Quantitative analysis of many-body localization in Sachdev-Ye-Kitaev type models

YITP workshop "Recent progress in theoretical physics based on quantum information theory"

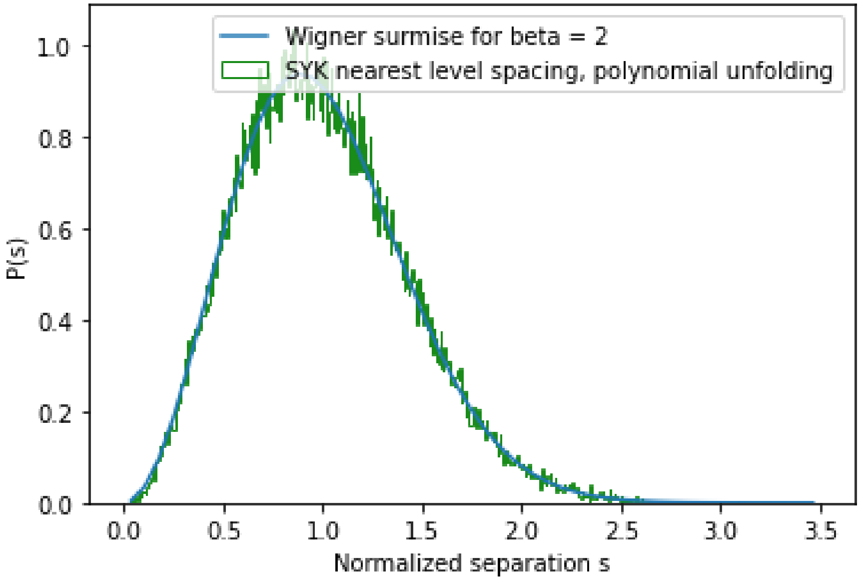
1 March 2021

Masaki Tezuka

(Department of Physics, Kyoto University)

Plan

- Sachdev-Ye-Kitaev model $\hat{H} = \sum_{1 \le a<b<c<d \le N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$
  - Maximally chaotic quantum mechanical model
- SYK4+2 $\hat{H} = \sum_{1 \le a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + i \sum_{1 \le a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b$
  - Departure from chaotic behavior
- Quantitative analysis of Fock-space localization
  - Many-body transition point
  - Inverse participation ratio
  - Entanglement entropy



$$
\begin{aligned}
\mathcal{H} = \;& \chi_0\chi_5\chi_{19}\chi_{27} + \chi_0\chi_6\chi_{21}\chi_{23} - \chi_0\chi_9\chi_{14}\chi_{24} - \chi_0\chi_{14}\chi_{18}\chi_{30} - \chi_0\chi_{14}\chi_{20}\chi_{25} - \chi_1\chi_2\chi_{16}\chi_{22} \\
& + \chi_1\chi_{18}\chi_{22}\chi_{23} + \chi_2\chi_4\chi_5\chi_{15} + \chi_2\chi_{13}\chi_{16}\chi_{21} + \chi_2\chi_{14}\chi_{19}\chi_{24} + \chi_2\chi_{20}\chi_{27}\chi_{33} + \chi_2\chi_{22}\chi_{31}\chi_{32} \\
& + \chi_3\chi_4\chi_5\chi_{29} - \chi_3\chi_8\chi_{14}\chi_{28} - \chi_3\chi_8\chi_{29}\chi_{31} + \chi_3\chi_{21}\chi_{26}\chi_{29} - \chi_3\chi_{22}\chi_{25}\chi_{33} + \chi_4\chi_7\chi_{13}\chi_{30} \\
& - \chi_4\chi_9\chi_{14}\chi_{17} - \chi_5\chi_6\chi_{17}\chi_{29} + \chi_5\chi_{12}\chi_{29}\chi_{31} - \chi_5\chi_{13}\chi_{19}\chi_{24} - \chi_5\chi_{14}\chi_{22}\chi_{31} - \chi_5\chi_{17}\chi_{31}\chi_{33} \\
& + \chi_5\chi_{20}\chi_{30}\chi_{31} - \chi_6\chi_{23}\chi_{27}\chi_{29} + \chi_7\chi_{12}\chi_{13}\chi_{18} + \chi_8\chi_{10}\chi_{24}\chi_{28} - \chi_9\chi_{12}\chi_{20}\chi_{33} + \chi_{10}\chi_{11}\chi_{28}\chi_{32} \\
& + \chi_{10}\chi_{21}\chi_{27}\chi_{29} - \chi_{12}\chi_{20}\chi_{22}\chi_{24} + \chi_{14}\chi_{17}\chi_{26}\chi_{27} - \chi_{15}\chi_{24}\chi_{26}\chi_{27} - \chi_{16}\chi_{18}\chi_{23}\chi_{27} - \chi_{18}\chi_{24}\chi_{30}\chi_{32}
\end{aligned}
$$

Publications and collaborators

- Sachdev-Ye-Kitaev model
  - Proposal for experiment: PTEP 2017, 083I01 and arXiv:1709.07189
    - with Ippei Danshita and Masanori Hanada
  - Black Holes and Random Matrices: JHEP 1705(2017)118
    - with J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, and A. Streicher
- SYK4+2
  - Chaotic-integrable transition: PRL **120**, 241603 (2018)
    - with Antonio M. García-García, Bruno Loureiro, and Aurelio Romero-Bermúdez
  - Characterization of quantum chaos: JHEP 1904(2019)082 and Phys. Rev. E **102**, 022213 (2020)
    - with Hrant Gharibyan, M. Hanada, and Brian Swingle
  - Related setups:
    - [short-range interactions] Phys. Rev. B **99**, 054202 (2019) with A. M. García-García
    - Phys. Lett. B **795**, 230 (2019) and J. Phys. A **54**, 095401 (2021) with Pak Hang Chris Lau, Chen-Te Ma, and Jeff Murugan
- Quantitative analysis of Fock-space localization in SYK4+2
  - Many-body transition point and inverse participation ratio
    - Phys. Rev. Research **3**, 013023 (2021) with Felipe Monteiro, Tobias Micklitz, and Alexander Altland
  - Entanglement entropy
    - arXiv:2012.07884 with F. Monteiro, A. Altland, David A. Huse, and T. Micklitz

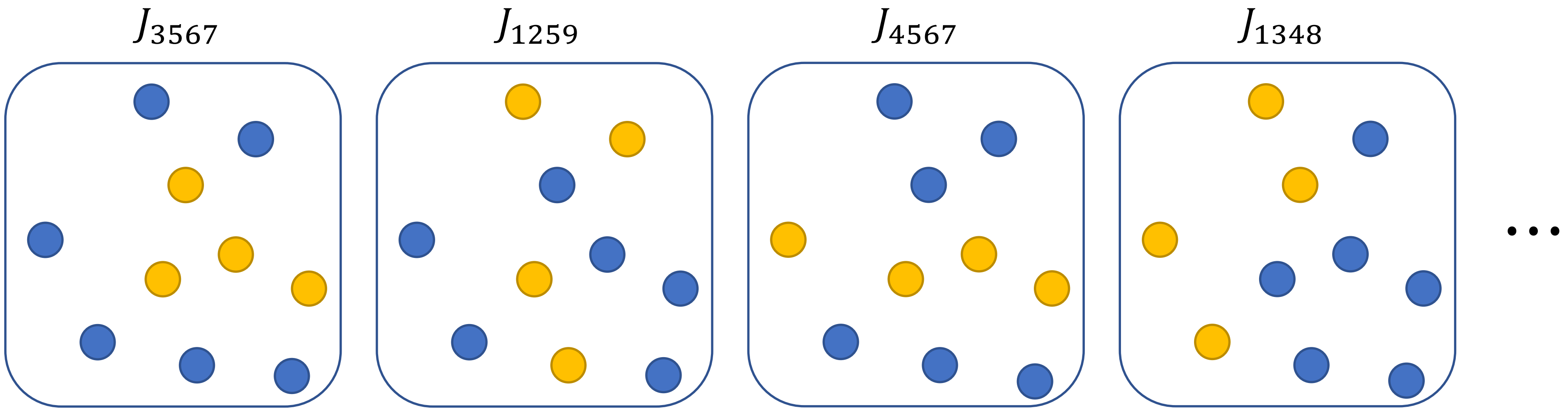
Sachdev-Ye-Kitaev (SYK) model

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \leq a < b < c < d \leq N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

[A. Kitaev: talks at KITP (Feb 12, Apr 7 and May 27, 2015)]

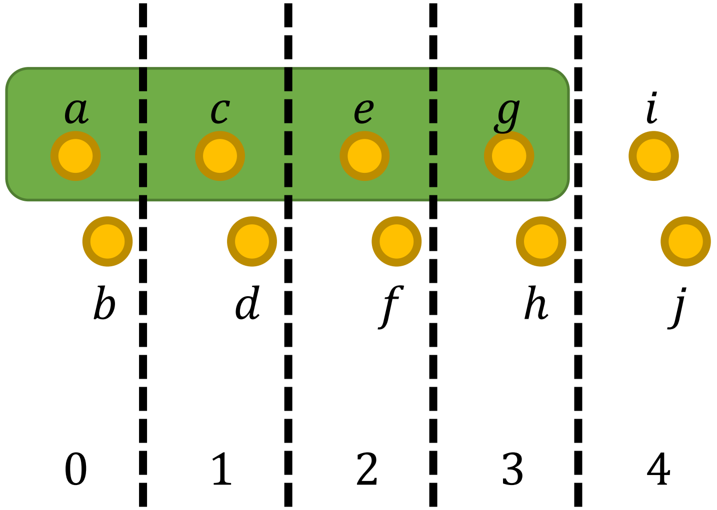
$\hat{\chi}_{a=1,2,\dots,N}$: N Majorana fermions ($\{\hat{\chi}_a, \hat{\chi}_b\} = \delta_{ab}$)

J_{abcd} : independent Gaussian random couplings ($\overline{J_{abcd}{}^2} = J^2(=1)$, $\overline{J_{abcd}} = 0$)



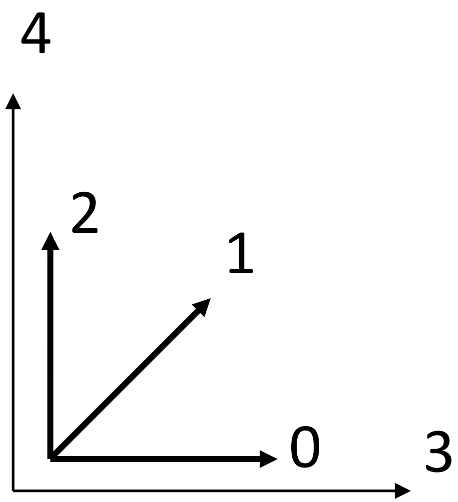
One term of the 10-Majorana fermion $\mathrm{SYK}_{q=4}$

$\chi_a \chi_c \chi_e \chi_g$

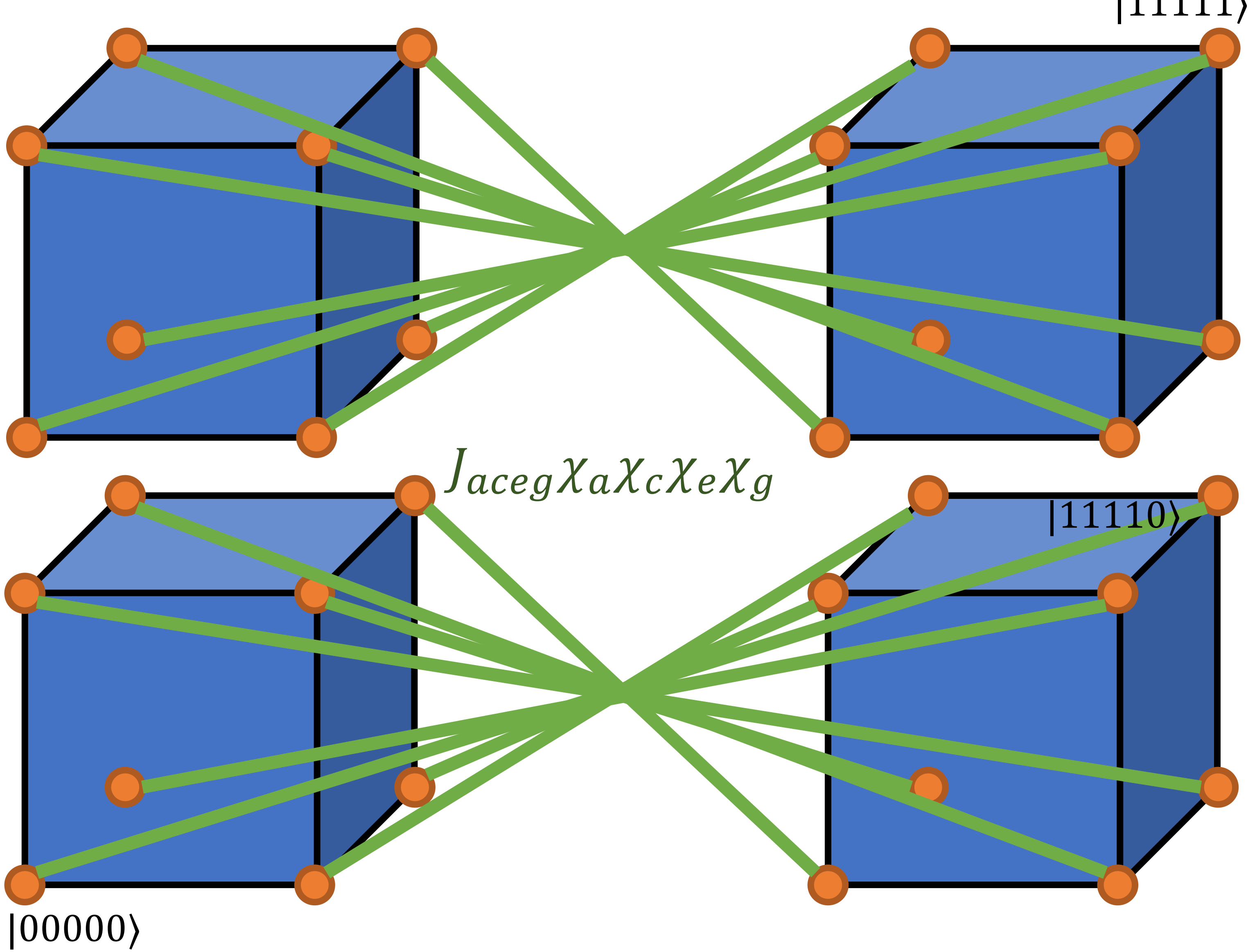


5 qubits

$\binom{10}{4} = 210$ terms



32-state Fock space: 5-dimensional hypercube



Sachdev-Ye-Kitaev model

N Majorana- or Dirac- fermions randomly coupled to each other

[Majorana version]

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a < b < c < d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

[A. Kitaev: talks at KITP (2015)]

[Dirac version]

$$\hat{H} = \frac{1}{(2N)^{3/2}} \sum_{ij;kl} J_{ij;kl} \hat{c}_i{}^{\dagger} \hat{c}_j{}^{\dagger} \hat{c}_k \hat{c}_l$$

[A. Kitaev's talk]
[S. Sachdev: PRX **5**, 041025 (2015)]

cf. SY model [S. Sachdev and J. Ye, 1993]

Studied for long time in the nuclear theory context

- [French and Wong, Phys. Lett. B **33**, 449 (1970)]
- [Bohigas and Flores, Phys. Lett. B **34**, 261 (1971)]

"Two-body Random Ensemble"

N: number of fermions

SYK: Solvable in the $N \gg 1$ limit (after sample average $\langle \cdots \rangle_{\{J\}}$)

Non-perturbative Hamiltonian = 0,

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a<b<c<d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

as perturbation

Free two-point function

$$G_{0,ij}(t) = -\langle \mathrm{T} \psi_i(t) \psi_j(0) \rangle = -\frac{1}{2} \mathrm{sgn}(t) \delta_{ij}$$

$\langle {J_{abcd}}^2 \rangle_{\{J\}} = J^2$, Gaussian distribution

$\langle J_{abcd} J_{abce} \rangle_{\{J\}} = 0$ if $d \neq e$ ➔ Most diagrams average to zero

Only "melon-type" diagrams survive sample averaging

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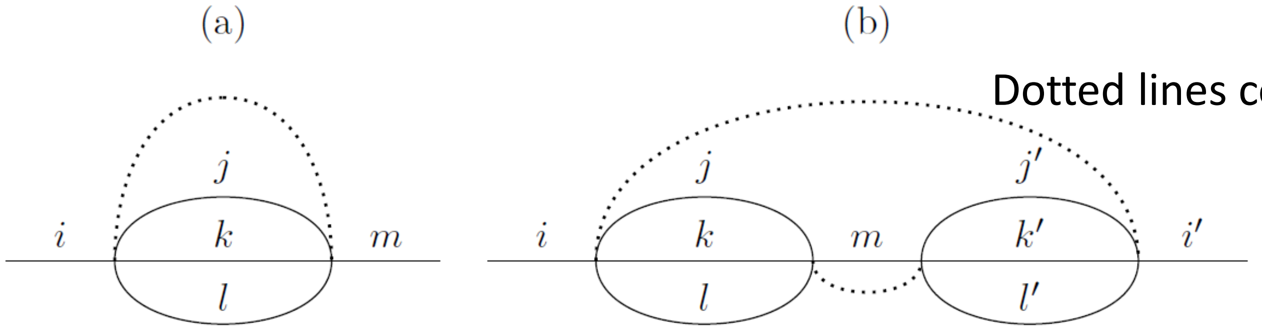


Dotted lines connect same couplings

$O(1)$ melon

$O(N^{-2})$ not melon

Feynman diagrams

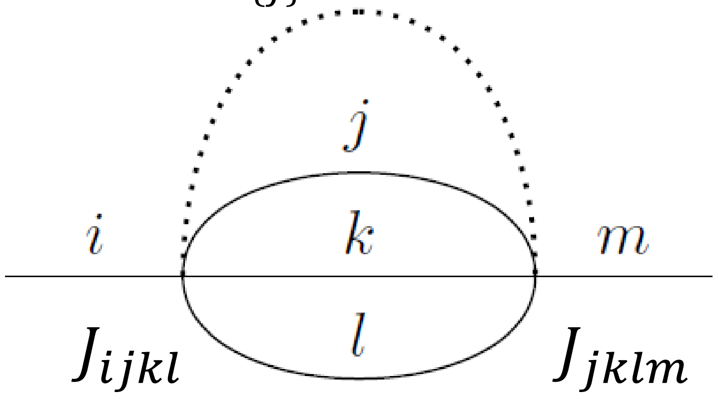
[J. Polchinski and V. Rosenhaus, JHEP 1604 (2016) 001]
[J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)]

