Towards understanding cosmological correlators from boundary perspective

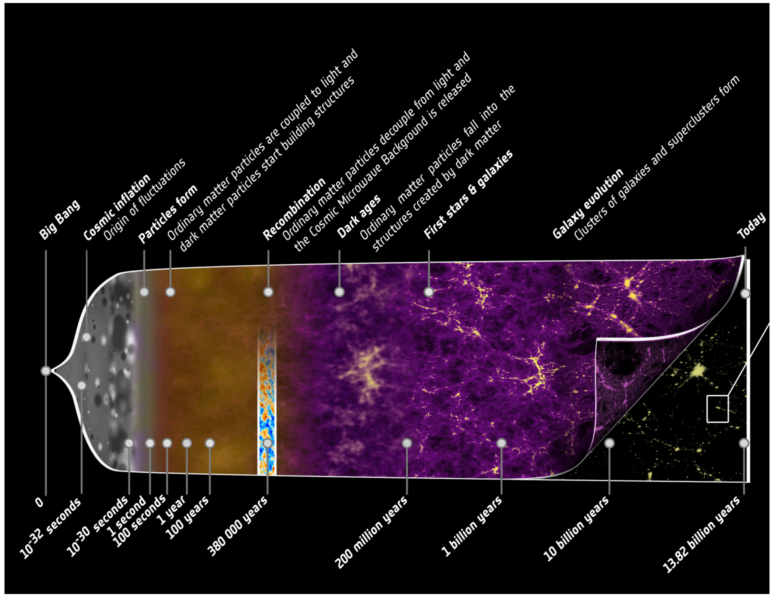
Yuko Urakawa (Bielefeld Univ./Nagoya Univ.)

Acceleration just after "beginning" of Universe

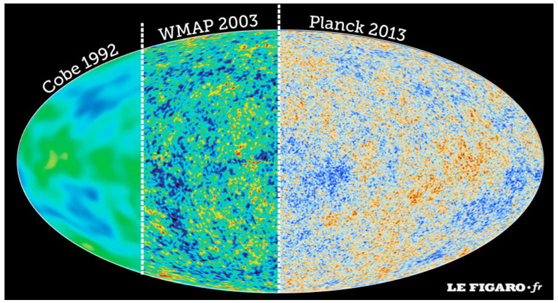
Cosmic inflation

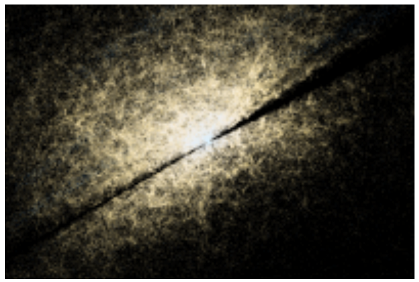
Recall Yasusada's talk

$$H^2 = \left(\frac{1}{a(t)}\frac{da(t)}{dt}\right)^2 = \frac{8\pi G}{c^2}\rho = \frac{\Lambda c^2}{3} \longrightarrow a(t) = a_i e^{\sqrt{\frac{\Lambda}{3}}c(t-t_i)}$$



inflation (almost) established





Observations

Cosmic inflation as natural laboratory

$H_{\rm inf} < 2.7\times10^{-5}\, M_{\rm pl} \sim 6.6 \times 10^{13}$ GeV PLANCK18

Natural laboratory to experiment with high energy physics (HEP)

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

UV modification of gravity

- Radiative corrections

$$G_{\mu\nu} = \frac{8\pi G}{c^4}\langle T_{\mu\nu}\rangle \qquad \langle T_{\mu\nu}\rangle = \frac{k_2}{2990\pi^2}\left(R_{\mu\rho}R^{\rho}{}_{\nu} - \frac{2}{3}RR_{\mu\nu} + \frac{1}{4}g_{\mu\nu}R^2 - \frac{1}{2}g_{\mu\nu}R_{\rho\sigma}R^{\rho\sigma}\right) + \cdots$$

Davies (77) → Starobinsky (80)

- GR in 4 dim is not UV complete

e.g., Horava (09), string theory….

BSM physics

- inflaton

- spectator fields

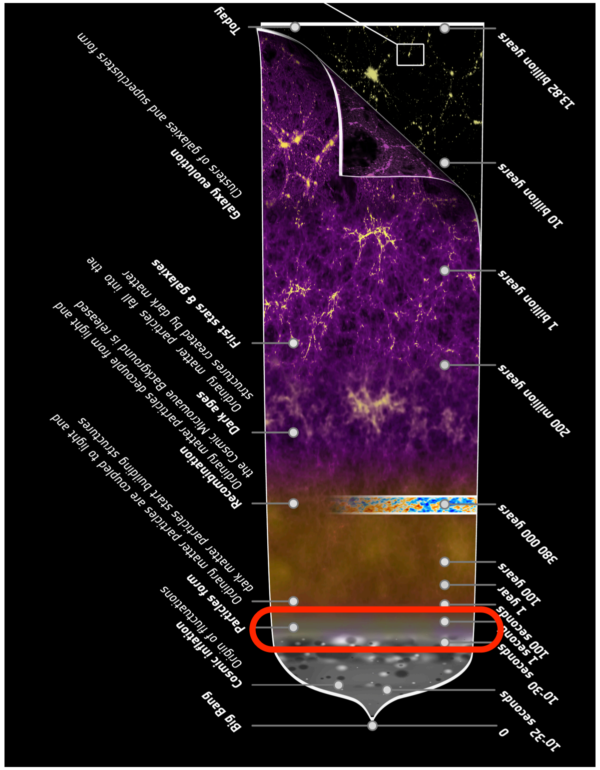
mass m << H or m > H

spin s=0, 1, 2, …..

