From the black hole conundrum to the structure of quantum gravity

Yasunori Nomura

UC Berkeley; LBNL; Kavli IPMU

Two pillars of modern physics

- Quantum mechanics
- General relativity

→ not get along well

Two classes of problems

- **At ~ $\boldsymbol{\ell_P}$**, theoretical control of quantum field theory (point particles in continuous spacetime) is lost. → string theory

(ℓ_P: Planck length)

- There seems to be a structural problem even **at long distances** when gravitational effects become so significant to form a horizon.

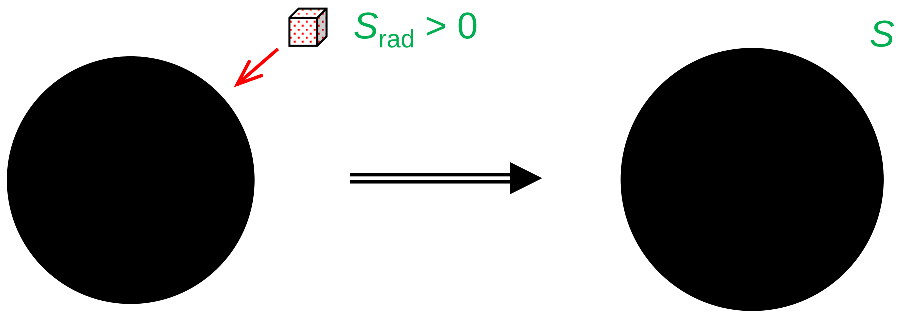
… black hole information problem

What is it?

… has to do with the third pillar

- Statistical mechanics

What happens if matter falls into a black hole?



... $\Delta S < 0$!?

A proposal [Bekenstein, 1973]

photo: APS

The entropy of a BH is proportional to its horizon area.

$$S_{\text{BH}} = \frac{A}{4G_{\text{N}}}$$

Note: $G_{\text{N}} = \ell_{\text{P}}^2 \sim (10^{-33}\,\text{cm})^2 \;\rightarrow\;$ huge entropy

Indeed, $\Delta\left(\frac{A}{4G_{\text{N}}} + S_{\text{matter}}\right) \geq 0$

Does this make sense?

$$\frac{A}{4G_{\text{N}}} = 4\pi G_{\text{N}} M^2$$

$$\frac{\partial S}{\partial E} = \frac{1}{T} \quad \rightarrow \text{ finite temperature}$$

M

Doesn't a BH only absorb stuff?

Black holes radiate [Hawking, 1974]

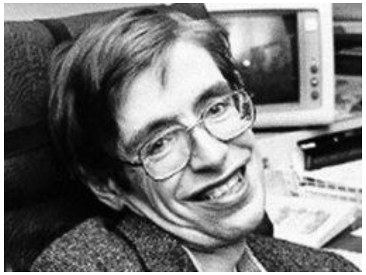


photo: NASA

The horizon is “smooth.”

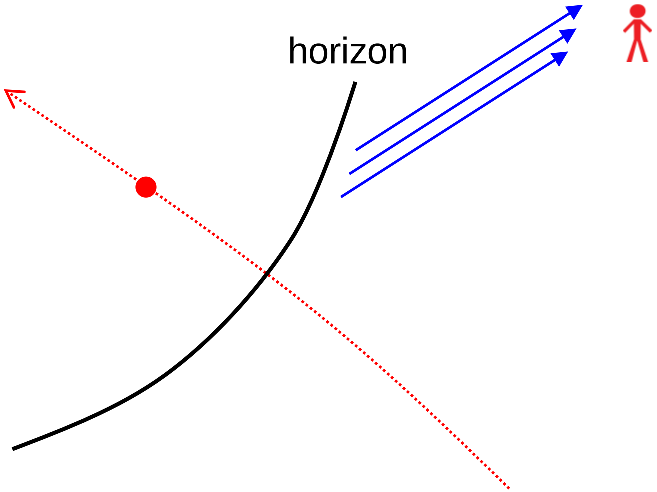
$\Downarrow$ Quantum mechanical effect

Hawking temperature

There must be radiation corresponding to $T_{\mathrm{H}} \sim \dfrac{1}{8\pi M G_{\mathrm{N}}}$.

BHs are thermodynamic objects.

→ Spacetime is composed of microscopic d.o.f.s!



Black holes radiate [Hawking, 1974]

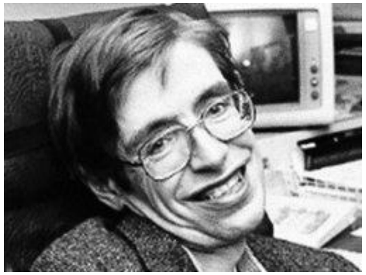


photo: NASA

The horizon is "smooth."

$\Downarrow$ Quantum mechanical effect

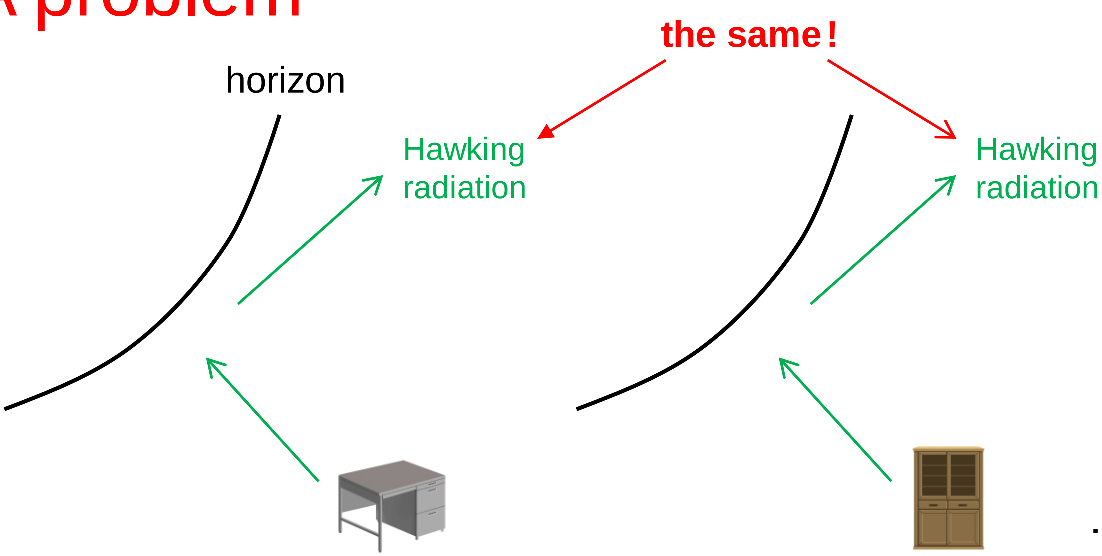
There must be radiation corresponding to $T_{\mathrm{H}} \sim \dfrac{1}{8\pi M G_{\mathrm{N}}}$.

Hawking temperature

BHs are thermodynamic objects.

$\rightarrow$ Spacetime is composed of microscopic d.o.f.s!

A problem



The time evolution is **not** one-to-one!
(not unitary)

… (the original form of)
BH information problem

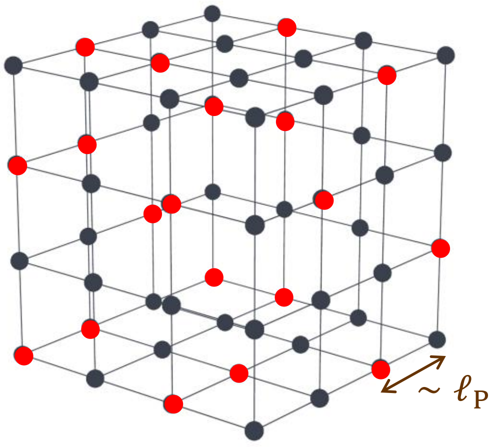
Holography

A clue comes from the BH physics itself.

A BH is the highest entropy state of the region,

and still $S \propto \frac{A}{\ell_{\mathrm{P}}^2}$

Strange!



$$S \sim \ln 2^{V/\ell_{\mathrm{P}}^3} \propto \frac{V}{\ell_{\mathrm{P}}^3} \ggg \frac{A}{\ell_{\mathrm{P}}^2}$$

The concept that spacetime exits down to $\sim \ell_{\mathrm{P}}$ is an illusion!

→ suggests that there is a formulation of quantum gravity in spacetime **one less dimension** than the naïve one.

AdS/CFT correspondence [Maldacena, 1997]

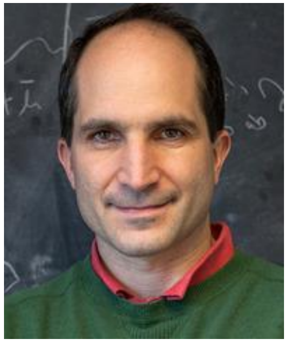
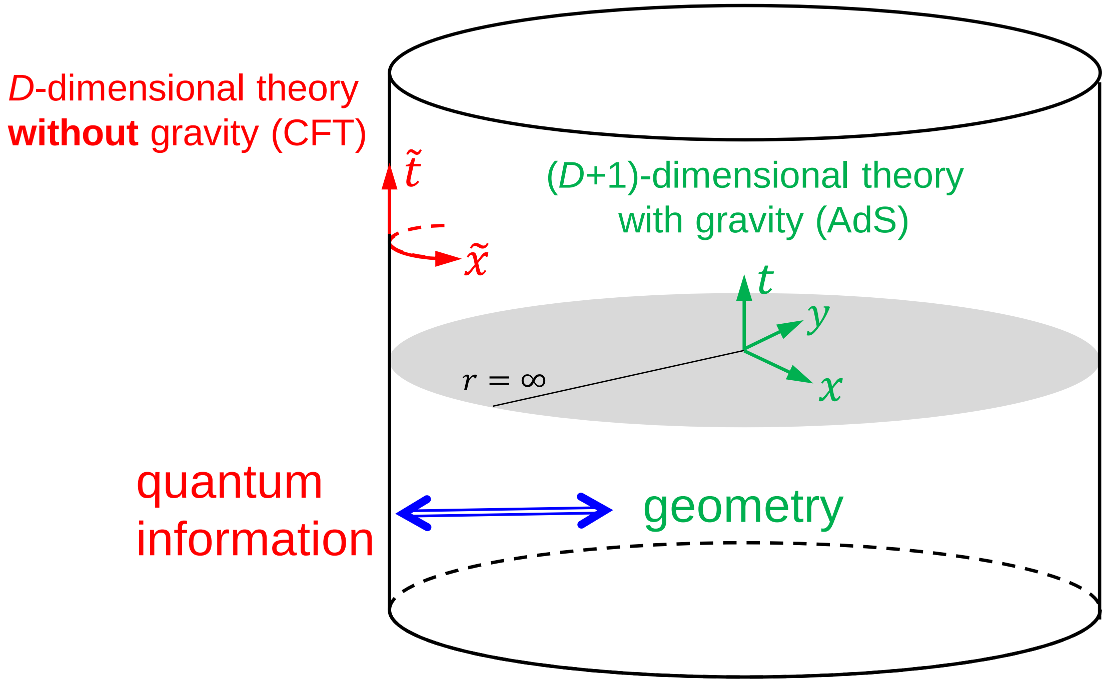
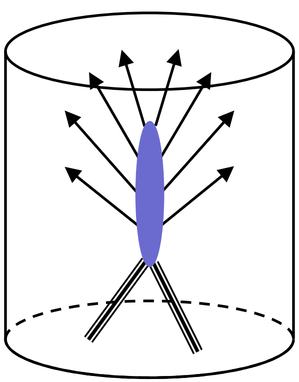


photo: IAS



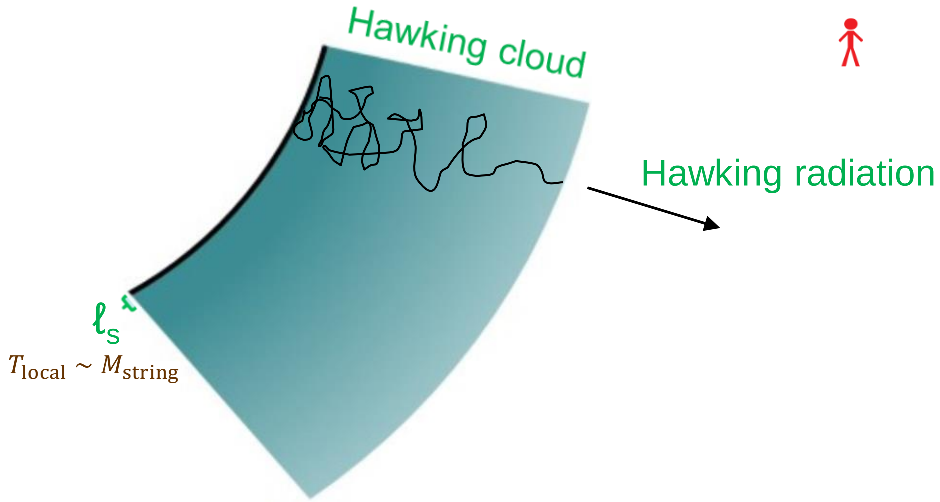
BH evolution **must be** unitary.



