Comments on the final state proposal & the gravitational path integral

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@ YITP 2021

based on eg. Stanford @ KITP, Hardy Maxfield ...

Outline:

1) Recap of Info Paradox
2) Review of the final state proposal
3) Lessons from the gravity path integral
4) How to fix the final state proposal.

Info Paradox

Hawking Radiation

$$|\Phi_d\rangle_{\tilde{r}r} = \frac{1}{\sqrt{d}} \sum_{j=1}^{d} |j\rangle_{\tilde{r}} |j\rangle_r$$

total state on Cauchy slice: $|\psi\rangle_{M\tilde{r}r} = |i\rangle_M |\Phi_d\rangle_{\tilde{r}r}$

→ Evolution is not unitary: $|i\rangle_M \to \rho_r = \frac{1}{d} \mathbb{1}_r$

e.g. compute purity: $\mathrm{tr}\,\rho^2 = \frac{1}{d}\,\mathrm{tr}\,\rho$

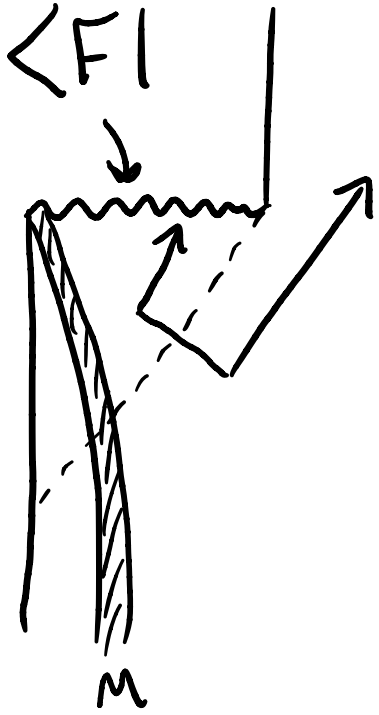
→ Need to move things from sing. to the outside!

Final State

[Horowitz, Maldacena]

→ Idea: post-selection @ sing. can do this!

→ If Big bang sing. has an initial state, why not future sing. too?



not normalized!

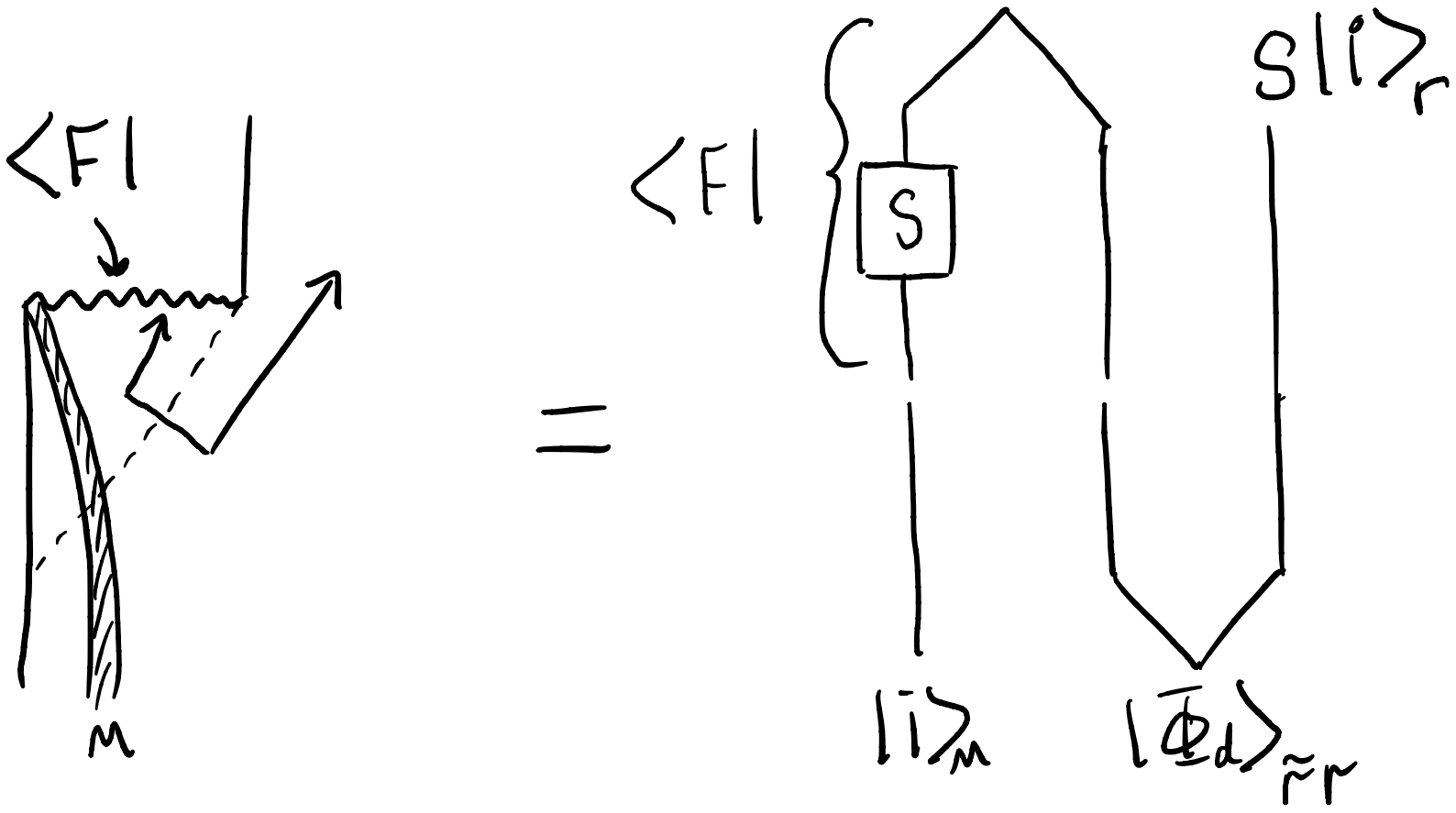
$$_{M\tilde{F}}\langle F| = d\,{}_{M\tilde{F}}\langle \Phi|\,(S\otimes \mathbb{1})$$

↕ unitary

$$_{M\tilde{F}}\langle F|\Psi\rangle = \sum_K S_{Ki}\,|K\rangle_F$$

$$\Rightarrow |i\rangle_M \to \sum_K S_{Ki}\,|K\rangle_F$$

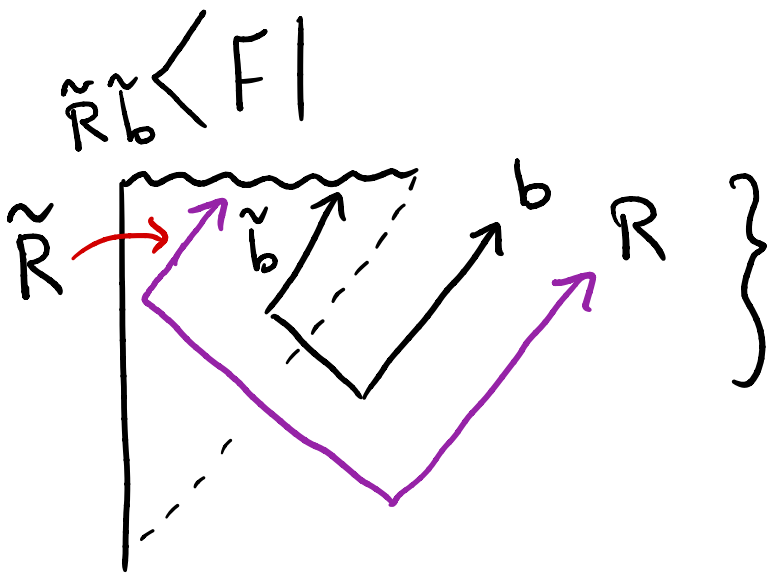
Quantum Circuit:



→ Gives unitary evaporation!

