PAGE CURVE IN MOVING MIRROR SETUP

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Based on [arXiv:2011.12005]
collaboration with
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Summary

[Akal, YK, Shiba, Takayanagi, Wei]

Page curve from BCFT?

Conclusion

- Simple setup that mimics black hole evaporation
  - Moving mirror (BCFT with radiation) can be obtained by standard BCFT with a particular conformal map
    (point: calculation is easy)
  - EE in moving mirror completely reproduces Page curve.

Future direction

- Relation between AdS/BCFT and braneworld holography
- New type of quench in CFT
  ⇒Application to condensed matter ?

Developments about Page Curve

CFT_d: non-gravitational bath
CFT_d on AdS_d: gravitational region

Setup:
AdS_d & CFT_d are glued along the (asymptotic) boundary

[Penington], [Almheiri, Engelhardt, Marolf, Maxfield], [Almheiri, Mahajan, Maldacena, Zhao]

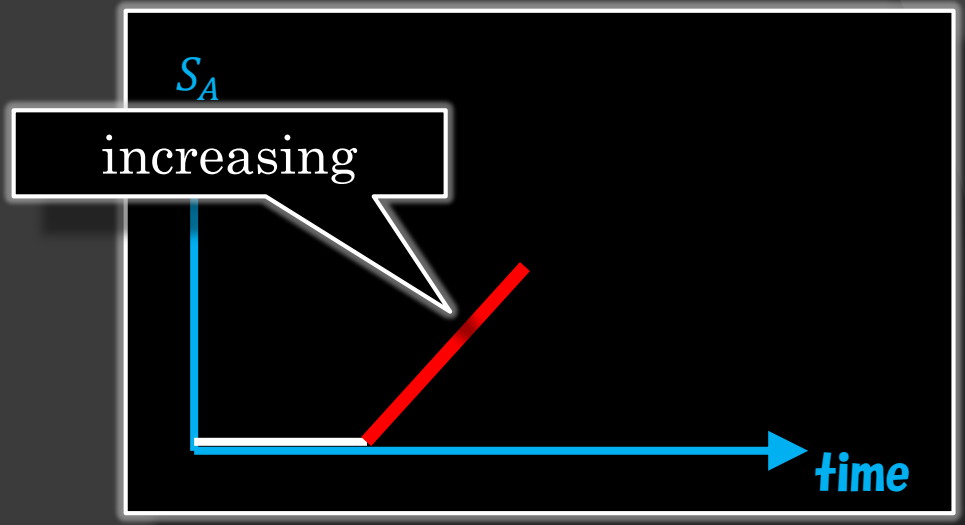
This AdS_d is dynamical.
Light can go through asymptotic boundary.
We can discuss the Page curve in this setup.

Developments about Page Curve

Physical expectation:

Entanglement Entropy between CFT_d and AdS_d

Developments about Page Curve

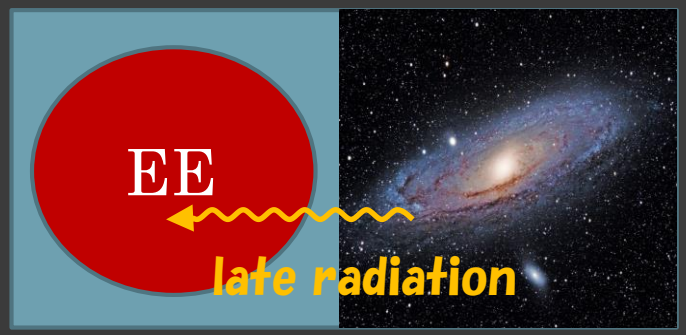


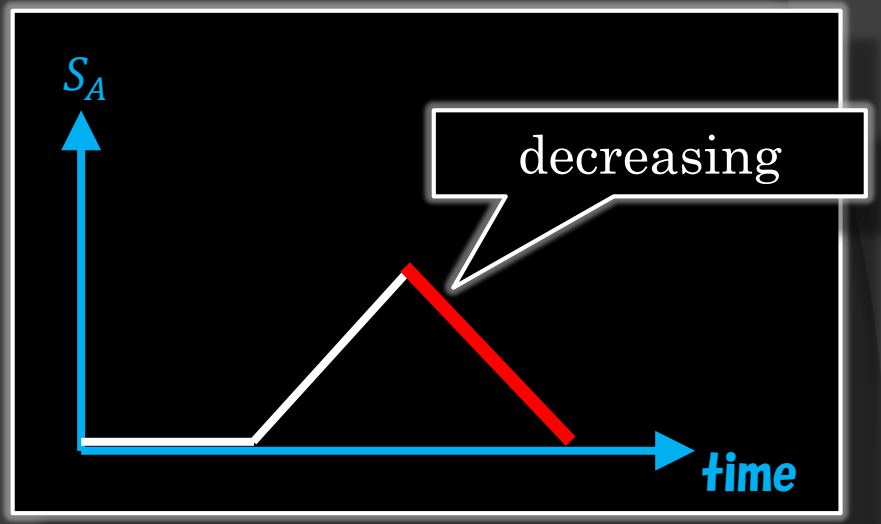
Physical expectation:

Entanglement Entropy between CFT_d and AdS_d

- **early radiation** increases S_A

Developments about Page Curve



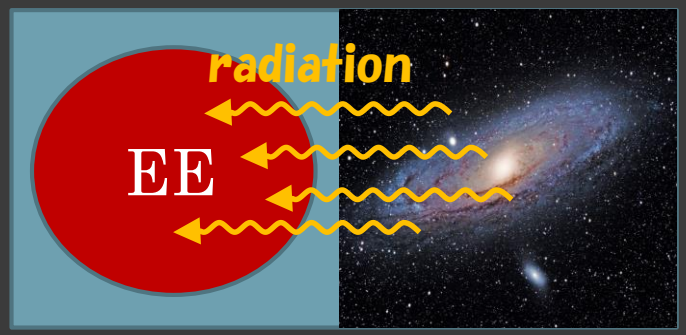


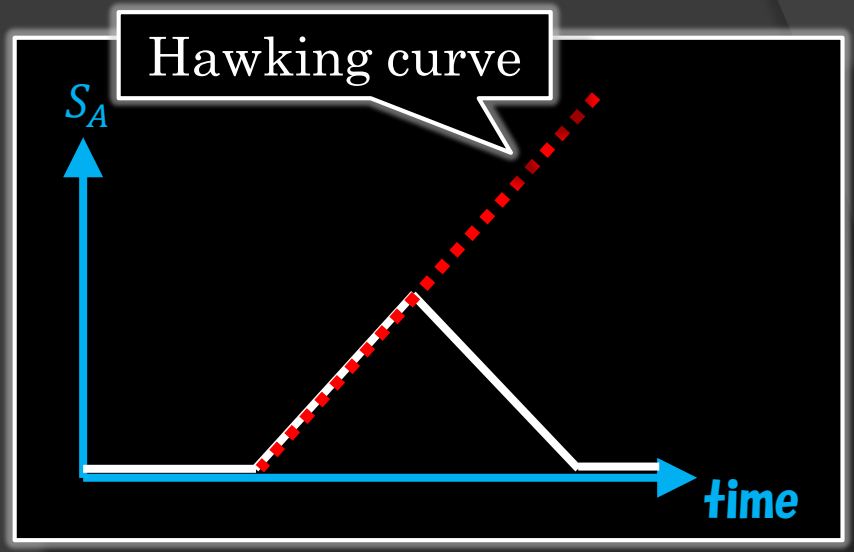
Physical expectation:

