H. Lin, J. Maldacena, Y. Zhao
arXiv:1904.12820

L. Susskind arXiv:1904.12819

Operator Size and Symmetry

Ying Zhao

IAS

Operator size

D. Roberts, D. Stanford, A. Streicher arXiv:1812.02633

X.-L. Qi, A. Streicher arXiv:1810.11958

- Number of simple operators affected

- Operator size at infinite temperature

$$S_\infty(\mathcal{O}) = \frac{1}{2}\sum_{i=1}^{N} 2^{-\frac{N}{2}} \mathrm{tr}([\mathcal{O},\psi_i]^\dagger[\mathcal{O},\psi_i])$$

$$S_\infty(\psi_1) = 1$$

$$\psi_1(u) = \sum_k \sum_{i_1<\ldots<i_k} c_{i_1\ldots i_k}\left(2^{\frac{k}{2}}\psi_{i_1}\ldots\psi_{i_k}\right) \qquad S_\infty(\psi_1(u)) = \sum_k \sum_{i_1<\ldots<i_k} k|\sqrt{2}c_{i_1\ldots i_k}|^2$$

- Size operator

$$\hat{S} = \sum_j i\psi_l^j\psi_r^j + \frac{N}{2} \qquad S_\infty(\mathcal{O}) = \langle I|\mathcal{O}_r^\dagger \hat{S}\mathcal{O}_r|I\rangle$$

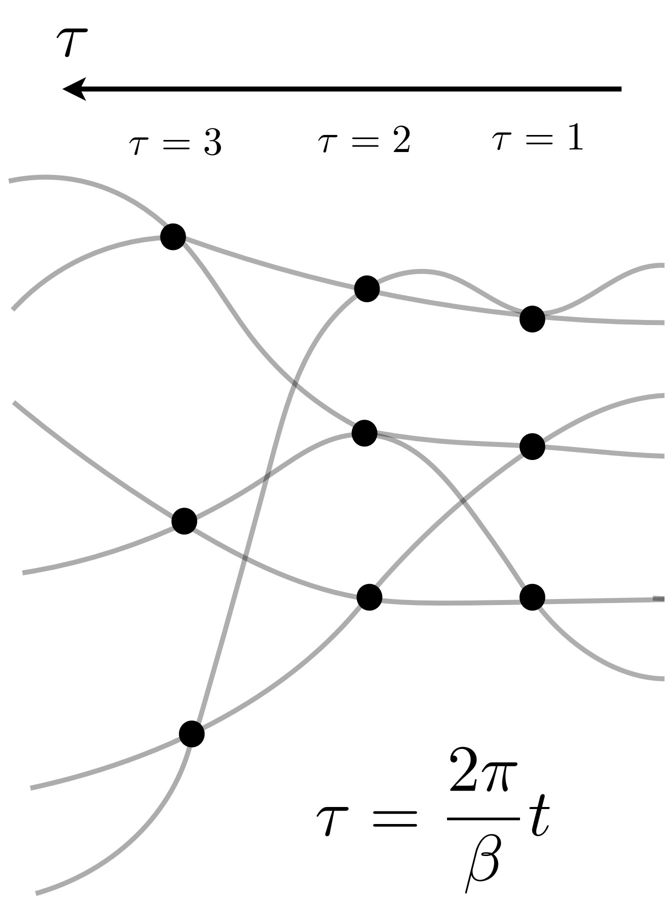
Falling toward black holes

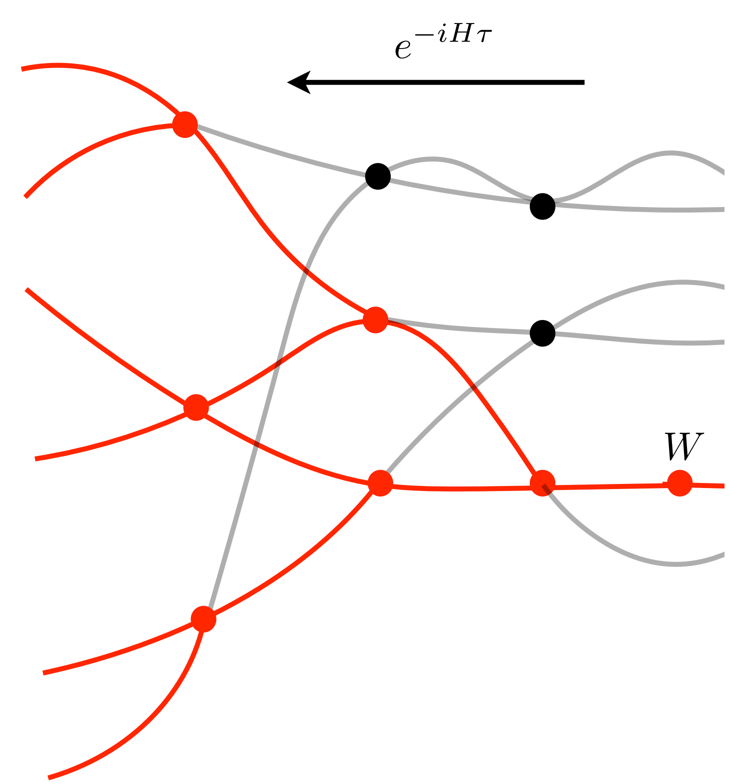
Y. Sekino, L. Susskind arXiv:0808.2096
L. Susskind arXiv:1802.01198