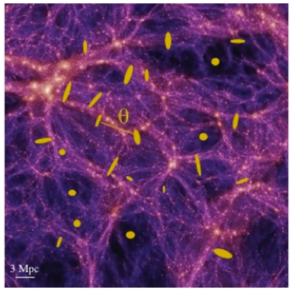
eg. s=1, axionic inflation

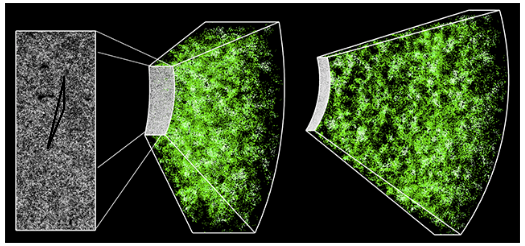
Cosmological correlators

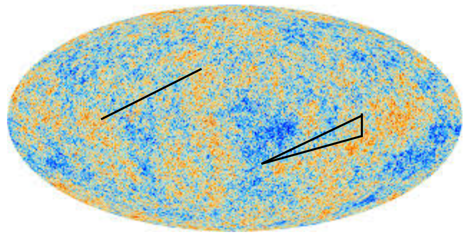
Understanding



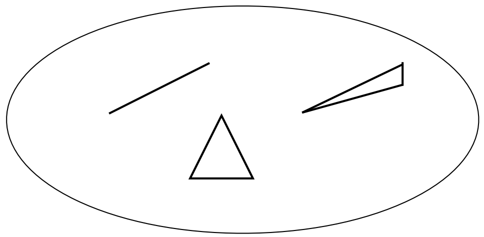
Understanding







at reheating surface



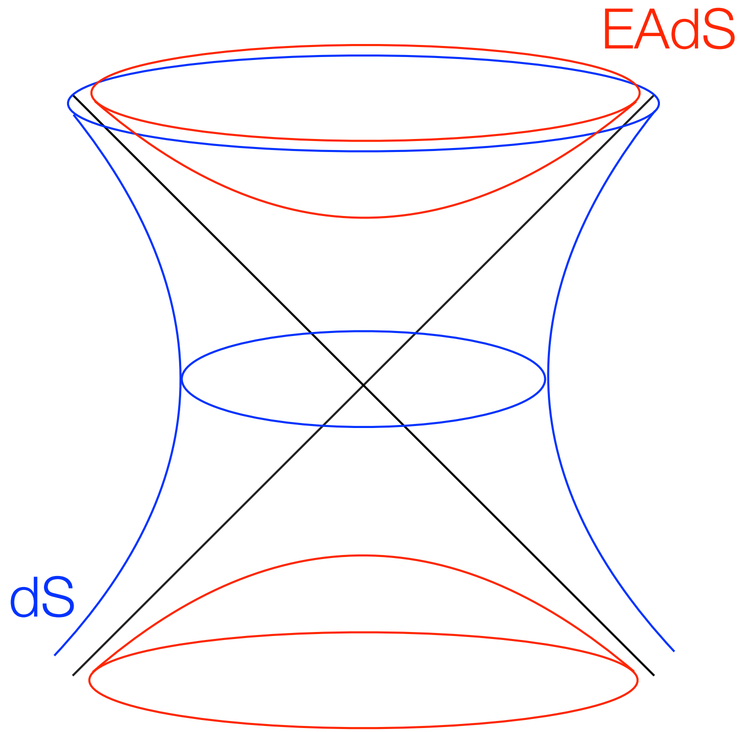
de Sitter (dS) spacetime

dS and EAdS are both embedded (d+1)-dim hyperboloid

$$\epsilon_- X_-^2 + \epsilon_0 X_0^2 + \sum_{i=1}^{d} X_i^2 = \epsilon_- l^2$$

in (d+1, 1) flat space

$$ds^2 = \epsilon_- dX_-^2 + \epsilon_0 dX_0^2 + \sum_{i=1}^{d} dX_i^2$$

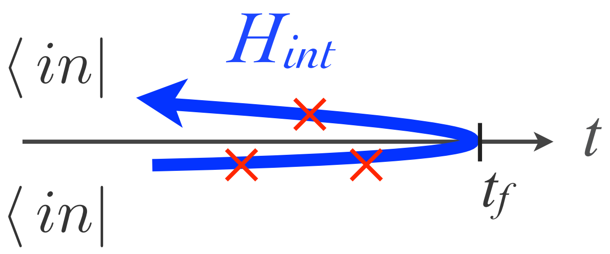


$$\begin{cases} \text{EAdS} & \epsilon_- = -1,\ \epsilon_0 = 1 \\ \text{dS} & \epsilon_- = 1,\ \epsilon_0 = -1 \end{cases}$$

c.f. AdS $\epsilon_- = \epsilon_0 = -1$

In-in formalism and BD vacuum (Euclidean vac.)

cosmological correlators are expectation values



$$G_F(x_1, x_2),\ G_D(x_1, x_2),\ G^{\pm}(x_1, x_2)$$

<u>Enclidean vacuum (Bunch-Davies vac., Adiabatic vac.,….)</u>

All the *n*-point fns. $\langle\phi(x_1)\ldots\ldots\phi(x_{n-1})\phi(x_n)\rangle$ should become regular in the limit $t_i \to -\infty(1 \pm i\varepsilon)$

free-level: dS invariant + Hadamard B. Allen(85)

4D AdS and dS

Anti de Sitter (AdS)

Vacuum with $\Lambda < 0$

in $R^{2,3}$ (-,-, +, +, +) SO(2,3)

$$-X_0^2 - X_1^2 + \sum_{a=2,3,4} X_a^2 = -l^2$$

$$ds^2 = l_{\mathrm{AdS}}^2 \left(\frac{-dt^2 + dx^2 + dy^2 + dz^2}{z^2} \right)$$

de Sitter (dS)

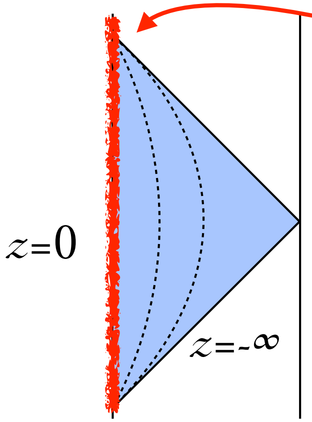
Vacuum with $\Lambda > 0$

in $R^{1,4}$ (-,+, +, +, +) SO(1,4)

$$-X_0^2 + X_1^2 + \sum_{a=2,3,4} X_a^2 = l^2$$

$$ds^2 = l_{\mathrm{dS}}^2 \left(\frac{-d\eta^2 + dx^2 + dy^2 + dw^2}{\eta^2} \right)$$

Boundary

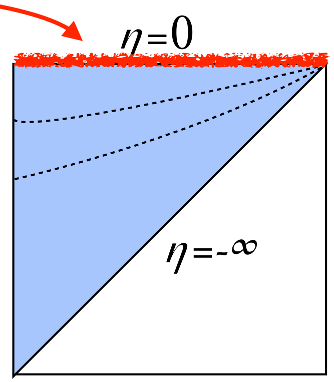


z:const, R^3

$$l_{\mathrm{AdS}} \to i l_{\mathrm{dS}}$$

$$z \to i\eta$$

$$t \to iw$$



η:const, R^3

dS/CFT conjecture

Strominger(01), Witten(01), Maldacena(02),...

dS and AdS are mathematically similar, yet physically different.

- Holographic direction is timelike, CFT lives on spatial boundary.

- Dual CFT is non-unitary.

e.g., $(\Box - m^2)\phi = 0$

4dim AdS $\Delta = \frac{3}{2} \pm \sqrt{\frac{9}{4} + m^2 l_{\rm AdS}^2}$

4dim dS $\Delta = \frac{3}{2} \pm \sqrt{\frac{9}{4} - m^2 l_{\rm dS}^2}$

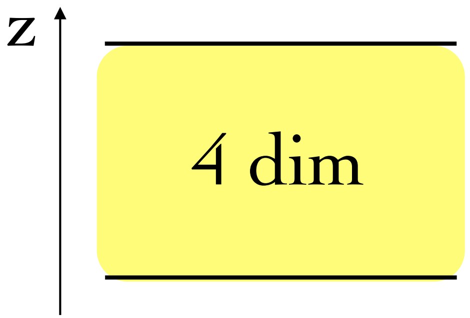
Principal series in dS ~ Below BF found in AdS Isono, Liu, Noumi (20)

- Lack of concrete examples.

… yet, Anninos, Hartman, Strominger(11)

- Relation with area law? (...yet Tetsuya's talk)

Bulk to Boundary



z=z1 holographic plane → CFT

$$\phi(z,\,k) = \mathcal{G}_k(z,\,z_1)\hat{\phi}(k)$$

z=z2 trivial B.C. (BD vacuum for dS)

using eom for 4-dim bulk theory $\delta S \sim \mathcal{L} dz\Big|_{z=z_2}^{z=z_1}$

- Massless free scalar field on EAdS

with $Z \sim e^{-S_{\rm EAdS}}$ $\langle \mathcal{O}(\mathbf{k})\mathcal{O}(\mathbf{k}')\rangle \sim \delta(\mathbf{k}+\mathbf{k}')l_{\rm AdS}^2 k^3$

- Massless free scalar field on dS

with $Z \sim e^{iS_{\rm dS}}$ $\langle \mathcal{O}(\mathbf{k})\mathcal{O}(\mathbf{k}')\rangle \sim \delta(\mathbf{k}+\mathbf{k}')l_{\rm dS}^2 (-k^3)$

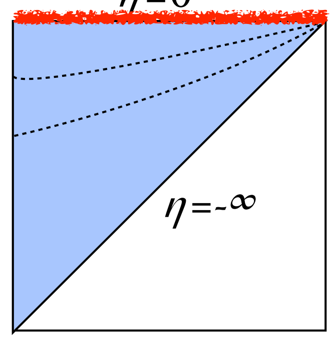
$$l_{\rm dS} = i\, l_{\rm AdS}$$

Maldacena (02)