Entanglement Entropy between CFT_d and AdS_d

- early radiation increases S_A
- **late radiation** decreases S_A

Developments about Page Curve





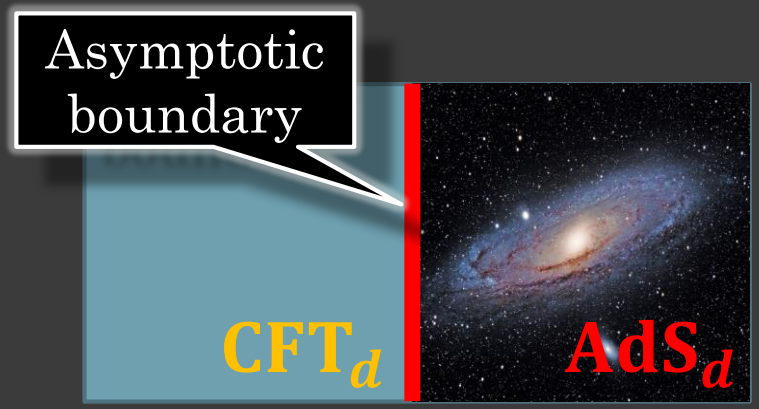
Physical expectation:

⇒ increasing & decreasing, called as **Page curve**

Naïve calculation (by Hawking):

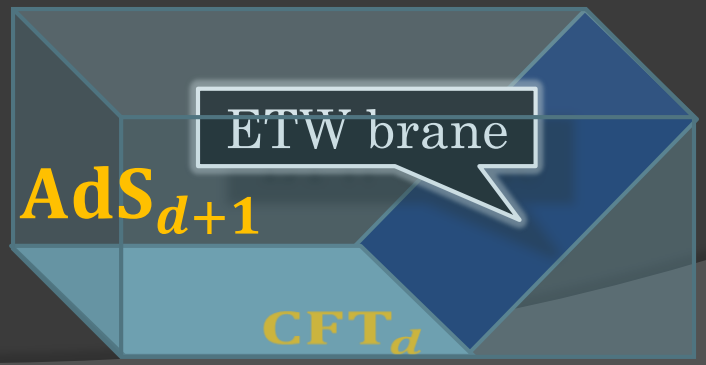
⇒ infinitely increasing (problematic!)

Developments about Page Curve

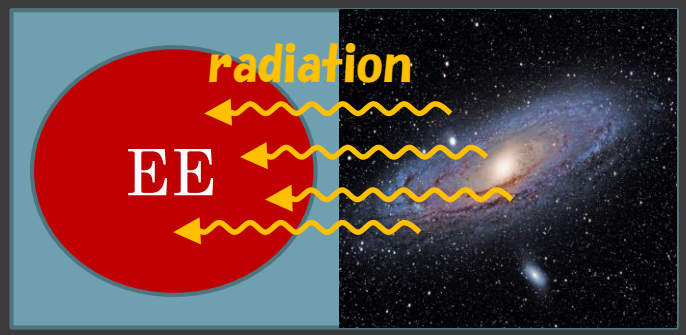


Braneworld holography

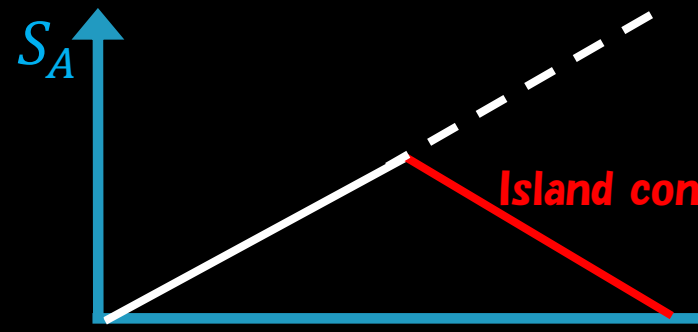
[Randall, Sandrum]



Developments about Page Curve

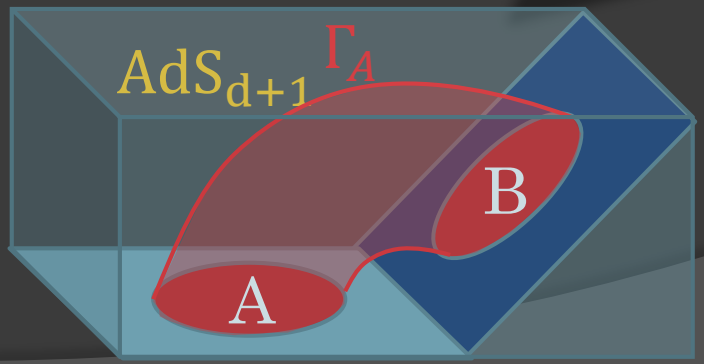


We can find the contribution from **Island**
(effective quantum contribution),
which reproduce the Page curve



Braneworld holography

[Randall, Sandrum]