- Scrambling   Local perturbation spreads

Size growth $\delta S\, e^{\frac{2\pi}{\beta} t}$

Scrambling time $\delta S e^{\frac{2\pi}{\beta} t_*} = S, \qquad t_* = \frac{\beta}{2\pi} \log \frac{S}{\delta S}$

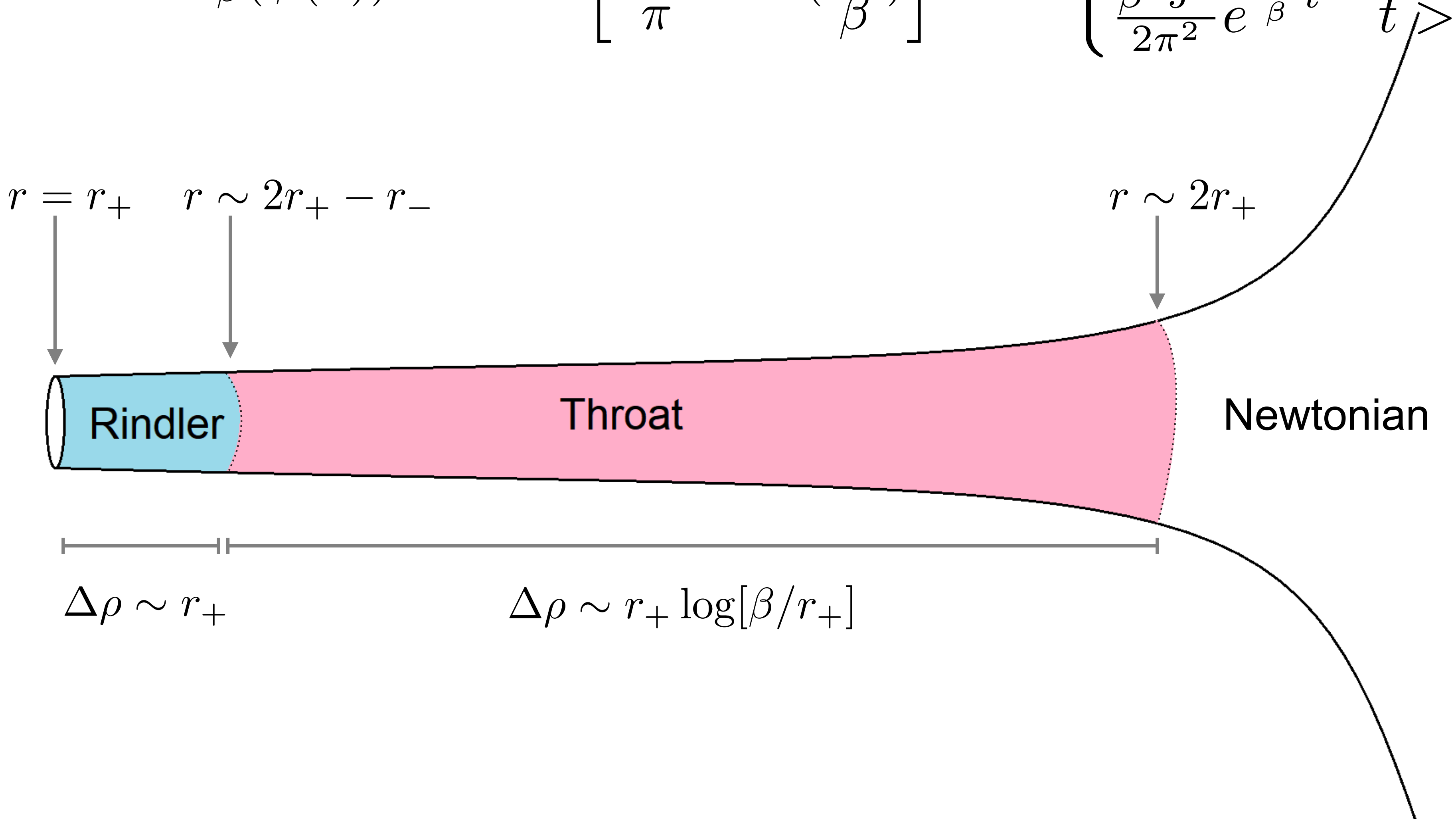




Operator size at finite temperature

X.-L. Qi, A. Streicher arXiv:1810.11958

- $S_\beta(\mathcal{O}) = \delta_\beta^{-1}\left[S_\infty(\mathcal{O}\rho^{\frac{1}{2}}) - S_\infty(\rho^{\frac{1}{2}})\right]$
- $S_\beta(\mathcal{O}) = \delta_\beta^{-1}\left[\frac{\langle TFD|\mathcal{O}_r^\dagger \hat{S}\mathcal{O}_r|TFD\rangle}{\langle TFD|\mathcal{O}_r^\dagger \mathcal{O}_r|TFD\rangle} - \langle TFD|\hat{S}|TFD\rangle\right]$ $\qquad \delta_\beta = 2G(\frac{\beta}{2})$
