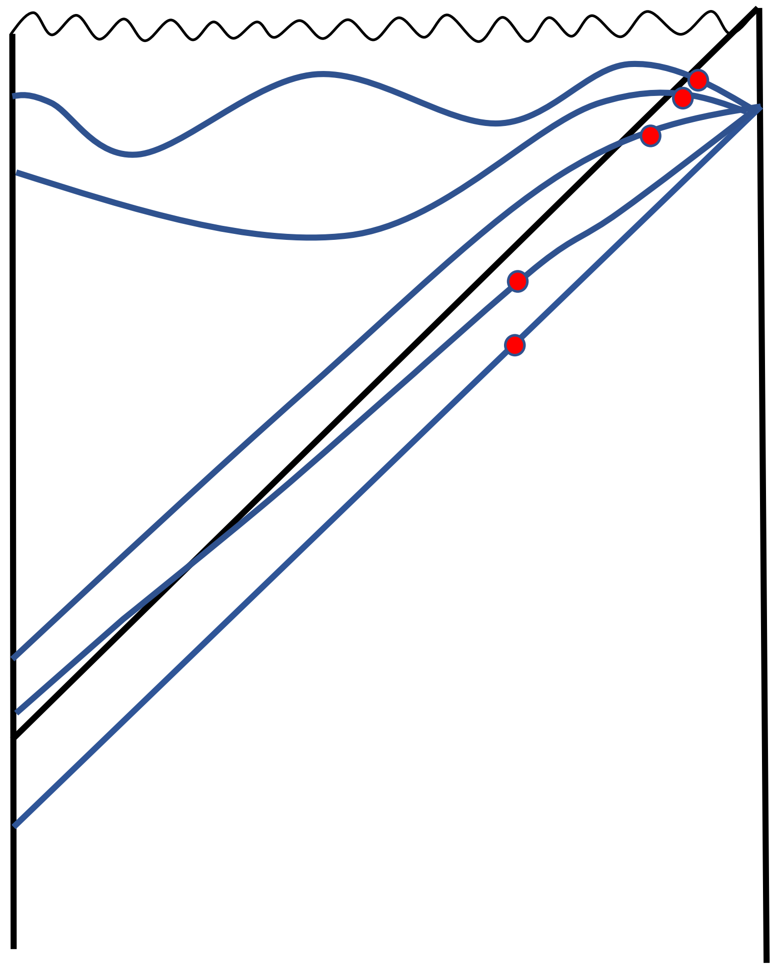
After the Page Time

In **any** Cauchy slice, there exists a surface that

a) lies entirely outside the event horizon

b) has area only slightly larger than the horizon area

O(1) bulk entropy



The generalised entropy of this surface is less than empty surface

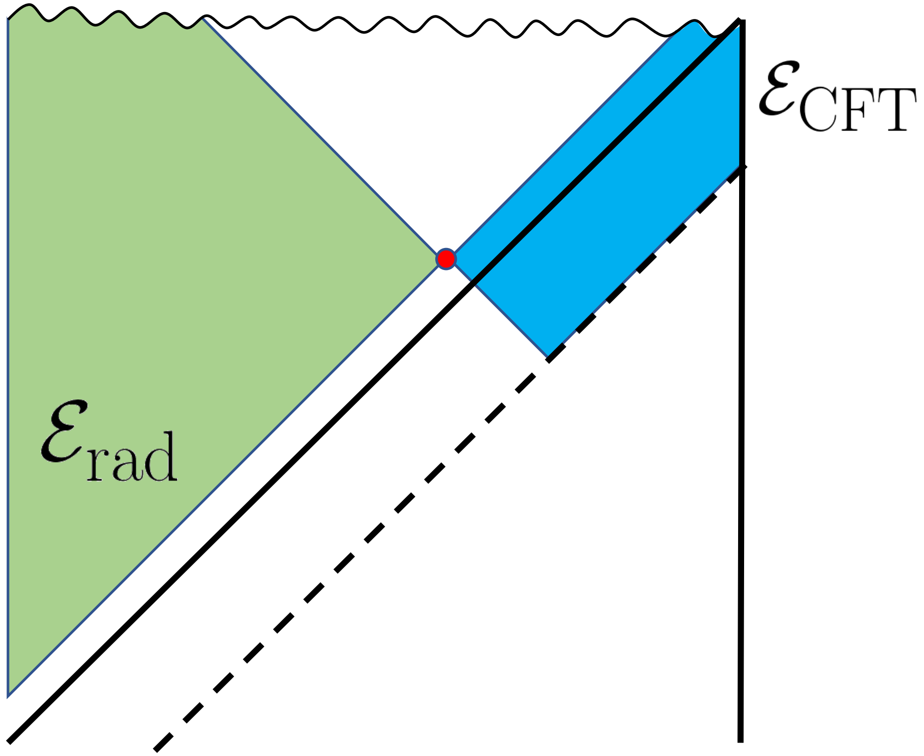
Quantum maximin $\Rightarrow$ **quantum RT surface cannot be empty**

To be justified at the end if I have time

A claim

There exists a **non-empty quantum extremal surface** that lies just inside the event horizon, exactly the **scrambling time** in the past

