Entanglement and Geometry from subalgebras of the Virasoro algebra

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+ ongoing work

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Holography

Geometry ⟷ Quantum states

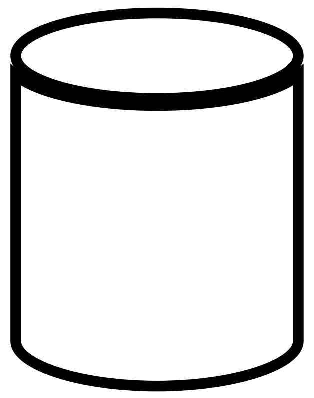
Emergent

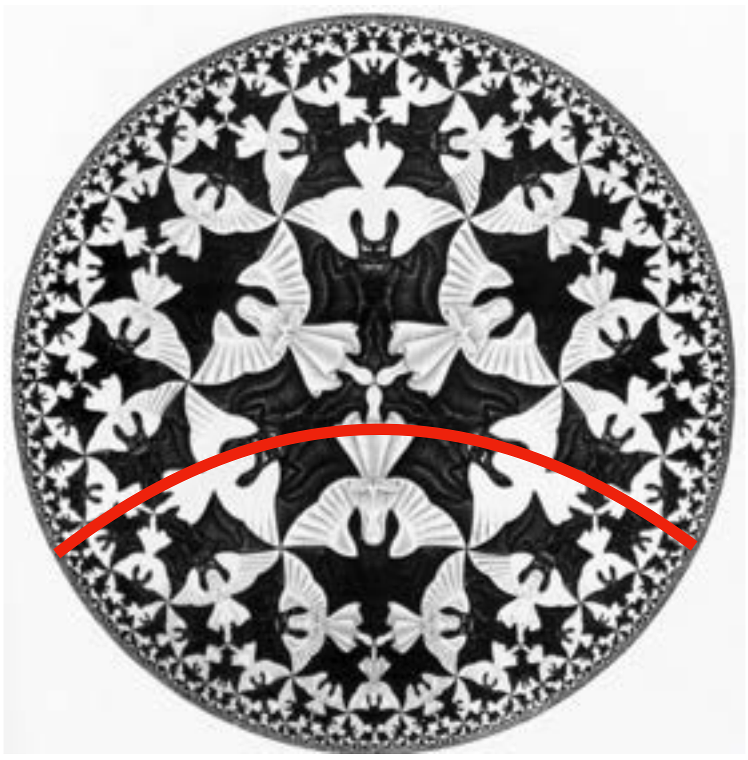
[Maldacena '97]

[Ryu, Takayanagi '06]

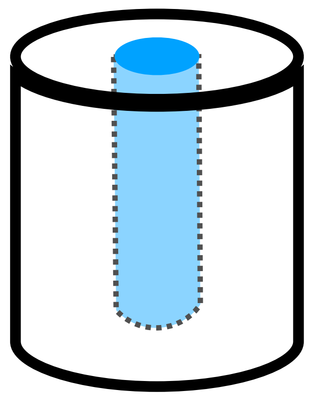
AdS

CFT





$|vac\rangle$



$\rho_{thermal}$

Holography ?

$|coherent\rangle$

Coherent states are ubiquitous

Displacement operator

$$|coherent\rangle = D(z)\,|vac\rangle$$

- Quantum information, quantum optics...,

$$D(z) = e^{za^\dagger - z^* a}$$

- Quantum quenches, (global, local)

[Calabrese, Cardy, Caputa, Tamaoka, Ryu, Takayanagi,...]

- Operator growth and chaos

[Balasubramanian, Caputa, Magan, Patramanis,...]

- Circuit complexity

[Chapman, Ge, Policastro, Zukowski, ...]

- ...