Boundary to Bulk (dS)

- Wave function on future boundary

Maldacena (02)



$$\Psi[\hat{\phi}] \sim Z[\hat{\phi}] = \langle e^{-\hat{\phi}\cdot\mathcal{O}} \rangle$$

Bulk correlators

$$\langle \hat{\phi}(\mathbf{k}_1)\cdots\hat{\phi}(\mathbf{k}_n)\rangle = \int D\hat{\phi}\,\hat{\phi}(\mathbf{k}_1)\cdots\hat{\phi}(\mathbf{k}_n)|\Psi[\hat{\phi}]|^2$$

e.g. $\langle \hat{\phi}(\mathbf{k})\hat{\phi}(-\mathbf{k})\rangle' = -\dfrac{1}{2\mathrm{Re}[\langle \mathcal{O}(\mathbf{k})\mathcal{O}(-\mathbf{k})\rangle']}$

<u>Harrison-Zeldovich spectrum</u> (← 0th approx. of CMB spectrum)

$$\langle \mathcal{O}(\mathbf{k})\mathcal{O}(-\mathbf{k})\rangle' \propto k^{6-\Delta} \propto k^3 \longrightarrow \langle \hat{\phi}(\mathbf{k})\hat{\phi}(-\mathbf{k})\rangle' \propto 1/k^3$$

↑ $\Delta_+ = \frac{3}{2} + \sqrt{\frac{9}{4} - m^2 l_{\mathrm{dS}}^2} \sim 3$

c.f. Inflation w/deformed CFT

McFadden, Skenderis (09, 10, 11,...) Bzowski et al. (12), Schalm et al. (12)
Garriga & Y.U. (13, 16), Garriga, Skenderis, Y.U (14), ...

Cosmological bootstrap

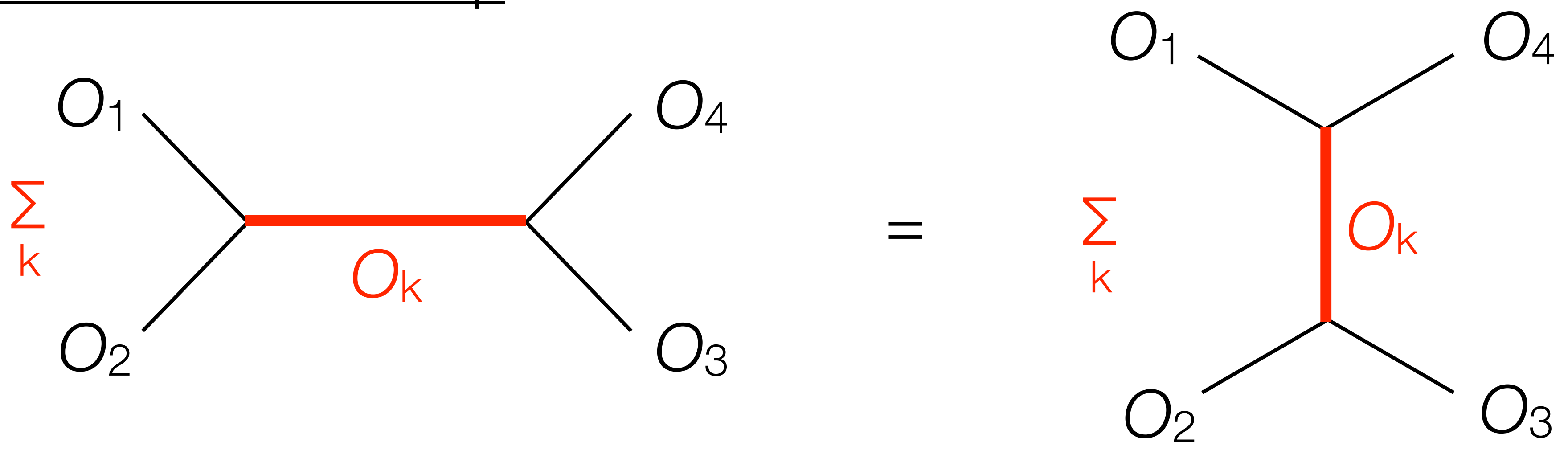
Arkani-Hamed, Baumann, Lee, & Pimental JHEP 04 (20) 105

Baumann, Pueyo, Joyce, Lee, & Pimental JHEP 12 (20) 204

Baumann, Pueyo, Joyce, Lee, & Pimental 2005.04234

Cosmological bootstrap

Conformal bootstrap



(Goal of) Cosmological bootstrap

Arkani-Hamed, Baumann, Lee, & Pimentel (18)

- Imposing de Sitter symmetry.
- Imposing correct UV energy singularities.

(+ Choosing Bunch-Davies vacuum, …..)

⟶ Possible (4pt) exchange diagrams

Scattering amplitude

- Getting around redundancies in Lagrangian.
- Bootstrapping scattering amplitude from physical principles.

<u>Massless spinning particles</u>

Compute $$A(1^{h_1}\cdots n^{h_n}) = e^{h_1}_{\mu_1}\ldots e^{h_n}_{\mu_n} A^{\mu_1\cdots\mu_n},$$

using spin helicity variables $[ij] = \tilde{\lambda}_{i\dot{\alpha}}\tilde{\lambda}_{j\dot{\beta}}\epsilon^{\dot{\alpha}\dot{\beta}},\ \langle ij\rangle = \lambda_{i\alpha}\lambda_{j\beta}\epsilon^{\alpha\beta}$

(MHV) Gluon scattering amplitude

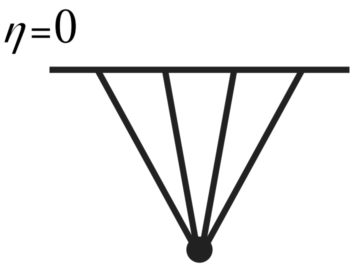
$$A(\cdots i^-\cdots j^-\cdots) = \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

Parke & Taylor (86)

How to connect with UV?

1) Identify contact diagrams from UV singularity

Maldacena & Pimental (11)
Arkani-Hamed & Maldacena (15)



$$\sim \int_{-\infty}^{0} d\eta\, \eta^{p-1} e^{ik_t\eta} A(k_1, k_2, k_3, k_4) \to \frac{A_{\text{flat}}}{k_t^p}$$

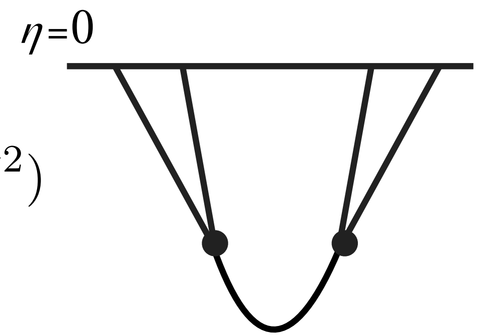
$$k_t \equiv \sum_n |\mathbf{k}_n| \to 0$$

2) Compute exchange diagrams from seed 1)

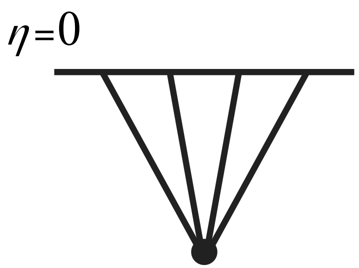
Arkani-Hamed et al. (18)

