Holographic entanglement entropy of deSitter braneworld

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Contents

1. Introduction

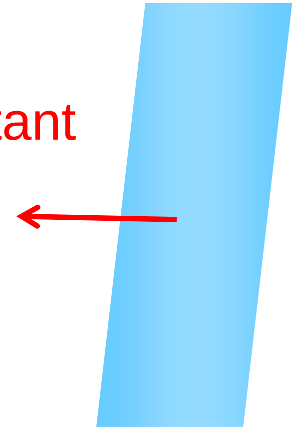
Braneworld

- Higher dimensional model inspired by string theory

Simplest model: Randall-Sundrum(RS) 1999

5-dim. bulk spacetime with negative cosmological constant

(5-dim. anti-deSitter spacetime)



Gravity associated with brane is fully taken into account

brane: 4-dim. spacetime

(4-dim. Minkowski spacetime)

-applicable to cosmology/black hole

-similar setup to adS/CFT (⇒ braneworld holography)

New lights to braneworld

Holographic entanglement entropy Ryu & Takayanagi 2006

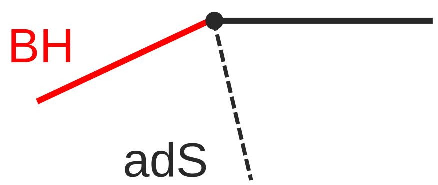
Island formula for Hawking radiation

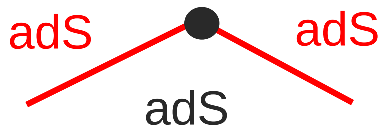
Penington 2020, Almheiri et al 2019,2020,…

co-dimension 2 holography

Akal, Kusuki, Takayanagi & Wei 2020

Hybrid formulation of adS/CFT and braneworld





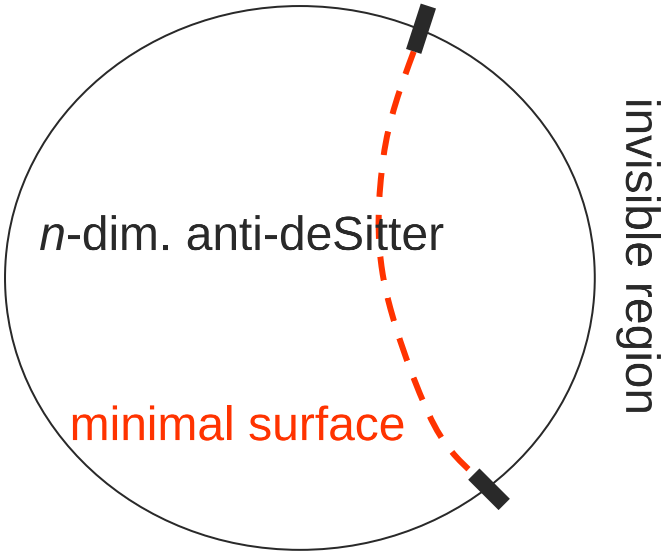
Revisit braneworld model with those in mind

Holographic entanglement entropy

in adS/CFT setup

Ryu-Takayanagi formula(2006)

$$S_{\text{ent}} = \frac{\text{area of minimal surface}}{4G_n}$$



Holographic entanglement entropy in braneworld?

-black hole on brane Emparam 2006

-deSitter spacetime on brane

Iwashita, Kobayashi, Shiromizu & Yoshino 2006

deSitter spacetime

-solution to Einstein equation with positive cosmological constant

-maximally symmetric, Lorentz group

-exponentially expanding universe

⇨ cosmological horizon

analogy to Hawking-Bekenstein formula for black hole entropy

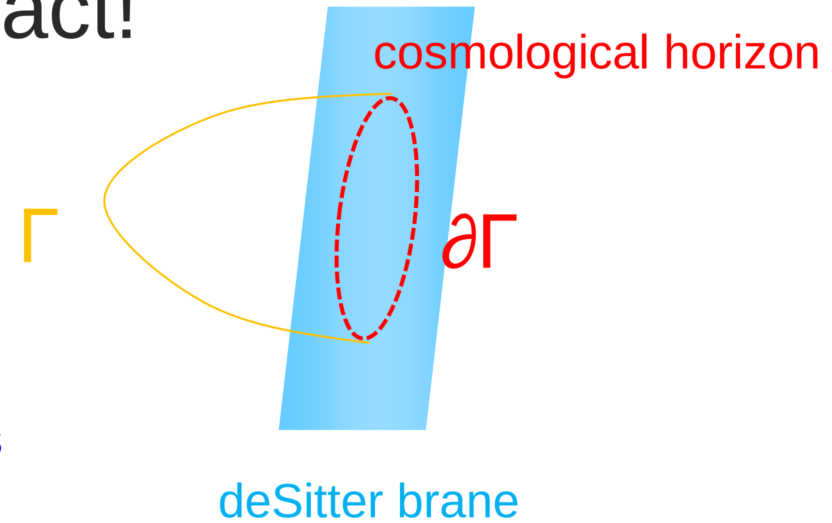
$$S = \frac{c^3 A_{\rm horizon}}{4G\hbar}$$

