Information Scrambling, Quantum Error Correction, and Information Paradox with Symmetry

Yoshifumi NAKATA

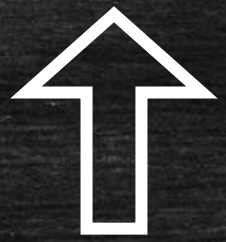
The University of Tokyo & JST PRESTO

All will be extensively updated soon!

arXiv:1903.05796 by E. Wakakuwa and YN
arXiv:2011.00668 by YN, E. Wakakuwa, and H. Yamasaki,
arXiv:2007.00895 by YN, E. Wakakuwa, and M. Koashi.

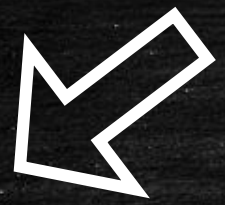
Quantum Information Science

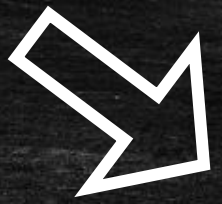
Quantum Error Correction



"Decoupling" approach

Scrambling





Black Hole
(Holography)

Fundamental Physics

Quantum Chaos

Strongly-correlated physics

Outline of this talk

1. Introduction
   1. Scrambling and Decoupling
2. Generalization of decoupling arXiv:1903.05796 by E. Wakakuwa and YN

3. Quantum Error Correction (QEC)
   1. Hybrid capacity theorem
4. QEC in Quantum Chaotic System

Quantum Information & Strongly-correlated physics

arXiv:2011.00668 by YN, E. Wakakuwa, and H. Yamasaki,

5. Hayden-Preskill protocol with symmetry

Fundamental Physics

arXiv:2007.00895
by YN, E. Wakakuwa, and M. Koashi.

6. Summary

Outline of this talk

1. Introduction
   1. Scrambling and Decoupling
2. Generalization of decoupling arXiv:1903.05796 by E. Wakakuwa and YN

3. Quantum Error Correction (QEC)
   1. Hybrid capacity theorem
4. QEC in Quantum Chaotic System

Quantum Information & Strongly-correlated physics

arXiv:2011.00668 by YN, E. Wakakuwa, and H. Yamasaki,

5. Hayden-Preskill protocol with symmetry

Fundamental Physics

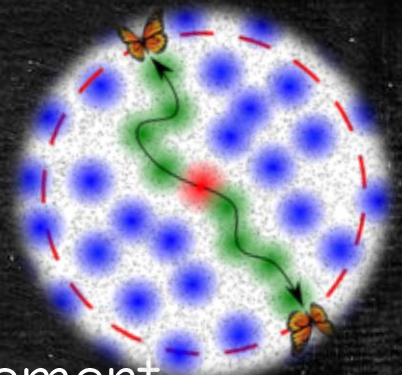
arXiv:2007.00895

by YN, E. Wakakuwa, and M. Koashi

6. Summary

What we mean by "scrambling" in this talk 2

- **Scrambling ≈"chaotic" unitary dynamics.**
  - Scrambling "delocalizes" information to the whole system.



- **Many different definitions:**
  - Haar random unitary, Unitary 2-design, OTOC, Operator Entanglement,……

Quantum Information

Fundamental Physics
Strongly-correlated physics

- **In this talk, scrambling = unitary 2-design (>≈OTOC)** [Roberts & Yoshida ('07)].
  - Relatively strong definition of "scrambling".

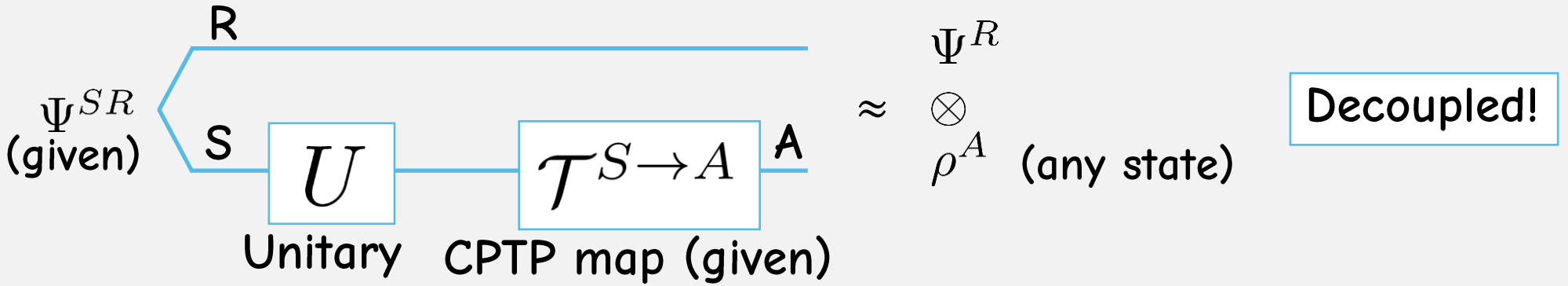
An important property of scrambling

- Why care about scrambling (= unitary 2-design) in QI?
  - Scrambling is related to "*decoupling*", the key task in QI theory.
  - The "decoupling" unitary can be used to encode information

Scrambling as a good encoder!!

What is decoupling?

Task: to find a unitary U such that the resulting state is a product state.



Decoupled!

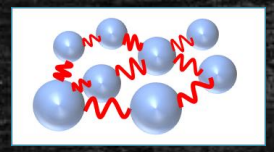
If U is scrambling, we can achieve decoupling (if $(\Psi, \mathcal{T})$ satisfies a certain condition).

Decoupling – a brief history –

Decoupling approach [Devetak & Winter ('04)]

One-shot decoupling theorem [Dupuis et al ('10)]

Scrambling



Quantum Thermodynamics
[del Rio et al ('16)]

Statistical physics

Quantum Error Correction
[Aveyesinghe, Devetak, Hayden, & Winter ('09)]

Quantum Information Science

Black hole information paradox
[Hayden & Preskill ('07)]

Fundamental Physics

Scrambling & decoupling are key tools
to understand quantum information in many-body systems.

nice to further explore decoupling in physics.....

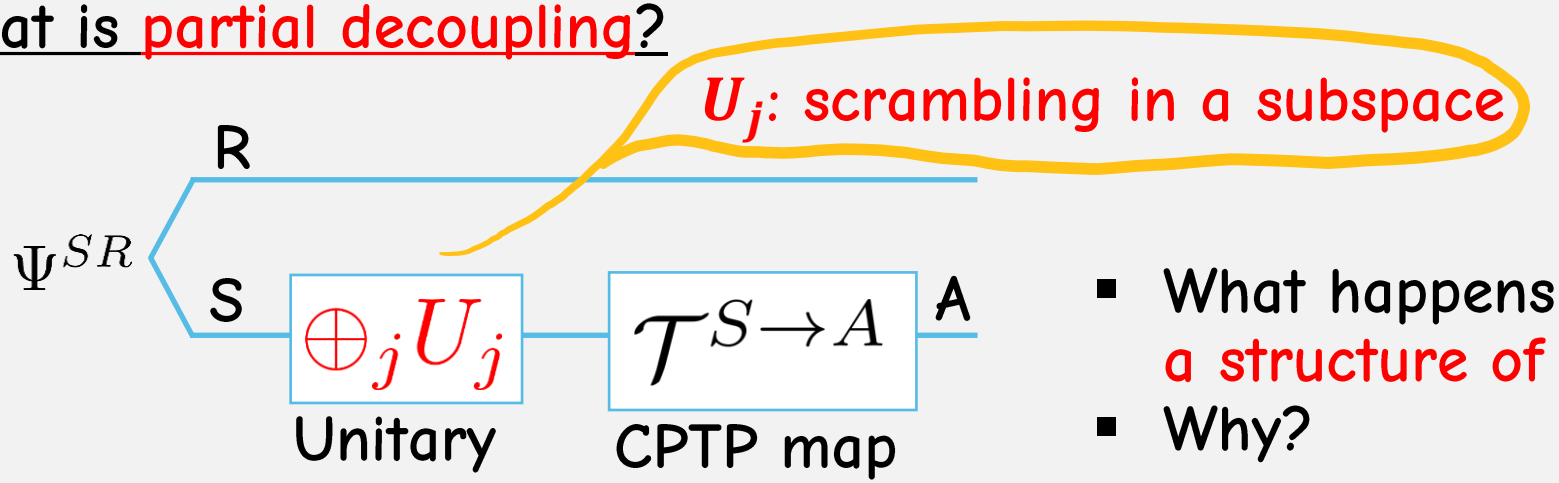
Partial Decoupling

"Partial" decoupling approach

One-shot partial decoupling theorem
[Wakakuwa & YN ('19)]

Special form of scrambling

What is partial decoupling?