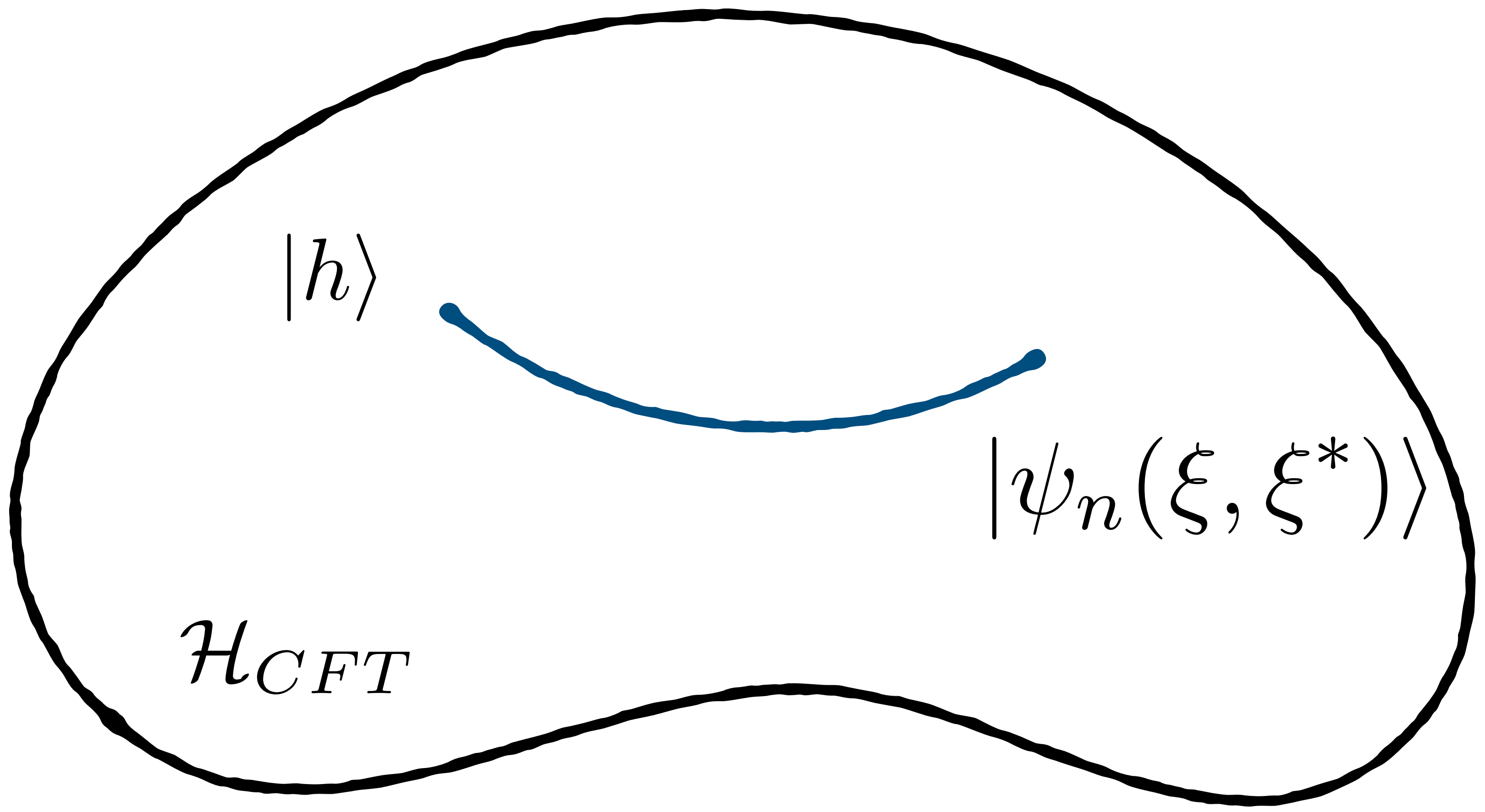
} **Generalized Coherent States:**

à la Gilmore-Peremolov -> Lie groups

[Peremolov, 1972, Gilmore 1972]

Our coherent states: SL(2) coherent states

$$|\psi_n(\xi,\xi^*)\rangle = \exp(\xi L_{-n} - \xi^* L_n)\,|h\rangle \qquad \xi = \rho e^{i\theta}$$



To make sl(2) algebra manifestly,

$$\tilde{L}_1 = \frac{L_n}{n}\,, \quad \tilde{L}_{-1} = \frac{L_{-n}}{n}\,, \quad \tilde{L}_0 = \frac{L_0 + c(n^2-1)/24}{n}\,.$$

[Witten 1988]

$$SL^{(n)}(2,\mathbb{R})$$

There are infinitely many sl(2) subalgebras in the full Virasoro algebra.

In this talk:

- The gravitational dual of this class of coherent states
- Dynamical properties of the entanglement in the coherent background

I will focus on the study in $n=1$ case, the general n case is similar.

Gravitational dual

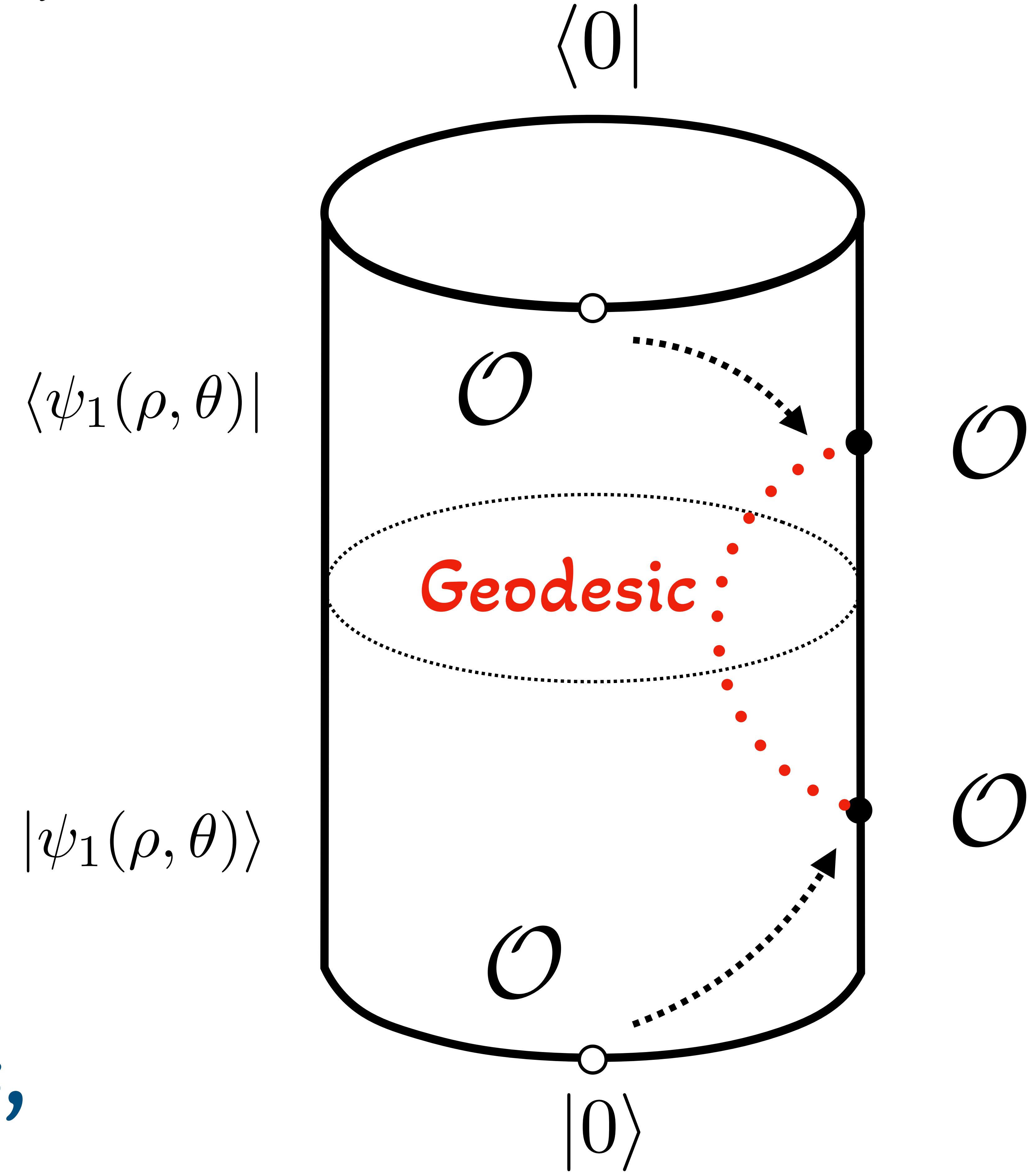
Observation: operator interpretation, using the properties of the global conformal generators,