→ However, there are issues...

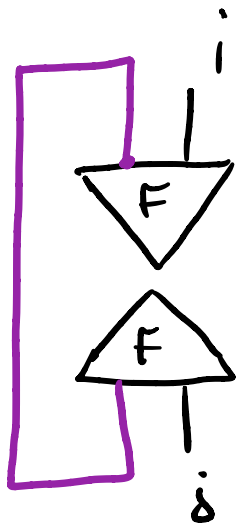
Final State: Fully entangled bh. [Bousso Stanford]



$$|\psi\rangle = |\Phi_2\rangle_{b\tilde{b}} \, |\Phi_d\rangle_{R\tilde{R}}$$

not normalized

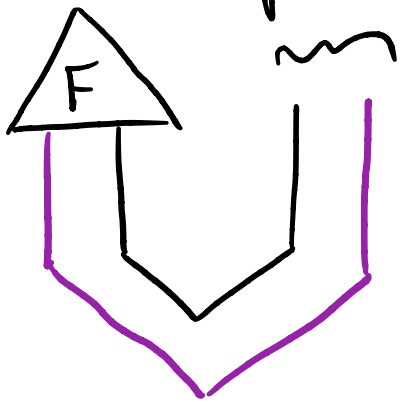
$\tilde{R}\tilde{b}\langle F|$: $\tilde{b}$ is maximally mixed. $\rho^F_{\tilde{b}} =$



$= \delta_{ij} \times d$

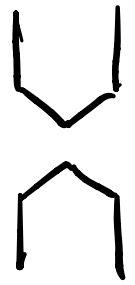
pure state.

$\langle F|\psi\rangle$:



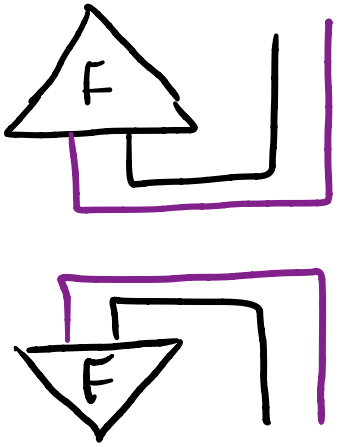
$\times \dfrac{1}{\sqrt{2d}}$

Problem w/ AMPS measurements

$\Pi_{\tilde{b}b} = \frac{1}{2}$ 

"Smooth Horizon"

&

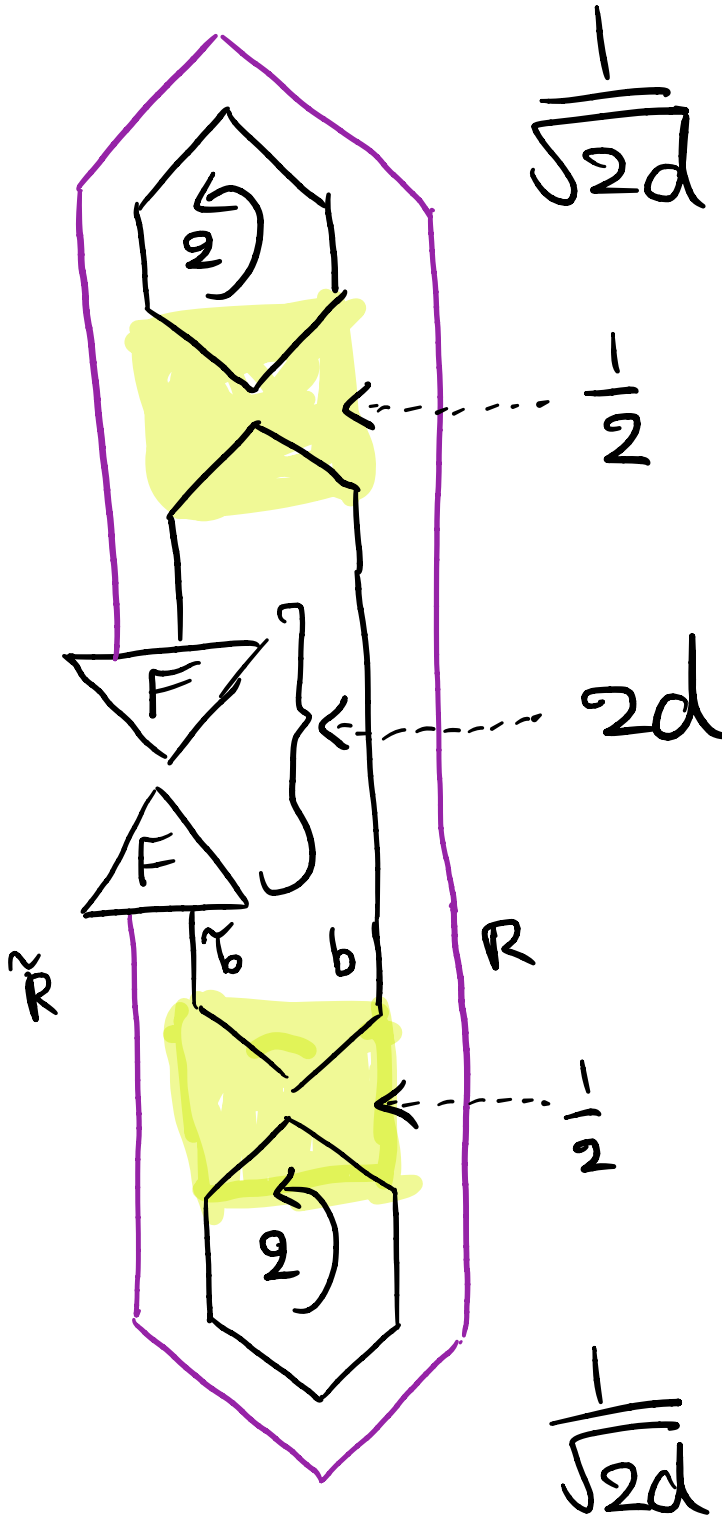
$\Pi_{bR} = \frac{1}{2d}$ 

"bR in the prepared pure state"

$$P_r(\Pi) = \langle \psi | \Pi | F \rangle\langle F | \Pi | \psi \rangle$$

$$\{ \text{Prob:} \quad tr\left(\Pi \cdots \Pi \, \rho \, \Pi \cdots \Pi \, \rho_f \right) \}$$

