Entanglement between two disjoint universes

Tomonori Ugajin (Kyoto)
Hakubi center

"Recent progress in theoretical physics based on QI"

03/01/2021

Based on

- "Entanglement between two disjoint universes"
  2008.05274

- "Islands in de Sitter space" 2008.05275

with Vijay Balasubramanian (PENN)

Arjun Kar (PENN → UBC)

+ WIP with A. Miyata (Tokyo)

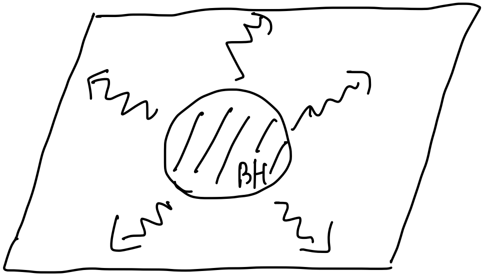
Introduction

- Today, I would like to talk about black hole and its information loss paradox

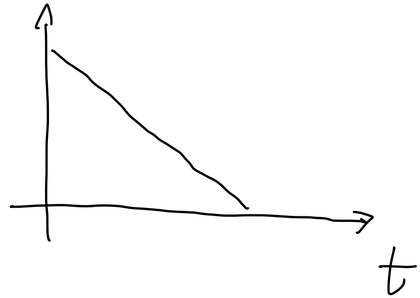
$\boxed{\text{What is information loss problem?}}$

- Quantum mechanically, a black hole emits approximately thermal radiation. (Hawking rad)

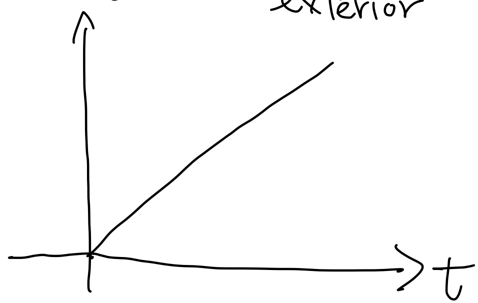
Hawking Radiation



Size of the black hole, S_{BH}

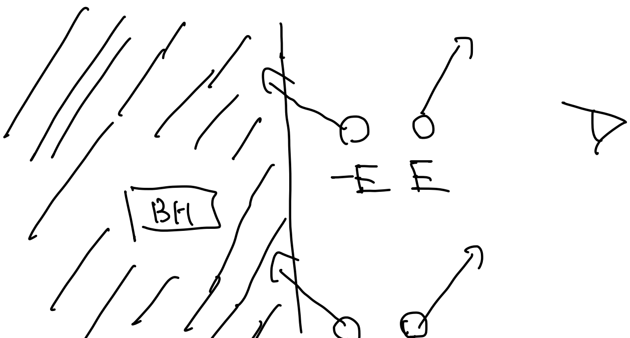


Entanglement between Interior and exterior



ex : replica trick

Horizon



-E E : Pair creations in the QFT vacuum.

- These two plots are in tension.

- In quantum gravity, the dimension of the Hilbert space of the black hole is given by the entropy of the BH

$$\dim \mathcal{H}_{BH} = e^{S_{BH}},$$

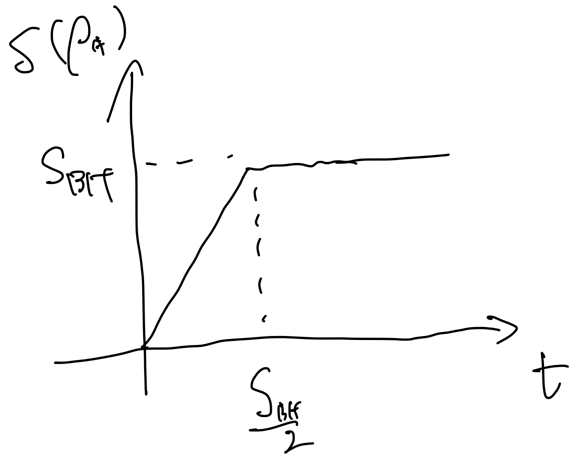
- The maximal entanglement which $\mathcal{H}_{BH}$ can store is bounded by S_{BH}.

