Tensionless limit of AdS_3/CFT_2 from integrability

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Based on arXiv:2303.02120 (letter) and another one

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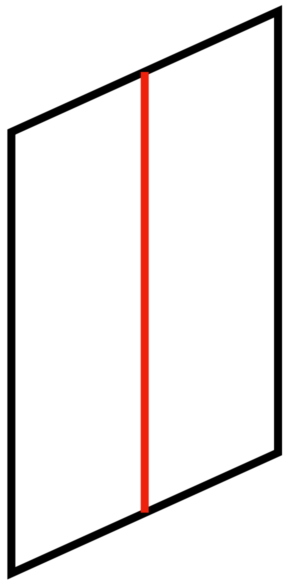
AdS$_3$/CFT$_2$ correspondence

- NSNS flux (F1-NS5) → Dual to $\mathcal{N}$=(4,4) symmetric orbifold CFT

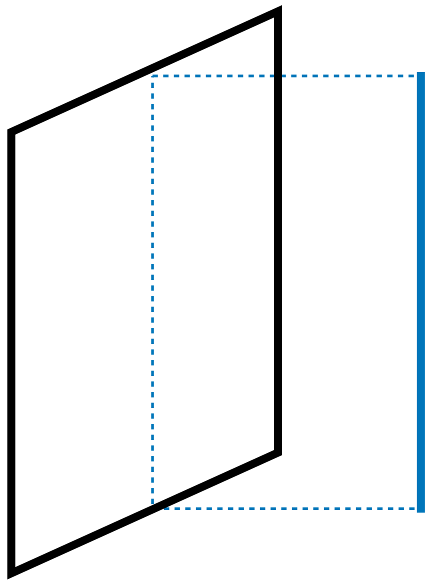
[Gaberdiel, Eberhardt, Gopakumar, 1911.00378]

- RR flux (D1-D5) → Dual CFT is unidentified, which is suitable for computing non-BPS spectrum at a fixed string-duality frame

Higgs branch



Coulomb branch



Wrapping D5 on T^4 → $AdS_3 \times S^3 \times T^4$ in the near horizon limit

Classical integrability

- GS superstring in $AdS_3 \times S^3 \times T^4$ is classically integrable, because "coset κ-gauge" → supercoset + free T^4 SCFTs

$$\frac{PSU(1,1|2)^2}{SU(1,1) \times SU(2)} \supset AdS_3 \times S^3$$

[Babichenko, Stefański, Zarembo, 0912.1723]

- But this gauge choice is singular, use the light-cone κ-gauge
- In LC gauge we find explicit interactions between massive modes from $AdS_3 \times S^3$ and massless modes in T^4

[Borsato, Ohlsson Sax, Sfondrini, Stefański, 1406.0453]

GS action in AdS$_3$ × S^3 × T^4

- Use the general GS action for IIB superstring

$$\mathcal{L}_{GS} = \mathcal{L}_B + \mathcal{L}_F + \mathcal{L}_{WZ}$$

$$\mathcal{L}_B = -\frac{1}{2}\left(\gamma^{\alpha\beta} G_{mn}\partial_\alpha X^m \partial_\beta X^n + \epsilon^{\alpha\beta} B_{mn}\partial_\alpha X^m \partial_\beta X^n\right)$$

$$\mathcal{L}_F = -i\gamma^{\alpha\beta}\bar{\tilde{\theta}}_I \not{E}_\alpha \left(\delta^{IJ} D_\beta + \frac{1}{48}\sigma_3^{IJ} \underbrace{\not{F}}_{\text{RR}} \not{E}_\beta + \frac{1}{8}\sigma_1^{IJ} \underbrace{\not{H}_\beta}_{\text{NSNS}}\right)\tilde{\theta}_J$$

$$\mathcal{L}_{WZ} = i\epsilon^{\alpha\beta}\bar{\tilde{\theta}}_I \sigma_1^{IJ} \not{E}_\alpha \left(\delta^{JK} D_\beta + \frac{1}{48}\sigma_3^{JK} \not{F} \not{E}_\beta + \frac{1}{8}\sigma_1^{JK} \not{H}_\beta\right)\tilde{\theta}_K$$

- Fluxes and string tension: k is quantized, g is continuous

$$F_3 = \frac{g}{T}\left(\Omega_{AdS_3} + \Omega_{S^3}\right), \quad H_3 = \frac{k}{2\pi T}\left(\Omega_{AdS_3} + \Omega_{S^3}\right), \quad T = \frac{R^2}{2\pi\alpha'} = \sqrt{g^2 + \frac{k^2}{4\pi^2}}$$