$$|\psi_1(\xi,\xi^*)\rangle = \frac{\mathcal{O}(u)}{\sqrt{\langle \mathcal{O}^\dagger(u)\mathcal{O}(u)\rangle}}|0\rangle\ , \quad u=\tanh\rho e^{i\theta}$$

The density matrix for coherent states on the cylinder, c.w. below the BH threshold

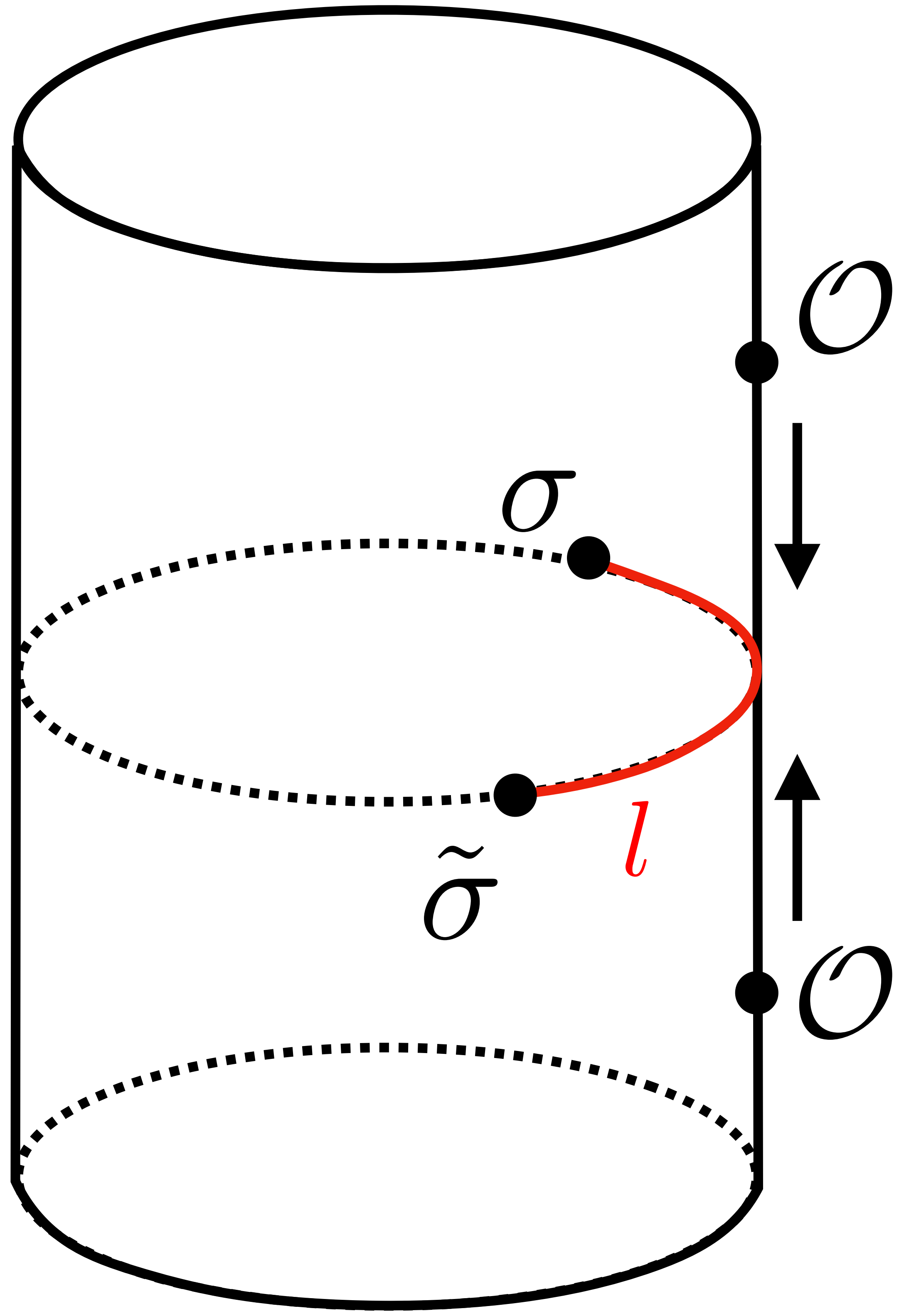
$$\rho_{coh} = \frac{\mathcal{O}(u)\,|0\rangle\langle 0|\,\mathcal{O}^\dagger(u)}{\langle \mathcal{O}^\dagger(u)\mathcal{O}(u)\rangle} \qquad h_{\mathcal{O}} < \frac{c}{24}$$