$$P_\sigma\left(\prod_{\tilde{b}b}\right) =$$



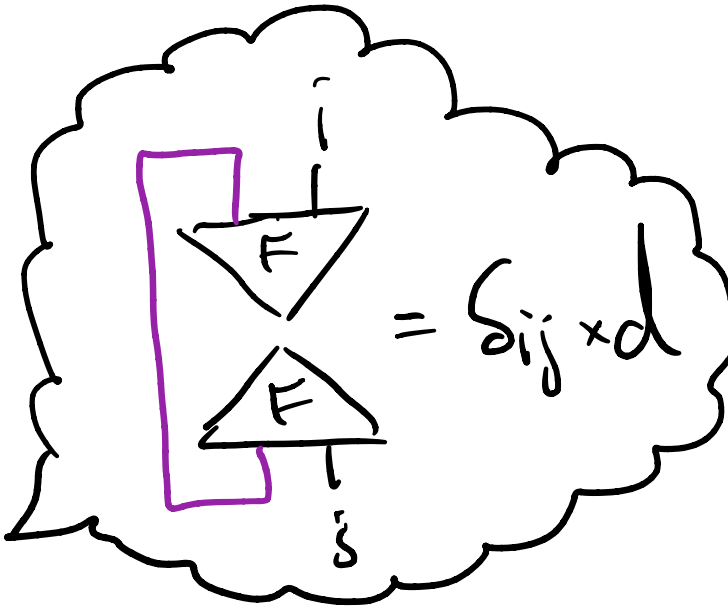
$\frac{1}{\sqrt{2d}}$

$\frac{1}{2}$

$2d$

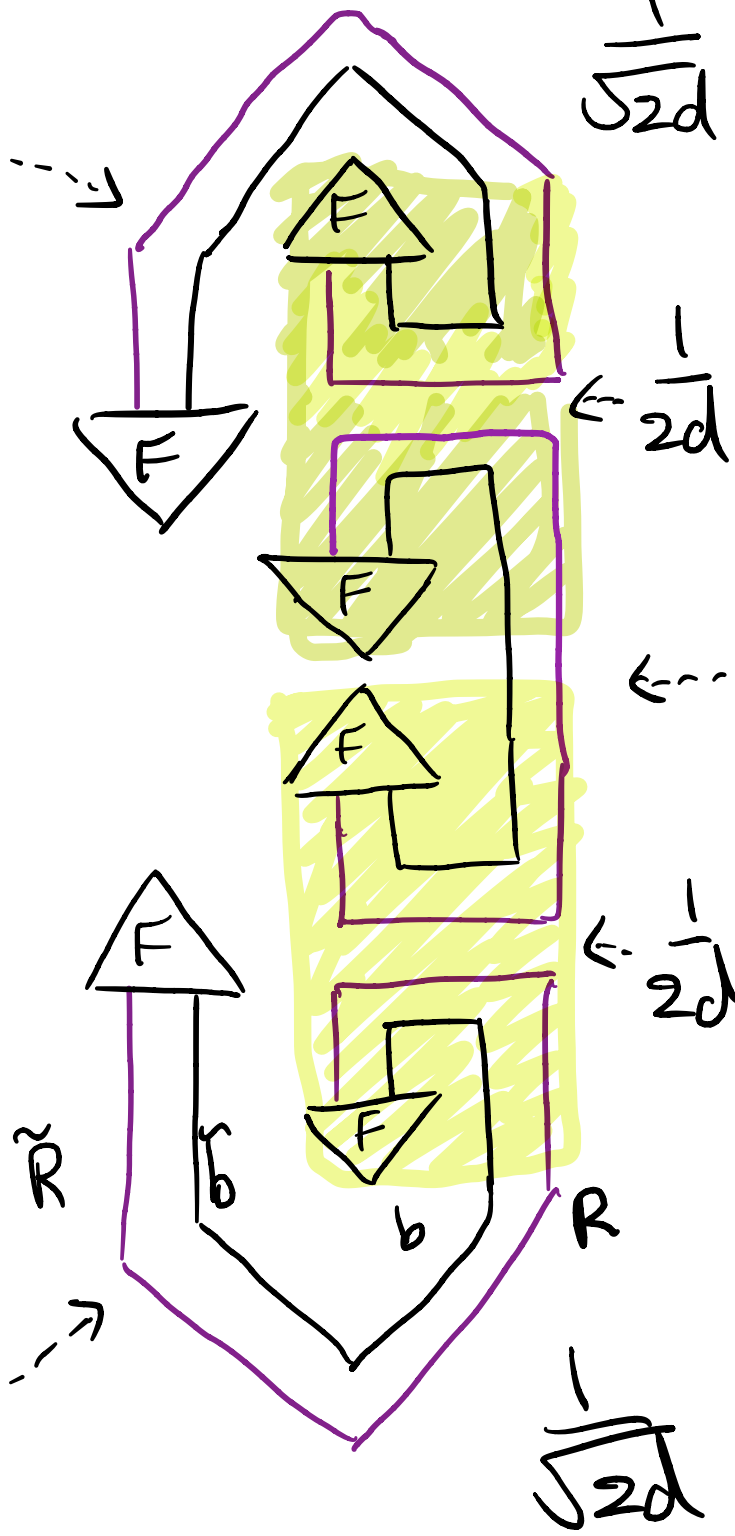
$\frac{1}{2}$

$\frac{1}{\sqrt{2d}}$



$= \underline{1}$ ✓

loop
2d



$\frac{1}{\sqrt{2d}}$

$\leftarrow \frac{1}{2d}$

$\leftarrow 2d$

$\leftarrow \frac{1}{2d}$

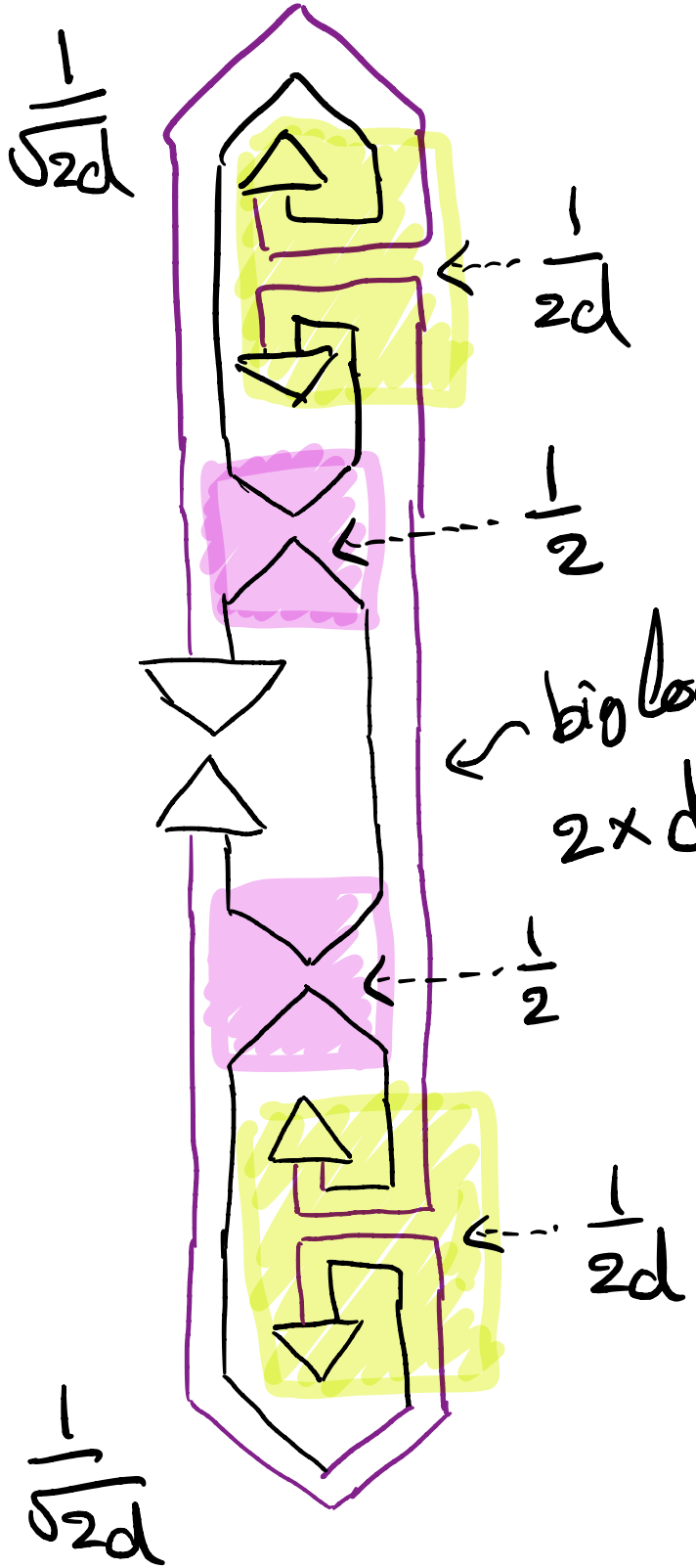
$\frac{1}{\sqrt{2d}}$

$P_r\left(\prod_{bR}\right) =$

$= 1$ ✓

2d

$$P_r\left(\tilde{\Pi}_{\tilde{b}b}\ \Pi_{bR}\right) =$$



$$= \frac{1}{2^4} \neq 1!$$

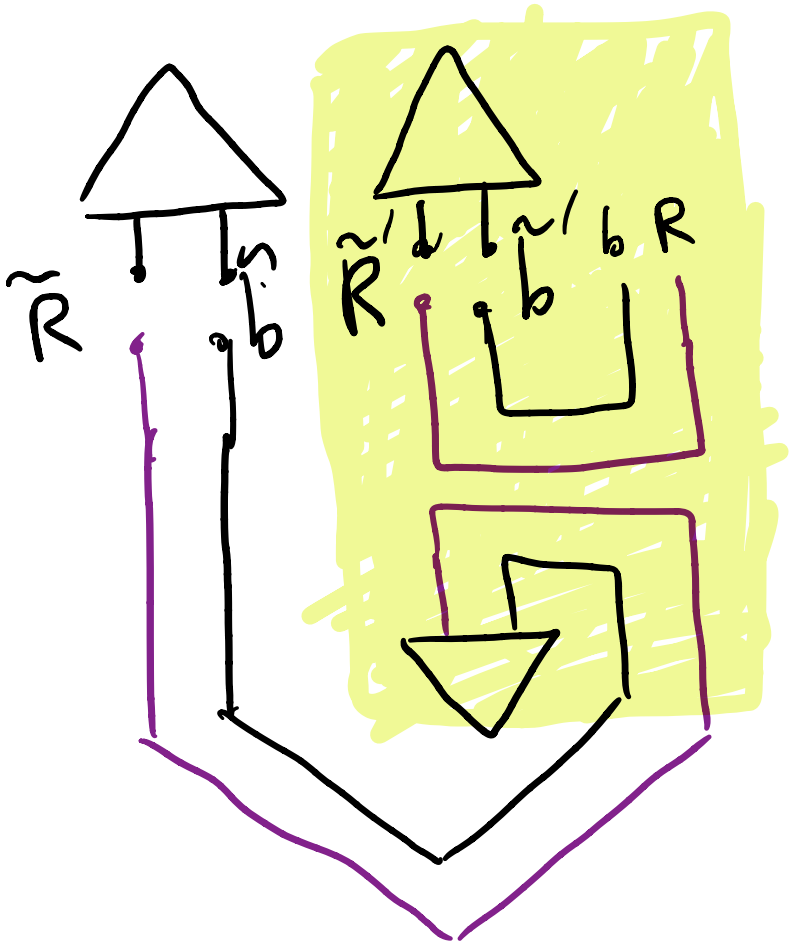
$$Pr\left(\prod_{\tilde{b}b}\right) = Pr\left(\prod_{bR}\right) = 1, \quad Pr\left(\prod_{\tilde{b}b}\prod_{bR}\right) = \frac{1}{2^4}$$