[Cvetič, Lü, Pope, Stelle, hep-th/9907202] [Wulff, 1304.6422] [Berkovits, Vafa, Witten, hep-th/9902098]

GS action in $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$

- The kinetic term rescaled as

$$ds^2 = -\left(\frac{1+\frac{\vec{z}^2}{4}}{1-\frac{\vec{z}^2}{4}}\right)^2 dt^2 + \frac{d\vec{z}^2}{(1-\frac{\vec{z}^2}{4})^2} + \left(\frac{1-\frac{\vec{y}^2}{4}}{1+\frac{\vec{y}^2}{4}}\right)^2 d\phi^2 + \frac{d\vec{y}^2}{(1+\frac{\vec{y}^2}{4})^2} + \sum_{i=6}^{9} dX_i dX_i$$

- Uniform light-cone gauge, take the decompactification limit

$$x^+ = \frac{1}{2}(\phi + t) = \tau, \qquad P_- = \int_{-r}^{r} d\sigma\, p_- = 4r \to \infty$$

- Central extension of the residual global symmetry

$$\mathfrak{psu}(1,1|2)_L \oplus \mathfrak{psu}(1,1|2)_R \;\to\; \mathfrak{psu}(1|1)^4 \;\to\; \mathfrak{psu}(1|1)^4_{c.e.}$$

- Symmetry of T^4

$$\mathfrak{so}(4) = \mathfrak{su}(2)_\bullet \oplus \mathfrak{su}(2)_\circ\,, \quad \mathfrak{su}(2)_\bullet \curvearrowright \{Q^{\dot{a}}, S_{\dot{a}}\},\ (\dot{a} = 1,2)$$

[Borsato, Ohlsson Sax, Sfondrini, Stefański, 1406.0453] [Lloyd, Ohlsson Sax, Sfondrini, Stefański, 1410.0866]

Integrable spin chain

- Consider an integrable spin chain which has the same symmetry as the decompactified gauge-fixed GS action

$$\mathfrak{psu}(1,1|2)_L \oplus \mathfrak{psu}(1,1|2)_R \quad \to \quad \mathfrak{psu}(1|1)^4_{c.e.} \;\; (\oplus \mathfrak{su}(2)_\bullet \oplus \mathfrak{su}(2)_\circ)$$

$$\{\mathfrak{Q}_L{}^{\dot a}, \mathfrak{S}_{L\dot b}\} = \frac{1}{2}\delta^{\dot a}{}_{\dot b}\,(H+M), \qquad \{\mathfrak{Q}_L{}^{\dot a}, \mathfrak{Q}_{R\dot b}\} = \delta^{\dot a}{}_{\dot b}\,C$$

$$\{\mathfrak{Q}_{R\dot a}, \mathfrak{S}_R{}^{\dot b}\} = \frac{1}{2}\delta_{\dot a}{}^{\dot b}\,(H-M), \qquad \{\mathfrak{S}_{L\dot a}, \mathfrak{S}_R{}^{\dot b}\} = \delta_{\dot a}{}^{\dot b}\,\overline{C}$$

- Shortening conditions for one-particle states are (pure RR)

$$H^2 = M^2 + 4C\overline{C} \quad \Rightarrow \quad E(p) = \sqrt{m^2 + 4h(T)^2 \sin^2\frac{p}{2}}, \quad \underbrace{(m \in \mathbb{Z})}_{\text{mass}}$$

[Beisert, Zwiebel, 0707.1031] [Borsato, Ohlsson Sax, Sfondrini, Torrielli, 1303.5995]

Integrable spin chain

- **The S-matrix between two fundamental excitations are almost determined by the symmetry, to all orders of h**

$$\mathbb{S} = S_0[\hat{S}_{\mathfrak{su}(1|1)} \otimes \hat{S}_{\mathfrak{su}(1|1)}]$$

