ER=EPR revisited: on the entropy of a wormhole

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QIST - It for Qubit workshop

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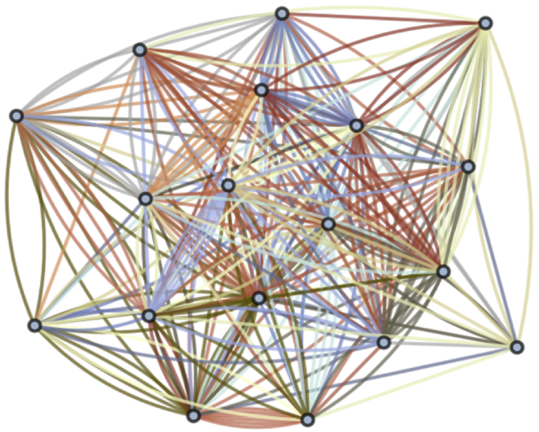
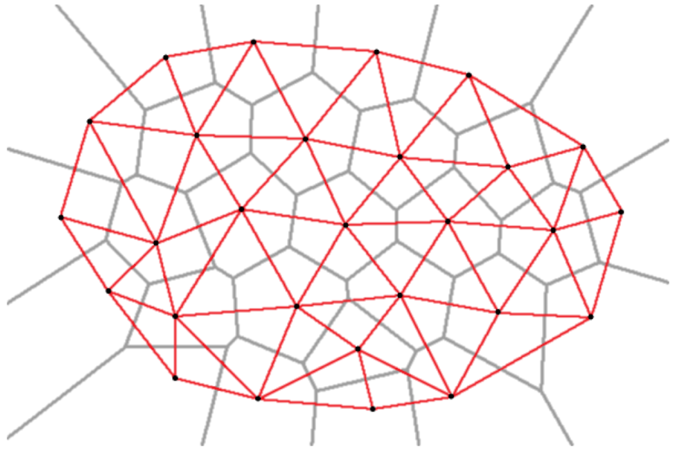
The search for the theory on quantum gravity is guided by basic principles:

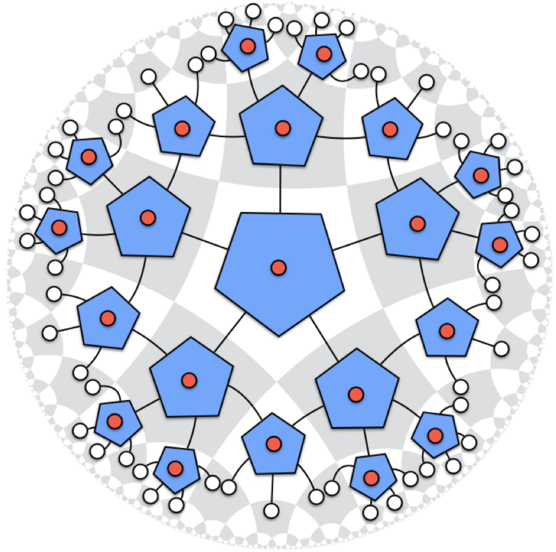
- holography: $S = \frac{1}{4}$ Area
- quantum information theory
- thermodynamics
- locality and causality
- space-time dynamics
- .....

Common Sense

Some helpful tools: many body QM, CFT bootstrap, large N, tensor networks, ….







The Bekenstein-Hawking relation

$$S = \frac{A}{4G_N} \tag{1}$$

forms one of the central guiding principles in the search for a quantum theory of gravity. The precise meaning and realm of applicability of (1), however, are still only partly understood. The two most well-formulated interpretations (1) are that it quantifies:

i) the number of microstates of a one-sided black hole with given macroscopic properties

ii) the microscopic entanglement* across the event horizon connecting the two sides of an eternal black hole.

These interpretations are quite different but both find support via AdS/CFT.

