Time without a Hamiltonian, the curious case of supersymmetric extremal black holes.

Kyoto Symposium
September 2022

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https://arxiv.org/abs/2207.00407

https://arxiv.org/abs/2207.00408

Useful discussions with Joaquin Turiaci and Vladimir Narovlansky

I was asked to give an extended introduction.

So we will first review some general ideas

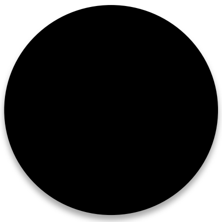
Black holes

- We think that black holes, as seen from the outside, can be described as quantum systems.

“Central dogma”

A black hole as a quantum system

- A black hole seen from the outside can be described as a quantum system with S degrees of freedom (qubits). S = Area/4 ($l_p = 1$)
- It evolves according to unitary evolution, as seen from outside.



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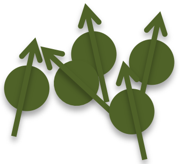
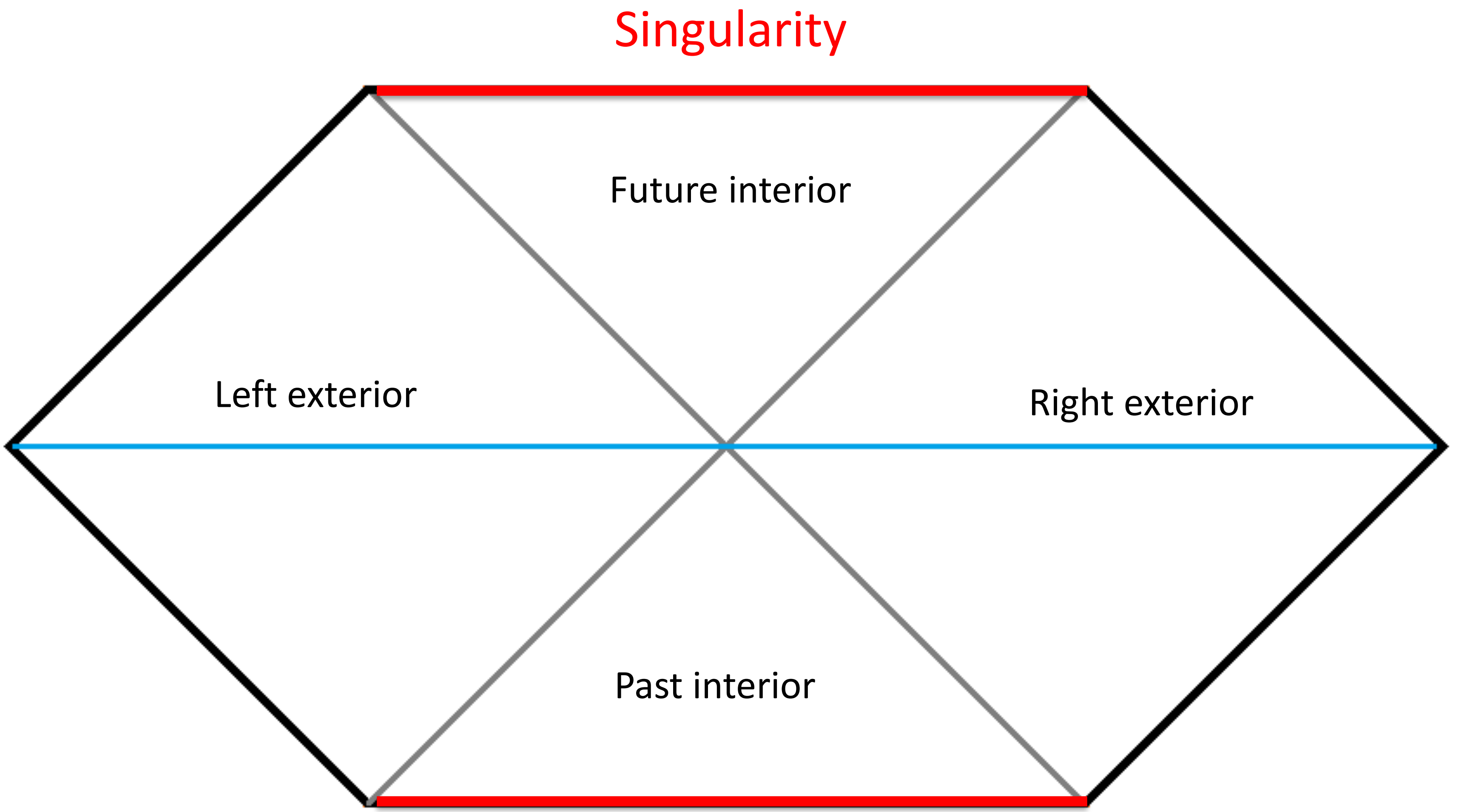


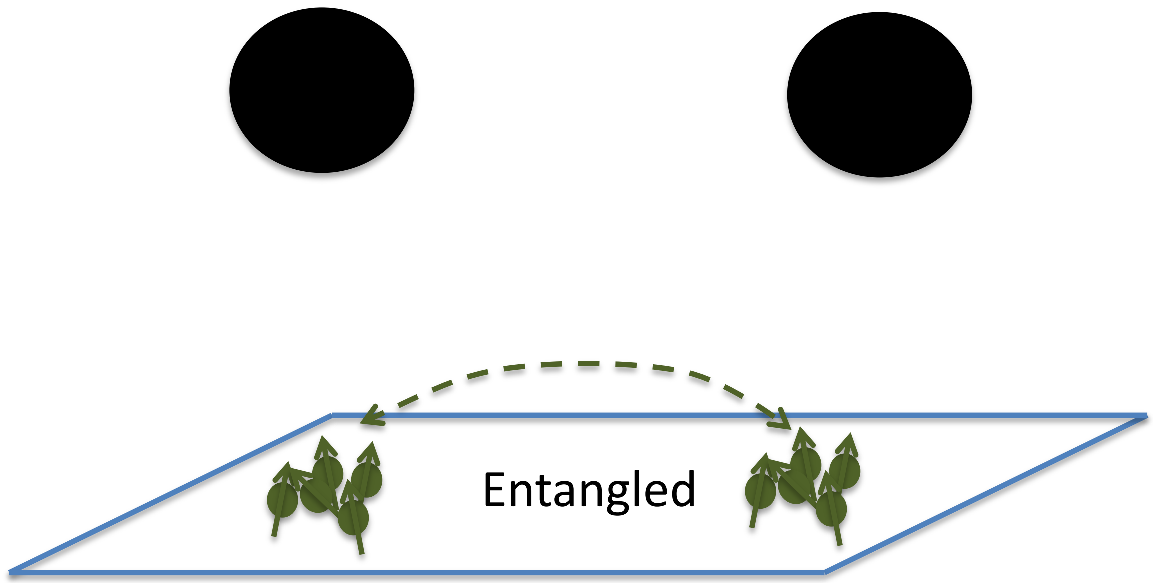
The Geometry of the simplest black hole solution



This solution contains two black holes

How should we interpret it?

Replace each black hole by a quantum system



In a particular entangled state

W. Israel
JM
JM, Susskind

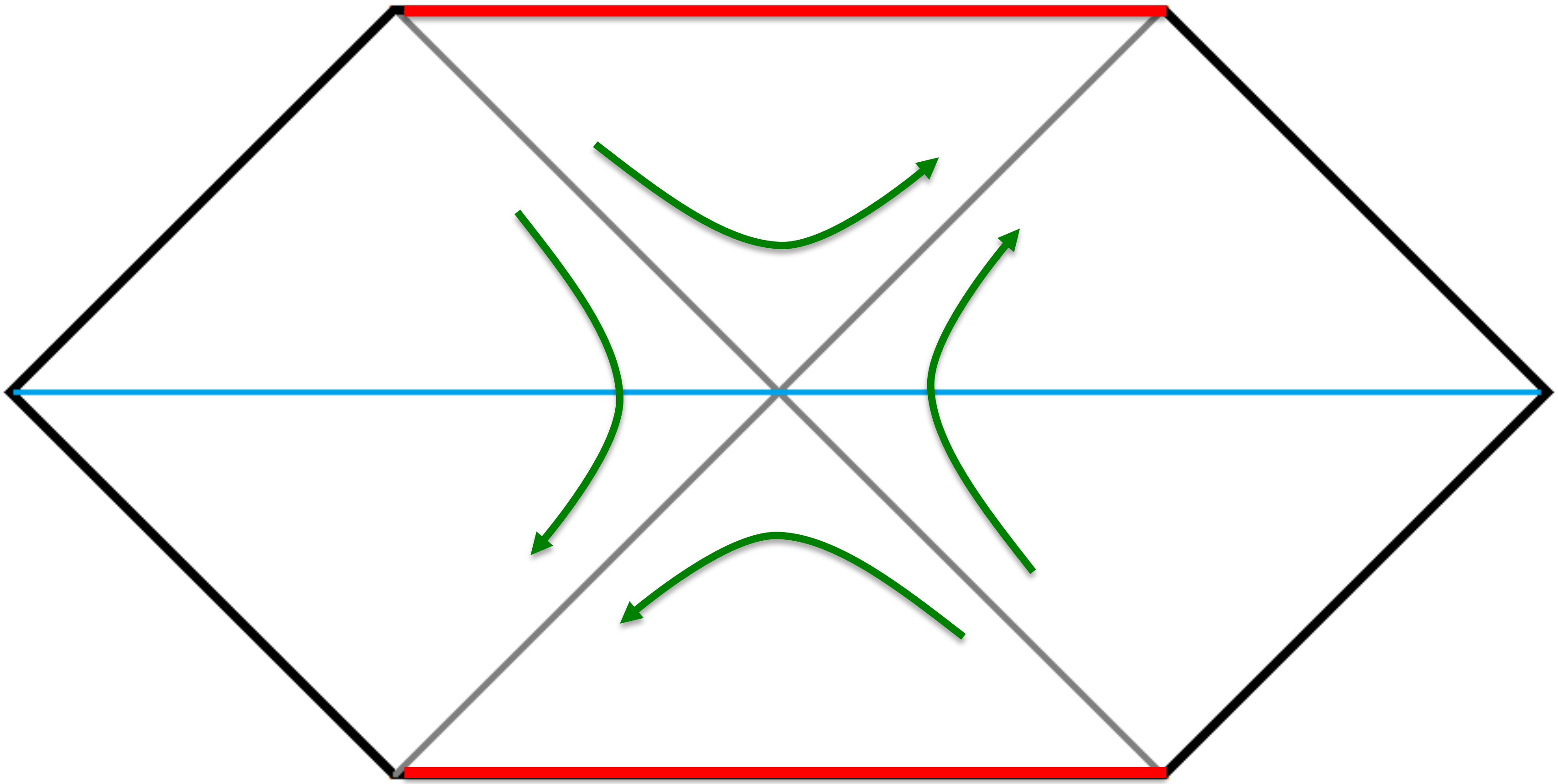
ER = EPR

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

(state at t=0)