- **The momenta of multi-particle states are quantized by asymptotic Bethe Ansatz equations at finite and large J**

$$(\pm 1) = e^{ip_{k,A}R} \prod_{\{l,B\}} S^{AB}(u_{k,A}, v_{l,B})$$

$$E_J(p) = \sum_i \sqrt{m_i^2 + 4h^2 \sin^2 \frac{p_i}{2}}$$

From spin chain to AdS/CFT

Energy spectrum of the integrable spin chain should match

- Spectrum of superstring at large h
- Spectrum of unknown large N CFT at small h

if one neglects wrapping corrections from massless $(m=0)$ modes

At finite J, the energy receives corrections from virtual particles

$$E_J(p) = \sum_i \sqrt{m_i^2 + 4h^2 \sin^2 \frac{p_i}{2}} + \boxed{\delta E_J}$$

Proposals for wrapping in AdS_3/CFT_2

- **Quantum Spectral Curve (QSC) without massless modes**

[Cavaglià, Gromov, Stefański, Torrielli, 2109.05500] [Ekhammar, Volin, 2109.06164]

- **Mirror Thermodynamic Bethe Ansatz (TBA) including massless**

[Frolov, Sfondrini, 2118.08898]

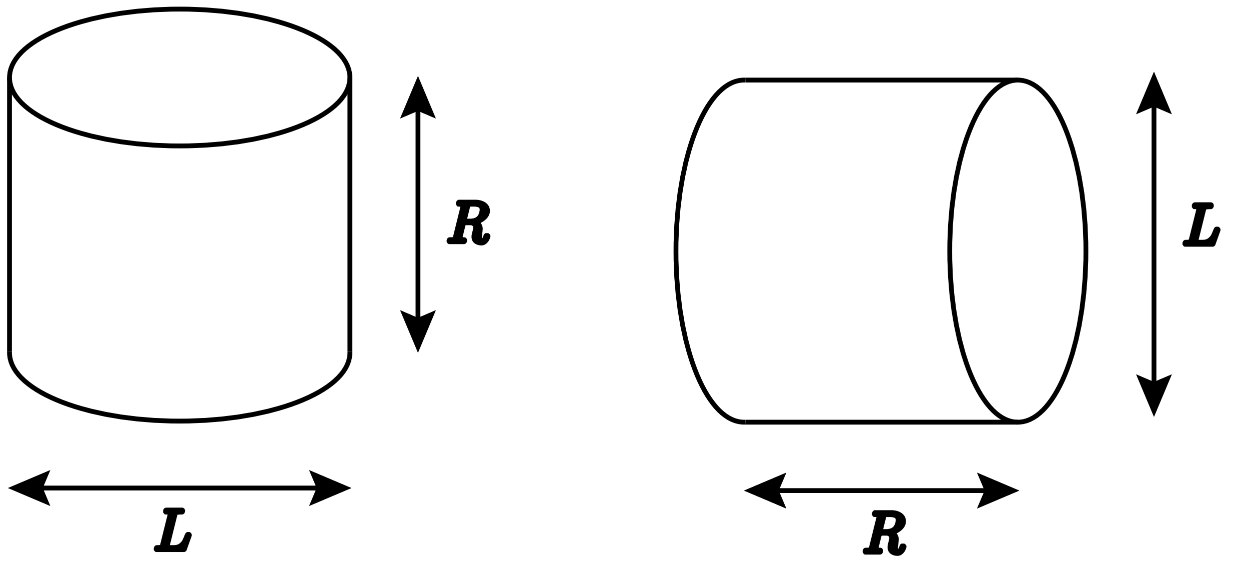
TBA predicts the spectrum of dual CFT & tensionless string
if small coupling $h(T)=0$ ⇔ tensionless limit ($T=0$)

Our goal: solve the mirror TBA for massless excited states at small h

Mirror trick and TBA

- **Equivalence of Euclidean torus partition functions**

[Zamolodchikov (1990)]
[Dorey, Tateo, hep-th/9607167]
[Frolov, Sfondrini, 2112.08898]