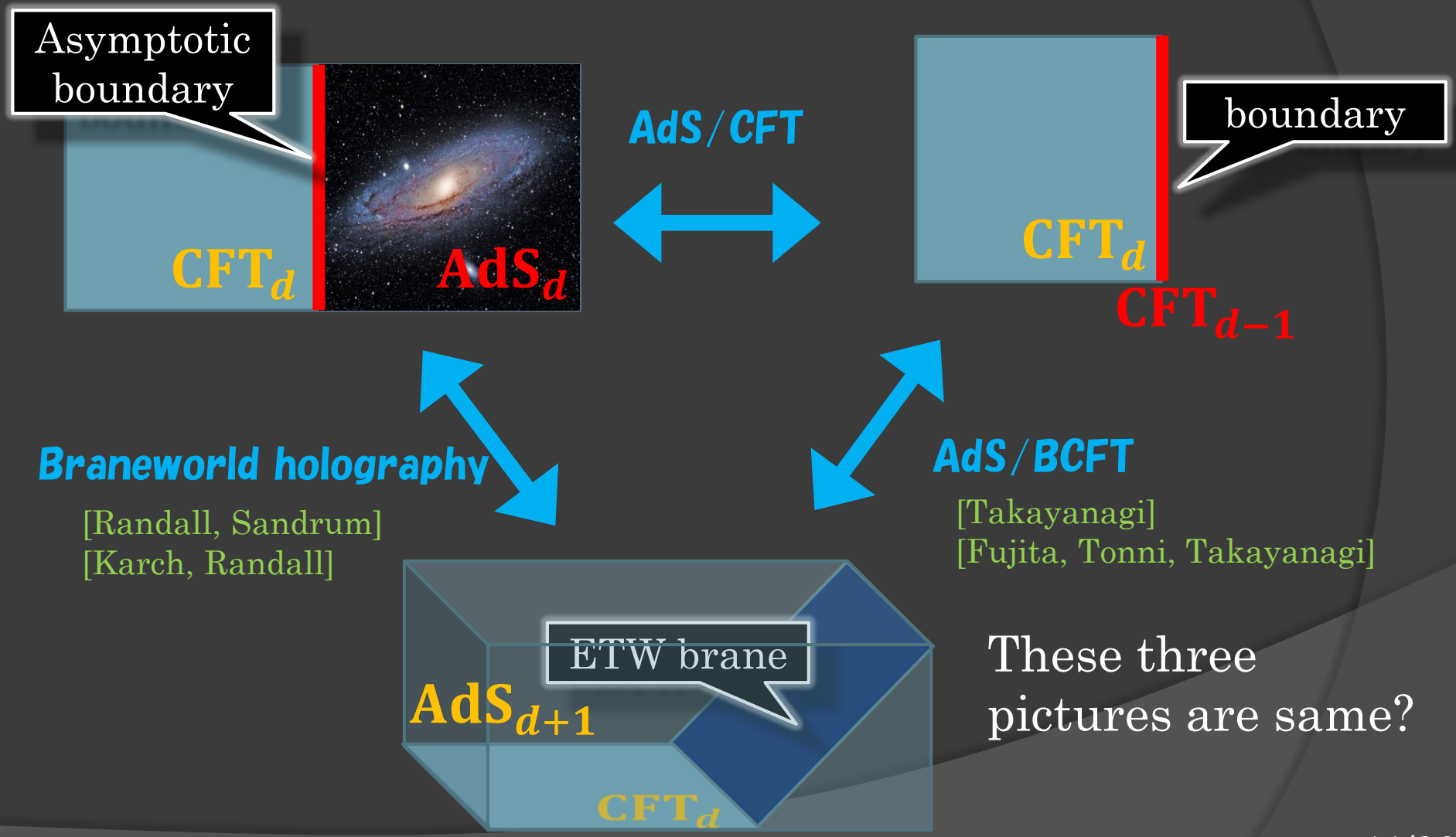
double holography
[Almheri, Mahajan, Maldacena, Zhao]

Developments about Page Curve

Setup:
AdS_d & CFT_d are glued along the (asymptotic) boundary

AdS/CFT correspondence:
$\mathrm{AdS}_d = \mathrm{CFT}_{d-1}$
This CFT_{d-1} can be thought of as boundary object of CFT_d

Developments about Page Curve

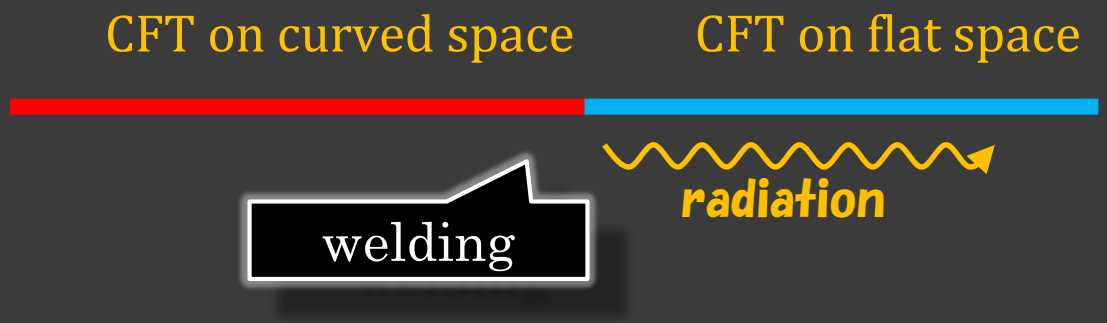


These three pictures are same?

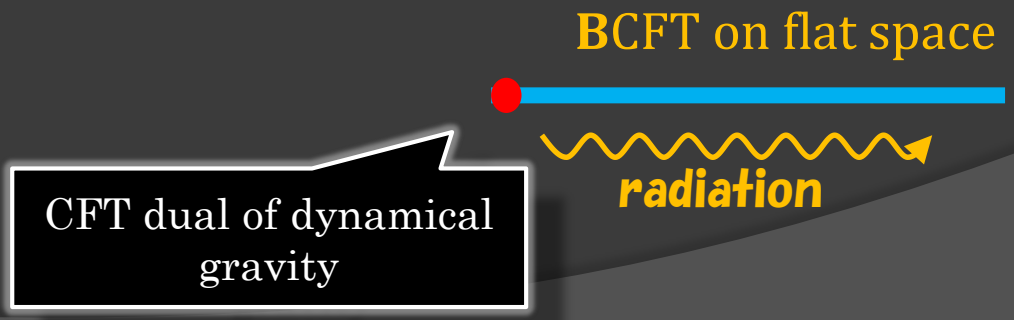
Page curve from CFT

- Conformal welding problem

[Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini]