After the Page time, this becomes the **quantum RT surface**

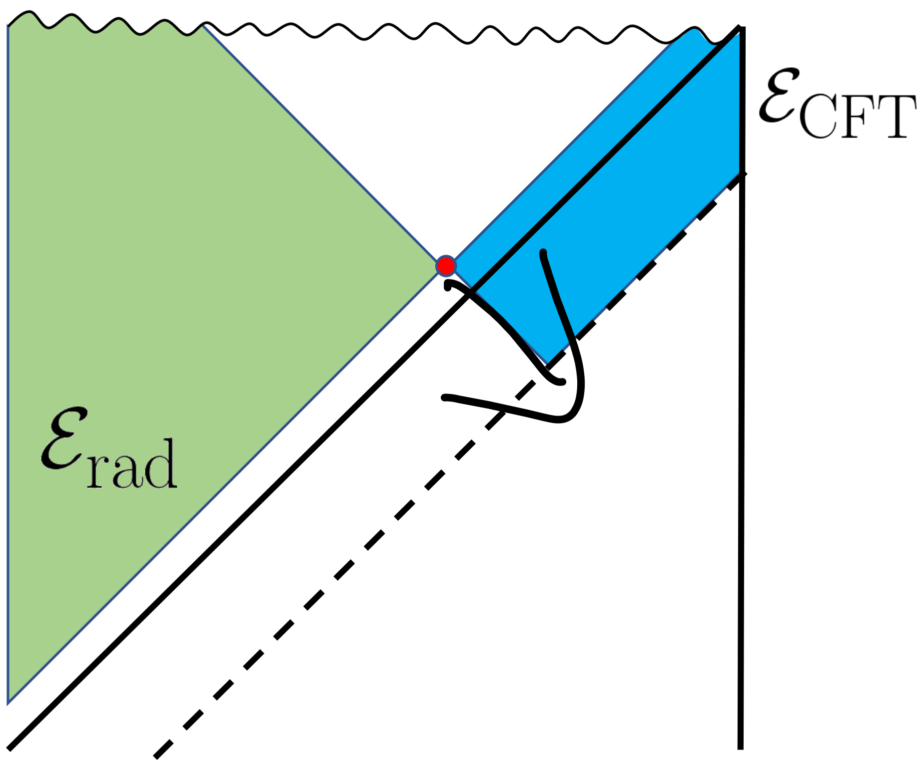


A claim

But wait what about the factor of $1/G_N$?

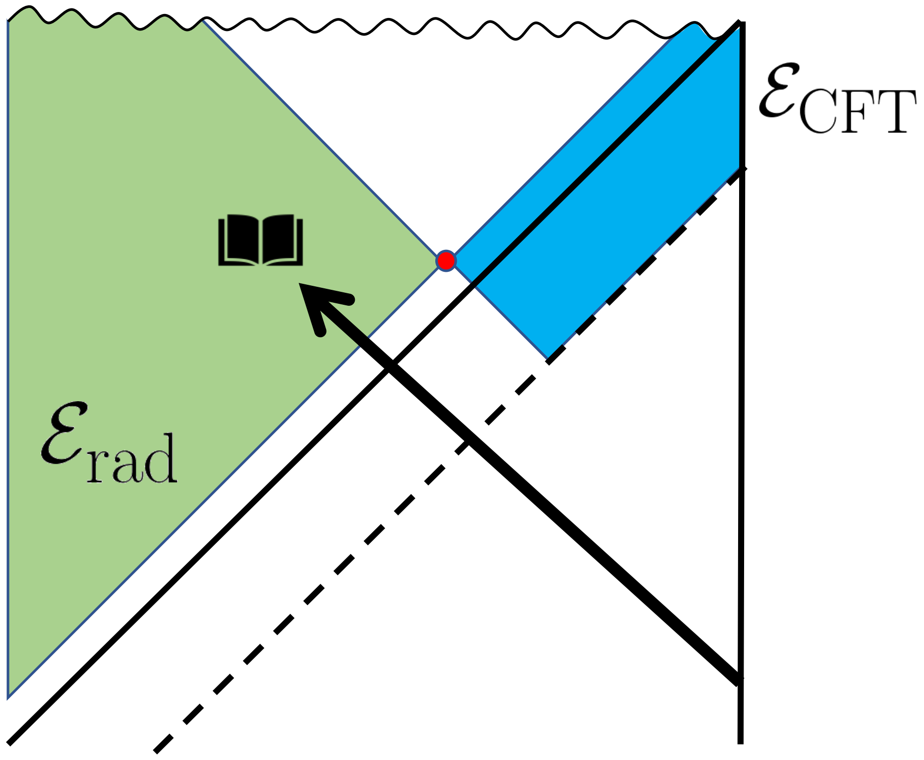
Intuition: moving the surface outwards increases its area, but decreases the bulk entropy. These effects cancel.

A shift of $\Delta r = O(G_N)$, one scrambling time in the past, has the same effect on the bulk entropy as a shift of $\Delta r = O(1)$ at the current time.