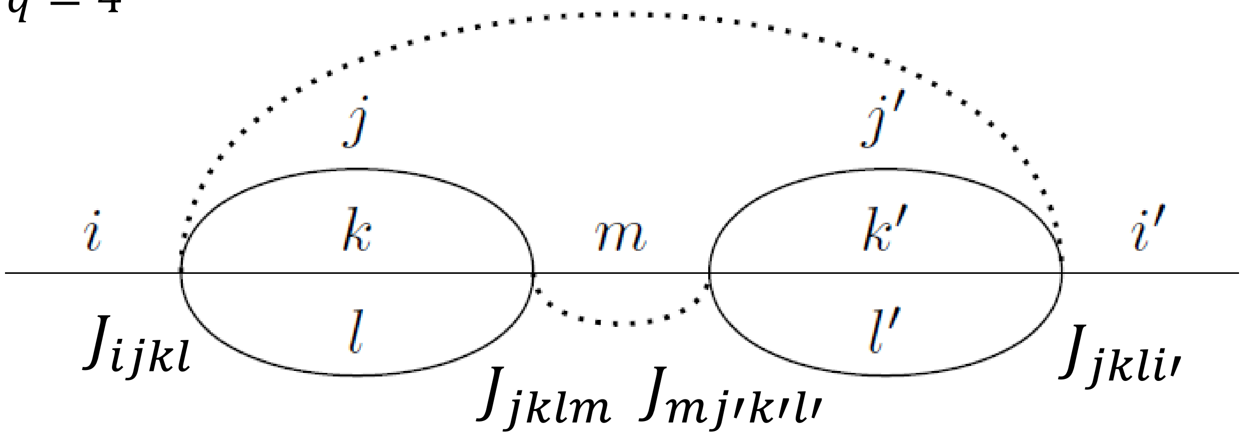
$$\hat{H}_{\mathrm{SYK4}} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a < b < c < d \le N} J_{abcd} \underbrace{\hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d}_{q\,=\,4}$$

$$\{\hat{\chi}_a, \hat{\chi}_b\} = \delta_{ab}$$

$$\langle J_{abcd} \rangle^2 = J^2 = 1$$

Sample average $\langle \cdots \rangle_{\{J\}}$





$$\sum_{jkl} \langle J_{ijkl} J_{jklm} \rangle_{\{J\}} = \frac{N^3}{3!} \delta_{im}$$

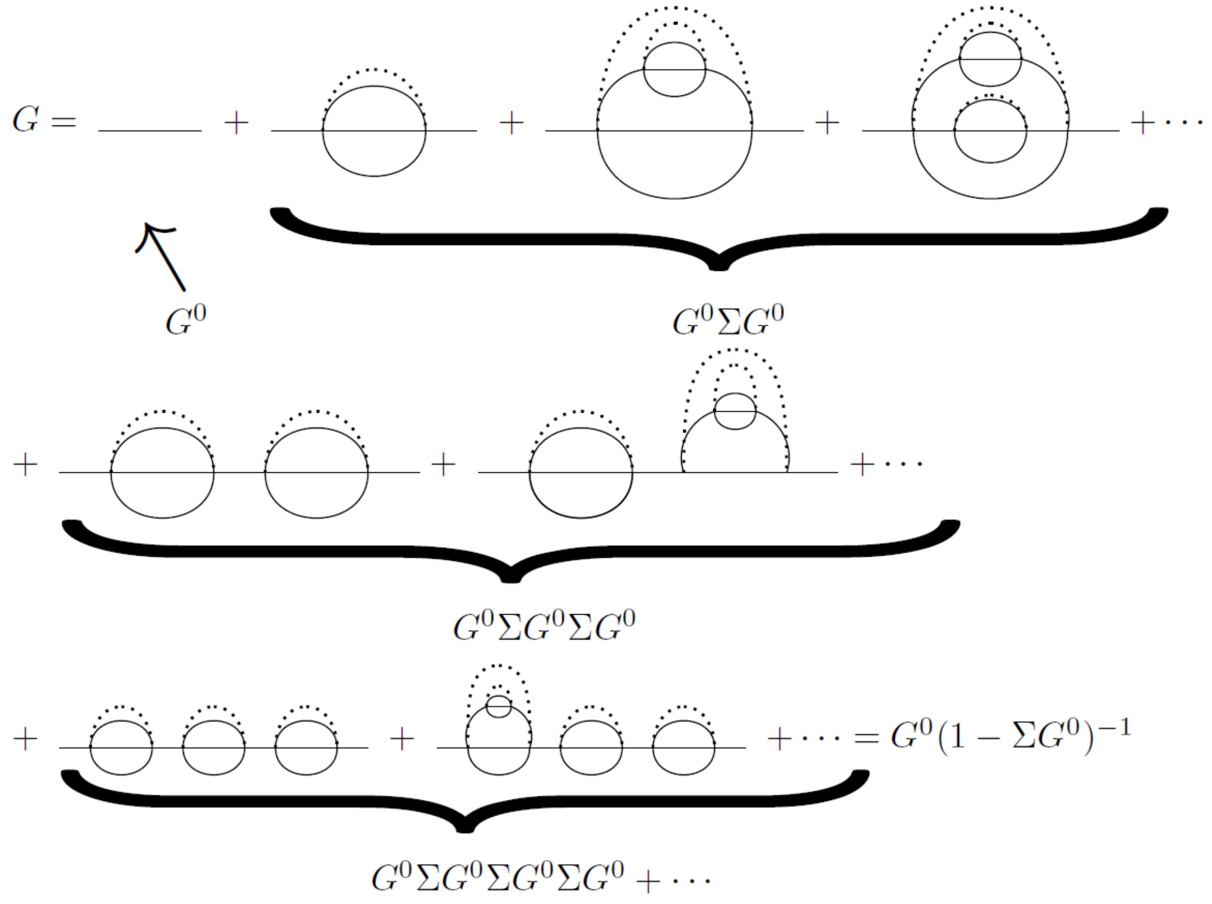
$$\sum_{m \neq i} \sum_{jklj'k'l'} \langle J_{ijkl} J_{jklm} J_{mj'k'l'} J_{j'k'l'i'} \rangle_{\{J\}} \propto N^4 \delta_{ii'}$$

➡ $O(N^0)$ contribution

➡ $O(N^{-2})$ contribution

Large-*N*: “Melon diagrams” dominate

Dominant diagrams in the $N \gg 1$ limit

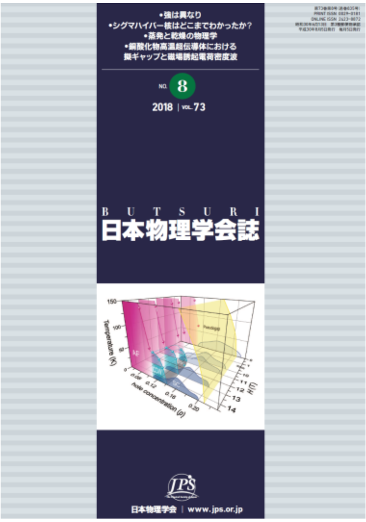


[Sachdev and Ye 1993],
[Parcollet and Georges 1999], …

$$G(1-\Sigma G_0) = G_0$$

$$G^{-1} = {G_0}^{-1} - \Sigma$$

$$G(i\omega)^{-1} = i\omega - \Sigma(i\omega)$$



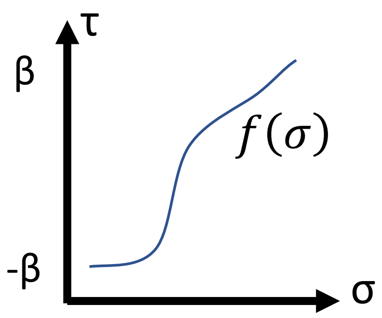
↑ Figure from [I. Danshita, M. Tezuka, and M. Hanada: Butsuri **73**(8), 569 (2018)]

Reparametrization degrees of freedom

$G(i\omega)^{-1} = i\omega - \Sigma(i\omega)$ Low energy ($\omega, T \ll J$): ignore $i\omega$

$$\int d\tau_2 G(\tau_1,\tau_2)\tilde{\Sigma}(\tau_2,\tau_3) = -\delta(\tau_1-\tau_3),$$

$$\tilde{\Sigma}(\tau_1,\tau_2) = -J^2[G(\tau_1,\tau_2)]^2 G(\tau_2,\tau_1)$$



Invariant under imaginary time reparametrization

$$\tau = f(\sigma),$$

$$G(\tau_1,\tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4}\frac{g(\sigma_1)}{g(\sigma_2)}G(\sigma_1,\sigma_2),$$

$$\tilde{\Sigma}(\tau_1,\tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-3/4}\frac{g(\sigma_1)}{g(\sigma_2)}\tilde{\Sigma}(\sigma_1,\sigma_2),$$

with f and g being arbitrary monotonic, differentiable functions.

emergent conformal gauge invariance

[S. Sachdev, Phys. Rev. X **5**, 041025 (2015)]

"System nearly invariant under a full reparametrization (Virasoro) symmetry, $NCFT_1$"

[J. Maldacena and D. Stanford, Phys. Rev. D **94**, 106002 (2016)]
Study of the Goldstone modes: *e.g.* [D. Bagrets, A. Altland, and A. Kamenev, Nucl. Phys. B **911**, 191 (2016)]

Saddle point solution

Obtain large-*N* saddle point solution
(in replica formalism; assume replica symmetry)

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2} \quad , \quad \Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}$$

Not invariant under arbitrary reparametrization,
but invariant under

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

Symmetry broken to $SL(2, R)$.
cf. isometry group of AdS_2
[see e.g. A. Strominger, hep-th/9809027]

as expected for a theory dual to 1+1d gravity

Jackiw-Teitelboim (JT) gravity: 1+1d dilaton gravity
near the horizon of a near-extremal black hole

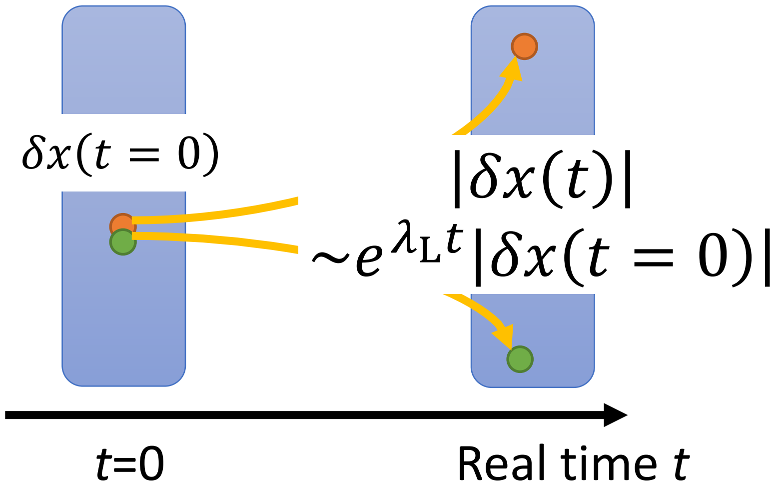
S. Sachdev, Phys. Rev. X **5**, 041025 (2015);
J. Maldacena and D. Stanford, Phys. Rev. D **94**, 106002 (2016)
Antal Jevicki, Kenta Suzuki, and Junggi Yoon, JHEP07(2016)007

Definition of Lyapunov exponent using out-of-time-order correlators (OTOC)

$$F(t) = \langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\rangle \quad W(t) = \mathrm{e}^{iHt} W \mathrm{e}^{-iHt}$$

Classical chaos:

Infinitesimally different initial coords



λ_{L}: Lyapunov exponent

$$\left(\frac{\partial x(t)}{\partial x(0)}\right)^2 = \{x(t), p(0)\}_{\mathrm{PB}}{}^2 \rightarrow e^{2\lambda_{\mathrm{L}} t}$$

Quantum dynamics:

$$C_T(t) = \langle [\hat{x}(t), \hat{p}(0)]^2 \rangle$$

For operators V and W, consider

$$C(t) = \langle |[W(t), V(t=0)]|^2 \rangle = \langle W^\dagger(t) V^\dagger(0) W(t) V(0) \rangle + \cdots$$

[Wiener 1938][Larkin & Ovchinnikov 1969]

OTOC $\sim e^{2\lambda_{\mathrm{L}} t}$ at long times, $\lambda_{\mathrm{L}} > 0$: chaotic

“Black holes are fastest quantum scramblers”

[P. Hayden and J. Preskill 2007] [Y. Sekino and L. Susskind 2008]

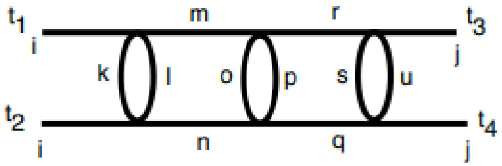
[Shenker and Stanford 2014]

$\lambda_{\mathrm{L}} \leq 2\pi k_{\mathrm{B}} T/\hbar$ (chaos bound)

[J. Maldacena, S. H. Shenker, and D. Stanford, JHEP08(2016)106]

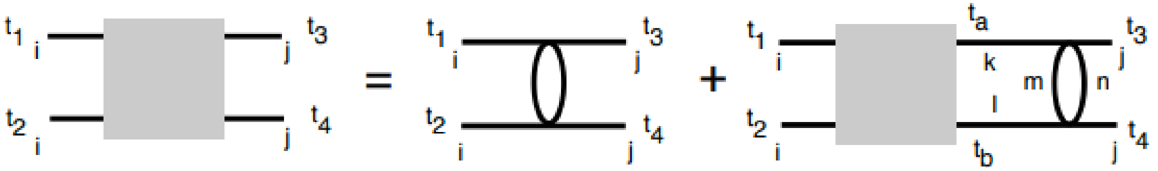
Out-of-time-ordered correlators (OTOCs)

$$\langle \hat{\chi}_i(t_1)\hat{\chi}_i(t_2)\hat{\chi}_j(t_3)\hat{\chi}_j(t_4)\rangle$$

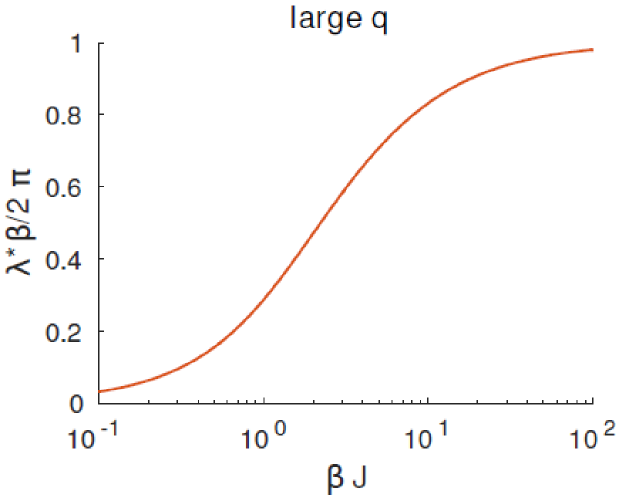


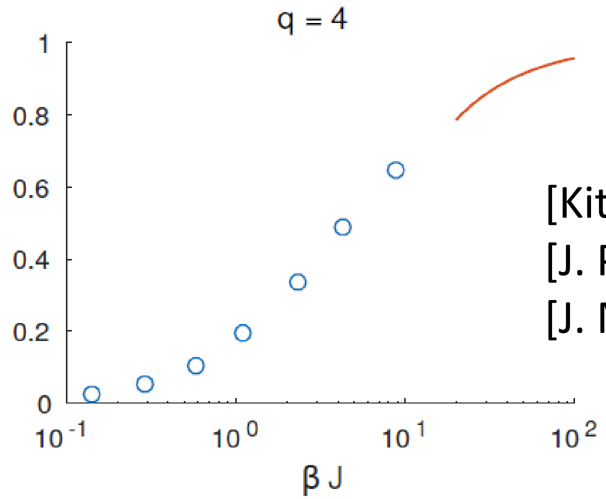
(a)

Regularized OTOC can be calculated for large-*N* SYK model, satisfies the chaos bound $\lambda_{\mathrm{L}} = 2\pi k_{\mathrm{B}}T/\hbar$ at low *T* limit



$$\Gamma(t_1,t_2,t_3,t_4) = \Gamma_0(t_1,t_2,t_3,t_4) + \int dt_a dt_b\, \Gamma(t_1,t_2,t_a,t_b)K(t_a,t_b,t_3,t_4)$$



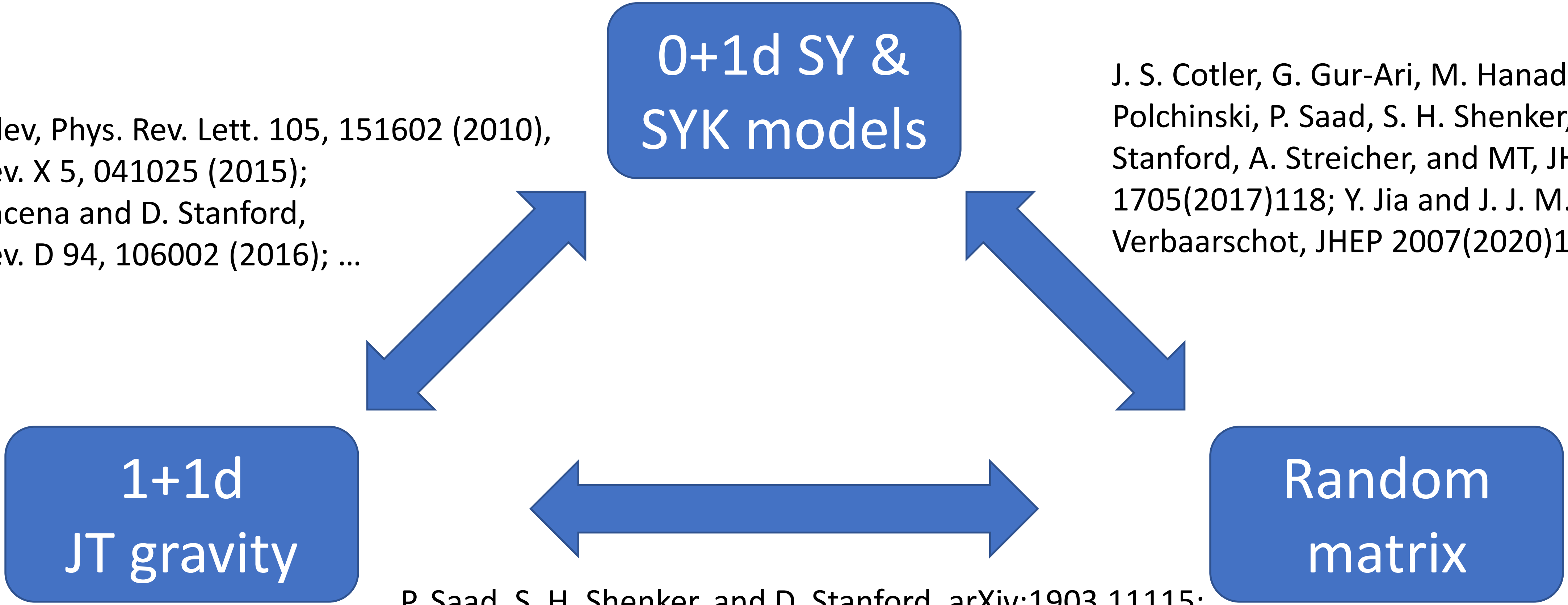


[Kitaev's talks]

[J. Polchinski and V. Rosenhaus, JHEP 1604 (2016) 001]

[J. Maldacena and D. Stanford, Phys. Rev. D **94**, 106002 (2016)]

Maximally chaotic systems



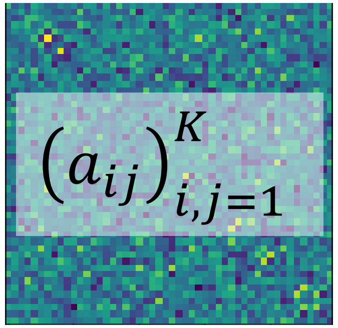
S. Sachdev, Phys. Rev. Lett. 105, 151602 (2010), Phys. Rev. X 5, 041025 (2015); J. Maldacena and D. Stanford, Phys. Rev. D 94, 106002 (2016); …

J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and MT, JHEP 1705(2017)118; Y. Jia and J. J. M. Verbaarschot, JHEP 2007(2020)193; …