=

A process in non-gravitational
(unitary) theory

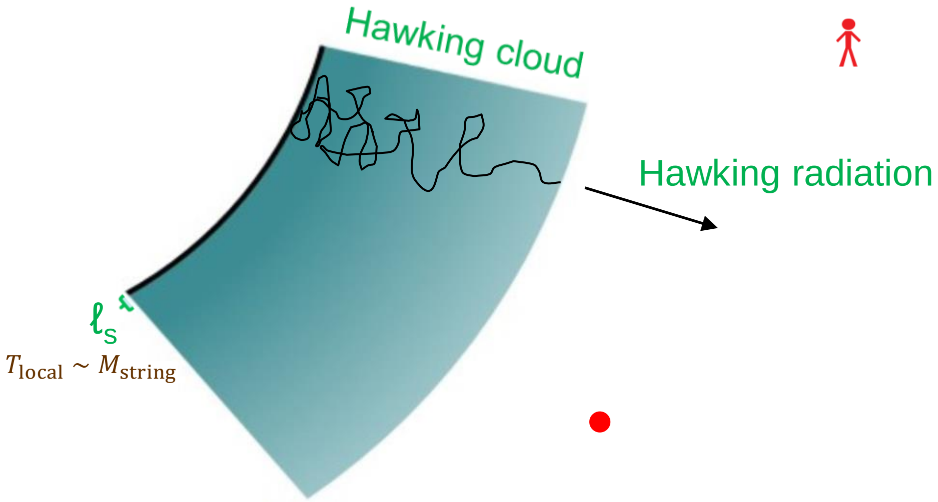
BH at the quantum level



The horizon behaves
as the surface of regular material.

… no issue with unitarity

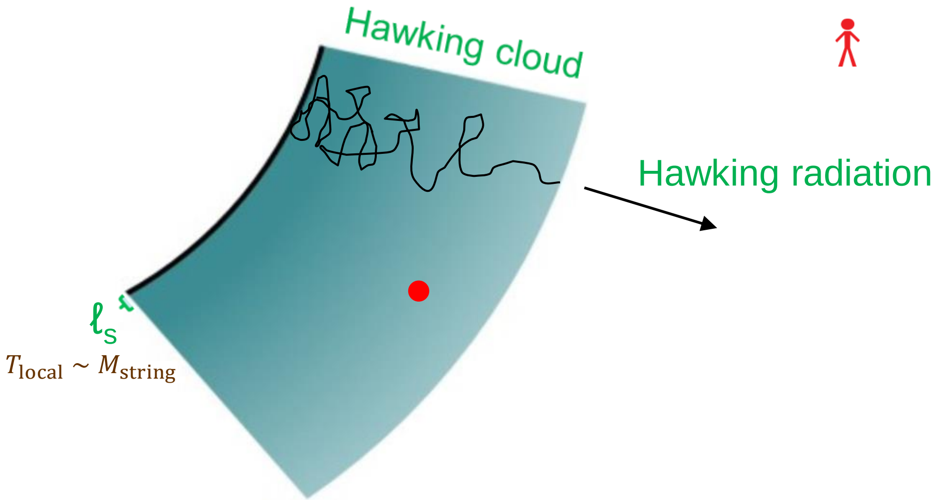
BH at the quantum level



The horizon behaves
as the surface of regular material.

… no issue with unitarity

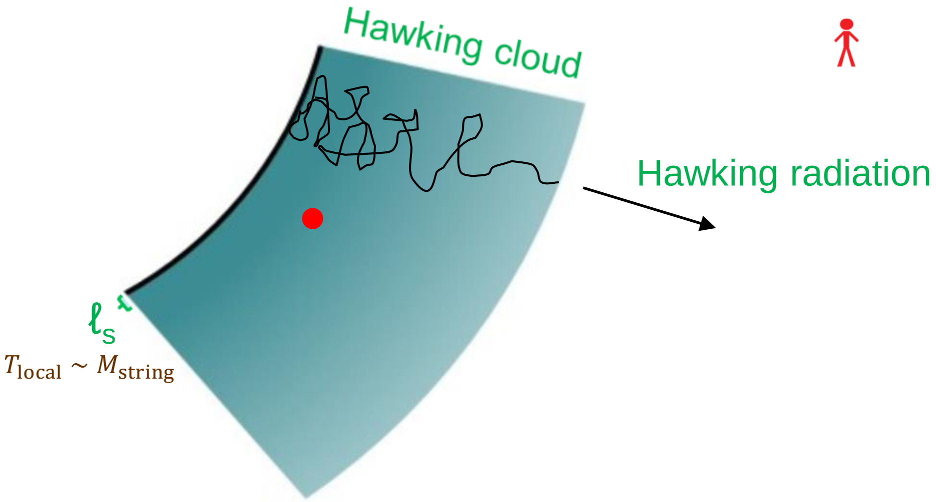
BH at the quantum level



The horizon behaves
as the surface of regular material.

… no issue with unitarity

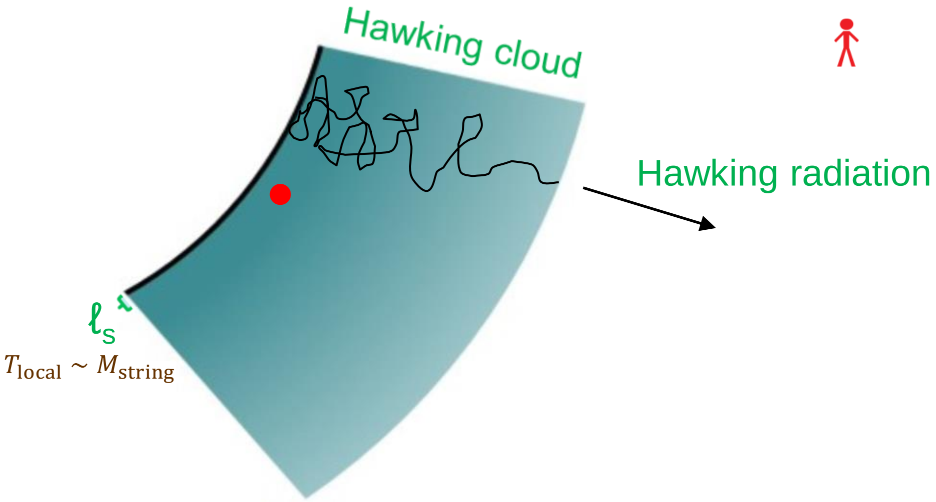
BH at the quantum level



The horizon behaves
as the surface of regular material.

… no issue with unitarity

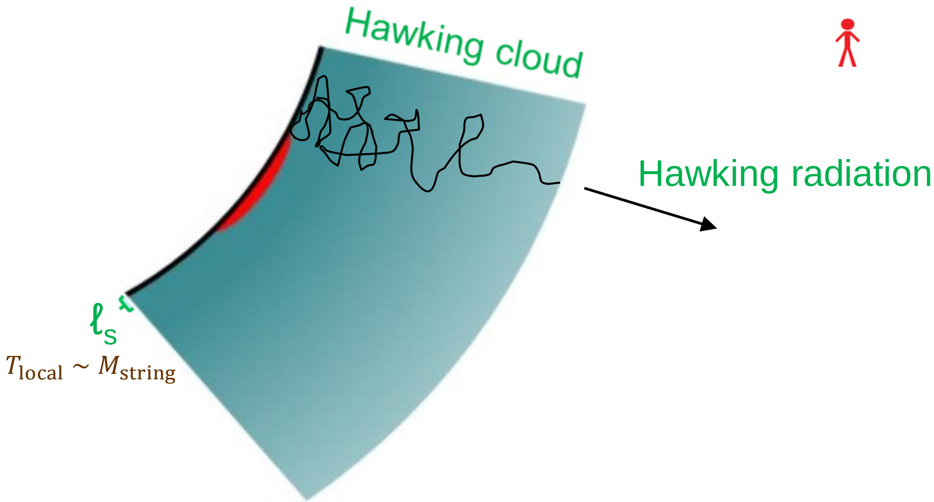
BH at the quantum level



The horizon behaves
as the surface of regular material.

… no issue with unitarity

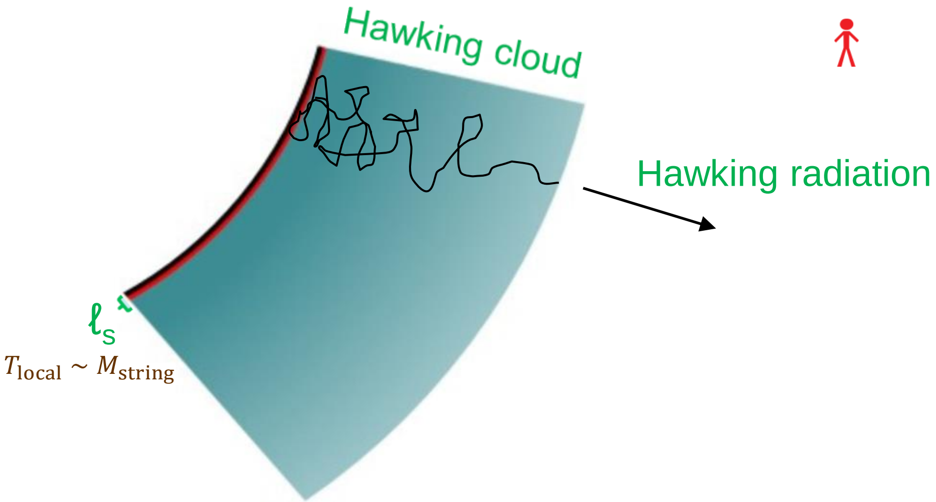
BH at the quantum level



The horizon behaves
as the surface of regular material.

… no issue with unitarity

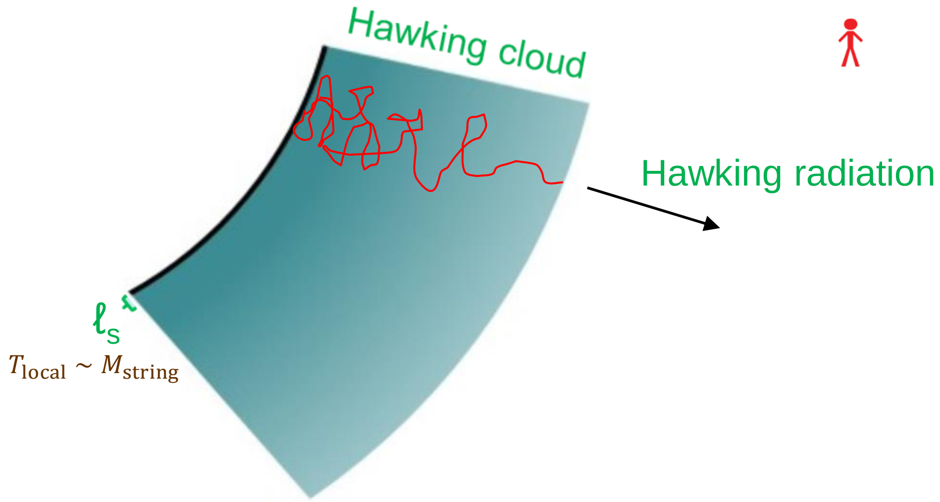
BH at the quantum level



The horizon behaves
as the surface of regular material.

… no issue with unitarity

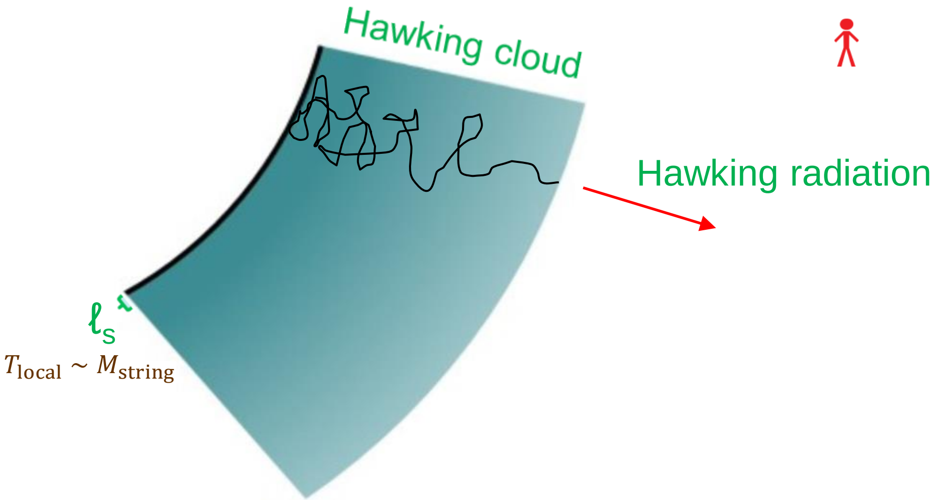
BH at the quantum level



The horizon behaves
as the surface of regular material.