- $S_\beta(\psi(u)) = 1 + 2\left[\frac{\beta\mathcal{J}}{\pi}\sinh(\frac{\pi t}{\beta})\right]^2 = \begin{cases} 2J^2t^2 & t < \beta \quad \text{Throat regime} \\ \frac{\beta^2 J^2}{2\pi^2} e^{\frac{2\pi}{\beta}t} & t > \beta \quad \text{Rindler regime} \end{cases}$

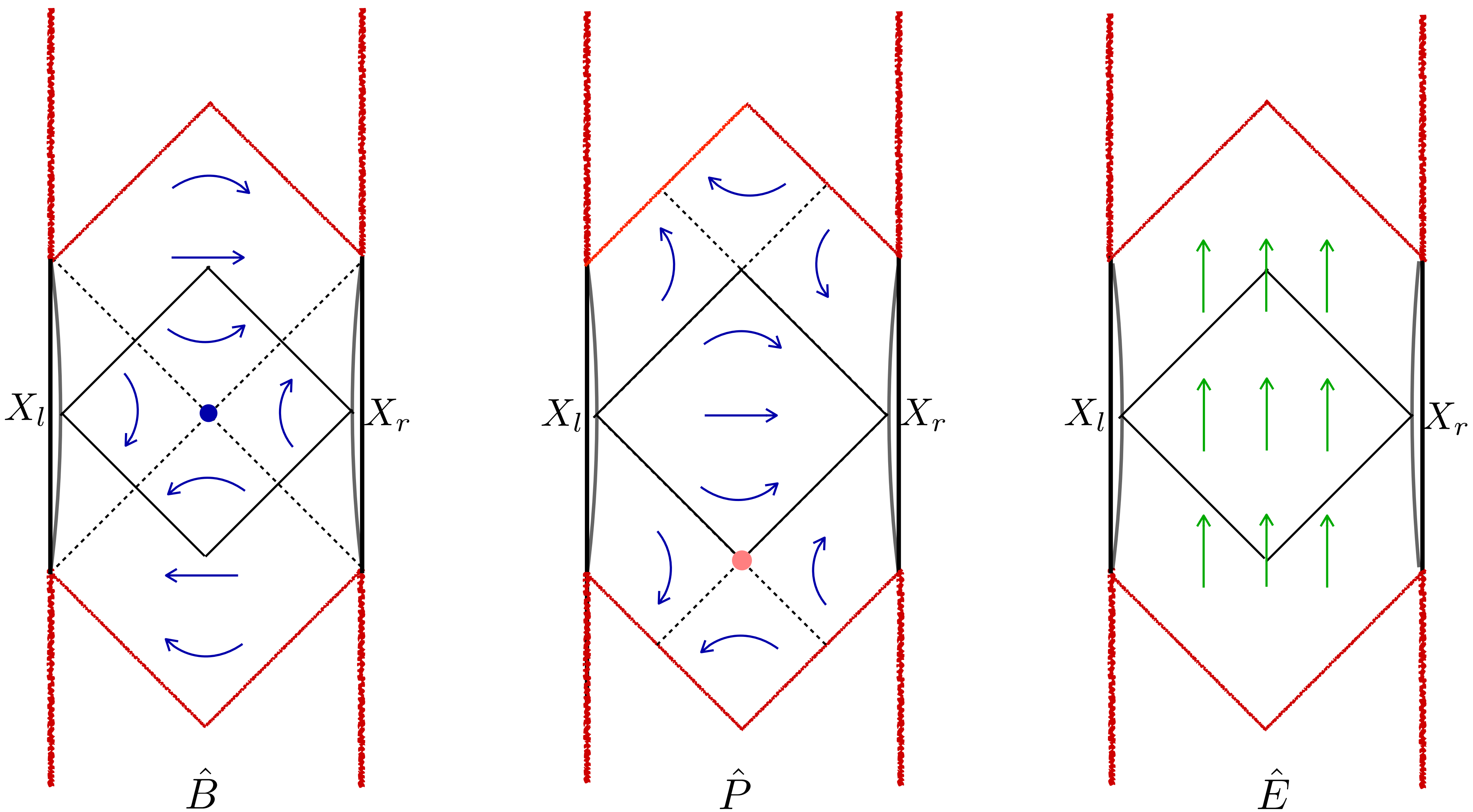


$\text{size} = P\tilde{\beta}$

A. Brown, H. Gharibyan, A. Streicher
L. Susskind, L. Thorlacius, Y. Zhao
arXiv:1804.04156

Symmetry generators in JT gravity

- $[\hat{P}, \hat{B}] = -i\hat{E}$ $\quad [\hat{P}, \hat{E}] = -i\hat{B}$ $\quad [\hat{B}, \hat{E}] = i\hat{P}$