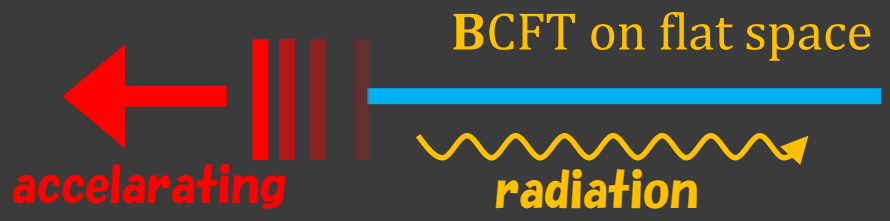


- **Our main interest = radiation from BCFT perspective**



Moving Mirror

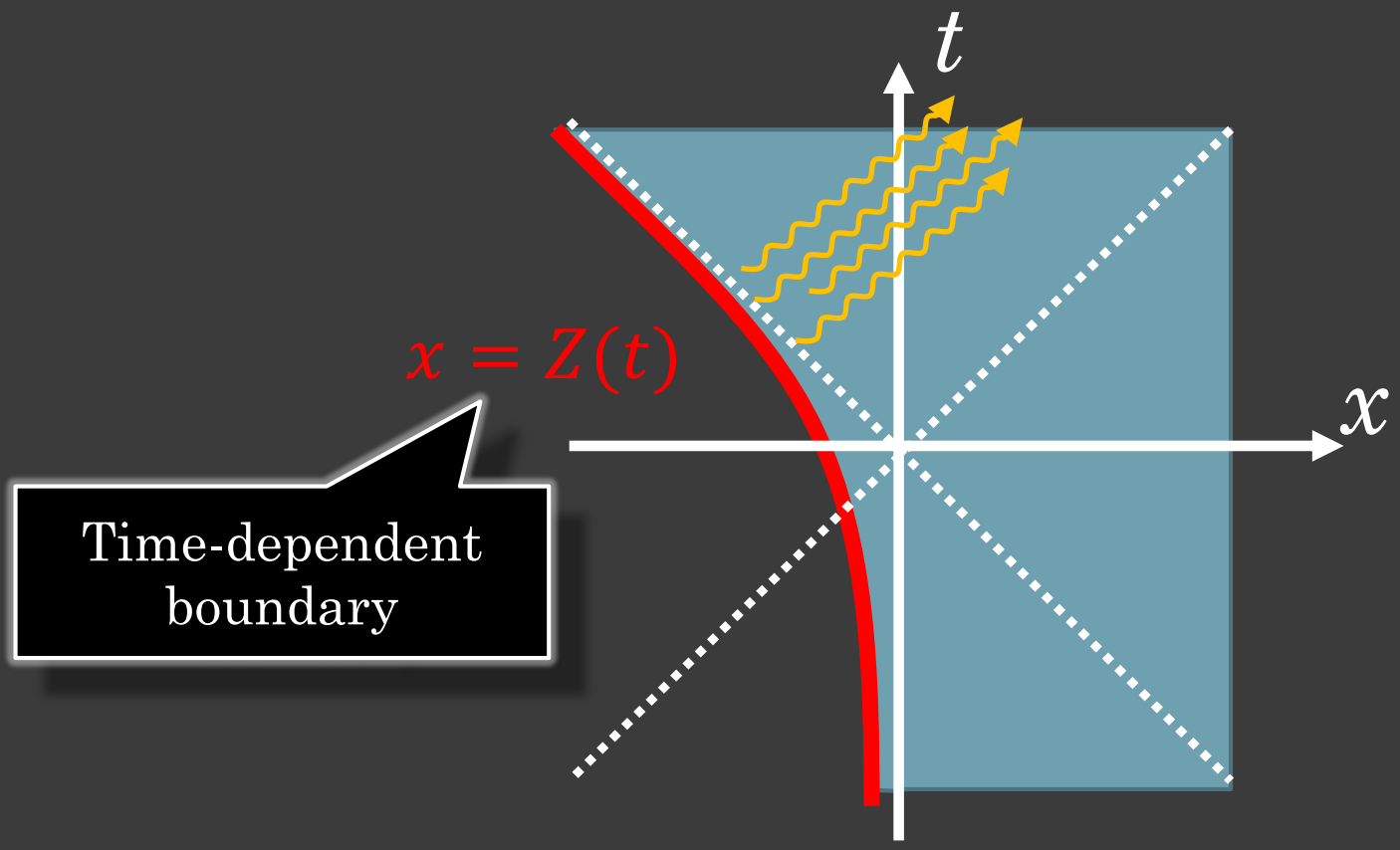
simple realization = **moving mirror** [Birrell Davies]



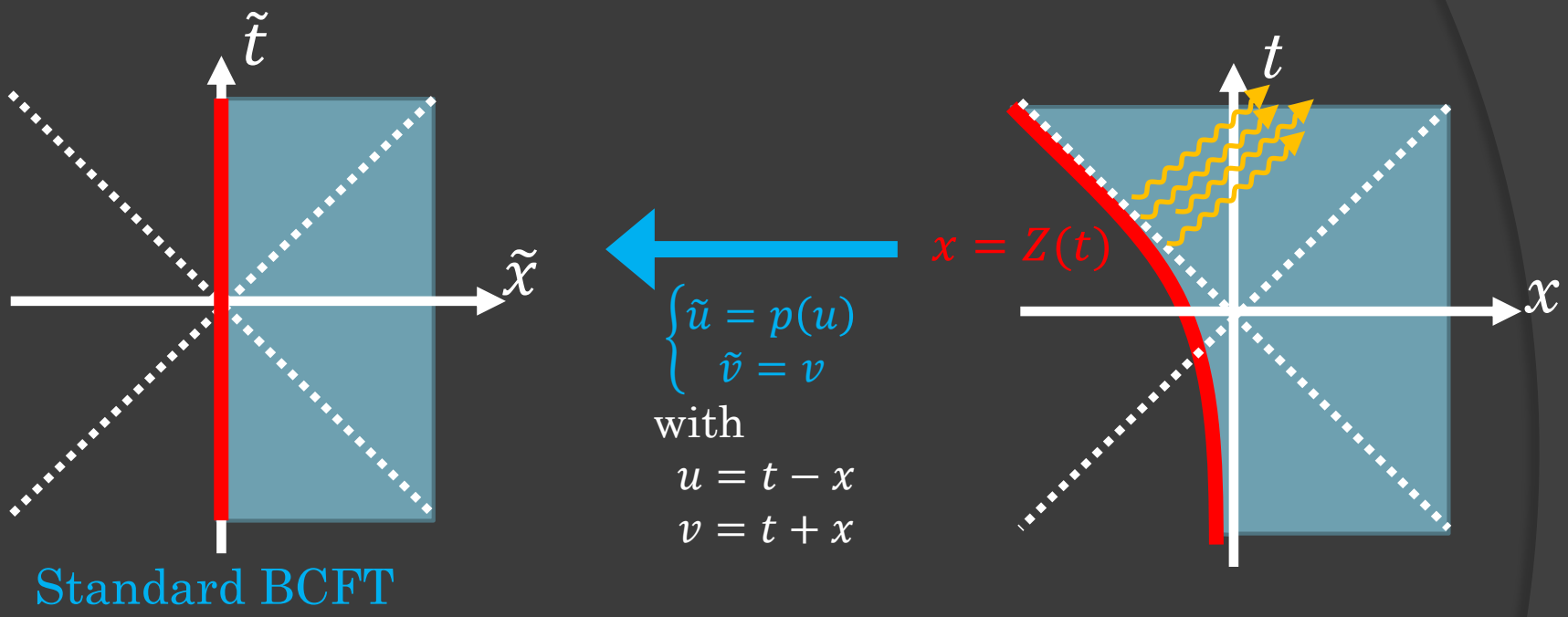
Question:
- How to describe moving mirror in CFT ?
- Time dependence of entropy of radiation ? (Page curve ?)
- Can we learn more about moving mirror by simplified calculation.

Interesting point:
- First step to model radiation setup from BCFT perspective
- Slight modification would be equivalent to JT+CFT bath (future work)

Moving mirror in CFT



Moving mirror in CFT

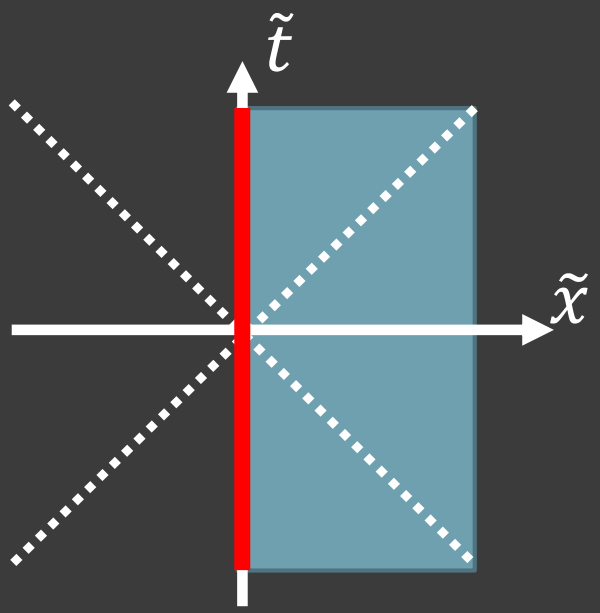


This map can be obtained as the solution to

$$t + Z(t) = p\big(t - Z(t)\big)$$

which comes from $\tilde{u} = \tilde{v}$ on the boundary.