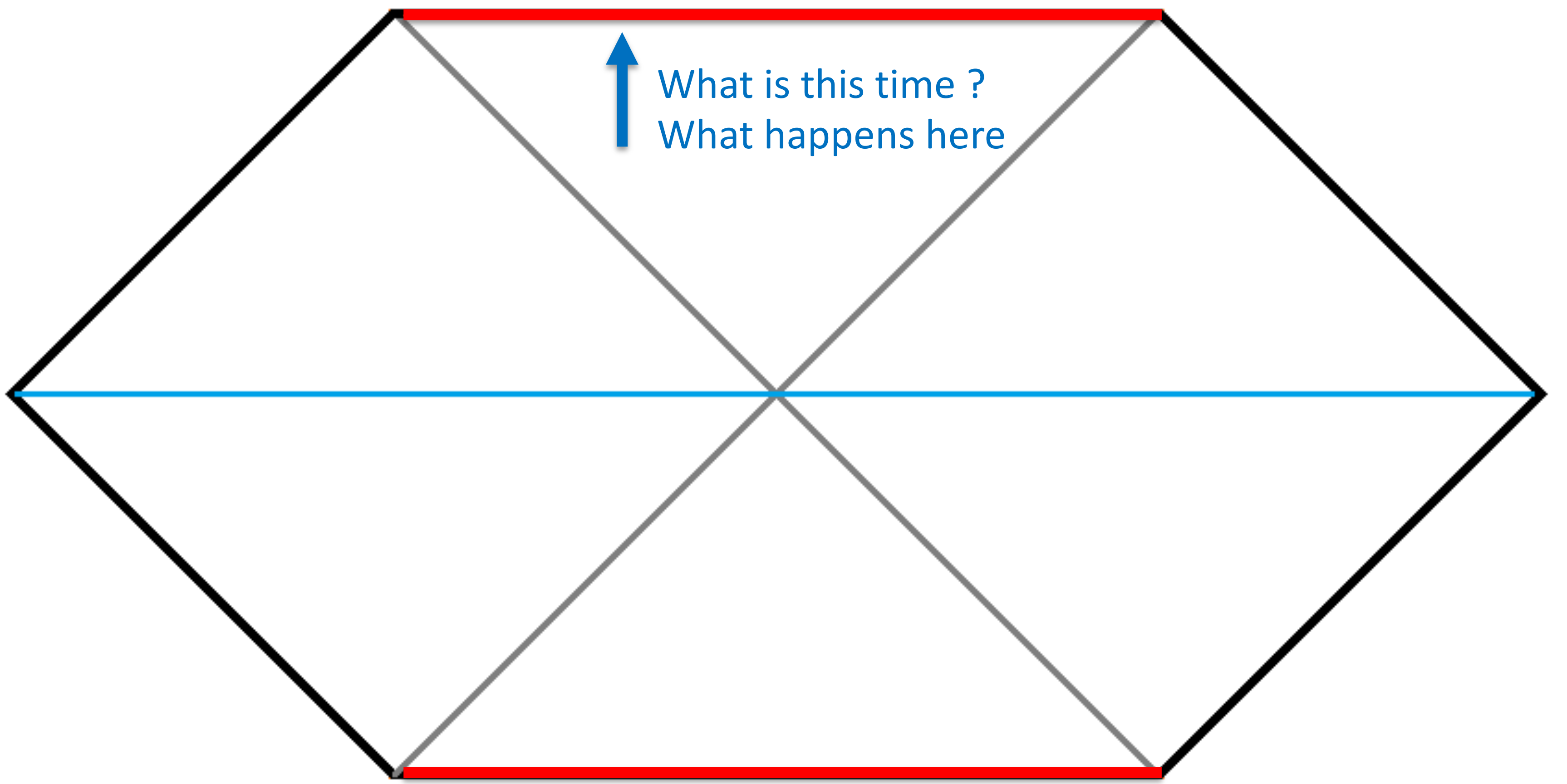
$$H_R - H_L$$

Acts as a boost at the middle



The most interesting question

Singularity



We will not answer this question…

We will try to analyze somewhat "simpler" problems…

As usual in physics, we will study problems with more symmetry

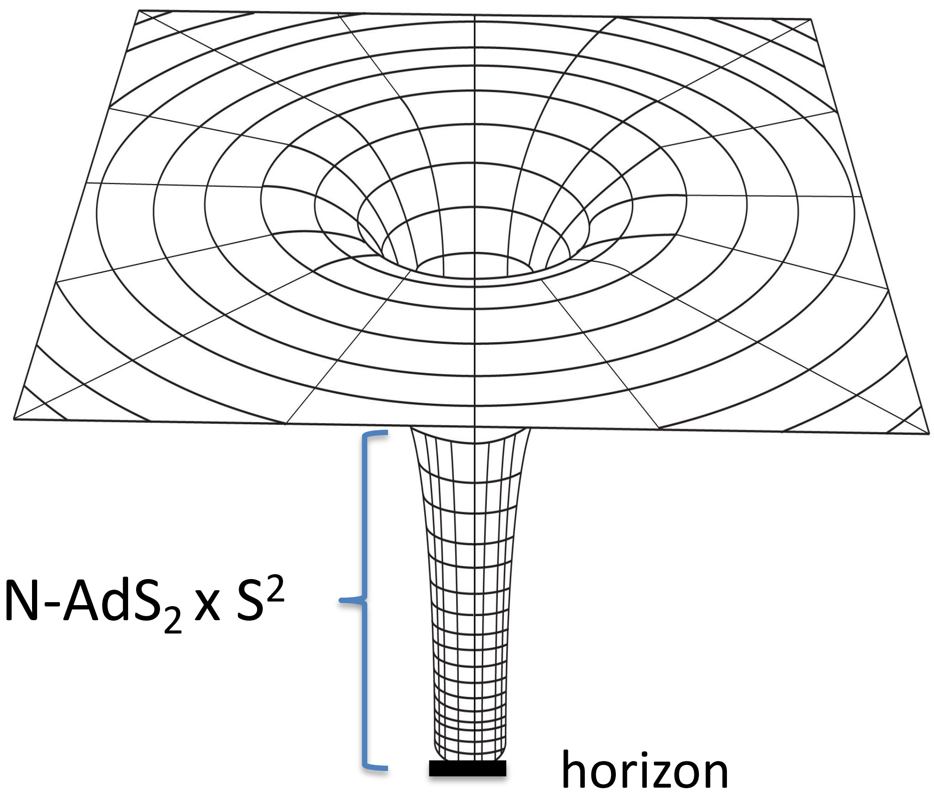
We will focus on near extremal black holes

and explain why they have more symmetry

Near extremal charged black holes

$$M \geq Q$$

$$M \sim Q$$



Focus on low energies

The quantum mechanical dual is a nearly critical system with a scaling (and conformal) symmetry

Approximate SL(2,R) symmetry = time translations + time rescalings + one more.

AdS_2 region	=	nearly critical QM system

N-AdS_2/$NCFT_1$ correspondence

The Sachdev-Ye-Kitaev model is an example of a quantum system with a nearly critical low energy regime

The SYK model

N Majorana fermions $\{\psi_i, \psi_j\} = \delta_{ij}$

Sachdev Ye Kitaev
Georges, Parcollet

$$H = \sum_{i_1, \cdots, i_4} J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$

Random couplings, gaussian distribution. $\langle J^2_{i_1 i_2 i_3 i_4} \rangle = J^2/N^3$

To leading order → treat J_{ijkl} as an additional field

J = dimensionful coupling. We will be interested in the strong coupling region

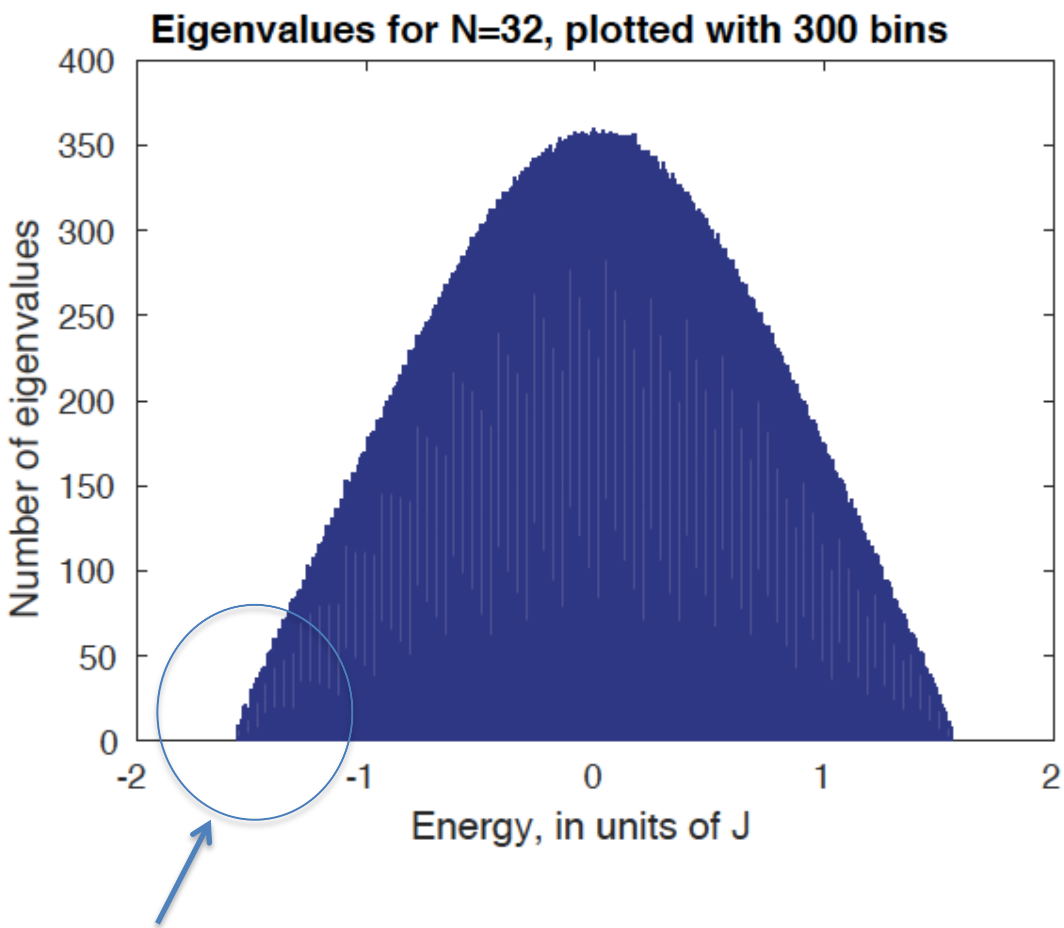
$$1 \ll \beta J, \ \tau J \ll N$$

Low, but not too low, energies.

It is nearly scale invariant in this regime.

Spectrum

D. Stanford



$$\dim_H = 2^{\frac{N}{2}}$$

Number of random couplings $\propto N^4 \ll 2^N$

(specific, but random J's)

Exponentially large number of states contributes to the low energy region we consider

- We can perform the large N summation of diagrams. → gives a scale invariant theory at low energies.
- 1/N corrections → there is a low action mode that becomes important at low energies. It can be exactly quantized. “Time superfluid mode” (Schwarzian action). Breaks the conformal symmetry.