… no issue with unitarity

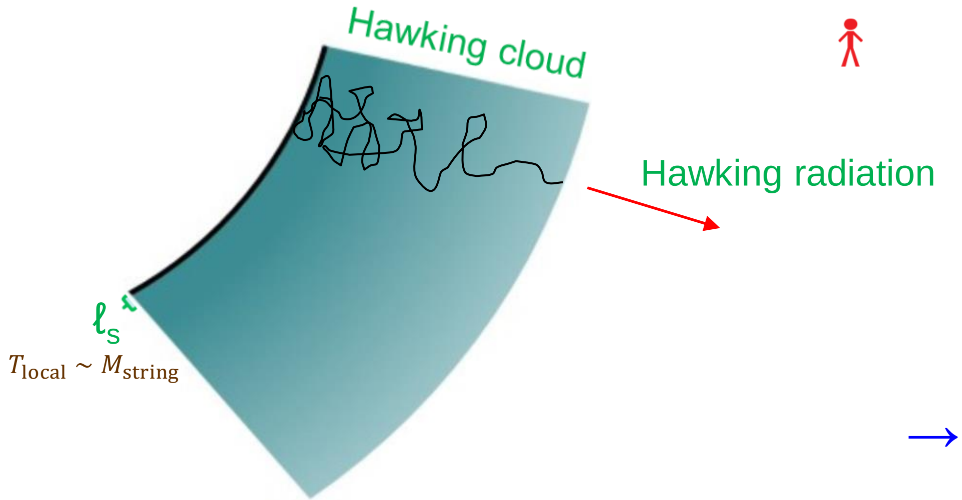
BH at the quantum level



The horizon behaves
as the surface of regular material.

… no issue with unitarity

BH at the quantum level

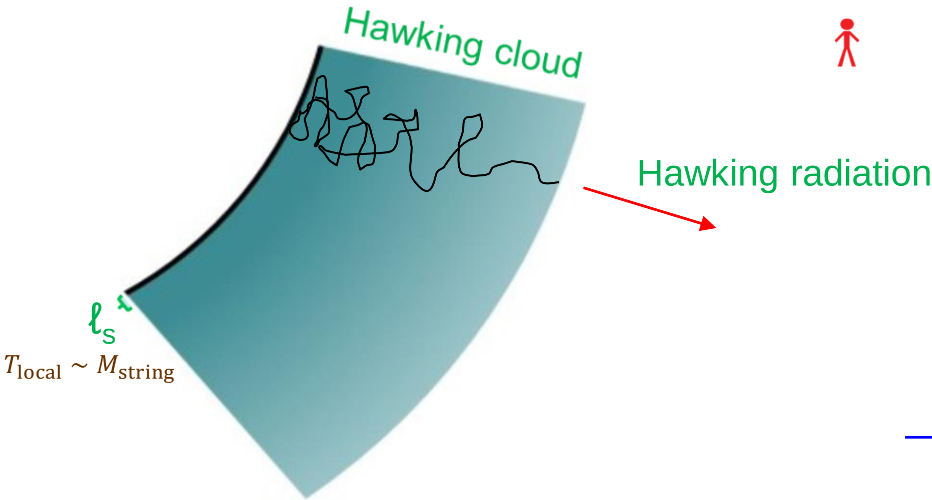


The horizon behaves
as the surface of regular material.

… no issue with unitarity

→ What about the interior?

BH at the quantum level

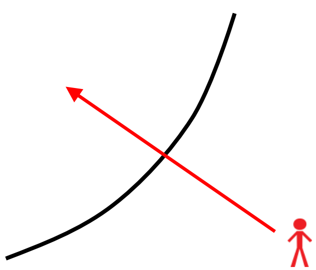


The horizon behaves
as the surface of regular material.

… no issue with unitarity

→ What about the interior?

Alternatively



$\xrightarrow[\text{Hawking's analysis}]{\qquad\qquad}$ information loss

→ What was wrong with Hawking's analysis?

Claim I :

In quantum gravity, a system with a BH (horizon) accommodates two very different descriptions.

These two descriptions, however, are physically equivalent.

Claim II :

Each description makes only one of QM (unitarity) and GR (interior spacetime) manifest.

Nevertheless, the theory is consistent with both; the properties of the one not chosen arises dynamically through subtle effects.

$\Longrightarrow$ We will discuss one of the description — unitary gauge construction — in detail.

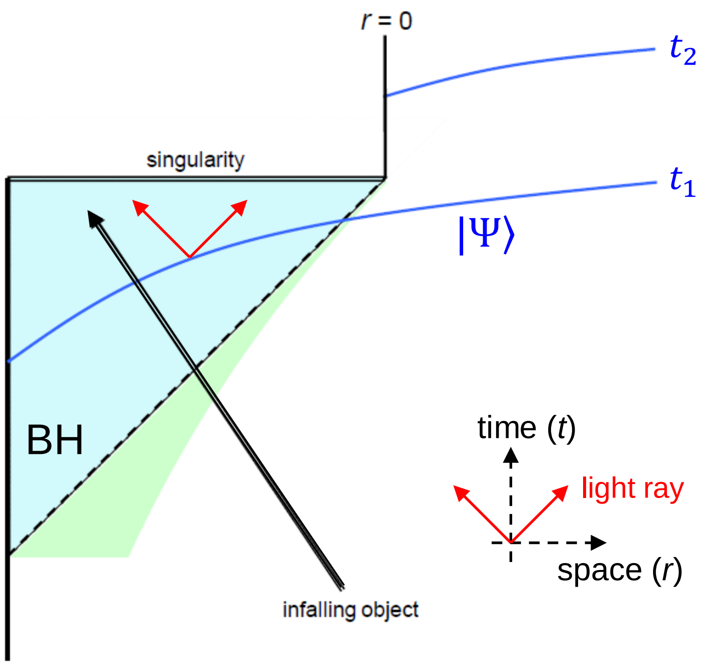
Y.N., “From the black hole conundrum to the structure of quantum gravity,” arXiv:2011.08707 [hep-th]

Y.N., “Black hole interior in unitary gauge construction,” arXiv:2010.15827 [hep-th]

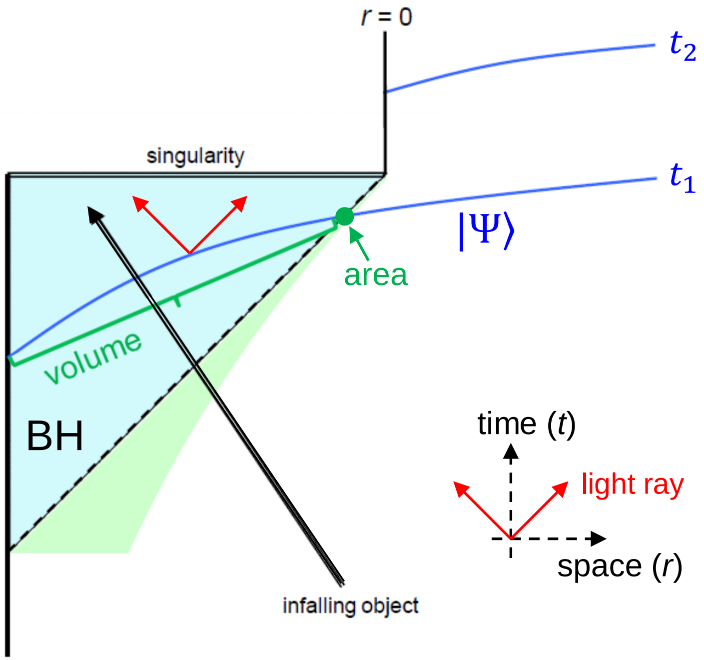
Picture based on Global Spacetime

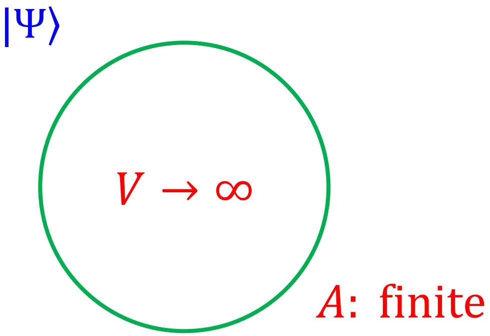
— replica wormholes —

Start with "global spacetime"



Start with "global spacetime"

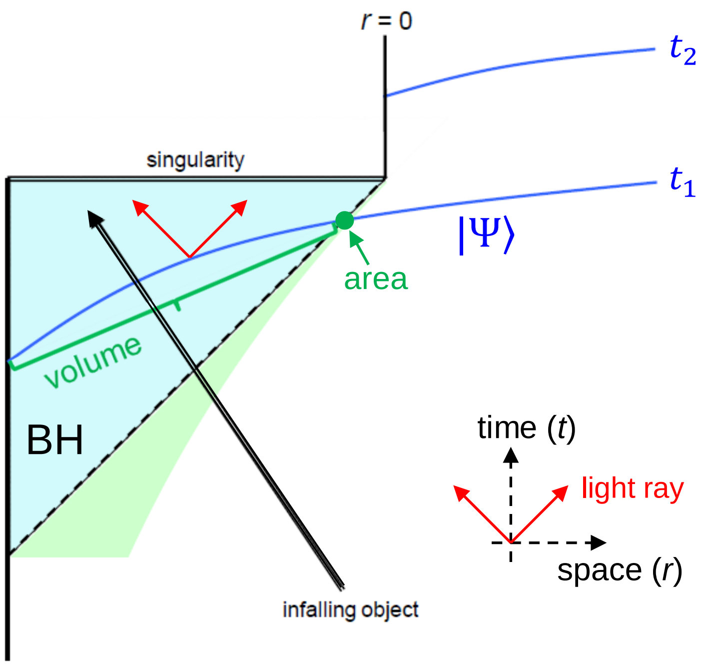


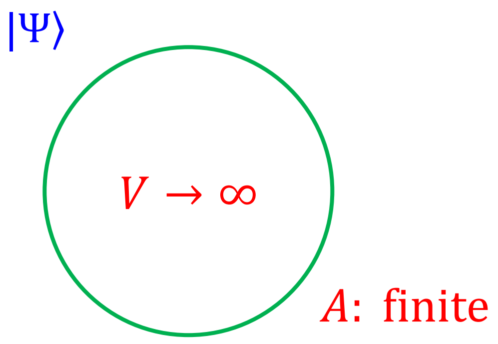


… at odds with $S = \frac{A}{4\ell_{\mathrm{P}}^2}$

Hugely redundant!

Start with "global spacetime"





… at odds with $S = \frac{A}{4\ell_P^2}$

Hugely redundant!

$$\langle\Psi_1|\Psi_2\rangle = 0 \quad \longrightarrow \quad \langle\Psi_1|\Psi_2\rangle \sim e^{-\frac{S}{2}}$$

semiclassical
(QFT in curved spacetime)

quantum gravity

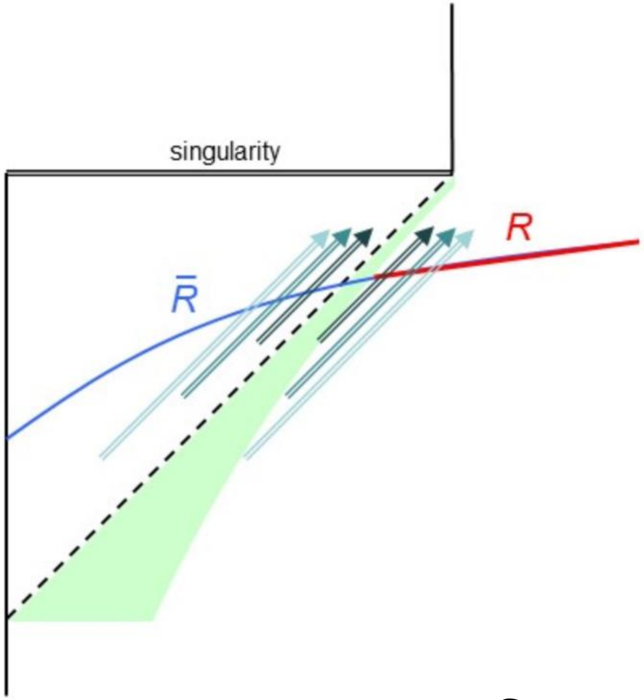
… only e^S independent states

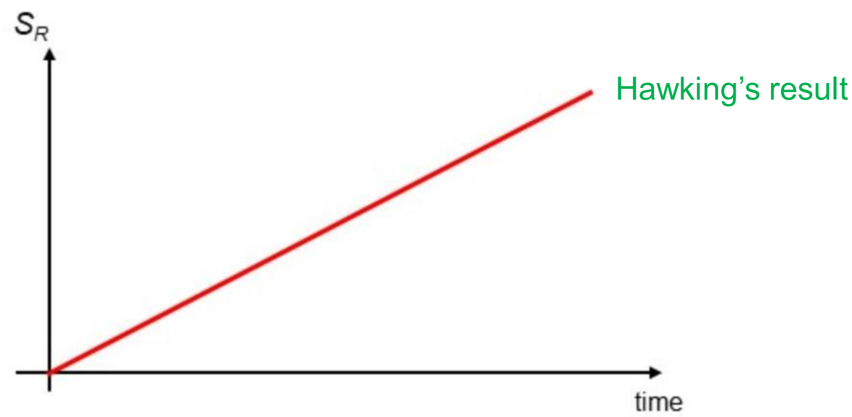
$$|\Psi\rangle = \sum_{i=1}^{e^S} c_i|\psi_i\rangle \qquad c_i \sim e^{-\frac{S}{2}}$$

$$\langle\Psi_1|\Psi_2\rangle = \sum_{i=1}^{e^S} c_{1,i}^* c_{2,i} \sim e^{\frac{S}{2}} e^{-S} \sim e^{-\frac{S}{2}}$$