WT identify of SCT

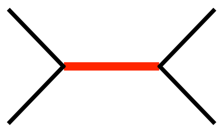
$$(\Delta_u + M^2)$$



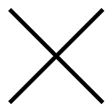
=



analogous to $(s - M^2)$



=



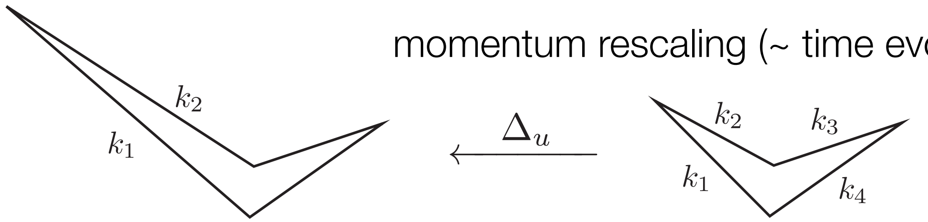
Cosmological colliders

Arkani-Hamed & Maldacena (15)
Arkani-Hamed et al. (18)

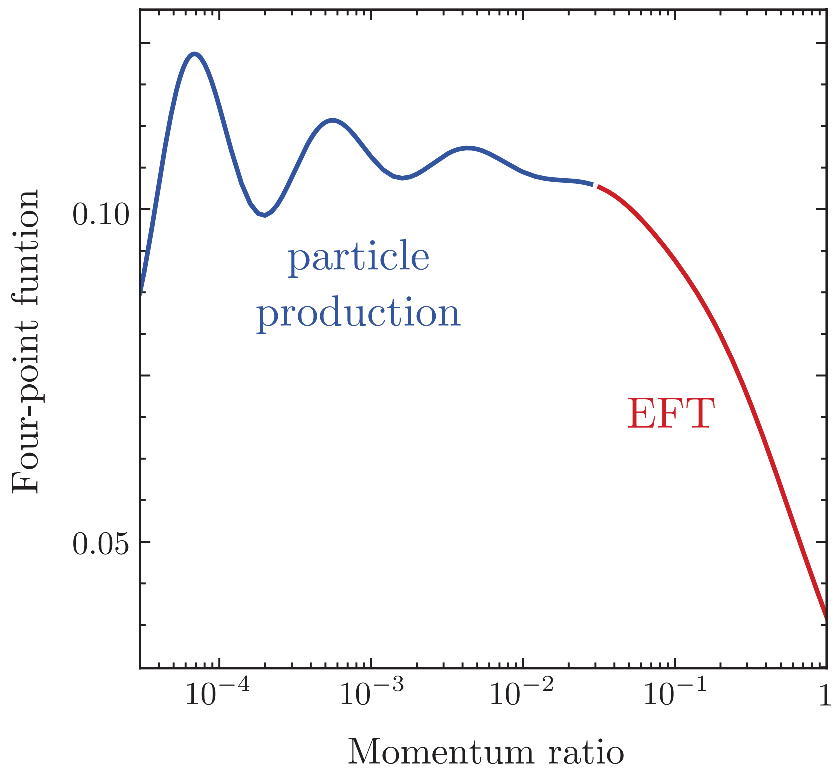
$$u \equiv \frac{s}{k_1 + k_2}\,, \qquad v \equiv \frac{s}{k_3 + k_4}$$

$$s \equiv |\mathbf{k}_1 + \mathbf{k}_2|$$

momentum rescaling (~ time evolution) time w/o time



$$\Delta_u \equiv u^2(1-u^2)\partial_u^2 - 2u^3\partial_u$$



ext. conformal scalar

Excitation of massive scalar Chen & Wang (09), Noumi, Yamaguchi, Yokoyama (12)…..,

Mellin space approach

Sleight JHEP 01 (20) 090

Sleight & Taronna JHEP 02(20) 098

Sleight & Taronna arXiv:2007.09993

Mellin transformation

Various advantages to analyze CFT (on $\boldsymbol{R}^d$). Mack(08)

encoding unitarity, causality, … Recall Toshifumi's talk

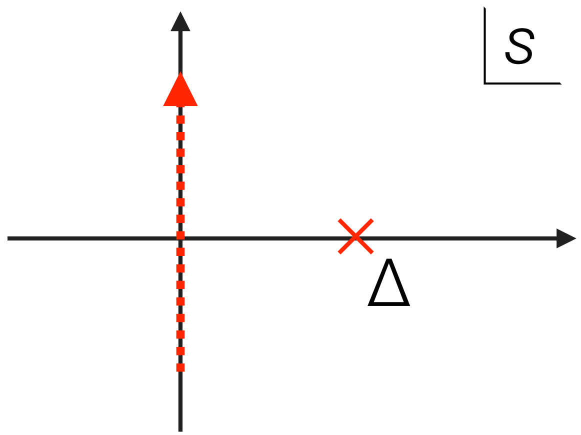
c.f. Fourier trans. 2pt fn. $1/(x^2)^\Delta \longleftrightarrow (p^2)^{\Delta-d/2}$ branch cut at origin

<u>Mellin transformation</u>

$$f(x) = \int_{-i\infty}^{i\infty} ds x^{-s} \tilde{f}(s)$$

ICTP lecture by Gopakumar

Picking out different scaling behaviors for power-law decomposition.



for $\tilde{f}(s) \sim \frac{1}{s-\Delta}$ $(\Delta>0)$

x >1 $f(x) \sim 1/x^\Delta$

Understanding dS from AdS

Sleight (19)
Sleight & Taronna (19)
Sleight & Taronna (20)

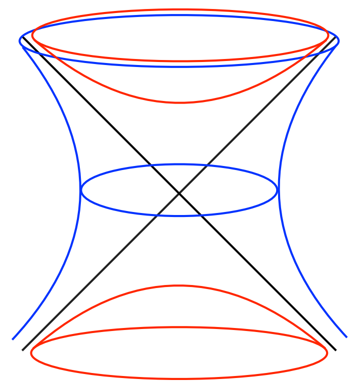
AdS 1) Well defined notion of Unitarity

2) Non-perturbative understanding of singularities

Using AdS as a guide to understand dS.

- Bulk-boundary/Bulk-Bulk propagator in

  Mellin-Barnes representation for dS.



dS ⟷ EAdS

analytic continuation

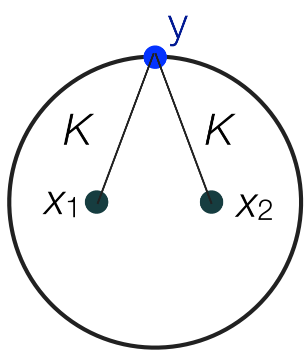
$z = -e^{\pm i\frac{\pi}{2}}\eta$

Mellin-Barnes (MB) representation

Sleight(19)
Sleight & Taronna(19)

Harmonic function in AdS $\left(\nabla^2_{\mathrm{AdS}} - m^2\right)\Omega_\nu\left(x_1, x_2\right) = 0$

$$\Omega_\nu\left(x_1, x_2\right) = \int_{\partial\mathrm{AdS}}$$



⟶ Fourier trans.