Entropy

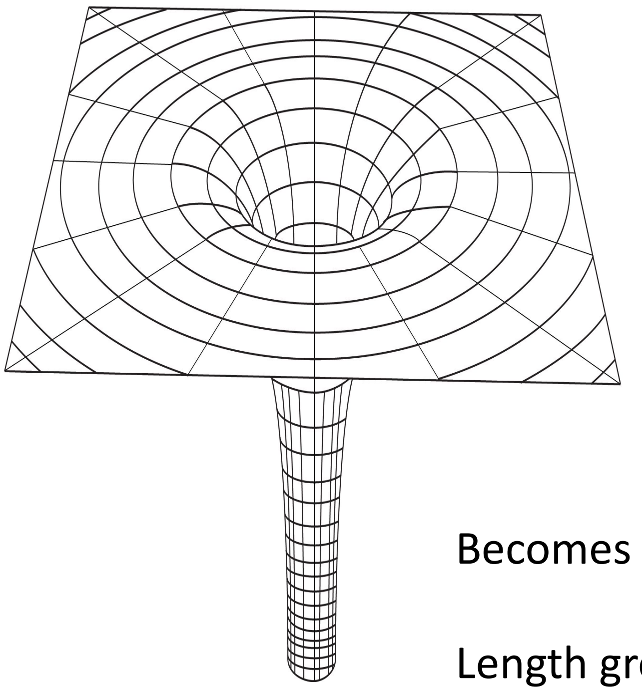
Large N. Constant plus linear in T correction

$$S = N\, s_0 + \frac{N\,T}{J} + \frac{3}{2}\log\left(\frac{T}{J}\right)$$

Conformal invariant

1/N correction. All orders → makes the density of states become small at low energies

Connection to Nearly AdS_2 gravity



Gravity action:

$$\log Z = -I = S_0 + T\,(constant)$$

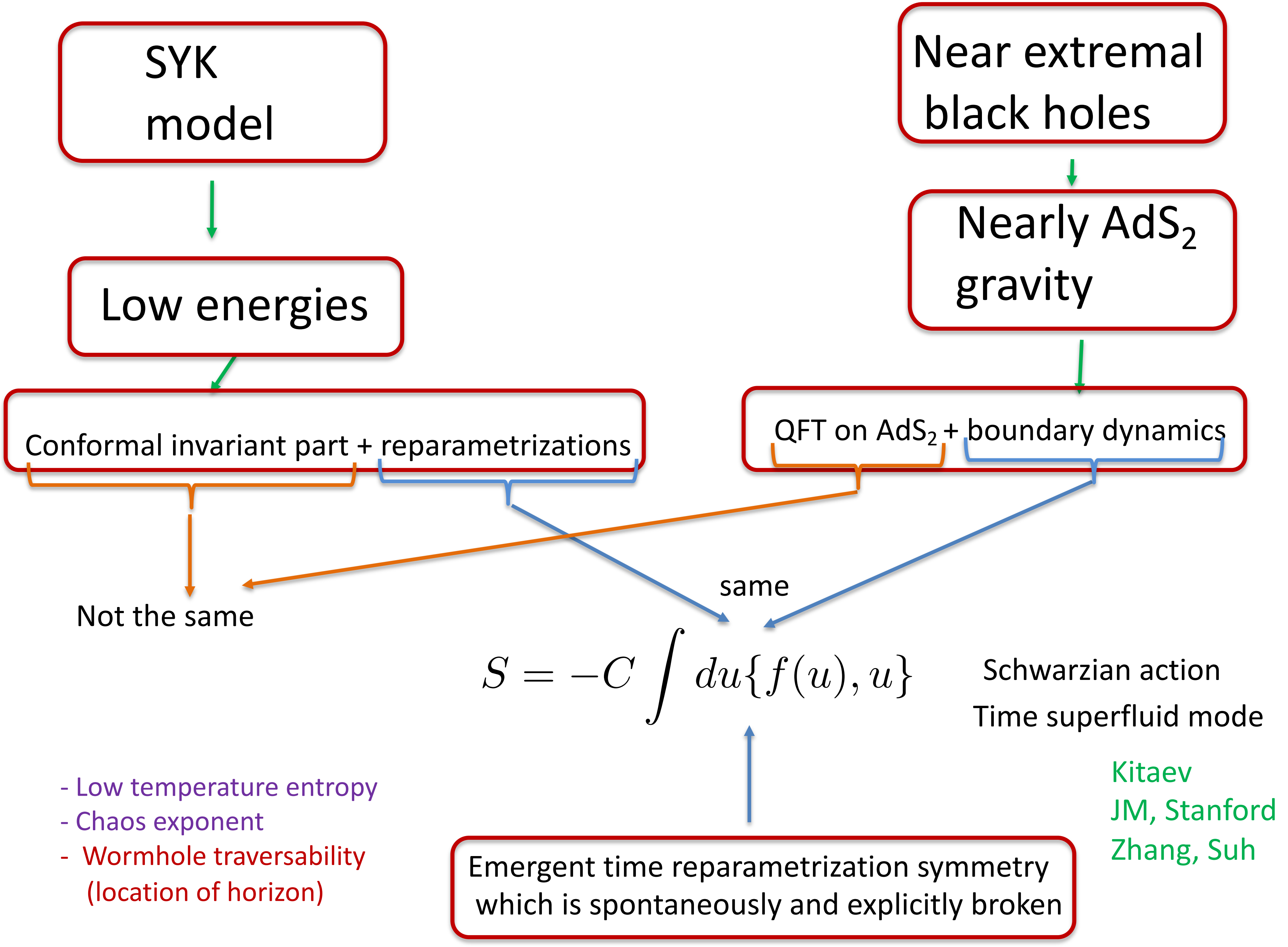
Becomes constant at low energy.

Length grows as 1/T (proper length goes as: – log T)

Very different geometries are having almost the same action →

There is a low action mode. → Same as the ``time superfluid mode''

What is the connection between AdS_2 and SYK ?


SYK model
Low energies
Conformal invariant part + reparametrizations
Near extremal black holes
Nearly AdS_2 gravity
QFT on AdS_2 + boundary dynamics
Not the same
same
$S = -C \int du \{ f(u), u \}$
Schwarzian action
Time superfluid mode
Kitaev
JM, Stanford
Zhang, Suh
- Low temperature entropy
- Chaos exponent
- Wormhole traversability (location of horizon)
Emergent time reparametrization symmetry which is spontaneously and explicitly broken

Yet one more simplification..

Supersymmetric black holes

- Supersymmetry: symmetry relating bosons and fermions → simplifies some computations.
- Black holes in supergravity theory
- Black holes with M=Q preserve some supersymmetries.
- Large number of exactly degenerate states giving rise to the extremal entropy of the black hole.

There is a supersymmetric SYK model

(we will discuss it later)

We will review results for the partition function

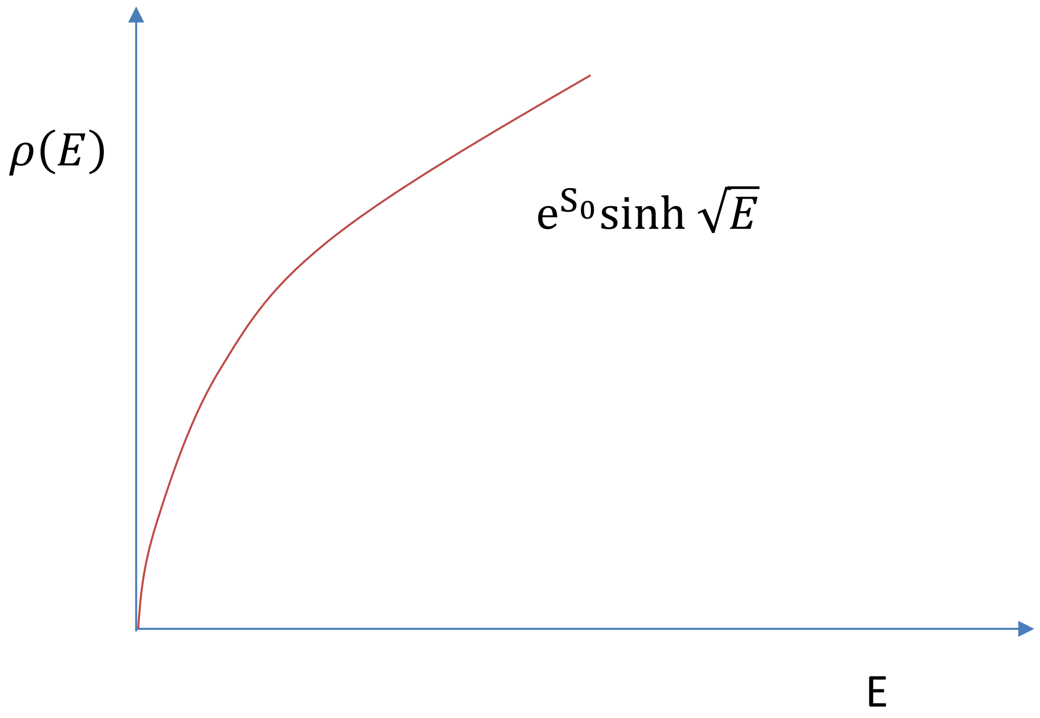
Results for the thermodynamics including the quantum corrections due to the boundary gravitons

=

the quantum mechanics of the time-superfluid mode.

Non-Supersymmetric

Bagrets, Altland, Kamenev
Stanford, Witten
Kitaev, Suh
Mertens Turiaci Verlinde



$$S = N\, s_0 + \frac{N\,T}{J} + \frac{3}{2}\log\left(\frac{T}{J}\right)$$

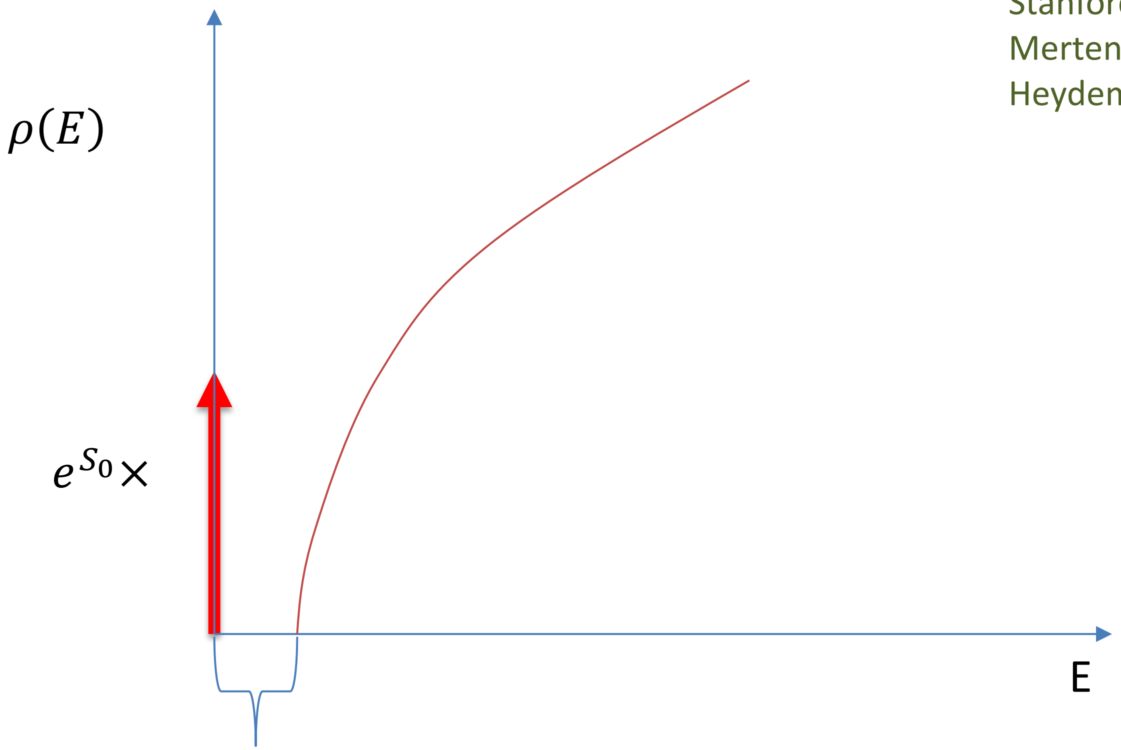
Entropy goes to zero at low energies.