⇨ deSitter entropy

Gibbons-Hawking 1977

deSitter braneworld: result

simplest, but non-trivial, exact!



Ryu-Takayanagi formula

$$S_{\rm ent} = {\rm ext}\left[\frac{{\rm Area}(\Gamma)}{4G}\right] \ ?$$

⇒ Ryu-Takayanagi with curvature corrections (Hung, Myers & Smolkin 2011)

in Einstein, Gauss-Bonnet, Lovelock

$$S_{\rm ent} = S_{\rm dS}$$

Hawking, Maldacena & Strominger 2000,
Iwashita, Kobayashi, Shiromizu & Yoshino 2006
Kushihara, Izumi & Shiromizu 2021

2. Review of braneworld

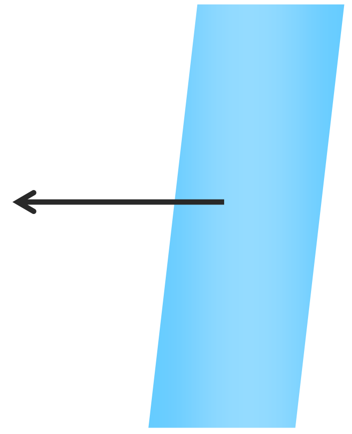
Randall-Sundrum II

Randall & Sundrum 1999

5-dim. bulk spacetime with negative cosmological constant

matter is confined on the brane

warped extra dimensions due to self-gravity of brane



brane: 4-dim. spacetime, tension

4D gravity is recovered

Randall & Sundrum 1999, Garriga & Tanaka 2000,
Giddings, Katz & Randall 2000

correction to Newton potential

$$V \simeq -\frac{GM}{r}\left(1 + O(\ell^2/r^2)\right)$$

$\ell \cdots$ adS curvature length

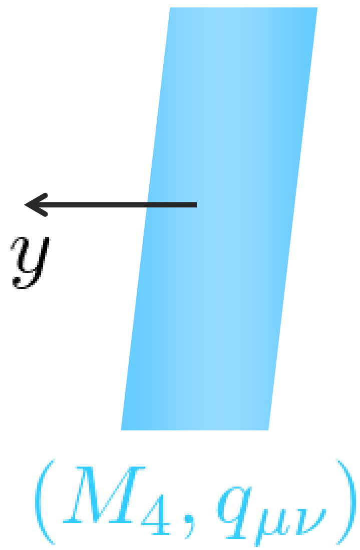
Geometrical projection

Shiromizu, Maeda & Sasaki 2000

From 5 to 4

$$R_{MN} - \frac{1}{2} g_{MN} R = \kappa^2 T_{MN} - \Lambda g_{MN}$$

$$T_{MN} = (-\sigma q_{MN} + \tau_{MN})\delta(y)$$



$${}^{(4)}R_{\mu\nu} - \frac{1}{2} q_{\mu\nu} {}^{(4)}R = -\Lambda_4 q_{\mu\nu} + 8\pi G \tau_{\mu\nu} + \kappa^4 \pi_{\mu\nu} - E_{\mu\nu}$$