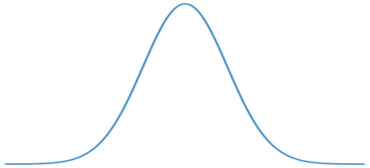
P. Saad, S. H. Shenker, and D. Stanford, arXiv:1903.11115; D. Stanford and E. Witten, arXiv:1907.03363; …

Gaussian random matrices

[Fidkowski and Kitaev 2010]
[You, Ludwig, and Xu 2017]



$a_{ij} = a_{ji}^*$

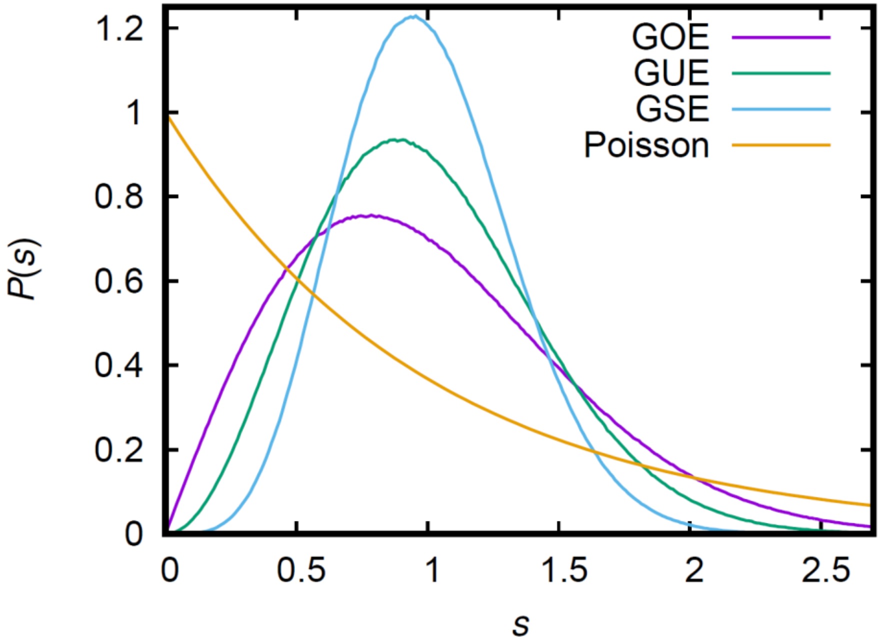


Gaussian distribution

Density $\propto e^{-\frac{\beta K}{4}\mathrm{Tr}H^2} = \exp\left(-\frac{\beta K}{4}\sum_{i,j}^{K}|a_{ij}|^2\right)$

Real (β=1): Gaussian Orthogonal Ensemble (GOE)
Complex (β=2): G. Unitary E. (GUE)
Quaternion (β=4): G. Symplectic E. (GSE)

Corresponds to Majorana SYK4 with
$N \equiv 0 \pmod 8$
$N \equiv 2, 6 \pmod 8$
$N \equiv 4 \pmod 8$



Joint distribution function for eigenvalues $\{e_j\}$

Level repulsion

$$p(e_1, e_2, \dots, e_K) \propto \prod_{1\le i<j\le K} |e_i - e_j|^{\beta} \prod_{i=1}^{K} e^{-\beta K e_i^2/4}$$

- $P(s)$: Distribution of normalized level separation $s_j = \frac{e_{j+1}-e_j}{\Delta(\bar{e})}$

  GOE/GUE/GSE: $P(s) \propto s^{\beta}$ at small s, has e^{-s^2} tail
  Uncorrelated: $P(s) = e^{-s}$ (Poisson distribution)

- $\langle r \rangle$: Average of neighboring gap ratio

$$r = \frac{\min(e_{i+1} - e_i\ ,\ \ e_{i+2} - e_{i+1})}{\max(e_{i+1} - e_i\ ,\ \ e_{i+2} - e_{i+1})}$$

	Uncorrelated	GOE	GUE	GSE
$\langle r \rangle$	2log 2 – 1 = 0.38629…	0.5307(1)	0.5996(1)	0.6744(1)

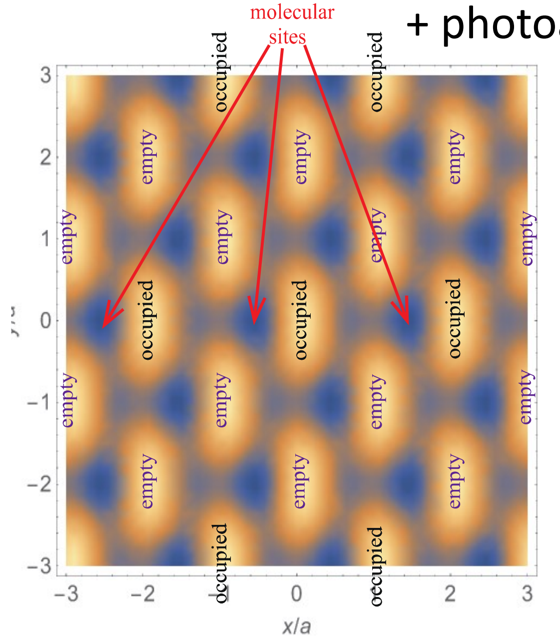
[Y. Y. Atas *et al.* PRL 2013]

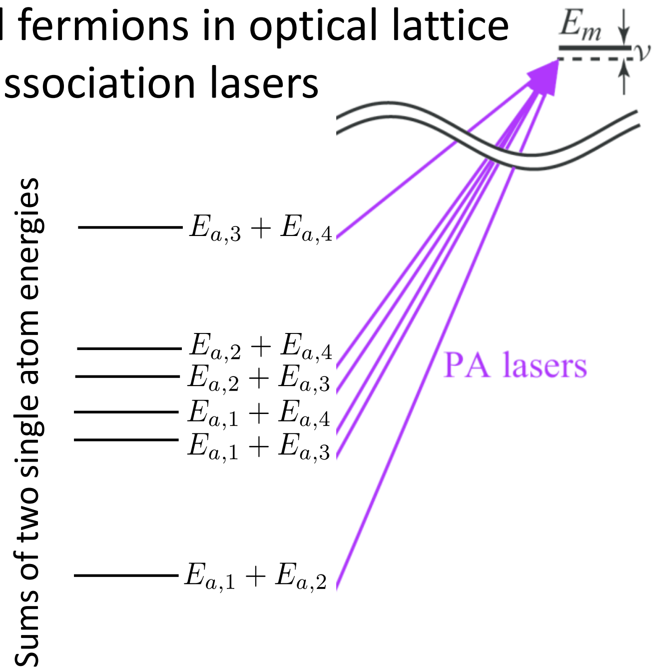
➔ SYK model: level correlation indistinguishable from corresponding Gaussian ensemble

Proposals for experimental realization

[I. Danshita, M. Hanada, MT: PTEP **2017**, 083I01 (2017)]

Ultracold fermions in optical lattice
+ photoassociation lasers





s: molecular levels

$$\hat{H}_{\rm m} = \sum_{s=1}^{n_s} \left\{ \nu_s \hat{m}_s^\dagger \hat{m}_s + \sum_{i,j} g_{s,ij} \left(\hat{m}_s^\dagger \hat{c}_i \hat{c}_j - \hat{m}_s \hat{c}_i^\dagger \hat{c}_j^\dagger \right) \right\}.$$

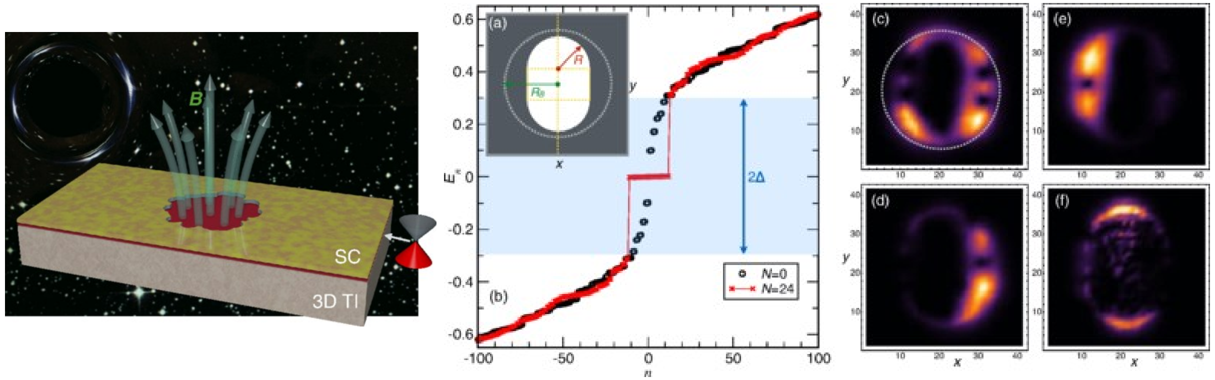
$|\nu_s| \gg |g_{s,ij}|$

$$\hat{H}_{\rm eff} = \sum_{s,i,j,k,l} \frac{g_{s,ij} g_{s,kl}}{\nu_s} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l.$$

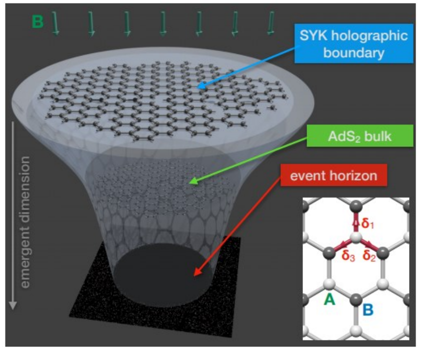
Quantum circuits [L. García-Álvarez et al., PRL 2017]

Majorana wire array [Chew, Essin, and Alicea, PRB 2017 (R)]

N quanta of magnetic flux through a nanoscale hole
[D. I. Pikulin and M. Franz, PRX **7**, 031006 (2017)]



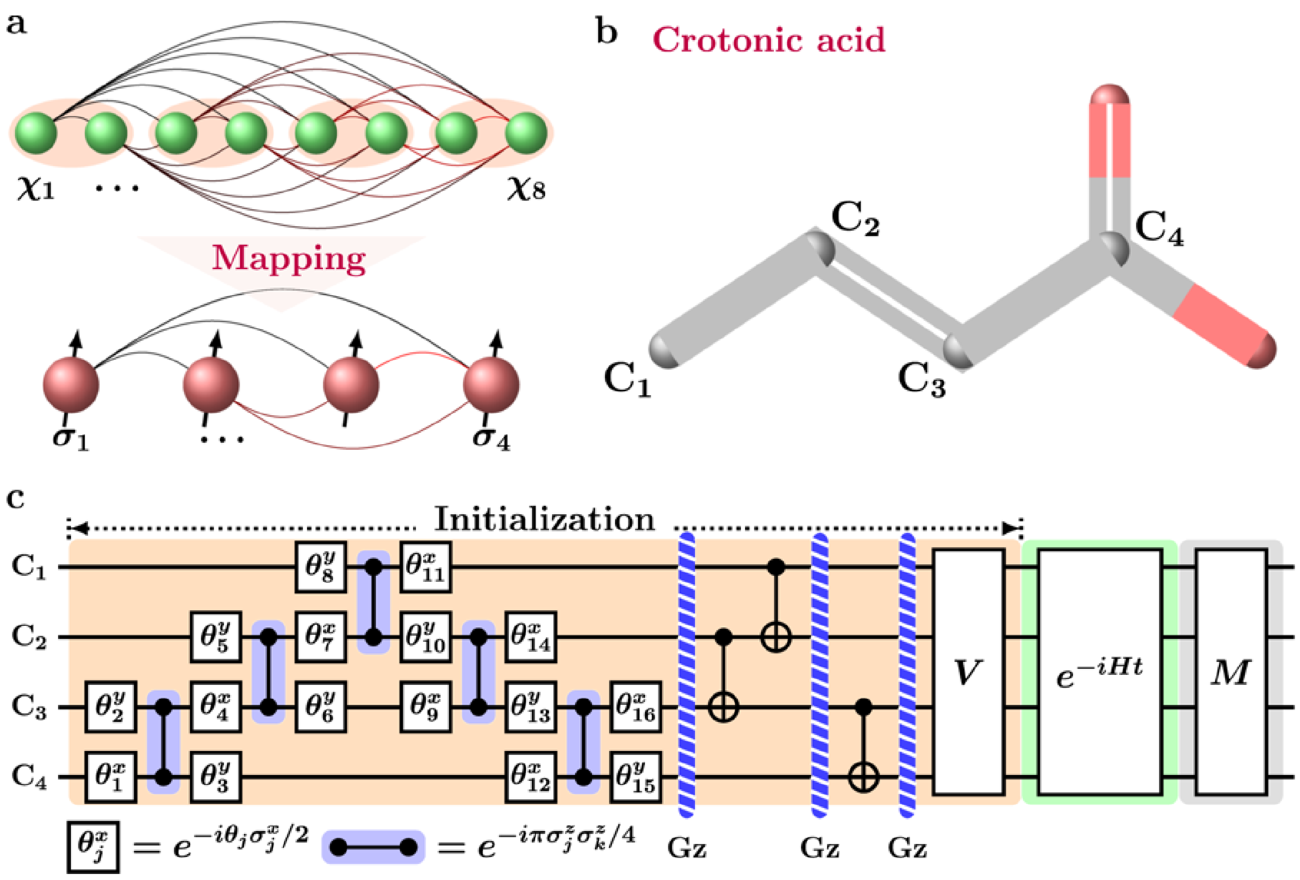
Graphene flake with an irregular boundary in magnetic field
[A. Chen et al., PRL **121**, 036403 (2018)]



Review: M. Franz and M. Rozali, “Mimicking black hole event horizons in atomic and solid-state systems”, Nature Reviews Materials **3**, 491 (2018)

NMR experiment for the SYK model

"Quantum simulation of the non-fermi-liquid state of Sachdev-Ye-Kitaev model"
Zhihuang Luo, Yi-Zhuang You, Jun Li, Chao-Ming Jian, Dawei Lu, Cenke Xu, Bei Zeng and Raymond Laflamme,
npj Quantum Information **5**, 53 (2019)

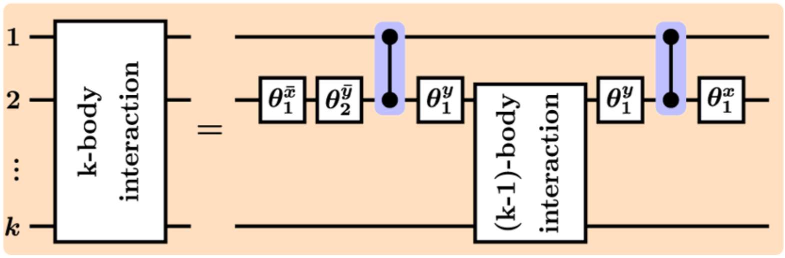


$$H = \frac{J_{ijkl}}{4!}\chi_i\chi_j\chi_k\chi_l + \frac{\mu}{4}C_{ij}C_{kl}\chi_i\chi_j\chi_k\chi_l$$

$$\chi_{2i-1} = \frac{1}{\sqrt{2}}\sigma_x^1\sigma_x^2\cdots\sigma_x^{i-1}\sigma_z^i,\ \chi_{2i} = \frac{1}{\sqrt{2}}\sigma_x^1\sigma_x^2\cdots\sigma_x^{i-1}\sigma_y^i.$$

$$H = \sum_{s=1}^{70} H_s = \sum_{s=1}^{70} a^s_{ijkl}\sigma^1_{\alpha_i}\sigma^2_{\alpha_j}\sigma^3_{\alpha_k}\sigma^4_{\alpha_l}$$

$$e^{-iH\tau} = \left(\prod_{s=1}^{70} e^{-iH_s\tau/n}\right)^n + \sum_{s<s'}\frac{[H_s, H_{s'}]\tau^2}{2n} + O(|a|^3\tau^3/n^2),$$



A. M. García-García, A. Romero-Bermúdez, B. Loureiro, and MT, Phys. Rev. Lett. **120**, 241603 (2018)
also see: reply (arXiv:2007.06121) in press to comment (J. Kim and X. Cao, arXiv:2004.05313).

SYK_{4+2}

Q.: Minimum requirements for chaotic behavior? (→ gravity interpretation?)
Study a simple model with analytical + numerical methods

$$\hat{H} = \underbrace{\sum_{1\le a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d}_{\mathrm{SYK}_4} + \underbrace{i\sum_{1\le a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b}_{\mathrm{SYK}_2}$$

Gaussian random couplings
J_{abcd}: average 0, standard deviation $\frac{\sqrt{6}J}{N^{3/2}}$
K_{ab}: average 0, standard deviation $\frac{K}{\sqrt{N}}$

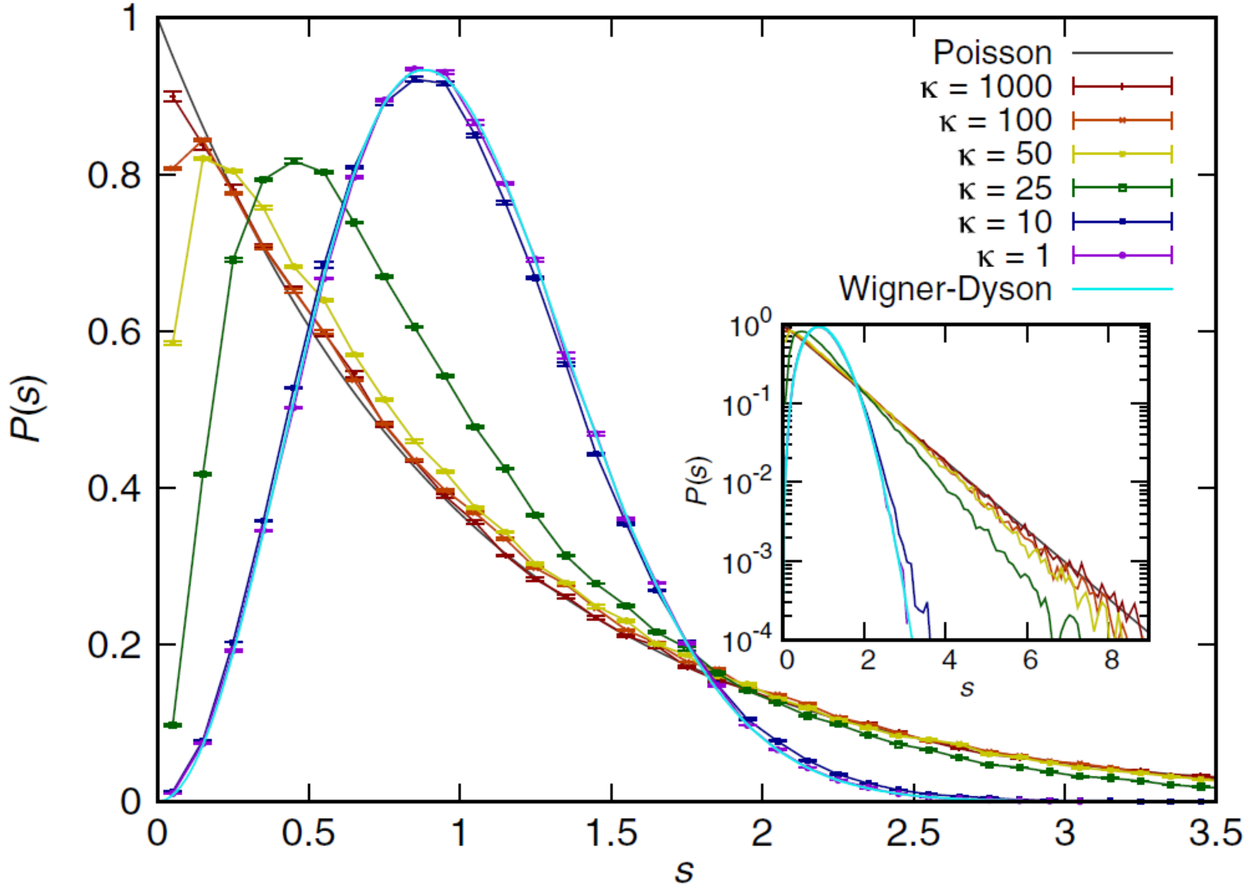
$J = 1$: unit of energy

SYK_4 as unperturbed Hamiltonian,
K controls the strength of SYK_2 (one-body random term, solvable)

