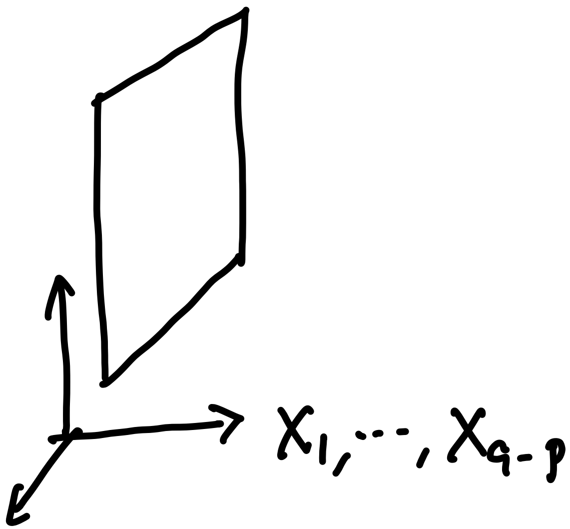
Holographic Geometry from Matrix Degrees of Freedom

Masanori Hanada
花田 政範.

$$\left.\begin{array}{ll} \text{Matrix model} & X_1, \cdots, X_9 \\ \text{4d SYM} & X_1, \cdots, X_6 \\ \text{(p+1)-d SYM} & X_1, \cdots, X_{9-p} \end{array}\right\} \quad \begin{array}{c} N \times N \text{ Hermitian matrices} \\ \text{(scalar fields).} \end{array}$$



$$X_I = \begin{pmatrix} X_I^{11} & & X_I^{ij} \\ & \ddots & \\ X_I^{ij} & & X_I^{NN} \end{pmatrix}$$

X_I^{ii} diagonal $\longrightarrow$ location of D-brane

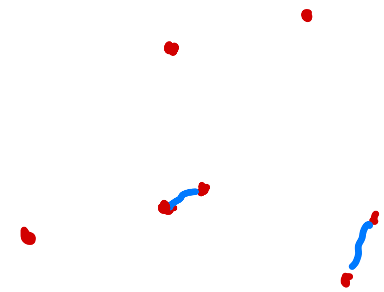
X_I^{ij} off-diagonal $\longrightarrow$ open strings

Potential energy $V(X) = -\frac{g^2}{4} \mathrm{Tr}[X_I, X_J]^2$

low energy $\Rightarrow$ $[X_I, X_J] \simeq 0$

(almost) simultaneously diagonalizable.

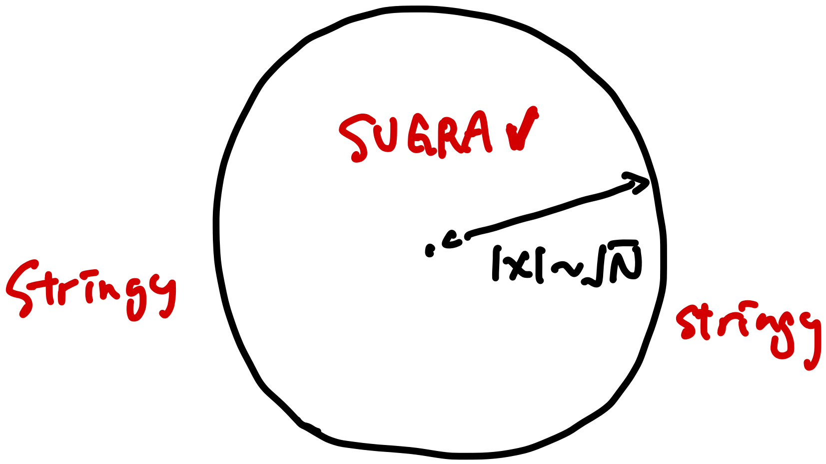
$$X = \begin{pmatrix} X^{11} & & \simeq 0 \\ & \ddots & \\ \simeq 0 & & X^{NN} \end{pmatrix}$$