Low energy limit with supersymmetry

- Time superfluid mode + fermionic partners.
- Different quantum properties at low energies.

$\mathcal{N}$=2, 4 supersymmetry

Stanford, Witten
Mertens Turiaci Verlinde
Heydeman, Iliesiu, Turiaci, Zhao



$$E_{gap} \sim \frac{E_{UV}}{S_e} \sim \frac{1}{r_e\, S_e} \sim \frac{1}{\phi_r} \sim T_0$$

$$S = -\phi_r \int du\{t(u), u\} + \text{partners}$$

What happens in the gravity solution at very low energies?

- Energies << gap → only ground states survive.
- 3rd Law is not obeyed → due to extra symmetry.
- Large degeneracy.
- Frozen system?

First a comment

Their entropies match beautifully with index computations…

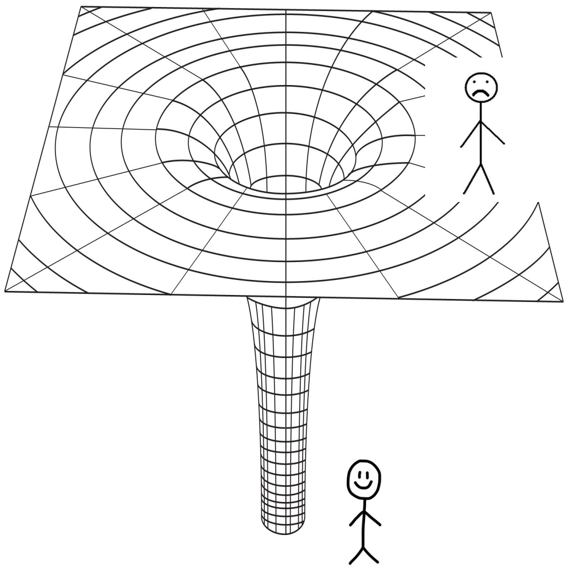
Strominger-Vafa, 1996.

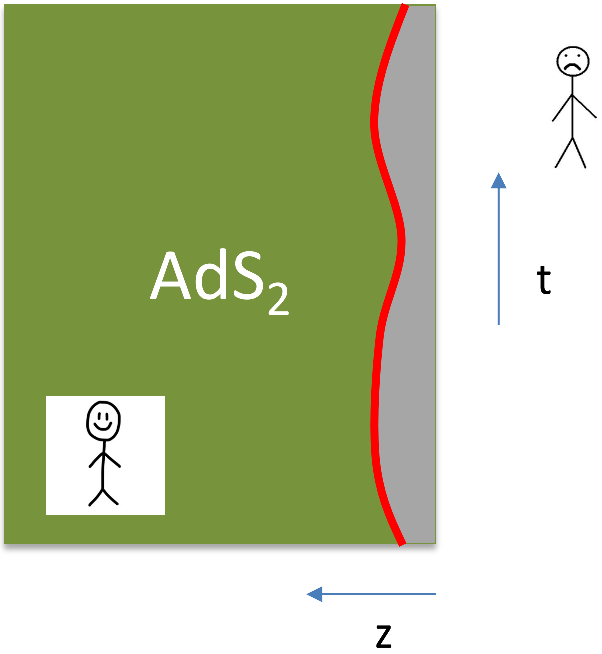
…

Dabholkar, Gomis, Murthy
Iliesiu, Murthy, Turiaci

A closer look at gravity when we take the low energy limit

The low energy limit





- The quantum fluctuations affect the length of the throat.
- These fluctuations blur the physics for the observer looking from outside.
- But the inside observer seems happy, living in a large spacetime.

What more could we say?

There are many questions that remain

There is more to a black hole than its entropy!

What about the details of is AdS_2 near horizon geometry?

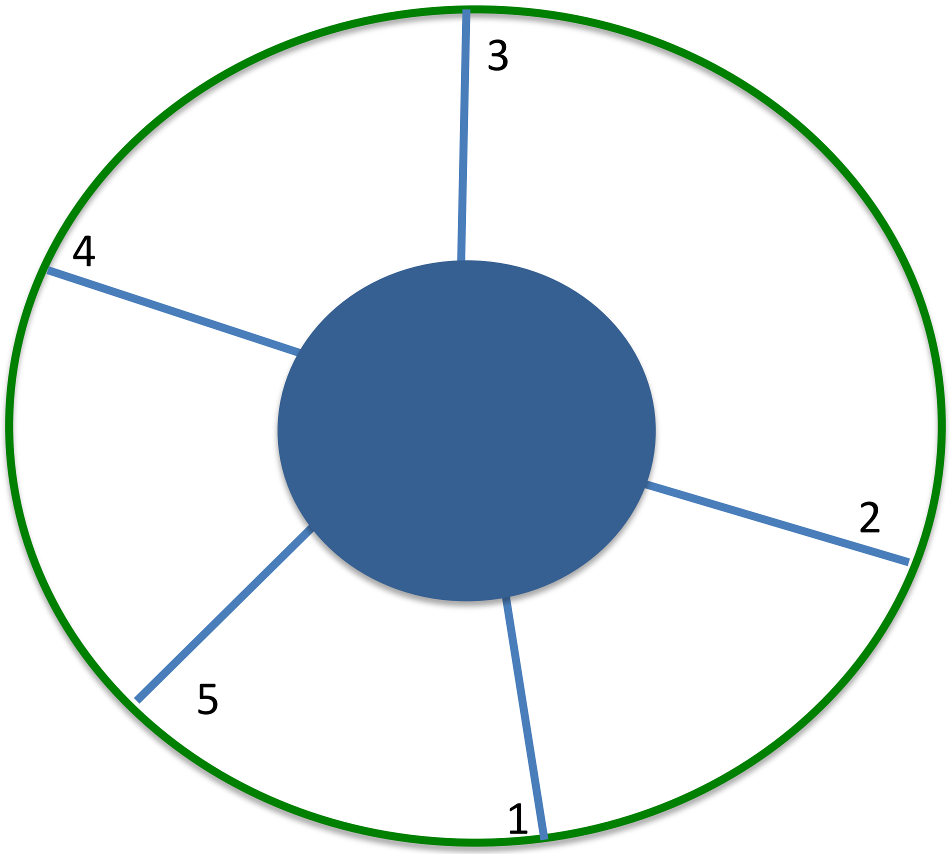
Is there an AdS_2/CFT_1 ?

There is more information in this AdS_2 theory:

Its correlation functions

Our main technical results involve computing some of them.

There are many previous results computing AdS correlators in AdS_D , D>2



Witten diagrams

But, in two dimensions the situation is a bit harder. And it has been understood only within the last few years.

The new feature is that there is a gravitational mode that becomes strongly coupled at low energies. Even for large N (or large Q).

The boundary mode dynamics

- To understand it, it is important to understand the asymptotic symmetries of AdS_2
- SL(2) → full time reparametrizations $f(\tau)$
- These symmetries are spontaneously broken.
- They are also explicitly broken by the boundary conditions

$$I = -\phi_r \int dt \{ f(t), t \}$$

$$\{f, t\} = \frac{f'''}{f'} - \frac{3}{2} \frac{f''^2}{f'^2}$$

- Governs the boundary mode dynamics.

The boundary mode dynamics with SUSY

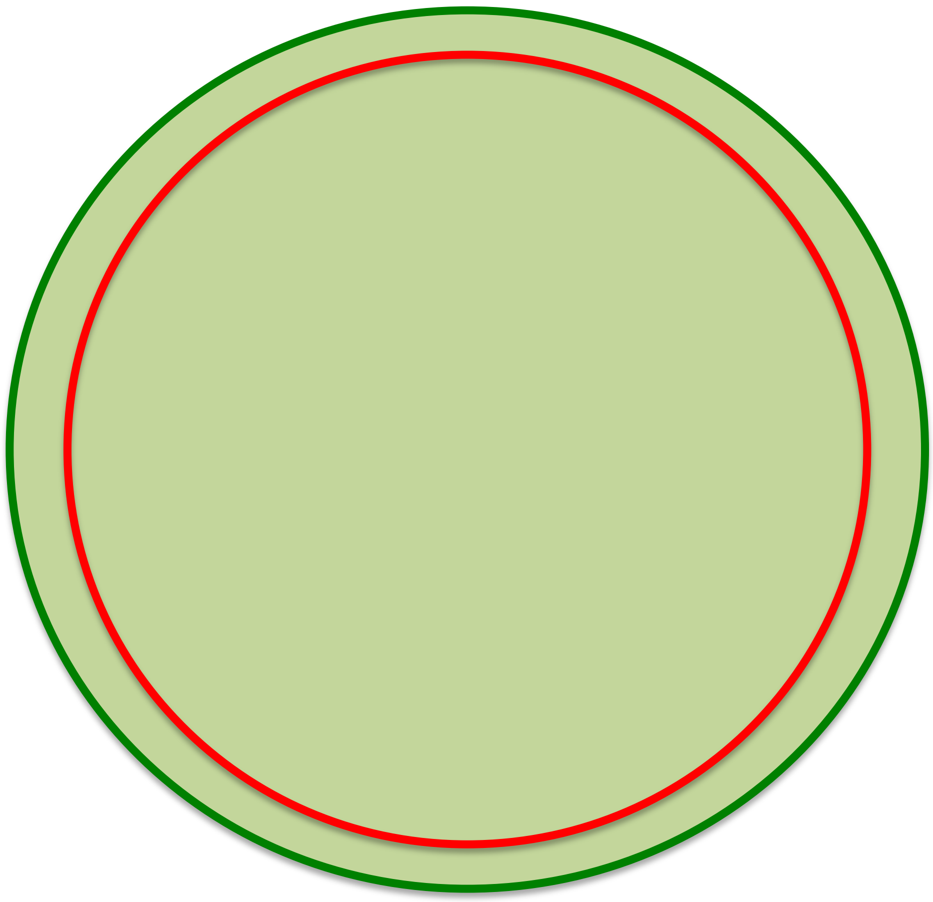
- OSp(2|2) → full time super-reparametrizations.

$$I = -\phi_r \int dt \{f(t), t\} + \text{partners}$$

- At very long times → the symmetries are restored!.
- The extremely low energy theory has H=0, and is topological, no time dependence.
- Restoration of the symmetry by quantum effects.