Hayden-Preskill

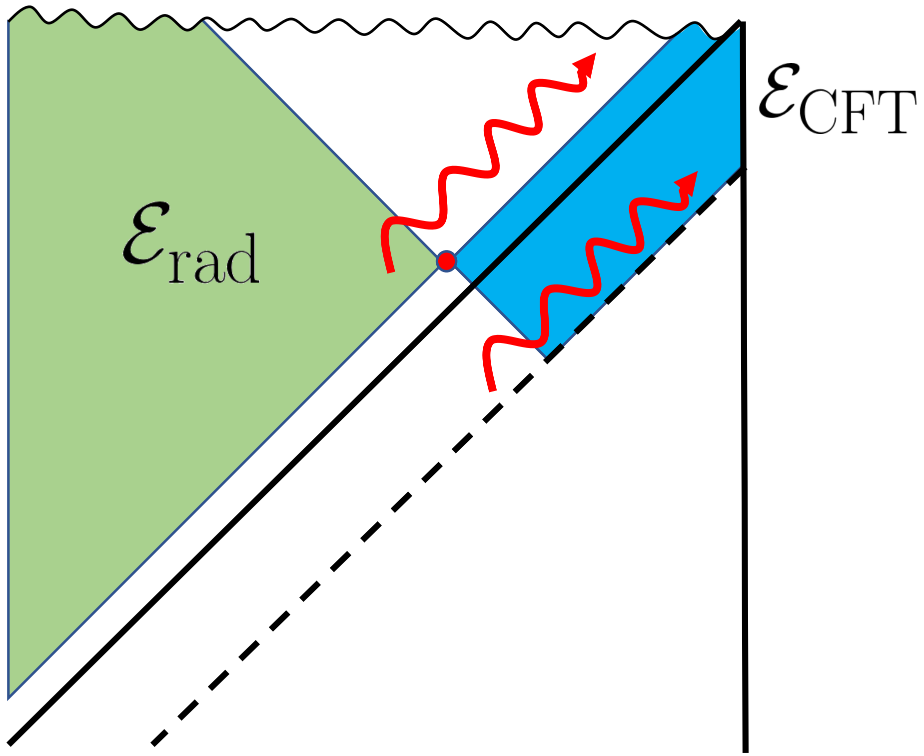
After the Page time, a diary, thrown into the black hole more than the **scrambling time** in the past, will be in the entanglement wedge of $\mathcal{H}_{\mathrm{rad}}$



The Page Curve

Ryu-Takayanagi formula $\Rightarrow$ the Page curve

Entanglement wedge reconstruction **explains** the Page curve: Hawking modes entangled with partner modes encoded in Hawking radiation. **No firewall!**



Decrease in entanglement **strictly less** than the entropy of the new Hawking radiation because the RT surface is **strictly inside** the event horizon

GSL is a **strict inequality**

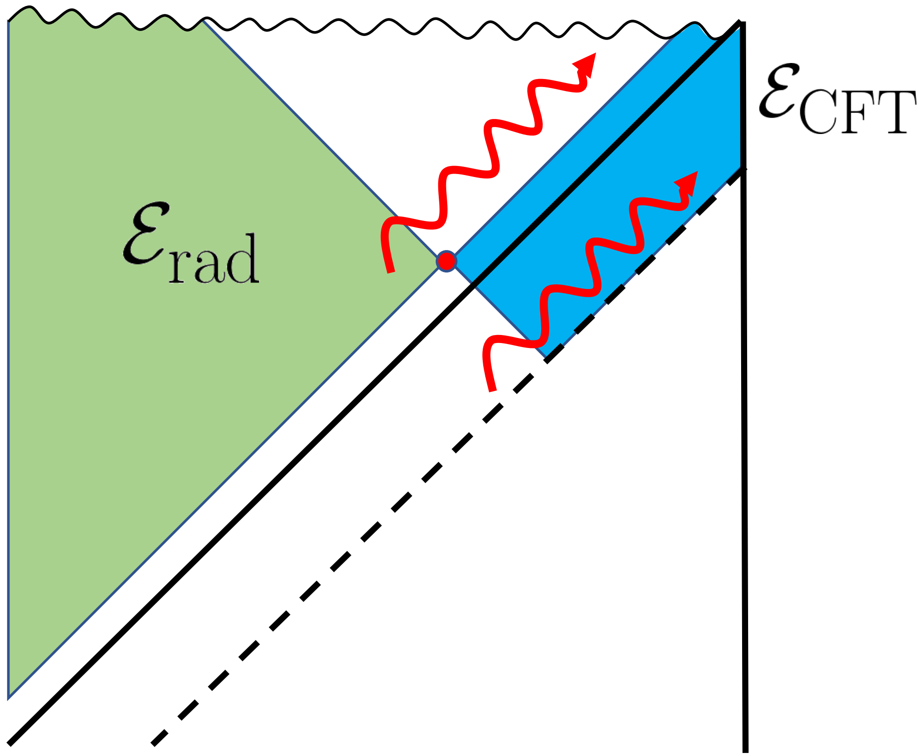
Exact quantitative agreement between the Page curve and the bulk entanglement structure

Always true as a consequence of RT surface being an extremum of the generalised entropy

How does the information get out?

The entanglement between the Hawking radiation and the interior does not depend on the **initial state** of the black hole or **any diary** that was thrown in

We've explained how the final state of $\mathcal{H}_{\rm rad}$ can be pure, but not how it can ***encode the information*** that was thrown into the black hole (as implied by entanglement wedge reconstruction).



Missing the final piece of the puzzle: state dependence

State Dependence

One can show using **approximate operator algebra quantum error correction** that a bulk operator can only be reconstructed on B *within a given code subspace* if it is contained in the entanglement wedge of B for all states **including mixed states** in that code subspace.