Approximate symmetry generators in SYK

J. Maldacena, X.-L. Qi, arXiv:1804.00491

- $$\hat{B} = \frac{\beta}{2\pi}(H_r - H_l)$$

$$\hat{E} = \frac{\beta}{2\pi}\left[H_r + H_l + i\mu\sum_j \psi_l^j\psi_r^j - \langle H_r + H_l + i\mu\sum_j \psi_l^j\psi_r^j\rangle_{\mathrm{TFD}}\right]$$

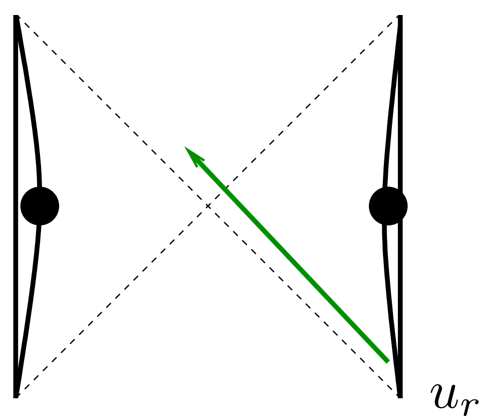
$$\hat{P} = -i[\hat{B},\hat{E}] = -i\mu\frac{\beta^2}{4\pi^2}\sum_j\left(\psi_l^j\dot{\psi}_r^j - \dot{\psi}_l^j\psi_r^j\right) \qquad \frac{\mu}{\mathcal{J}} = \frac{\alpha_s}{\Delta}\left(\frac{2\pi}{\beta\mathcal{J}}\right)^2\frac{1}{G(\frac{\beta}{2})}$$

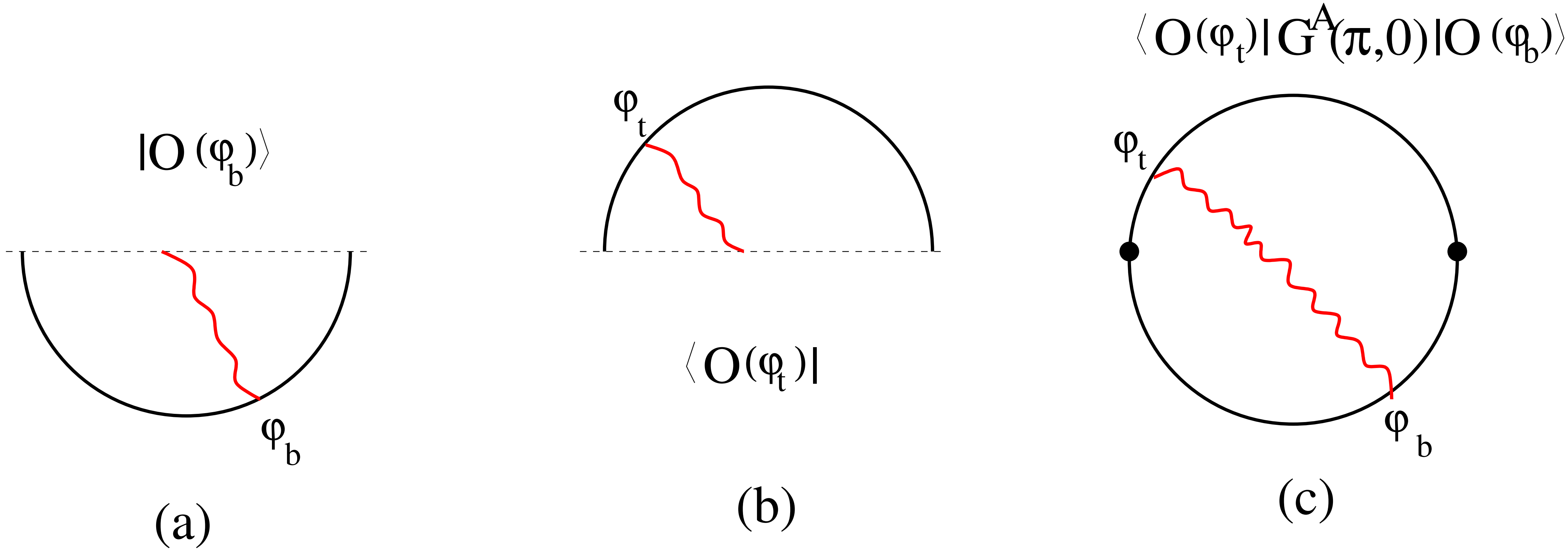
- $$\boxed{S_\beta(\mathcal{U}_r) = \frac{\Delta}{2\alpha_s}\frac{\beta\mathcal{J}}{2\pi}\langle TFD|\mathcal{U}_r^\dagger(\hat{E}-\hat{B})\mathcal{U}_r|TFD\rangle}$$