$$S_{EE} < \log \dim \mathcal{H}_{BH} = S_{BH}$$

The Page Curve

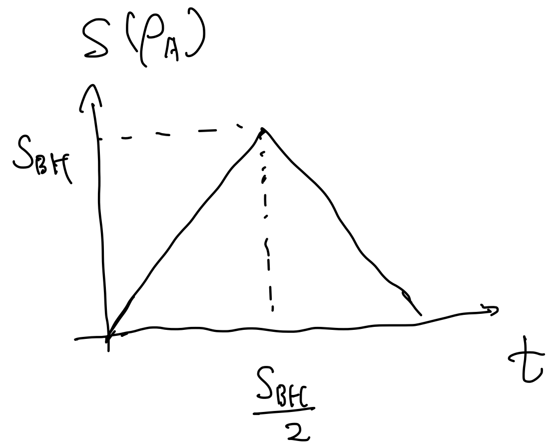
Instead, Page argued that in a unitary theory, the entanglement entropy should behave in the following way,

(1)



or

(2)



$\Rightarrow$ How do we understand the late time part of these curves from gravity point of view?

Recent developments

- Usual rule (replica trick) for entropy computation in QFT gets modified when gravity is dynamical → island formula
  [Almheiri, Maldacena ...]
  [Shenker Stanford Penington Yang]

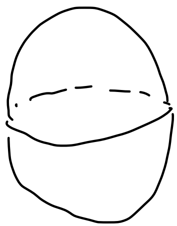
- Island formula can successfully reproduce the page curve for black holes.
  [Penington]
  [Almheiri Maldacena ...]

Introduction

In this talk, we will discuss a version of island formula, applicable to any black hole.

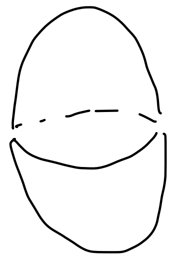
Set up

We start from two 1+1 dimensional disjoint universes



the Universe A

CFT_A



the Universe B

CFT_B + Gravity

semi classical JT gravity

$$H_{tot} = H_A \otimes H_B$$

The Setup

- We can choose the gravitating universe (universe B) to be Asymptotically AdS, dS, or flat in this setup.

- We will place black holes to the gravitating universe.

- A similar setup was considered in a recent paper by Hartman Jiang Shaghoulian.

Gravity action S_{grav}

- We choose JT gravity action in 2d

$$S_{grav}[g_{\mu\nu}, \Phi] = \frac{1}{16\pi G}\int dx^2 \sqrt{-g}\, \Phi\,(R+\Lambda) + \cdots$$

- Comes from 4d near extremal BH

$$ds^2 = \underbrace{-f(r)dt^2 + \frac{dr^2}{f(r)}}_{g_{\mu\nu}} + \underbrace{r^2}_{\Phi}\, d\Omega_2^2$$

- The value of Φ at the horizon = BH entropy

The setup (2)

On the bipertite system $H_A \otimes H_B$ we consider the following (TFD like) state.

$$\boxed{|\psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_{i=1}^{\infty} e^{-\frac{\beta E_i}{2}} |i\rangle_A \otimes |\psi_i\rangle_B}$$

$|\psi_i\rangle$: an energy eigen state of CFT_B

$$\hat{H}_{CFT} |\psi_i\rangle = E_i |\psi_i\rangle$$

- By tuning β, we can tune the entanglement between the two

The set up (3)

- We are interested in the entanglement entropy between the two universes.