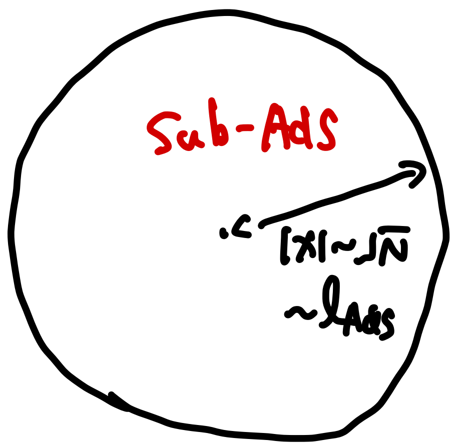
D-branes

+

a little bit of strings



matrix model

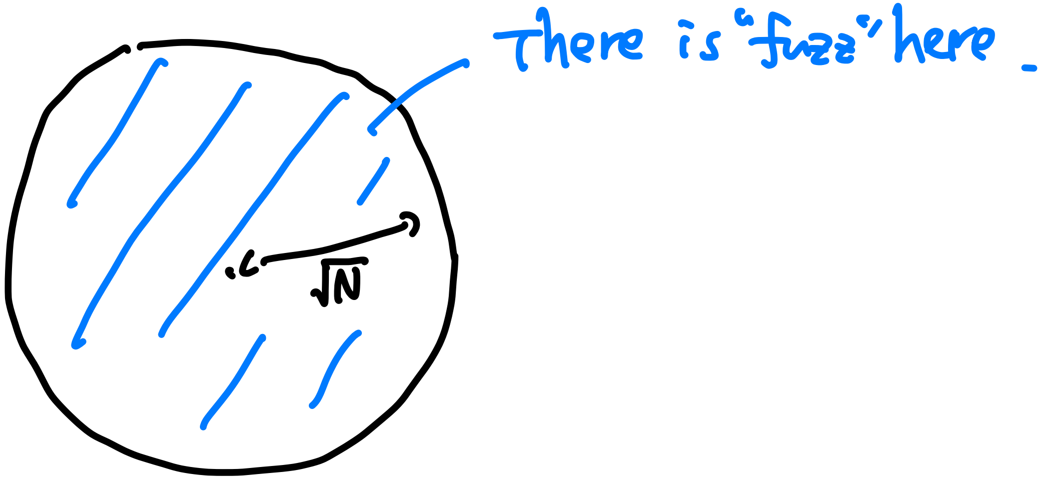


4d SYM on S^3

$\boxed{|X| \lesssim \sqrt{N} \text{ is important for holography}}$

Puzzle

Polchinski, Susskind, ...
(1998 —)



D-brane bound state delocalizes ...?

$$\langle \mathrm{Tr}\, X_I^2 \rangle \sim N^2$$
$$\langle \mathrm{Tr}\, [X_I, X_J]^2 \rangle \sim N^3$$

← 't Hooft counting.

- eigenvalues of $X_I \sim \sqrt{N}$
- Almost maximally non-commutative

Maybe strong dynamics resolve the issue? → NO.

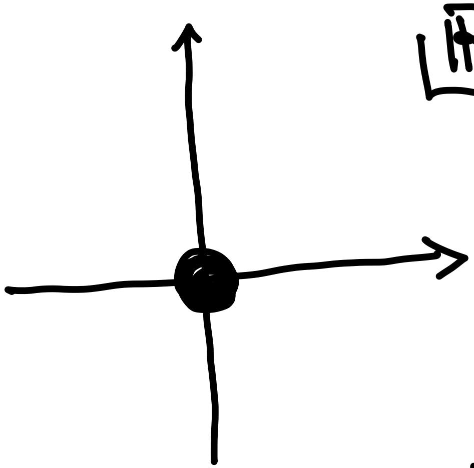
Resolution of the Puzzle

MH, 2021

- The same "puzzle" exists in the Gaussian matrix model.

ground state : $\langle X | g.s. \rangle \sim e^{-\mathrm{Tr} X_I^2}$

$\mathbb{R}^{9N^2}$ (for matrix model)



$\boxed{\text{SU(N)-invariant wave packet}}$

$\mathcal{H} = \{ |x\rangle \mid x \in \mathbb{R}^{9N^2} \}$

$\hat{X}_I^{ij} |x\rangle = X_I^{ij} |x\rangle$

What was wrong?

$|x\rangle \longrightarrow |UxU^{-1}\rangle$ not SU(N)-inv.

But

$|g.s\rangle = \int dx\, |x\rangle \langle x | g.s\rangle \longrightarrow |g.s.\rangle$ SU(N)-inv.

Meaning of "eigenvalue" must be reconsidered.

