Lorentzian spacetime wormholes

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Outline

In a nutshell, I will introduce a moduli space of **semiclassical Lorentzian wormholes** in AdS and check through various examples that integrating over this moduli space is (gauge) **equivalent** to analytically continuing the results of Euclidean path integral with spacetime wormholes.

Motivation and introduction to Lorentzian topology change

Boundary predictions as **precision checks** on our proposal

Lorentzian wormholes using Louko-Sorkin **crotches** and slits in AdS

Constrained instantons explain why singular **slit spacetimes** contribute

Example 1. spectral form factor slits near the horizon on the **double cone**

Example 2. two-point function at late times

Example 3. **firewall probability** rejuvenating the two-sided black hole

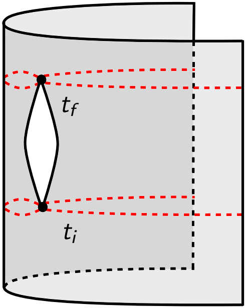
Motivation and introduction to Lorentzian topology change

Many physically relevant questions about black holes intrinsically involve **real times** (obviously), such as the fate of an **infalling observer**, the nature of the black hole **interior** and the resolution of the singularity.

Recent progress bases on computing Euclidean wormhole amplitudes suggests that **topology change** is relevant for answering those questions. In particular topology change often becomes important when exponentially **long times** or **interiors** are involved. When one asks complex questions, competition with topological suppression can arise (Almheiri, Lin, Stanford, Yang, Iliesiu, Mezei, Sarosi . . .).

We want to understand what those Euclidean answers really *mean* for a **real time observer** (such as ourselves). In other words, we think it is important to understand **topology change via Lorentzian spacetimes**, in order to truly understand black hole physics in our universe. Lorentzian topology changing spacetimes were also discussed recently by (Marolf, Maxfield, Collin-Ellerin, Dong, Rangamani, Wang, Tajdini, Rath, Usatyuk . . .) which were **sources of inspiration** for what follows.

Lorentzian topology change can be slightly more tricky than its Euclidean twin brother, for the following reason.
We usually imagine **baby universes** that detach from the parent at some time t_i and reattach at some time t_f (Giddings, Strominger ...)



$\sim T^2 e^{-2S_{\text{inst}}}$

At these times **the spatial metric** at some d−2 sphere (or two points in 2d) **vanishes** $\sqrt{g} = 0$, because the sphere shrinks to one point.
In other words the **metric is singular** at these special locations.

So to allow for topology change we should entertain Lorentzian **metrics with singularities**, where the metric is not invertible (Louko, Sorkin).

To appreciate that this is really **not optional** consider 2d JT gravity on some closed manifold. The Gauss-Bonnet theorem says

$$\int \mathrm{d}^2 x \sqrt{g} R = -8\pi(g-1) \quad \textbf{real}$$

Assuming $R+2=0$ everywhere this makes a contradiction (unless $g=1$), because for Lorentzian metrics $\sqrt{g}$ is imaginary.
Therefore $R+2=0$ **smooth spacetime is not enough**, and one should allow for the aforementioned singular points (surfaces).

As shown by (Louko, Sorkin) those points (surfaces) have the property

$$\sqrt{g} R \supset -4\pi \sum_{\text{crotches}} \delta(x - x_c)$$

We will have one such **singular points marking the birth and death of baby universes**, which reproduced the correct Euler character $\chi = -2$ (number of baby universes) $+ \ldots$. Much more details later!

Our goal is to find the moduli space of semiclassical Lorentzian wormholes with such singularities in AdS and **check through** various **examples** that integrating over this moduli space is (gauge) equivalent to analytically continuing the results of Euclidean path integral with wormholes.

Boundary predictions

Before getting to the Lorentzian wormholes, I will first introduce those examples. In particular I will give two rather non-trivial **predictions** that the Lorentzian wormholes should reproduce.
The boundary prediction is **universal**, the Euclidean calculations have only been done in 2d.

The first example is the **spectral form factor**

$$Z(\beta+\mathrm{i}T,\beta-\mathrm{i}T) = \mathrm{Tr}\Big(e^{-(\beta+\mathrm{i}T)H}\Big)\,\mathrm{Tr}\Big(e^{-(\beta-\mathrm{i}T)H}\Big)$$
$$= \sum_{i=1}^{\dim(H)}\sum_{j=1}^{\dim(H)} e^{-\beta(E_i+E_j)}e^{-\mathrm{i}T(E_i-E_j)}$$

After some **time averaging** one finds for **chaotic quantum systems** a universal late-time profile. For instance via periodic orbits (Haake book).

$$Z(\beta+\mathrm{i}T,\beta-\mathrm{i}T)_{\mathrm{conn}} = \int_0^\infty \mathrm{d}E\, e^{-2\beta E}\min(\rho(E), T/2\pi)\,,\quad \rho(E)=e^{S(E)}$$