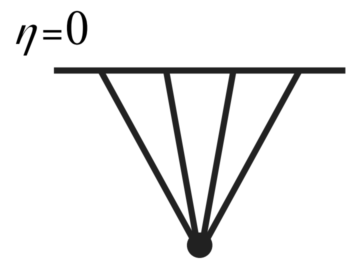
$$\Omega_{\nu,\vec{k}}(z_1; z_2) = \frac{\nu^2}{\pi} K_{\frac{d}{2}+i\nu}(z_1, \vec{k}) K_{\frac{d}{2}-i\nu}(z_2, -\vec{k})$$

K: bulk-boundary propagator

Expressing K in MB rep, Performing analytic continuation $z = -e^{\pm i\frac{\pi}{2}}\eta$

MB rep. of Wightman fn. for BD vac. in dS

$$G_{\mathbf{k}}(\eta_1, \eta_2) \propto (\eta_1\eta_2)^{d/2} \prod_{i=1}^{2} \int_{-i\infty}^{i\infty} \frac{du_i}{2\pi i} (\cdots) \prod_{j=1}^{2} \underbrace{\Gamma(u_j + i\frac{\nu}{2})\Gamma(u_j - i\frac{\nu}{2})}_{\text{poles}} (-\frac{k\eta_j}{2})^{-2u_j}$$



- bulk-boundary(bulk) propagator by sending $\eta_1 \to 0$
- contact diagram by integrating bulk-boundary propagator

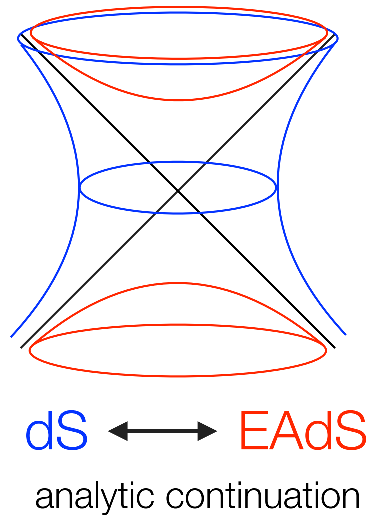
Understanding dS from AdS

Sleight (19)
Sleight & Taronna (19)
Sleight & Taronna (20)

AdS 1) Well defined notion of Unitarity

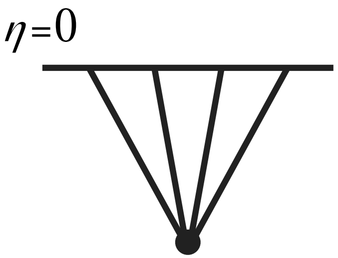
2) Non-perturbative understanding of singularities

Using AdS as a guide to understand dS.

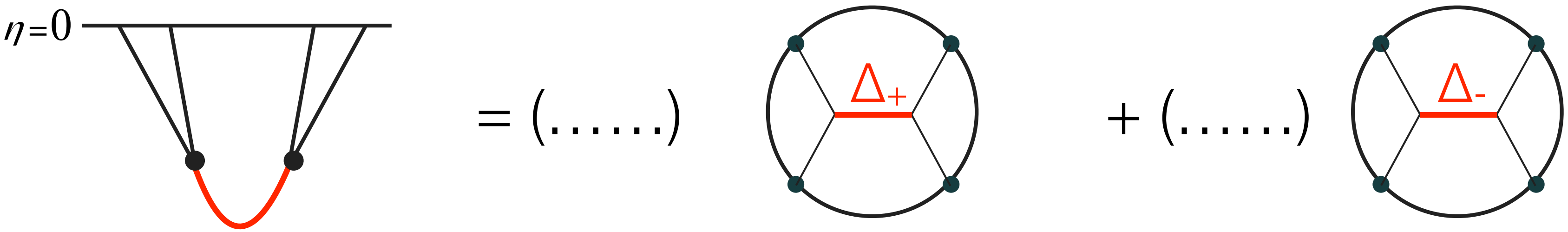


* external scalar field is general. Recall Arkani-Hamed et al. (18)

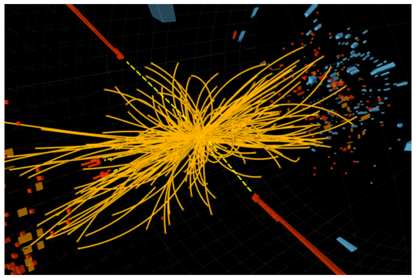
- Propagators in Mellin-Barnes rep. for dS.
- Contact interaction



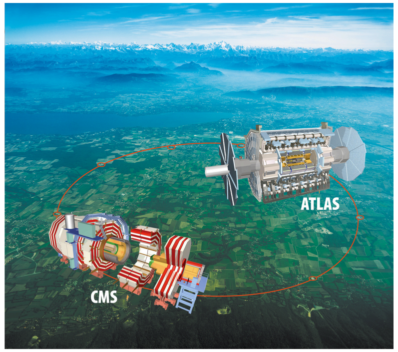
- Relation between exchange diagrams for dS and AdS



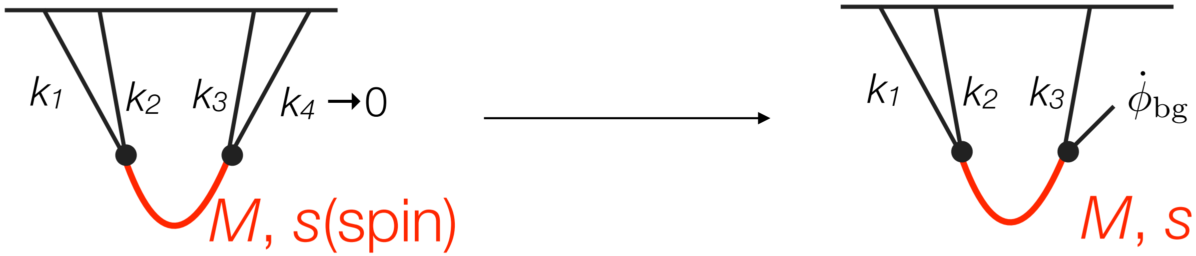
<u>Detectability of spinning particle</u>



✔



3pt fn. from spinning particle



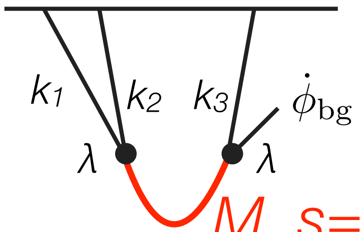
$k_3/k_1, k_3/k_2 << 1$

Particle creation

$\langle\delta\Phi\delta\Phi\delta\Phi\rangle \propto (\mathbf{k}_1 \cdot \mathbf{k}_3)^s$

Arkani-Hamed & Maldacena (15)

Minimum extension from dS



$$\sim \lambda^2 \dot{\phi}_{\rm bg}\, e^{-M/T_{\rm H}} \left(\frac{k_1}{k_3}\right)^{\Delta} (\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_3)^s \mathcal{P}(k_1)\mathcal{P}(k_3)$$

Arkani-Hamed & Maldacena (15)

$k_3/k_1, k_3/k_2 << 1$

$$\Delta = \frac{3}{2} \pm i\sqrt{\left(\frac{M}{H}\right)^2 - \left(s - \frac{1}{2}\right)^2}$$

$\mathcal{P}(k)$:power spectrum

1) Boltzmann suppression

Higuchi bound $M > O(H)$

2) Dilution of massive fields

~~dS~~ Bordin et al. (18)

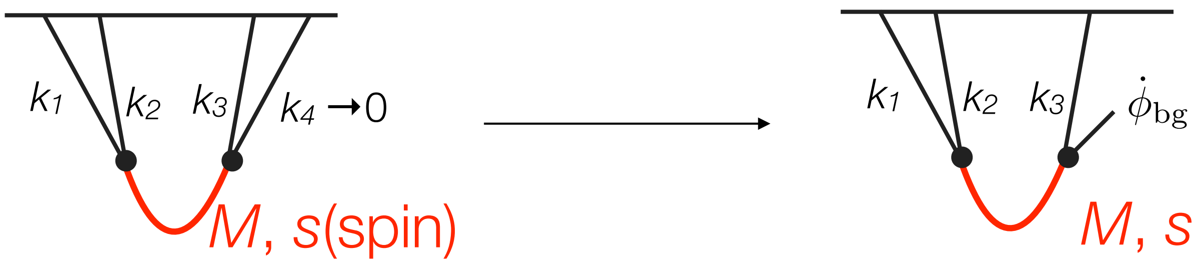
global ~~rotation~~ Kehagias & Riotto (17)

3) Weak coupling

Large coupling tends to make 1) more severe.