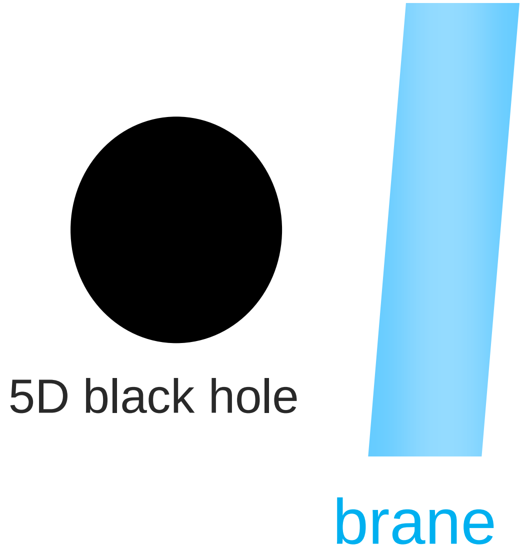
$$\begin{cases} \Lambda_4 = \dfrac{\Lambda}{2} + \dfrac{\kappa^4 \sigma^2}{12} \\ 8\pi G = \dfrac{\kappa^4 \sigma}{6} \\ \pi_{\mu\nu} = -\dfrac{1}{4} \tau_{\mu\alpha} \tau_\nu^\alpha + \dfrac{1}{12} \tau \tau_{\mu\nu} + \dfrac{1}{8} q_{\mu\nu} \tau_{\alpha\beta} \tau^{\alpha\beta} - \dfrac{1}{24} q_{\mu\nu} \tau^2 \\ E_{\mu\nu} = C_{\mu y \nu y} \end{cases}$$

a part of 5-dimensional Weyl tensor

$E_{\mu\nu}$

$$ {}^{(4)}R_{\mu\nu} - \frac{1}{2} q_{\mu\nu} {}^{(4)}R = -\Lambda_4 q_{\mu\nu} + 8\pi G \tau_{\mu\nu} + \kappa^4 \pi_{\mu\nu} - E_{\mu\nu} $$

Binetury, Deffayet, Ellwanger & Langlois 2000,
Ida 2000, Mukohyama 2000,
Mukohyama, Shiromizu &Maeda 2000, …



$E_{\mu\nu}$ ~ holographic 4D radiation

proportional to black hole mass

Braneworld holography

Gubser 2001, Giddings, Katz & Randall 2000,
Hawking, Hertog & Reall 2000, Shiromizu & Ida 2001,
Koyama & Soda 2001, Soda & Kanno 2002,…

$$
{}^{(4)}R = -8\pi G\tau - \frac{\kappa^2}{4}\left(\tau_{\mu\nu}\tau^{\mu\nu} - \frac{1}{3}\tau^2\right)
$$

$$
\simeq -8\pi G\tau - \frac{\ell^2}{4}\left({}^{(4)}R_{\mu\nu}{}^{(4)}R^{\mu\nu} - \frac{1}{3}{}^{(4)}R^2\right)
$$

trace anomaly of CFT

⇒ 4D gravity+matter+holographic CFT

Black hole on brane

Exact solution?

-not discovered in 5D.

only numerical studies

Shiromizu & Shibata 2000, Wiseman 2002&2003,
Kudoh, Tanaka & Nakamura 2003, Kudoh 2004, Tanahashi & Tanaka 2008,
Yoshino 2009, Figueras & Wiseman 2011, Wang & Choptuik 2016

-discovered in 4D.

Emparan, Horowitz & Myers 2000

deSitter on brane

Nihei(1999), Kaloper(1999), Garriga & Sasaki(2000),…

$$ds^2 = dr^2 + (\ell H)^2 \sinh^2(r/\ell)[-dt^2 + H^{-2}\cosh^2(Ht)d\Omega_{n-2}^2]$$

(n-1)-dimensional deSitter spacetime in complete chart

$$= dr^2 + (\ell H)^2 \sinh^2(r/\ell)[-(1-H^2\rho^2)dT^2 + (1-H^2\rho^2)^{-1}d\rho^2 + \rho^2 d\Omega_{n-3}^2]$$

(n-1)-dimensional deSitter spacetime in static chart

$$\Lambda = -\frac{(n-1)(n-2)}{\ell^2}$$ $\ell \cdots$ adS curvature length

$r = r_0$ brane location

$H^{-1} = \ell \sinh(r_0/\ell)$ *H*: expansion rate

$$\frac{n-2}{\ell}\cosh(r_0/\ell) = 4\pi G_n \sigma$$ σ: brane tension

cosmological horizon

deSitter spacetime

$$ds^2 = -(1 - H^2\rho^2)dT^2 + (1 - H^2\rho^2)^{-1}d\rho^2 + \rho^2 d\Omega_2^2$$

$\rho = H^{-1}$ cosmological horizon

cf) Schwarzschild black hole

$$ds^2 = -\left(1 - \frac{2Gm}{r}\right)dt^2 + \left(1 - \frac{2Gm}{r}\right)^{-1}dr^2 + r^2 d\Omega_2^2$$

$r = 2Gm$ event horizon (*m*: mass)

$$S_{\text{Bekenstein-Hawking}} = \frac{A_{\text{horizon}}}{4G} \sim \left(\frac{\text{horizon size}}{l_{\text{pl}}}\right)^2$$

4. Holographic entanglement entropy in deSitter braneworld

Holographic entanglement entropy v.s. deSitter entropy

Braneworld holography

Holographic entanglement entropy in braneworld?