$$\boxed{S(\rho_A) = -\operatorname{tr} \rho_A \log \rho_A}$$

- The reduced density matrix on the $\boxed{\text{universe } A}$ is

$$\rho_A = \operatorname{tr}_B |\psi\rangle\langle\psi|$$

$$\boxed{p_i = \frac{e^{-\beta E_i}}{Z(\beta)}}$$

$$= \sum_{i,j} \sqrt{p_i p_j}\ {}_B\langle\psi_i|\psi_j\rangle_B\ |i\rangle_A\langle j|$$

- When $G_N = 0$ $\Rightarrow$ $S(\rho_A) = S_{\text{thermal}}(\beta)$
- We imagine computing this entropy by replica trick

$$S(\rho_A) = -\mathrm{tr}\, \rho_A \log \rho_A$$
$$= -\frac{\partial}{\partial n} \underbrace{\log \mathrm{tr}(\rho_A^n)}\Big|_{n=1}$$

This is computed by a gravitational path integral.

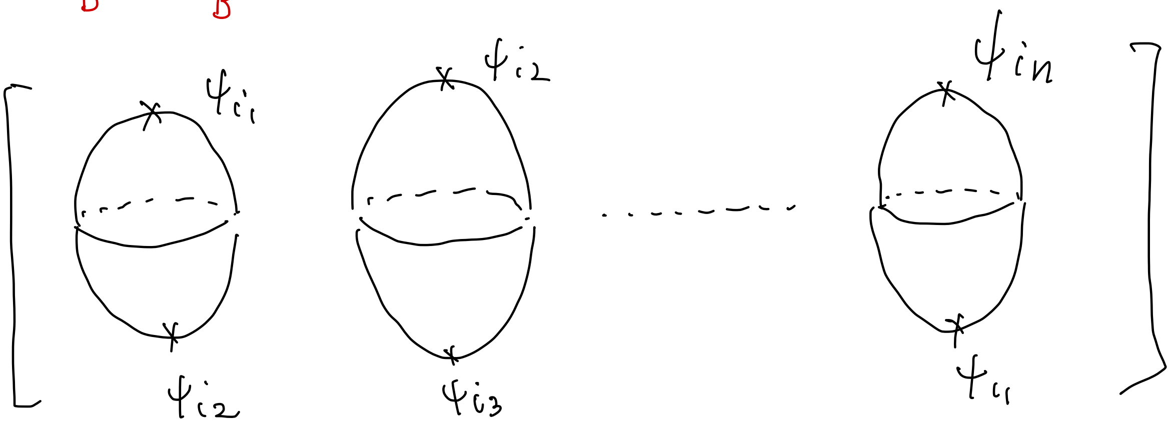
The Rényi entropy

$\mathrm{tr}(\rho_A^n)$ is computed by a path integral on $\boxed{n \text{ copies of } B}$

$$\mathrm{tr}\,\rho_A^n = \sum_{i_1\cdots i_n} p_{i_1}\cdots p_{i_n}\ {}_B\langle\psi_{i_1}|\psi_{i_2}\rangle_B \cdots\cdots {}_B\langle\psi_{i_n}|\psi_{i_1}\rangle_B$$

$${}_B\langle\psi_{i_1}|\psi_{i_2}\rangle_B \cdots\ {}_B\langle\psi_{i_n}|\psi_{i_1}\rangle_B$$

$= \mathrm{PI}$



QFT + gravity (1)

- when the QFT is coupled to gravity, we need to integrate over metrics, which satisfy boundary conditions,

$$Z_{QFT+gravity} = \int Dg_{\mu\nu}\, Z_{QFT}[g_{\mu\nu}]\, e^{-S_G[g_{\mu\nu}]}$$

The action of gravitational sector.