→ Statistics of outside measurement depend on whether future measurement will take place!

→ Address this prob. using intuition from gravity path integral.

Note: $\langle F | \Pi_{bR} | \psi \rangle_{\tilde{R}\tilde{b}bR}$

$$= \sum_{\tilde{R}'\tilde{b}'} \langle F | \langle F | \psi \rangle_{\tilde{R}'\tilde{b}'Rb} \langle \psi | F \rangle_{\tilde{R}'\tilde{b}'} | \psi \rangle_{\tilde{R}\tilde{b}bR}$$

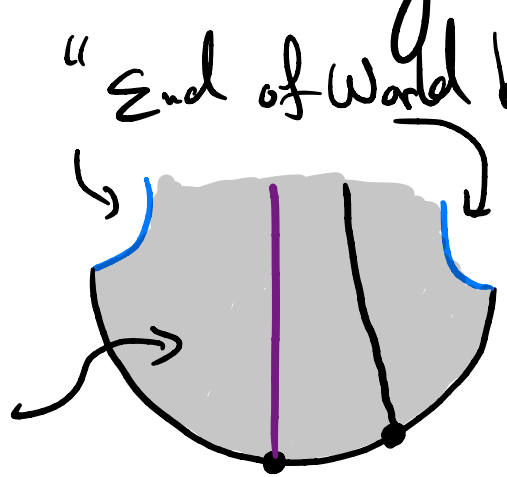


There are two interiors w/ two final ~~states~~, which is the 'real' one?

$\tilde{R}\tilde{b}$ vs $\tilde{R}'\tilde{b}'$

Gravitational Path Integral.

Closed Universe:

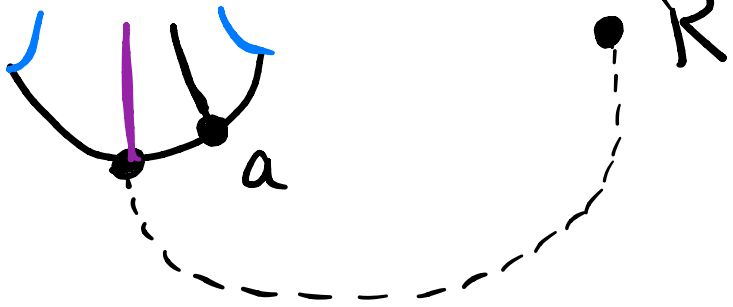


$= |\Psi_{ia}\rangle_{CU}$

eg. δT, $\mathcal{E}AdS$.