If it is only in the entanglement wedge for **pure states** in that code space, only a **state-dependent** reconstruction will be possible.

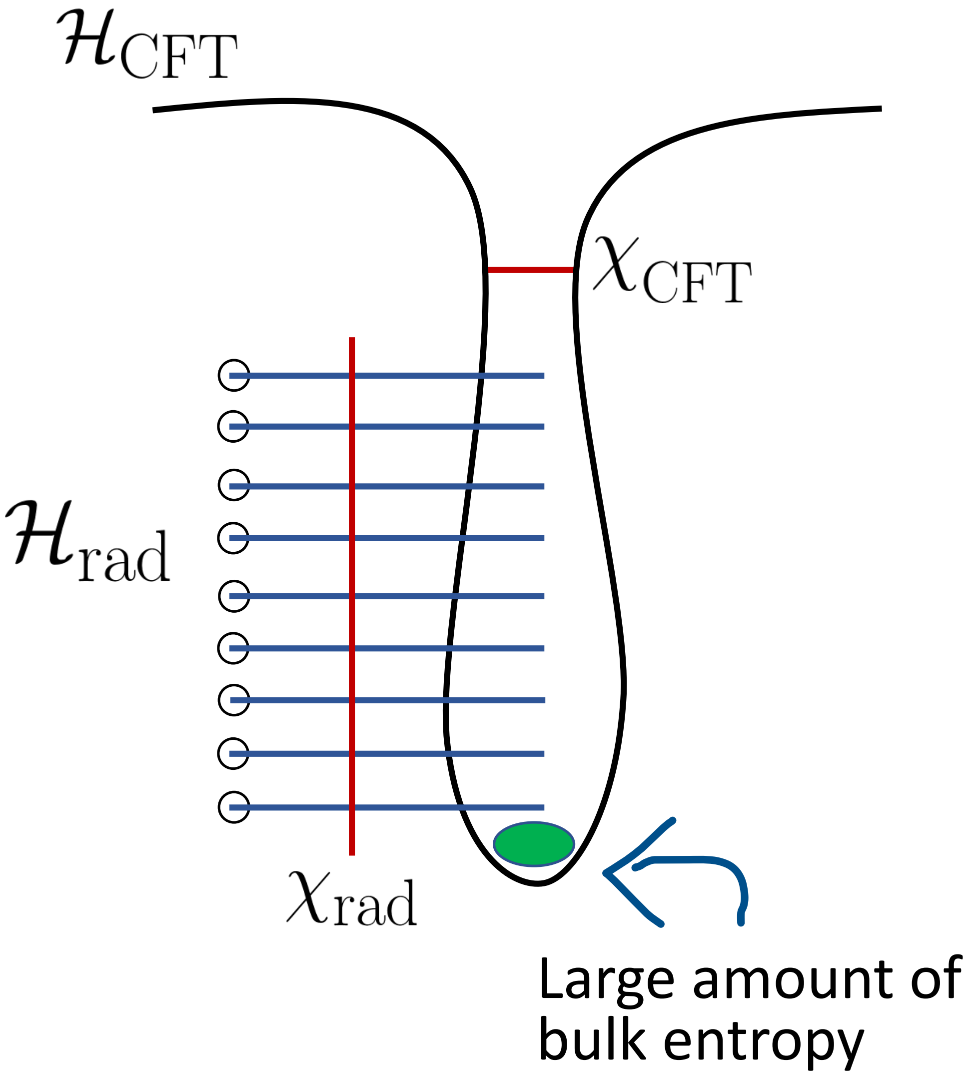
Hayden + GP, 2018
*arXiv:*1807.06041

(Partially) Unknown Initial Microstates

Interior can only be reconstructed from $\mathcal{H}_{\text{rad}}$ if the initial state is known to be in a **sufficiently small code space**. Otherwise it will not be in the entanglement wedge for sufficiently mixed states.

$$S_{\text{code}} < S_{\text{rad}} - S_{BH}$$

Agrees with toy models!