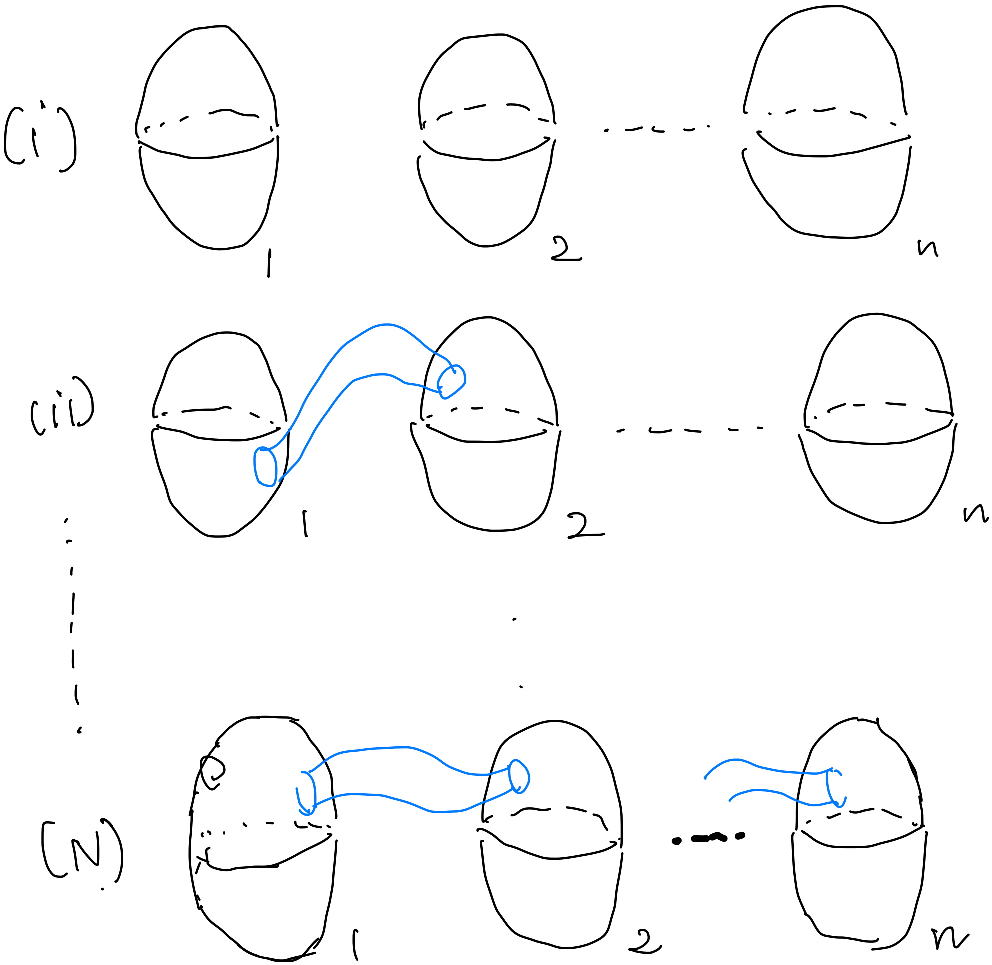
QFT + Gravity (2)

- We will consider the semi classical limit $G_N \to 0$, where the path integral can be evaluated by saddle point appoximation,

$$Z = \int Dg_{\mu\nu} \, Z_{QFT}[g_{\mu\nu}] \, e^{-S_G[g_{\mu\nu}]}$$

$$= \sum_{g_c} Z_{QFT}[g_c] \, e^{-S_G[g_c]} \, , \quad \frac{\delta}{\delta g}\left(S_G - \log Z_{QFT}\right) = 0$$

Many Saddles Satifying the bdy cond



(Fully disconnected = QFT replica mfd)

(U1 and U2 are connected by an Euclidean wormhole)

(Fully Connected.)