Wang & Xianyu (19, 20)

3pt fn. from spinning particle



$k_3/k_1, k_3/k_2 << 1$

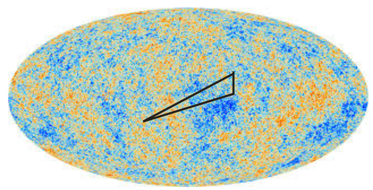
Particle creation

$\langle \delta\Phi\delta\Phi\delta\Phi \rangle \propto (\mathbf{k}_1 \cdot \mathbf{k}_3)^s$

Arkani-Hamed & Maldacena (15)

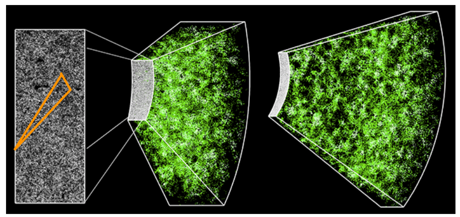
1) Cosmic microwave background $\langle \delta T \delta T \delta T \rangle$

PLANCK18, Bartolo et al. (17), Franciolini et al. (18), Bordin & Cabass (19)



2) Galaxy number count $\langle \delta n \, \delta n \, \delta n \rangle$

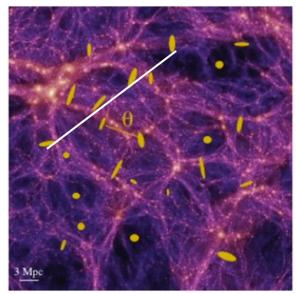
Moradinezhad et al (18[1], 18[2])



3) Galaxy shape correlation <(shape) (shape)>

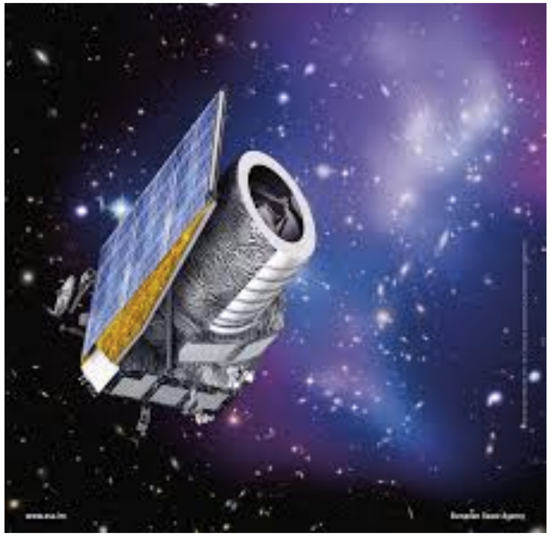
Schmidt et al (15, 16), Kogai, Matsubara, Nishizawa, Y.U. (18)

Kogai, Akitsu, Schmidt, Y.U. (20)

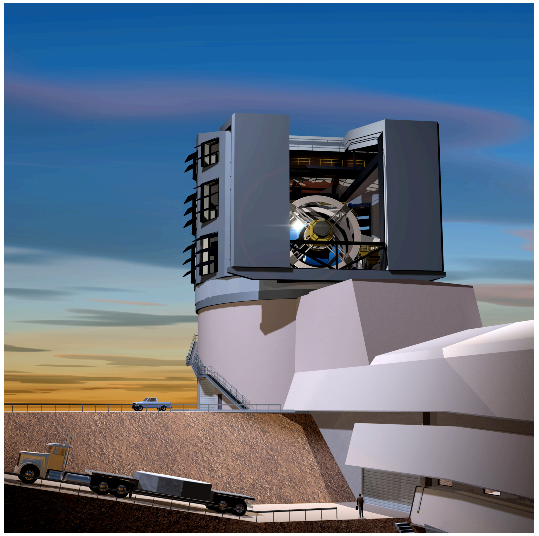


LargeScaleStructure surveys in the next decade

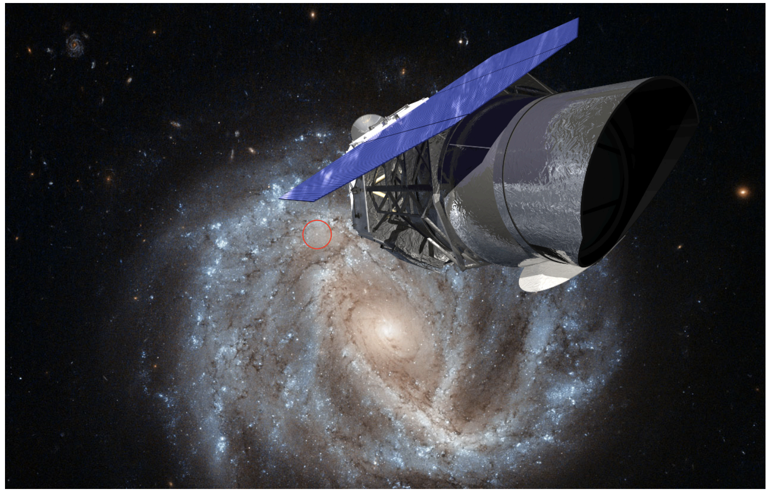
Wide and **Deep** survey missions



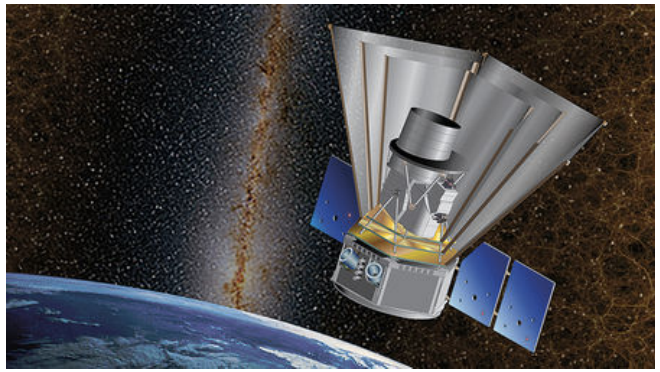
Euclid launch 2020



LSST science operation mid2020s



Roman space telescope launch mid2020s



SPHREx launch 2023

Galaxy imaging survey as spin sensitive detector

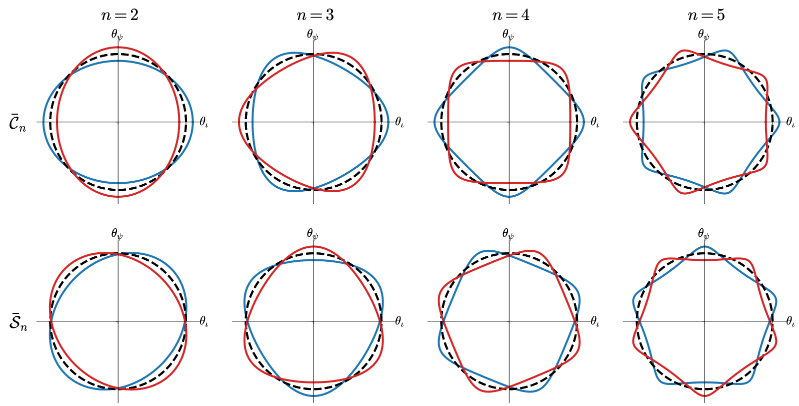
Galaxy shape traces tidal force $\Phi[\partial\partial\Phi]^{TL}$, $\Phi[\partial\partial\partial\Phi]^{TL}$, $\Phi[\partial\partial\partial\partial\Phi]^{TL}$, …

Angular dep. PNG of spin-s particle $B_\Phi(\boldsymbol{k}_S, \boldsymbol{k}_L) \propto \mathcal{P}_s(\hat{\boldsymbol{k}}_L \cdot \hat{\boldsymbol{k}}_S)$

Arkani-Hamed & Maldacena (15)

Kogai, Akitsu, Schmidt, & Y.U. (20)

decomposition



Imprints of PNG from spin s

s=2 s=3 s=4 s=5

<u>LSST like Forecast</u> s=4 "massless" O(10), massive O(10^5) needs deeper survey

from galaxy bispectrum $\sigma(\mathcal{A}_4) \sim 21$ Moradinezhad Dizgah et al. (18)

What about w/o de Sitter symmetry?

w/dS, studies of dS are still useful??

gδN formalism

T.Tanaka & Y.U. 2101.05707

w/Takahiro Tanaka (Kyoto, YITP)

Large scale evolution

~~dS~~, global ~~rotation~~, etc…, often become significant at large scales.

t		
late	$\partial/\partial_t \sim H \gg \partial/\partial x^i_{\rm phys}$	} large scale evolution
	$\sim$	
early	$\ll$	} w/dS sym.