$$S_\beta(\mathcal{U}) = \delta_\beta^{-1}\left[\langle TFD|\mathcal{U}_r^\dagger\hat{S}\mathcal{U}_r|TFD\rangle - \langle TFD|\hat{S}|TFD\rangle\right] \qquad \delta_\beta = 2G(\frac{\beta}{2})$$

$$\hat{S} = \sum_j i\psi_l^j\psi_r^j + \frac{N}{2}$$

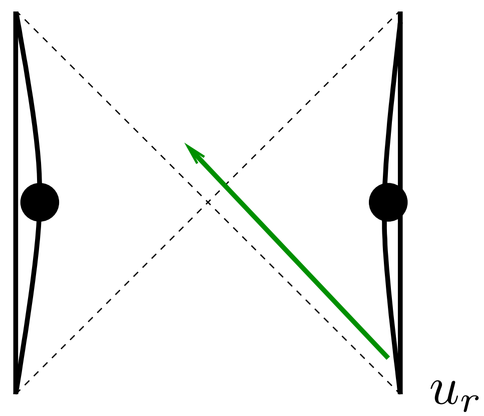
Example 1

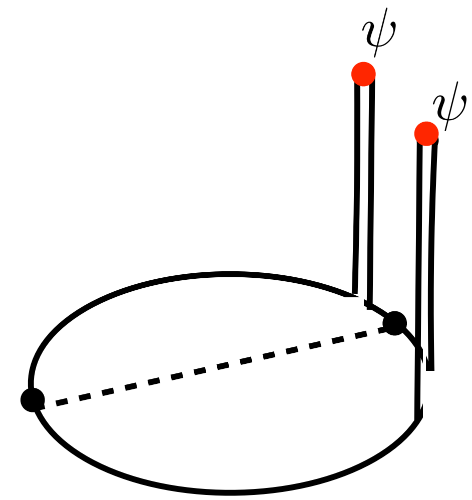




$$\left(\hat{B},\hat{P},\hat{E}\right)=\left(\Delta\frac{1}{\tan\frac{\varphi_t-\varphi_b}{2}},-i\Delta\frac{\sin\frac{\varphi_t+\varphi_b}{2}}{\sin\frac{\varphi_t-\varphi_b}{2}},\Delta\frac{\cos\frac{\varphi_t+\varphi_b}{2}}{\sin\frac{\varphi_t-\varphi_b}{2}}\right)$$