Review of Nearly-AdS_2 gravity (Euclidean) correlators

Euclidean black hole



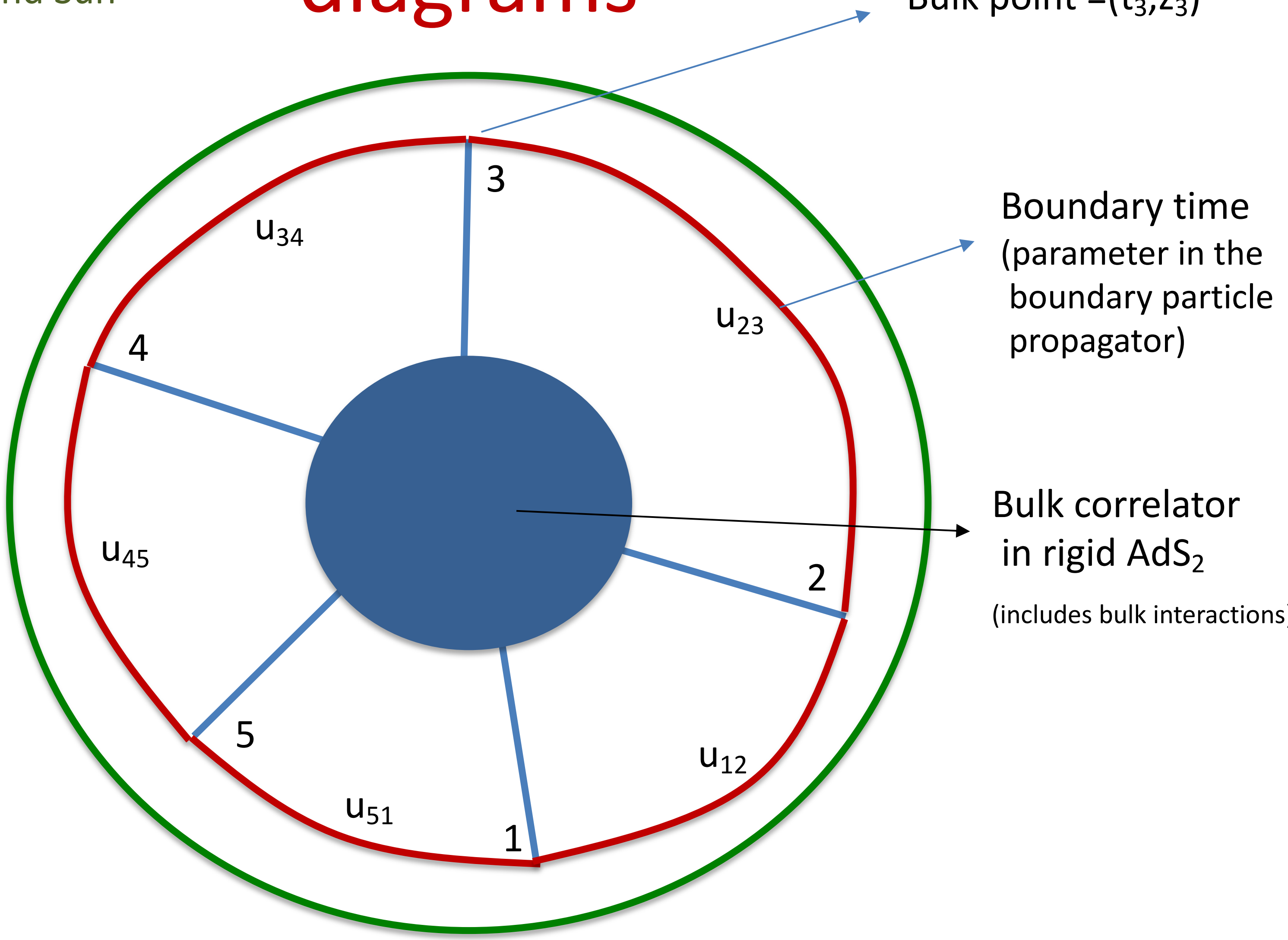
Boundary = circle of length β

Nearly AdS_2 gravity

- Matter fields moving in a rigid AdS_2 spacetime.
- The boundary becomes dynamical and behaves as a particle moving in AdS_2
- The quantum mechanics of this boundary particle can be exactly solved. Z. Yang, Kitaev and Suh
- It behaves as a non-relativistic particle moving in AdS_2 with an ``electric field''.

Quantum gravity from Witten-like diagrams

Z. Yang, Kitaev and Suh



Z. Yang, Kitaev and Suh

$$\langle O(u_1)\cdots O(u_n)\rangle = (\text{Boundary Particle})(\text{Correlator in AdS}_2)$$

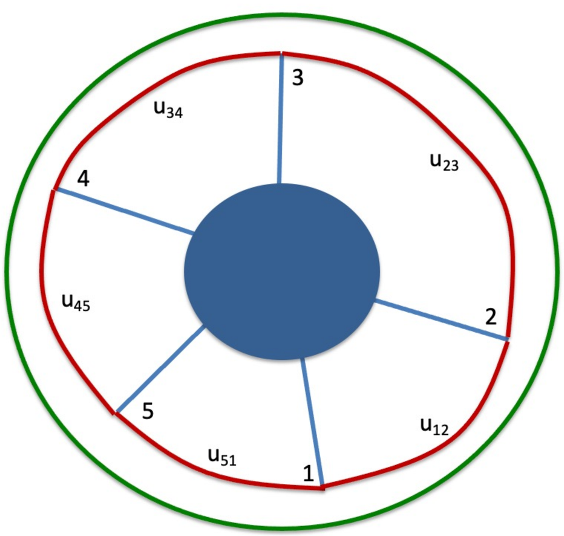
$$\langle O(u_1)\cdots O(u_n)\rangle = \int \frac{\prod_i dx_i dz_i/z_i^2}{\text{Vol}(SL(2))} P(\vec{x}_i, \vec{x}_{i+1}; u_{i,i+1}) \prod_i z_i^{\Delta_i} \langle O(x_1)\cdots O(x_n)\rangle$$

QFT in AdS_2 correlators

They simplify because we are near the boundary

Boundary particle propagator.

(We will review later how they are computed)



The N=2 case

- We have a similar expression.
- At low energies, or $u_{ij} \gg \beta_0$
- The propagator becomes independent of u.
- Only the zero energy states contribute.
- The correlator becomes topological.

$$\langle O(u_1)\cdots O(u_n)\rangle = \int \frac{\prod_i dx_i dz_i d^2\theta_i/z_i}{\mathrm{Vol}(SU(1,1|1))} P_0(\vec{x}_i,\vec{x}_{i+1}) \prod_i z_i^{\Delta_i} \langle O(x_1,\theta_1,\bar{\theta}_1)\cdots O(x_n,\theta_n,\bar{\theta}_n)\rangle$$

Independent of the u_i . Depends on the order.

$$\langle O(u_1)\cdots O(u_n)\rangle = \text{number} = F(\Delta_i, g_i)$$

How can we have a theory with no Hamiltonian?

- The structure is in the form of the observables (simple operators).
- Ground states + some simple operators.

- We have looked at Euclidean correlators, but in the bulk we can also have a bulk Lorentzian continuation.

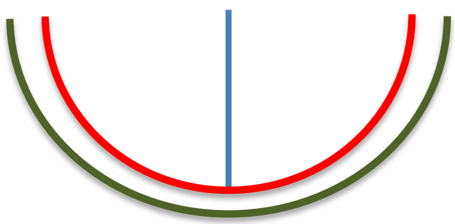
Infrared operators

$$\hat{O} = P_0 O P_0 \sim \lim_{u\to\infty} e^{-uH} O e^{-uH}$$

Projector on to the microstates.

O is not BPS.
But $\hat{O}$ is BPS.

O's are simple in the UV theory. But $\hat{O}$ is complicated due to the projector P_0, which depends on the flow and characterizes how the ground states are embedded in the full Hilbert space.



= bulk picture

The two point function in the N=2 susy theory

- We can compute the two point function using a variety of methods.
  - The chord diagram technique in N=2 SYK.

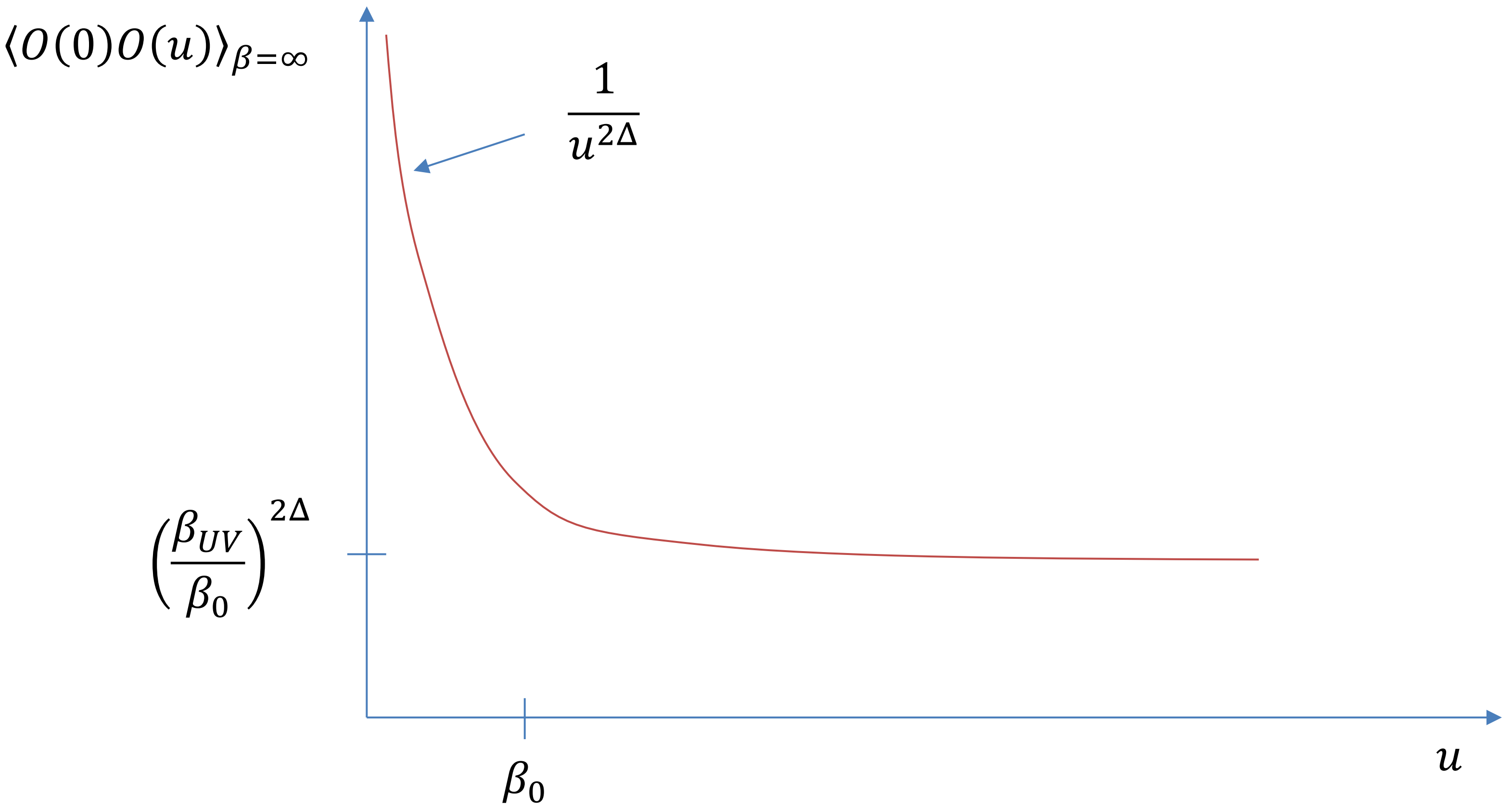
    Berkooz, Brukner, Narovlansky, Raz

  - The super-Liouville approach. As in Mertens, Turiaci, Verlinde

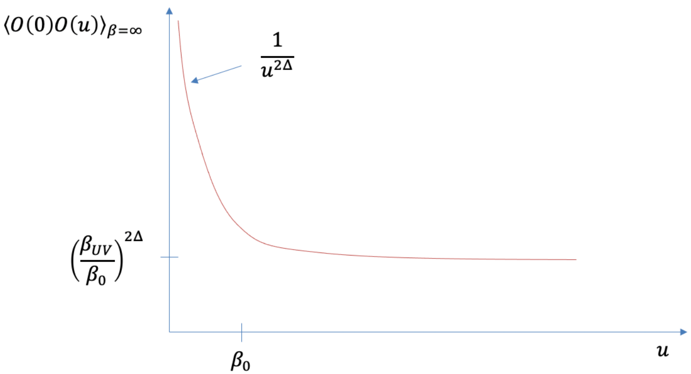
  - Using the boundary propagators.