$\to e^{e^S}$ approximately orthogonal states

Unitarity of Hawking evaporation

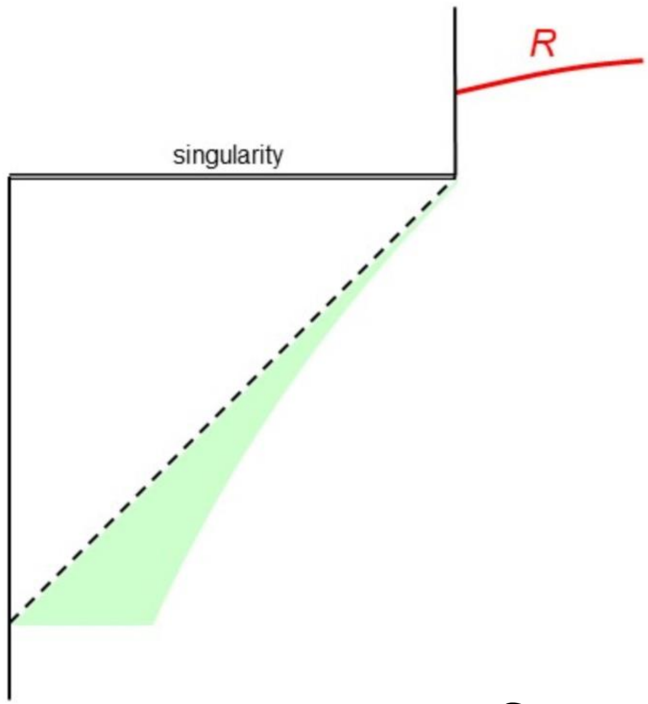


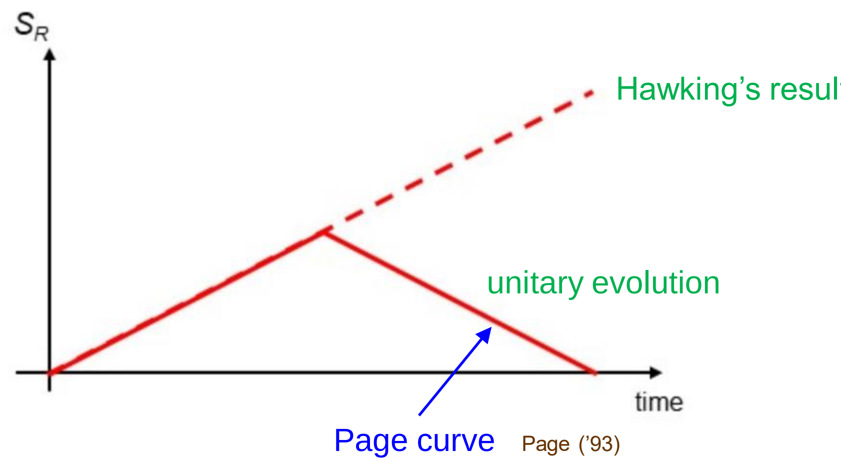


$$S_R = -\mathrm{Tr}[\rho_R \ln \rho_R] \qquad (\rho_R = \mathrm{Tr}_{\bar{R}} |\Psi\rangle\langle\Psi|)$$

~ the # of EPR particles in R whose partners are in $\bar{R}$

Unitarity of Hawking evaporation





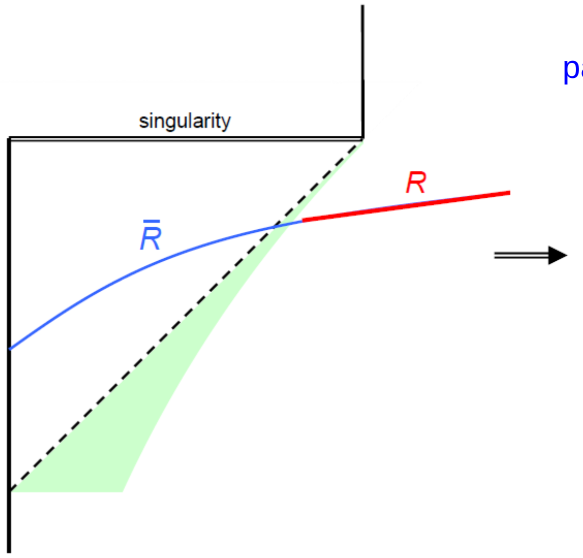
$$S_R = -\mathrm{Tr}[\rho_R \ln \rho_R] \qquad (\rho_R = \mathrm{Tr}_{\bar{R}} |\Psi\rangle\langle\Psi|)$$

~ the # of EPR particles in R whose partners are in $\bar{R}$

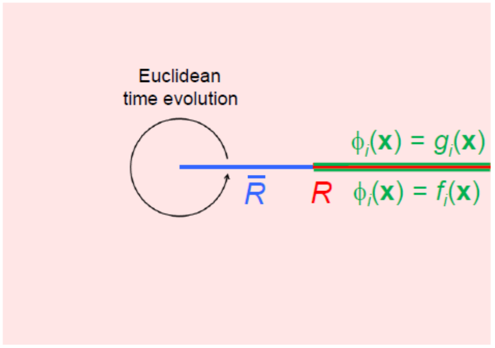
→ How to get this curve?

Page curve from replica wormholes

Penington ('19); Almheiri, Engelhardt, Marolf, Maxfield ('19); …
Penington, Shenker, Stanford, Yang ('19);
Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini ('19)



path integral

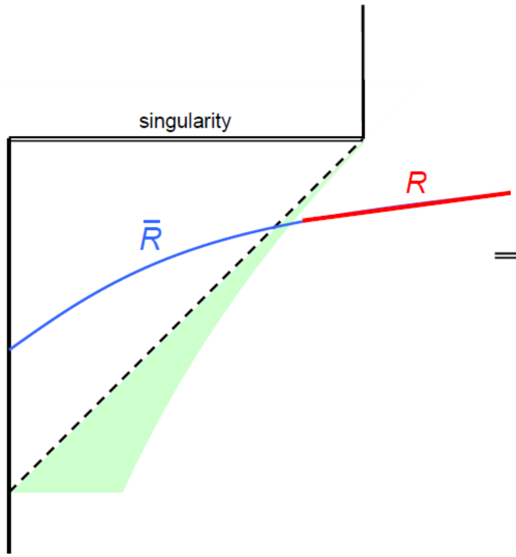


$\rightarrow \rho_R = \rho_R[f_i(\mathbf{x}), g_i(\mathbf{x})]$ (~ coefficient of $|g_i(\mathbf{x})\rangle\langle f_i(\mathbf{x})|$)

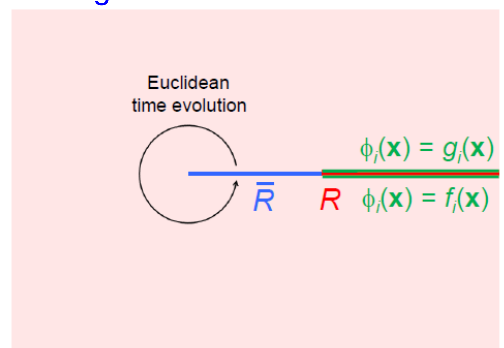
$$\boxed{S_R \equiv -\mathrm{Tr}[\rho_R \ln \rho_R] = \lim_{n\to 1} \frac{1}{1-n} \ln \mathrm{Tr}[\rho_R^n]}$$

Page curve from replica wormholes

Penington ('19); Almheiri, Engelhardt, Marolf, Maxfield ('19); …
Penington, Shenker, Stanford, Yang ('19);
Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini ('19)



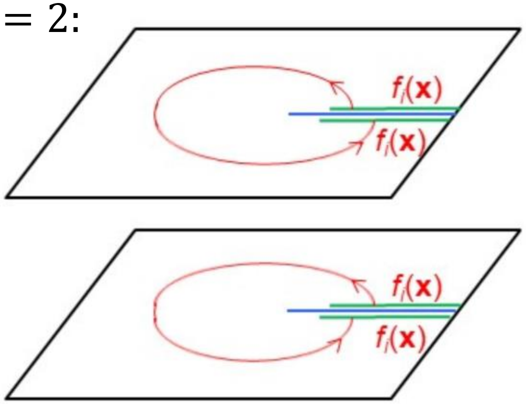
path integral

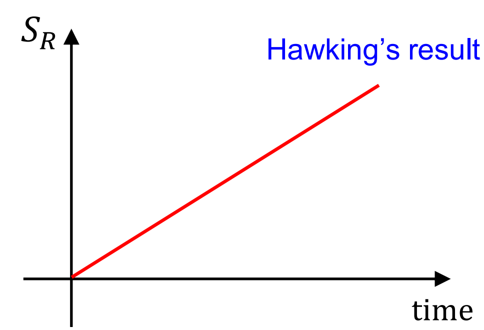


$\rightarrow \rho_R = \rho_R[f_i(\mathbf{x}), g_i(\mathbf{x})]$ (~ coefficient of $|g_i(\mathbf{x})\rangle\langle f_i(\mathbf{x})|$)

$$\boxed{S_R \equiv -\mathrm{Tr}[\rho_R \ln \rho_R] \; = \; \lim_{n\to 1} \frac{1}{1-n} \ln \mathrm{Tr}[\rho_R^n]}$$

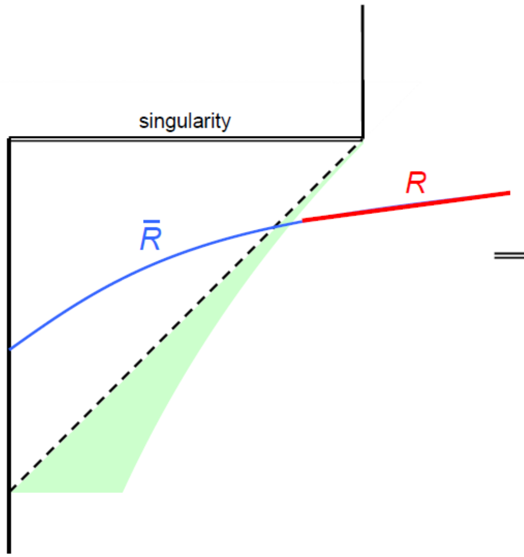
$n = 2$:



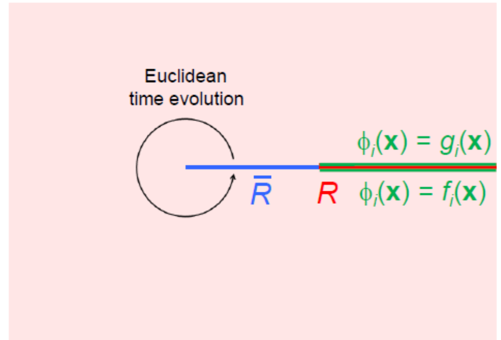


Page curve from replica wormholes

Penington ('19); Almheiri, Engelhardt, Marolf, Maxfield ('19); …
Penington, Shenker, Stanford, Yang ('19);
Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini ('19)



path integral

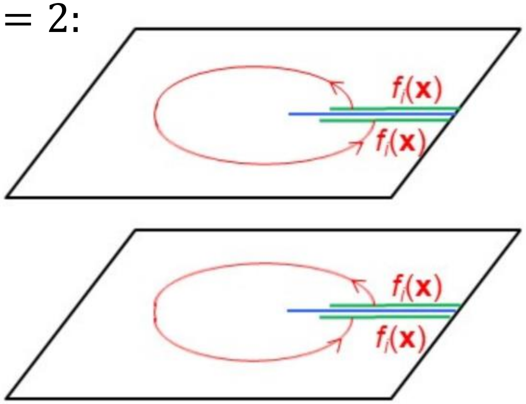


$\to \rho_R = \rho_R[f_i(\mathbf{x}), g_i(\mathbf{x})]$ (~ coefficient of $|g_i(\mathbf{x})\rangle\langle f_i(\mathbf{x})|$)

$$\boxed{S_R \equiv -\mathrm{Tr}[\rho_R \ln \rho_R] \;=\; \lim_{n\to 1} \frac{1}{1-n} \ln \mathrm{Tr}[\rho_R^n]}$$

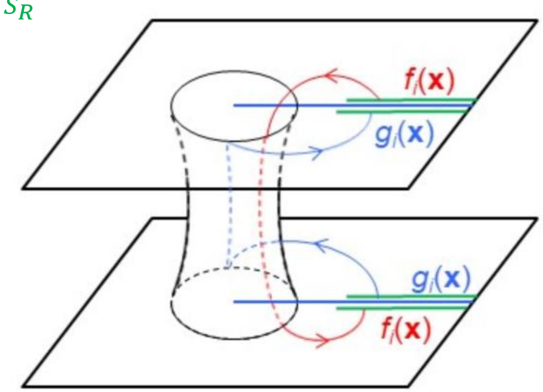
$n = 2$:

$e^{2S_{\mathrm{BH}}} e^{S_R}$

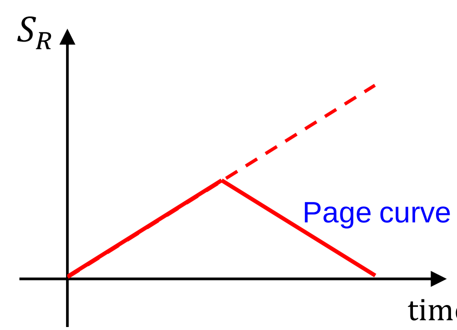


$+$

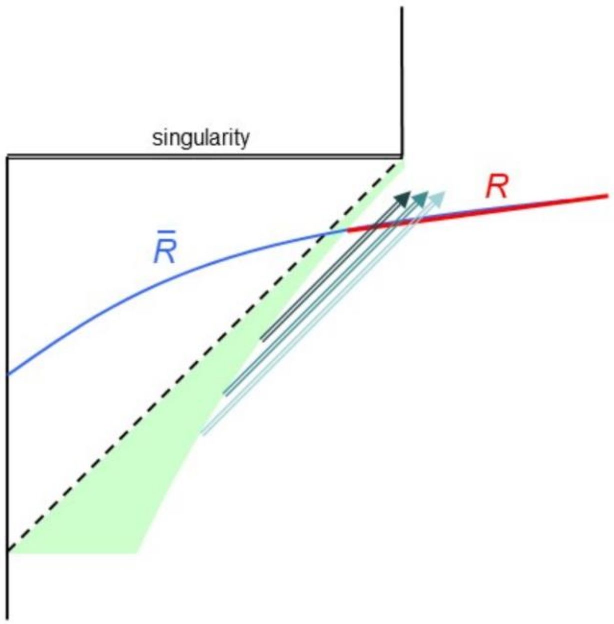
$e^{S_{\mathrm{BH}}} e^{2S_R}$



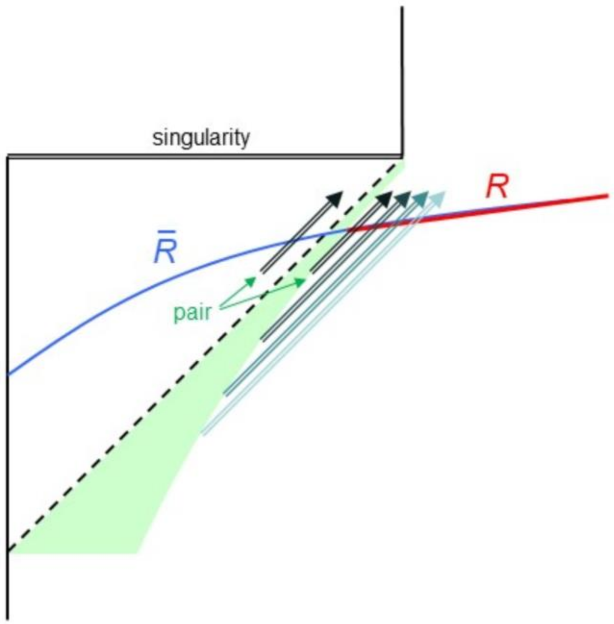
replica wormhole (nonperturbative effect)



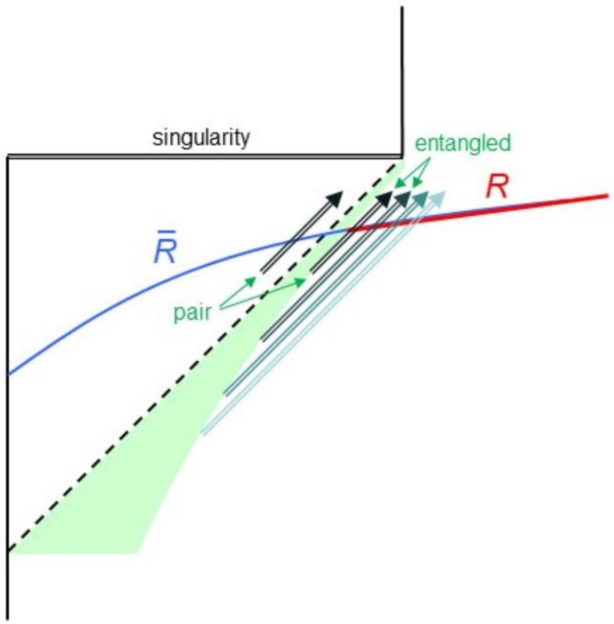
Redundancy in the Hawking process



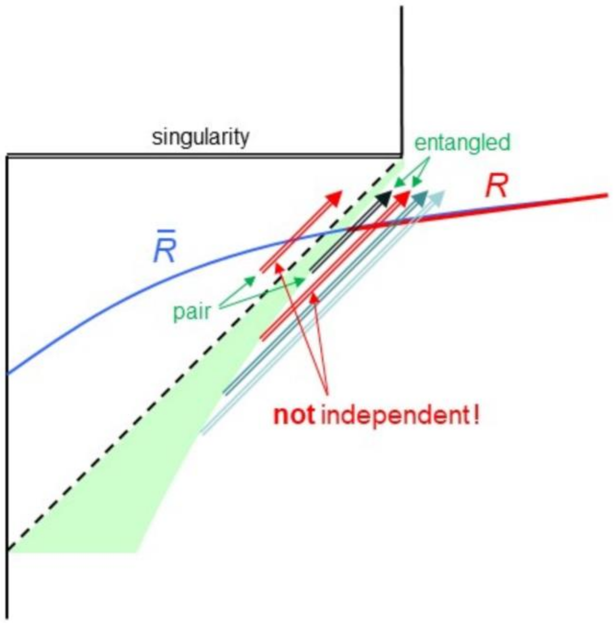
Redundancy in the Hawking process



Redundancy in the Hawking process



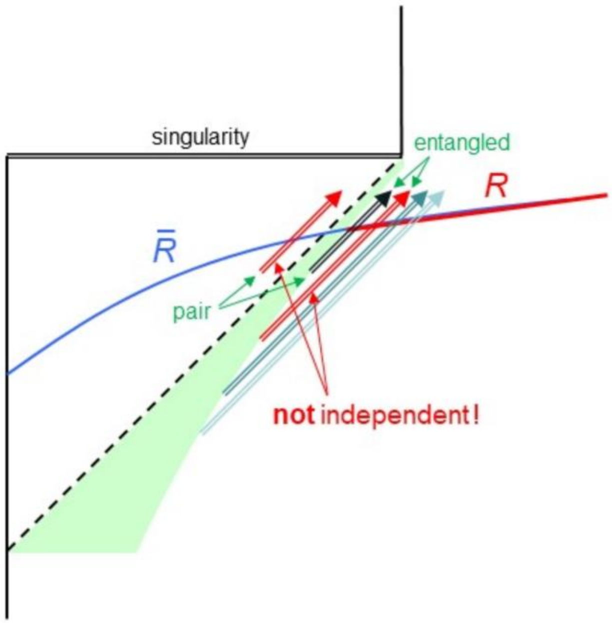
Redundancy in the Hawking process



$\rightarrow$ Hawking radiation emitted earlier is
not independent of the interior d.o.f.s!

…; Maldacena, Susskind ('13); …

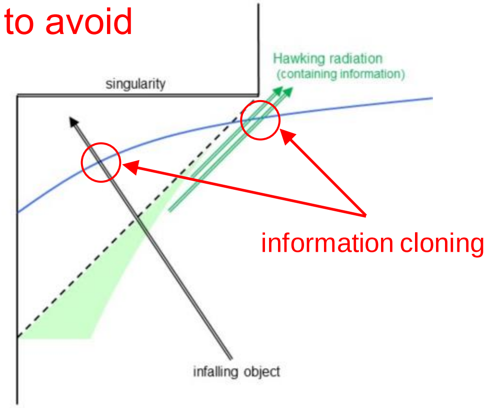
Redundancy in the Hawking process



$\rightarrow$ Hawking radiation emitted earlier is
not independent of the interior d.o.f.s!

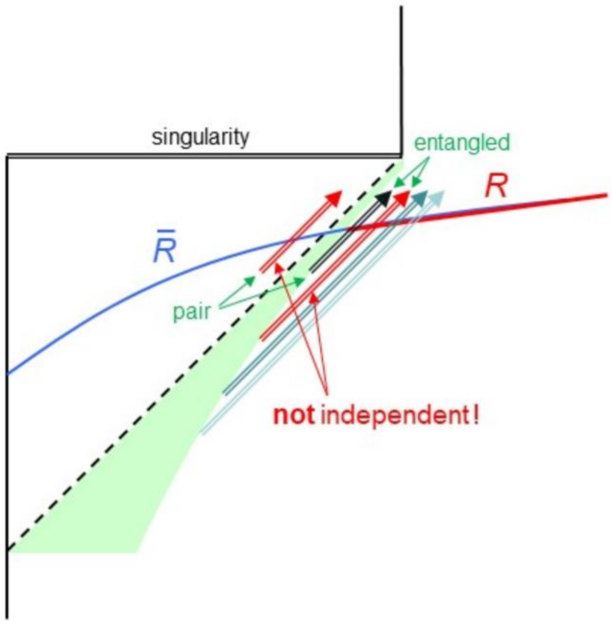
…; Maldacena, Susskind ('13); …

- needed to avoid

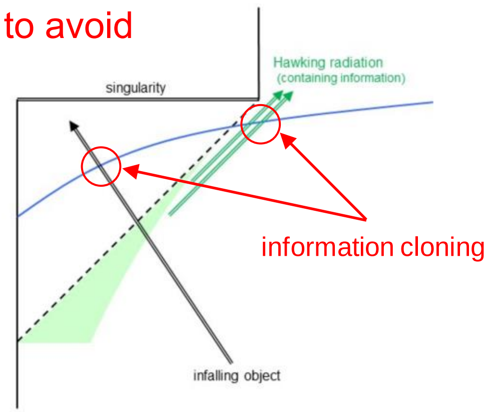


- consistent because of causality

Redundancy in the Hawking process



- needed to avoid



- consistent because of causality

→ Hawking radiation emitted earlier is
not independent of the interior d.o.f.s!

…; Maldacena, Susskind ('13); …

Global spacetime
(embracing the **interior**)

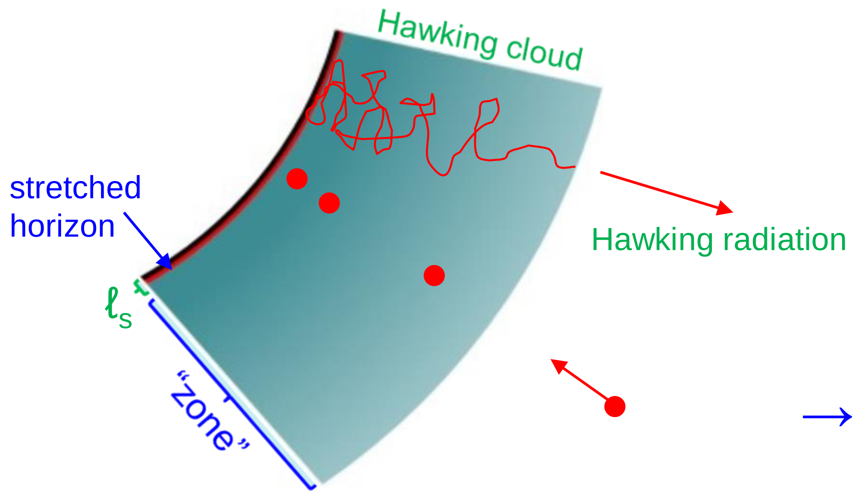
Replica wormholes
(nonperturbative effects of gravity)

Page curve
(signifying **unitarity**)

Picture based on Holography

— unitary gauge construction —

Start with a "distant" (holographic) description



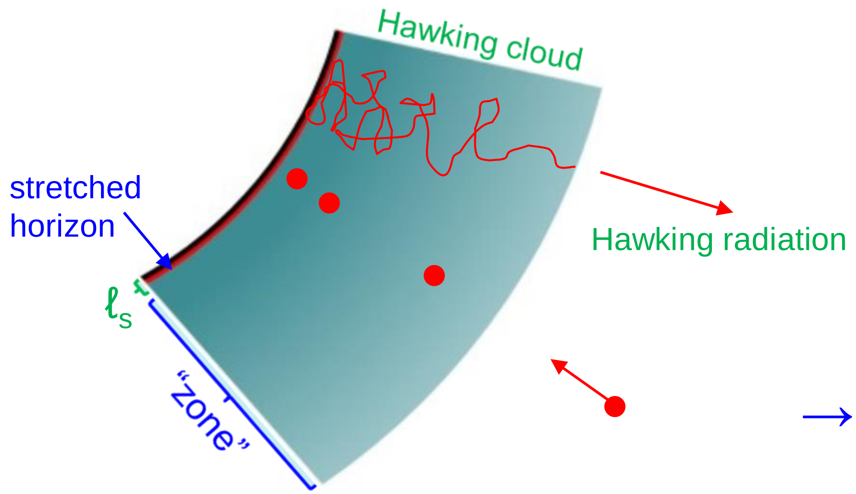
The d.o.f.s outside the horizon comprise the **entire** system.