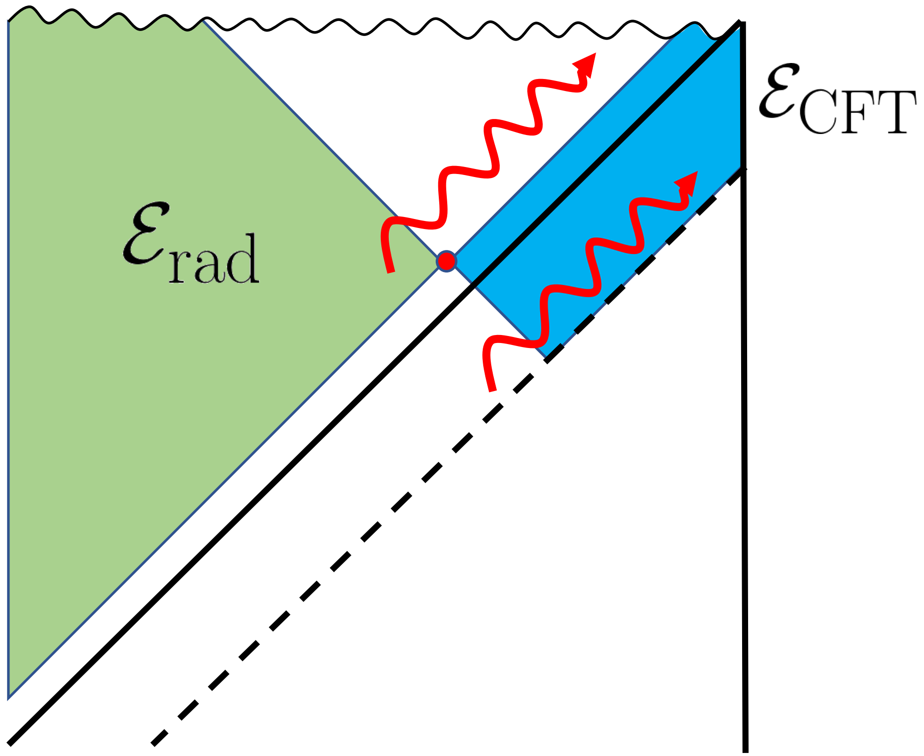


How does the information get out?

Entanglement between Hawking radiation interior is independent of the initial black hole microstate.

But the encoding of the interior in $\mathcal{H}_{\mathrm{rad}}$ depends on the initial state.

Hence, the Hawking radiation provides new information about the initial state to an observer with access only to $\mathcal{H}_{\mathrm{rad}}$



Conclusions

- There is a phase transition in the **quantum RT surface** of an evaporating black hole at **exactly the Page time**. The new RT surface lies just inside the event horizon, one scrambling time in the past.
- This explains the **Page curve** using the **RT formula**
- It also explains the **Hayden-Preskill decoding criterion** using **entanglement wedge reconstruction**
- Entanglement wedge reconstruction also provides the **mechanism** that makes the Page curve consistent with the bulk entanglement structure, without a firewall paradox
- Similarly, the **state dependence** of the entanglement wedge reconstruction provides the mechanism by which information is able to escape the black hole
- There would still be paradoxes if entanglement wedge reconstruction were meant to be exact, but these are avoided by **non-perturbatively small corrections**

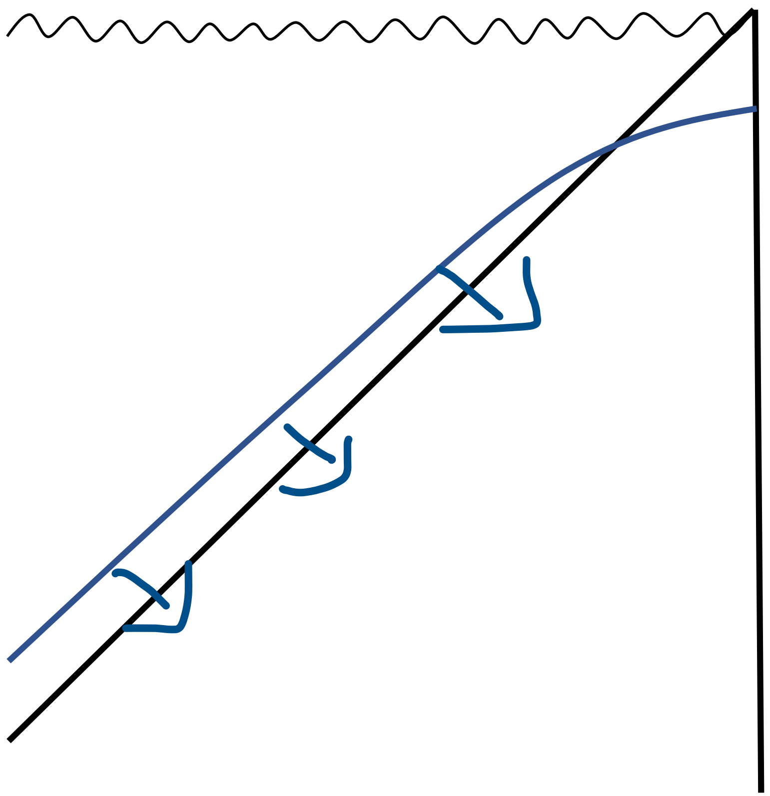
OK. Time to justify that claim

The 'Classical Maximin Surface'

Warm up: assume bulk entropy is locally constant and use the maximin prescription.

Area always decreases along ingoing lightcones.

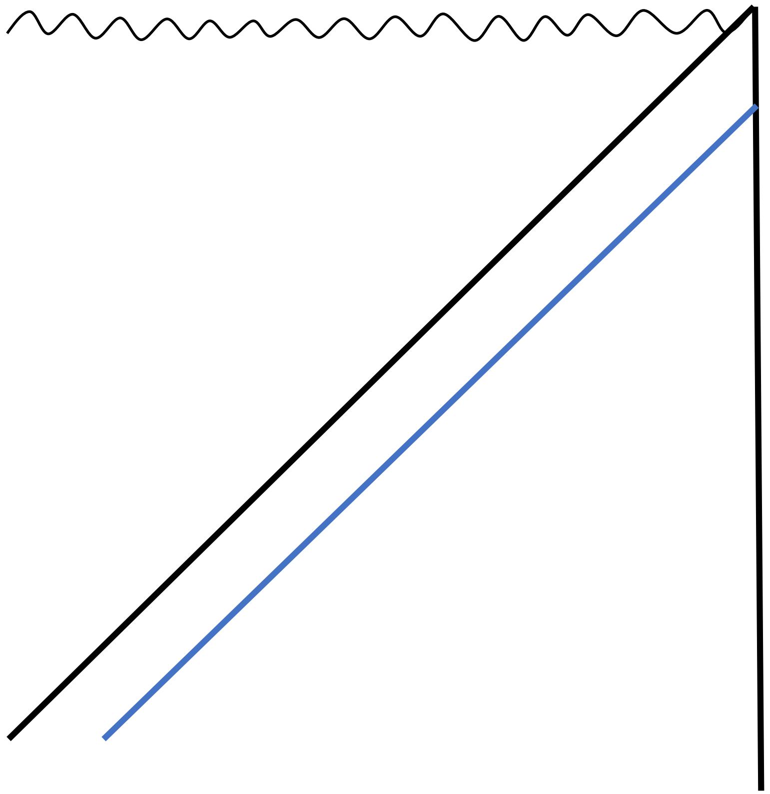
Maximising Cauchy surface = **past lightcone** of the current boundary