Here we take (GUE) $N \equiv 2 \pmod 4$

Both terms respect charge parity in complex fermion description
➔ Full numerical exact diagonalization (ED) of $2^{N/2-1}$-dimensional matrix, $N \lesssim 34$ possible

RMT-like behavior lost as SYK2 term is introduced



N=30, Central 10 % of eigenvalues

$P(s)$: level spacing distribution

Ratio of consecutive level spacing $E_{i+1} - E_i$
to the local mean level spacing Δ
(requires unfolding of the spectrum)

SYK_4 limit (small *K*):
Obeys random matrix theory (RMT)
(GUE (Gaussian Unitary Ensemble) if $N \equiv 2 \pmod 4$)

SYK_2 (large *K*): Poisson (e^{-S})

Also see: T. Nosaka, D. Rosa, and J. Yoon, JHEP **1809**, 041 (2018) for other symmetry cases
cf. A. V. Lunkin, K. S. Tikhonov, and M. V. Feigel'man, PRL 121, 236601 (2018); Y. Yu-Xiang, F. Sun, J. Ye, and W. M. Liu, 1809.07577, …

PRL **120**, 241603 (2018)

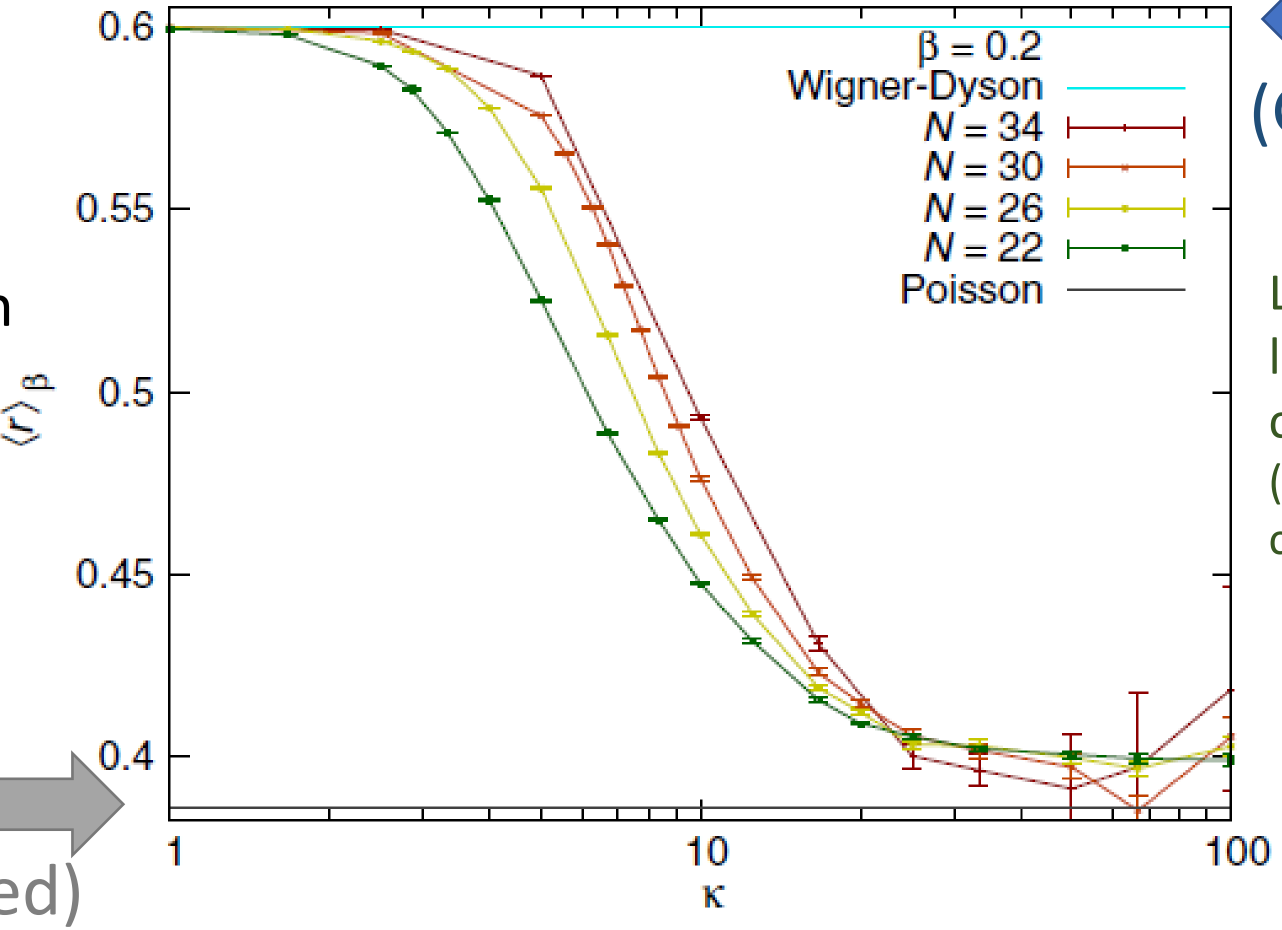
$\mathrm{SYK}_{q\geq 4} + \mathrm{SYK}_2$: breakdown of chaos

$$\hat{H} = \sum_{1\leq a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + i\sum_{1\leq a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b$$

SYK_4 SYK_2

K_{ab}: standard deviation $= \kappa/\sqrt{N}$

Averaged ratio between neighboring energy level separations



GUE (Gaussian Unitary Ensemble)

Poisson (uncorrelated)

Lyapunov exponent calculated in the large-*N* limit: also deviates from the chaos bound, approaches zero at low *T* (see also our reply 2007.06121 to a comment, PRL in press)

We consider N Majorana fermions with normalization $\{\hat{\chi}_a, \hat{\chi}_b\} = \delta_{ab}$ here

Deviation from Gaussian random matrix as SYK_2 component is introduced

Many-body localization

ETH: "(almost) all eigenstates are thermal (expectation values of operators = microcanonical average)"

- Anderson localization: concept in non-interacting systems
  - Localization of wavefunctions due to scatterings at impurities
  - Many experiments in cold atom gases, optical fibers, etc.
- MBL: does localization occur in interacting systems?

  [Gornyi, Mirlin, Polyakov 2005, Basko, Aleiner, Altshuler 2006, Oganesyan and Huse 2007, … many others]
  - Memory of initial conditions remains accessible at long times
  - Reduced density matrix on a subsystem does not approach a thermal one
  - Energy eigenstates do not obey Eigenstate Thermalization Hypothesis (ETH)
  - Area law, rather than volume law, of entanglement entropy
  - "Standard model": spin-1/2 Heisenberg model + random field in *z* direction
    - Much debate on the location of the localization transition

$$\hat{H} = \sum_i^N \widehat{S_i} \cdot \widehat{S_{i+1}} + \sum_i^N h_i \widehat{S_i^z}$$

$h_i \in [-h, h]$ uniform distribution

F. Monteiro, T. Micklitz, MT, and A. Altland, Phys. Rev. Research **3**, 013023 (2021)

Our model and choice of basis

$$\mathrm{SYK}_4 + \delta\,\mathrm{SYK}_2$$

$$\hat{H} = -\sum_{1\le a<b<c<d}^{N=2N_\mathrm{D}} J'_{abcd}\hat{\psi}_a\hat{\psi}_b\hat{\psi}_c\hat{\psi}_d + i\sum_{1\le a<b}^{N} K_{ab}\hat{\psi}_a\hat{\psi}_b$$

Block-diagonalize the SYK_2 part
(the skew-symmetric matrix (K_{ab}) has eigenvalues $\pm v_j$)

$$\hat{H} = -\sum_{1\le a<b<c<d}^{2N_\mathrm{D}} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + i\sum_{1\le j\le N}^{2N_\mathrm{D}} v_j\hat{\chi}_{2j-1}\hat{\chi}_{2j}$$

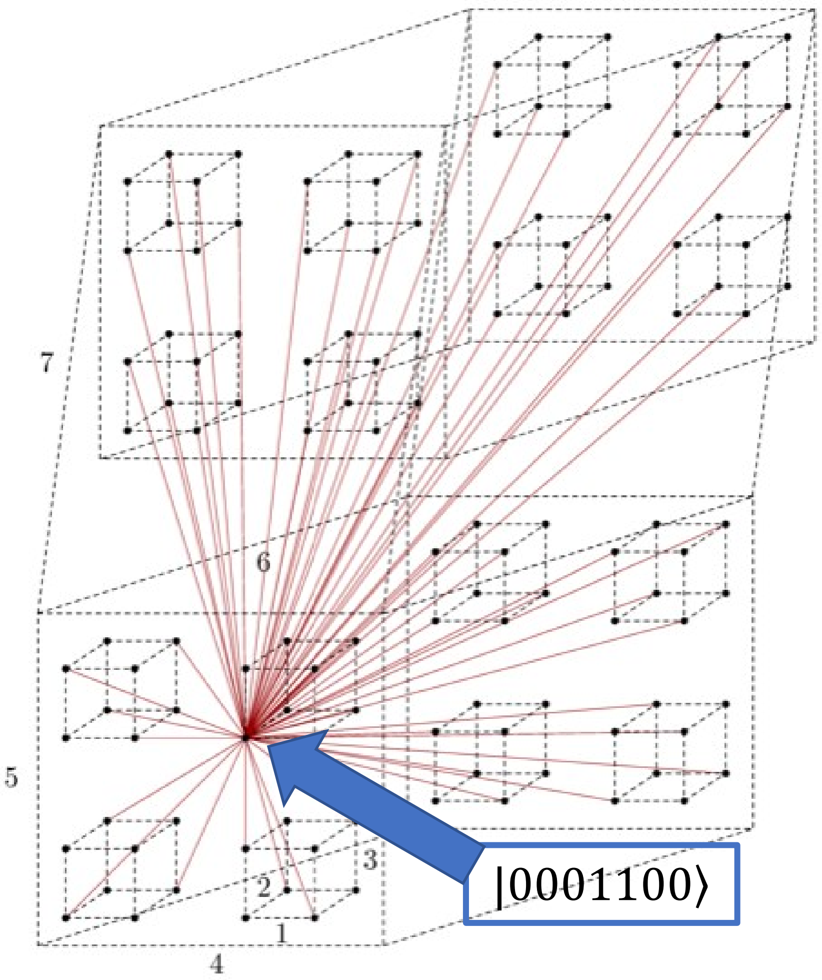
Normalization of J_{abcd}, v_j :
SYK_4 bandwidth = 1,
Width of v_j distribution = δ

We choose $\{\hat{\psi}_a,\hat{\psi}_b\} = \{\hat{\chi}_a,\hat{\chi}_b\} = 2\delta_{ab}$ as the normalization for the $N = 2N_\mathrm{D}$ Majorana fermions. For $\hat{c}_j = \frac{1}{2}\left(\hat{\chi}_{2j-1} + \mathrm{i}\hat{\chi}_{2\mathrm{j}}\right)$ we have $\{\hat{c}_i,\hat{c}_j^\dagger\} = \delta_{ij}$.

F. Monteiro, T. Micklitz, MT, and A. Altland, Phys. Rev. Research **3**, 013023 (2021)

Our model and choice of basis

$N = 2N_{\mathrm{D}} = 14$: $2^7 = 128$ states



Basis diagonalizing the complex fermion number operators $\hat{n}_j = \hat{c}_j^\dagger \hat{c}_j$ → Sites: the $2^{N_{\mathrm{D}}}$ vertices of an N_{D}-dim. hypercube.

$$\hat{c}_j = \frac{1}{2}\left(\hat{\chi}_{2j-1} + \mathrm{i}\hat{\chi}_{2\mathrm{j}}\right)$$

$$\widehat{H} = -\sum_{1\le a<b<c<d}^{2N_{\mathrm{D}}} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + i\sum_{1\le j\le N}^{N_{\mathrm{D}}} v_j\hat{\chi}_{2j-1}\hat{\chi}_{2j}$$

$$= -\sum_{1\le a<b<c<d}^{2N_{\mathrm{D}}} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + \sum_{1\le j\le N}^{N_{\mathrm{D}}} v_j\left(2\hat{n}_j - 1\right)$$

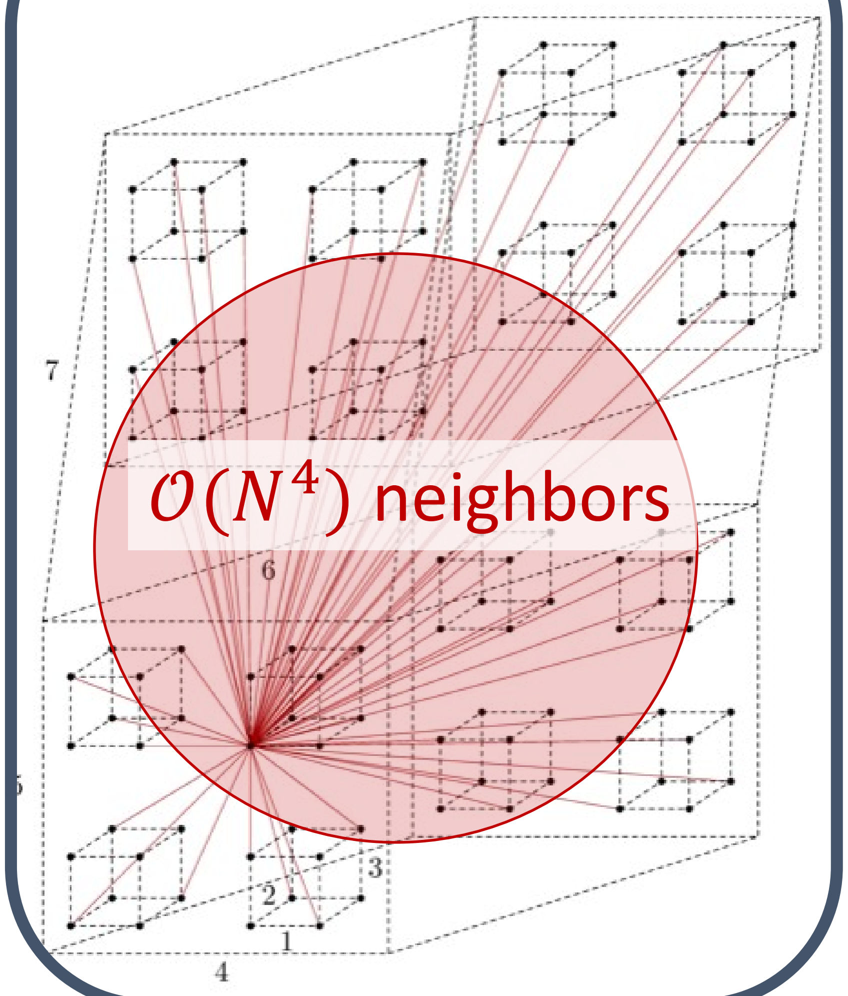
Each term of SYK_4 connects vertices with distance = 0, 2, 4.

For $N = 14$, each vertex is directly connected with 1 (distance=0, itself) + 21 (distance=2) + 35 (distance=4) vertices out of the possible $2^N = 128$ (64 per parity).

F. Monteiro, T. Micklitz, MT, and A. Altland, Phys. Rev. Research **3**, 013023 (2021)

Our model and choice of basis

2^{N_D} Fock states



Basis diagonalizing the complex fermion number operators
$\hat{n}_j = \hat{c}_j^\dagger \hat{c}_j$ → Sites: the 2^{N_D} vertices of an N_D-dim. hypercube.

$$\mathrm{SYK}_4 + \delta\ \mathrm{SYK}_2$$

$$\hat{H} = -\sum_{1\le a<b<c<d}^{2N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + \sum_{1\le j\le N}^{N} v_j(2\hat{n}_j - 1)$$

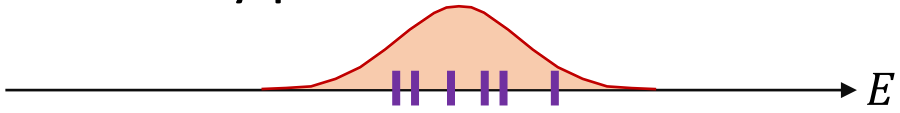
Each term of SYK_4 connects vertices with distance = 0, 2, 4.

For $N = 34$, each vertex is directly connected with
1 (distance=0, itself) + 136 (distance=2) + 2380 (distance=4)
vertices out of the possible $2^{N/2} = 131072$ (65536 per parity).