The second interpretation ii) is supported via the Ryu-Takayanagi formula

and by the proposed holographic interpretation of the CFT thermofield double

$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\frac{\beta}{2}E_n} |n\rangle_L |n\rangle_R$$

$$Z = \sum_n e^{-\beta E_n}$$

as the quantum state of a two-sided black hole connected via an ER bridge.

There are several unsatisfactory aspect to the claim that
a two-sided black hole is uniquely described by the TFD:

The TFD is an idealized theoretical construct: it is a unique pure state with zero vN entropy, while two-sided black hole is a macroscopic object which under normal circumstances would carry a large amount of entropy.

So let's ask the question:

How much entropy does a two sided black hole contain?

Clearly it is bounded by: $0 < S < A/4G_N$

In this talk I will argue that

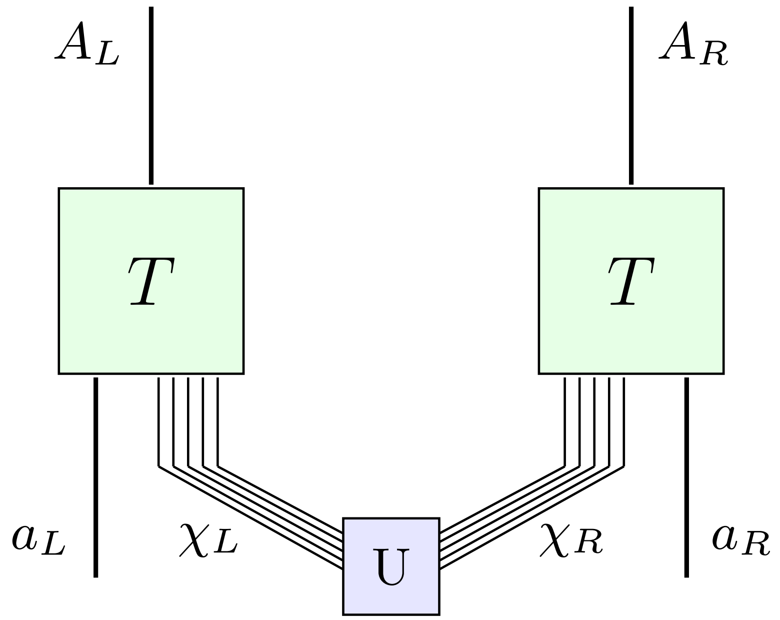
iii) *a space-time with an ER bridge with cross sectional area A can carry a macroscopic amount of quantum information with entropy equal to* $S_{BH} = A/4G_N$.

=> a new holographic conjecture!

Since any macroscopic object usually decoheres into a mixed state, a corollary of this assertion is that

iii') *the local space-time region of an ER bridge is typically in a mixed state with entropy* $S_{BH} = A/4G_N$.

The reasoning makes use of AdS/CFT and the assumption that space-time behaves like a quantum error correcting code – i.e. that the Hilbert space of the effective QFT on a given classical space-time lies within a small code subspace of the full microscopic Hilbert space. The key insight that leads to statement iii) is that the Poincaré recurrence time as seen from within the code subspace is much shorter than the Poincaré recurrence time of the microscopic theory.



Consider a time evolved TFD state

$$|\mathrm{TFD}(t_L, t_R)\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-i(t_L+t_R)E_n} e^{-\frac{\beta}{2}E_n} |n\rangle_L |n\rangle_R$$

Or more abstractly, we can define a `basis' of generalized TFD states

$$|\mathrm{TFD}\rangle_\alpha = \frac{1}{\sqrt{Z}} \sum_n e^{i\alpha_n} e^{-\frac{\beta}{2}E_n} |n\rangle_L |n\rangle_R$$

$${}_\beta\langle\mathrm{TFD}|\mathrm{TFD}\rangle_\alpha = \frac{1}{Z} \sum_n e^{i(\alpha_n-\beta_n)-\beta E_n} = \delta_{\alpha\beta} + \mathcal{O}(e^{-S/2})$$