Gravitational picture: A massive particle threads the insertion points of the operators, i.e., conical singularity in the bulk.



Dynamics probed by the entanglement

As one parameter in the coherent states increases, this corresponds to the situation where the two operators are moving towards each other. Such a dynamics can be probed by the entanglement entropy.



We can study from both the holographic side and CFT side:

- Holographically: Ryu-Takayanagi formula, regulated geodesic length

[Hubeny, Rangamani, Takayanagi, '07]

[Roberts '12]

$$S^{\text{Holo}} = \frac{L_\gamma}{4G}$$

- CFT method: treating the moving operators as a part of the background, together with two twist operators, 4-pt function, dominated by HHLL block, leading contribution has a closed form

[Hartman, '13]

[Asplund, Bernamonti, Galli, Hartman, '14]

[Caputa, Simon, Štikonas, Takayanagi, '14]

$$\text{Tr}\rho_A^q \propto \langle\psi(\rho,\theta)|\, \sigma(z_1,\bar{z}_1)\tilde{\sigma}(z_2,\bar{z}_2)\,|\psi(\rho,\theta)\rangle$$

$$S(z_1,z_2) = -\lim_{q\to 1}\partial_q \text{Tr}\rho_A^q$$

Punchline: They match !

CFT way : Replica trick and HHLL block

$$h_\sigma = \frac{c}{24}(q - \frac{1}{q})$$

Replica trick -> Insertion of twist operator

[Calabrese, Cardy '09]

$$S(z_1, z_2) = -\lim_{q\to 1} \partial_q \mathrm{Tr}\rho_A^q\,, \quad \mathrm{Tr}\rho_A^q \propto \langle\psi(\rho,\theta)|\,\sigma(z_1,\bar{z}_1)\tilde{\sigma}(z_2,\bar{z}_2)\,|\psi(\rho,\theta)\rangle$$

Twist operator in coherent states: 4-pt function

$$\langle\psi(\rho,\theta)|\,\sigma(z_1,\bar{z}_1)\tilde{\sigma}(z_2,\bar{z}_2)\,|\psi(\rho,\theta)\rangle = \frac{\langle\mathcal{O}(u',\bar{u}')\sigma(z_1,\bar{z}_1)\tilde{\sigma}(z_2,\bar{z}_2)\mathcal{O}(u,\bar{u})\rangle}{\langle\mathcal{O}(u',\bar{u}')\mathcal{O}(u,\bar{u})\rangle}$$

Conformal mapping: fix three points

$$u' \to \infty\,, \quad z_1 \to 1\,, \quad z_2 \to \eta = \frac{(u'-z_1)(z_2-u)}{(u'-z_2)(z_1-u)}\,, \quad u \to 0$$

$$\frac{\langle \mathcal{O}(u',\bar{u}')\sigma(z_1,\bar{z}_1)\tilde{\sigma}(z_2,\bar{z}_2)\mathcal{O}(u,\bar{u})\rangle}{\langle \mathcal{O}(u',\bar{u}')\mathcal{O}(u,\bar{u})\rangle} = G(\eta,\bar{\eta})(1-\eta)^{2h_\sigma}(1-\bar{\eta})^{2\bar{h}_\sigma}\langle \sigma(z_1,\bar{z}_1)\tilde{\sigma}(z_2,\bar{z}_2)\rangle$$

HHLL approximation: large c + sparse spectrum of light operators

$$\Delta << c$$

$$h_{\mathcal{O}} \lesssim \frac{c}{24}$$

Identity block domination

[Fitzpatrick, Kaplan, Walters, '14+'15]

[Hartman, '13]

[Perlmutter '15]

$$G(\eta,\bar{\eta}) = \left|\alpha^{2h_\sigma}(1-\eta^\alpha)^{-2h_\sigma}\eta^{(\alpha-1)h_\sigma}\right|^2 + O\left((h_\sigma/c)^2\right)$$