More generally:

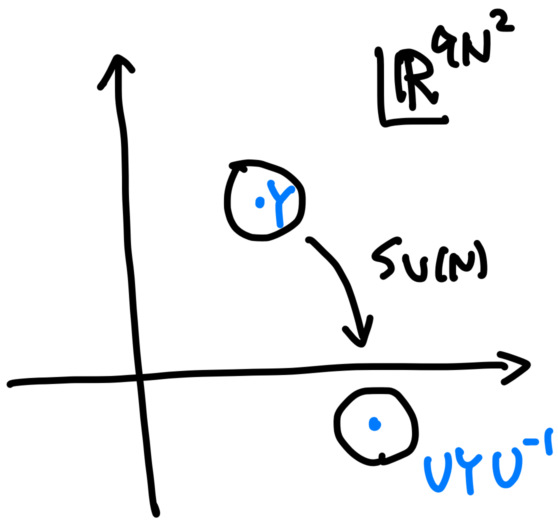
$|Y, Q\rangle$: low-energy wave packet around $Y \in \mathbb{R}^{9N^2}$ in coordinate basis and $Q \in \mathbb{R}^{9N^2}$ in momentum basis

coordinate eigenstate $|X\rangle$ has infinite energy

$\langle X|\hat{H}|X\rangle \geq \langle X|\frac{\hat{P}^2}{2}|X\rangle = \infty$

No information of low-energy configuration!

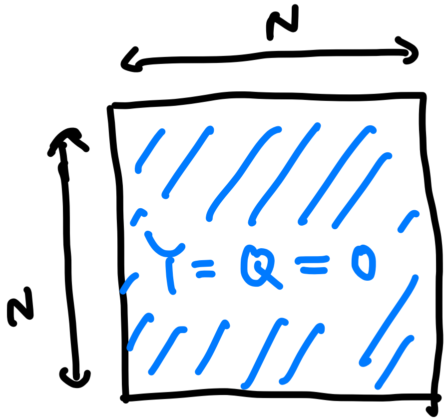
$$|g.s.\rangle = |Y=0, Q=0\rangle$$



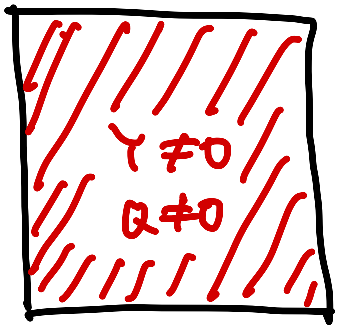
$$|Y, Q\rangle \longrightarrow |UYU^{-1}, UQU^{-1}\rangle$$

- We can diagnalize Y or Q.
- At low energy, $[Y_I, Y_J] \simeq 0$.
- We can show that Y describes D-branes & open strings

$$|Y, Q\rangle = \int d^{9N^2}X \; |X\rangle\langle X|Y, Q\rangle$$

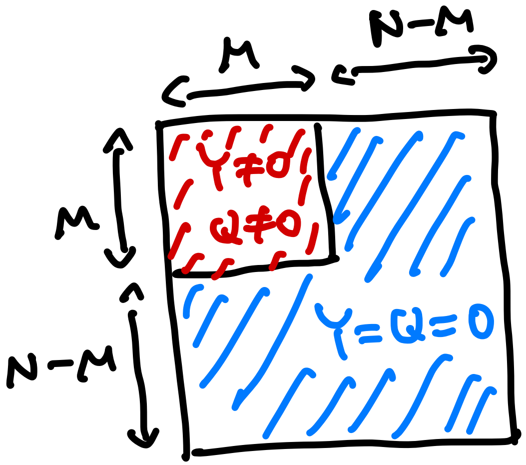


ground state



$\mathrm{tr}\,Y^2 \sim \mathrm{tr}\,Q^2 \sim N^2$

large black hole $\left(\sum_{Y,Q} C_{Y,Q} |Y,Q\rangle \right)$



Small black hole

(Banks-Fischler-Klebanov-Susskind 1996)