The ‘Classical Maximin Surface’

$$ds^2 = -\left(1 + \frac{r^2}{l^2} - \frac{16\pi G_N M(v)}{(d-1)\Omega_{d-1} r^{d-2}}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$

Ingoing Vaidya metric



The 'Classical Maximin Surface'

$\beta, \frac{dr_s}{dv}$ constant

Apparent horizon

$$ds^2 = -\frac{4\pi}{\beta}(r - r_s(v))dv^2 + 2dvdr + r^2 d\Omega^2$$

$$r_{\text{hor}} = r_s - \frac{\beta}{2\pi}\left|\frac{dr_s}{dv}\right|$$

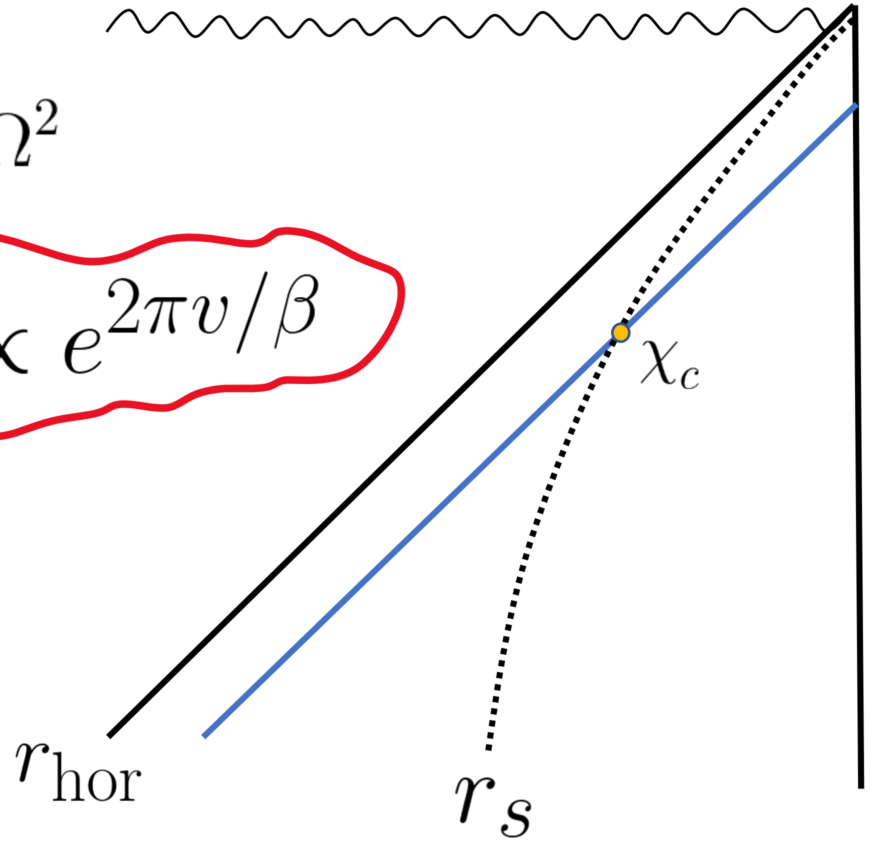
Event horizon

$O(G_N)$

$$r_{l.c} - r_{\text{hor}} \propto e^{2\pi v/\beta}$$

$\boxed{\text{Scrambling time!}}$

$$v = -\frac{\beta}{2\pi}\log S_{BH} + O(\beta)$$



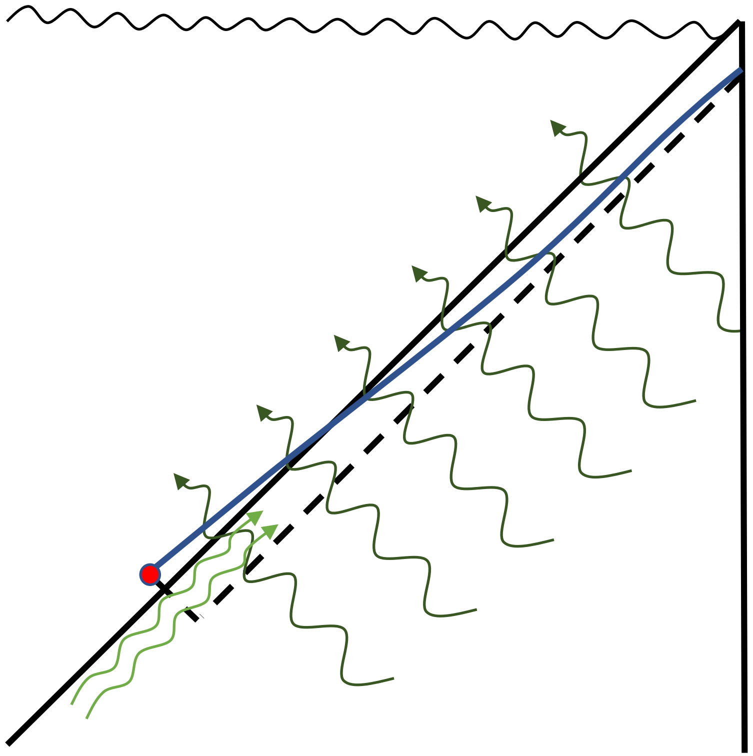
Outside horizon?