The resulting entropy

Actual value of $S(\rho_A)$ is computed by the minimum

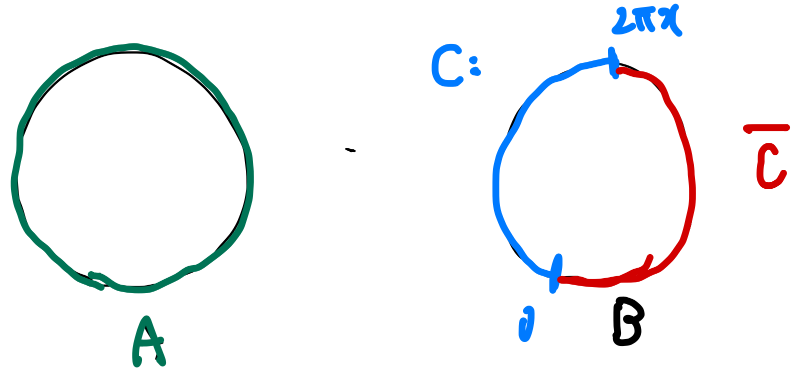
$$\boxed{S(\rho_A) = \min\left\{ S_{\substack{\text{no}\\\text{island}}},\ S_{\text{island}} \right\}}$$

$$S_{\text{no island}} = S_{CFT}(\beta) : \text{ CFT thermal entropy}$$

$$= \frac{c}{3}\left(\frac{\pi}{\beta}L\right) \quad = \text{Hawking's result}$$

$= \text{QFT result of the entropy}$

S_{island} :



$$\boxed{S_{\text{island}} = \underset{C}{\text{Ext}}\left[\Phi(\partial AC) + S_\beta^{CFT}(AC) - S_{vac}^{CFT}(AC)\right]}$$

where

$S_\beta^{CFT}(AC)$: the CFT entanglement entropy of $|TFD\rangle$ on AC

$$\Phi(\partial AC) = \Phi(0) + \Phi(2\pi x)$$

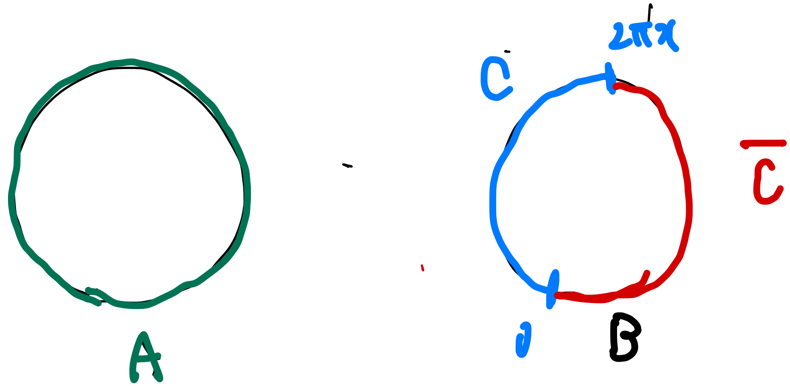
: The sum of dilaton values at the boundary

- This region C in the gravitating universe is called island.

- Since the state is pure on AB,

$$S_{\text{island}} = \underset{\bar{C}}{\text{Ext}} \left[\Phi(\partial \bar{C}) + S_\beta(\bar{C}) - S_{\text{vac}}(\bar{C}) \right]$$

Summary



- $$|TFD\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_i e^{-\frac{\beta E_i}{2}} |i\rangle_A \otimes |\psi_i\rangle_B$$

- $$S(\rho_A) = \min\left\{ S_{\text{no island}},\ S_{\text{island}} \right\}$$

↳ Only in gravity
coming from the wormhole

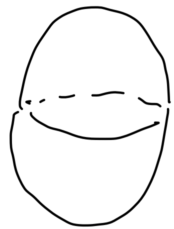
$$S_{\text{no island}} = S_{CFT}(\beta), \qquad S_{\text{island}} = \underset{\bar{C}}{\mathrm{Ext}} \left[\Phi(\partial \bar{C}) + S_\beta(\bar{C}) - S_{\text{vac}}(\bar{C}) \right]$$