Example 1



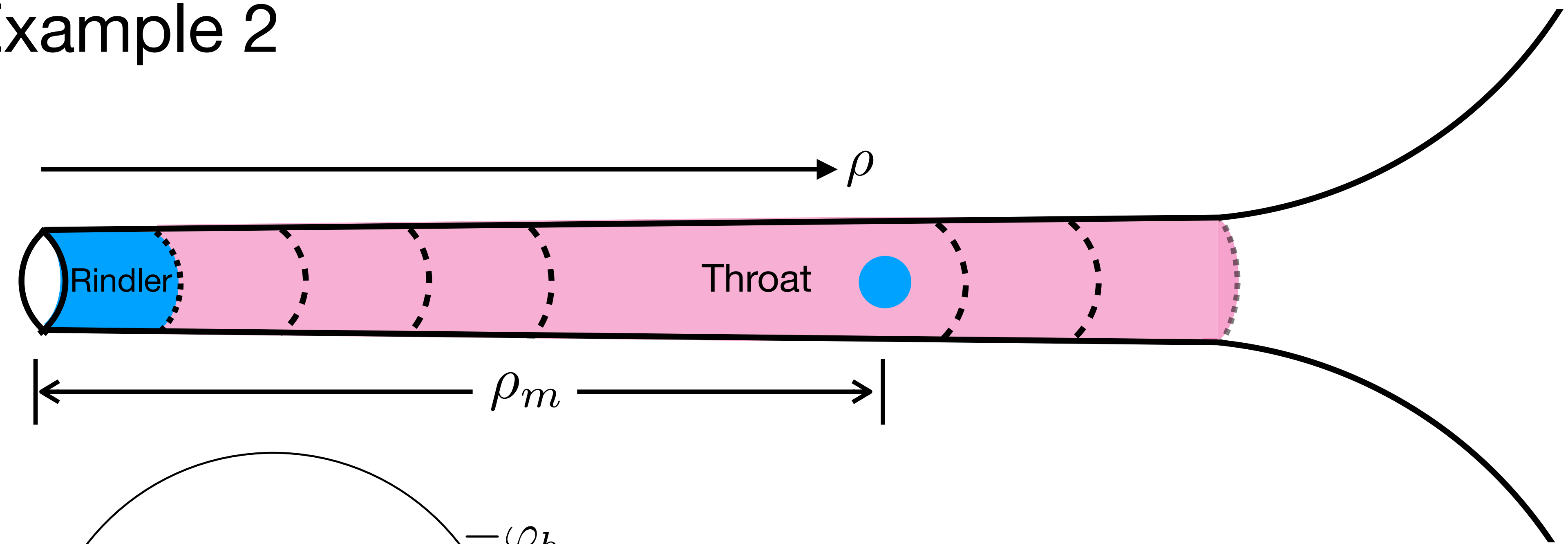


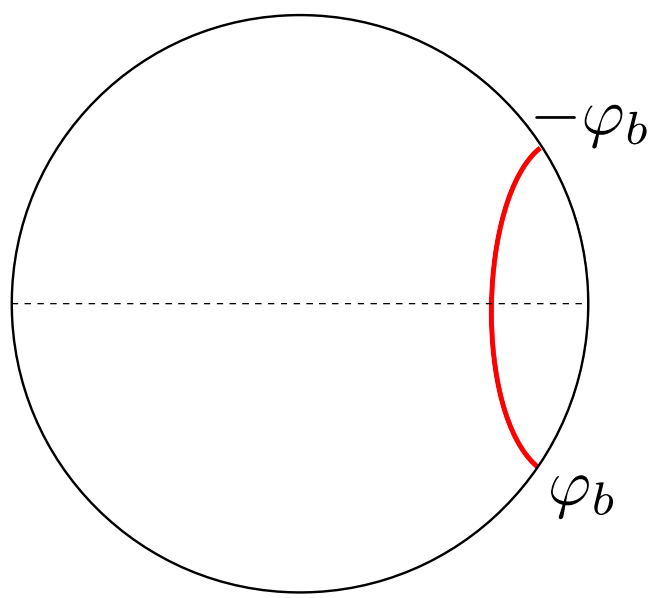
$$\varphi_b = \frac{2\pi}{\beta}\left(-\frac{1}{2\mathcal{J}} + iu_r\right) \qquad \varphi_t = \frac{2\pi}{\beta}\left(\frac{1}{2\mathcal{J}} + iu_r\right)$$

$$\left(\hat{B}, \hat{P}, \hat{E}\right) \approx \Delta \frac{2\beta\mathcal{J}}{2\pi}\left(1, \sinh(\frac{2\pi}{\beta}u_r), \cosh(\frac{2\pi}{\beta}u_r)\right)$$

$$S_\beta(\psi(u_r)) = \frac{\Delta}{2\alpha_s}\frac{\beta\mathcal{J}}{2\pi}(\hat{E} - \hat{B}) = 2\frac{\Delta^2}{\alpha_s}\left(\frac{\beta\mathcal{J}}{2\pi}\right)^2 \sinh^2\left(\frac{\pi}{\beta}u_r\right)$$

Example 2



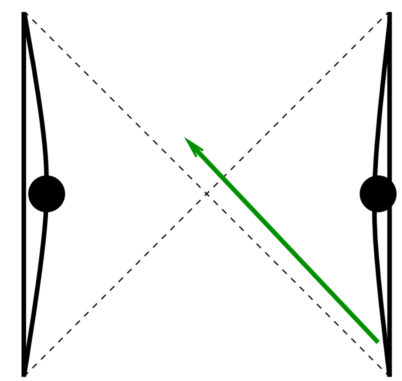


$$\rho_m = -\log\left(\tan(\frac{\varphi_t}{2})\right)$$

$$\left(\hat{B}, \hat{P}, \hat{E}\right) = (\Delta \sinh \rho_m, 0, \Delta \cosh \rho_m)$$

$$S_\beta(\rho_m) = \frac{\Delta^2}{2\alpha_s} \frac{\beta \mathcal{J}}{2\pi} e^{-\rho_m} \quad \Longrightarrow \tilde{\beta} \mathcal{J}$$