→ The evolution is unitary.

→ How does the "interior" emerge?

Papadodimas, Raju ('12–'15); Verlinde, Verlinde ('12–'13); Y.N., Sanches, Varela, Weinberg ('12–'15); …
Y.N. ('19, 20)

Start with a “distant” (holographic) description



The d.o.f.s outside the horizon comprise the **entire** system.

→ The evolution is unitary.

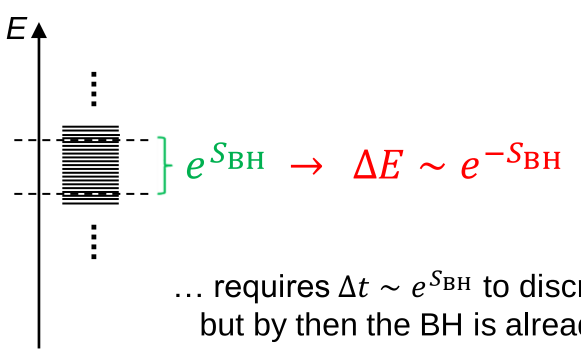
→ How does the “interior” emerge?

Papadodimas, Raju (’12–’15); Verlinde, Verlinde (’12–’13);
Y.N., Sanches, Varela, Weinberg (’12–’15); …
Y.N. (’19, 20)

Key features Y.N. (’19, 20)

— defining characteristics of BHs

(I) Exponentially dense spectrum



… requires $\Delta t \sim e^{S_{\rm BH}}$ to discriminate them,
but by then the BH is already evaporated.

Relevant modes:

$$\begin{cases} \text{zone} & \begin{cases} \text{hard: } \omega \gtrsim T_{\rm H} & \text{(objects)} \\ \text{soft: } \omega \lesssim T_{\rm H} & \text{(cloud)} \end{cases} \\ \text{far} & \end{cases}$$

(II) Dynamics at the stretched horizon

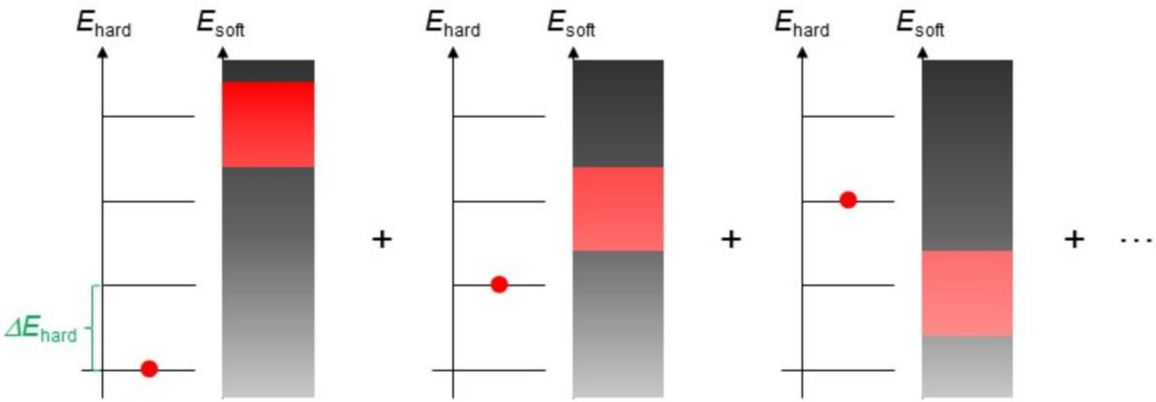
$T_{\mathrm{local}} \sim M_{\mathrm{string}}$

… string dynamics

- quantum chaos Maldacena, Shenker, Stanford ('15)
- fast scrambling Hayden, Preskil ('07); Sekino, Susskind ('08)
- universal Banks, Seiberg ('10); …; Harlow, Ooguri ('18)

(*e.g.* no global symmetry)

→ "ultimate" thermalization in the zone

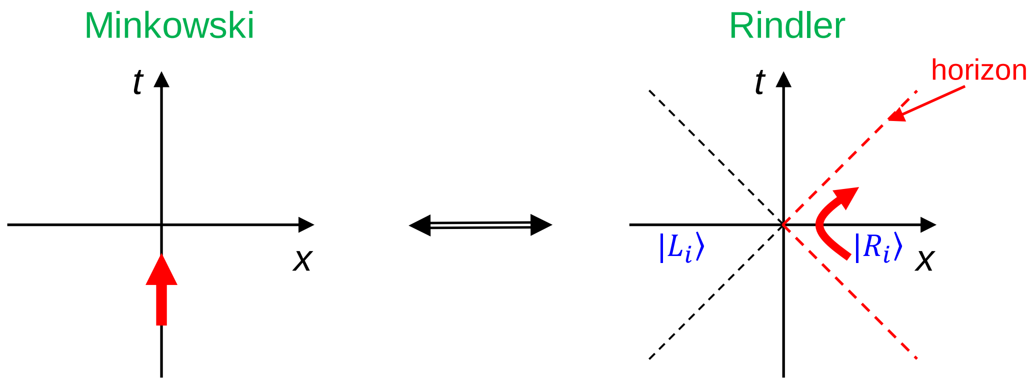


$$\sum_{i=1}^{e^{S_{\mathrm{bh}}(M)}} c_i^{(0)} |0\rangle|\Psi_i^{(0)}\rangle + \sum_{i=1}^{e^{S_{\mathrm{bh}}(M-\Delta E)}} c_i^{(1)} |1\rangle|\Psi_i^{(1)}\rangle + \sum_{i=1}^{e^{S_{\mathrm{bh}}(M-2\Delta E)}} c_i^{(2)} |2\rangle|\Psi_i^{(2)}\rangle + \cdots$$

$$\xrightarrow[c_i^{(0)} \sim c_i^{(1)} \sim c_i^{(2)} \sim \cdots]{} \propto |0\rangle\langle 0| + e^{-\frac{\Delta E}{T_{\mathrm{H}}}} |1\rangle\langle 1| + e^{-\frac{2\Delta E}{T_{\mathrm{H}}}} |2\rangle\langle 2| + \cdots$$

… **universal** across all low energy species

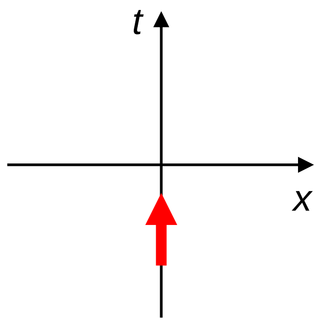
Emergence of the interior: Basic picture



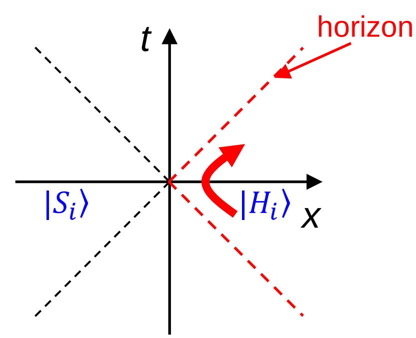
$|0\rangle$ $\quad\quad$ $\propto \sum_i e^{-\frac{E_i}{T}} |R_i\rangle |L_i\rangle$ … thermal state Fulling ('73); Davies ('75); Unruh ('76)

Emergence of the interior: Basic picture

Minkowski



Rindler



$$|0\rangle \qquad \propto \sum_i e^{-\frac{E_i}{T}} |R_i\rangle\,|L_i\rangle$$

… thermal state Fulling ('73); Davies ('75); Unruh ('76)

Near empty

Interior spacetime

(An object thrown "sees" interior spacetime)

frame change

$$|\Psi_{\rm BH}\rangle \propto \sum_i e^{-\frac{E_i}{T_{\rm H}}} |H_i\rangle\, \overbrace{|S_i\rangle}$$

Hard mode states

… play the role of the mirror partners

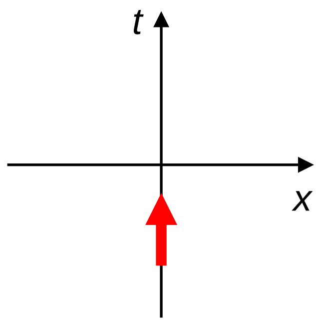
Coarse-grained soft mode states

(representing their collective excitations)

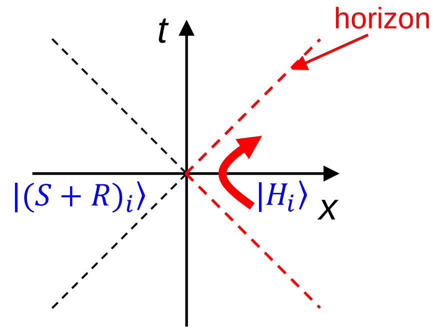
… **universally** thermal

Emergence of the interior: Basic picture

Minkowski



Rindler



$$|0\rangle \qquad \Longleftrightarrow \qquad \propto \sum_i e^{-\frac{E_i}{T}} |R_i\rangle |L_i\rangle$$

… thermal state Fulling ('73); Davies ('75); Unruh ('76)

Near empty
Interior spacetime

(An object thrown "sees" interior spacetime)

frame change

$$\Longleftrightarrow \quad |\Psi_{\mathrm{BH}}\rangle \propto \sum_i e^{-\frac{E_i}{T_{\mathrm{H}}}} |H_i\rangle \overbrace{|S_i\rangle}$$

Hard mode states: $|H_i\rangle$

… play the role of the mirror partners

Coarse-grained soft mode states
(representing their collective excitations)

… **universally** thermal

At late times, the BH is entangled with radiation

$$|\Psi_{\mathrm{BH}}\rangle \propto \sum_i e^{-\frac{E_i}{T_{\mathrm{H}}}} |H_i\rangle \overbrace{|(S+R)_i\rangle}$$

Hard mode states: $|H_i\rangle$

… play the role of the mirror partners

Coarse-grained soft mode **and** radiation states
(representing collective excitations of these modes)

… Interior d.o.f.s involve early Hawking radiation.

More details and math behind them

- Black hole **vacuum** state at time *t*

Soft mode … the density of states: $e^{S_{\rm BH}(E_{\rm soft})}$

$$|\Psi_{A,0}(M)\rangle = \sum_n \sum_{i_n=1}^{e^{S_{\rm bh}(M-E_n)}} \sum_{a=1}^{e^{S_{\rm rad}}} c^A_{n i_n a} \underbrace{|\{n_\alpha\}\rangle}_{\text{Hard mode}} \overbrace{|\psi^{(n)}_{i_n}\rangle}^{\text{Soft mode}} \underbrace{|\phi_a\rangle}_{\text{Far mode (radiation)}}$$

index specifying microstate: $A = 1, \cdots, e^{S_{\rm tot}}$

$$\Big(\ \langle\{m_\alpha\}|\{n_\alpha\}\rangle = \delta_{mn}, \quad \langle\psi^{(m)}_{i_m}|\psi^{(n)}_{j_n}\rangle = \delta_{mn}\delta_{i_m j_n}, \quad \langle\phi_a|\phi_b\rangle = \delta_{ab}\ \Big)$$

$$e^{S_{\rm tot}} \equiv \sum_n e^{S_{\rm bh}(M-E_n)} e^{S_{\rm rad}} = z\, e^{S_{\rm bh}(M)+S_{\rm rad}} \quad \Big(\ z \equiv \sum_n e^{-\frac{E_n}{T_{\rm H}}}\ \Big)$$

Complete thermalization by (redshifted) string dynamics

$$\langle c^A_{n i_n a}\rangle = 0, \quad \sqrt{\langle |c^A_{n i_n a}|^2\rangle} = \frac{1}{e^{\frac{1}{2}S_{\rm tot}}} \Longrightarrow {\rm Tr}_{\rm soft}|\Psi_{A,0}(M)\rangle\langle\Psi_{A,0}(M)| = \frac{1}{z}\sum_n e^{-\frac{E_n}{T_{\rm H}}} |\{n_\alpha\}\rangle\langle\{n_\alpha\}| \otimes \rho_\phi$$

… thermal density matrix for the hard modes

Excitations in the zone:

$$b_\gamma = \sum_n \sqrt{n_\gamma}\, |\{n_\alpha - \delta_{\alpha\gamma}\}\rangle\langle\{n_\alpha\}|$$

$$b^\dagger_\gamma = \sum_n \sqrt{n_\gamma + 1}\, |\{n_\alpha + \delta_{\alpha\gamma}\}\rangle\langle\{n_\alpha\}|$$

… standard annihilation and creation operators

What about the interior?