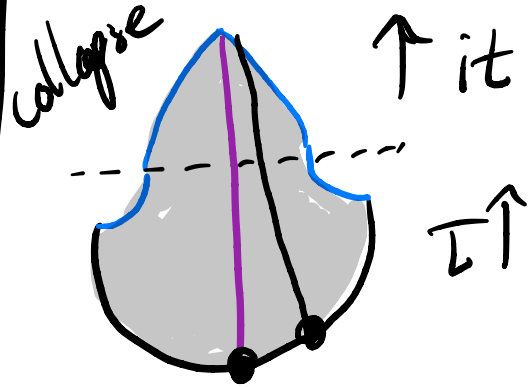
Particle sources.

Entangle w/ reference:

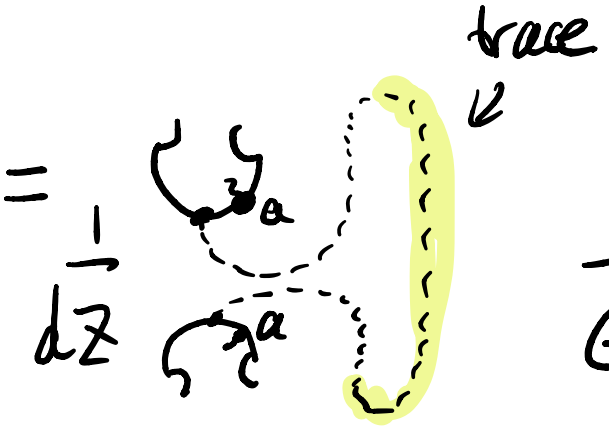


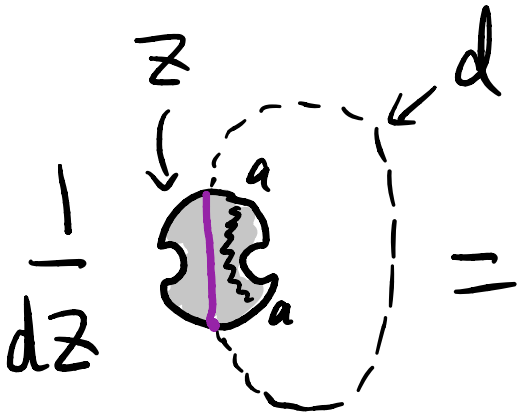
$$|\Psi_a\rangle_{CU} = \frac{1}{\sqrt{dz}} \sum_{i=1}^{d} |\psi_{ia}\rangle_{CU} |i\rangle_R$$

Aside:
in Lorentzian time:

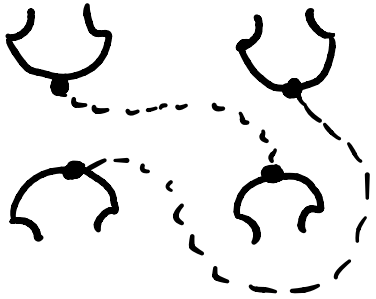
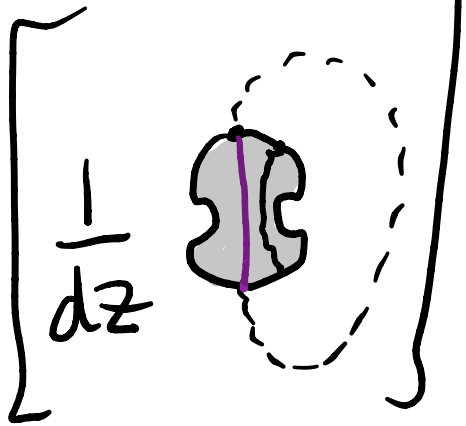


Purity of R

$\mathrm{tr}\,\rho_R =$ 

$\xrightarrow[\text{GPI}]{}$ 

$= 1$

$\mathrm{tr}\,\rho_R^2 =$ 
$\xrightarrow[\text{GPI}]{}$ 
$+ \frac{1}{d} \cdots$

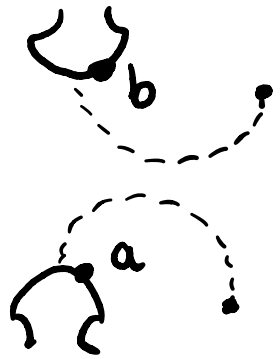
$= 1 + \cdots$

$\Rightarrow$ R is pure.

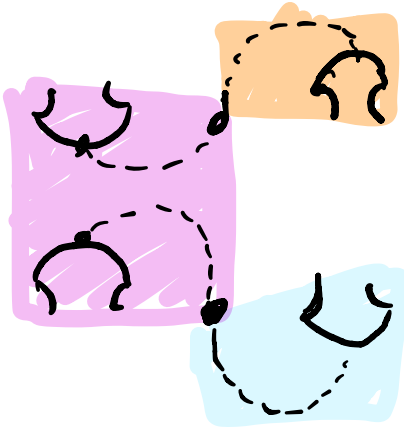
Interior $\in R$ [Penington et al]

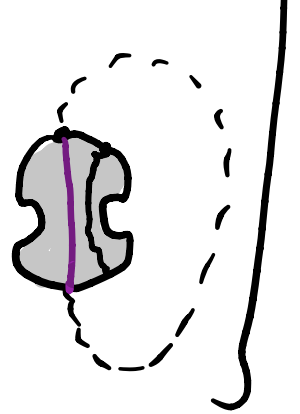
→ Operators on R can affect the closed universe.

e.g. "Petz map"

→ Our case: $\mathcal{O}^R_{ba} = \frac{1}{dz}$  takes CU w/ a to b

Claim: $\langle \Psi_b | \mathcal{O}^R_{ba} | \Psi_a \rangle = 1$

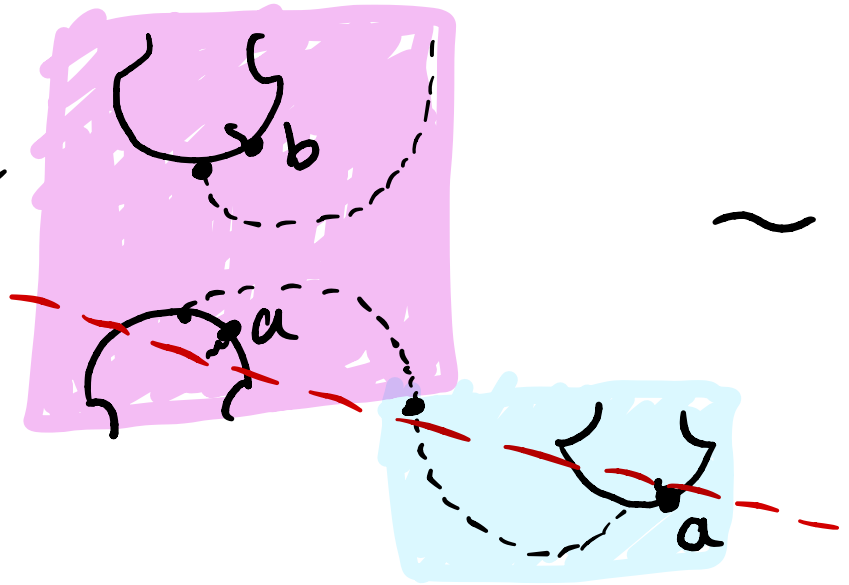
Check: $\frac{1}{(dz)^2}$  $= \left[\frac{1}{dz} \right]^2 + \frac{1}{d} \cdots$



$= 1 + \cdots$

→ We effectively have:

$\mathcal{O}^R_{ba} |\Psi_a\rangle_{CUR}$ ~



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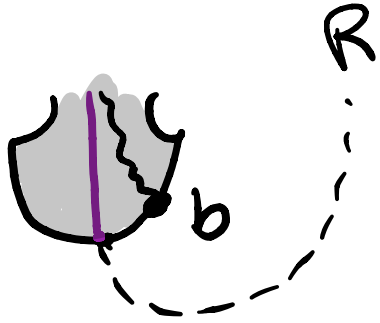
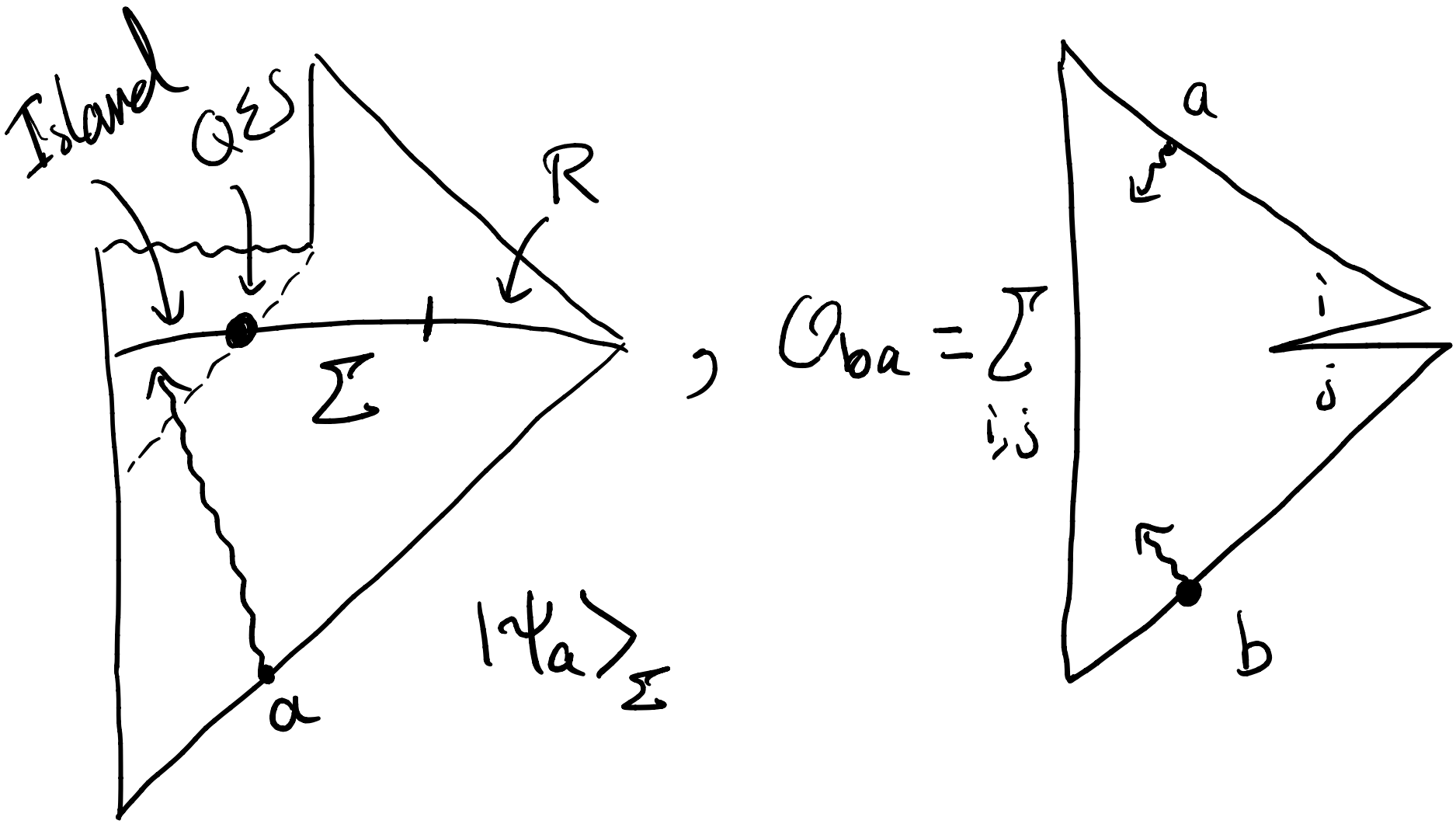
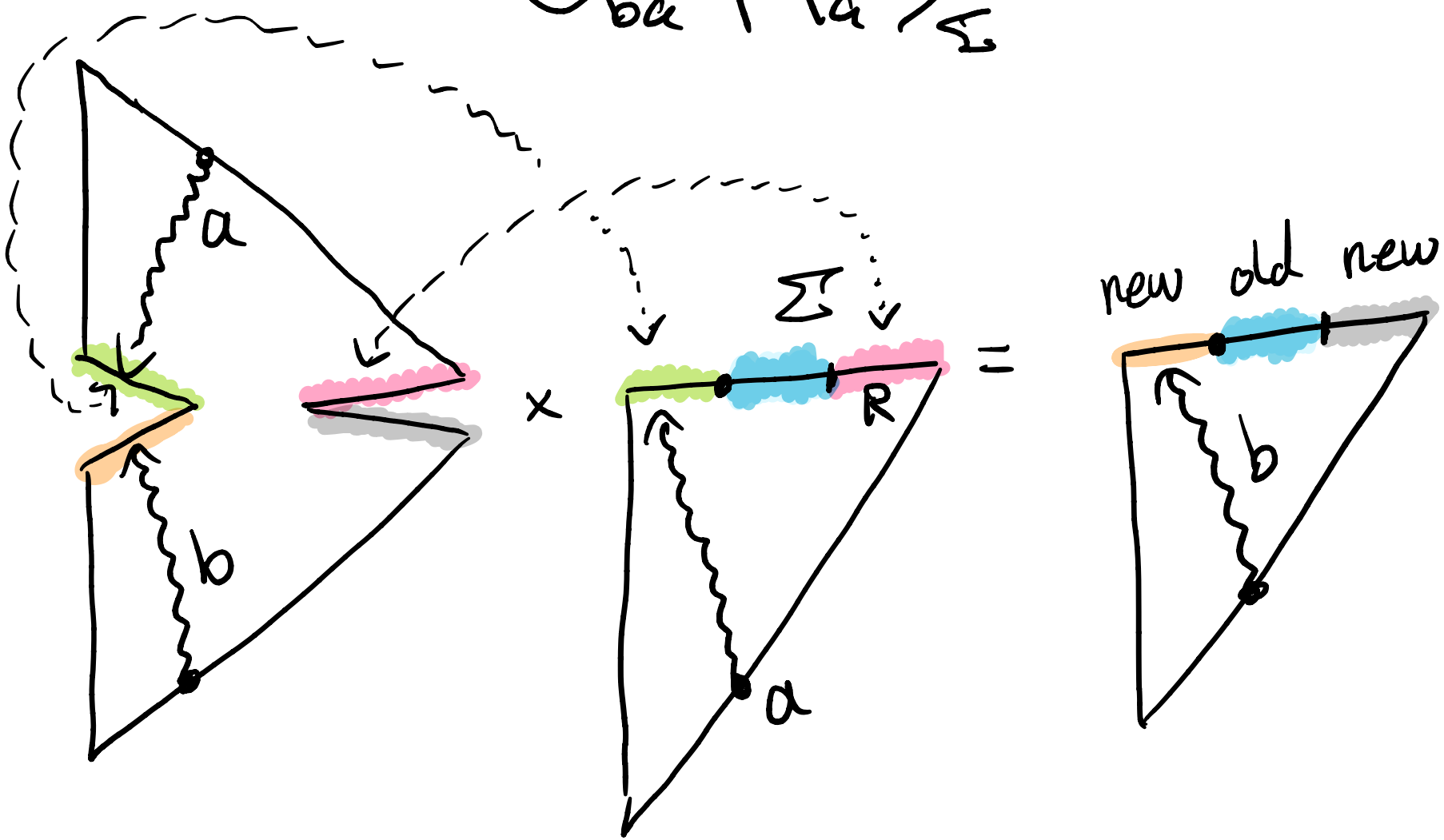