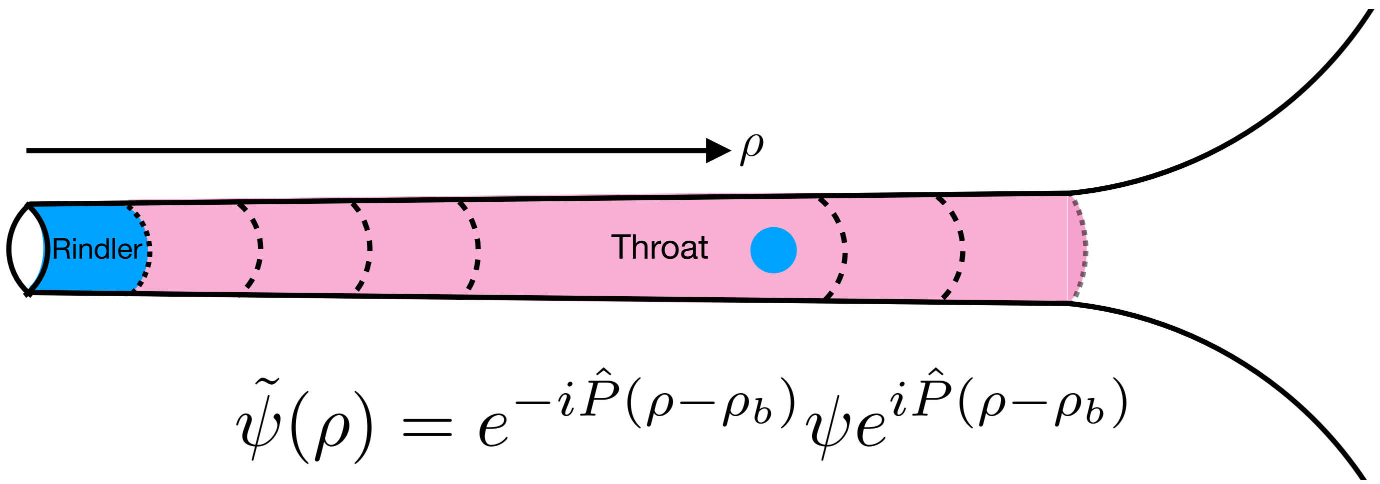
Size growth from SL(2) algebra



- Example 1 revisited

$$\frac{\partial S_\beta(\mathcal{U}(u_r))}{\partial u_r} = -i\frac{\Delta}{2\alpha_s}\frac{\beta\mathcal{J}}{2\pi}\langle\mathcal{U}_r(u_r)^\dagger\left[\frac{2\pi}{\beta}\hat{B},\hat{E}-\hat{B}\right]\mathcal{U}_r(u_r)\rangle = \frac{\Delta}{2\alpha_s}\mathcal{J}\langle\mathcal{U}_r(u_r)^\dagger\hat{P}\mathcal{U}_r(u_r)\rangle$$

- Example 2 revisited



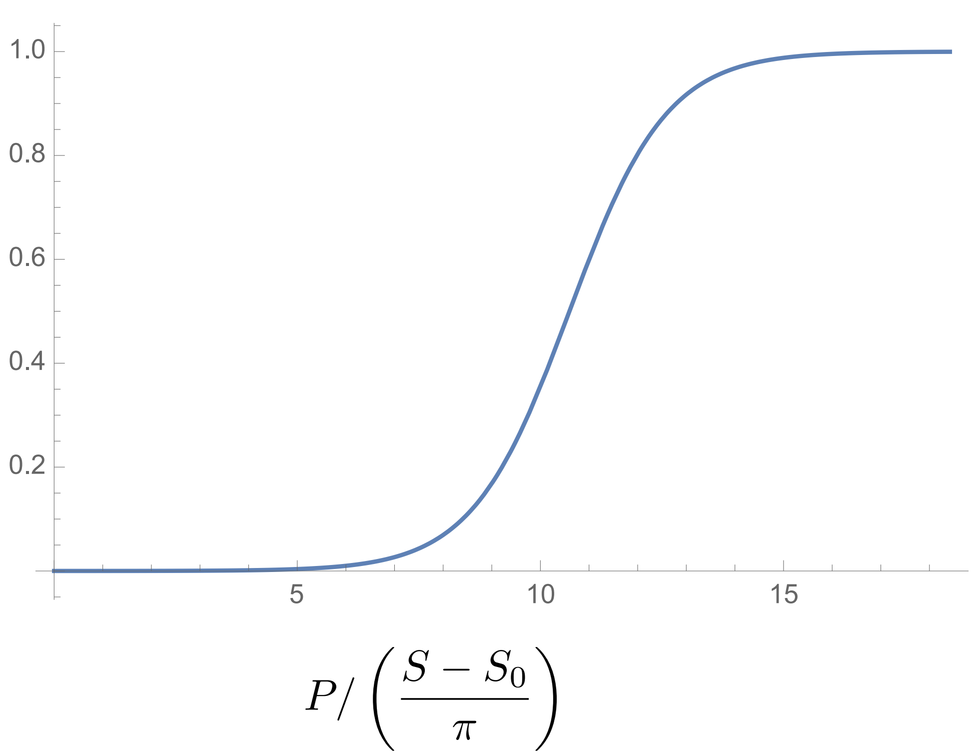
$$\tilde{\psi}(\rho) = e^{-i\hat{P}(\rho-\rho_b)}\psi e^{i\hat{P}(\rho-\rho_b)}$$