More details and math behind them

- Black hole **vacuum** state at time *t*

$$|\Psi_{A,0}(M)\rangle = \sum_n \boxed{\sum_{i_n=1}^{e^{S_{\rm bh}(M-E_n)}} \sum_{a=1}^{e^{S_{\rm rad}}} c^A_{n i_n a} \boxed{|\{n_\alpha\}\rangle} \boxed{|\psi^{(n)}_{i_n}\rangle|\phi_a\rangle}}$$

index specifying microstate: $A = 1, \cdots, e^{S_{\rm tot}}$

$$\left(\langle\{m_\alpha\}|\{n_\alpha\}\rangle = \delta_{mn}, \quad \langle\psi^{(m)}_{i_m}|\psi^{(n)}_{j_n}\rangle = \delta_{mn}\delta_{i_m j_n}, \quad \langle\phi_a|\phi_b\rangle = \delta_{ab} \right)$$

$$e^{S_{\rm tot}} \equiv \sum_n e^{S_{\rm bh}(M-E_n)} e^{S_{\rm rad}} = z\, e^{S_{\rm bh}(M)+S_{\rm rad}} \quad \left(z \equiv \sum_n e^{-\frac{E_n}{T_{\rm H}}} \right)$$

Complete thermalization by (redshifted) string dynamics

$$\langle c^A_{n i_n a}\rangle = 0, \quad \sqrt{\langle |c^A_{n i_n a}|^2\rangle} = \frac{1}{e^{\frac{1}{2}S_{\rm tot}}} \quad \Longrightarrow \quad {\rm Tr}_{\rm soft}|\Psi_{A,0}(M)\rangle\langle\Psi_{A,0}(M)| = \frac{1}{z}\sum_n e^{-\frac{E_n}{T_{\rm H}}}|\{n_\alpha\}\rangle\langle\{n_\alpha\}| \otimes \rho_\phi$$

… thermal density matrix for the hard modes

Mirror microstates **for each *A***

cf. Papadodimas, Raju ('12–'15); Verlinde, Verlinde ('12–'13); Y.N., Sanches, Varela, Weinberg ('12–'15)

$$\|\{n_\alpha\}_A\rangle\!\rangle = \alpha^A_n \sum_{i_n=1}^{e^{S_{\rm bh}(M-E_n)}} \sum_{a=1}^{e^{S_{\rm rad}}} c^A_{n i_n a}|\psi^{(n)}_{i_n}\rangle|\phi_a\rangle \quad \Longrightarrow \quad |\Psi_{A,0}(M)\rangle = \frac{1}{\sqrt{z}}\sum_n e^{-\frac{E_n}{2T_{\rm H}}}|\{n_\alpha\}\rangle\|\{n_\alpha\}_A\rangle\!\rangle$$

… **thermofield double form**

normalization: $$\alpha^A_n = \frac{1}{\sqrt{\sum_{i_n=1}^{e^{S_{\rm bh}(M-E_n)}} \sum_{a=1}^{e^{S_{\rm rad}}} c^{A*}_{n i_n a} c^A_{n i_n a}}} = \sqrt{z}\, e^{\frac{E_n}{2T_{\rm H}}}\left(1 - \frac{1}{2}\varepsilon^{AA}_n\right)$$

exponentially small correction

Mirror microstates **for each *A***

$$\|\{n_\alpha\}_A\rangle\rangle = \alpha_n^A \sum_{i_n=1}^{e^{S_{\rm bh}(M-E_n)}} \sum_{a=1}^{e^{S_{\rm rad}}} c^A_{n i_n a} |\psi^{(n)}_{i_n}\rangle |\phi_a\rangle \quad \Longrightarrow \quad |\Psi_{A,0}(M)\rangle = \frac{1}{\sqrt{z}} \sum_n e^{-\frac{E_n}{2T_{\rm H}}} |\{n_\alpha\}\rangle \|\{n_\alpha\}_A\rangle\rangle$$

… **thermofield double form**

normalization: $$\alpha_n^A = \frac{1}{\sqrt{\sum_{i_n=1}^{e^{S_{\rm bh}(M-E_n)}} \sum_{a=1}^{e^{S_{\rm rad}}} c^{A*}_{n i_n a} c^A_{n i_n a}}} = \sqrt{z}\, e^{\frac{E_n}{2T_{\rm H}}} \left(1 - \frac{1}{2}\varepsilon_n^{AA}\right)$$

exponentially small correction

Mirror microstates **for each *A***

$$\|\{n_\alpha\}_A\rangle\rangle = \alpha_n^A \sum_{i_n=1}^{e^{S_{\rm bh}(M-E_n)}} \sum_{a=1}^{e^{S_{\rm rad}}} c_{n i_n a}^A |\psi_{i_n}^{(n)}\rangle|\phi_a\rangle \Longrightarrow |\Psi_{A,0}(M)\rangle = \frac{1}{\sqrt{z}}\sum_n e^{-\frac{E_n}{2T_{\rm H}}}|\{n_\alpha\}\rangle\|\{n_\alpha\}_A\rangle\rangle$$

… thermofield double form

normalization: $$\alpha_n^A = \frac{1}{\sqrt{\sum_{i_n=1}^{e^{S_{\rm bh}(M-E_n)}}\sum_{a=1}^{e^{S_{\rm rad}}} c_{n i_n a}^{A*} c_{n i_n a}^A}}$$

$$= \sqrt{z}\, e^{\frac{E_n}{2T_{\rm H}}}\left(1 - \frac{1}{2}\varepsilon_n^{AA}\right)$$

exponentially small correction

Mirror microstates **for each *A***

$$\|\{n_\alpha\}_A\rangle\!\rangle = \alpha_n^A \sum_{i_n=1}^{e^{S_{\rm bh}(M-E_n)}} \sum_{a=1}^{e^{S_{\rm rad}}} c^A_{n i_n a} |\psi^{(n)}_{i_n}\rangle |\phi_a\rangle \Longrightarrow |\Psi_{A,0}(M)\rangle = \frac{1}{\sqrt{z}} \sum_n e^{-\frac{E_n}{2T_{\rm H}}} |\{n_\alpha\}\rangle \|\{n_\alpha\}_A\rangle\!\rangle$$

… **thermofield double form**

normalization: $$\alpha_n^A = \frac{1}{\sqrt{\sum_{i_n=1}^{e^{S_{\rm bh}(M-E_n)}} \sum_{a=1}^{e^{S_{\rm rad}}} c^{A*}_{n i_n a} c^A_{n i_n a}}} = \sqrt{z}\, e^{\frac{E_n}{2T_{\rm H}}} \left(1 - \frac{1}{2}\varepsilon_n^{AA}\right)$$

exponentially small correction

Mirror operators **for each microstate** (representing collective excitations of soft and far modes)

$$\tilde{b}_\gamma^A = \sum_n \sqrt{n_\gamma}\, \|\{n_\alpha - \delta_{\alpha\gamma}\}_A\rangle\!\rangle \langle\!\langle \{n_\alpha\}_A\|$$
$$\tilde{b}_\gamma^{A\dagger} = \sum_n \sqrt{n_\gamma + 1}\, \|\{n_\alpha + \delta_{\alpha\gamma}\}_A\rangle\!\rangle \langle\!\langle \{n_\alpha\}_A\|$$

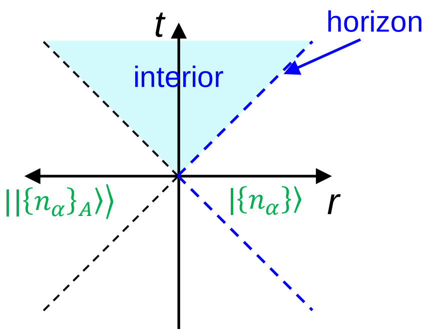
… satisfy the commutation relation of ann./cre. operators
up to exponentially suppressed corrections of $\sim e^{-S}$.

Infalling mode operators for each microstate

$$a_\xi^A = \sum_\gamma \left(\alpha_{\xi\gamma} b_\gamma + \beta_{\xi\gamma} b_\gamma^\dagger + \zeta_{\xi\gamma} \tilde{b}_\gamma^A + \eta_{\xi\gamma} \tilde{b}_\gamma^{A\dagger}\right)$$
$$a_\xi^{A\dagger} = \sum_\gamma \left(\beta^*_{\xi\gamma} b_\gamma + \alpha^*_{\xi\gamma} b_\gamma^\dagger + \eta^*_{\xi\gamma} \tilde{b}_\gamma^A + \zeta^*_{\xi\gamma} \tilde{b}_\gamma^{A\dagger}\right)$$

standard Bogoliubov coefficients

… describe interior spacetime
for **the hard modes** (objects).



Sufficient? $\rightarrow$ No

Hard modes (a falling object) may be entangled with soft modes.

$$|\Psi(t_*)\rangle = \sum_{A=1}^{S_{\rm tot}} \sum_I d_{AI}(t_*) |\Psi_{A,I}(M)\rangle$$

$\rightarrow$ Which infalling micro-operators should we use?

Global promotion

$$\mathcal{M} = \left\{ \sum_{A=1}^{e^{S_{\rm tot}}} a_A |\Psi_{A,0}(M)\rangle \,\middle|\, a_A \in \mathbb{C},\ \sum_{A=1}^{e^{S_{\rm tot}}} |a_A|^2 = 1 \right\}$$ … space of vacuum microstates

$$\tilde{\mathcal{M}} = \left\{ \sum_{A'=1}^{e^{S_{\rm eff}}} a_{A'} |\Psi_{A',0}(M)\rangle \,\middle|\, a_{A'} \in \mathbb{C},\ \sum_{A'=1}^{e^{S_{\rm eff}}} |a_{A'}|^2 = 1 \right\}$$ … subspace of $\mathcal{M}$ with $S_{\rm eff} < S_{\rm bh}(M) + S_{\rm rad}$

Globally promoted operators:

$$\tilde{\mathcal{B}}_\gamma = \sum_{A'=1}^{e^{S_{\rm eff}}} \tilde{b}_\gamma^{A'}, \quad \tilde{\mathcal{B}}_\gamma^\dagger = \sum_{A'=1}^{e^{S_{\rm eff}}} \tilde{b}_\gamma^{A'\dagger}$$

$$\Longrightarrow \quad \mathcal{A}_\xi = \sum_\gamma \left(\alpha_{\xi\gamma} b_\gamma + \beta_{\xi\gamma} b_\gamma^\dagger + \zeta_{\xi\gamma} \tilde{\mathcal{B}}_\gamma + \eta_{\xi\gamma} \tilde{\mathcal{B}}_\gamma^\dagger \right)$$

$$\mathcal{A}_\xi^\dagger = \sum_\gamma \left(\beta_{\xi\gamma}^* b_\gamma + \alpha_{\xi\gamma}^* b_\gamma^\dagger + \eta_{\xi\gamma}^* \tilde{\mathcal{B}}_\gamma + \zeta_{\xi\gamma}^* \tilde{\mathcal{B}}_\gamma^\dagger \right)$$

… satisfy the correct algebra for **any** vacuum state in $\tilde{\mathcal{M}}$.

(up to exponentially suppressed corrections)

Sufficient? $\rightarrow$ No

Hard modes (a falling object) may be entangled with soft modes.

$$|\Psi(t_*)\rangle = \sum_{A=1}^{S_{\rm tot}} \sum_{I} d_{AI}(t_*) |\Psi_{A,I}(M)\rangle \xrightarrow[\text{decomposition}]{\text{Schmidt}} |\Psi(t_*)\rangle = \sum_{I=1}^{\mathcal{K}} g_I |\Psi_{A(I),I}(M)\rangle$$

where $\mathcal{K} \leq S_{\rm exc} < S_{\rm bh}(M) + S_{\rm rad}$

$\Downarrow$ One can always choose

$$\boxed{\tilde{\mathcal{M}} \supseteq \tilde{V}\big[|\Psi(t_*)\rangle\big]}$$

where $\tilde{V}\big[|\Psi(t_*)\rangle\big] = \mathrm{span}\big(\{|\Psi_{A(I),0}(M)\rangle\}\big)$

Global promotion

$$\mathcal{M} = \left\{ \sum_{A=1}^{e^{S_{\rm tot}}} a_A |\Psi_{A,0}(M)\rangle \,\middle|\, a_A \in \mathbb{C}, \sum_{A=1}^{e^{S_{\rm tot}}} |a_A|^2 = 1 \right\}$$

… space of vacuum microstates

$$\tilde{\mathcal{M}} = \left\{ \sum_{A'=1}^{e^{S_{\rm eff}}} a_{A'} |\Psi_{A',0}(M)\rangle \,\middle|\, a_{A'} \in \mathbb{C}, \sum_{A'=1}^{e^{S_{\rm eff}}} |a_{A'}|^2 = 1 \right\}$$

… subspace of $\mathcal{M}$ with $S_{\rm eff} < S_{\rm bh}(M) + S_{\rm rad}$

Globally promoted operators:

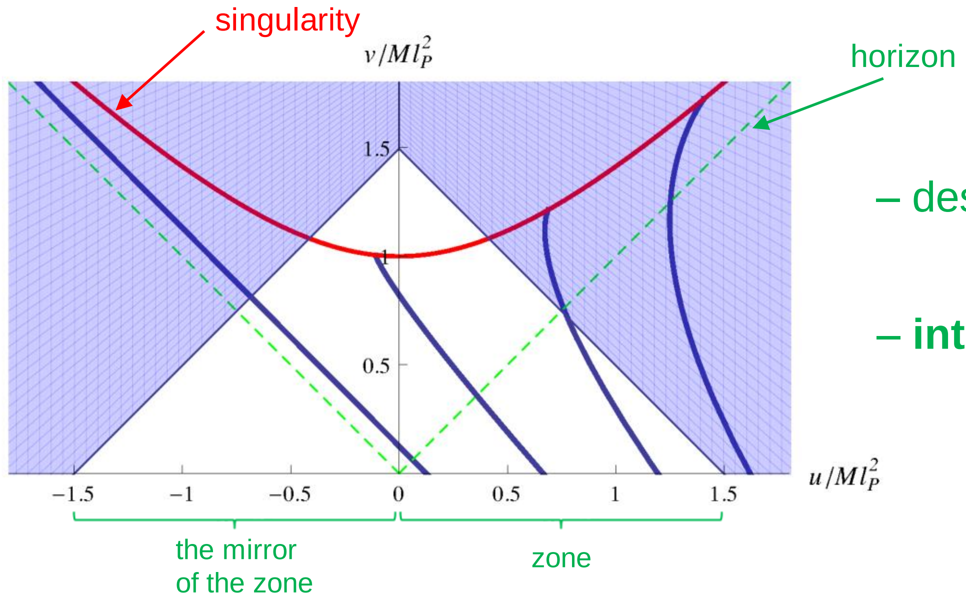
$$\tilde{\mathcal{B}}_\gamma = \sum_{A'=1}^{e^{S_{\rm eff}}} \tilde{b}_\gamma^{A'}, \quad \tilde{\mathcal{B}}_\gamma^\dagger = \sum_{A'=1}^{e^{S_{\rm eff}}} \tilde{b}_\gamma^{A'\dagger}$$

$$\Longrightarrow \boxed{\begin{aligned} \mathcal{A}_\xi &= \sum_\gamma \big(\alpha_{\xi\gamma} b_\gamma + \beta_{\xi\gamma} b_\gamma^\dagger + \zeta_{\xi\gamma} \tilde{\mathcal{B}}_\gamma + \eta_{\xi\gamma} \tilde{\mathcal{B}}_\gamma^\dagger\big) \\ \mathcal{A}_\xi^\dagger &= \sum_\gamma \big(\beta_{\xi\gamma}^* b_\gamma + \alpha_{\xi\gamma}^* b_\gamma^\dagger + \eta_{\xi\gamma}^* \tilde{\mathcal{B}}_\gamma + \zeta_{\xi\gamma}^* \tilde{\mathcal{B}}_\gamma^\dagger\big) \end{aligned}}$$

… satisfy the correct algebra
for **any** vacuum state in $\tilde{\mathcal{M}}$.
(up to exponentially suppressed corrections)

Effective theory of the interior

… erected at each time t (as measured in the asymptotic region)



– describes only a **limited** region of spacetime

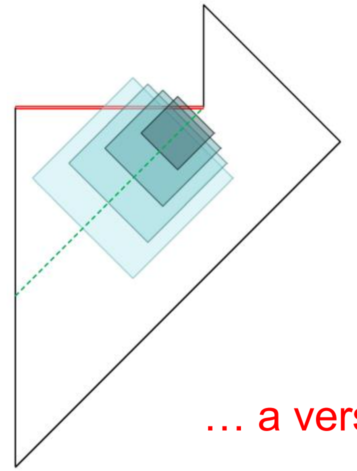
(causal region of the zone and its mirror at t)

– **intrinsically** semiclassical

(coarse-grained; the unique infalling vacuum)

→ no cloning problem !

Describing a large interior region requires **multiple** effective theories erected at different times.



… a version of complementarity

Susskind, Thorlacius Uglum ('93)

Comments

– Intrinsic ambiguity

Infalling mode operators are not strictly orthogonal to $\tilde{\mathcal{M}}$.

$\Longrightarrow$ ambiguity of the procedure of erecting the effective theory of order $\epsilon = \max\left\{ \frac{1}{e^{\frac{1}{2}\{S_{\rm bh}(M)+S_{\rm rad}\}}}, \frac{e^{S_{\rm eff}}}{e^{S_{\rm bh}(M)+S_{\rm rad}}} \right\}$

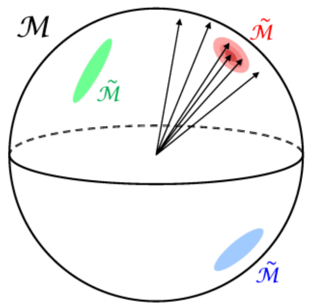
… manifestation of the fact that the theory has only a finite # of d.o.f.s.

– State dependence

$\tilde{\mathcal{M}}$, of dimension $e^{S_{\rm eff}}$, can be taken large,

$S_{\rm eff} = c\,\{S_{\rm bh}(M) + S_{\rm rad}\}$ for any c (< 1)

unless c is exponentially close to 1.



$\Longrightarrow$ A single set of global operators cannot cover the entire $\mathcal{M}$.

… state dependence

Papadodimas, Raju ('13–'15)

– Young black holes

For a young BH, $S_{\rm bh} > S_{\rm rad}$, infalling operators can be taken to act only on soft modes, using the Petz map.

This option is **not** available for an old BH.
→ **must** involve early radiation.

$$\tilde{\mathcal{O}}^A[\mathcal{O}] = \sum_\kappa \sum_\lambda \mathcal{O}_{\kappa\lambda}\, \alpha_\kappa^A \alpha_\lambda^{A*} \sum_{i_\kappa, i'_\kappa=1}^{e^{S_{\rm bh}(M-E_\kappa)}} \sum_{j_\lambda, j'_\lambda=1}^{e^{S_{\rm bh}(M-E_\lambda)}} \sum_{a=1}^{e^{S_{\rm rad}}} X^{(\kappa,A)}_{i'_\kappa i_\kappa} c^A_{\kappa i_\kappa a} c^{A*}_{\lambda j_\lambda a} X^{(\lambda,A)}_{j_\lambda j'_\lambda} |\psi^{(\kappa)}_{i'_\kappa}\rangle\langle\psi^{(\lambda)}_{j'_\lambda}| \quad \left(X^{(\kappa,A)}_{i_\kappa j_\kappa} = e^{-\frac{1}{2}S_{\rm rad}} \sum_{a=1}^{e^{S_{\rm rad}}} \frac{c^A_{\kappa i_\kappa a} c^{A*}_{\kappa j_\kappa a}}{\left| \alpha_\kappa^A \sum_{k_\kappa=1}^{e^{S_{\rm bh}(M-E_\kappa)}} c^{A*}_{\kappa k_\kappa a} c^A_{\kappa k_\kappa a} \right|^2} \right)$$

Summary

— unitary gauge construction —

Distant description (manifestly **unitary**) $\xrightarrow[\text{Collective phenomena}]{}$ **Interior** spacetime (effective emergence)

— It is crucial for the string dynamics, $T_{\text{loc}} \sim M_{\text{string}}$,
to lead to "ultimate" (universal) thermalization.

… the defining characteristic for BHs in this description

(→ A similar construction does **not** work for the surface of regular material.)

— Emergence of the interior does not require
detailed knowledge about the UV physics.

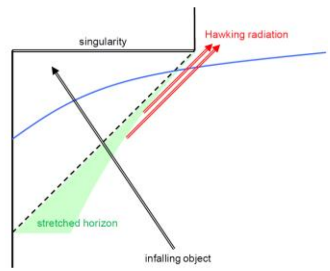
… Some basic features (quantum chaos, fast scrambling, universality) are sufficient.

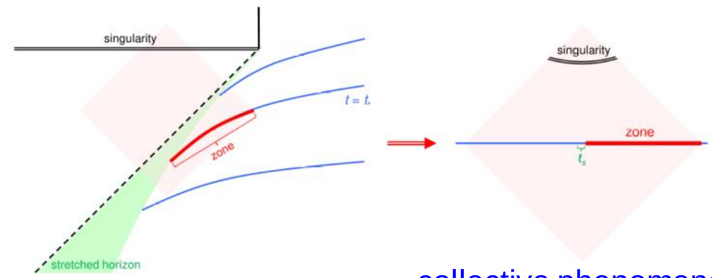
Structure of Quantum Gravity

	Global spacetime **— General relativity —**	Unitary / holographic **— Quantum mechanics —**

- Interior

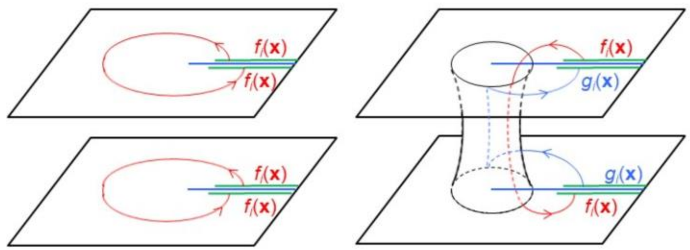
Evident





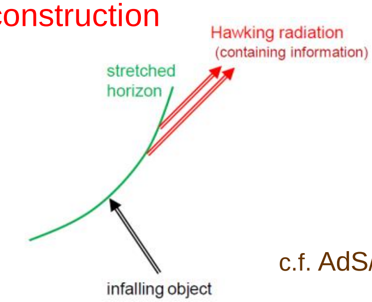
collective phenomenon

- Unitarity



replica wormholes

By construction



c.f. AdS/CFT

- Apparent violation of BH entropy

huge interior spatial volume at late times

semiclassically orthogonal states in fact have $\langle \Psi_1 | \Psi_2 \rangle \sim e^{-S_{\rm BH}/2}$

$\rightarrow$ $e^{S_{\rm BH}}$ states (+ null states)

Effective theory of the interior has a finite maximal volume.

Hilbert space of dimension $e^{S_{\rm BH}}$ can host $e^{e^{S_{\rm BH}}}$ approximately orthogonal states.

- Ensemble nature

Wormhole contributions

$\rightarrow$ "statistical" results

$\langle \psi_I | \psi_J \rangle = 0,\ |\langle \psi_I | \psi_J \rangle|^2 \sim e^{-S_{\rm BH}/2} \neq 0$

Penington, Shenker, Stanford, Yang ('19)

Wormholes calculate (incoherent) average

Averaging in soft mode states

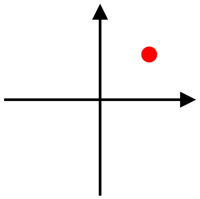
$\int dU_{\rm soft} \langle \psi_{U(i)} | \psi_{U(j)} \rangle = 0,$

$\int dU_{\rm soft} |\langle \psi_{U(i)} | \psi_{U(j)} \rangle|^2 \sim e^{-S_{\rm BH}/2}$

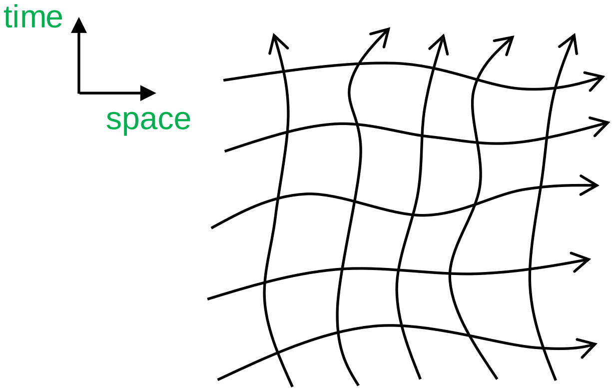
Langhoff, Y.N. ('20); Chakravarty ('20); Pollack, Rozali, Sully Wakeham ('20)

Redundancies of the description

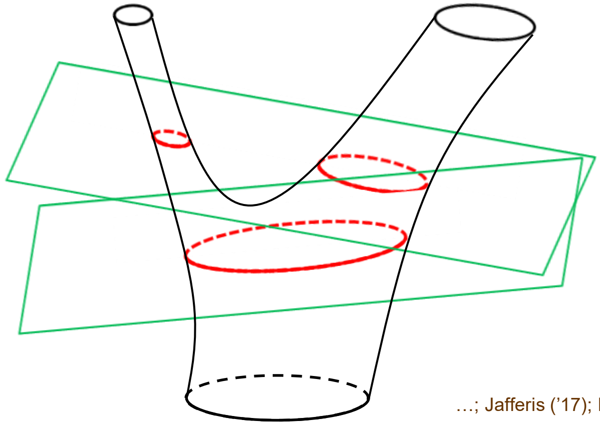
- General covariance (perturbative)



$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \leftrightarrow \begin{pmatrix} r \\ \theta \end{pmatrix} = \begin{pmatrix} \sqrt{2} \\ \pi/4 \end{pmatrix} \leftrightarrow \ \dots$$



- Nonperturbative redundancies



…; Jafferis ('17); Marolf, Maxfield ('20); …

… allows for making (only) one of the two pillars manifest,

but the theory still accommodates both of them (QM + GR).

Summary

Black hole conundrum

$\Downarrow$

Structure of quantum gravity

$\supset$ Quantm mechanics & General relativity, but in a subtle manner!

… only one of them being manifest

Global spacetime

- interior — evident
- unitarity — nonperturbative gravity

… path integral (GR friendly)

Unitary / holographic

- unitarity — by construction
- interior — collective phenomenon

… operator (QM friendly)

$\Longrightarrow$ Lower energy physics without details of microscopic physics

And yet, we want to understand the microscopic theory of quantum gravity

… string theory, quantum information science, …