- What happens if the scrambling has a structure of direct-sum?
- Why?

would be nice to explore more potential of decoupling.....

Partial Decoupling

"Partial" decoupling approach

Special form of scrambling

One-shot partial decoupling theorem
[Wakakuwa & YN ('19)]

Quantum Error Correction (QEC)
-- New capacity theorem --
[Wakakuwa, YN, & Yamasaki ('20)]

Quantum Information Science

QEC in
Quantum Chaos

Q many-body physics

Hayden-Preskill protocol
with symmetry
[YN, Wakakuwa & Koashi ('20)]

Fundamental Physics

Main message:
Partial decoupling as a new tool to study QI in many-body systems.

Partial Decoupling

"Partial" decoupling approach

One-shot partial decoupling theorem

[Wakakuwa & YN ('19)]

Special form of scrambling

2nd part of the talk

Quantum Error Correction (QEC)

-- New capacity theorem --

[Wakakuwa, YN, & Yamasaki ('20)]

Quantum Information Science

QEC in Quantum Chaos

Q many-body physics

Hayden-Preskill protocol with symmetry

[YN, Wakakuwa & Koashi ('20)]

Fundamental Physics

Partial decoupling

Quantum information in many-body

The last part of the talk

Outline of this talk

1. Introduction
   1. Scrambling and Decoupling
   2. Generalization of decoupling

3. Quantum Error Correction (QEC)
   1. Hybrid capacity theorem
4. QEC in Quantum Chaotic System

Quantum Information & Strongly-correlated physics

5. Hayden-Preskill protocol with symmetry

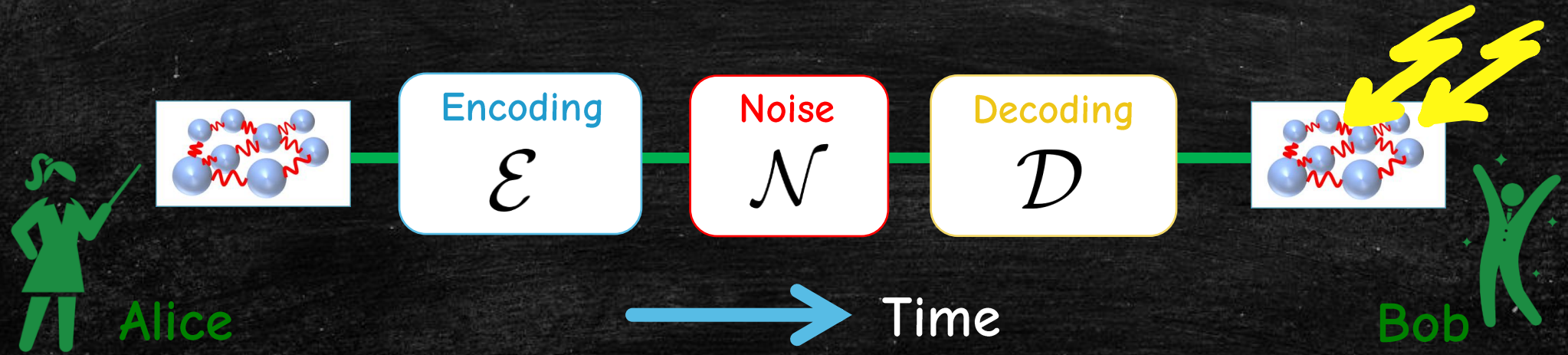
Fundamental Physics

6. Summary

What is Quantum Error Correction (QEC)?

- QEC is a method to protect information from noise.
  - Given noise $\mathcal{N}$, to find a pair of "encoding" map $\mathcal{E}$ and "decoding" map $\mathcal{D}$ s.t.

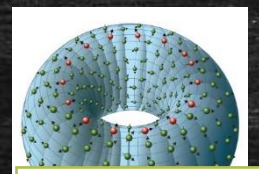
$$\mathcal{D} \circ \mathcal{N} \circ \mathcal{E} \approx \text{identity}$$

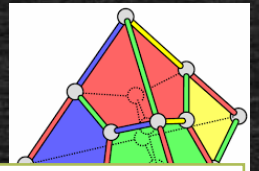


Two approaches in QEC

1. Practical approach: explicitly constructing encoder and decoder.

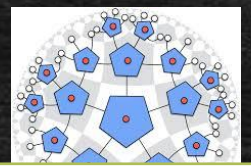
E.g.)





Topological orders

Toric code Color code



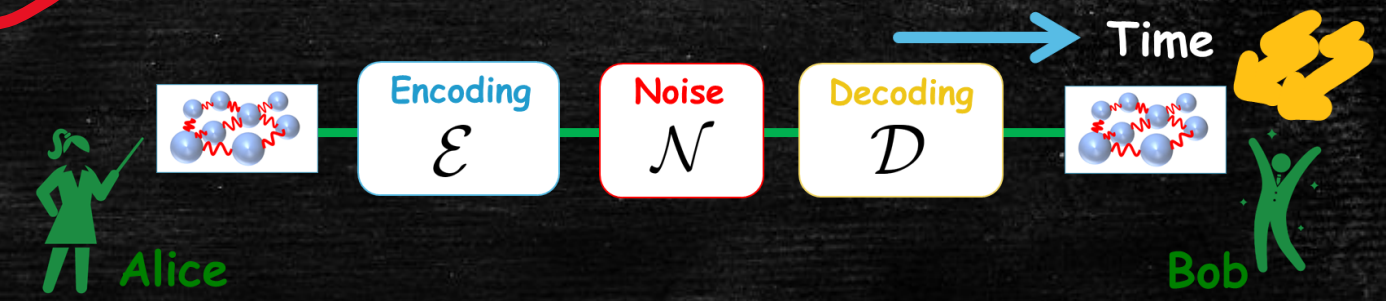
AdS/CFT

HAPPY code

Also offer interesting toy models of physical phenomena.

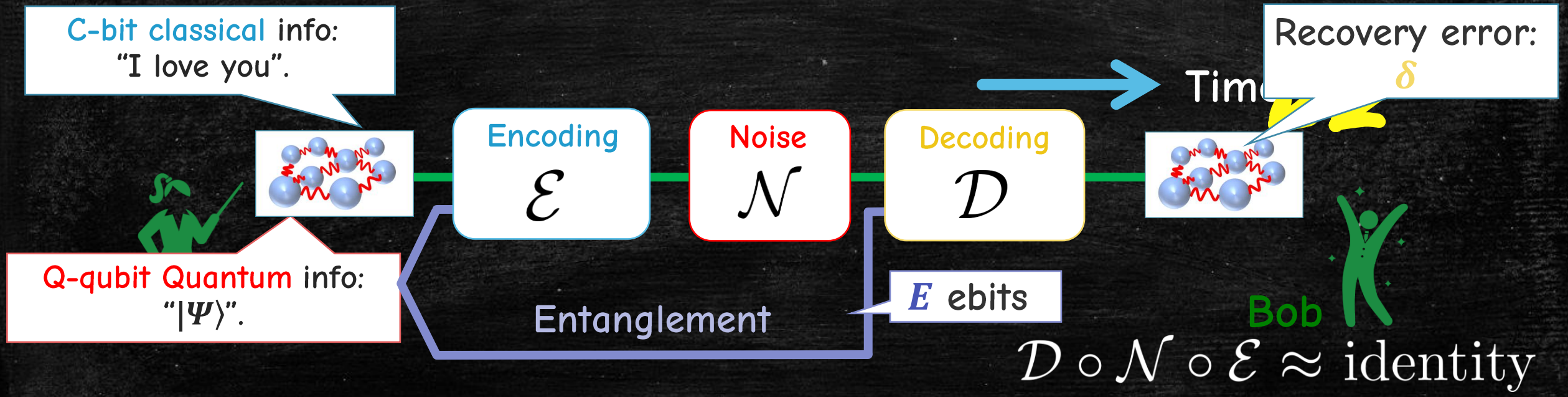
2. Fundamental approach: find the fundamental limit of QEC.
   - How much info. can be, in principle, protected from a given noise $\mathcal{N}$?

Scrambling comes into the play!!