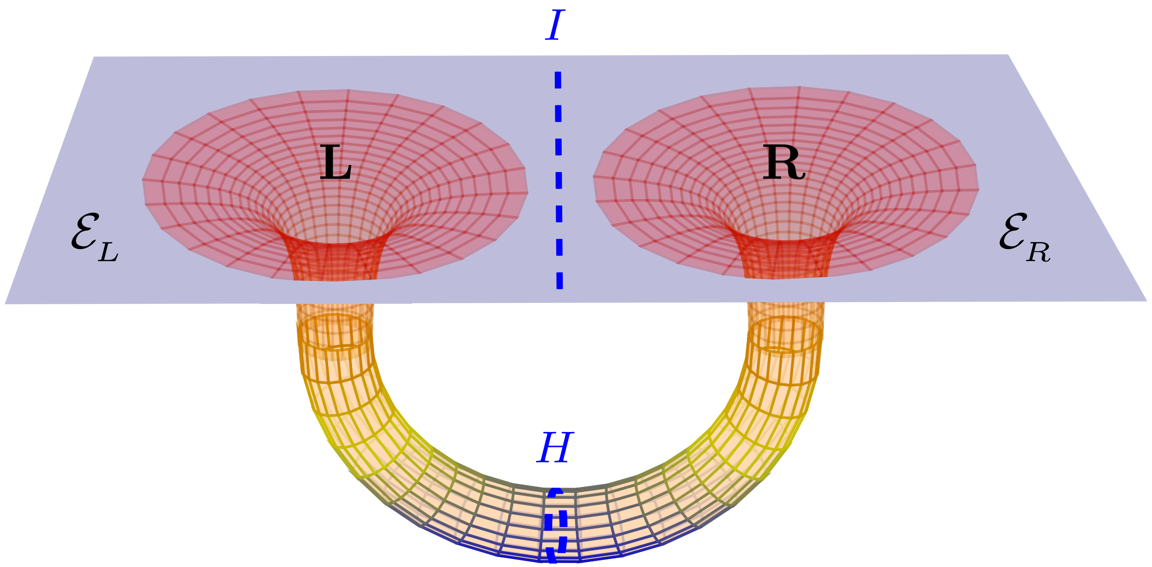
$$Z[\alpha] \equiv |\langle \mathrm{TFD}|\mathrm{TFD}\rangle_\alpha|^2 = \frac{1}{Z}\sum_{m,n} e^{-\beta(E_n+E_m)+i(\alpha_n-\alpha_m)}$$

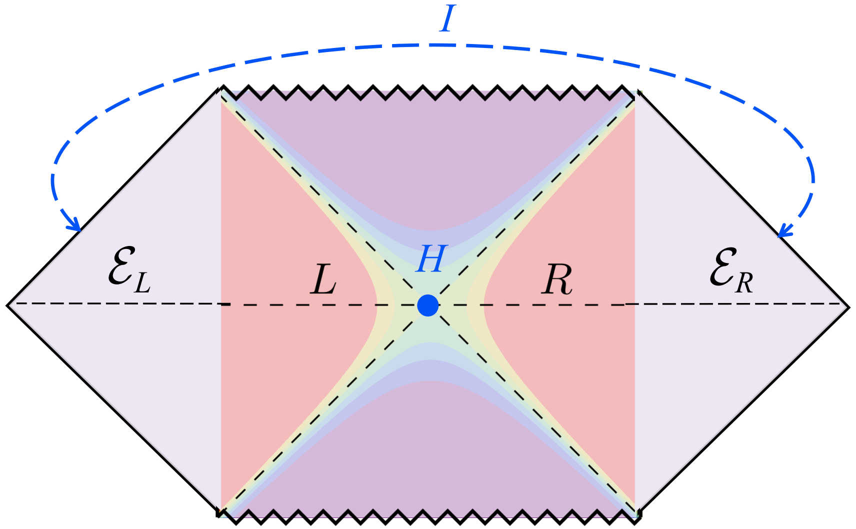
Spectral form factor

$-\log(Z[\alpha])$ = measure of `complexity'