Ground state at finite volume evolving for a long period of time

Gas of all types of particles at large volume and finite temperature

$$e^{-RE_0(L)} = \lim_{R\to\infty} e^{-\boxed{\tilde{\mathcal{F}}(R)}}$$

Exact energy ($L = J$) ← Mirror free energy ← TBA equations

Mirror trick and TBA

- **Equivalence of Euclidean torus partition functions**

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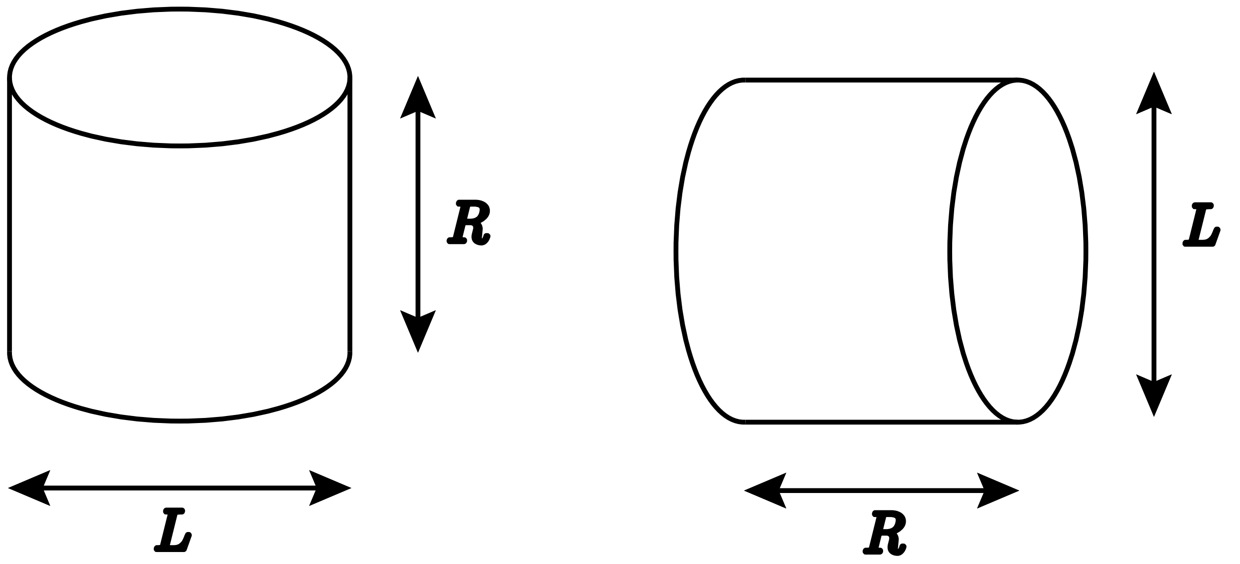

Ground state at finite volume evolving for a long period of time

Gas of all types of particles at large volume and finite temperature

$$e^{-RE_0(L)} = \lim_{R\to\infty} e^{-\tilde{\mathcal{F}}(R)}$$

We assume that the massless contributions to the mirror free energy at the large R saddle-point is negligible

Mirror trick and TBA

- **Equivalence of Euclidean torus partition functions**

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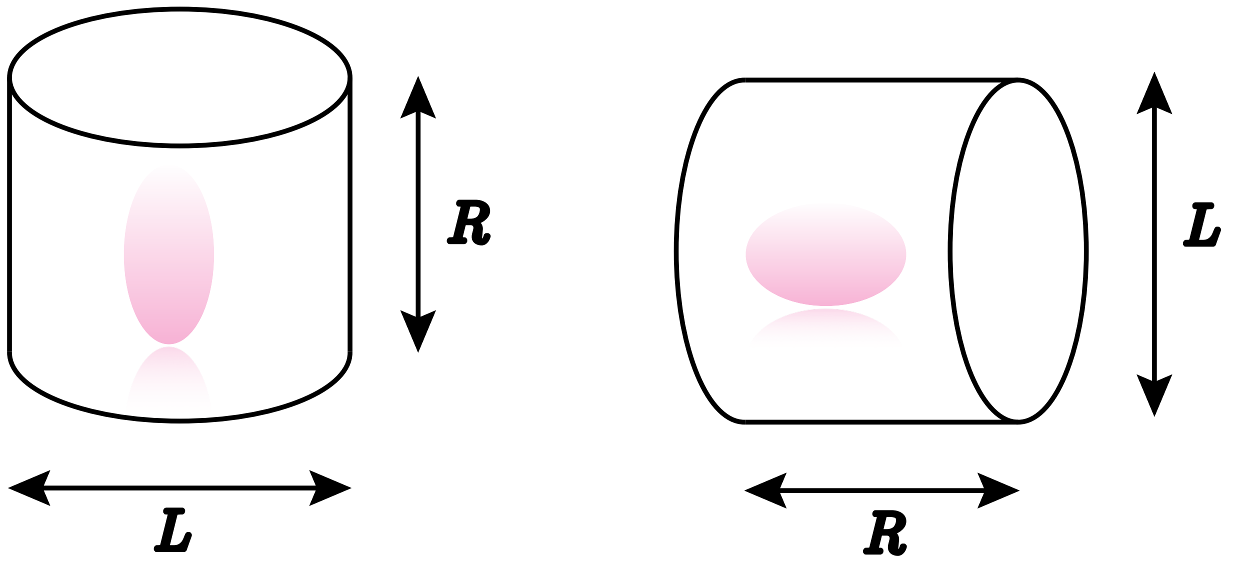