Four regimes of disorder strengths

I

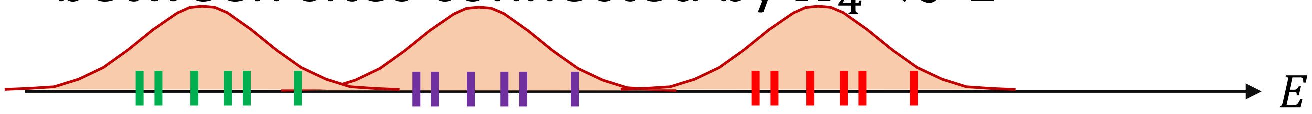
- Typical energy difference between arbitrary pair of sites $\lesssim 1$



$N_{\mathrm{D}}^{-1/2}$

II

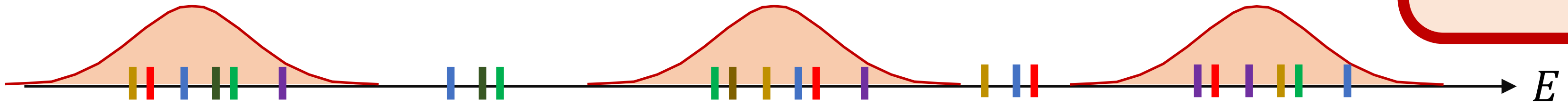
- Typical energy difference > 1, but difference between sites connected by $\hat{H}_4 \lesssim 1$



1

III

- Difference between sites connected by $\hat{H}_4 > 1$



δ_{c}

IV

- Fock space localization ($\delta_{\mathrm{c}} \sim N_{\mathrm{D}}^2 \ln N_{\mathrm{D}}$ for Bethe lattice)

[Altshuler, Gefen, Kamenev, and Levitov, PRL **78**, 2803 (1997)]

δ

$$\hat{H} = \hat{H}_4 + \hat{H}_2 \qquad \hat{H}_2 = \sum_{1 \le j \le N}^{N} v_j (2\hat{n}_j - 1)$$

width of v_j dist. = δ

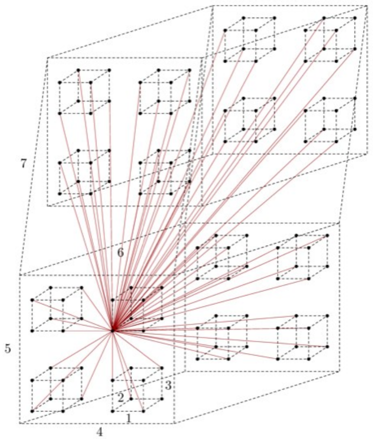
Site energy of site #m:

$$\epsilon_{(m = \sum_{1 \le j \le N}^{N} 2^{j-1} n_j)} = \sum_{1 \le j \le N}^{N} (-1)^{n_j - 1} v_j$$

Width of ϵ_m dist. $= \sqrt{N_{\mathrm{D}}}\,\delta$

$$\hat{H}_4 = -\sum_{1 \le a < b < c < d}^{2N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

SYK_4 bandwidth = 1



Diagnostic quantities: Moments of wave functions and spectral two-point correlation function

- Moments of eigenstate wave functions

$$I_q = \nu^{-1} \sum_{n,\psi} \left\langle |\langle\psi|n\rangle|^{2q} \delta(E_\psi) \right\rangle_J$$

with average density of states at band center

$$\nu = \nu(E \simeq 0), \nu(E) = \sum_{\psi} \left\langle \delta(E - E_\psi) \right\rangle_J$$

➔ Parametrizes localization, allows comparison with numerics

$I_2 = \nu^{-1} \sum_{n,\psi} \left\langle |\langle\psi|n\rangle|^{4} \delta(E_\psi) \right\rangle_J$:

inverse participation ratio (IPR), $\frac{1}{D} \leq I_2 \leq 1$

Equal weights

Single non-zero element

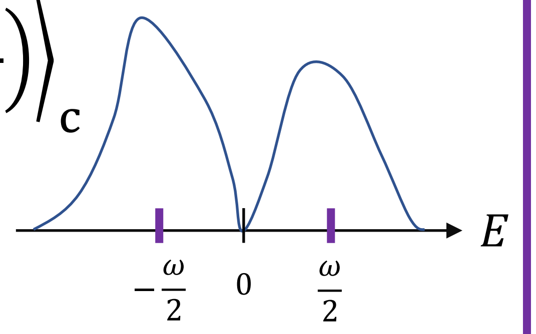
D: dimension of $\{|n\rangle\} = 2^{N-1}$

- Spectral two-point correlation function

$$K(\omega) = \nu^{-2} \left\langle \nu\left(\frac{\omega}{2}\right) \nu\left(-\frac{\omega}{2}\right) \right\rangle_c$$

c: connected part

$\langle AB \rangle_c = \langle AB \rangle_J - \langle A \rangle_J \langle B \rangle_J$



➔ Reflects level repulsion if the spectrum is random matrix-like

We calculate these quantities for large N and compare against numerical results

Analytical results

Method: Exact matrix integral representation of I_q and $K(\omega)$; mapping to a supersymmetric sigma model; saddle point equations; effective medium approximation

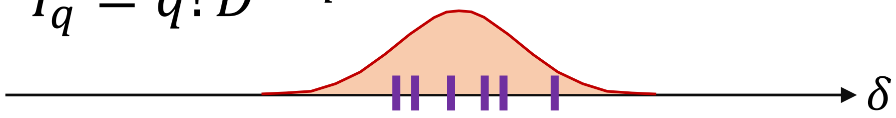
PRR 3, 013023 (2021)

$(N_{\mathrm{D}} = \frac{N}{2}, c = O(1), D = 2^{N_{\mathrm{D}}-1})$

I

- I: Average density of states (ADoS) at band center $\nu = cD$
- $I_q = q!\, D^{1-q}$

Fully delocalized

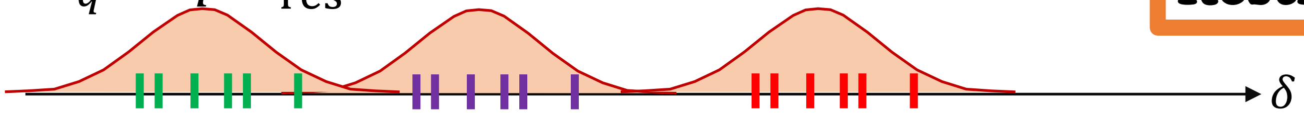


$N_{\mathrm{D}}^{-1/2}$

II

- II: ADoS $\nu = \frac{cD}{\sqrt{N_{\mathrm{D}}}\delta}$, spread of wave functions $D_{\mathrm{res}} \simeq \frac{D}{\sqrt{N_{\mathrm{D}}}\delta}$
- $I_q = q!\, D_{\mathrm{res}}^{1-q}$

Restricted



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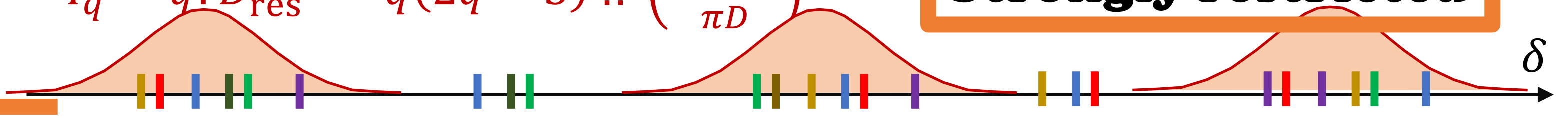
III

- III: ADoS $\nu = \frac{cD}{\sqrt{N_{\mathrm{D}}}\delta}$, spread of wave functions $D_{\mathrm{res}} \simeq \frac{D}{\sqrt{N_{\mathrm{D}}}\delta^2}$
- $I_q = q!\, D_{\mathrm{res}}^{1-q} = q(2q-3)!! \left(\frac{4\sqrt{N}\delta^2}{\pi D}\right)^{q-1}$

Strongly restricted



δ_{c}

IV

$$\delta_{\mathrm{c}} = \frac{N_{\mathrm{D}}^2}{4\sqrt{3}} \log N_{\mathrm{D}} \text{ for large } N$$

- IV: All eigenstates localized to $\mathcal{O}(1)$ sites

δ

Eigenenergy spectral statistics (for odd N case for simplicity)

$$\widetilde{K}(s) = 1 - \frac{\sin^2 s}{s^2} + \delta\left(\frac{s}{\pi}\right),$$

$s = \pi\omega\nu$ in I, II, III : **agrees with Gaussian Unitary Ensemble (GUE)**

IV: **Poisson statistics**

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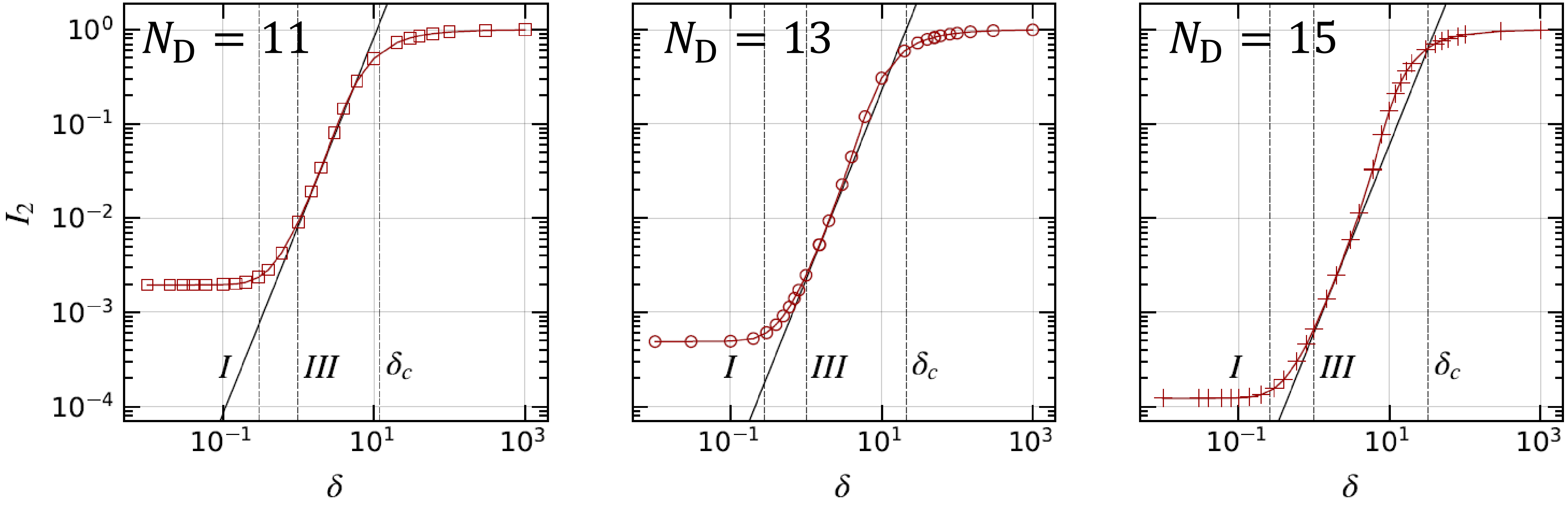
Inverse participation ratio vs prediction for III

IPR $I_2 =$ average of $\sum_n |\langle\psi|n\rangle|^4$ for normalized ψ, $\frac{1}{D} \leq I_2 \leq 1$

Equal weights

Single non-zero element



$$I_q = \frac{q(2q-3)\,!!}{\delta^{2(1-q)}}\left(\frac{\pi D}{4\sqrt{N_D}}\right)^{1-q} = q(2q-3)\,!!\,\left(\frac{4\sqrt{N_D}\delta^2}{2^{N-1}\pi}\right)^{q-1} \text{ in III}$$

Central 1/7 of the energy spectrum

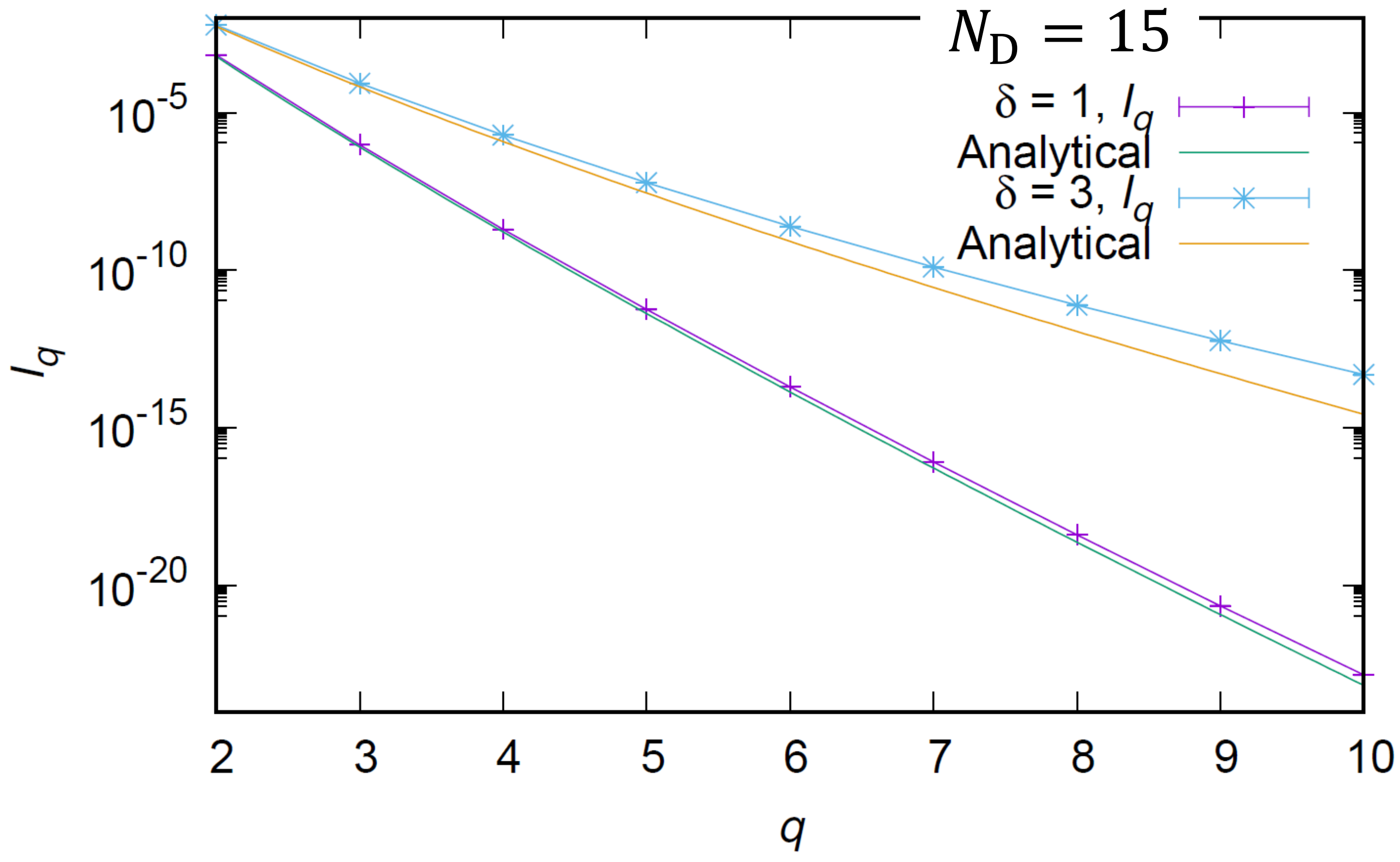
Higher moments of eigenvectors

PRR 3, 013023 (2021)

Analytical prediction:

$$I_q = \frac{q(2q-3)\,!!}{\delta^{2(1-q)}}\left(\frac{\pi D}{4\sqrt{N_\mathrm{D}}}\right)^{1-q} = q(2q-3)\,!!\,\left(\frac{4\sqrt{N_\mathrm{D}}\delta^2}{2^{N-1}\pi}\right)^{q-1} \text{ in III}$$

$$I_q = \nu^{-1}\sum_{n,\psi}\langle|\langle\psi|n\rangle|^{2q}\delta(E_\psi)\rangle_J$$



Good agreement up to large q for $\delta \sim 1$

Central 1/7 of the energy spectrum

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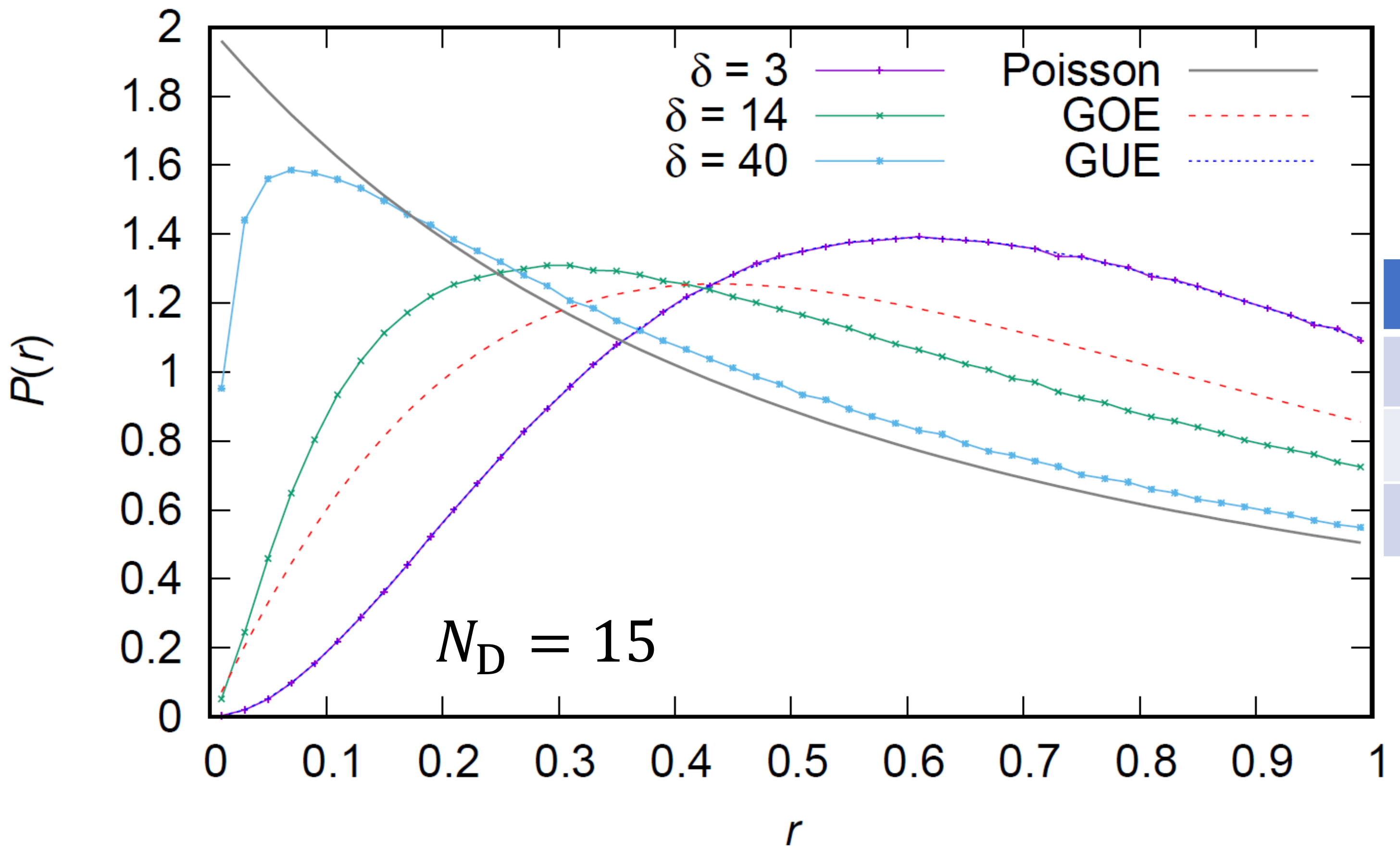
Spectral statistics: gap ratio distribution



$(\delta_c = \frac{Z}{\sqrt{2\rho}} W(2Z\sqrt{\pi}) = 38.47)$

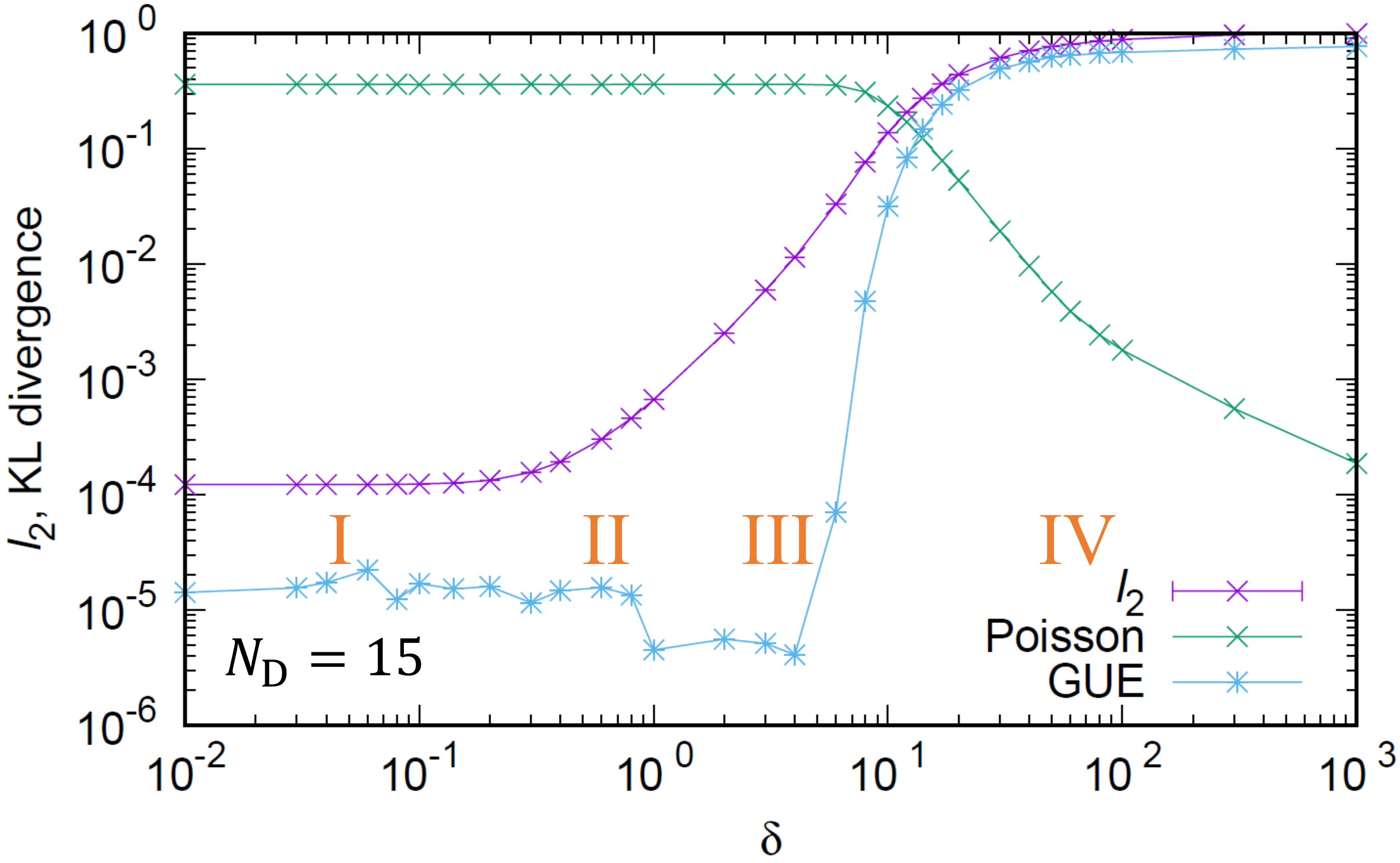
Measure difference by Kullback-Leibler (KL) divergence: $D_{\mathrm{KL}}(P||Q) = \sum_x P(x) \log \frac{P(x)}{Q(x)}$.