Determining the dilaton profile

$$|\psi\rangle = \frac{1}{\sqrt{Z}} \sum e^{-\frac{\beta E}{2}} |i\rangle_A \otimes |\psi_i\rangle_B$$

$$\langle\psi|T_B^{CFT}|\psi\rangle$$

Universe A



Universe B

: JT gravity : Φ

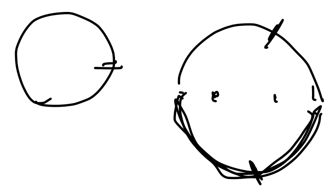
+

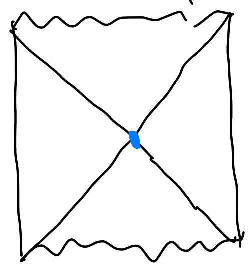
CFT $\quad T_B^{CFT}$

$$\Downarrow$$

$$\nabla_\mu\nabla_\nu\Phi - g_{\mu\nu}\nabla^2\Phi = \langle T_{\mu\nu}\rangle \Leftarrow \langle\psi|T_{00}|\psi\rangle = \frac{c}{\beta^2}$$

AdS black hole





- Initially ($\beta \to \infty$) : an AdS eternal BH on B

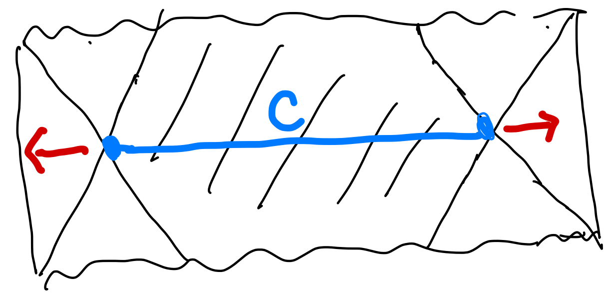
$\boxed{\langle \psi | T | \psi \rangle = 0}$

$\Downarrow$

- As we increase the entanglement :

$\boxed{\langle \psi | T | \psi \rangle \neq 0}$

[Bak ......]



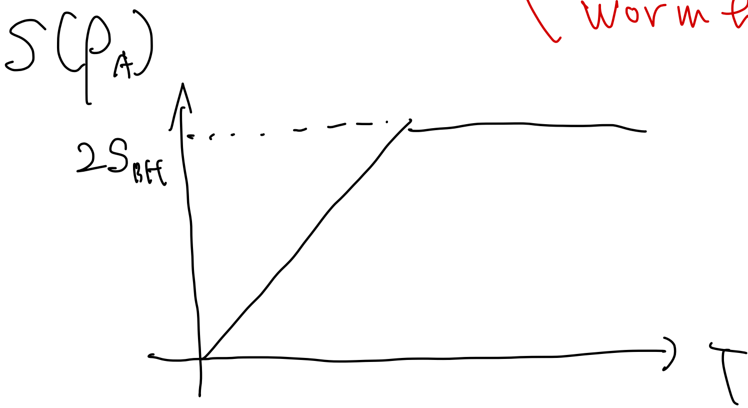
$\beta \to 0$

horizons approach AdS bdys.

The black hole develops a large Causal shadow region

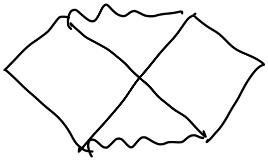
In Summary

$$S(\rho_A) = \begin{cases} S_{\text{no island}} = S_{QFT}(\beta) & \text{, low temp} \\ \quad \text{(Disconnected)} & \\ S_{\text{island}} = 2\,S_{BH} & \text{: high temperature} \\ \quad \text{(fully connected wormhole)} & \end{cases}$$