Moving mirror in CFT



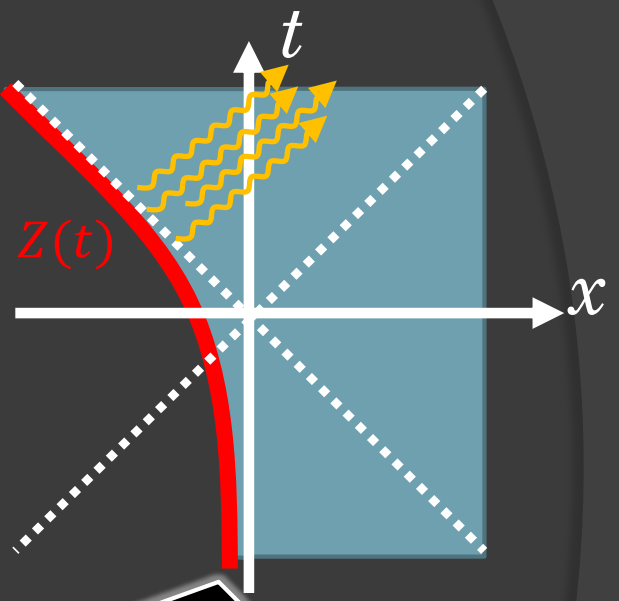
Standard BCFT

$$\begin{cases} \tilde{u} = p(u) \\ \tilde{v} = v \end{cases}$$

with

$u = t - x$

$v = t + x$

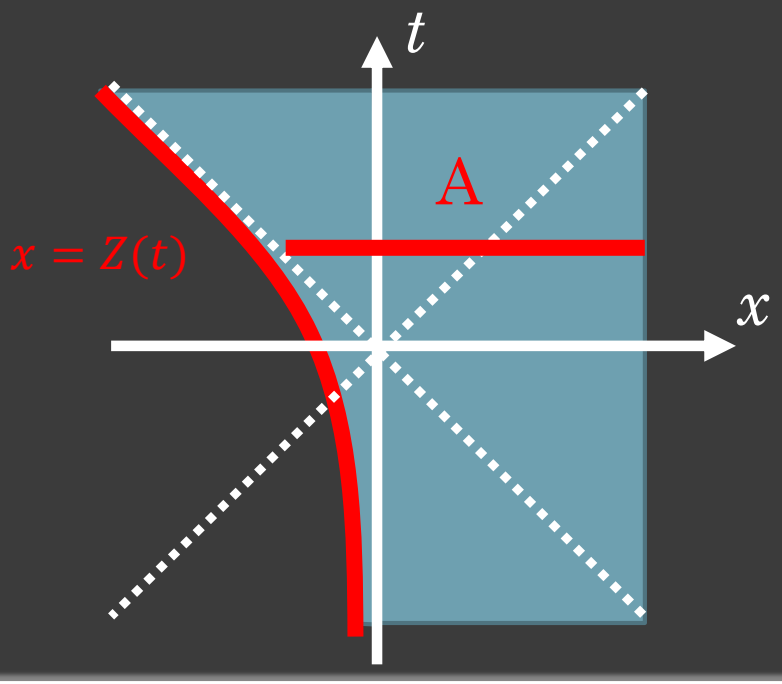


Non-zero energy flux on the boundary

$$T_{tx} = \frac{c}{24}\left(\frac{3}{2}\left(\frac{p''(u)}{p'(u)}\right)^2 - \frac{p'''(u)}{p'(u)}\right)$$

like JT gravity + bath setup.

Page curve



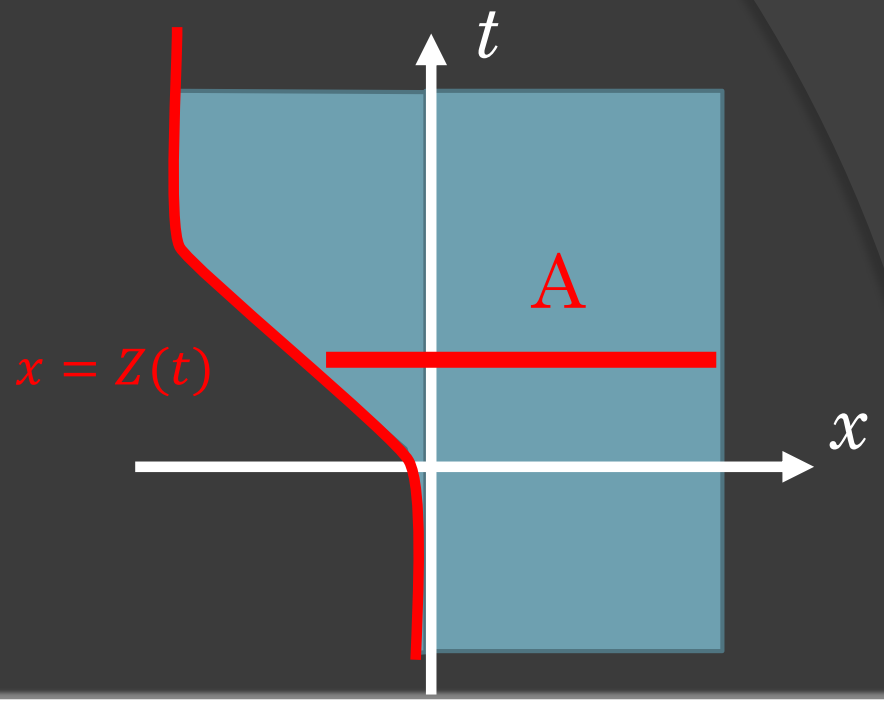
Our interest:
Entropy of radiation

In moving mirror setup,
this is EE for subsystem
$A = [Z(t) + \delta, \infty]$

- EE in standard BCFT is calculable (just correlator of twist fields).
- EE in **moving mirror** can be mapped from standard BCFT by $p(u)$.