δ	$D_{\mathrm{KL}}(P(\delta, r)\|\|P_{\mathrm{Poisson}}(r))$	$D_{\mathrm{KL}}(P(\delta, r)\|\|P_{\mathrm{GUE}}(r))$
3	0.3608	5×10^{-6}
14	0.1234	0.1463
40	0.0096	0.5705

$$r = \frac{\min(E_{i+1} - E_i\,,\ E_{i+2} - E_{i+1})}{\max(E_{i+1} - E_i\,,\ E_{i+2} - E_{i+1})}$$

PRR 3, 013023 (2021)

Departure from random matrix $P(r)$ occurs after I_2 has grown significantly

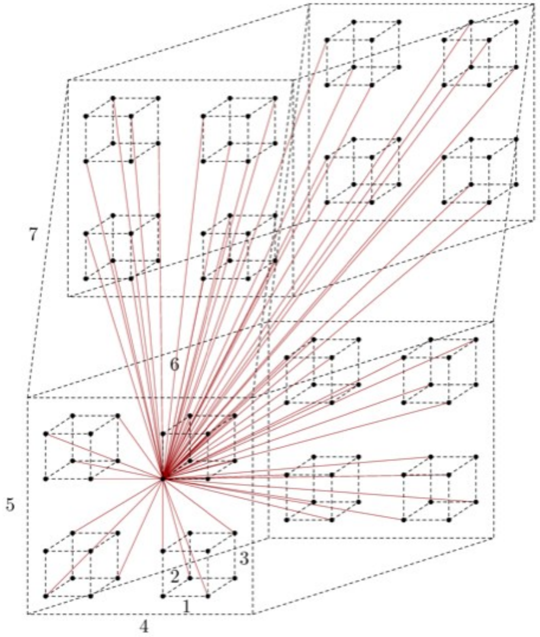


Summary so far…

Felipe Monteiro, Tobias Micklitz, Masaki Tezuka, and Alexander Altland, Phys. Rev. Research 3, 013023 (2021) arXiv:2005.12809

Fock space localization in many-body quantum systems

Sachdev-Ye-Kitaev model as tractable system



Analytical estimate of inverse participation ratio, spectral statistics

Numerical calculation of inverse participation ratio, energy spectrum correlation

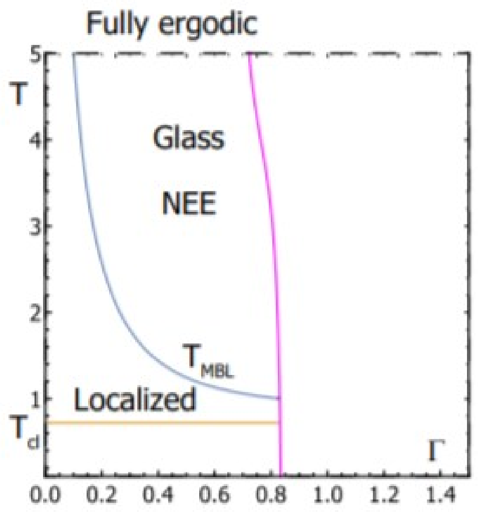
Four regimes (I: ergodic, II: localization starts, III: localization rapidly progresses, IV: MBL) found in $SYK_4 + \delta\, SYK_2$ system (in SYK_2-diagonal basis);
I, II, III are chaotic while IV is not

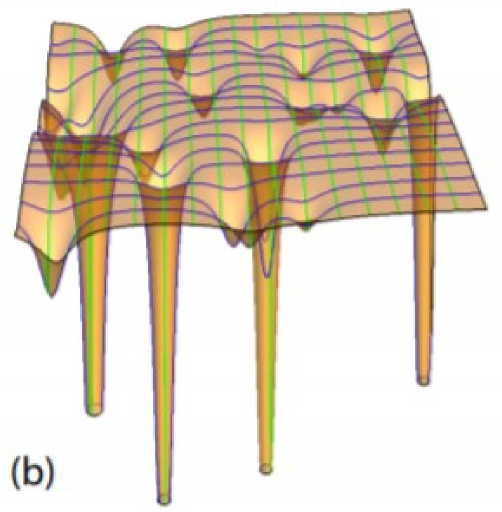
Prediction for momenta of eigenstate wavefunctions I_q is verified by **parameter free comparison**, and **energy spectrum statistics** is consistent with GUE/Poisson transition well after entering regime III

➔ Behavior of the entanglement entropy?

Physics just outside MBL (regions II & III)?

- Thermal phase smoothly connected to extended states (as those in translationally invariant models)?
- Non-ergodic extended (NEE) states discussed for several models (Bethe lattice, random regular graphs, disordered Josephson junction chains, …)



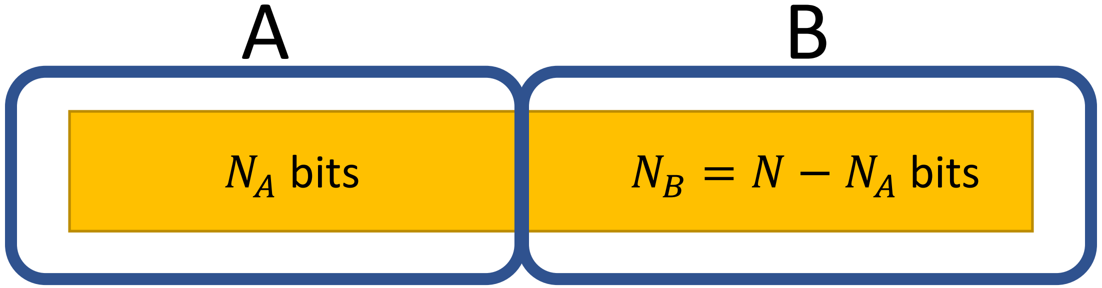


"golf course" potential energy landscape

"Non-ergodic extended phase of the Quantum Random Energy Model"
[L. Faoro, M. V. Feigel'man, L. Ioffe, Ann. Phys. **409**, 167916 (2019)]

arXiv:2012.07884

Evaluation of entanglement entropy



Zero-energy eigenstate $|\psi\rangle$, density matrix $\rho = |\psi\rangle\langle\psi|$

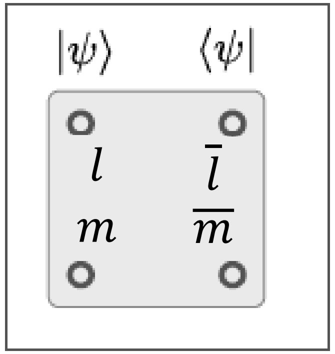
Reduced density matrix $\rho_A = \mathrm{tr}_B \rho$

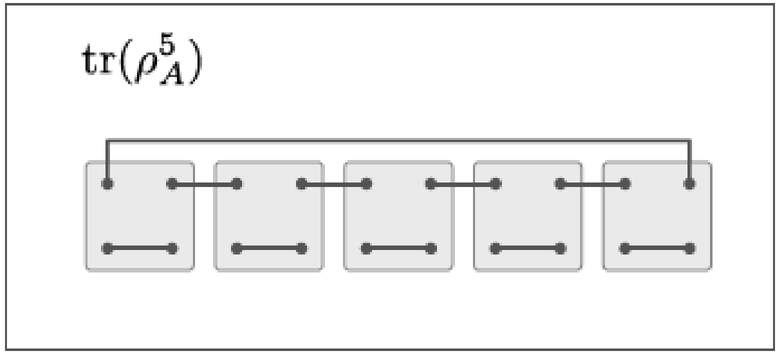
Entanglement entropy $S_A = -\mathrm{tr}_A(\rho_A \ln \rho_A)$

Fock space $\mathcal{F} = \mathcal{F}_A \otimes \mathcal{F}_B$

$n = (l, m)$

Evaluate disorder averaged moments $M_r = \langle \mathrm{tr}_A(\rho_A^r) \rangle$, $S_A = -\partial_r M_r|_{r=1}$.





$\mathcal{N} = (n^1, n^2, \ldots, n^r), \mathcal{N}_A = (l^1, l^2, \ldots, l^r), \mathcal{N}_B = (m^1, m^2, \ldots, m^r)$

$$\rho_A^r = \sum_{\substack{l^1,\ldots,l^r \\ m^1,\ldots,m^r}} \psi^{(l^1,m^1)}\, \overline{\psi}^{(l^2,m^1)} \psi^{(l^2,m^2)}\, \overline{\psi}^{(l^3,m^2)} \cdots \psi^{(l^r,m^r)}\, \overline{\psi}^{(l^1,m^r)}$$

arXiv:2012.07884

Evaluation of power of reduced density matrix

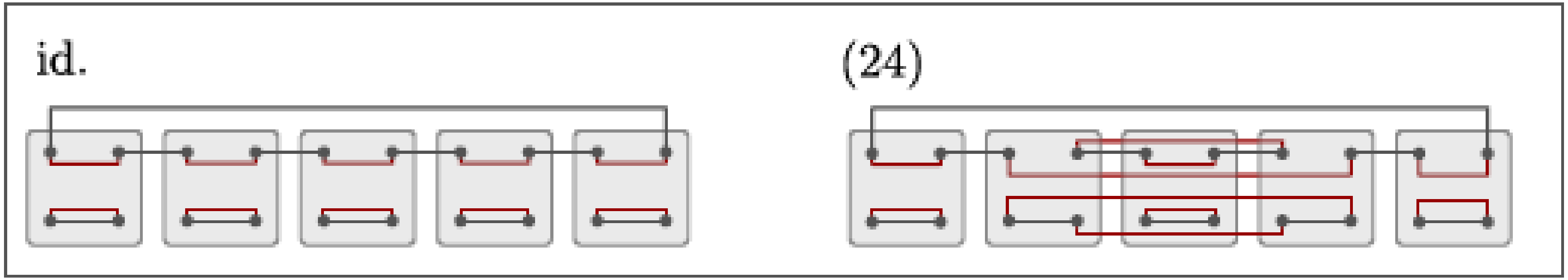
$$n^1 \qquad \overline{n}^1 \qquad n^2 \qquad \overline{n}^2$$

$$\rho_A^r = \sum_{\substack{l^1,\dots,l^r \\ m^1,\dots,m^r}} \psi^{(l^1,m^1)}\,\overline{\psi}^{(l^2,m^1)}\,\psi^{(l^2,m^2)}\,\overline{\psi}^{(l^3,m^2)} \cdots \psi^{(l^r,m^r)}\,\overline{\psi}^{(l^1,m^r)}$$

For this sum to survive disorder averaging,

$\mathcal{N} = (n^1, n^2, \dots, n^r)$ and $\overline{\mathcal{N}} = (\overline{n}^1, \overline{n}^2, \dots, \overline{n}^r)$ should be equal as sets,

$\mathcal{N}^i = \overline{\mathcal{N}}^{\sigma(i)}$



$n^1 = \overline{n}^1, n^2 = \overline{n}^2, n^3 = \overline{n}^3, n^4 = \overline{n}^4, n^5 = \overline{n}^5$

$n^1 = \overline{n}^1, \boldsymbol{n^2 = \overline{n}^4}, n^3 = \overline{n}^3, \boldsymbol{n^4 = \overline{n}^2}, n^5 = \overline{n}^5$

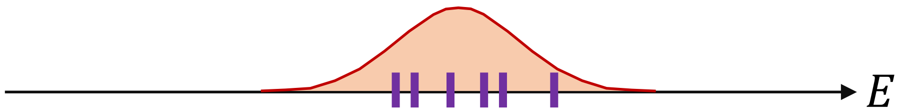
$$M_r = \langle \mathrm{tr}_A(\rho_A^r) \rangle = \sum_{\sigma} \sum_{\mathcal{N}} \prod_{i=1}^{r} \left\langle \left| \psi_{n^i} \right|^2 \right\rangle \delta_{\mathcal{N}_A,(\sigma\circ\tau)\mathcal{N}_A}\, \delta_{\mathcal{N}_B,\sigma\mathcal{N}_B}$$

Analytical results

PRR 3, 013023 (2021)

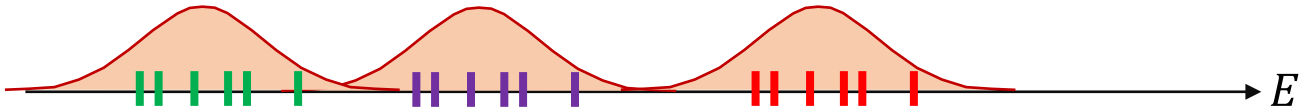
I

- I: Uniform distribution of wave functions, $\nu_n = \nu$



$N_{\mathrm{D}}^{-1/2}$

II



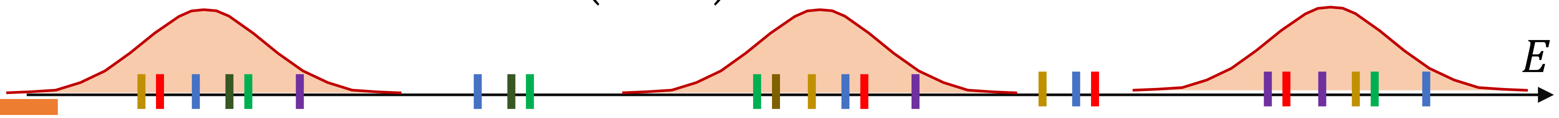
Nearest neighbors remain energetically close, $\delta \ll \Delta_4$, and level broadening $\kappa = \Delta_4$

$\Delta_4 = \mathcal{O}(1)$

- II, III: Global DoS $D\nu \approx \frac{D}{\sqrt{2\pi N_{\mathrm{D}}\delta}}$

III

Only $\mathcal{O}\left(\left(\frac{\Delta_4}{\delta}\right)^2\right)$ of nearest neighbors remain in resonance, broadening reduced to $\kappa \sim \Delta_4^2/\delta$



δ_{c}

$\delta_{\mathrm{c}} = \frac{N_{\mathrm{D}}^2}{4\sqrt{3}} \log N_{\mathrm{D}}$ for large N

IV

- IV: All eigenstates localized to $\mathcal{O}(1)$ sites

δ

arXiv:2012.07884

Regime I: maximally random case

$D_{A(B)} = 2^{N_{A(B)}-1}$

$M_r = \langle \mathrm{tr}_A(\rho_A^r)\rangle,\ S_A = -\partial_r M_r|_{r=1}$

Uniform distribution of wave functions, $\nu_n = \nu$

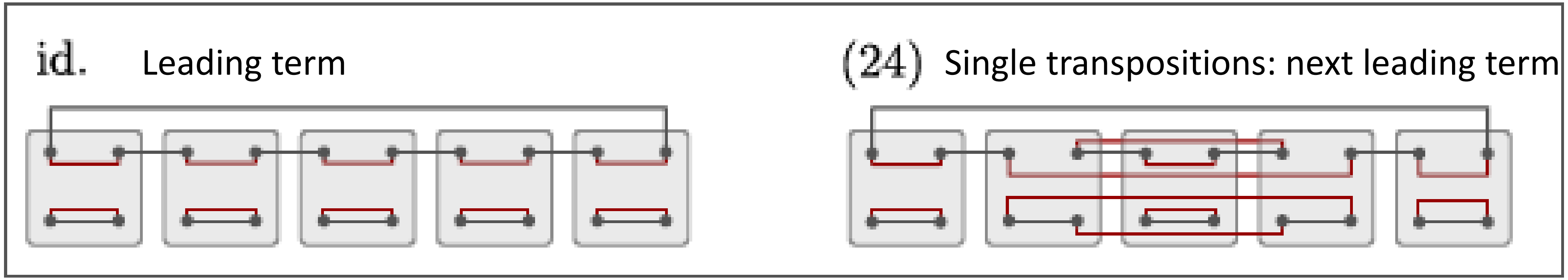
$$M_r \approx D_A^{1-r} + \binom{r}{2} D_A^{2-r} D_B^{-1}$$

Up to single transpositions

Difference from the thermal value $S_{\mathrm{th}} = \ln D_A$

$$S_A - S_{\mathrm{th}} = -\frac{D_A}{2D_B}$$

Exponentially small if $N_A \ll N_B$;
S_A very close to the thermal value



uniform

$$M_r = \langle \mathrm{tr}_A(\rho_A^r)\rangle = \sum_{\sigma}\sum_{\mathcal{N}}\prod_{i=1}^{r}\left\langle\left|\psi_{n^i}\right|^2\right\rangle \delta_{\mathcal{N}_A,(\sigma\circ\tau)\mathcal{N}_A}\,\delta_{\mathcal{N}_B,\sigma\mathcal{N}_B}$$

arXiv:2012.07884

Regimes II and III: reduced effective dimension

- Assume ergodicity and calculate S_A
- Energy shell: extended cluster of resonant sites (width κ) embedded in the Fock space
- Neighboring sites of n: energy $v_m = v_n \pm \mathcal{O}(\delta)$, much more likely to be in the same shell because $\delta \ll \Delta_2 = \sqrt{N_\mathrm{D}}\delta$

Additional assumptions

- Exponentially large number of sites → self averaging
  (sum over site energies = average over approx. Gaussian distributed contributions of subsystem energies to the total energy)
- Total energy $E \sim E_A + E_B$

➔ Up to single transpositions (justified in $1 \ll N_A \ll N_\mathrm{D}$ & replica limit):

$$S_A - S_\mathrm{th} = -\frac{1}{2}\ln\left(\frac{N_\mathrm{D}}{N_B}\right) + \frac{N_A}{2N_D} - \sqrt{\frac{N_\mathrm{D}}{2N_A}\frac{D_A}{2D_B}}$$