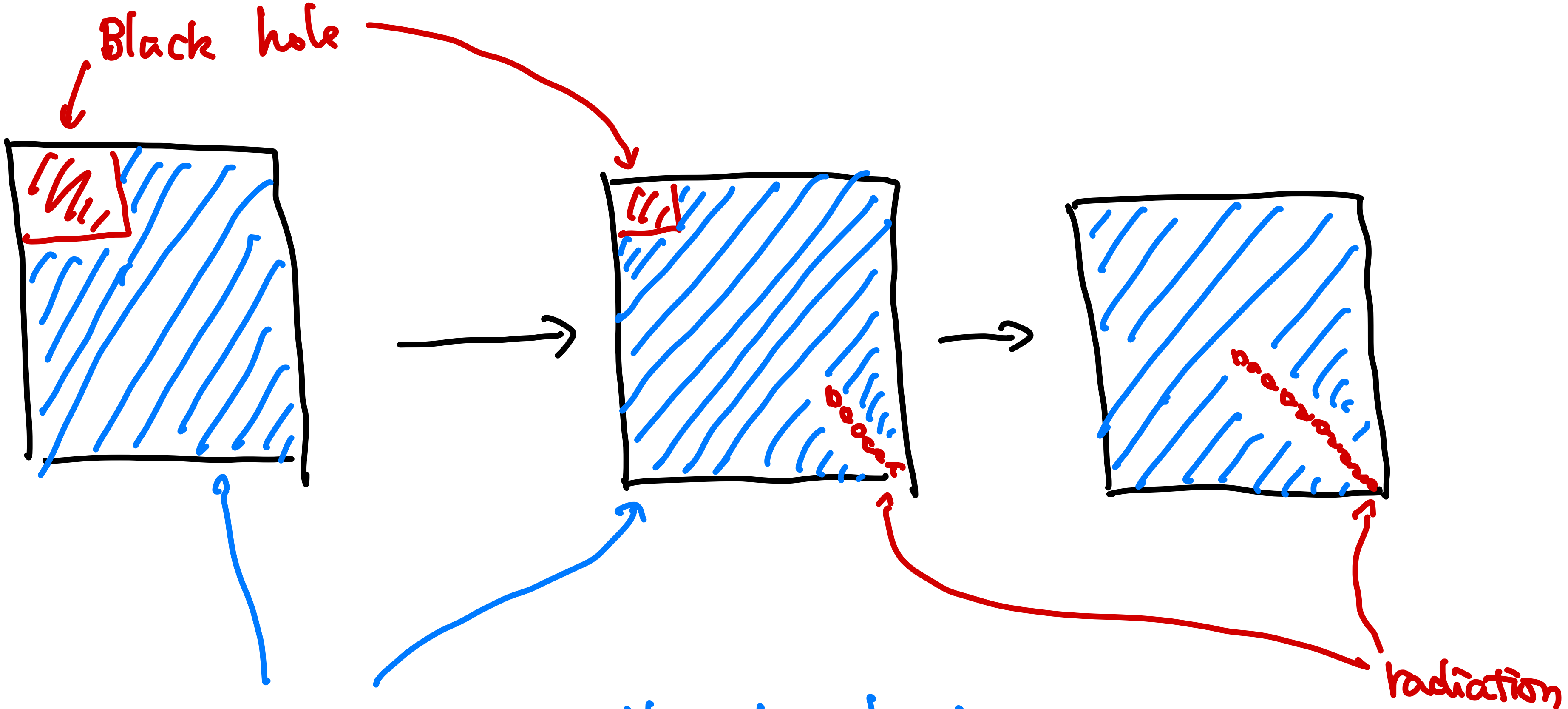
MH-Maltz 2016

Berenstein 2018

...

Partial deconfinement

($SU(M) \subset SU(N)$ deconfines)



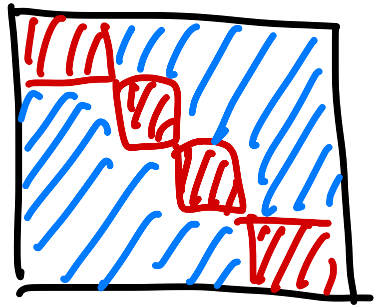
Confined sector resembles the island.

(Gautam - MH - Jevicki - Peng, 2022)

Black Hole Evaporation

Large-N limit

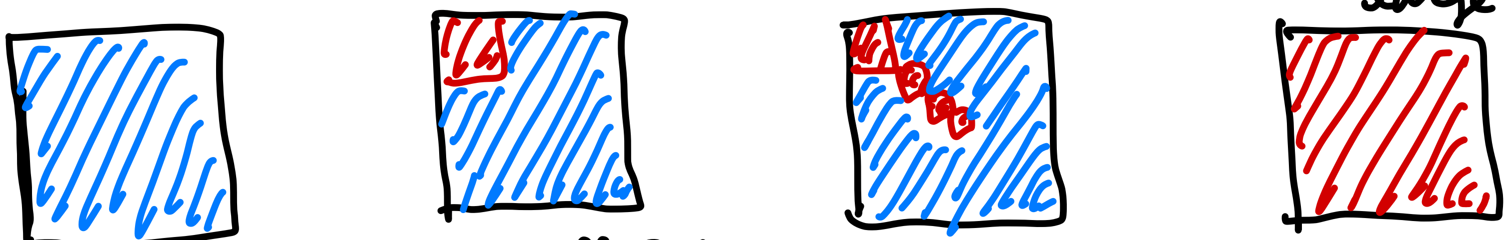
$\simeq$ 2nd Quantization



Banks–Fischler–Shenker–Susskind
1996

$\boxed{\text{Similar picture in Maldacena-type duality}}$

large BH.



small BH

Vacuum is also naturally described. $Y = Q = 0$

4d SYM $\to$ AdS vacuum

Matrix model $\to$ BPS 0-brane,
or its uplift to M (near the origin)