Writing this out and using integration by parts gives

$$\int_0^{E(T)} \mathrm{d}E\, e^{-2\beta E}\, \rho(E) + \frac{T}{2\pi}\int_{E(T)}^{\infty} \mathrm{d}E\, e^{-2\beta E} = \frac{1}{2\beta}\int_0^{T/2\pi} \mathrm{d}\rho\, e^{-2\beta E(\rho)}$$

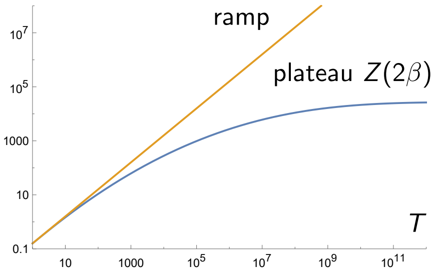
$$\sim \sum_{g=0}^{\infty} T^{2g+1} \oint_0 \frac{\mathrm{d}\rho}{\rho^{2g+1}} e^{-2\beta E}$$

Given the suppression by powers of $1/\rho^2 = e^{-2S(E)}$ it is tempting to identify this with some type of **instanton expansion**. After integration by parts and using a **semiclassical approximation** one obtains

$$\sim \sum_{g=0}^{\infty} T^{2g+1} \int_{\dots}^{\infty} \mathrm{d}E\, e^{-2gS(E)}\, e^{-2\beta E}\,, \quad e^{S(E)} = e^{A(E)/4G}$$

Here $A(E)$ is the area of black holes in your theory with ADM energy E and the **entropy** $S(E)$ seems to play the role of an **instanton action.** One nice thing about this equation is that the right hand-side as a whole follows from perturbative periodic orbit (quantum chaos) considerations (Saad, Stanford, Yang, Yao).

This expansion is **convergent**.
In particular for $T \to \infty$ this goes to a non-zero constant **plateau** $Z(2\beta)$.

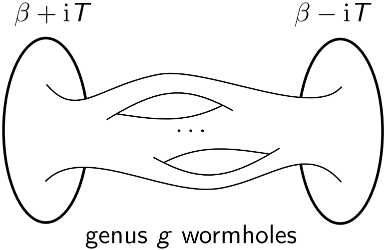


The first litmus test for the Lorentzian wormhole solutions that we will discuss, will be to **reproduce this expansion**.

Let me mention that until recently it was not even clear that a Euclidean gravity calculation could reproduce this expansion.
In gravity, one computes the spectral form factor by path integrating over **wormhole geometries**, with two asymptotically AdS boundaries.

Building on work by (Okuyama,Sakai on Airy gravity) and (Saad, Stanford, Yang, Yao) we found that the sum over **Euclidean wormhole** geometries in 2d

$$Z_g(\beta+\mathrm{i}T, \beta-\mathrm{i}T)_{\text{conn}} =$$



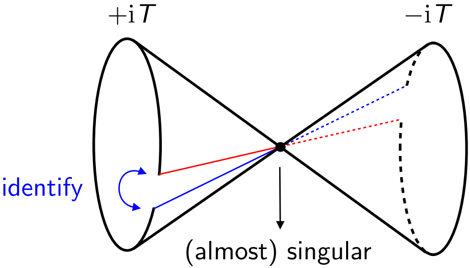
$$\sim T^{2g+1}$$

indeed reproduces this expansion.
We work in the **late time limit** $T \to \infty$ and $e^{S_0} \to \infty$ with their ratio fixed. This double-scaled regime is sometimes called the τ-scaling limit, and it is a crucial ingredient in making the expansion **convergent**.

I will not discuss this Euclidean story here, but feel free to ask afterwards.

Let us emphasize that the only input here has been chaotic behavior in the double scaling limit, leading to the sine kernel and $\min(\rho(E), T/2\pi)$. Since black holes in any dimension are chaotic, this equation should hold **universally**. Within individual charge sectors.
Much like the **double cone** explaining the ramp (Saad, Shenker, Stanford)

$$Z_0(+\mathrm{i}T, -\mathrm{i}T)_{\text{conn}} =$$

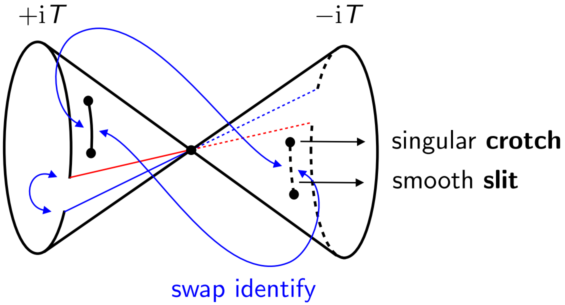


$$\sim T$$

I will review this double cone in more detail later.
our Lorentzian semiclassical wormholes exist indeed for any theory, in line with this claimed universality. You should think of them as **higher genus** versions of the **double cone**.

In particular our solutions look for instance like

$$Z_1(+\mathrm{i}\,T, -\mathrm{i}\,T)_{\text{conn}} =$$



$$\sim T^3 e^{-2S_{\text{inst}}}$$

This is just a teaser, much more details will follow.

You may think at this point, Andreas, these higher genus corrections are super specific things, **why** in the world **should anyone care**. Since I think motivation is everything, let me repeat the point.