Ground state at finite volume evolving for a long period of time

Gas of all types of particles at large volume and finite temperature

$$e^{-RE_0(L)} = \lim_{R\to\infty} e^{-\tilde{\mathcal{F}}(R)}$$

It is expected that the excited-state energy at finite volume can be obtained by the "contour deformation" of TBA

Our setup

- Length L = even
- Excite a pair of massless particles (massive particles contribute at higher orders in h)
- ($L/2$) distinct solutions (mode numbers) for each L
- Take the small coupling (h=0) limit

The TBA equations simplify;
two Y-functions convoluted by Cauchy kernel

TBA at small string tension

- **Cauchy kernel** $$s(\gamma) = \frac{1}{2\pi i}\frac{\partial \log S(\gamma)}{\partial \gamma}\,, \quad S(\gamma) = -i \tanh\left(\frac{\gamma}{2} - \frac{i\pi}{4}\right)$$

- **Mirror TBA equations for massless & auxiliary particles**

$$\log Y_0 = -L\tilde{E}_0 + \log[(1+Y_0)^2(1-Y)^4] * s - \sum_j \log S(\gamma_{*j} - \gamma)$$

$$\log Y = \log[(1+Y_0)^2] * s - \sum_j \log S(\gamma_{*j} - \gamma)$$

- **Solve TBA and compute the exact energy**

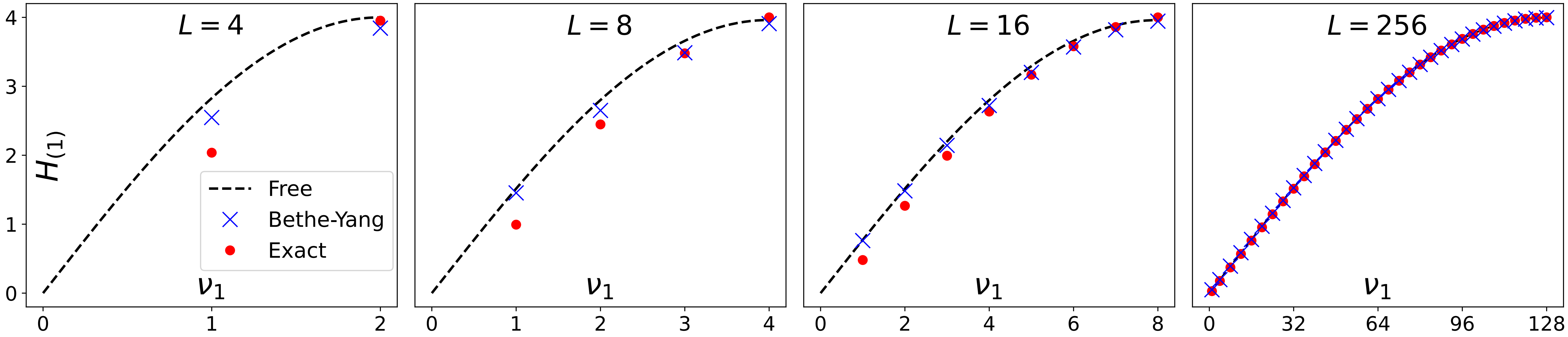
$$E = \sum_j \left|2h \sin\frac{p_{*j}}{2}\right| - \int \frac{d\tilde{p}_0}{2\pi} \log(1+Y_0)$$

Numerical solutions

$$E = \sum_j \left| 2h \sin \frac{p_{*j}}{2} \right| - \int \frac{d\tilde{p}_0}{2\pi} \log(1 + Y_0)$$

Both terms are $O(h)$

- Wrapping corrections are parametrically small, $O(1/L)$
- The coefficients may be transcendental numbers



Conclusion and Outlook

- **Integrability methods in AdS_3/CFT_2**
- **Numerically solved excited-state TBA with massless particles**
- **$O(1/L)$ contributions to the energy**

- **How to fully justify TBA**
- **Identify the dual CFT**
- **Blackhole (TBA for the temperature of Hagedorn transition)**