Substracting the vacuum contribution, the physics is governed by the crossratio

$$\Delta S(\eta) = S(z_1, z_2) - S^{vac}(z_1, z_2) = \frac{c}{6} \ln \left| \frac{(1-\eta^\alpha)^2}{(1-\eta)^2 \alpha^2 \eta^{\alpha-1}} \right| = \frac{c}{3} \ln \left| \frac{1}{\alpha} \frac{\sinh(\frac{\alpha \ln \eta}{2})}{\sinh \frac{\ln \eta}{2}} \right|$$

Let us use the following configuration

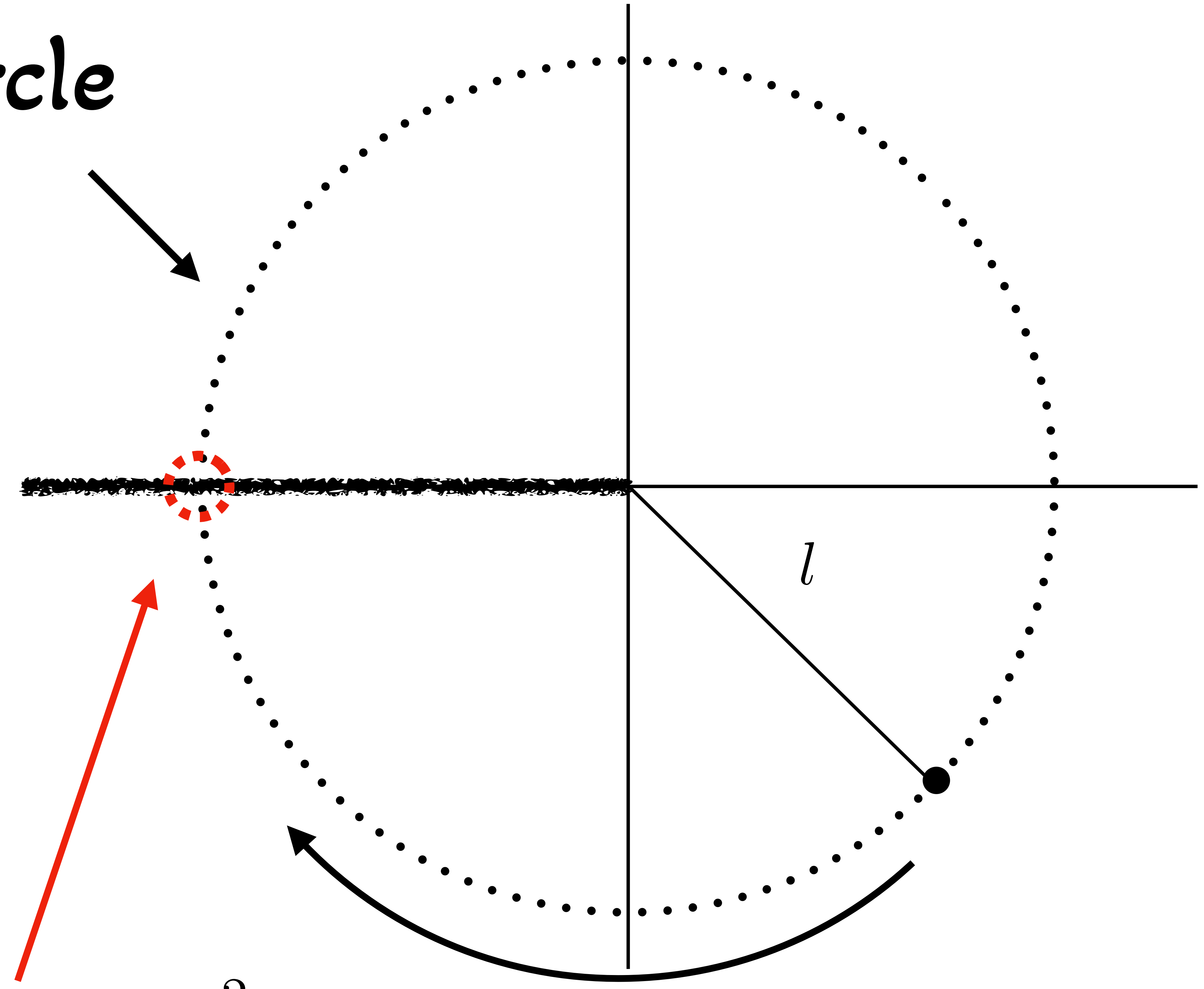
$$u' = \coth \rho \,, \quad z_1 = r e^{il/2} \,, \quad z_2 = r e^{-il/2} \,, \quad u = \tanh \rho$$

The crossratio

[Caputa, Ge '22]