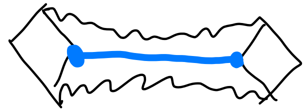
$\boxed{\text{Page curve for AdS BH}}$

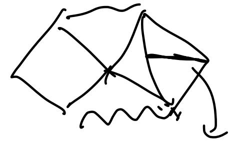
Black holes in flat space



$\beta \to 0$

$\Rightarrow$





A

$$\boxed{\langle T \rangle = 0}$$

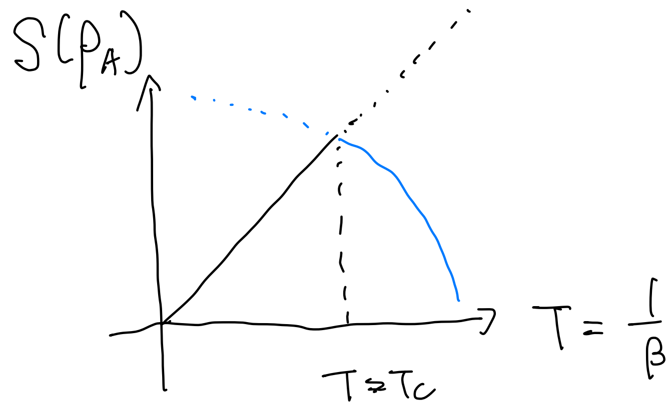
$$\boxed{\langle T \rangle = \frac{c}{\beta^2}}$$

- As we increase the entanglement, $\beta \to 0$
  the area of the event horizon decreases

$$\boxed{S_{BH} = \phi_0 - \frac{a}{\beta^4}} = S_{island} \times \frac{1}{2}$$

Black holes in flat space

$$S(\rho_A) = \min\{ S_{\text{no island}}, S_{\text{island}} \} = \min\{ S_{CFT}(\beta), 2S_{BH} \}$$



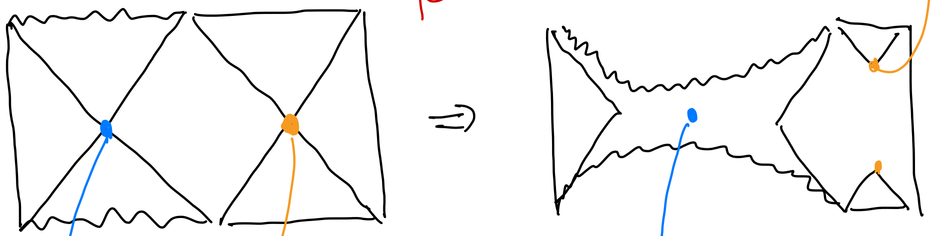
$S(\rho_A)$ is decreasing on $T > T_c$

$\Rightarrow$ Consistent with the Page curve of evapolating BH

de Sitter black hole

$\beta \to 0$

Area of cH fixed

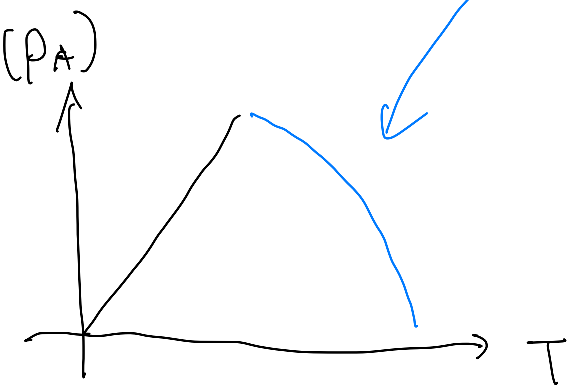


BH horizon

Cosmological horizon

Apparent horizon whose area keep decreasing

$$S_{AH} \sim \phi_0 - \frac{c}{\beta^2}$$



Entropy is decreasing because the dS BH is evaporating

Outlook

- We discussed entropy calculation in the presence of gravity in a simplified setting

$\boxed{\text{To do's}}$

- $n \neq 1$ Rényi calculation
- Generalization to relative entropy
- Time dependence
- Microscopic understanding ...

Thank you!!