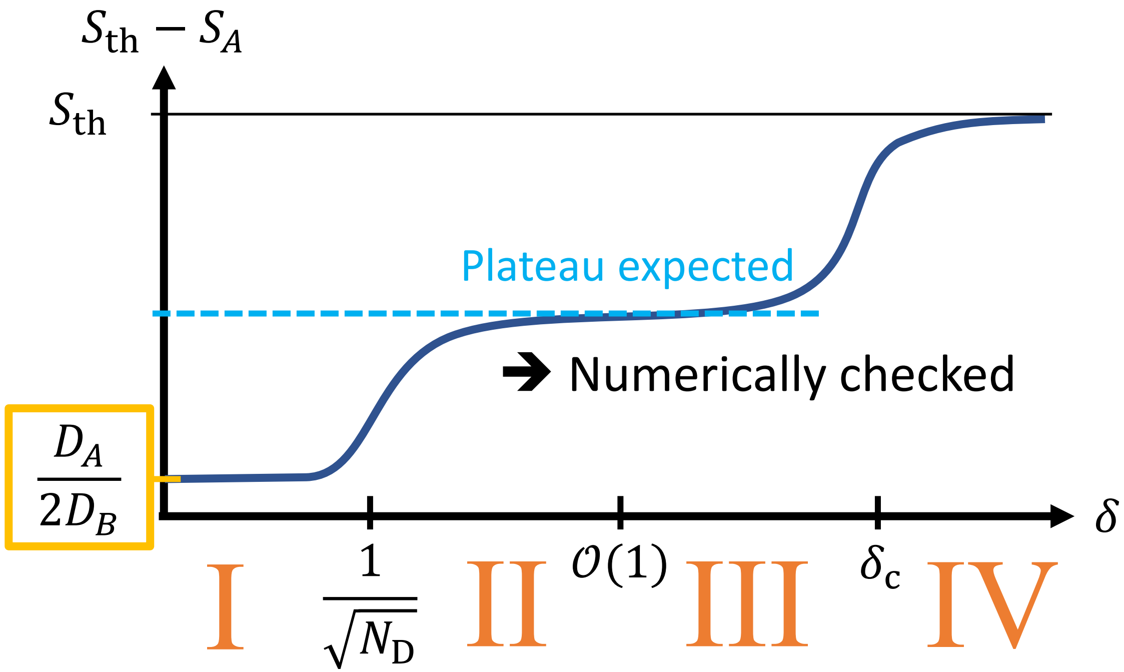
in Regimes II, III
$\left(\frac{1}{\sqrt{N_\mathrm{D}}} \ll \delta < \delta_\mathrm{c} \sim N_\mathrm{D}^2 \ln N_\mathrm{D}\right)$

$$S_A - S_\mathrm{th} = -\frac{D_A}{2D_B}$$

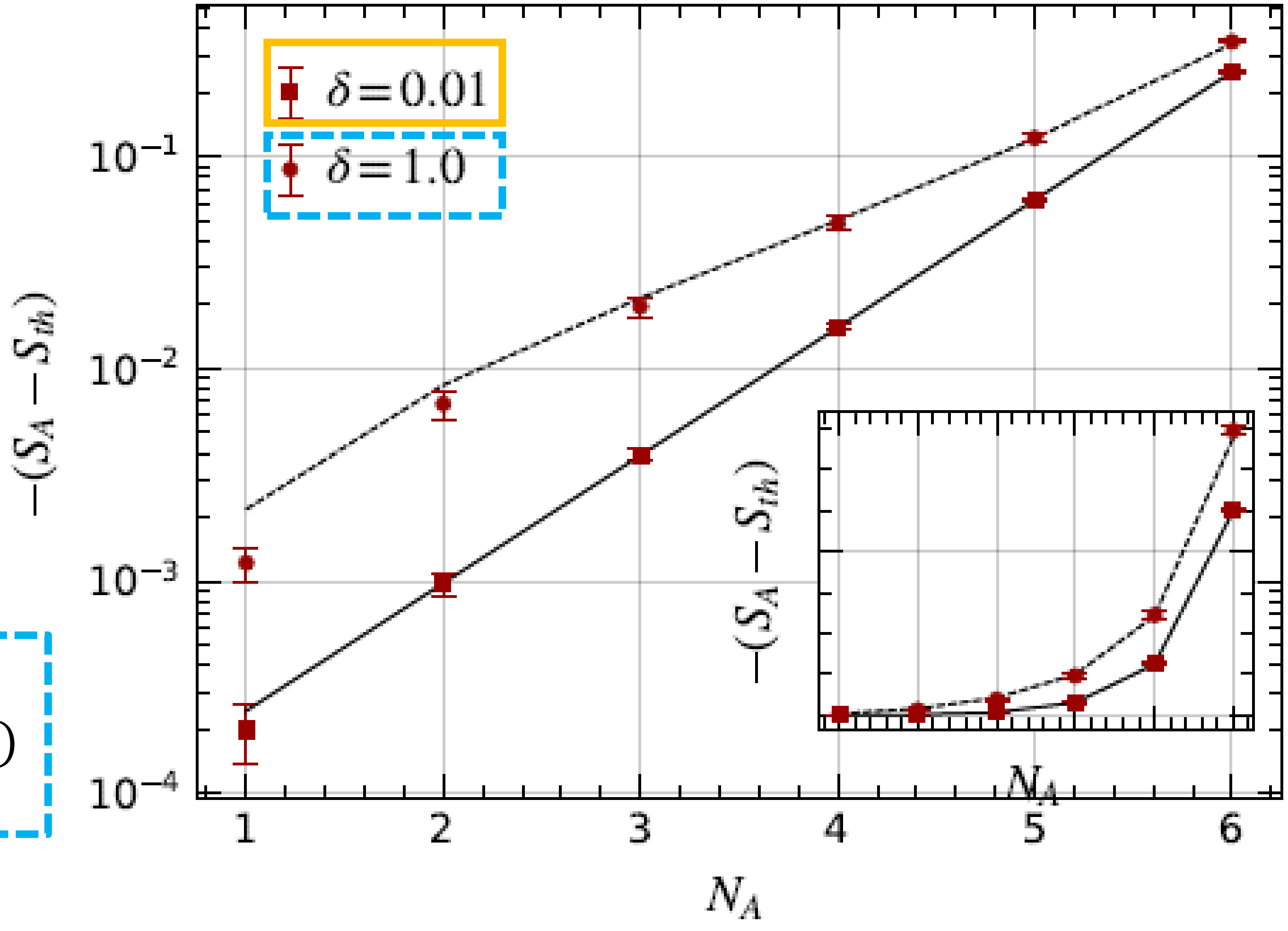
in Regime I

arXiv:2012.07884

Offset from the thermal value



$N_{\rm D} = 14$ ($N = 28$ Majorana fermions)



$$S_A - S_{\rm th} = -\frac{1}{2}\ln\left(\frac{N_{\rm D}}{N_B}\right) + \frac{N_A}{2N_D} - \sqrt{\frac{N_{\rm D}}{2N_A}\frac{D_A}{2D_B}} \ (< 0)$$

in Regimes II, III ($\frac{1}{\sqrt{N_{\rm D}}} \ll \delta < \delta_{\rm c} \sim N_{\rm D}^2 \ln N_{\rm D}$)

Phys. Rev. Research 3, 013023 (2021) arXiv:2005.12809

arXiv:2012.07884

Summary of the main part

- The Sachdev-Ye-Kitaev (SYK) model: quantum mechanical model realizing chaos bound (~ random matrix, black holes)
- Several experimental proposals, small systems realized

- SYK_{4+2}: analytically tractable model for many-body localization (MBL)
  - Fock space: (N/2)-dimensional hypercube
- Analytical results on eigenfunction moments and MBL point

  ➔ Agreement with numerical results without free paramters
- Evaluation of entanglement entropy S_A assuming ergodicity in energy shells

  ➔ Agreement between the numerical and analytical results

Question: simpler model with random matrix behavior?

Sparse (or pruned) SYK

$$H = \sum_{i<j<k<l} x_{ijkl} J_{ijkl} \chi_i \chi_j \chi_k \chi_l \,, x_{ijkl} = \begin{cases} 1 & \text{(probability } p\text{)} \\ 0 & \text{(probability } 1-p\text{)} \end{cases}, P(J_{ijkl}) = \frac{\exp\left(-\frac{J_{ijkl}^2}{2J^2}\right)}{\sqrt{2\pi J^2}}$$

$k = \binom{N}{4} p/N$: Number of non-zero x_{ijkl} per fermion

$k \sim 1$ enough for
- Random matrix-like behavior
- Large entropy per fermion at low T !

$$p \sim \frac{4!}{N^3} = \mathcal{O}(N^{-3})$$

- Talk by Brian Swingle at Simons Center (18 September 2019)
- "Sparse Sachdev-Ye-Kitaev model, quantum chaos and gravity duals" A. M. García-García, Y. Jia, D. Rosa, J. J. M. Verbaarschot, arXiv:2007.13837
- "A Sparse Model of Quantum Holography" S. Xu, L. Susskind, Y. Su, and B. Swingle, arXiv:2008.02303

The product of two Gaussians = Gaussian

The product of two (Gaussians + $(1-p)\delta(x)$) = Gaussian + $(1-p')\delta(xx')$

Preliminary

Sparse (or pruned) SYK with interaction $= \pm 1$

The product of two ± 1s = ± 1

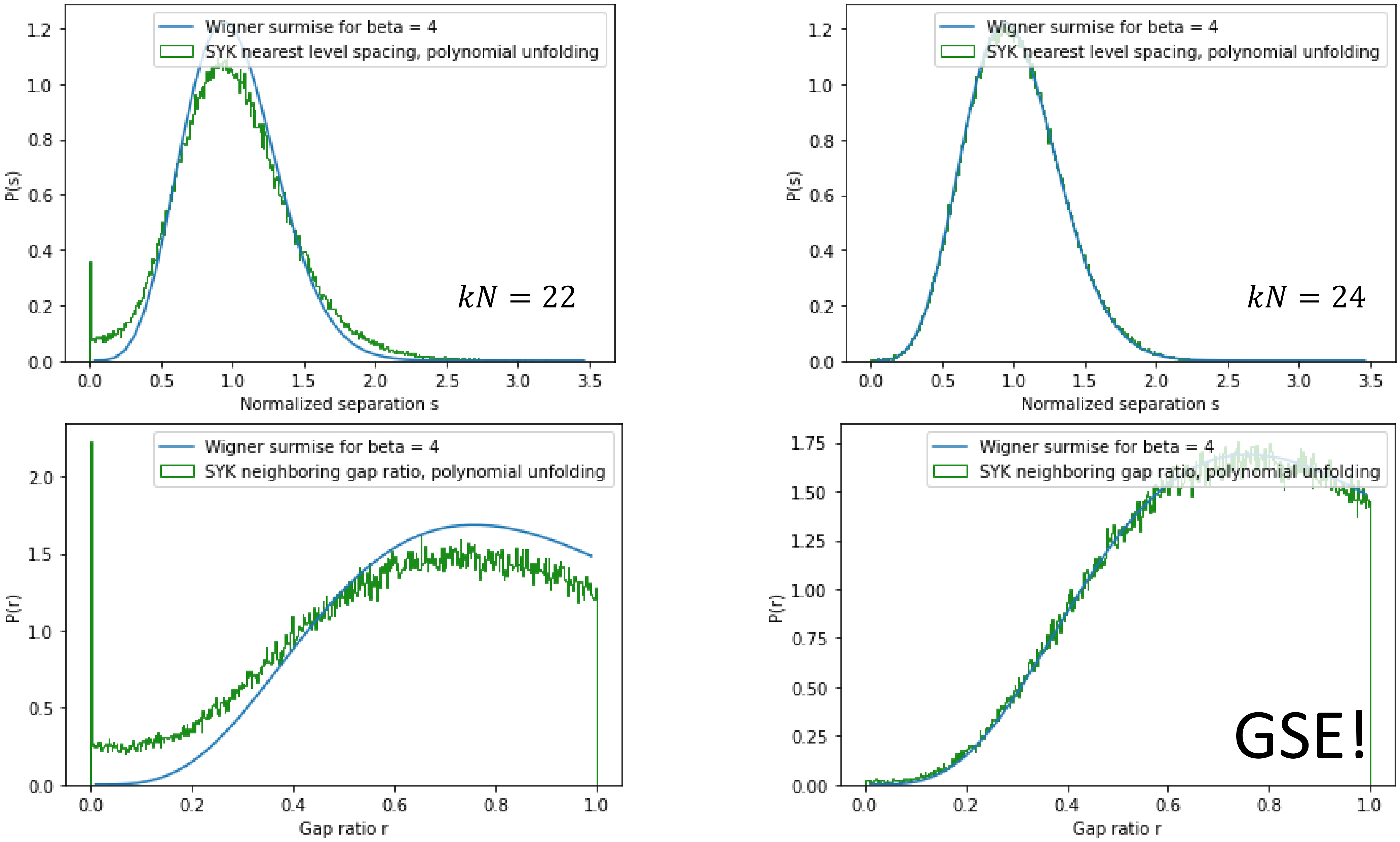
The product of two $(p\delta(x^2-1)+(1-p)\delta(x))$ = $(p'\delta(x^2-1)+(1-p')\delta(x))$

$$H = \sum_{i<j<k<l} x_{ijkl}\chi_i\chi_j\chi_k\chi_l\,,\; x_{ijkl} = \begin{cases} 1 & (\text{probability } p/2) \\ -1 & (\text{probability } p/2) \\ 0 & (\text{probability } 1-p) \end{cases}$$

Random-matrix statistics for $k = \binom{N}{4}p/N \gtrsim 1$!