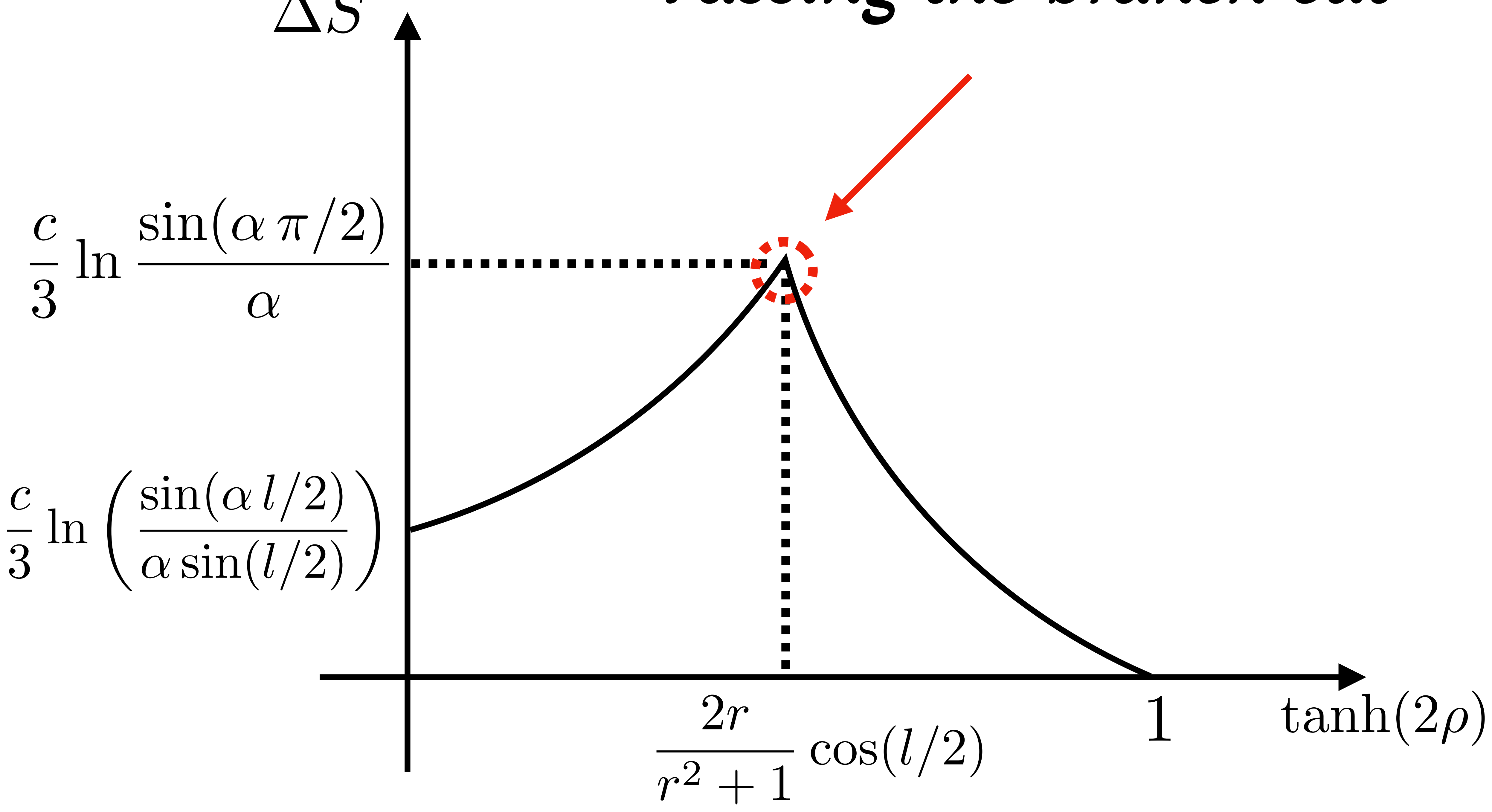
$$\eta = \frac{\left(-r\cosh\rho + e^{\frac{il}{2}}\sinh\rho\right)\left(-\cosh\rho + e^{\frac{il}{2}} r\sinh\rho\right)}{\left(\sinh\rho - e^{\frac{il}{2}} r\cosh\rho\right)\left(r\sinh\rho - e^{\frac{il}{2}}\cosh\rho\right)}$$