= measure of time passed since TFD

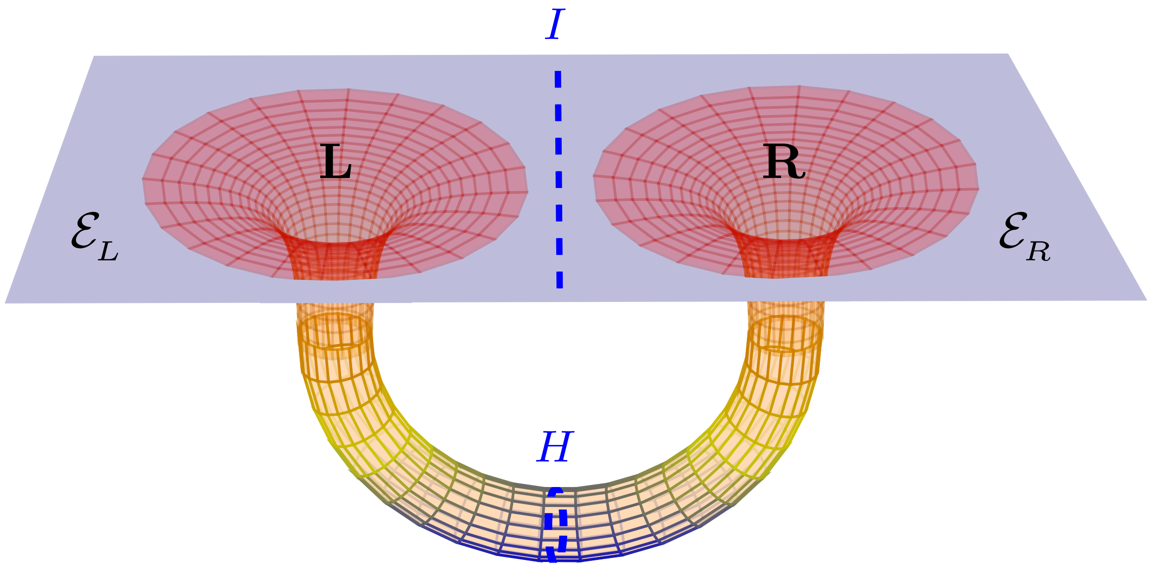
$$ds^2_{d+1} = -f(x)^2 dt^2 + g_{ij}(x) dx^i dx^j \qquad (5)$$

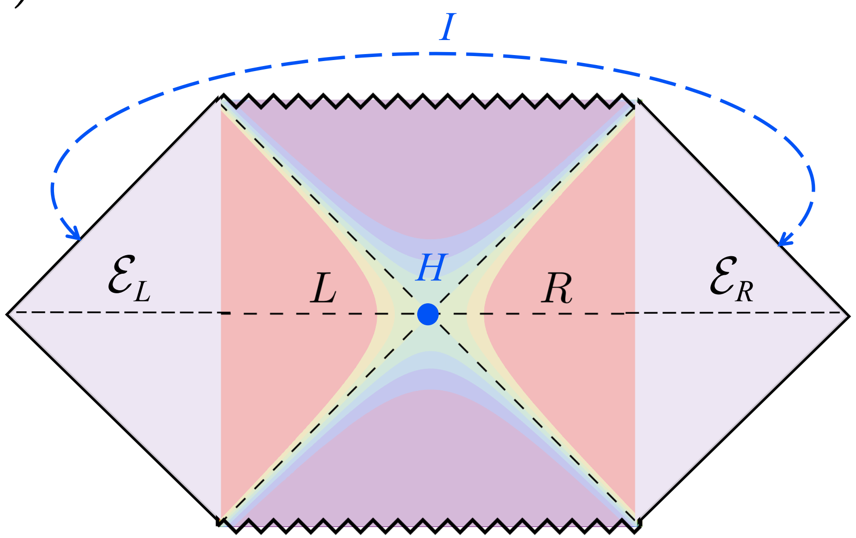




Here $f(x)$ vanishes, and thus flips sign, at the horizon. As a result, the space-time (5) does not admit a global time-like Killing vector: the locally defined Killing vector $\frac{\partial}{\partial t}$ switches sign when passing through the horizon H, and thus comes back in the opposite direction when transported around the non-contractible cycle in Fig 1.