→ The action of this special operator on R removed the old interior & created a new one!

Another (quick) example:



$O_{ba}|\psi_a\rangle_\Sigma$

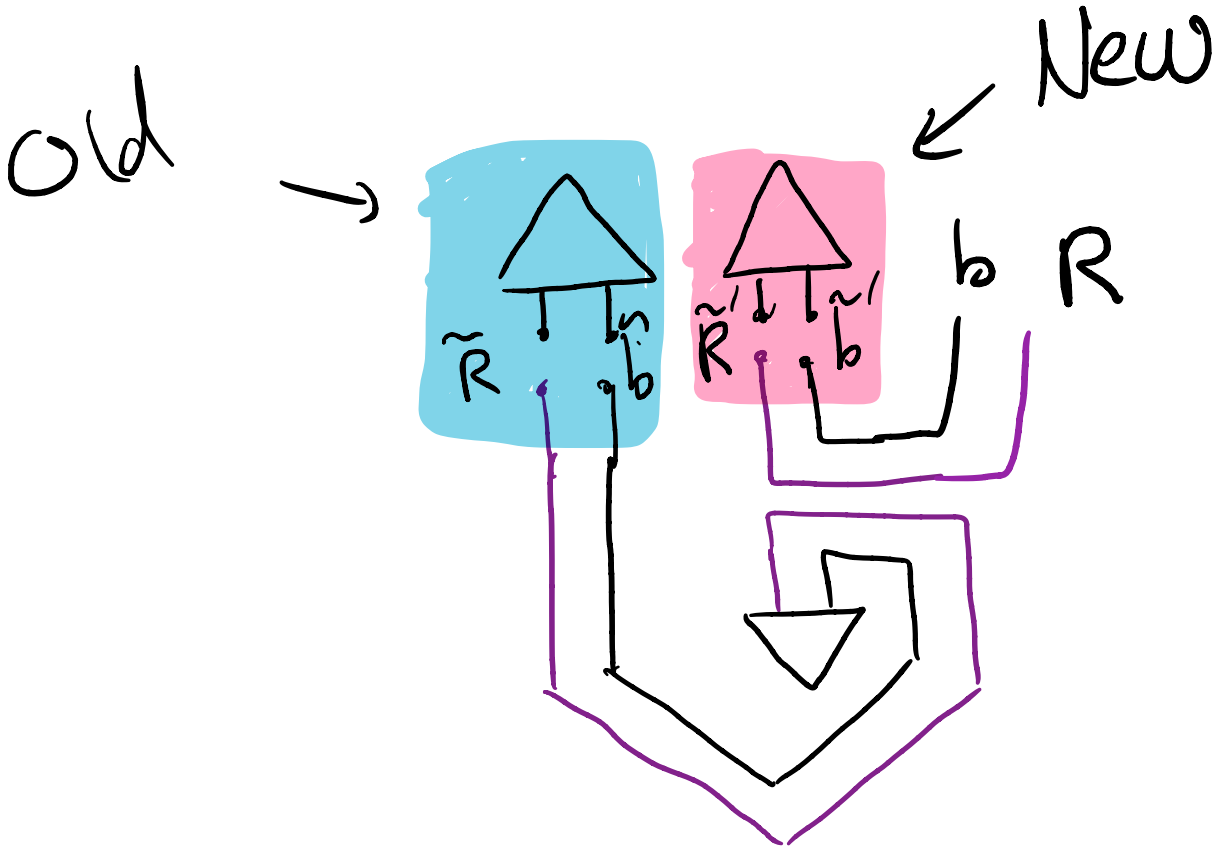


Interior (island) is replaced by new interior from the operator.

Back to the Final State...