Quantitative setting of QEC

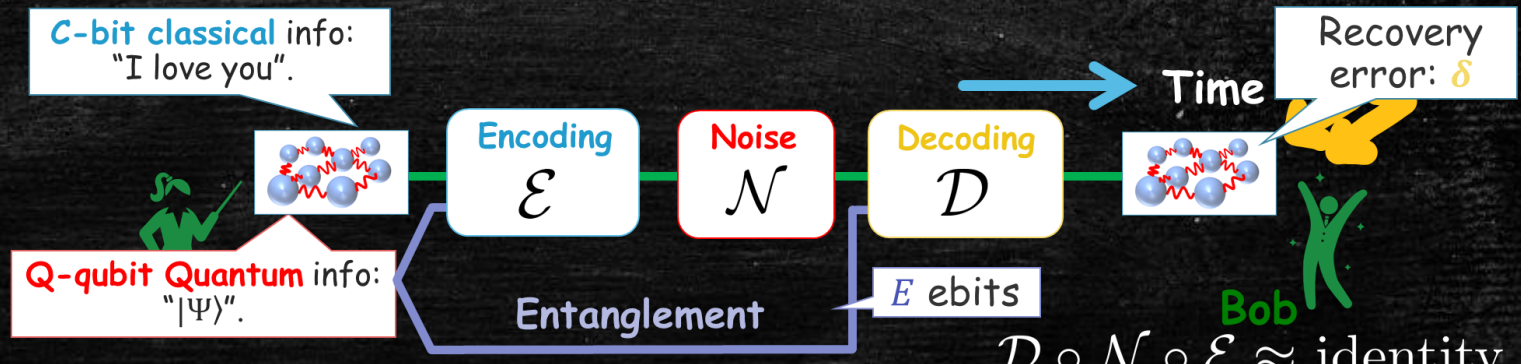
- Various situations of QEC.
  - Is the information *classical* or *quantum*?
  - Is additional resource, e.g., *entanglement*, available?
- What is the trade-off between (C, Q, E, δ)?



Fundamental limit of QEC

Trade-off between (C, Q, E, δ)?

- **Capacity theorems** give answers!
  - HSW theorem ('97), LSD theorem ('97~'05), Hsieh & Wilde ('10) (i.i.d setting).
  - Mosonyi, Datta, Dupuis, Berta, Anshu, a lot of ……
- General case $(0 < C, Q, E < \infty)$ remains unexplored.
  - Based on the partial decoupling, we proved **"hybrid" capacity theorem**.

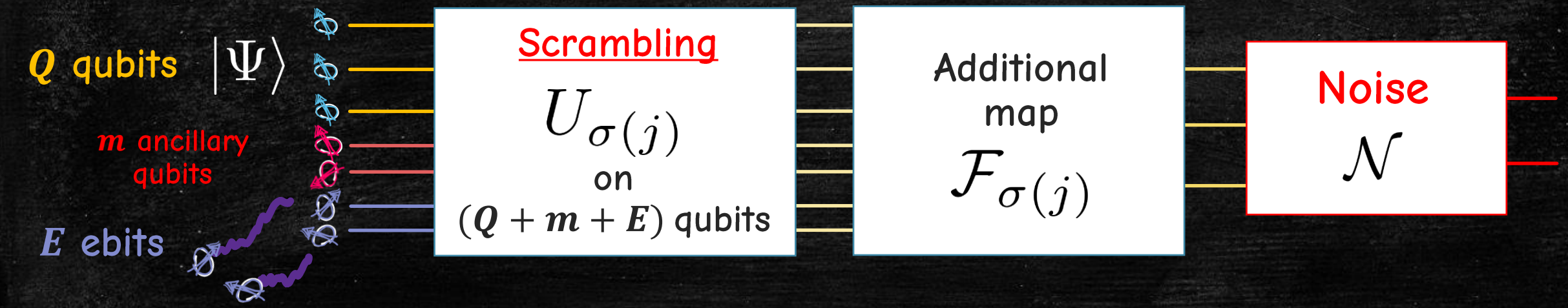


Hybrid capacity theorem 1

Trade-off between (C, Q, E, δ)?

- **Specific encoding:**
  1. Embed the alphabet $j \in \{1, \dots, 2^C\}$ into $\{1, \dots, J\}$, where $J \geq 2^C$, and apply a random permutation σ to obtain $\sigma(j)$.
  2. Attach ancillary qubits of m qubits.
  3. Apply **scrambling** on (Q+m+E) qubits and an additional map $\mathcal{F}$, dep. on $\sigma(j)$.

C bits: $j \in \{1, \dots, 2^C\}$ → $\sigma(j) \in \{1, \dots, J\}$



Hybrid capacity theorem 2

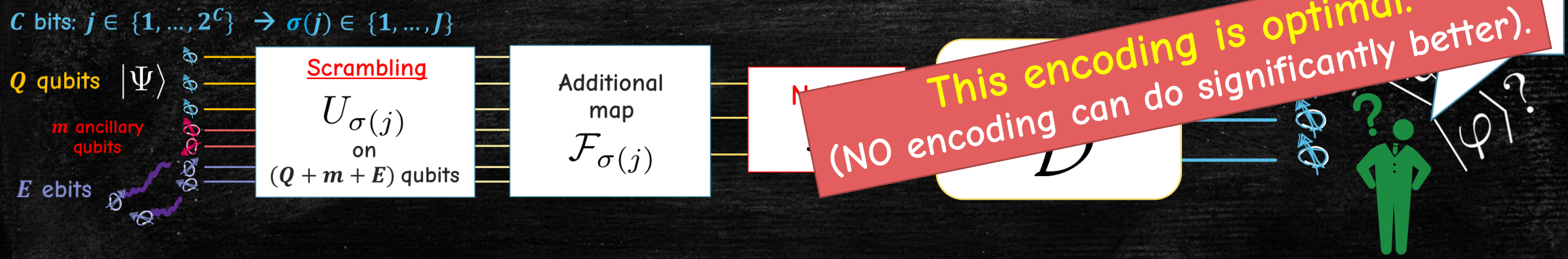
Trade-off between (C, Q, E, δ)?

Conditional entropy

There exists a decoding map s.t.

$$\delta \lesssim \sqrt{\delta_1^{1/2} + \delta_2^{1/2}} \quad \text{where} \quad \begin{aligned} \delta_1 &\approx 2^{C+Q-E+H_{\max}(S_c S_r|B)_{\varrho_{\mathcal{N}}} - \log[J]} \\ \delta_2 &\approx 2^{Q-E+H_{\max}(S_r|BS_c)_{\varrho_{\mathcal{N}}}} \end{aligned}$$

and, $\varrho_{\mathcal{N}}$ is constructed from $\mathcal{F}_j$ and the noise $\mathcal{N}$.



Hybrid capacity theorem 3

C-bit classical and Q-qubit quantum information can be protected from the noise $\mathcal{N}$ ($\delta \ll 1$) if and only if

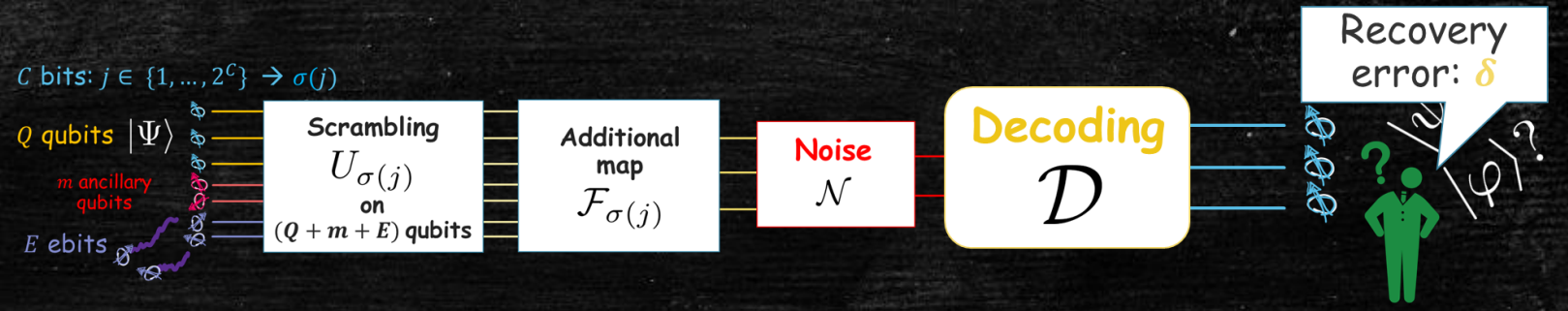
$$C + Q \lesssim E - \min_{\mathcal{F},J}\{H_{\max}(S_c S_r|B)_{\varrho_{\mathcal{N}}} - \log[J]\}$$

$$Q \lesssim E - \min_{\mathcal{F}}\{H_{\max}(S_r|BS_c)_{\varrho_{\mathcal{N}}}\}$$

Hybrid capacity theorem (informal)

1. The fundamental limit is characterized by conditional entropies of $\varrho_{\mathcal{N}}$.
2. Encoding by "random permutation + scrambling + $\mathcal{F}$" is optimal!!