$$ds^2_{d+1} = -f(x)^2 dt^2 + g_{ij}(x) dx^i dx^j .$$





$$\left.\begin{array}{ll} t_L = t_R & \text{at } I \\ t_L = 2t - t_R & \text{at } H \end{array}\right\} \Leftrightarrow \; t_L = t_R = t$$

$$\left.\begin{array}{ll} t_L = -t_R & \text{at } H \\ t_L = t_R - 2t & \text{at } I \end{array}\right\} \Longrightarrow \; -t_L = t_R = t$$

Define the unitary operators U_α

$$|\mathrm{TFD}\rangle_\alpha = U_\alpha |\mathrm{TFD}\rangle$$

Consider the set of $\bar{\alpha}$'s such that

$$[U_{\bar{\alpha}}, O_{\mathrm{code}}] = 0$$

We can think of the $U_{\bar{\alpha}}$'s as time evolution operators over multiples of the Poincaré time of the code subspace.

The $\bar{\alpha}$ label superselection sectors = irreps of the QFT operator algebra

CFT Poincare recurrence time is much bigger than the QFT recurrence time

$$\tau_{cft} \sim \exp(e^{S_{BH}}) \;>>\; \tau_{qft} \sim \exp(e^{S_{qft}}) \,.$$

=> General state = incoherent superposition of time-evolved TFD states

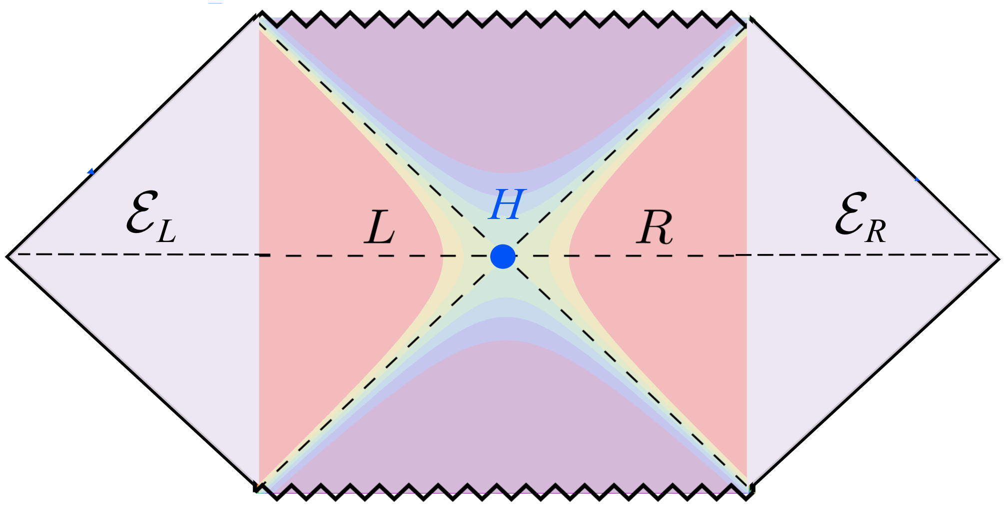
$$\bar{\rho}_{\mathrm{TMD}} = \frac{1}{N} \sum_{\bar{\boldsymbol{\alpha}}} |\mathrm{TFD}\rangle_{\bar{\alpha}} \; {}_{\bar{\alpha}}\langle \mathrm{TFD}|$$