We first discuss some qualitative features of the answer

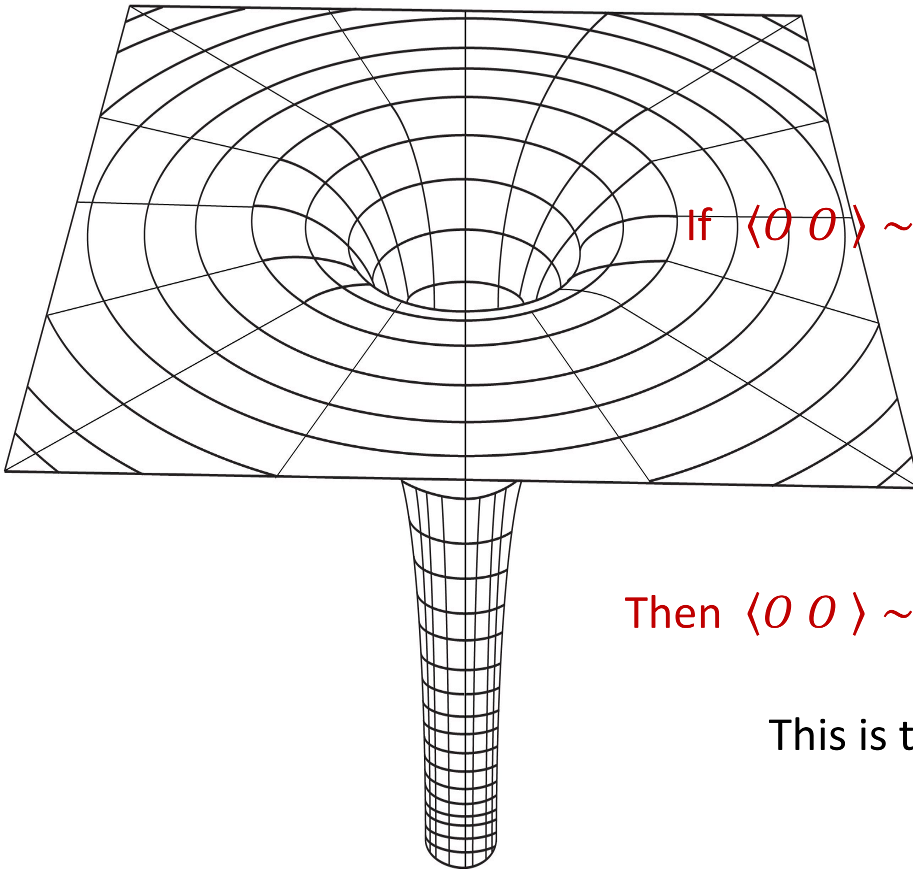
The two point function at zero temperature



$$\beta_{UV} \sim r_e \sim \frac{1}{J}$$



- It connects the shorter distance limit to the long distance, exactly AdS_2 regime.
- It is non-zero.
- This non-zero value has a power law suppression (power of the entropy) relative to its natural UV value.



If $\langle O\ O \rangle \sim 1$

Operator normalized so that its two point function is one in the region outside the black hole.

Then $\langle O\ O \rangle \sim \left(\frac{r_e}{\beta_0}\right)^{2\Delta} \sim \frac{1}{S_e^{2\Delta}}$

This is the value at very long times.

Typical values of matrix elements

The two point function is telling us information about the average value of the matrix elements of the operator in a microstate

Raju, Shrivastava

$$\langle OO \rangle_{IR} = e^{-S_0} Tr[\hat{O}\hat{O}] = e^{-S_0} \sum_{ij} |O_{ij}|^2$$

Typical values of eigenvalues

We could diagonalize the (Hermitian) operator O, $O_{\alpha,\beta} \sim o_\alpha \delta_{\alpha,\beta}$

$$\langle OO \rangle_{IR} = e^{-S_0} Tr[\hat{O}\hat{O}] = e^{-S_0} \sum_\alpha o_\alpha^2$$

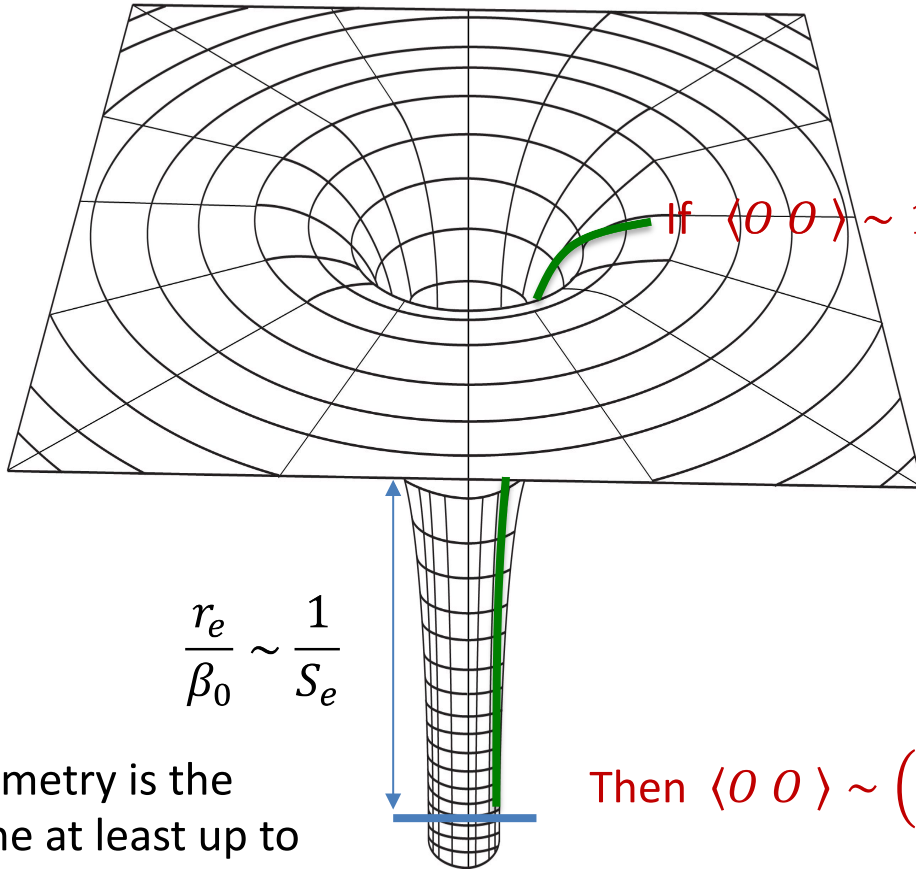
The two point function is giving us size of the typical eigenvalues of the operator.

This is the typical value of O in the basis that diagonalizes O.

This is larger than the typical value of the one point function of O on a random quantum states

$$\int d\psi \left[\langle \psi | O | \psi \rangle \right]^2 = e^{-S_0} \langle OO \rangle$$

This gives an interesting implication for where the geometry can start differing for various microstates in this basis



If $\langle O\ O \rangle \sim 1$

Universal factor coming from the propagation of the particle in AdS_2

$$\frac{r_e}{\beta_0} \sim \frac{1}{S_e}$$

The geometry is the usual one at least up to this point

Then $\langle O\ O \rangle \sim \left(\frac{r_e}{\beta_0}\right)^{2\Delta} \sim \frac{1}{S_e^{2\Delta}}$

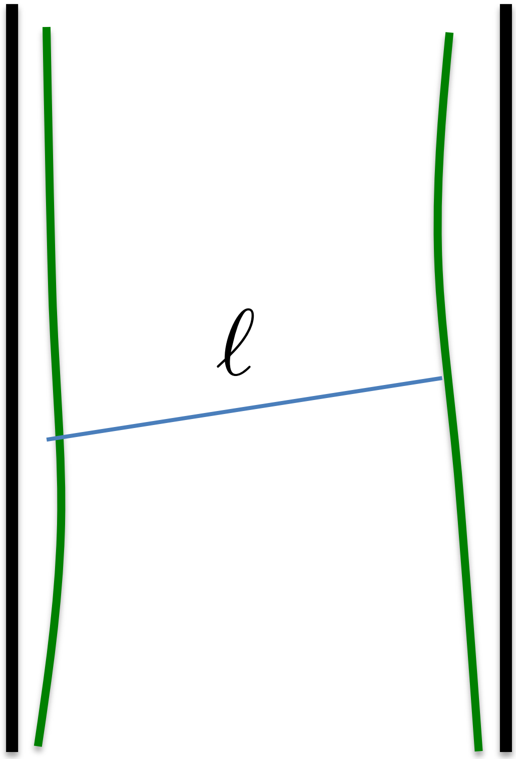
The geometry might be different from this point forwards for the different microstates in this basis where we diagonalize O.

This gives us a constraint on what we should expect for individual microstates.

Another implication…

Some details on the computation of the two point function using the Liouville method

Mertens, Turiaci, Verlinde



Basic variable of the two sided problem:
the distance.

This is a gauge invariant coordinate for the wormhole.
Kuchar

It turns out that its action is a Liouville like action

$$\int du[\dot{\ell}^2 + e^{-\ell}]$$

Harlow-Jafferis, Lin

With supersymmetry → Super Liouville theory.

Naively we would try to consider an N=2 superLiouville theory. However, we need an N=4 one because we have 2 SUSYs on the left and 2 SUSYs on the right.

Some more details on the computation of the 2pt function

- With the super-Liouville method.
- N=2 Superliouville theory in 2d → N=2 Schwarzian in 1d.