By using this, we can study QEC in quantum chaos.



Outline of this talk

1. Introduction
   1. Scrambling and Decoupling
   2. Generalization of decoupling

3. Quantum Error Correction (QEC)
   1. Hybrid capacity theorem

4. QEC in Quantum Chaotic System

5. Hayden-Preskill protocol with symmetry

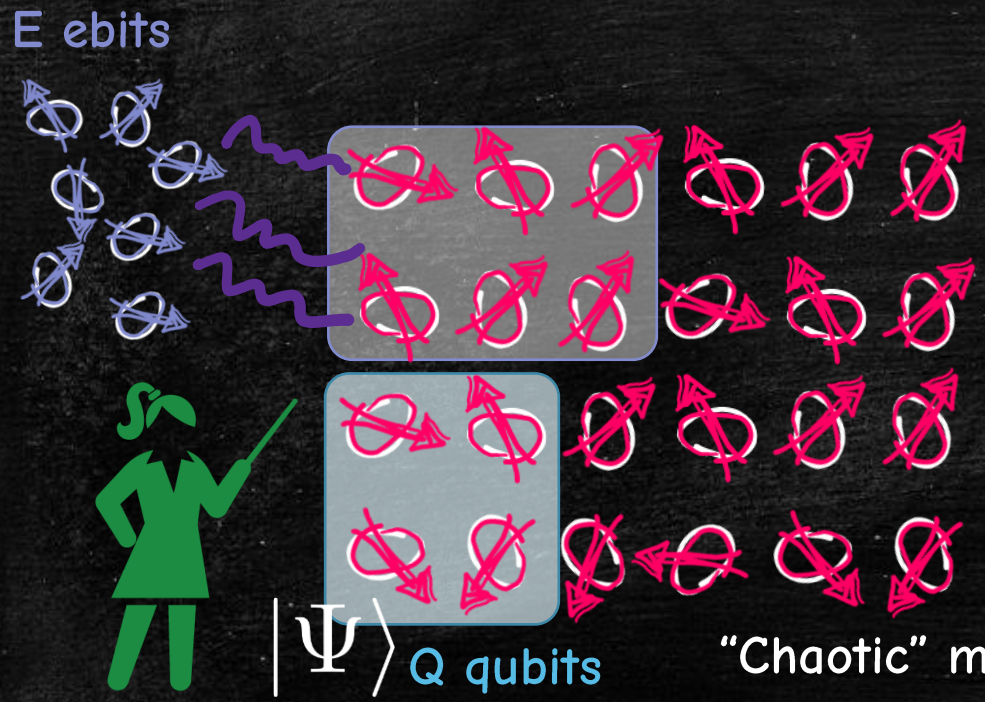
6. Summary

Quantum Information & Strongly-correlated physics

Fundamental Physics

QEC in "Quantum Chaos": setting 1

- Thought experiment: QEC in "chaos"
  1. Alice encodes a quantum state $|\Psi\rangle$ to a (part of) many-body system.

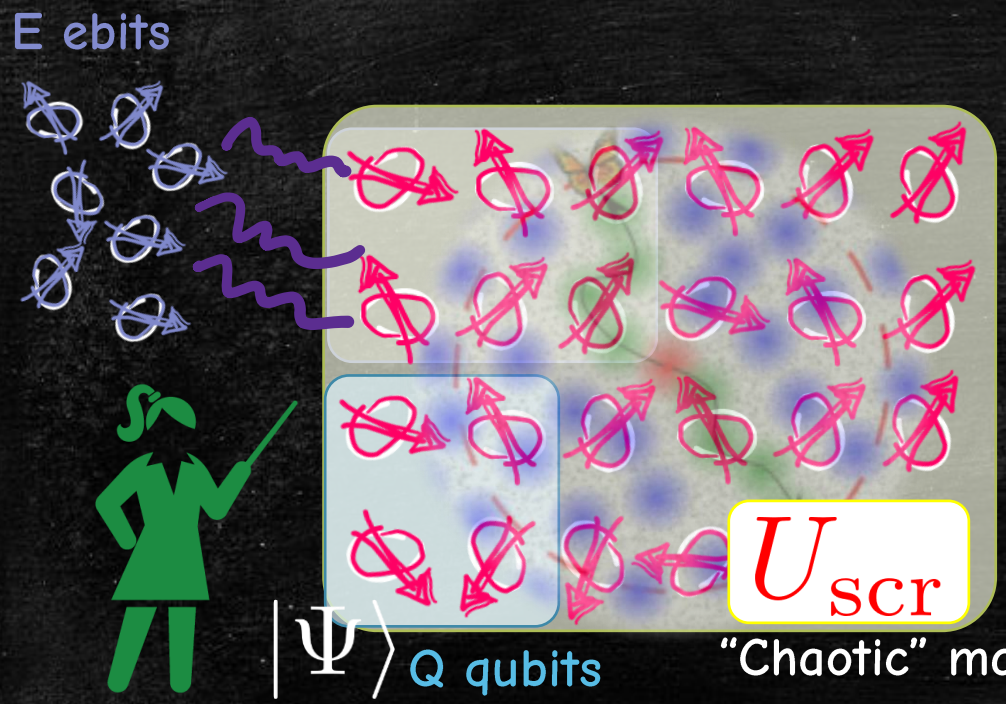


"Chaotic" many-body system of N qubits

- Some of the other spins (E qubits) are entangled with the qubits outside.
- The rest qubits are pure states, e.g., $|0\rangle^{\otimes(N-Q-E)}$.

QEC in "Quantum Chaos": setting 2

- Thought experiment: QEC in "chaos"
  1. Alice encodes a quantum state $|\Psi\rangle$ to a (part of) many-body system.
  2. Hamiltonian dynamics U_{scr} (= scrambling)



QEC in "Quantum Chaos": setting 3

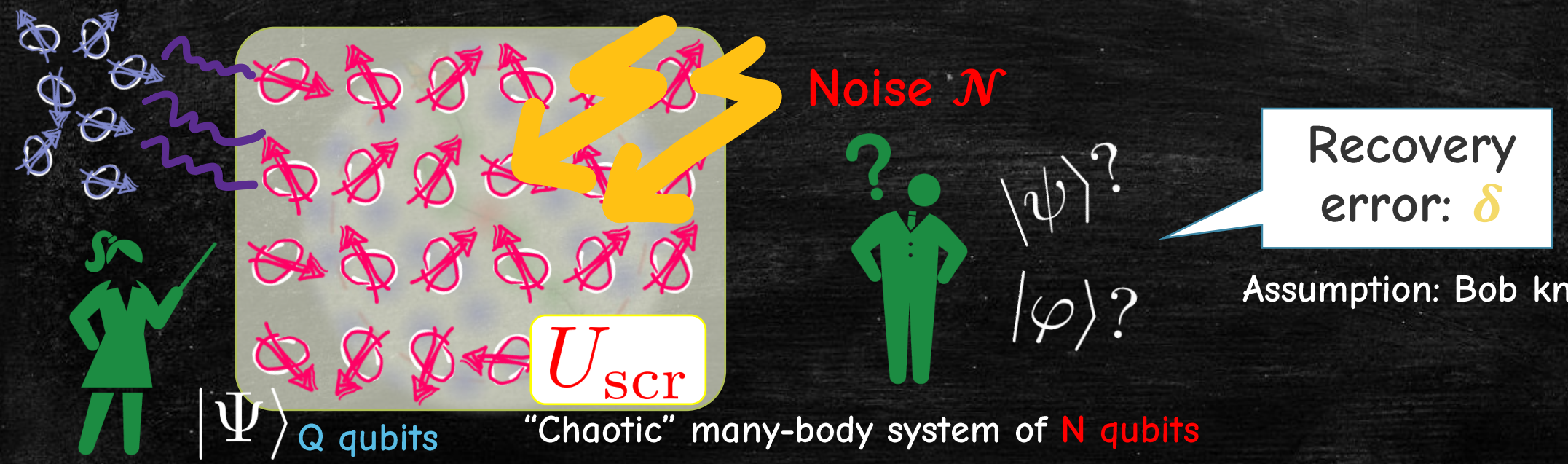
- Thought experiment: QEC in "chaos"
  1. Alice encodes a quantum state $|\Psi\rangle$ to a (part of) many-body system.
  2. Hamiltonian dynamics U_{scr} (= scrambling)
  3. Noise $\mathcal{N}$ occurs on the system

Noise $\mathcal{N}$

e.g.) thermal noise, partial trace, etc...