⇒Everything is fine. Let us consider **entropy of radiation**

Page curve



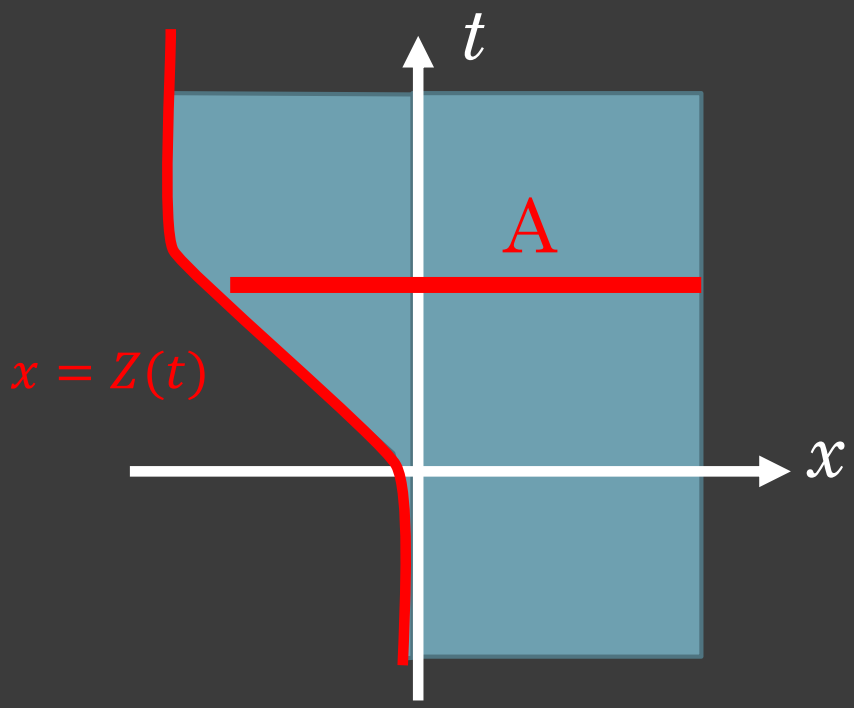
Moving mirror which mimics black hole evaporation,

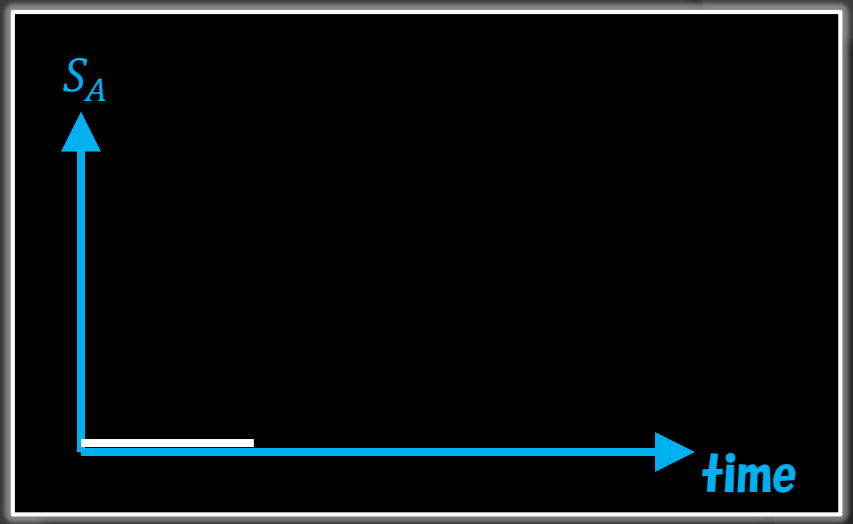
$$Z(t) \simeq \begin{cases} 0 & (t < 0) \\ -t & \left(0 < t < \frac{u_0}{2}\right) \\ -\frac{u_0}{2} & \left(\frac{u_0}{2} < t\right) \end{cases}$$

can be realized by

$$p(u) = -\beta \log\left(1 + \mathrm{e}^{-\frac{u}{\beta}}\right) + \beta \log\left(1 + \mathrm{e}^{\frac{u-u_0}{\beta}}\right)$$