Need to include quantum corrections

The Quantum Extremal Surface

As the surface approaches the past lightcone, the entropy of the **outgoing modes** will decrease by a (formally) infinite amount

Quantum extremal surface should be stabilised a small distance away from the past lightcone



Problem: Greybody factors mean that outgoing modes are entangled with later ingoing modes

(Temporary) Solution: Extract Hawking radiation from close to the horizon, before the reflection happens

The Quantum Extremal Surface

$$\boxed{\frac{dM}{dv} = -\frac{\pi c_{\text{evap}}}{12\beta^2}}$$

Entropy of ingoing modes is approximately constant

Entropy of outgoing modes is given by

Number of modes extracted

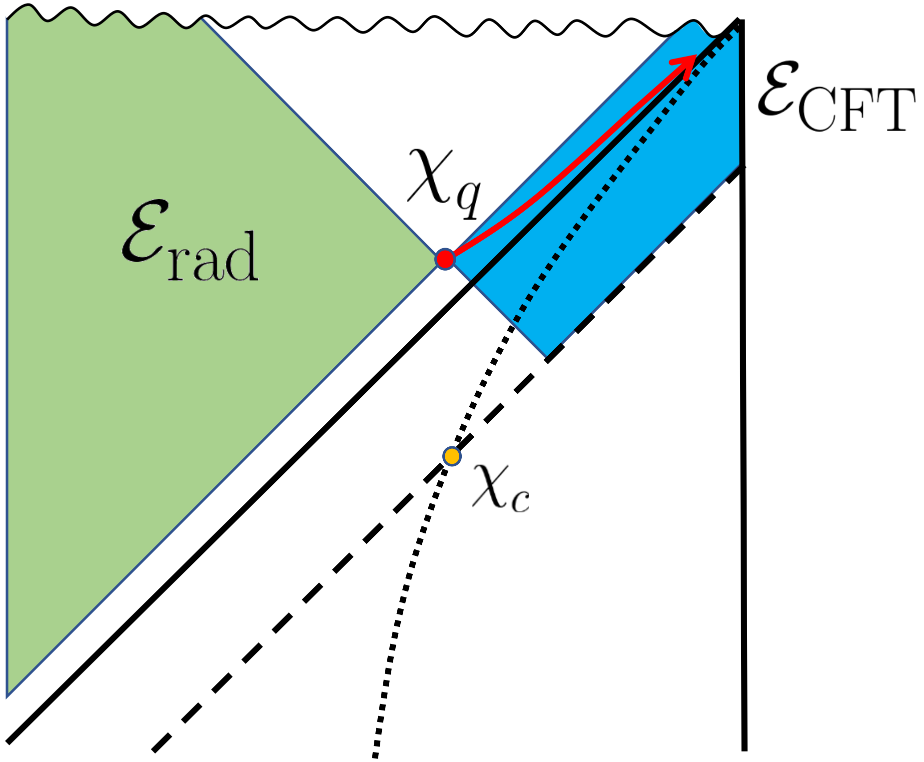
$$S_{\text{out}} = \frac{c_{\text{evap}}}{6} \log \frac{r_{lc}(v) - r}{\sqrt{\varepsilon_1 \varepsilon_2}}$$

Constant (unphysical) cut-off at the quantum extremal surface in units of r

Related to the fixed physical cut-off on the frequency of the extracted Hawking modes by:

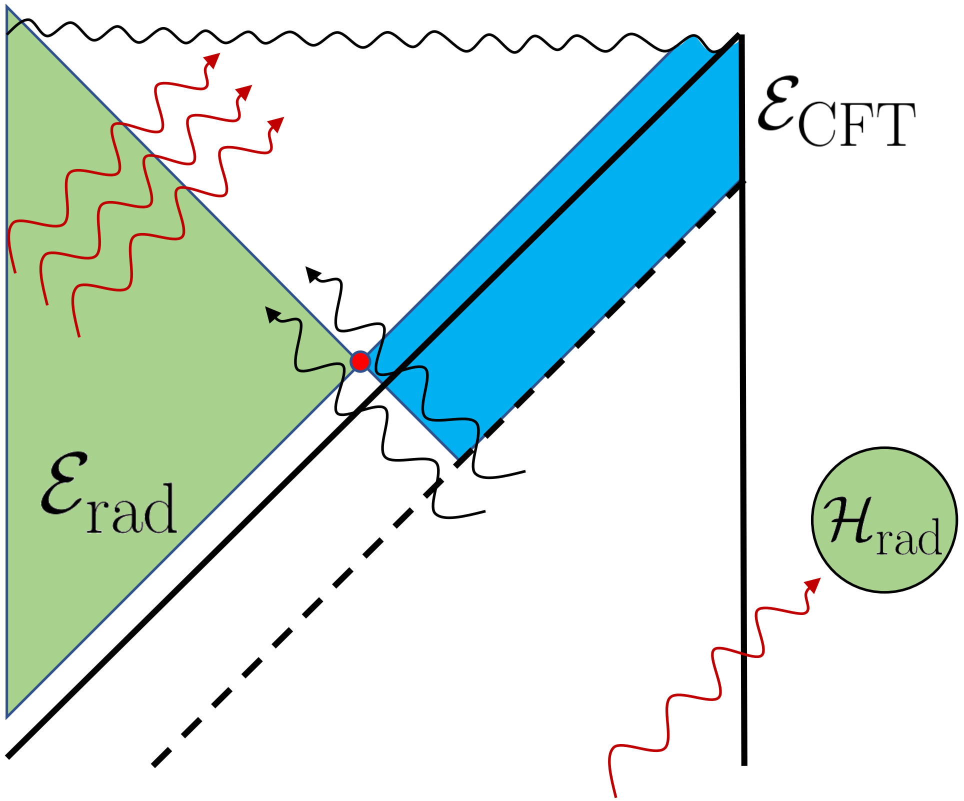
$$\varepsilon_2 = e^{\frac{2\pi v}{\beta}} \varepsilon_0$$

$$\boxed{\begin{aligned} r &= r_s - 2(r_s - r_{\text{hor}}) \\ v &= -\frac{\beta}{2\pi} \log \frac{S_{BH}}{c_{\text{evap}}} + O(\beta) \end{aligned}}$$



Greybody Factors

Infalling modes near quantum extremal surface are in a time-translation invariant mixed state (unentangled with any other modes in the entanglement wedge)



Greybody Factors

Substitute in U_0

Scrambling time!

$$U = (r_{\text{hor}} - r)e^{-2\pi v/\beta}$$

$$U = e^{-2\pi v/\beta}\frac{2G_N\beta}{\pi(d-1)r_s^{d-2}\Omega_{d-1}}\left[\beta\frac{dM}{dv} + \frac{c_{\text{evap}}\pi}{6\beta} - \frac{dS_{\text{in}}}{dv}\right]$$

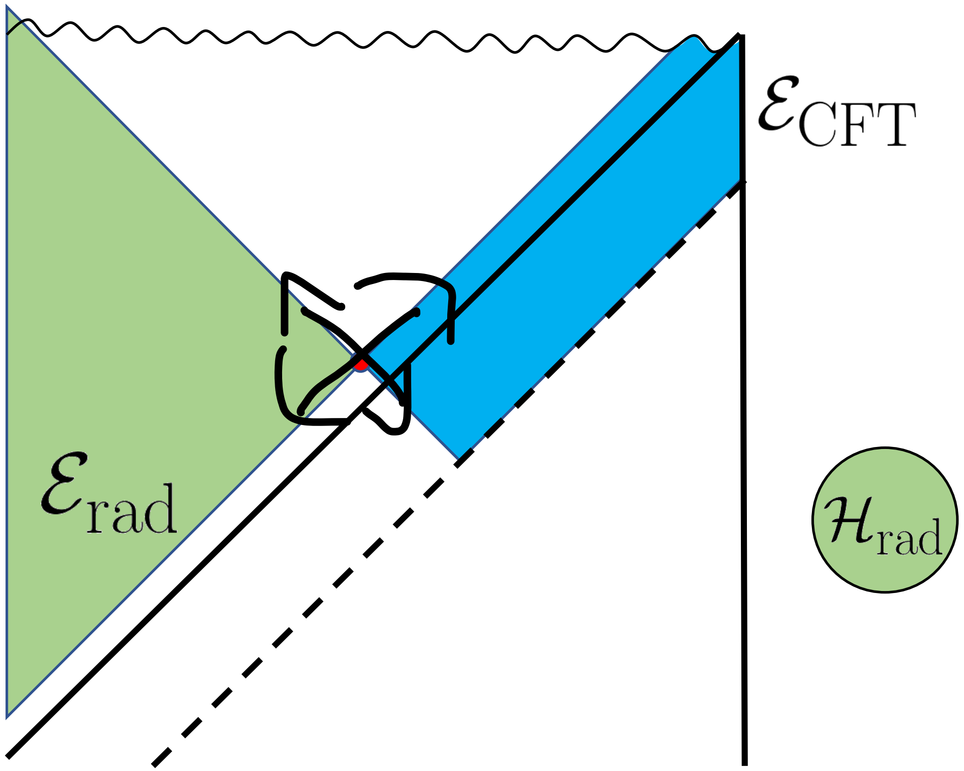
(Numerical) constants

Complicated function

$$U\frac{\partial S_{\text{bulk}}}{\partial U} = \underbrace{\frac{\beta}{2\pi}\left[\beta\frac{dM}{dv} + \frac{c_{\text{evap}}\pi}{6\beta} - \frac{dS_{\text{in}}}{dv}\right]}_{\text{Positive constant}}$$

$$\boxed{\begin{array}{ll} U = 0 & LHS < RHS \\ U \gg 0 & LHS > RHS \end{array}}$$

Must exist solution U_0



Thank you!