QEC in "Quantum Chaos": setting 4

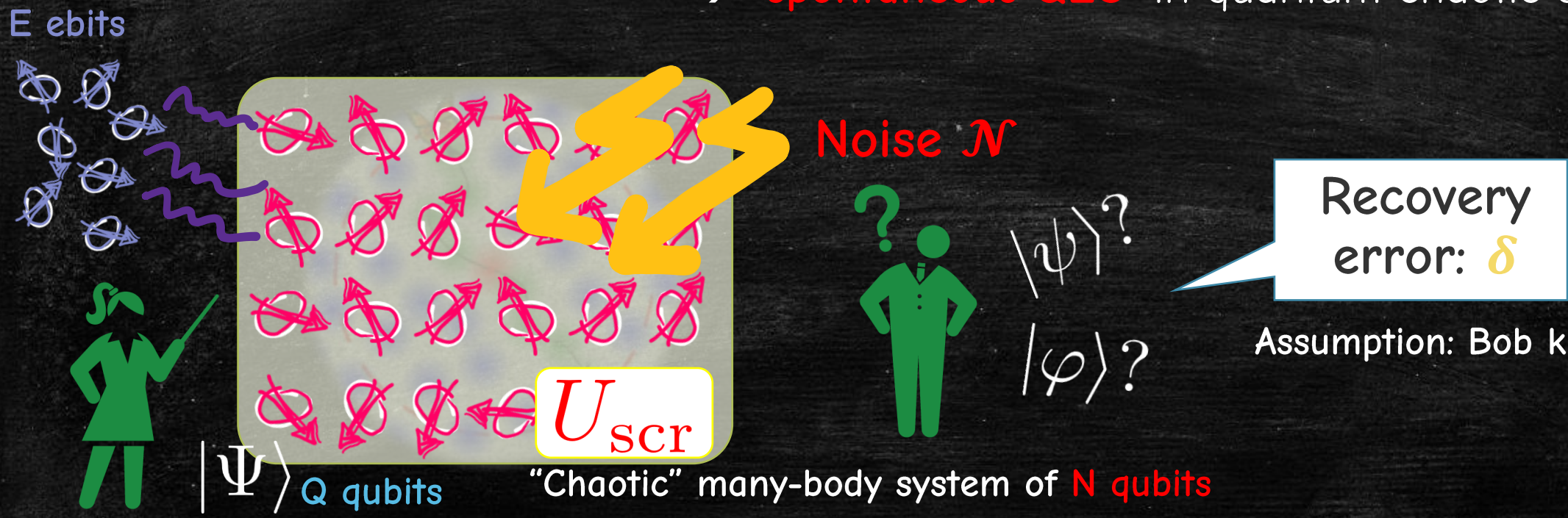
- **Thought experiment: QEC in "chaos"**
  1. Alice encodes a quantum state $|\Psi\rangle$ to a (part of) many-body system.
  2. Hamiltonian dynamics U_{scr} (= **scrambling**).
  3. Noise $\mathcal{N}$ occurs on the system.
  4. Bob tries to recover $|\Psi\rangle$ from the noisy system.



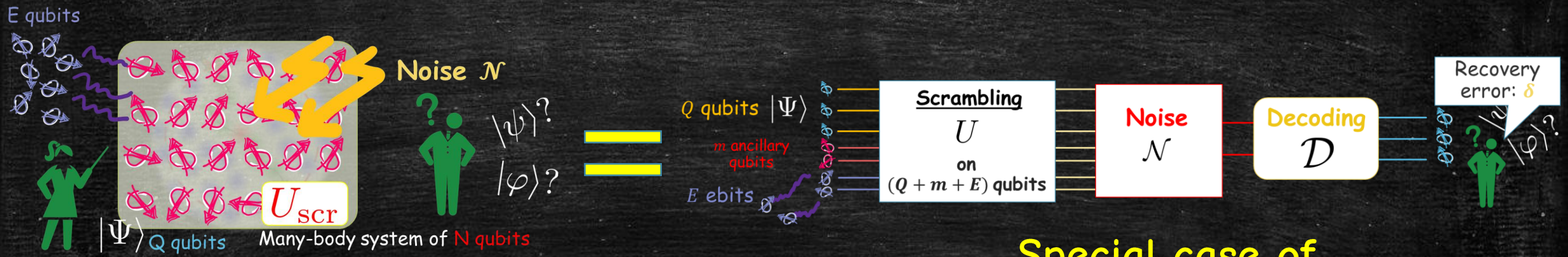
QEC in "Quantum Chaos": setting 5

- $\delta \ll 1$ means that...
  1. the state $|\Psi\rangle$ is protected from the noise $\mathcal{N}$.
  2. This should be due to the internal scrambling dynamics U_{scr}.

→ "spontaneous QEC" in quantum chaotic system.



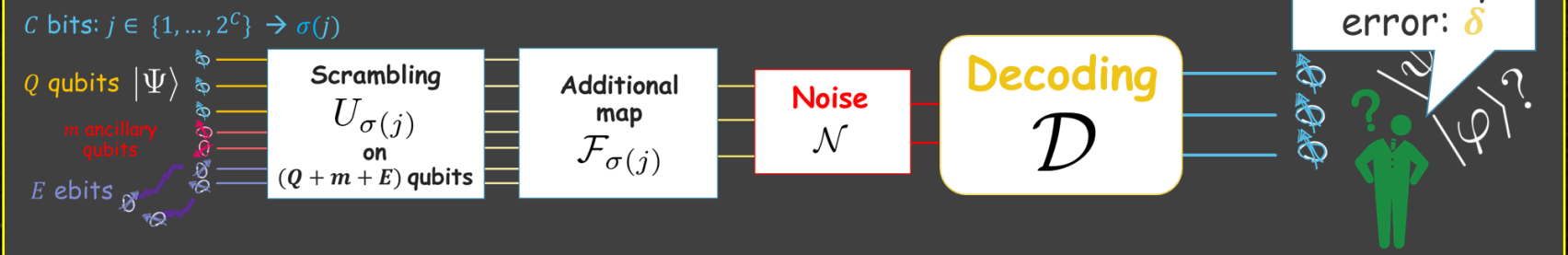
QEC in "Quantum Chaos" 1



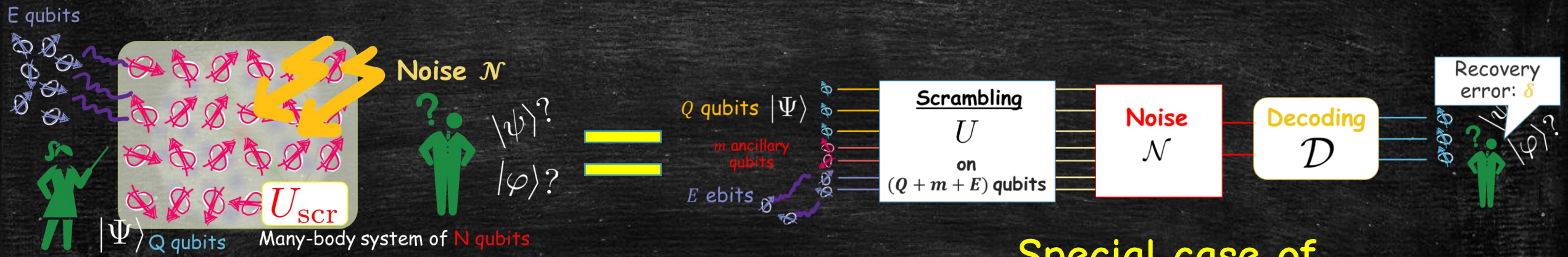
QEC in "Chaos"

Special case of the capacity theorem
(C=0 & $\mathcal{F}$=identity)

Encoding used in the capacity theorem



QEC in "Quantum Chaos" 1



QEC in "Chaos"

Special case of the capacity theorem
(C=0 & $\mathcal{F}$=identity)