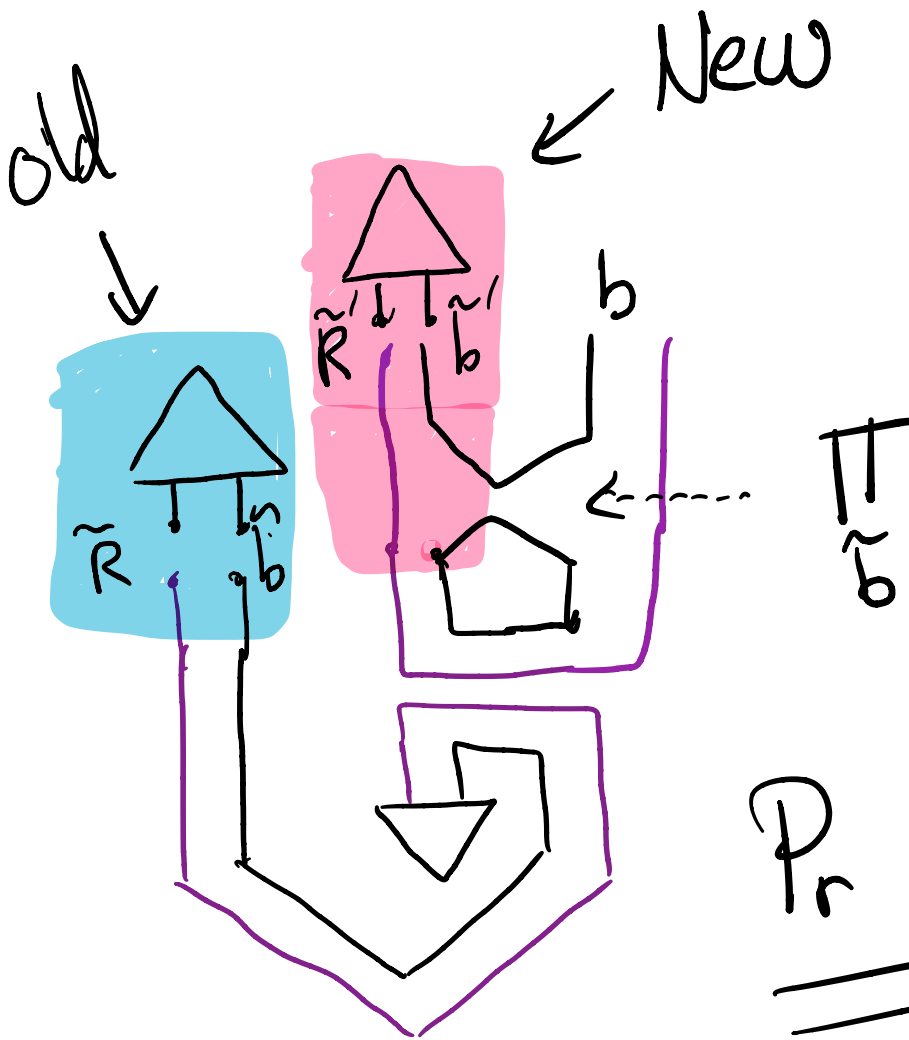
$$\langle F | \Pi | \Psi \rangle$$

$\tilde{R}\tilde{b}$ $\quad$ bR $\quad$ $\tilde{R}\tilde{b}bR$



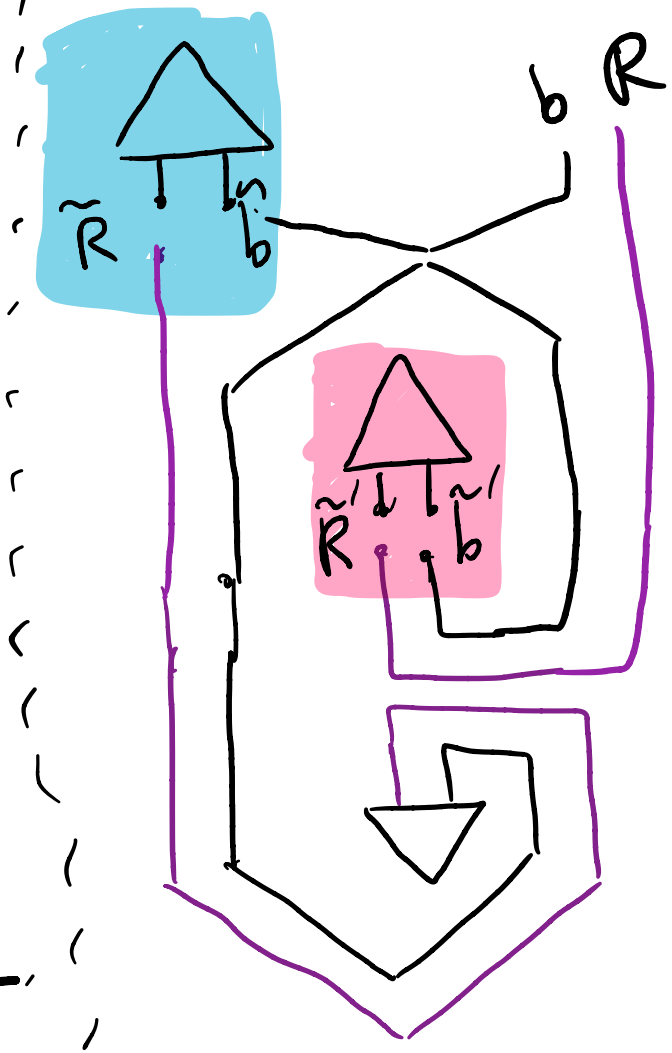
For AMPS, next measurement on NEW horizon.

$\Rightarrow \Pi_{b'b}$:

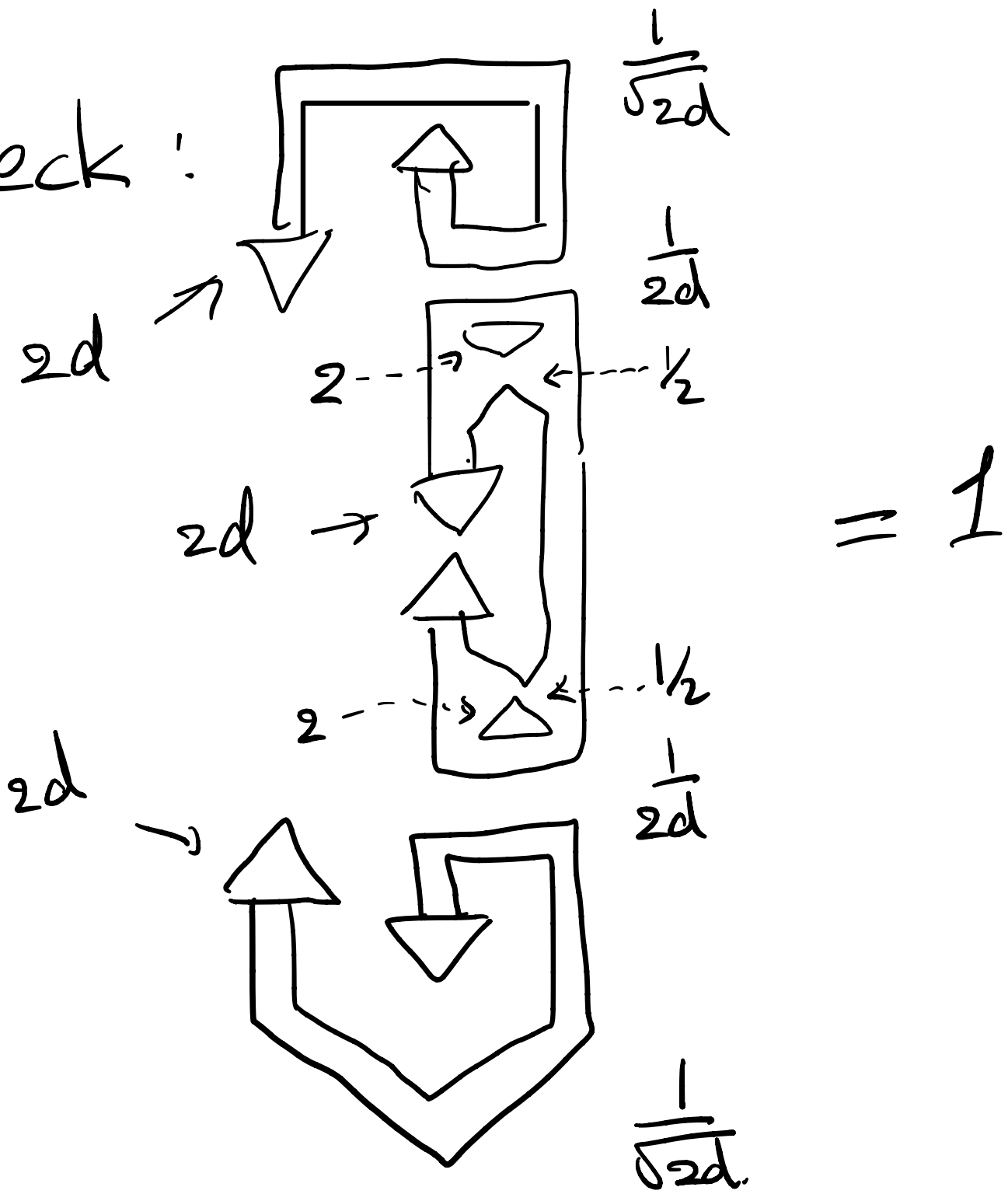


$P_r = 1$

VS B.S.



Check:



loops $\Big\{$ $= 1$

Conclusions...

→ Final state leads to acausality

→ $\gamma_b \in R$ in gravity path integral

→ leads to new rules on measuring γ_b after R has been measured.

→ Suggests a modification of final state:

New interior w/ New final state after modifying R!