Ryu-Takayanagi (+Jacobson-Myers) formula

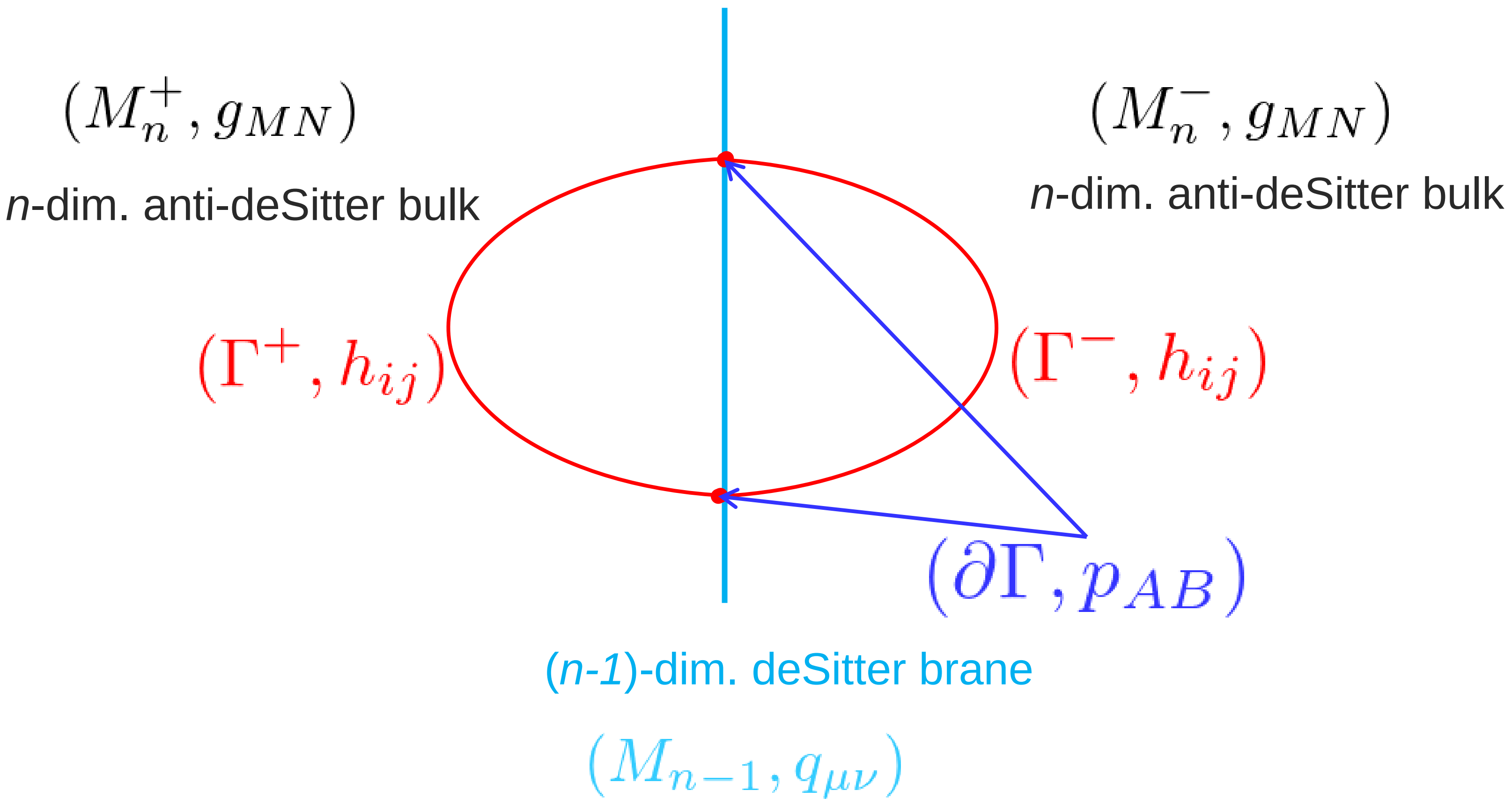
$$S_{\rm ent} = \frac{A_{n-2}}{4G_n}$$ area of minimal surface