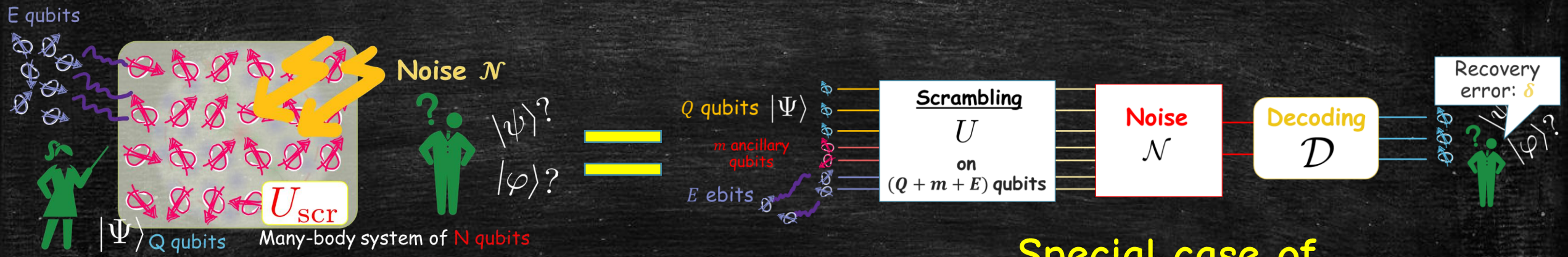
- From the theorem, we obtain

$$\delta \lesssim 2^{(Q-E+H_{\max}(S|B)_{\varrho_{\mathcal{N}}})/4} \quad \text{where} \quad \varrho_{\mathcal{N}} = (\mathrm{id} \otimes \mathcal{N})(\Phi)$$

Conditional entropy

Maximally entangled state

QEC in "Quantum Chaos" 2



QEC in "Chaos"

Special case of the capacity theorem

$$\delta \lesssim 2^{(Q-E+H_{\max}(S|B)_{\varrho_{\mathcal{N}}})/4} \quad \text{where} \quad \varrho_{\mathcal{N}} = (\mathrm{id} \otimes \mathcal{N})(\Phi)$$

- This formula is useful to study QEC property of quantum chaos.
- For typical "local" noises, encoding by scrambling seems close to optimal (preliminary).

→ (Prospect) spontaneous QEC is intrinsic in quantum chaos.

Outline of this talk

1. Introduction
   1. Scrambling and Decoupling
   2. Generalization of decoupling
3. Quantum Error Correction (QEC)
   1. Hybrid capacity theorem
4. QEC in Quantum Chaotic System

Quantum Information & Strongly-correlated physics

5. Hayden-Preskill protocol with symmetry

Fundamental Physics

6. Summary

Partial Decoupling

Special form of scrambling

"Partial" decoupling approach

One-shot partial decoupling theorem
[Wakakuwa & YN ('19)]

Hayden-Preskill protocol
with symmetry
[YN, Wakakuwa & Koashi ('20)]

Fundamental Physics

Quantum Error Correction (QEC)
New capacity theorem
[Wakakuwa, YN & Yamasaki ('20)]

Quantum Information Science

Quantum Chaos

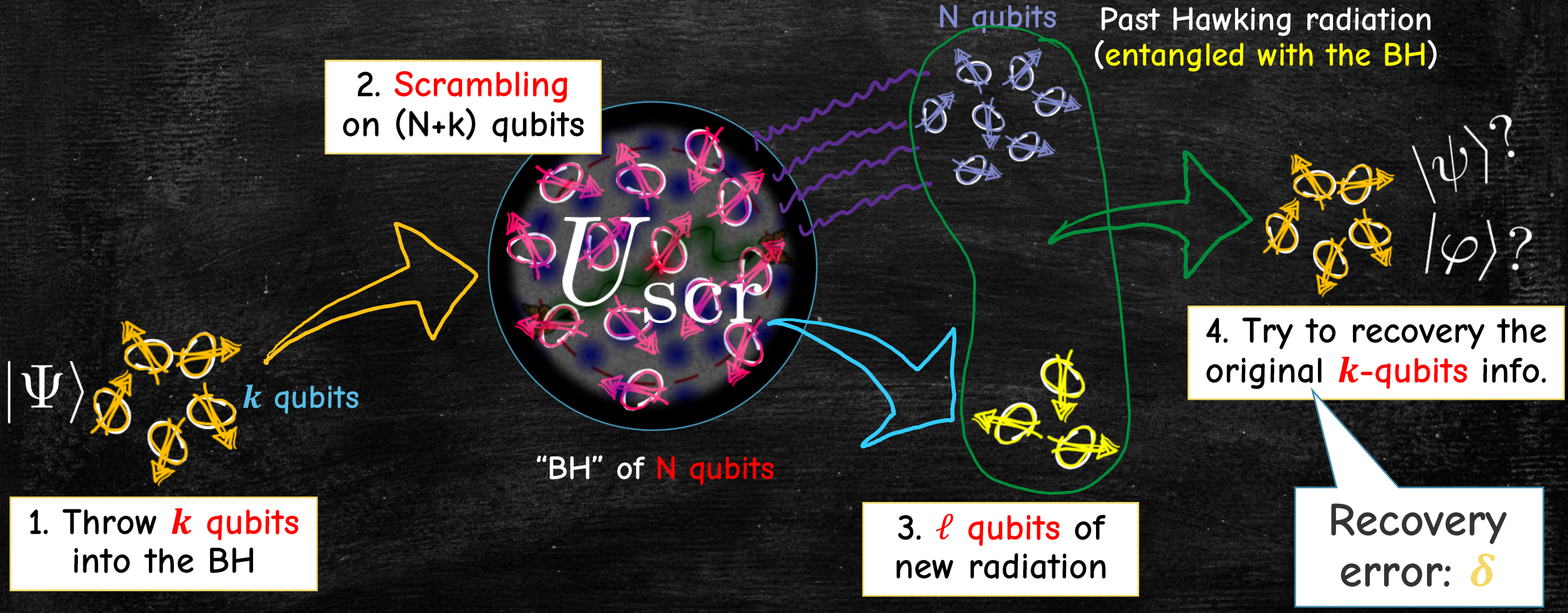
Q many-body physics

Partial decoupling is useful to investigate
"scrambling with symmetry".

Hayden-Preskill toy model: setting 1

- A toy model of the information paradox.
  - Proposed by Hayden & Preskill ('07).
  - A special case of the thought-experiment of "QEC in quantum chaos".

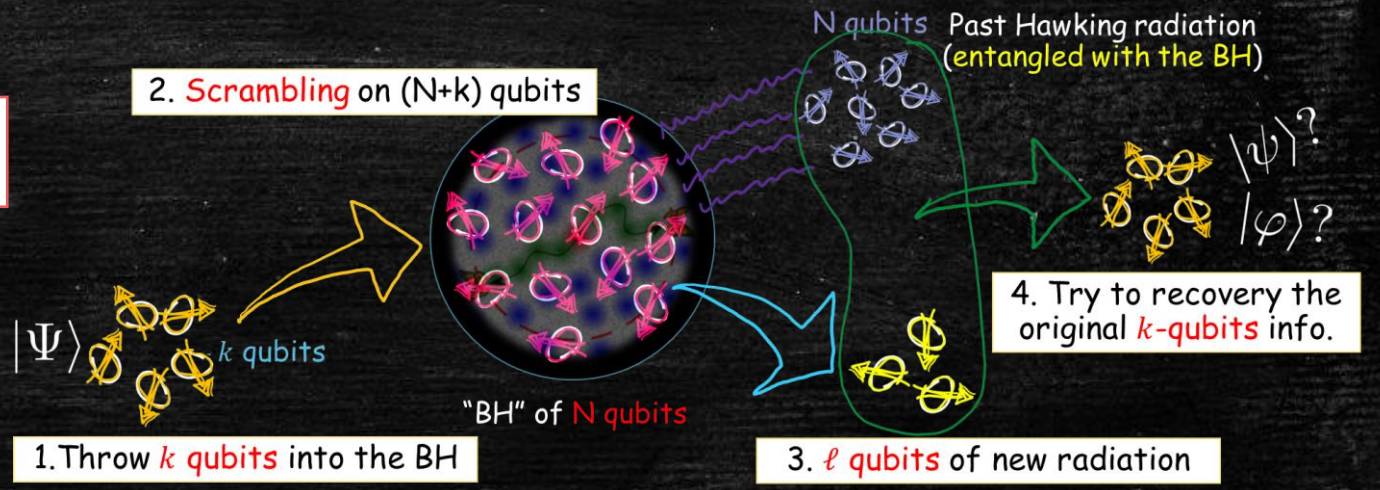
Hayden-Preskill toy model: setting 2



Hayden-Preskill toy model: result

- **A toy model of the information paradox.**
  - Proposed by Hayden & Preskill ('07).
  - A special case of the thought-experiment of "QEC in quantum chaos".
- **Quick leakage of the information: $\delta \leq 2^{k-\ell}$ (decoupling approach).**
  - k-qubit information can be recovered from $\ell >\approx k$ of new radiation!
  - BHs CANNOT hide any information due to scrambling.