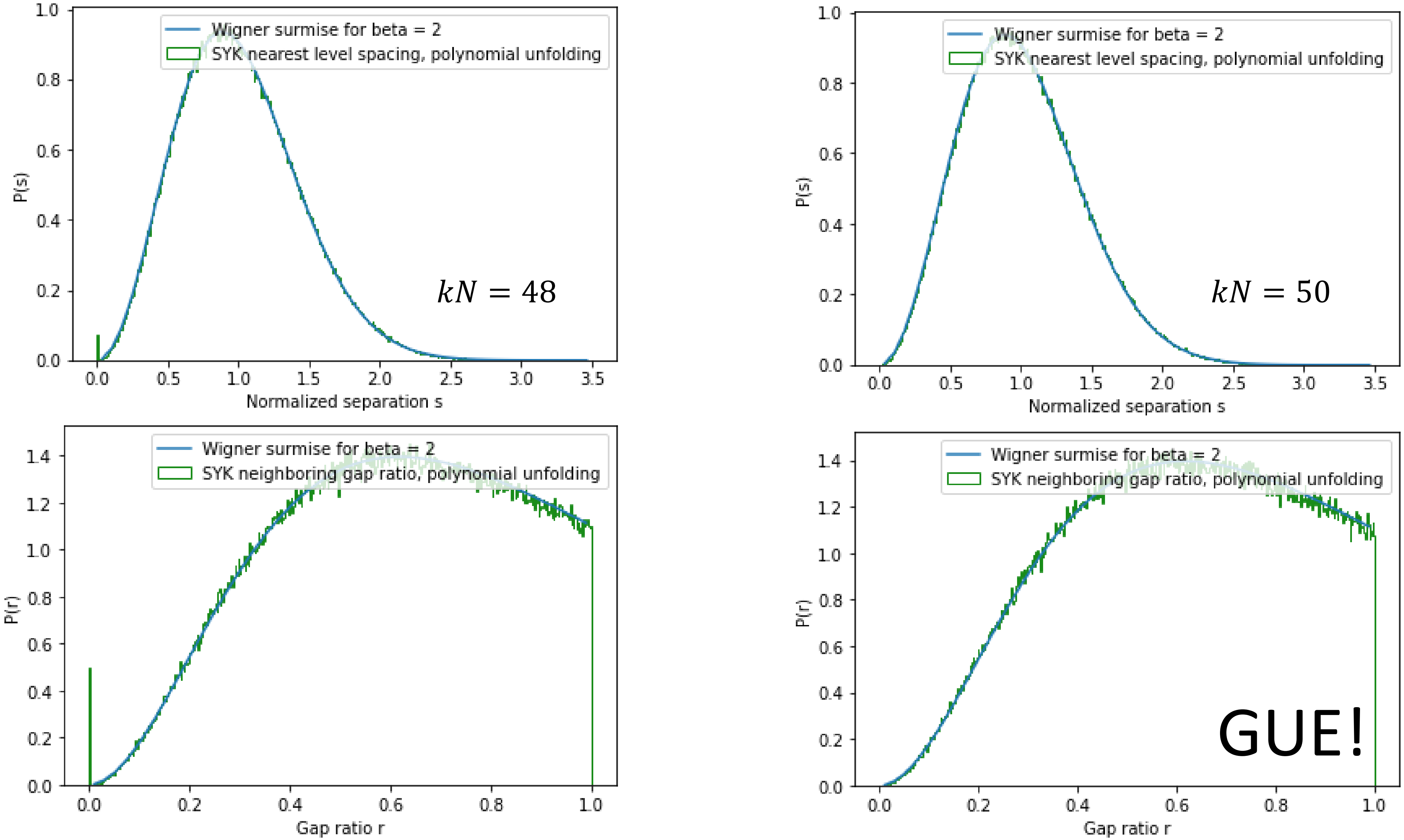
$N = 20, 1000$ samples

Preliminary



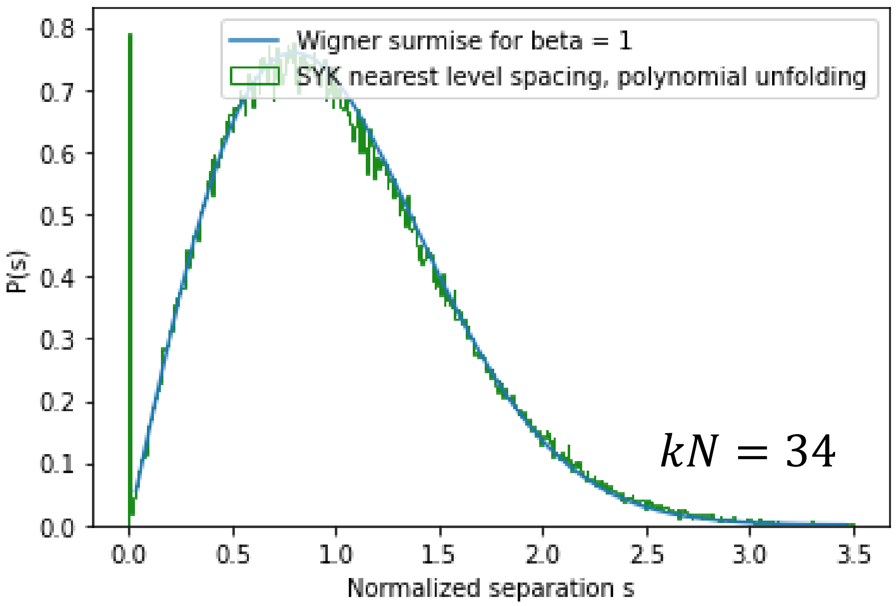
$N = 22, 1000$ samples

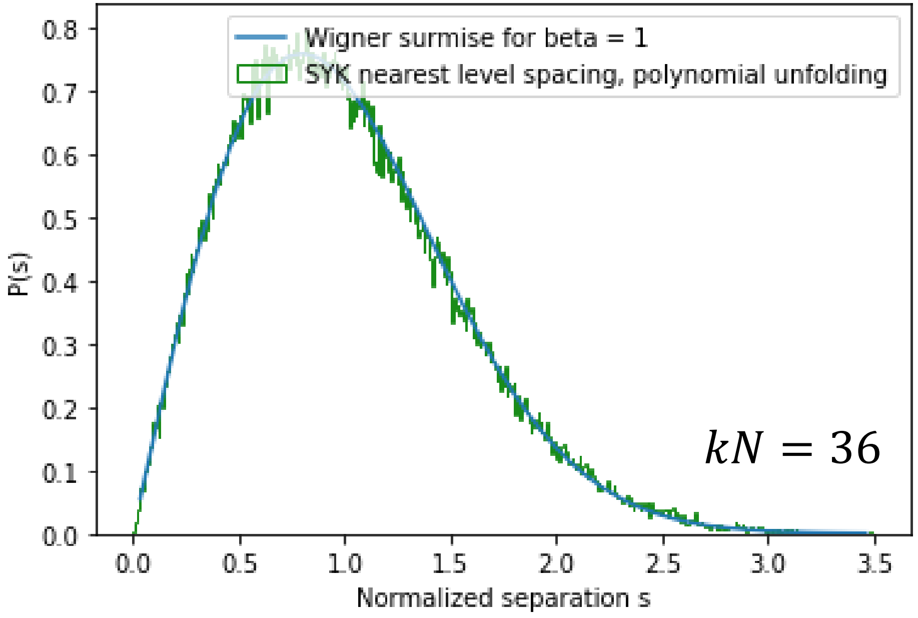
Preliminary

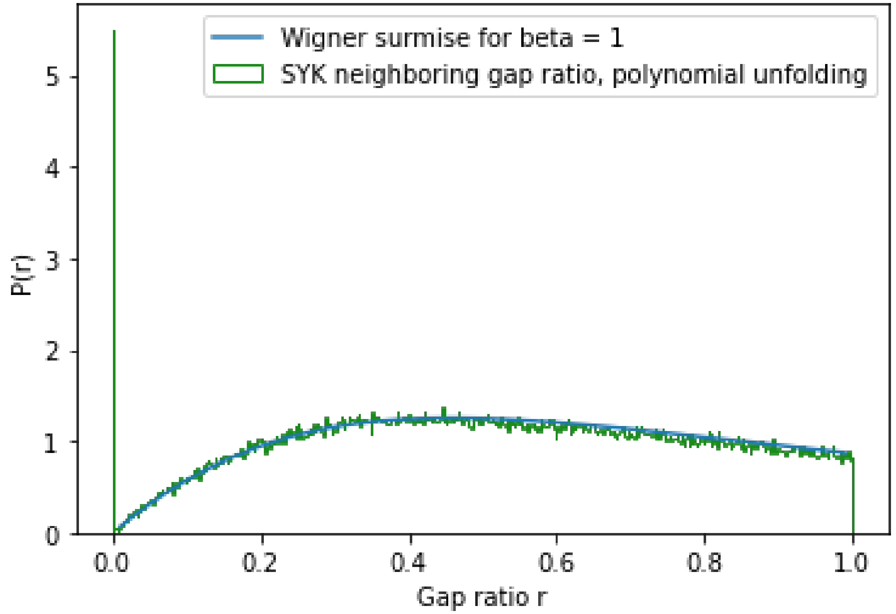


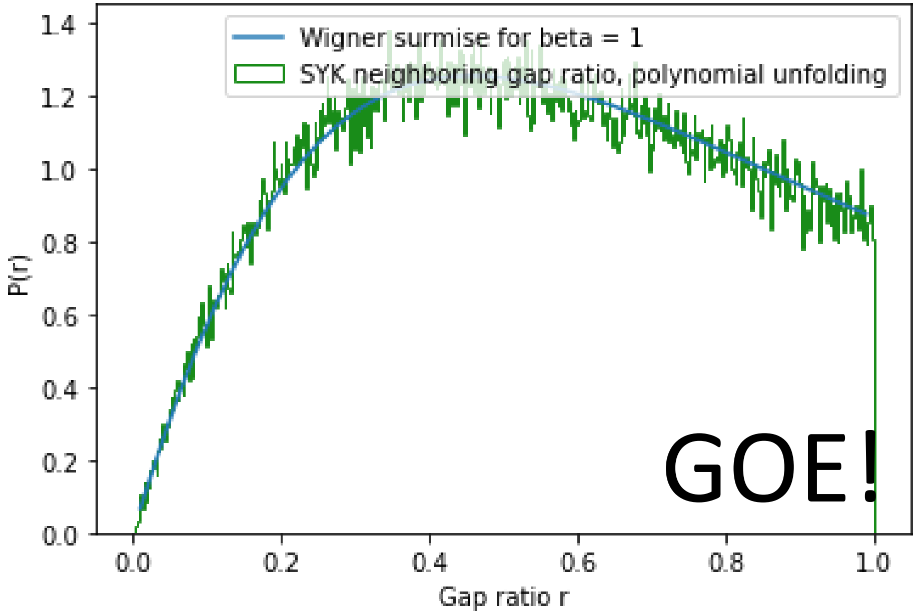
$N = 24$, 1000 samples

Preliminary



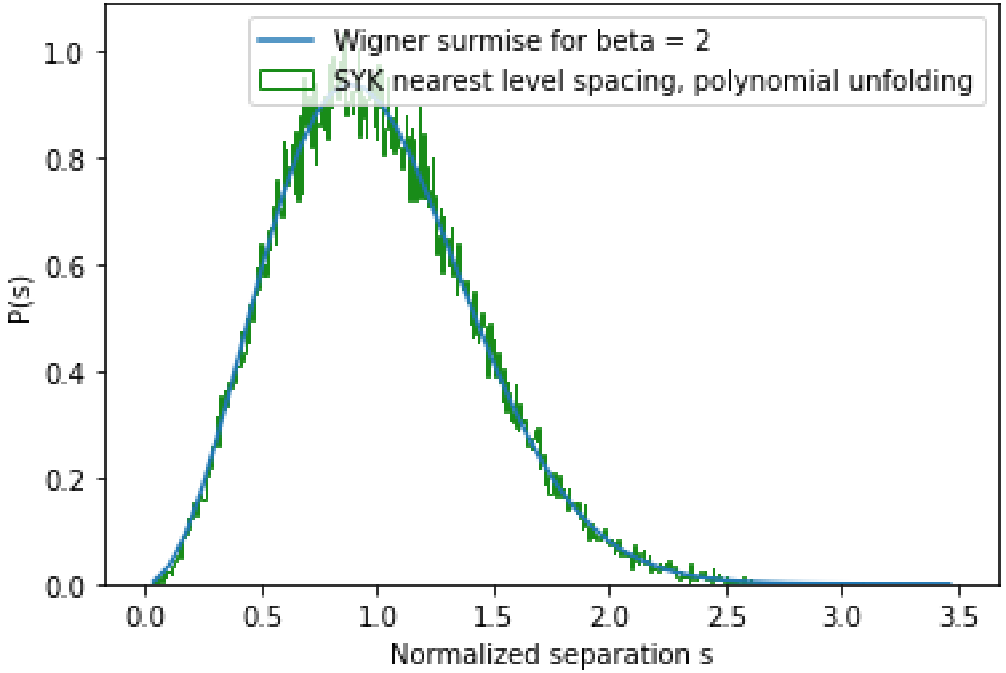


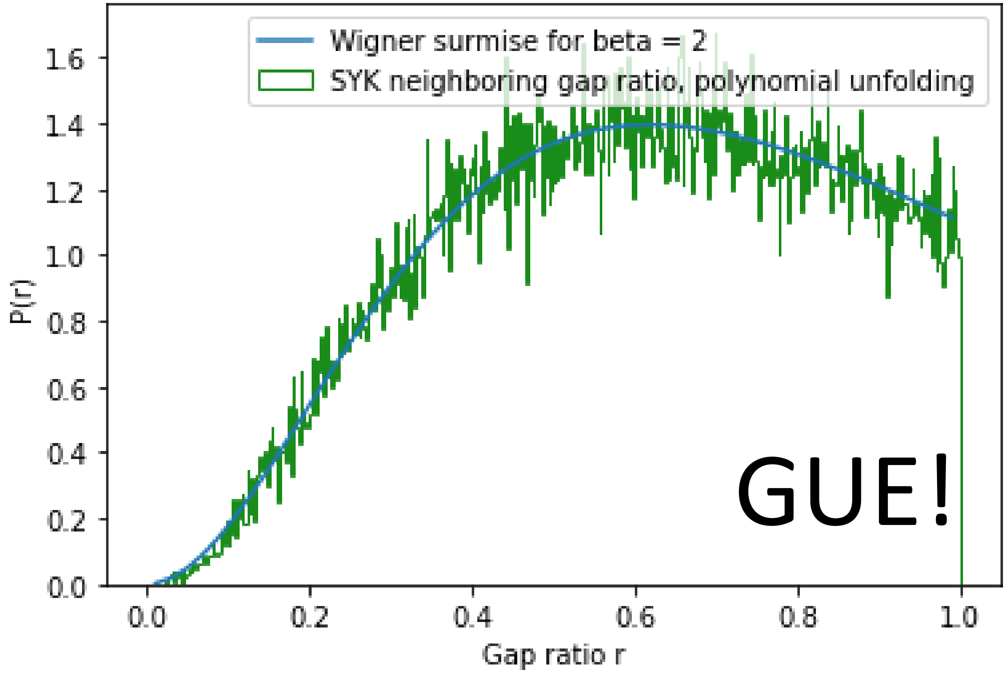




$N = 34, kN = 36, 1$ sample

Preliminary



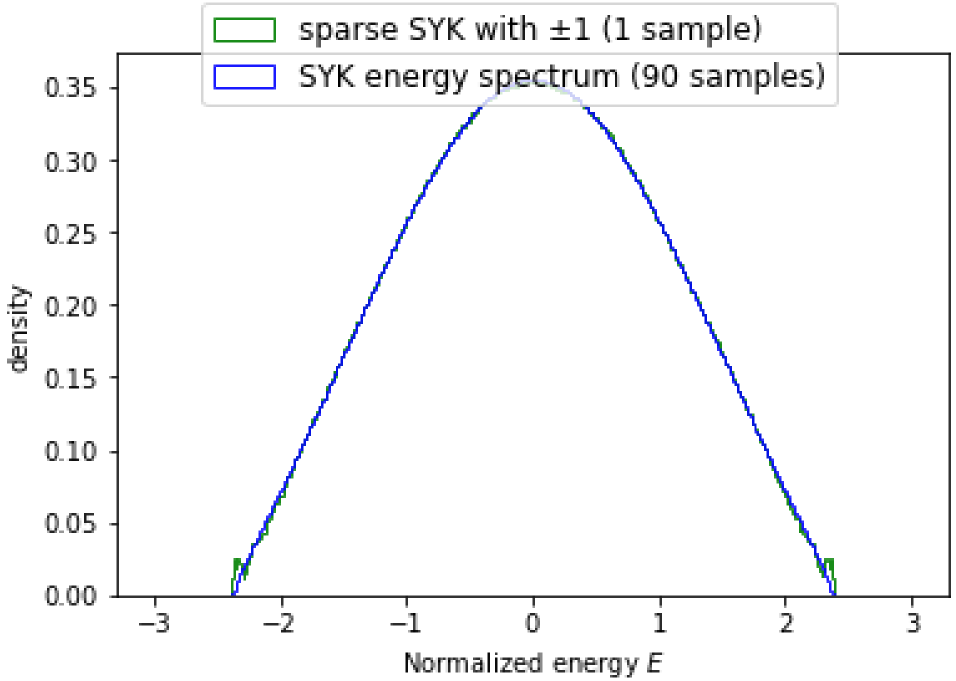


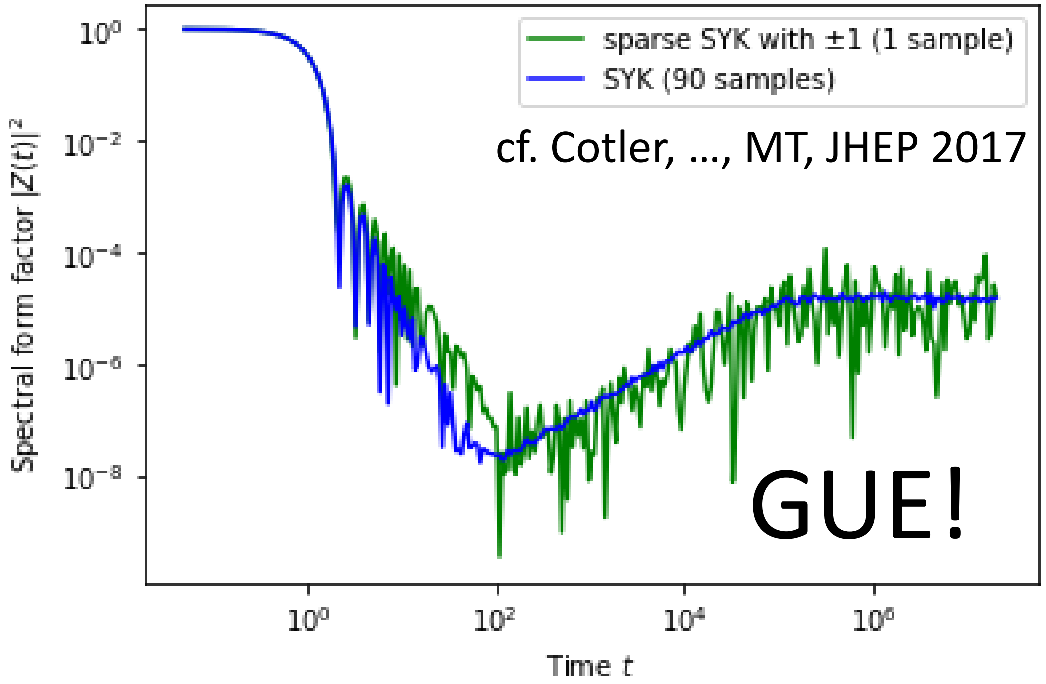
$$
\begin{aligned}
\mathcal{H} = \;& \chi 0\chi 5\chi 19\chi 27 + \chi 0\chi 6\chi 21\chi 23 - \chi 0\chi 9\chi 14\chi 24 - \chi 0\chi 14\chi 18\chi 30 - \chi 0\chi 14\chi 20\chi 25 - \chi 1\chi 2\chi 16\chi 22 \\
& + \chi 1\chi 18\chi 22\chi 23 + \chi 2\chi 4\chi 5\chi 15 + \chi 2\chi 13\chi 16\chi 21 + \chi 2\chi 14\chi 19\chi 24 + \chi 2\chi 20\chi 27\chi 33 + \chi 2\chi 22\chi 31\chi 32 \\
& + \chi 3\chi 4\chi 5\chi 29 - \chi 3\chi 8\chi 14\chi 28 - \chi 3\chi 8\chi 29\chi 31 + \chi 3\chi 21\chi 26\chi 29 - \chi 3\chi 22\chi 25\chi 33 + \chi 4\chi 7\chi 13\chi 30 \\
& - \chi 4\chi 9\chi 14\chi 17 - \chi 5\chi 6\chi 17\chi 29 + \chi 5\chi 12\chi 29\chi 31 - \chi 5\chi 13\chi 19\chi 24 - \chi 5\chi 14\chi 22\chi 31 - \chi 5\chi 17\chi 31\chi 33 \\
& + \chi 5\chi 20\chi 30\chi 31 - \chi 6\chi 23\chi 27\chi 29 + \chi 7\chi 12\chi 13\chi 18 + \chi 8\chi 10\chi 24\chi 28 - \chi 9\chi 12\chi 20\chi 33 + \chi 10\chi 11\chi 28\chi 32 \\
& + \chi 10\chi 21\chi 27\chi 29 - \chi 12\chi 20\chi 22\chi 24 + \chi 14\chi 17\chi 26\chi 27 - \chi 15\chi 24\chi 26\chi 27 - \chi 16\chi 18\chi 23\chi 27 - \chi 18\chi 24\chi 30\chi 32
\end{aligned}
$$

2^{16} dimensions/parity; Standard SYK: 46376 terms ➔ randomly chose 36, half +1, half -1

$N = 34, kN = 36, 1$ sample

Preliminary





$$
\begin{aligned}
\mathcal{H} = {} & \chi_0\chi_5\chi_{19}\chi_{27} + \chi_0\chi_6\chi_{21}\chi_{23} - \chi_0\chi_9\chi_{14}\chi_{24} - \chi_0\chi_{14}\chi_{18}\chi_{30} - \chi_0\chi_{14}\chi_{20}\chi_{25} - \chi_1\chi_2\chi_{16}\chi_{22} \\
& + \chi_1\chi_{18}\chi_{22}\chi_{23} + \chi_2\chi_4\chi_5\chi_{15} + \chi_2\chi_{13}\chi_{16}\chi_{21} + \chi_2\chi_{14}\chi_{19}\chi_{24} + \chi_2\chi_{20}\chi_{27}\chi_{33} + \chi_2\chi_{22}\chi_{31}\chi_{32} \\
& + \chi_3\chi_4\chi_5\chi_{29} - \chi_3\chi_8\chi_{14}\chi_{28} - \chi_3\chi_8\chi_{29}\chi_{31} + \chi_3\chi_{21}\chi_{26}\chi_{29} - \chi_3\chi_{22}\chi_{25}\chi_{33} + \chi_4\chi_7\chi_{13}\chi_{30} \\
& - \chi_4\chi_9\chi_{14}\chi_{17} - \chi_5\chi_6\chi_{17}\chi_{29} + \chi_5\chi_{12}\chi_{29}\chi_{31} - \chi_5\chi_{13}\chi_{19}\chi_{24} - \chi_5\chi_{14}\chi_{22}\chi_{31} - \chi_5\chi_{17}\chi_{31}\chi_{33} \\
& + \chi_5\chi_{20}\chi_{30}\chi_{31} - \chi_6\chi_{23}\chi_{27}\chi_{29} + \chi_7\chi_{12}\chi_{13}\chi_{18} + \chi_8\chi_{10}\chi_{24}\chi_{28} - \chi_9\chi_{12}\chi_{20}\chi_{33} + \chi_{10}\chi_{11}\chi_{28}\chi_{32} \\
& + \chi_{10}\chi_{21}\chi_{27}\chi_{29} - \chi_{12}\chi_{20}\chi_{22}\chi_{24} + \chi_{14}\chi_{17}\chi_{26}\chi_{27} - \chi_{15}\chi_{24}\chi_{26}\chi_{27} - \chi_{16}\chi_{18}\chi_{23}\chi_{27} - \chi_{18}\chi_{24}\chi_{30}\chi_{32}
\end{aligned}
$$

2^{16} dimensions/parity; Standard SYK: 46376 terms ➔ randomly chose 36, half +1, half -1

Phys. Rev. Research 3, 013023 (2021) arXiv:2005.12809

arXiv:2012.07884

Summary

- The Sachdev-Ye-Kitaev (SYK) model: quantum mechanical model realizing chaos bound (~ random matrix, black holes)
- Several experimental proposals, small systems realized
- A lot of possibilities for simplification? e.g. Sparse SYK with couplings $= \pm 1$

  ➔ Scrambling properties, holography??

- SYK_{4+2}: analytically tractable model for many-body localization (MBL)
  - Fock space: N-dimensional hypercube
- Analytical results on eigenfunction moments and MBL point

  ➔ Agreement with numerical results without free paramters
- Evaluation of entanglement entropy S_A assuming ergodicity in energy shells

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