deSitter entropy in braneworld

Euclidean path integral I_E in full spacetime

$$S_{\rm dS} = \beta\langle E\rangle - I_E = -I_E$$

Common setup



in Einstein

Hawking, Maldacena & Strominger (2001),
Iwashita, Kobayashi, Shiromizu & Yoshino (2006)

$$S_{\rm dS} = -I_E = \frac{1}{16\pi G_n} \int_{M_n^+ \cup M_n^-} d^n x \sqrt{g}(R - 2\Lambda) + \int_{M_{n-1}} d^{n-1} x \sqrt{q} \left(-\sigma + \frac{[K]^-}{8\pi G_n} \right)$$

$$= \frac{(n-2)\Omega_{n-1}\ell^{n-2}}{4\pi G_n} \int_0^{r_0/\ell} dx \sinh^{n-3} x$$

$$S_{\rm ent} = \frac{A_{\rm min}}{4G_n} = \frac{\Omega_{n-3}\ell^{n-2}}{2G_n} \int_0^{r_0/\ell} dx \sinh^{n-3} x$$

$$\Omega_{n-3} = \Omega_{n-1} \frac{n-2}{2\pi}$$

⇒ $$S_{\rm dS} = S_{\rm ent}$$

in Gauss-Bonnet

higher curvature terms

Maeda & Torii (2004),…

$$S = \frac{1}{16\pi G_n}\int_{M_n^+\cup M_n^-} d^n x\sqrt{-g}\Big(R - 2\Lambda + \frac{\beta\ell^2}{4}\mathcal{L}_{\rm GB}\Big) + \int_{M_{n-1}} d^{n-1}x\sqrt{-q}\Big(-\sigma + \frac{1}{16\pi G_n}[Q]^-\Big)$$

$$\begin{cases} \mathcal{L}_{\rm GB} = R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD} \\ Q = 2K + \beta\ell^2(J - 2^{(n-1)}G_{\mu\nu}K^{\mu\nu}) \end{cases}$$

Hung, Myers & Smolkin (2011)

$$S_{\rm JM} = \frac{1}{4G_n}\int_{\Gamma^+\cup\Gamma^-} d^{n-2}x\sqrt{h}\Big(1 + \frac{\beta\ell^2}{2}{}^{(n-2)}R\Big) + \frac{1}{2G_n}\int_{\partial\Gamma} d^{n-3}x\sqrt{p}\frac{\beta\ell^2}{2}[{}^{(n-3)}k]^-$$

variation

$$\begin{cases} {}^{(n-2)}k - \beta\ell^2\,{}^{(n-2)}G_{ij}\,{}^{(n-2)}k^{ij} = 0 & \text{(in bulk)} \\ \beta\left[{}^{(n-3)}k^{AB} - {}^{(n-3)}kp^{AB}\right]^-\,{}^{(n-2)}k_{AB} = 0 & \text{(on brane)} \end{cases}$$

HEE and deSitter entropy

Kushihara, Izumi & Shiromizu 2021

$$S_{\rm dS} = S_{\rm ent}$$
$$= \frac{(n-2)\ell^{n-2}}{4\pi G_n}\Omega_{n-1}\left(\left[1-\beta\frac{(n-2)(n-3)}{2}\right]\int_0^{r_0/\ell} dx \sinh^{n-3} x \right.$$
$$\left. +\beta(n-3)\sinh^{n-4}(r_0/\ell)\cosh(r_0/\ell)\right)$$

$$S_{\rm dS} = S_{\rm ent}$$

Lovelock setup

$$S = \frac{1}{16\pi G_n}\int_{M_n^+ \cup M_n^-} d^n x \sqrt{-g}\Big(-2\Lambda + \sum_m c_m \mathcal{L}_m\Big) + \int_{M_{n-1}} d^{n-1}x\sqrt{-q}\Big(-\sigma + \sum_m c_m \frac{[Q_m]^-}{16\pi G_n}\Big)$$

$$\begin{cases} \mathcal{L}_m = \dfrac{1}{2^m} g^{K_1 L_1 \ldots K_m L_m}_{M_1 N_1 \ldots M_m N_m} R_{K_1 L_1}{}^{M_1 N_1} \ldots R_{K_m L_m}{}^{M_m N_m} \\ Q_m = \dfrac{4m}{2^m}\displaystyle\int_0^1 ds\, q^{\alpha_1\beta_1\ldots\alpha_{m-1}\beta_{m-1}\alpha_m}_{\mu_1\nu_1\ldots\mu_{m-1}\nu_{m-1}\mu_m} \left({}^{(n-1)}R_{\alpha_1\beta_1}{}^{\mu_1\nu_1} - 2s^2 K^{\mu_1}_{\alpha_1} K^{\nu_1}_{\beta_1}\right) \end{cases}$$