What if the BH has symmetry?



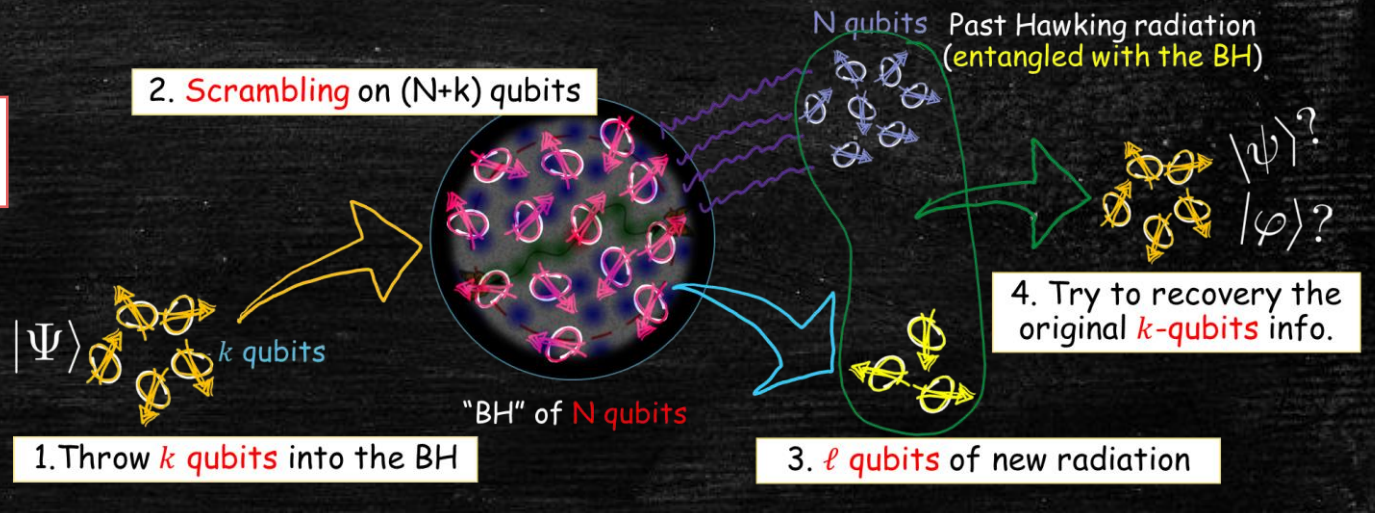
Why symmetry?

- Quantum BH has NO global symmetry [Harlow & Oguri ('19)].
  - The violation should be weak to be consistent with classical case?
  - Energy conservation should be there?
  - Studying exact symmetry may help better understanding.

Excuse

- **HP protocol with symmetry is interesting as a toy model.**
  - By introducing symmetry, a non-trivial relation b/t information leakage problem and thermodynamics shows up.

What if the BH has symmetry?

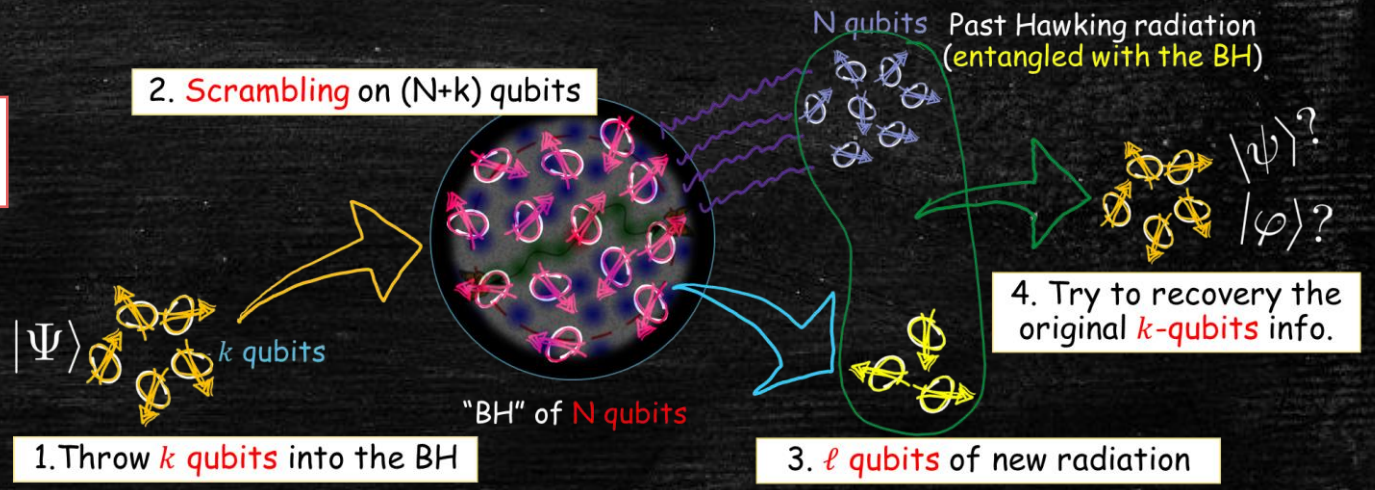


Hayden-Preskill with symmetry 1

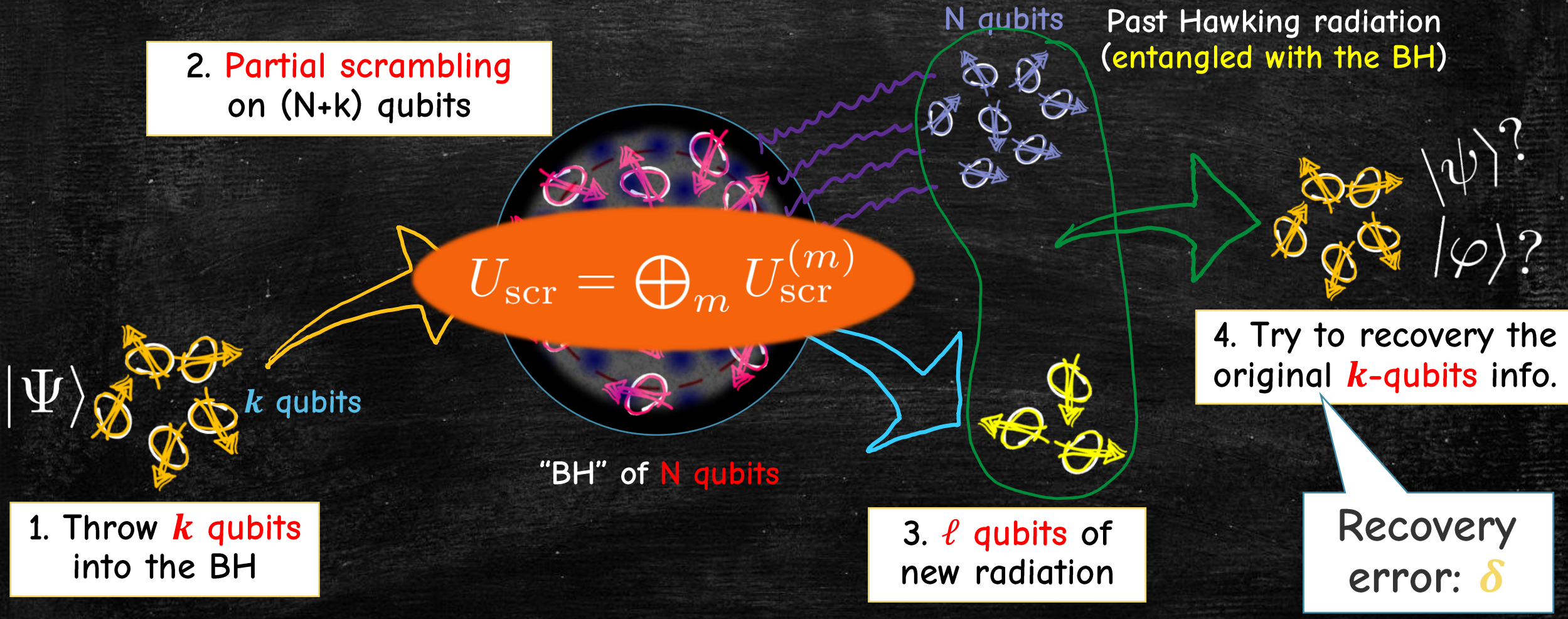
- Consequences of the symmetry
  - ∃conserved quantity that should NOT be scrambled.
- **Scrambling with symmetry:** $U_{\mathrm{scr}} = \bigoplus_m U_{\mathrm{scr}}^{(m)}$
  - For simplicity, abelian symmetry is assumed.
  - m labels the conserved quantity.