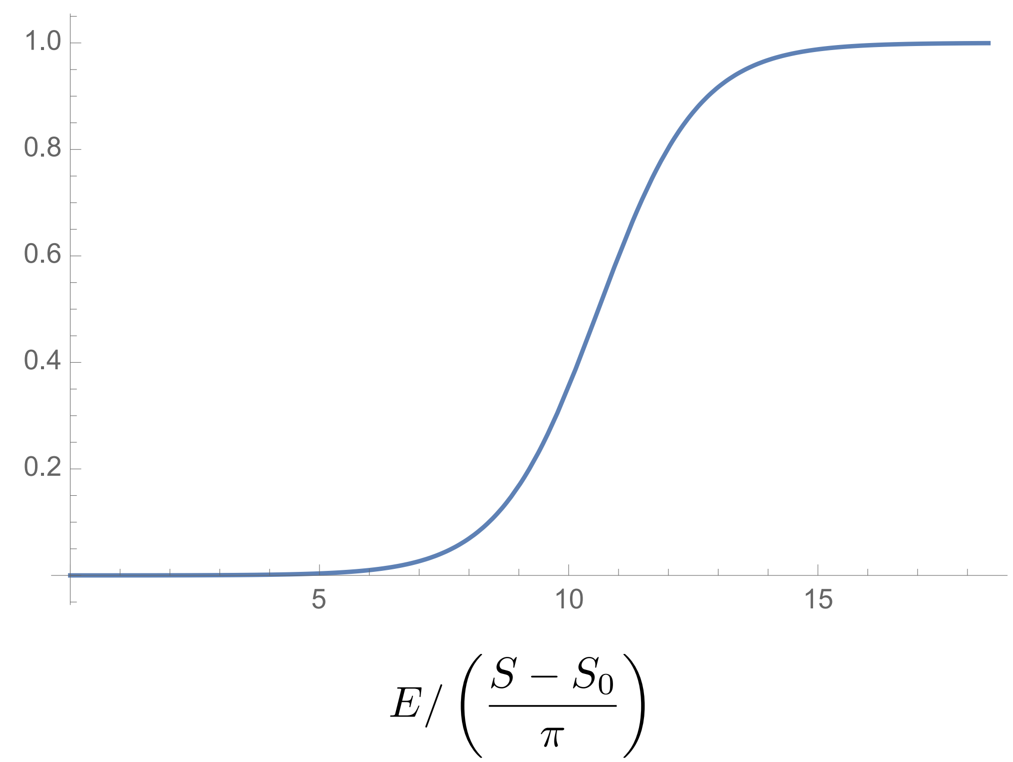
$$[\hat{P},\hat{E}-\hat{B}] = i(\hat{E}-\hat{B})$$

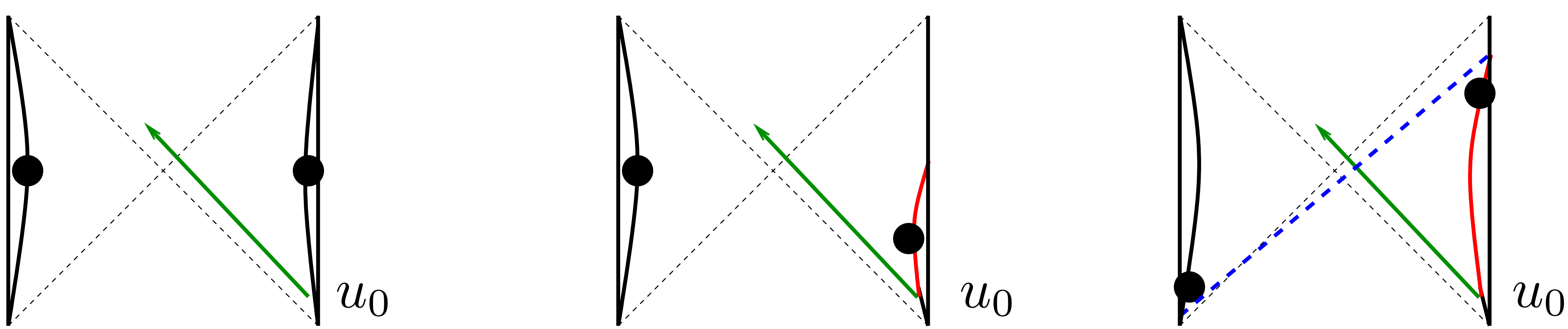
$$\frac{d}{d\rho}S_\beta(\tilde{\psi}(\rho)) = \frac{\Delta^2}{2\alpha_s}\frac{\beta\mathcal{J}}{2\pi}2\langle TFD|\tilde{\psi}(\rho)[i\hat{P},\hat{E}-\hat{B}]\tilde{\psi}(\rho)|TFD\rangle$$

$$= -S_\beta(\tilde{\psi}(\rho))$$

Saturation of symmetry generators







(a) (b) (c)

Thank you.