Unit circle



$$\cos(l/2) = \frac{1+r^2}{2r}\tanh(2\rho)$$

Passing the branch cut

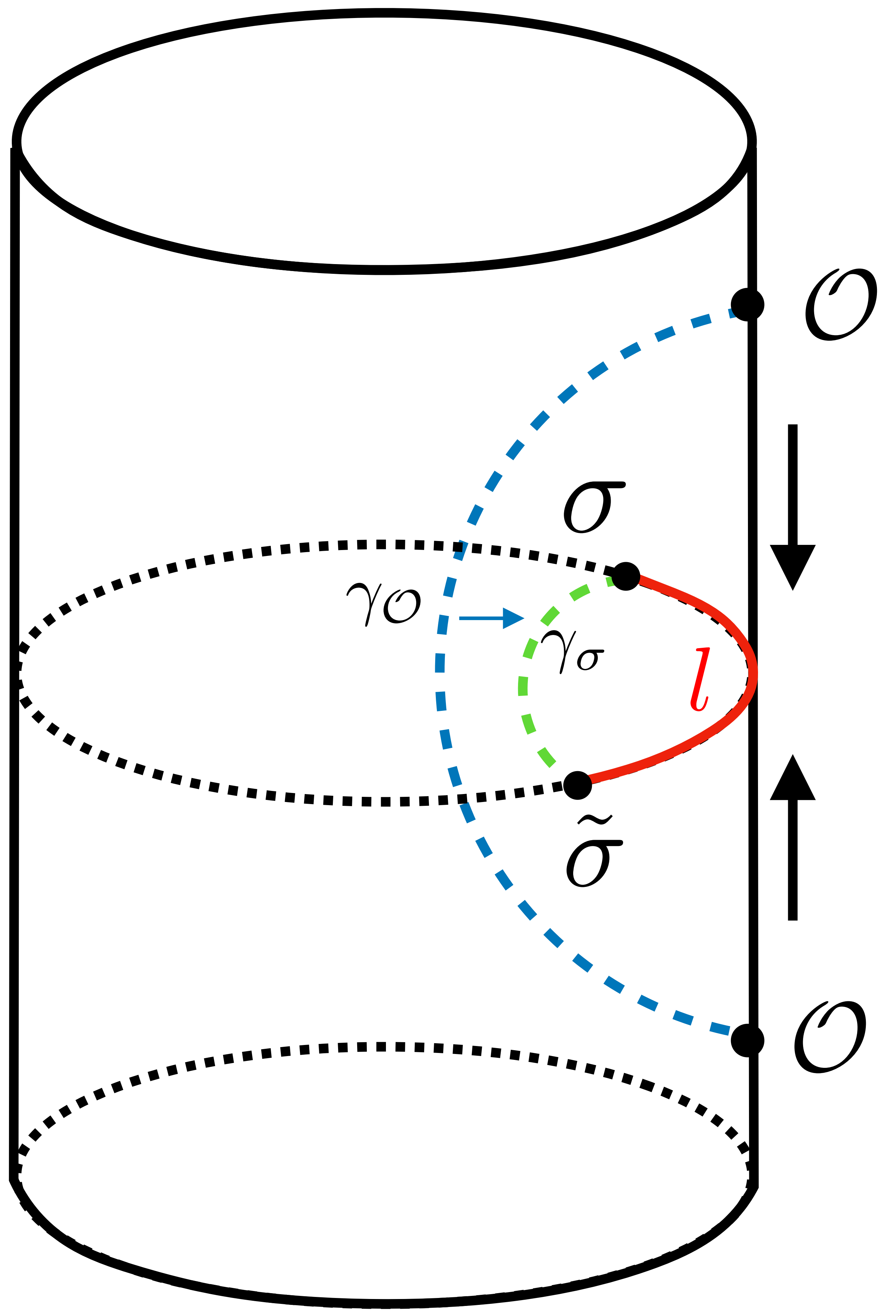


$$l < \pi$$

Such a kink can also be understood in terms of crossing of geodesics

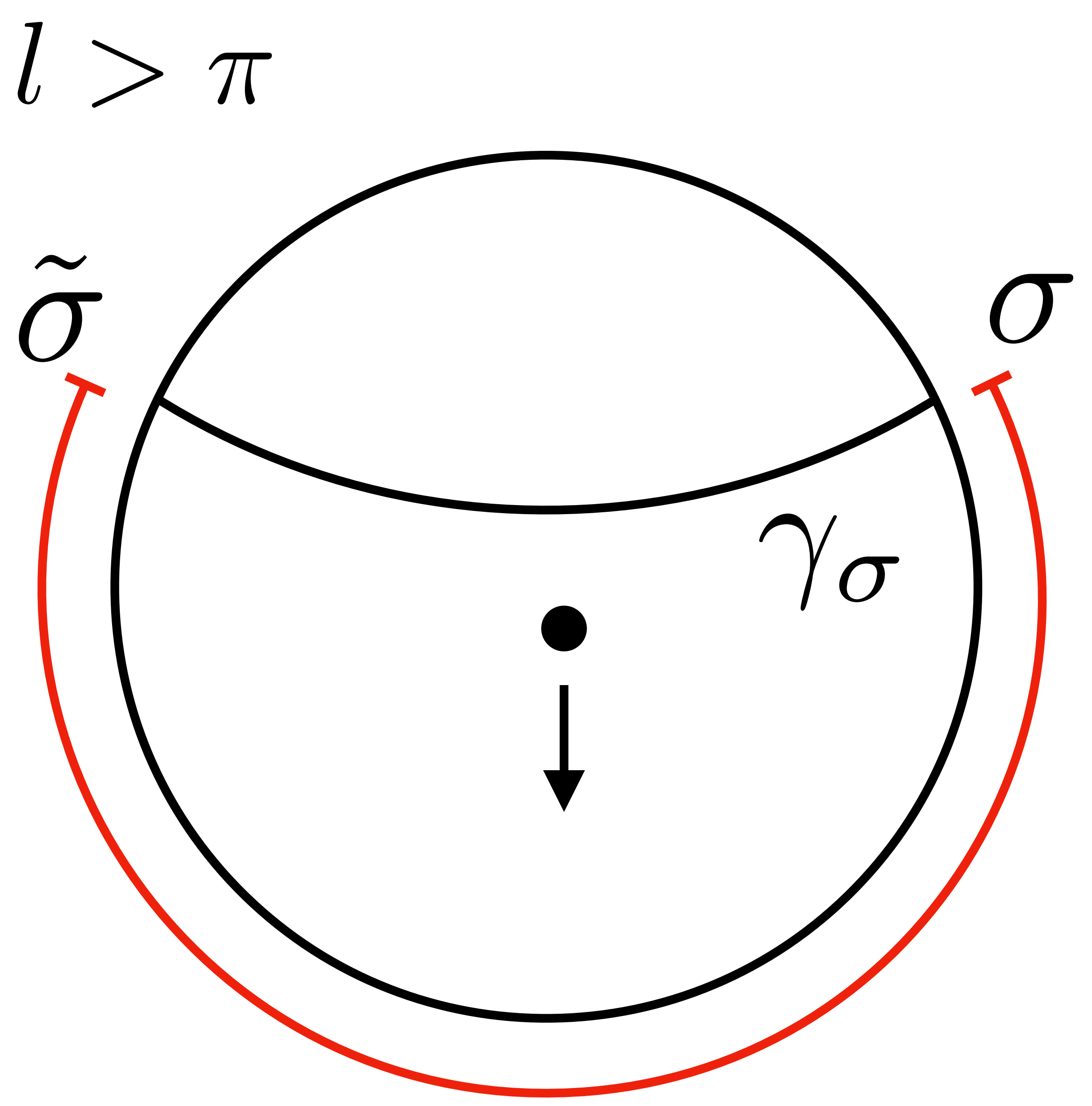
Understanding in three ways

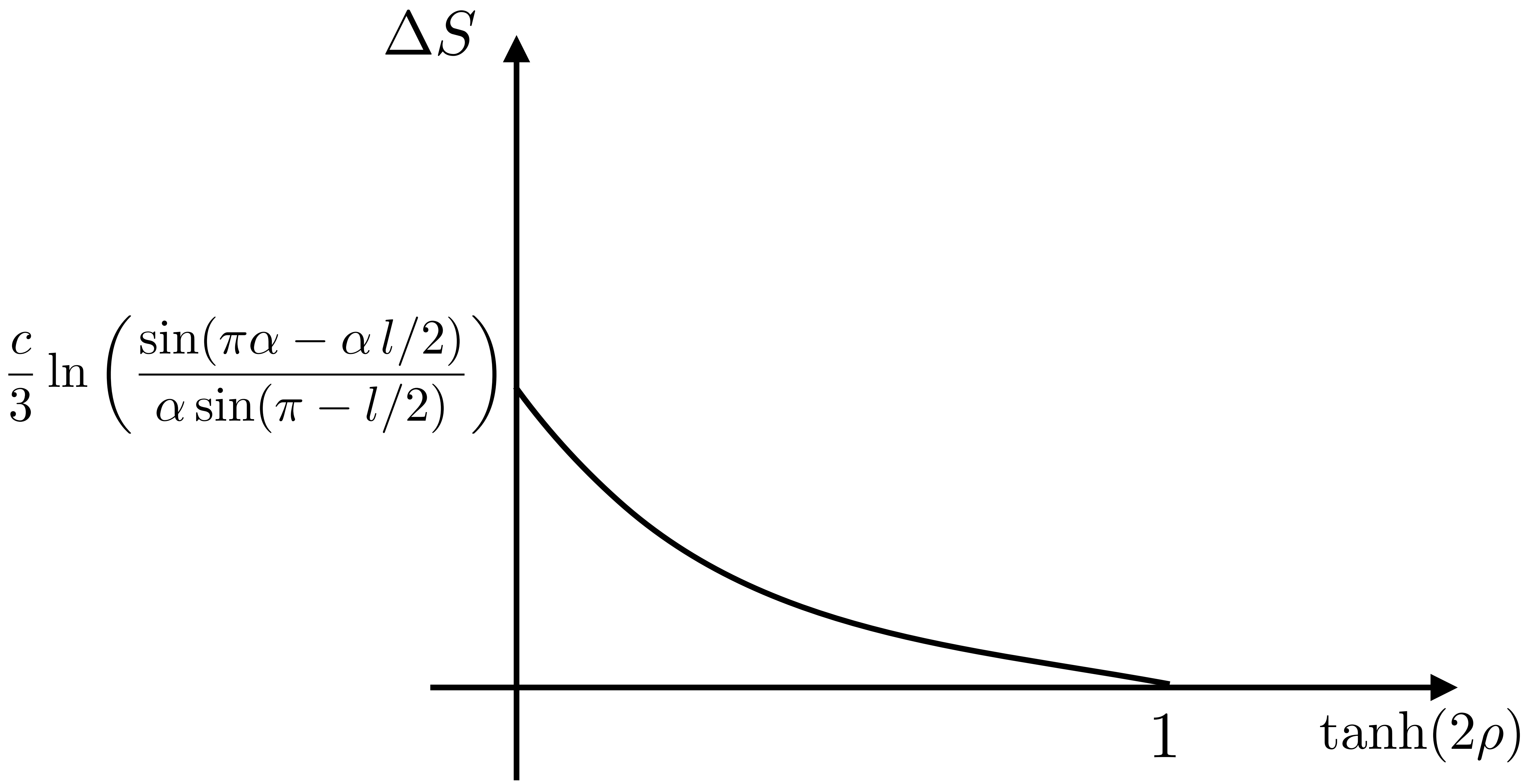
- Crossing the branch cut
- Kink in the EE
- Crossing of two geodesics



Result for l > π

[Caputa, Ge '22]





There is no crossing, no kink as a result.

To summarize:

As two operators are approaching each other, the entanglement for the subregion first increases, reaches the maximum and then decrease to the vacuum entanglement entropy. From gravity side, this can be understood in terms of the relative distance between the massive particle (classical matter) and the minimal surface. This explains the revival in the dynamics of the entanglement entropy.

Future directions:

- What is the real-time version ? Or how to understand in the Lorentzian signature? The timelike geodesic will be disanchored from the boundary; there will be new singularities when operators are null separated
- If the conformal weight exceeds the BH threshold $h>c/24$, bouncing black hole, more precise solution has to be explored ?

ありがとうございます！