Scrambling with a direct-sum structure!

What if the BH has symmetry?



Hayden-Preskill with symmetry 2

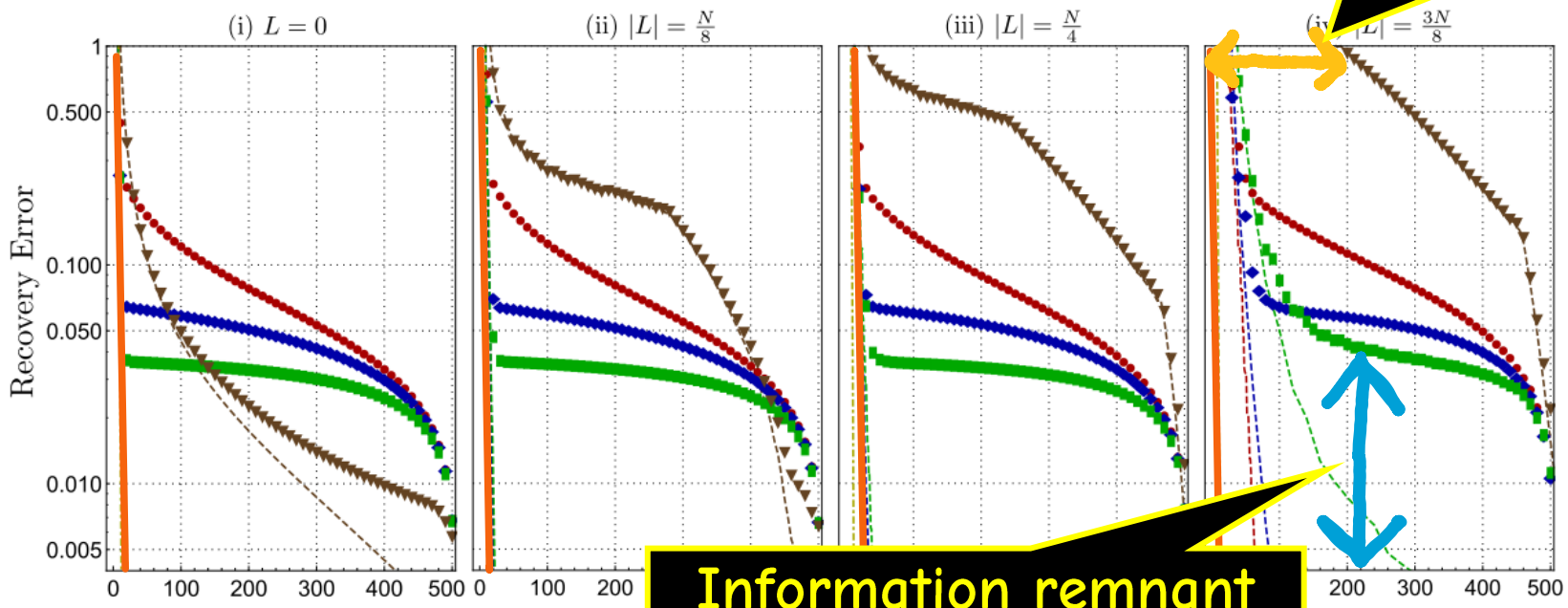


Hayden-Preskill with symmetry 3

- Using partial decoupling theorem, a general formula for the error δ is obtained!!

Recovery errors for various initial conditions

– Characterized by the value L of the angular momen… and its fluctuation δL.



Delay of information leakage

Information remnant

Non-symmetric case
(HP original model)

Red: $\delta L = 0.1\sqrt{N}$,
Blue: $\delta L = 0.3\sqrt{N}$,
Green: $\delta L = 0.9\sqrt{N}$,
Brown: $\delta L = 0.3\,N$.

ℓ: Number of new radiation

…rror: δ
…on
…BH)
$|\psi\rangle$?
$|\varphi\rangle$?
…very the
…its info.

Scrambling with symmetry:
interesting since it connects information leakage and thermodynamics.

- Using partial decoupling theorem, a general formula for the error δ is obtained!!

1. **Delay:** information leaks out more slowly than non-symmetric case.
   - Delay = information-theoretically trivial delay + non-trivial delay

Fully determined by thermodynamics of the BH.
e.g.) **thermal sensitivity** of the conserved quantity,
conjugate state function to the conserved quantity.

   - Indicates a relation between information leakage & thermodynamics.
   - "Information is physical: if you know physics, you know information".

2. **Information remnant:** some info. remains un-evapolated until the last moment.
   - Closely related to symmetry-breaking of the BH.

See also arXiv:2103.01876 by Tajima & Saito.

Outline of this talk

1. Introduction
   1. Scrambling and Decoupling
   2. Generalization of decoupling
3. Quantum Error Correction (QEC)
   1. Hybrid capacity theorem
4. QEC in Quantum Chaotic System

Quantum Information & Strongly-correlated physics

5. Hayden-Preskill protocol with symmetry

Fundamental Physics

6. Summary

Quantum Information Science

Quantum Error Correction

1. New capacity th'm for hybrid information

Fundamental limit of QEC

Generalization of decoupling.

0. "Partial decoupling"

"Decoupling" approach

QEC is intrinsic in chaotic system.

2. QEC in quantum chaos.

Scrambling

3. Hayden-Preskill with symmetry

Black Hole (Holography)

Fundamental Physics

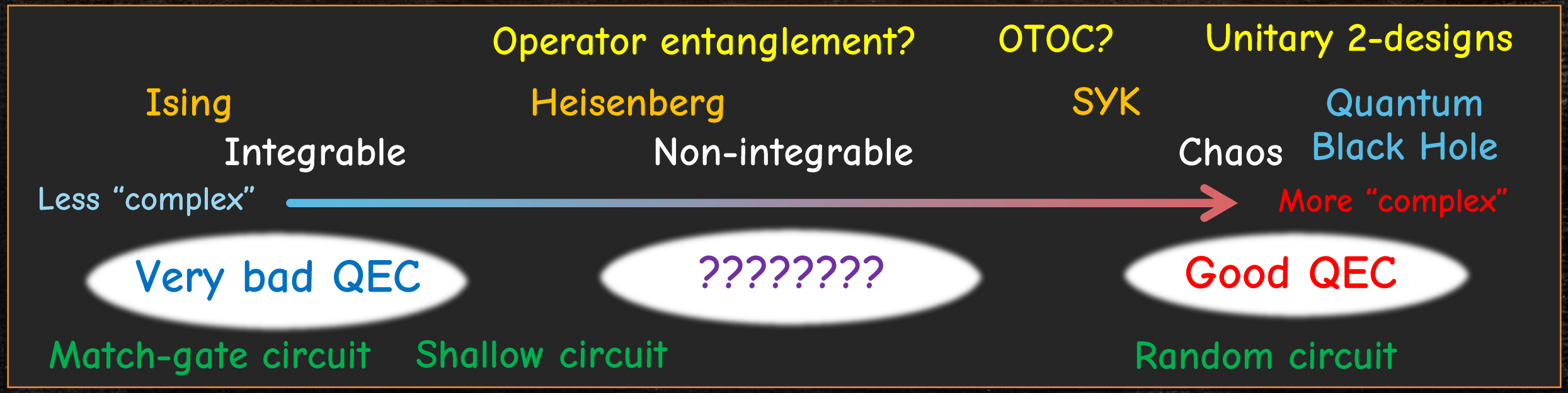
Relation to thermodynamics?

Quantum Chaos

Strongly-correlated physics

Future problems

- Does scrambling in a weaker sense reproduce the same results?
  - All are based on "unitary 2-designs" (>≈ OTOC, operator entanglement)
  - Worth checking "decoupling" in various many-body systems.
- More about scrambling with symmetry? e.g.) U(1) symmetry, hard & soft modes?



Thank you for your attention!!

Any question?

All will be extensively updated soon!

arXiv:1903.05796 by E. Wakakuwa and YN
arXiv:2011.00668 by YN, E. Wakakuwa, and H. Yamasaki,
arXiv:2007.00895 by YN, E. Wakakuwa, and M. Koashi.