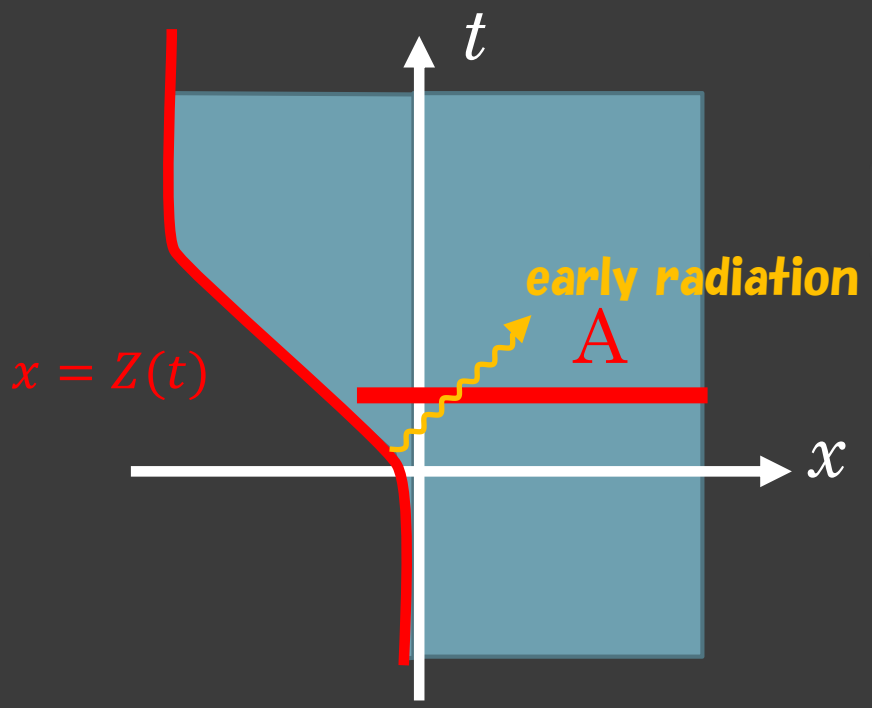
Page curve

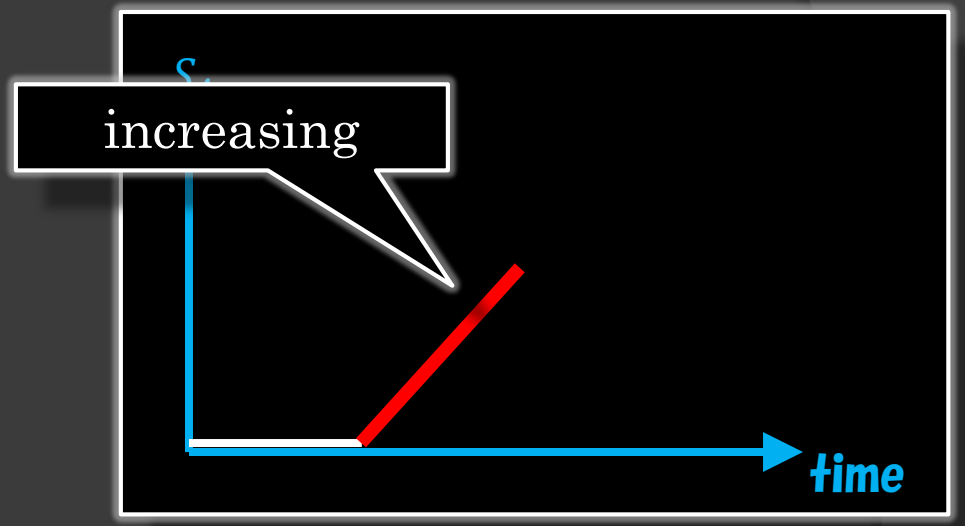




If this model mimics the evaporating black hole, we should find

Page curve

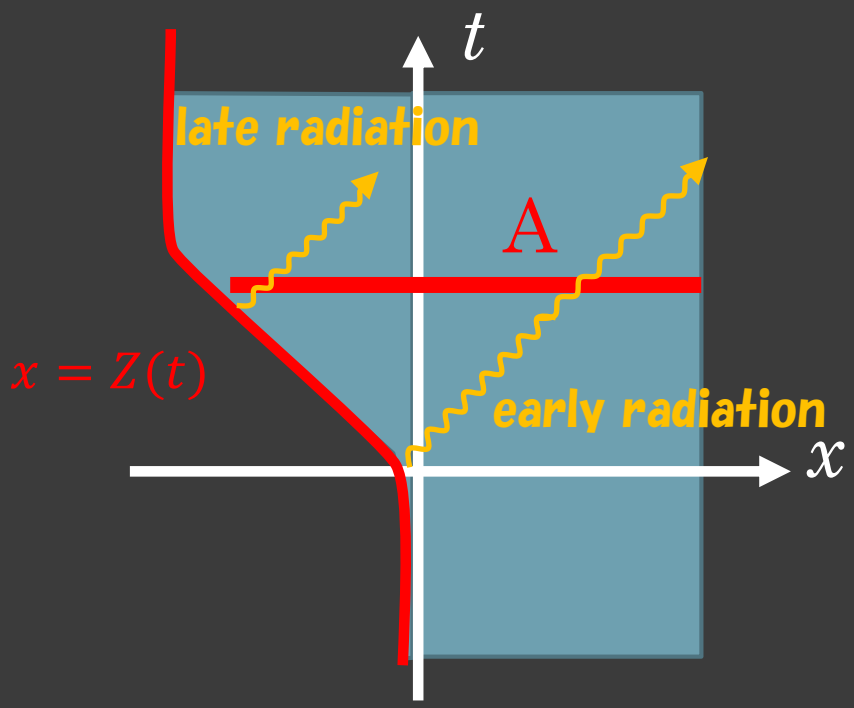


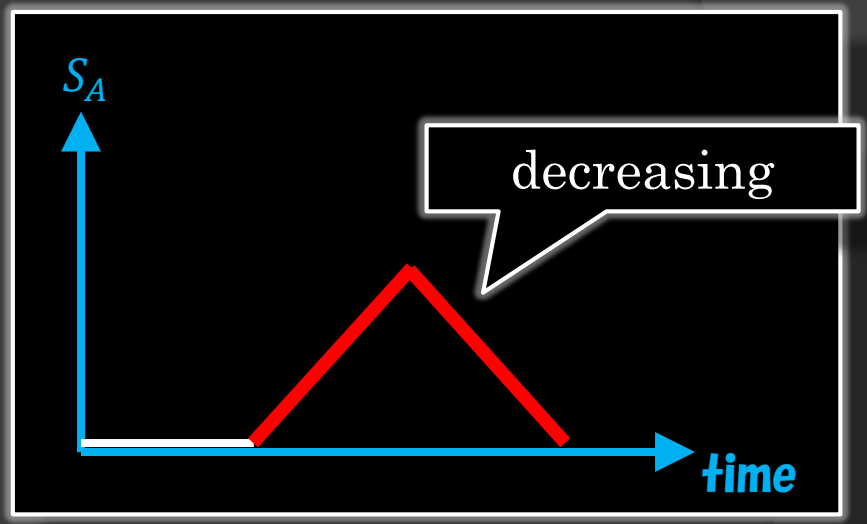


If this model mimics the evaporating black hole, we should find

- Increasing process
  ⇒ **early radiation**

Page curve

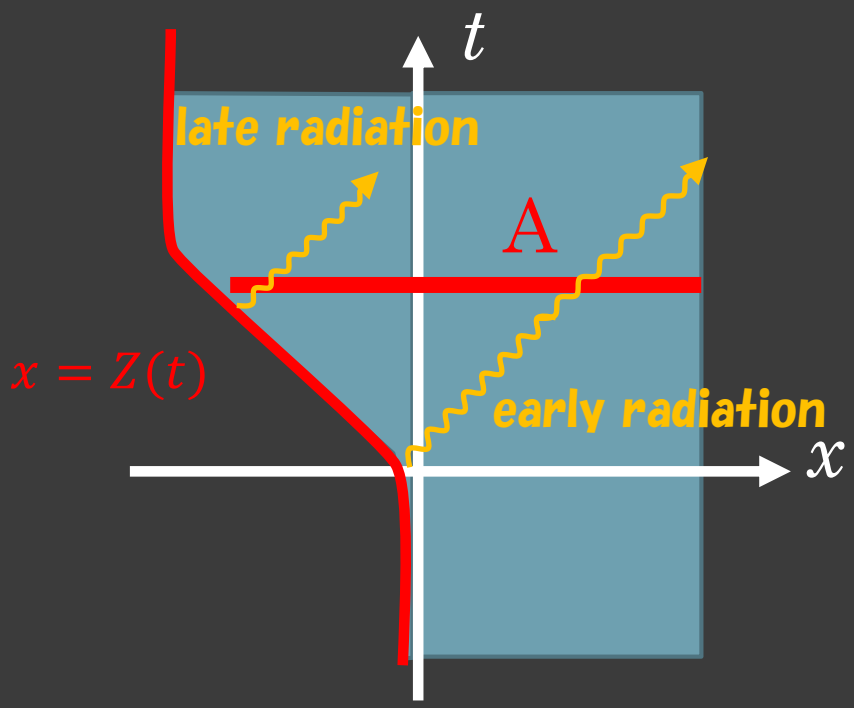


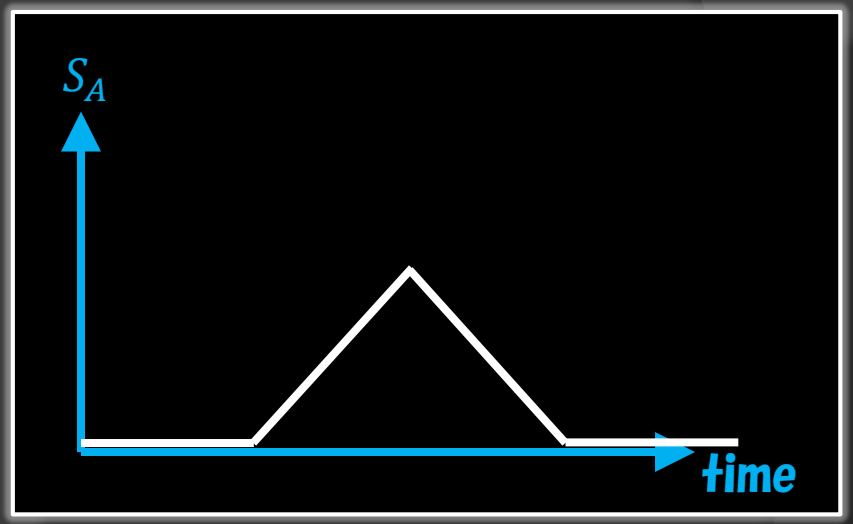


If this model mimics the evaporating black hole, we should find

- Increasing process
  ⇒ early radiation
- Decreasing process
  ⇒ **late radiation**