Large scale evolution **for scalar field system**

Gradient expansion (GE) → delta N formalism

Salopek&Bond (90)
Shibata & Sasaki (90),
Deruelle & Langlois (94),…

Starobinsky (82, 85),
Sasaki & Stewart (95),
Sasaki & Tanaka (98),
Lyth, Malik, & Sasaki (04),…

Large scale evolution

~~dS~~, global ~~rotation~~, etc…, often become significant at large scales.

t

late $\quad \partial/\partial_t \sim H \gg \partial/\partial x^i_{\rm phys}$ } large scale evolution

$\sim$

early $\quad \ll$ } w/dS sym.

Large scale evolution ~~**for scalar field system**~~

Gradient expansion (GE) → ~~δN formalism~~ g(eneralized)δN formalism

Salopek&Bond(90)
Shibata & Sasaki(90),
Deruelle & langlois (94),…

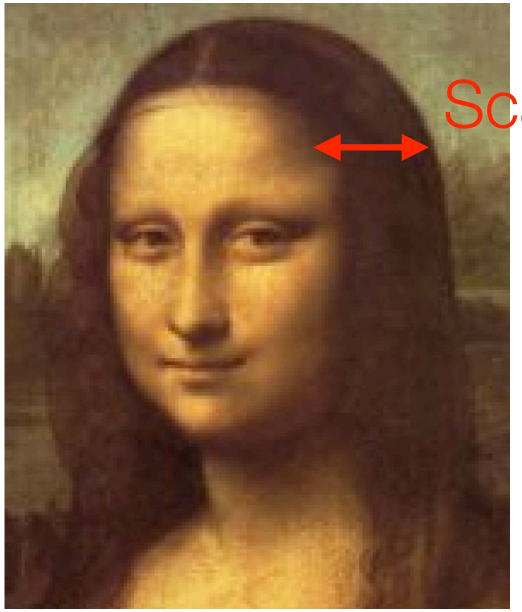
Starobinsky(82, 85),
Sasaki & Stewart (95),
Sasaki & Tanaka (98),
Lyth, Malik, & Sasaki (04),…

Tanaka & Y.U.(21)

Separate Universe approach ~ Mosaic art

Basic assumption in gradient expansion

size of mono color stone
= smoothing scale λ_s



Scale of interest

λ (>> λ_s)

Fine-grained view

Coarse-grained view

Let her get aged.

(☆) Her history in detailed view = Her history in coarse-grained view

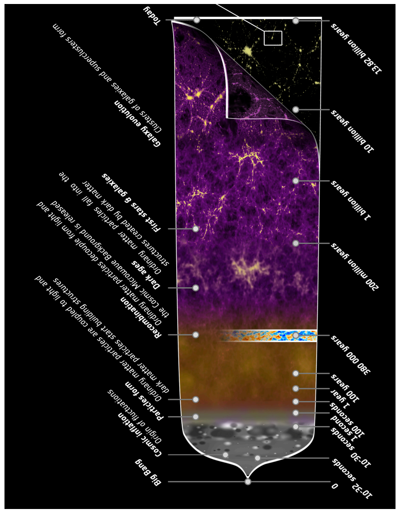
Separate universe

Salopek&Bond(90)

(☆) Evolution of inhomogeneous Universe
= Evolution of glued numerous homogeneous universes

Scale of interest λ >> Smoothing scale λ_S > Size of casual patch

~ c_S/H



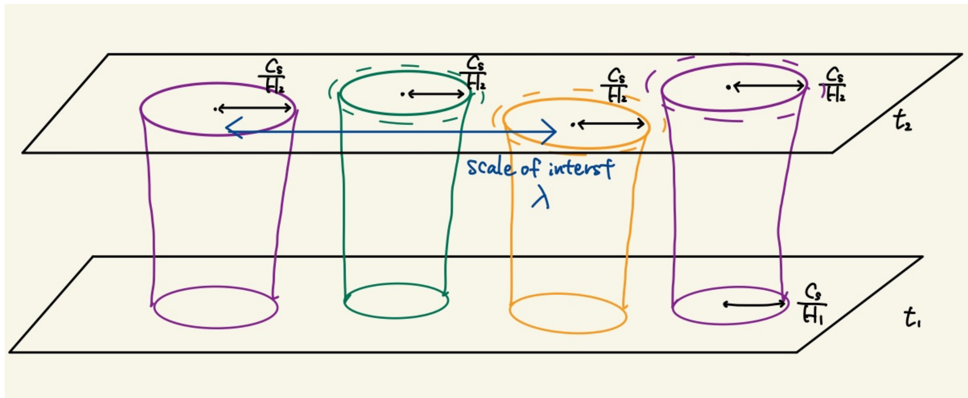
=

(☆)

t

late

early



Fine-grained view

Solving PDEs

Coarse-grained view

Solving ODEs
(Inhomogeneity: Different ICs)

Separate universe

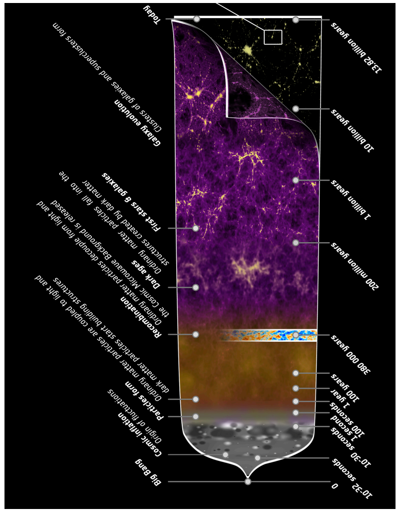
Salopek&Bond (90)

(☆) Evolution of inhomogeneous Universe

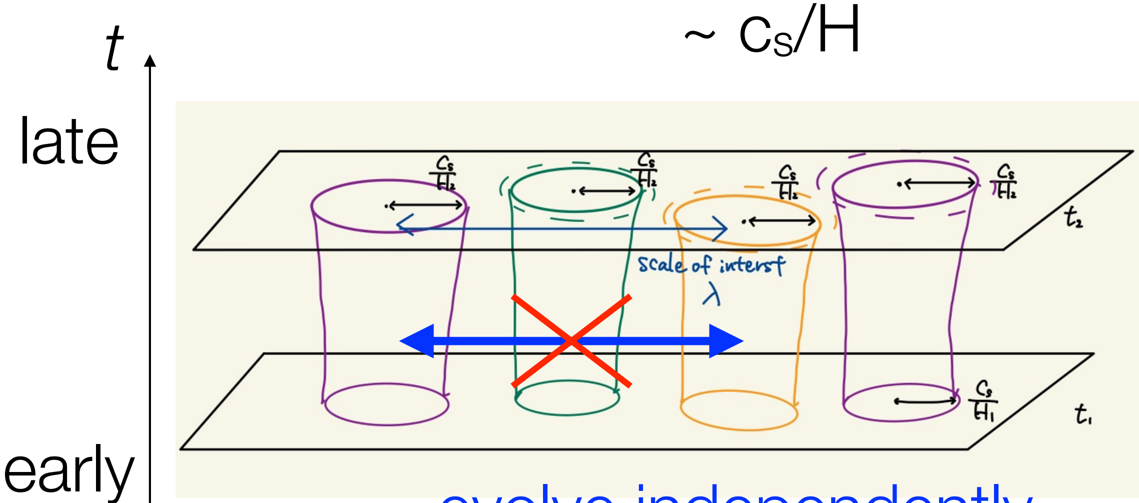
= Evolution of glued numerous homogeneous Universes