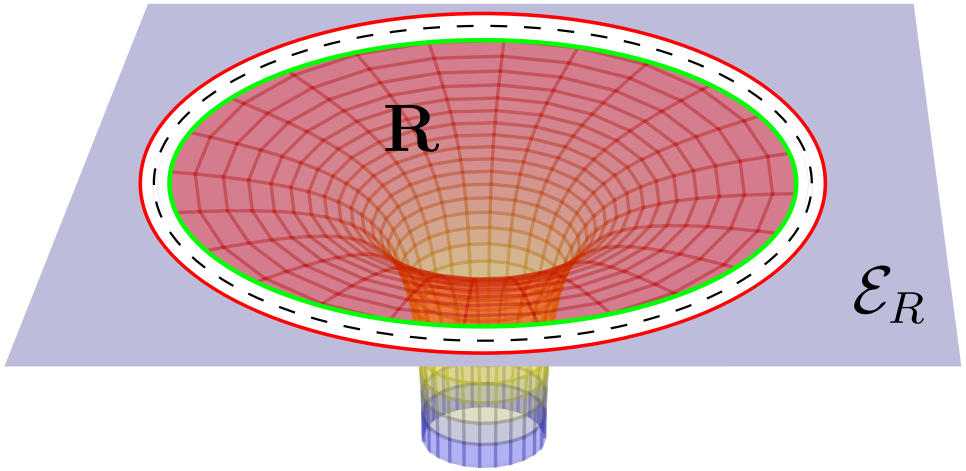
$$\rho_{\mathrm{TMD}} = \frac{1}{Z} \sum_n e^{-\beta E_n} \, |n\rangle_L \langle n| \otimes |n\rangle_R \langle n|$$

Thermal mixed double = incoherent superposition of time-evolved TFD states

$$\bar{\rho}_{\mathrm{TMD}} = \frac{1}{N} \sum_{\bar{\boldsymbol{\alpha}}} |\mathrm{TFD}\rangle_{\bar{\alpha}} \; {}_{\bar{\alpha}}\langle \mathrm{TFD}|$$

$$\frac{1}{N} \sum_{\bar{\boldsymbol{\alpha}}} e^{i(\bar{\alpha}_n - \bar{\alpha}_m)} = \delta_{nm}$$





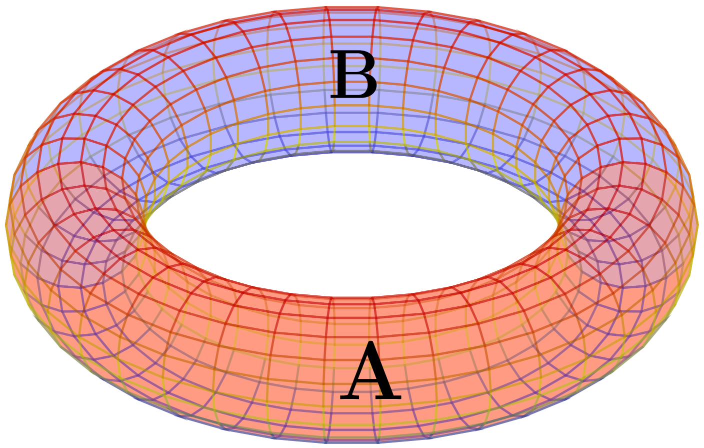
$$|\Psi\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\frac{\beta}{2} E_n} |n\rangle_L |n\rangle_R \, |n\rangle_{\mathcal{E}}$$

$$\rho_{\text{TMD}} = \frac{1}{Z} \sum_n e^{-\beta E_n} \, |n\rangle_L \langle n| \otimes |n\rangle_R \langle n|$$

entropy $S_{LR} = -\text{tr}\big(\rho_{\text{TMD}} \log \rho_{\text{TMD}}\big) = S_{BH}.$

Topological entanglement:

$$|\Psi\rangle = \sum_i \psi_i |R_i\rangle$$



$$S_A^{\rm top} = \sum_i \left(2|\psi_i|^2 \log S_0^i \; - \; |\psi_i|^2 \log|\psi_i|^2\right)$$

$$= - \int d\mu(i)\, |\tilde{\psi}_i|^2 \log|\tilde{\psi}_i|^2$$

$$\tilde{\psi}_i = \psi_i / S_0^i, \qquad \int d\mu(i) \ldots = \sum_i |S_0^i|^2 \ldots$$

i = Virasoro representations

$S_0^{\,i}$ = sinh(2πpbP) sinh(2πpP/b)

cf McGough, HV, 2012

$$\rho_{\mathrm{TMD}} = \frac{1}{Z} \sum_n e^{-\beta E_n} \, |n\rangle_L \langle n| \otimes |n\rangle_R \langle n|$$