Mertens, Turiaci, Verlinde

- Liouville quantum mechanics.

$$S = \int du \left[\frac{1}{4}\dot{\ell}^2 + \dot{a}^2 + \bar{\psi}_\pm \dot{\psi}_\pm + e^{-\ell/2 - ia}\psi_+\psi_- + e^{-\ell/2+ia}\bar{\psi}_+\bar{\psi}_- + e^{-\ell} \right]$$

- Find eigenfunctions.
- Build the Hartle-Hawking state (use input from the disk partition function)
- Compute $\langle e^{-\Delta \ell} \rangle, \; or \; \langle e^{-\Delta(\ell + 2\, i\, a)} \rangle$

The full answer is a bit long…

$$Tr\left[\, e^{-u\,H}\, O\, e^{-u'H}\, O\right] = \langle \psi | e^{-\Delta \ell} | \psi \rangle =$$

$$= \sum_r e^{-\frac{(r-1)^2}{4}(u+u')} \frac{2^{2+2\Delta}}{\pi^2 \hat{q}^2 \Gamma(2\Delta)} \int_0^\infty ds \int_0^\infty ds' e^{-4s^2 u - 4s'^2 u'}$$

$$\times \frac{s \sinh(2\pi s)}{E} \frac{s' \sinh(2\pi s')}{E'} 2\left(E + E' + 4\Delta^2\right) \Gamma(\Delta \pm is \pm is')$$

$$+ 2 \sum_r e^{-\frac{(r-1)^2}{4}u - \frac{(r-3)^2}{4}u'} \frac{2^{4+2\Delta}}{\pi^2 \hat{q}^2 \Gamma(2\Delta)} \int_0^\infty ds \int_0^\infty ds' e^{-4s^2 u - 4s'^2 u'}$$

$$\times \frac{s \sinh(2\pi s)}{\left(4s^2 + \frac{(r-1)^2}{4}\right)} \frac{s' \sinh(2\pi s')}{\left(4s'^2 + \frac{(r-3)^2}{4}\right)} \Gamma(\Delta + \frac{1}{2} \pm is \pm is')$$

$$+ 2 \sum_{|r|<1} e^{-\frac{(r-1)^2}{4}u} \frac{2^{2\Delta}}{\pi \hat{q}^2 \Gamma(2\Delta)} \cos(\frac{\pi r}{2}) \int_0^\infty ds e^{-4s^2 u}$$

$$\times \frac{s \sinh(2\pi s)}{\left(4s^2 + \frac{(r-1)^2}{4}\right)} \left(4s^2 + \frac{(r-1)^2}{4} + 2\Delta(r + 2\Delta - 1)\right) \Gamma(\Delta \pm \frac{(r-1)}{4} \pm is)$$

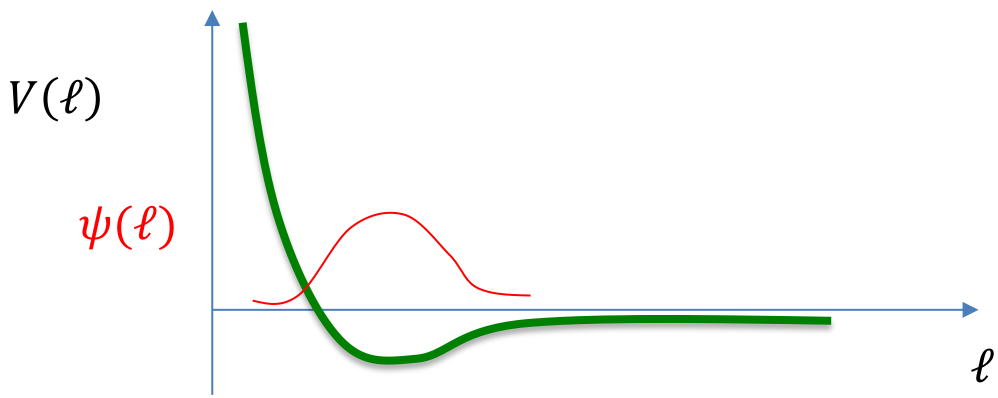
$$+ 2 \sum_{|r|<1} e^{-\frac{(r+1)^2}{4}u'} \frac{2^{2+2\Delta}}{\pi \hat{q}^2 \Gamma(2\Delta)} \cos(\frac{\pi r}{2}) \int_0^\infty ds e^{-4s^2 u'} \frac{s \sinh(2\pi s)}{\left(4s^2 + \frac{(r+1)^2}{4}\right)} \Gamma(\Delta + \frac{1}{2} \pm \frac{(r-1)}{4} \pm is)$$

$$+ \sum_{|r|<1} \frac{2^{2\Delta}\Delta}{\hat{q}^2} \cos(\frac{\pi r}{2})^2 \frac{\Gamma(\Delta)^2 \Gamma(\Delta + \frac{1}{2} \pm \frac{r}{2})}{\Gamma(2\Delta)}$$

$$S = \int du \left[\frac{1}{4}\dot{\ell}^2 + \dot{a}^2 + \bar{\psi}_\pm \dot{\psi}_\pm + e^{-\ell/2 - ia}\psi_+\psi_- + e^{-\ell/2+ia}\bar{\psi}_+\bar{\psi}_- + e^{-\ell} \right]$$

$$S = \int du \left[\frac{1}{4}\dot{\ell}^2 - e^{-\ell/2} + e^{-\ell} \right]$$

There is a zero energy normalizable ground state



For this state the length of the wormhole is bounded, and time independent.

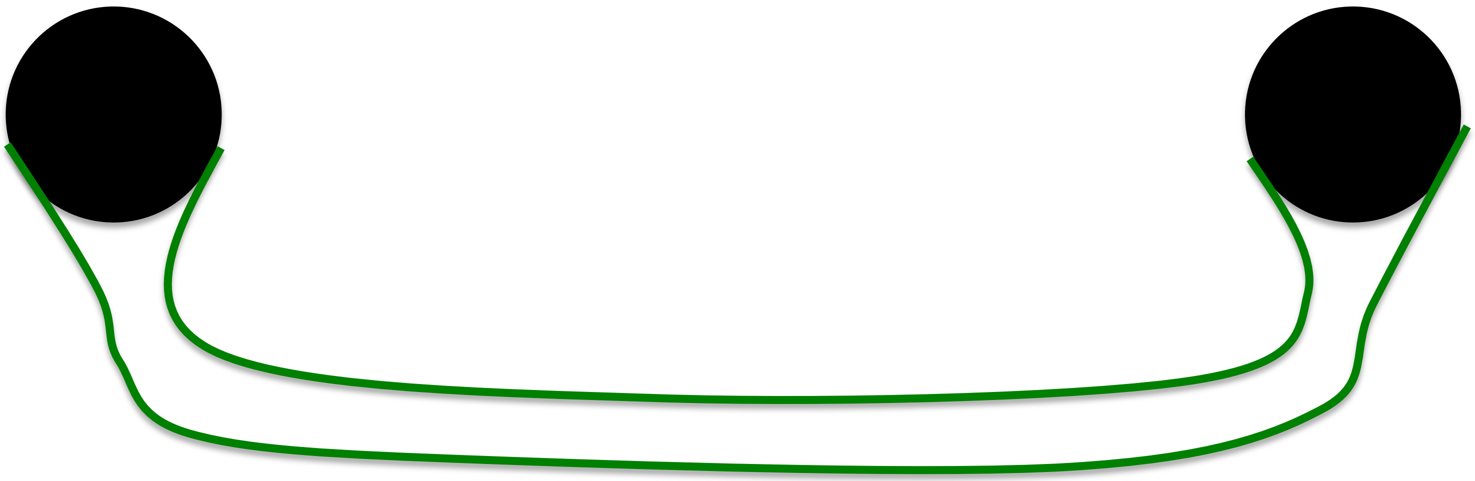
This is different from the naïve classical picture of an infinitely long throat, or

$$ds^2 = -r^2 dt^2 + \frac{dr^2}{r^2}$$

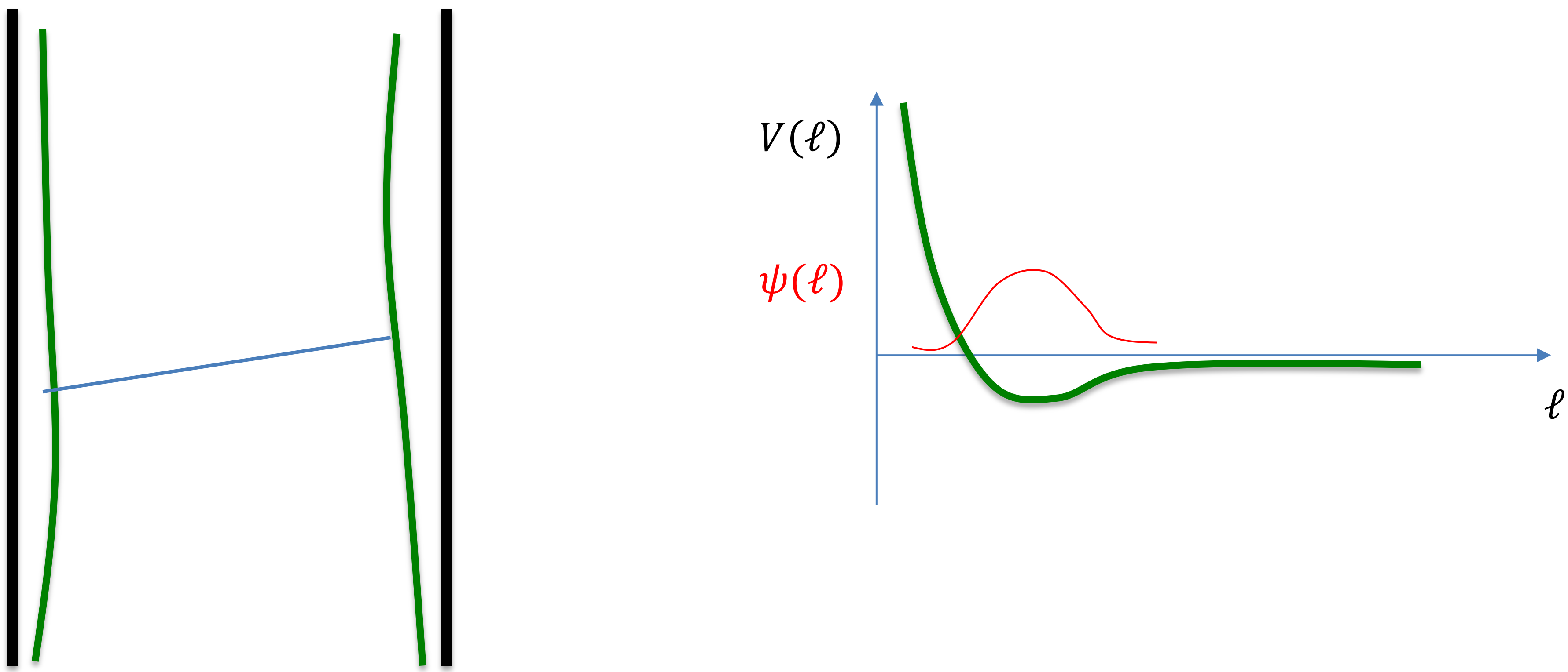
Let us emphasize the last point by asking:

Are there supersymmetric wormhole configurations?

SUSY ER= EPR ?



From the previous comments → Yes!


$V(\ell)$
$\psi(\ell)$
ℓ

Now we turn to the boundary particle propagator

We need it to construct more general correlators

Z. Yang, Kitaev Suh

The boundary particle formalism

- Boundary = Particle moving in AdS_2

Z. Yang, Kitaev Suh

The propagator, no SUSY

- Use group theory to construct it.
- $AdS_2 = SL(2)/U(1)$
- Somewhat similar to the problem of wavefunctions of particles in a magnetic field on S^2 , which are related to matrix elements of SU(2) group elements.