Page curve

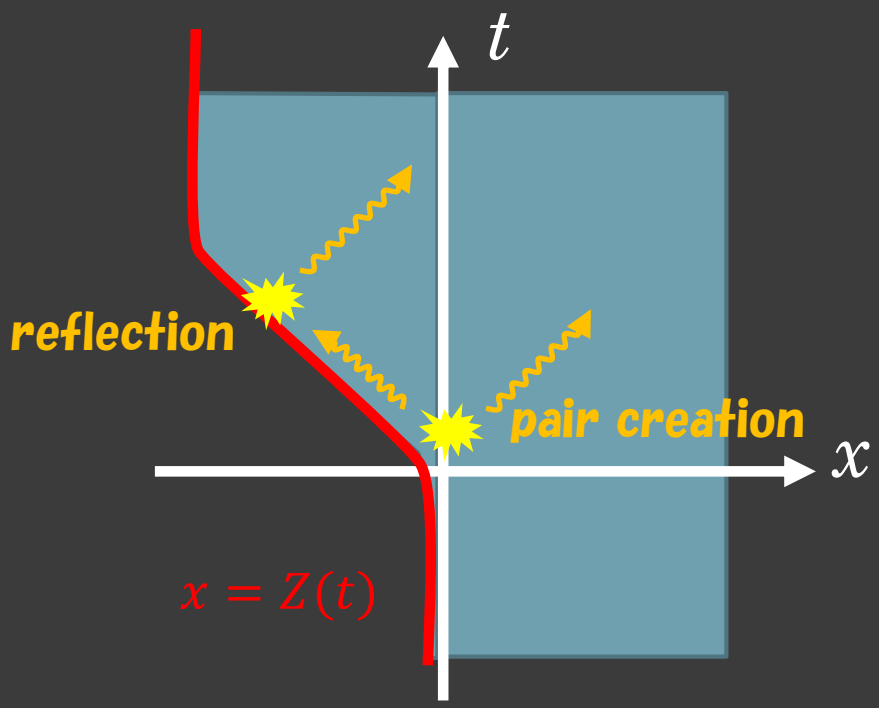


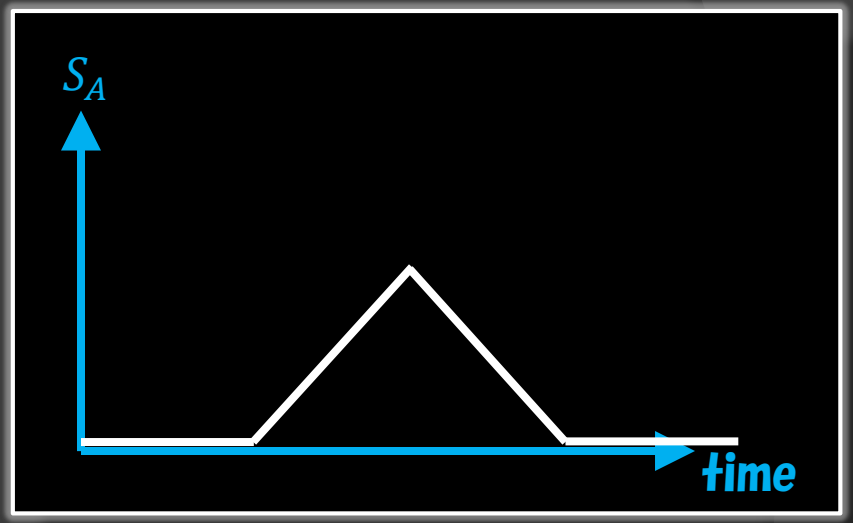


Our BCFT calculation completely reproduces this Page curve.

$$S_A \sim \begin{cases} \frac{c}{6\beta} t & (0 < t < t_P) \\ -\frac{c}{6\beta} t & (t_P < t) \end{cases}$$

Quasi particle picture





Clear quasi particle picture

(by mass less free scalar, EE in various setups, etc.)

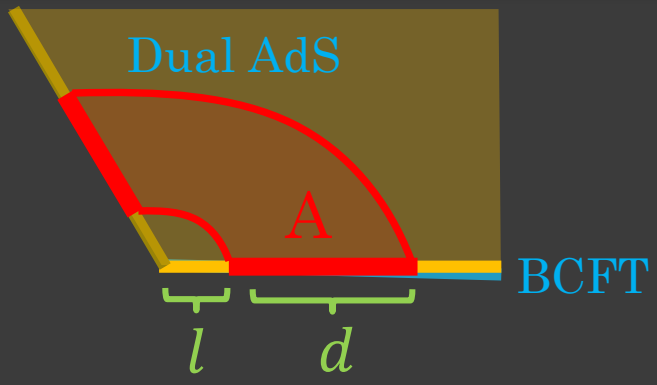
- **pair creation** occurs on $v + p(u) = 0$
- **reflection** happens on boundary

$\Rightarrow$ late radiation is really the partner of early radiation

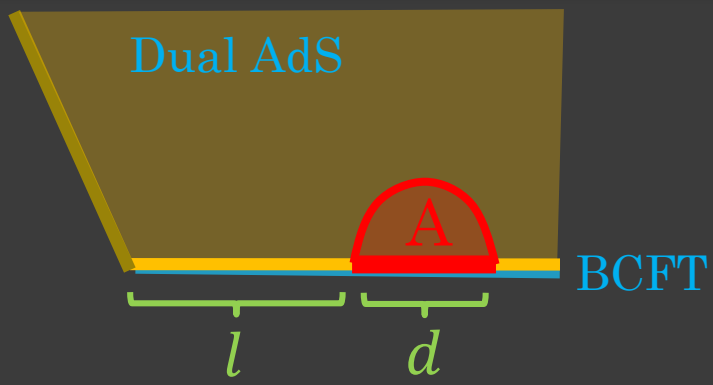
Holographic dual

Holographic EE in BCFT = Area of RT surface

(different point: RT surface can end on ETW brane)



Disconnected phase



Connected phase

Our CFT calculation is completely reproduced by ***disconnected phase*** (like island formula).

Note: earlier works of holographic analysis in [Bianchi, Smerlak], [Hotta, Sugita] correspond to connected phase.

Summary

[Akal, YK, Shiba, Takayanagi, Wei]

Page curve from BCFT?

Conclusion

- Simple setup that mimics black hole evaporation
  - Moving mirror (BCFT with radiation) can be obtained by standard BCFT with a particular conformal map (point: calculation is easy)
  - EE in moving mirror completely reproduces Page curve.
  - Clear quasiparticle picture can be given by BCFT calculation.
- Holographic calculation is consistent
  - Disconnected phase is dominant (like island formula)

Summary

[Akal, YK, Shiba, Takayanagi, Wei]

Page curve from BCFT?

Future directions

- Precise relation between moving mirror setup and JT+bath setup
  - relation between braneworld holography and AdS/BCFT
  - boundary state corresponding to evaporating BH
- New class of non-equilibrium setups
  - dynamics of information in this new type of quenches in CFT (application to condensed matter)
  - our generalized BCFT would provide other interesting setups other than Page curve (point: BCFT calculation is simple)
- Generalization to higher dimension