The three entropies of TMD are all equal:

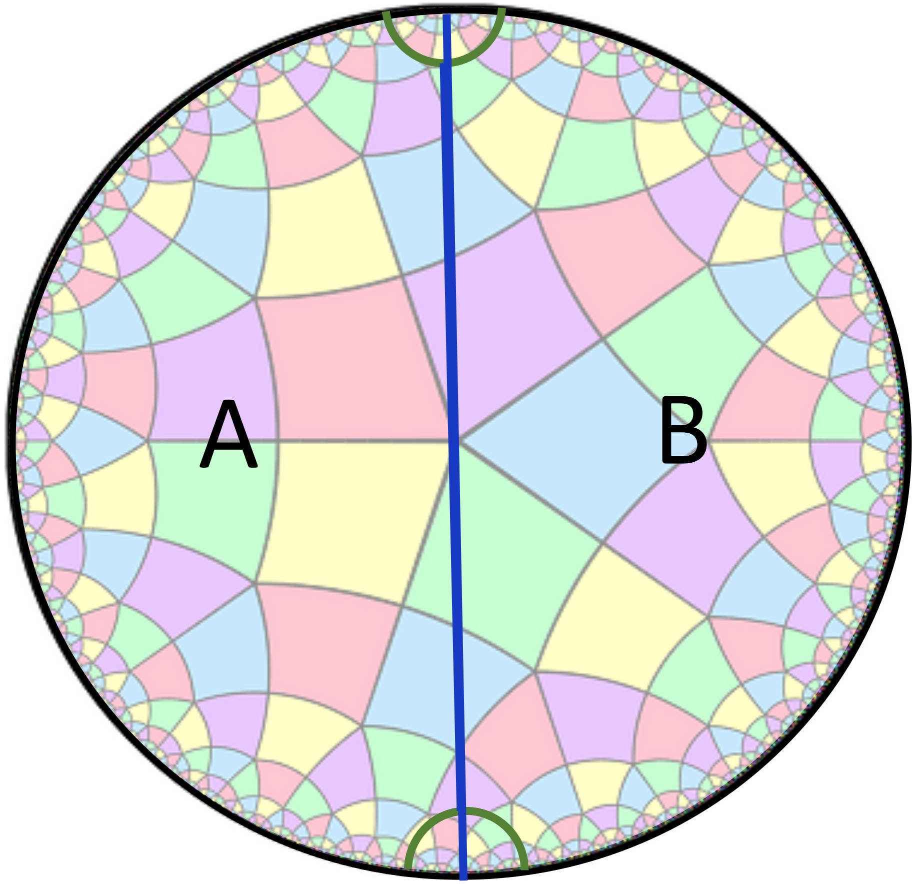
$$S_{LR} = S_L = S_R = S_{BH}$$

=> Mutual information $I_{LR} = \dfrac{S_L + S_R - S_{LR}}{2}$ is half as big as that of the TFD

$$I_{LR} = \begin{cases} S_{BH} & \text{for TFD} \\ S_{BH}/2 & \text{for TMD} \end{cases}$$

No true microscopic quantum entanglement!

What about AdS Rindler?



Must introduce UV cutoff and consider entanglement of purification $\mathcal{E} < S_{AB}$

Concluding comments:

- The microscopic entanglement that creates the fabric of space-time is encoded in hidden highly delocalized degrees of freedom.
- Local QFT entanglement is only a small contribution -- and this is all the unique quantum entanglement that is needed.
- The microscopic state of a local region of space does not need to be in unique or pure TFD like state.
- The QFT operators are defined via a QEC procedure, that acts within a code subspace.