The boundary super-particle formalism

- Boundary = Particle moving in AdS_2
- Symmetries:
  - Full symmetry under the $SU(1,1|1)= OSp(2|2)$ supergroup. This is a gauge symmetry.
  - N=2 worldline supersymmetry (Poincare)
  - Physical U(1) R symmetry.

(N=1 case: Fan, Mertens)

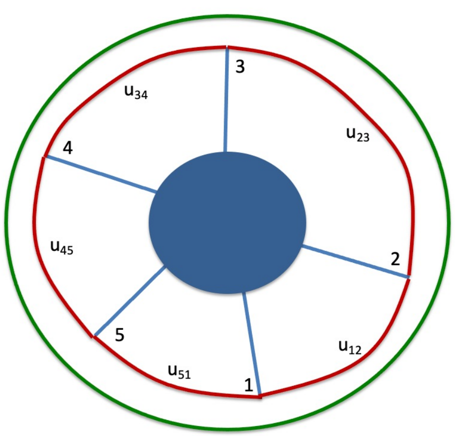
The answer is simpler in the $u \rightarrow$ infinity limit

Z. Yang, Kitaev Suh

Propagator for zero energy states

- $Q_1 P_0 = \overline{Q_1} P_0 = 0 \rightarrow H_1 P_0 = 0$
- This propagator will enables us to compute any correlator in AdS_2 at zero energy.

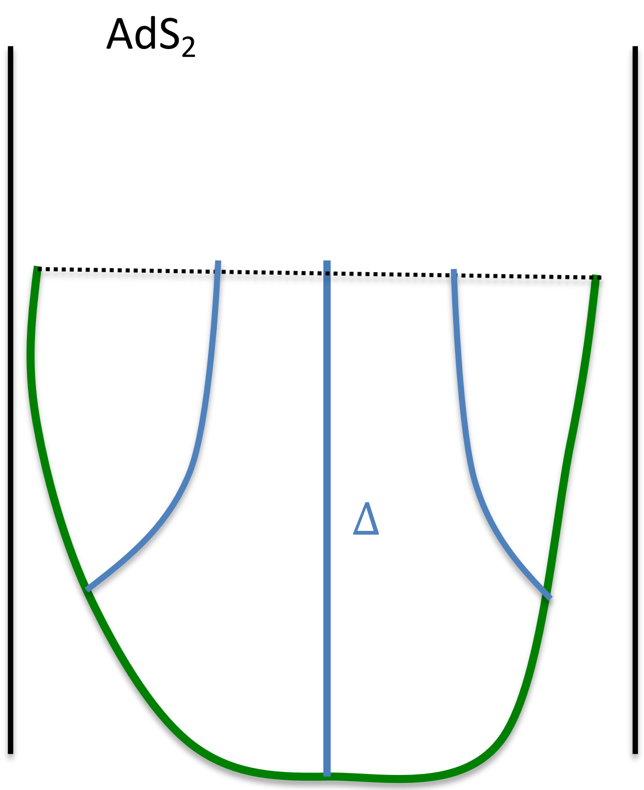
$$P_0 = \theta(x_{12}) \frac{(z_1 z_2)^{1/4}}{\sqrt{x_{12}}} \exp\left(-\frac{(\sqrt{z_1} + \sqrt{z_2})^2}{x_{12}} \right) (\text{+fermions})$$



$$\langle O_1 \cdots O_n \rangle \sim e^{-S_0} Tr\left[\hat{O}_1 \cdots \hat{O}_n \right]$$

We checked that it does obeys the composition law.

Filling the inside



The distance increases, but remains finite.

We could insert many particles.

The distance depends only on the total dimension.

The entanglement entropy between the two sides decreases.

$S = S_0$ - (finite)

Type II_1 algebra.

Similarities with dS: H=0, type II_1 in the semiclassical limit.

Chandrasekharan, Long, Penington, Witten

Chaos in operators

Saad, Shenker, Stanford

- Since H=0, no chaos from energy levels.
- One can argue that the IR operators, $\hat{O}$=POP, are random matrices, with some evidence for eigenvalue repulsion.
- → Chaos in operators, or their eigenvalues.

See also: Jafferis, Kolchmeyer, Mukhametzhanov, Sonner.

We can compare the super-Schwarzian answers against those of the $\mathcal{N}$=2 SYK model

$\mathcal{N}$=2 SYK model

Fu, Gaiotto, Sachdev, JM

- Similar to the SYK model.
- N complex fermions ψ^i
- Supercharge involves a product of three fermions with random couplings.
- $Q = \sum_{ijk} \psi^i \psi^j \psi^k$
- $H = \{Q, Q^\dagger\}$

$\mathcal{N}$=2 SYK model

- We can compute the number of ground states analytically and numerically. Fu, Gaiotto, Sachdev, JM
- Now, we can compute correlators.
- They indeed go to constants at long times, both for R-charged operators such as ψ^i as well as for neutral operators such as $\psi^{i\dagger}\psi^j$

Numerical computation of the energy gap

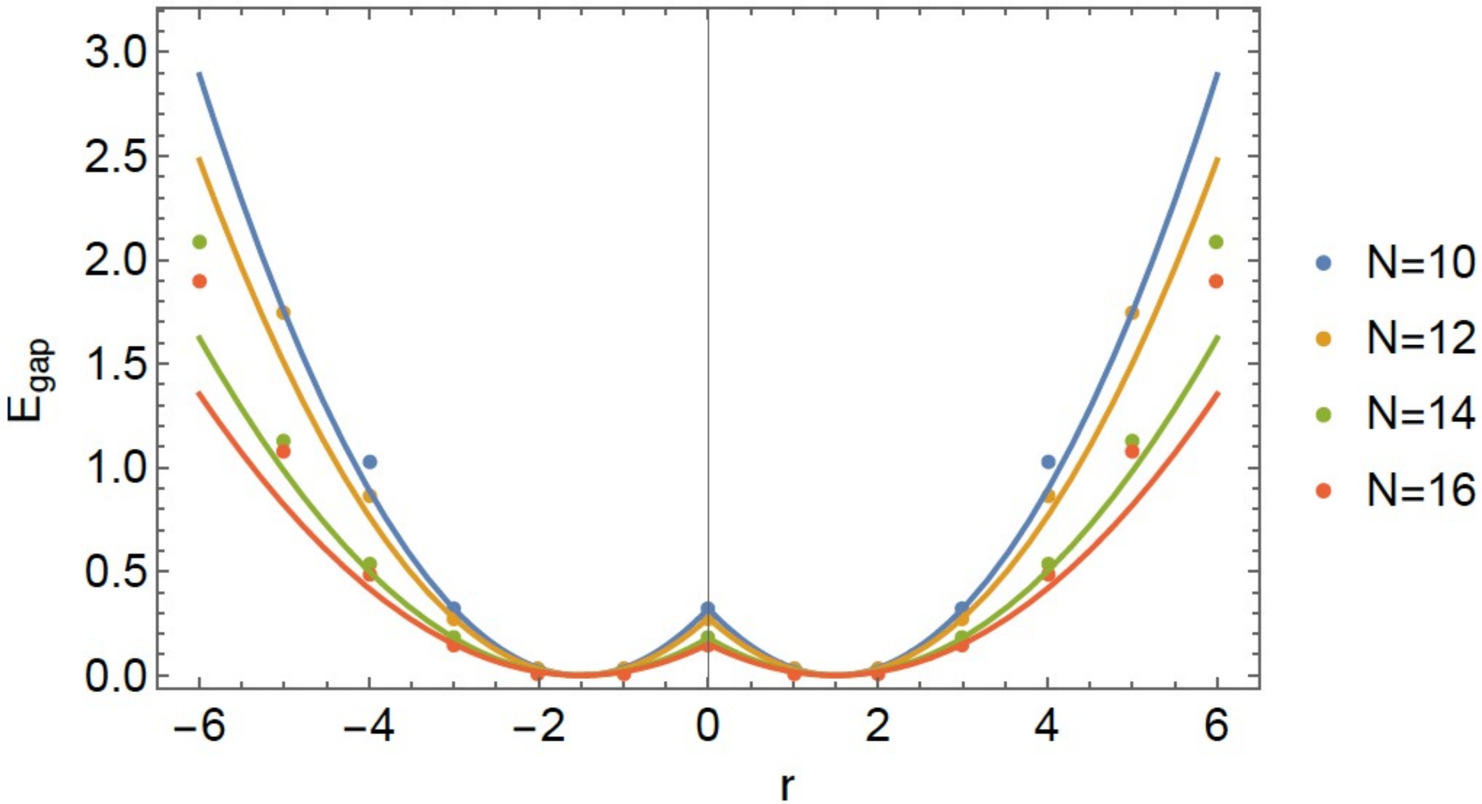


Figure 1: Gap as a function of R-charge for various values of N.

$$E_{gap} \propto \frac{1}{\phi_r}\left(\frac{|r|}{3} - \frac{1}{2}\right)^2 \propto \frac{J}{N}\left(\frac{|r|}{3} - \frac{1}{2}\right)^2$$

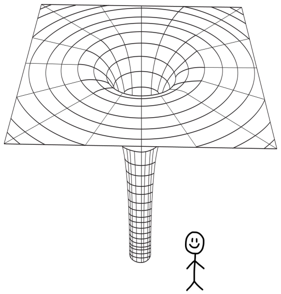
Constant two point function at long Euclidean times

Operator	R-charge	Schwarzian prediction	Numerical answer (N=16)
ψ_i	0	0.103	0.110 ± 0.005
	$-1/3$	0.103	0.110 ± 0.005
$\psi_i\psi_j$	$-1/3$	0.0213	0.024 ± 0.003
$\bar{\psi}_i\psi_j$	$-1/3$	0.0243	0.027 ± 0.001
	0	0.0754	0.079 ± 0.001
	$+1/3$	0.0243	0.027 ± 0.001

Conclusions

- AdS_2/TFT_1
- There is a bulk time but no boundary time. H=0.
- There are other interesting observables for SUSY AdS_2 : the correlators.
- We computed the two point function.
- We computed the zero energy propagator→ any correlator.
- These put constraints on how different various microstates can be.
- Good match to numerical SYK answers.

Future



- What is bulk time in this limit ?

- Can we get a gravity picture for the microstates?

Extra slides

N=1 case: Fan, Mertens

N=2 Propagator

Propagator:

- $P(1,2; u_{12}, \kappa_1, \overline{\kappa_1}, \kappa_2, \overline{\kappa_2}) = \langle 1| e^{\kappa_1 Q + \overline{\kappa_1}\, \bar{Q}}\, e^{\,u_{12} H} e^{\kappa_2 Q + \overline{\kappa_2}\, \bar{Q}} |2\rangle$
  $= e^{i\, q\left(\gamma_+^1 - \gamma_+^2 + \varphi(1,2)\right)}\, F(invariants, u_{12}, \kappa_1, \overline{\kappa_1}, \kappa_2, \overline{\kappa_2})$
- $i\partial_u P = H_1 P$
- $iD_{\kappa_1} P = Q_1 P, \qquad i\overline{D_{\overline{\kappa_1}}}\, P = \overline{Q_1}\, P$
- Q_1 is a Grassman odd differential operator. It is invariant under the left symmetries. We had to guess its form.