$$\ldots \left({}^{(n-1)}R_{\alpha_{m-1}\beta_{m-1}}{}^{\mu_{m-1}\nu_{m-1}} - 2s^2 K^{\mu_{m-1}}_{\alpha_{m-1}} K^{\nu_{m-1}}_{\beta_{m-1}}\right) K^{\mu_m}_{\alpha_m}$$

$$= \frac{4m}{2^m} q^{\alpha_1\beta_1\ldots\alpha_{m-1}\beta_{m-1}\alpha_m}_{\mu_1\nu_1\ldots\mu_{m-1}\nu_{m-1}\mu_m} \sum_{k=0}^{m-1} \binom{m-1}{k} \frac{(-2)^k}{2k+1} K^{\mu_1}_{\alpha_1} K^{\nu_1}_{\beta_1} \ldots K^{\mu_k}_{\alpha_k} K^{\nu_k}_{\beta_k} K^{\mu_m}_{\alpha_m}$$

$${}^{(n-1)}R_{\alpha_{k+1}\beta_{k+1}}{}^{\mu_{k+1}\nu_{k+1}} \ldots {}^{(n-1)}R_{\alpha_{m-1}\beta_{m-1}}{}^{\mu_{m-1}\nu_{m-1}}$$

HEE formula in Lovelock

$$S_{\rm JM} = \frac{1}{4G_n}\int_{\Gamma^+\cup\Gamma^-} d^{n-2}x\sqrt{h}\sum_m c_m\tilde{\mathcal{L}}_m + \frac{1}{4G_n}\int_{\partial\Gamma} d^{n-3}x\sqrt{p}\sum_m c_m[\tilde{Q}_m]^-$$

Hung, Myers & Smolkin (2011)

$$\begin{cases} \tilde{\mathcal{L}}_m = \dfrac{m}{2^{m-1}} h^{i_1 j_1 \ldots i_{m-1} j_{m-1}}_{k_1 l_1 \ldots k_{m-1} l_{m-1}} \, {}^{(n-2)}R_{i_1 j_1}{}^{k_1 l_1} \ldots {}^{(n-2)}R_{i_{m-1} j_{m-1}}{}^{k_{m-1} l_{m-1}} \\ \tilde{Q}_m = \dfrac{4m(m-1)}{2^{m-1}} \displaystyle\int_0^1 ds\, p^{A_1 B_1 \ldots A_{m-2} B_{m-2} A_{m-1}}_{C_1 D_1 \ldots C_{m-2} D_{m-2} C_{m-1}} \left({}^{(n-3)}R_{A_1 B_1}{}^{C_1 D_1} - 2s^2\, {}^{(n-3)}k^{C_1}_{A_1}\, {}^{(n-3)}k^{D_1}_{B_1}\right) \end{cases}$$

$$\ldots \left({}^{(n-2)}R_{A_{m-2} B_{m-2}}{}^{C_{m-2} D_{m-2}} - 2s^2\, {}^{(n-3)}k^{C_{m-2}}_{A_{m-2}}\, {}^{(n-3)}k^{D_{m-2}}_{B_{m-2}}\right) {}^{(n-3)}k^{C_{m-1}}_{A_{m-1}}$$

HEE and deSitter entropy

Kushihara, Izumi & Shiromizu 2021

$$\begin{aligned} S_{\mathrm{dS}} &= S_{\mathrm{ent}} \\ &= \frac{\Omega_{n-1}}{4\pi G_n} \sum_m m c_m \ell^{n-2m} \frac{(n-2)!}{(n-2m)!} \Bigg((-1)^m (n-1) \int_0^{r_0/\ell} dx \sinh^{n-1} x \\ &\qquad + (2m-1) \cosh(r_0/\ell) \sinh^{n-2m}(r_0/\ell) \int_0^1 ds \left(1 - s^2 \cosh^2(r_0/\ell)\right)^{m-1} \Bigg) \end{aligned}$$

$$S_{\mathrm{dS}} = S_{\mathrm{ent}}$$

5. Summary and issues

Summary

In deSitter braneworld

Holographic entanglement entropy
= deSitter entropy

Issues

- General formulation for holographic entanglement entropy in braneworld?
- Extension to co-dimension 2 braneworld?
- Revisit braneworld black hole?
- …

⇨ Hints for quantum gravity