Scale of interest λ >> Smoothing scale λ_S > Size of casual patch

~ c_S/H



= (☆)



evolve independently

When (☆) holds?

gδN formalism

Tanaka & Y.U. (21)

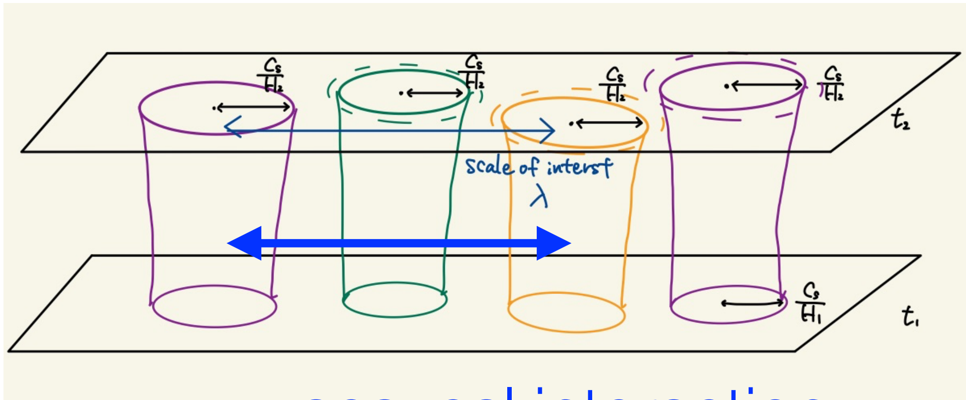
a local theory w/gravity, spatial Diff

fields in $\mathcal{L}$ ⟶ physical fields $\{\varphi_{phys}\}$

solving constraints ($\mathcal{C}$)

non-local

e.g., MC $\mathcal{H}_i \equiv \dfrac{\partial(N\sqrt{g}\mathcal{L})}{\partial N^i} = 0$ $\qquad \partial_i(\cdots) + (\cdots)\partial_i(\cdots) = 0$



acausal interaction

~~(☆)~~

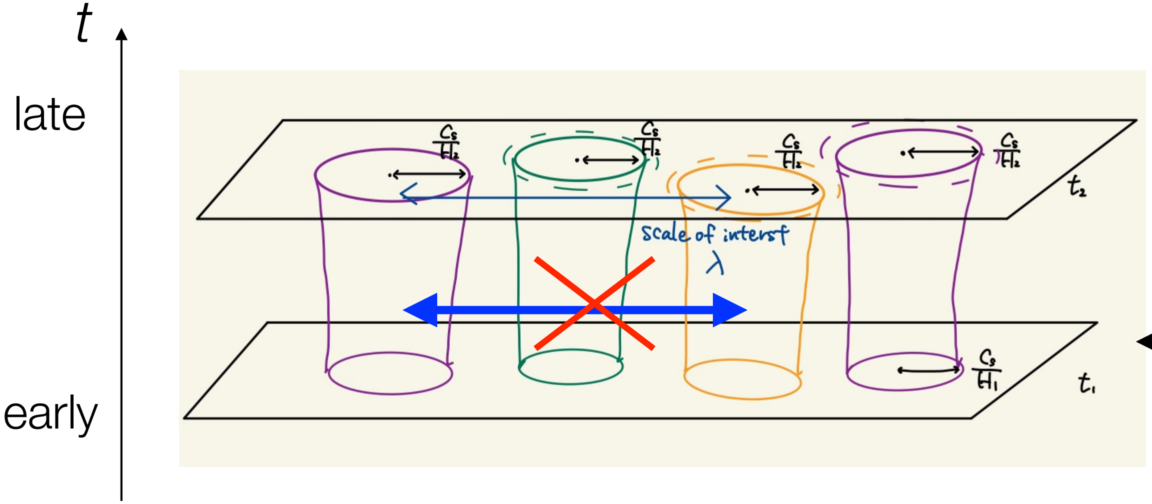
gδN formalism

Tanaka & Y.U. (21)

a local theory w/gravity, spatial Diff

fields in $\mathcal{L}$ ⟶ $\{\varphi_{phys}, \varphi_{auxi}\}$ ⟶ physical fields $\{\varphi_{phys}\}$

solving local C — solving nonlocal C — non-local



(☆) ✔

IC by $\{\varphi_{phys}, \varphi_{auxi}\}$

at $\partial/\partial_t \sim H \sim \partial/\partial x^i_{\rm phys}$

density pert. $\zeta(t_{\rm final}) = \zeta_{\rm hom}(t_{\rm final}; \{\varphi_{\rm phys}, \varphi_{\rm auxi}\})$

incl. spinning particles

GW $\gamma_{ij}(t_{\rm final}) = \gamma_{ij\,{\rm hom}}(t_{\rm final}; \{\varphi_{\rm phys}, \varphi_{\rm auxi}\})$

* Initial correlators should satisfy nonlocal C.

Infrared universality

N.B. non-local terms from MC decay with $1/V_{phys}$ for scalar field system

Sugiyama, Komatsu, & Futamase (12)   Garriga, Y.U., & Vernizzi (16)

↑ Noether charge for spatial Diff. invariance.   Tanaka & Y.U. (21)

if there is no other non-local C, approximate locality for $\{\varphi_{phys}\}$

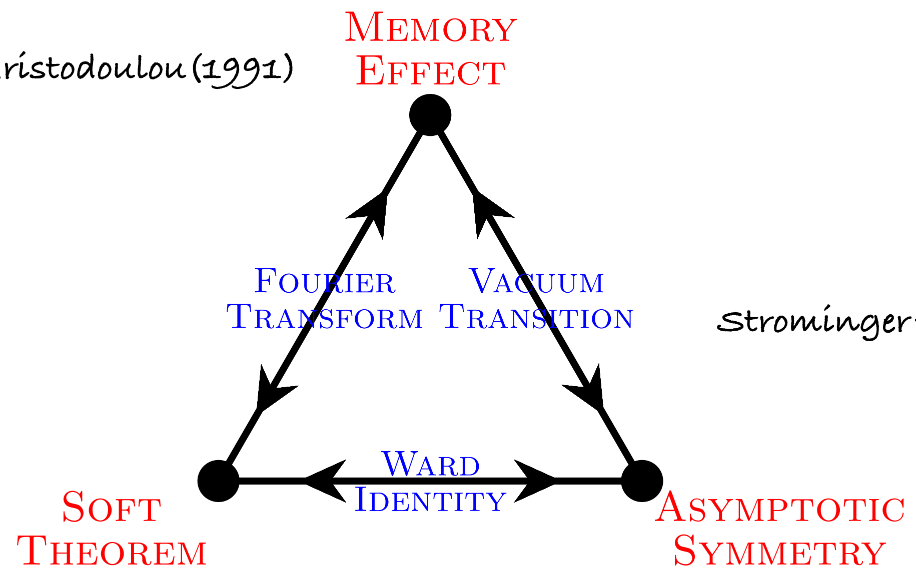
Similar structure for cosmological correlators yet w/nontrivial cond.

Tanaka & Y.U. (17, in prep)

* Condition for ∃WAM

Weinberg (02)

Christodoulou (1991)



Strominger+ (13,14, ...)

Cancellation of IR divergence

Faddeev-Kulish (1970)

Recall Hotta-san's talk

Bloch & Nordsieck (1937)
Weinberg (1965)

Bondi, Metzner, Sachs (1962)

Summary: Comprehensive studies of inflation