In truth, I do *not* think you should care about higher genus corrections to the spectral form factor.
But you should care about higher genus corrections to **physical observables**, such as correlation functions, which actually affect our observations in black hole backgrounds, and as we will discuss later even indicate what your **survival chances** are when you jump into an old (typical) black hole.
There is two layers to such questions, one what the actual answer is (a number) and two what the physical mechanism is that explains this answer. This mechanism is Lorentzian wormhole physics, and our goal is to understand how Lorentzian wormhole physics works.
It just turns out that the spectral form factor is the **simplest setup** in which Lorentzian wormhole physics plays a role, therefore we try to first understand what we can in this simplest example, before moving on to the **stuff we actually should care about**.

Moving on we shall do.

The second (closely related) example of a benchmark for our Lorentzian wormholes is the late time **two point correlation function**.
For this we consider **2d only**, but a generalization to higher dimensions presumably exists. See work by (Sonner, de Boer...).

After some time averaging one finds for **chaotic quantum systems** a universal late-time profile (Saad, Blommaert...)

$$\mathrm{Tr}\Big(\mathcal{O}\, e^{-(\beta/2+\mathrm{i}T)H}\mathcal{O}\, e^{-(\beta/2-\mathrm{i}T)H}\Big)$$
$$= \int_0^\infty \mathrm{d}E\, e^{-\beta E} \min(\rho(E), T/2\pi)\, e^{-S_0} \, |\mathcal{O}_{EE}|^2$$

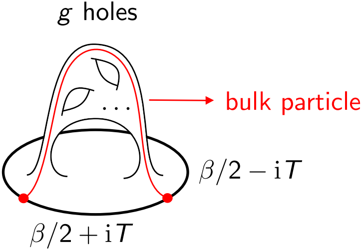
After some work one obtains from this the **semiclassical expansion**

$$\mathrm{Tr}\Big(\mathcal{O}\, e^{-(\beta/2+\mathrm{i}T)H}\mathcal{O}\, e^{-(\beta/2-\mathrm{i}T)H}\Big) = \big\langle e^{-\Delta\ell}\big\rangle$$

$$\sim \sum_{g=1}^{\infty} T^{2g-1}\int_{\dots}^{\infty} \mathrm{d}E\, e^{-\beta E}\, e^{-\Delta\ell(E)}\, e^{-(2g-1)S(E)}$$

Here $\ell(E)$ is the length of the ER bridge in the TFD at $T=0$.
This can be reproduced from Euclidean geometry (Saad, Blommaert. . .)

$$\mathrm{Tr}\Big(\mathcal{O}\, e^{-(\beta/2+\mathrm{i}T)H}\mathcal{O}\, e^{-(\beta/2-\mathrm{i}T)H}\Big)_g =$$



$$\sim\ T^{2g-1}$$

but later we will explain how this is **reproduced by Lorentzian wormhole solutions**.

Lorentzian wormholes general construction

The firewall setup (Stanford, Yang) I will introduce later, first I want to give some details about our Lorentzian geometries.

Now let us start constructing our Lorentzian wormhole solutions in AdS.

Let me remind you that according to (Louko, Sorkin) one can allow for Lorentzian topology change by including singular points (which they call **crotches**, for visual reasons which will soon become apparent) with the property that

$$\sqrt{g}R \supset -4\pi \sum_{\text{crotches}} \delta(x - x_c)$$

We will have $2g$ such **singular points marking the birth and death of baby universes**, which reproduced the correct Euler character $\chi = -2g$ (in the case of the spectral form factor which we consider first).

Let us first consider one such crotch in some more detail.
Next slide is a bit **technical** but **not essential** to follow.

As warm up consider the spacetime

$$\mathrm{d}s^2 = (x^2 + y^2)(\mathrm{d}x^2 + \mathrm{d}y^2) - 2(x\mathrm{d}x - y\mathrm{d}y)^2 \sim w\bar{w}\,\mathrm{d}w\mathrm{d}\bar{w}$$

This metric vanishes at $x = y = 0$ and to characterize this singularity (Louko, Sorkin) propose to regulate the singular point as follows

$$\mathrm{d}s^2 = (x^2 + y^2 + i\sigma)(\mathrm{d}x^2 + \mathrm{d}y^2) - 2(x\mathrm{d}x - y\mathrm{d}y)^2\,, \quad \sigma \to 0$$

They choose this regularization in order to have an allowable metric for all $\sigma > 0$ (a concept which they invented in their paper).
Outside of $x = 0 = y$ one can do the diffeomorphism $z \sim w^2$ or $u + \mathrm{i}v \sim (x + \mathrm{i}y)^2$ which reduces the metric to Lorentzian flat space

$$\mathrm{d}s^2 = -\mathrm{d}u^2 + \mathrm{d}v^2 \sim \mathrm{d}z\mathrm{d}\bar{z}$$

This **diffeomorphism** however is **singular** at the origin and via direct calculation for $\sigma \to 0$ one finds a delta function there with a *negative* sign

$$\sqrt{g}R = -4\pi\,\delta(x)$$

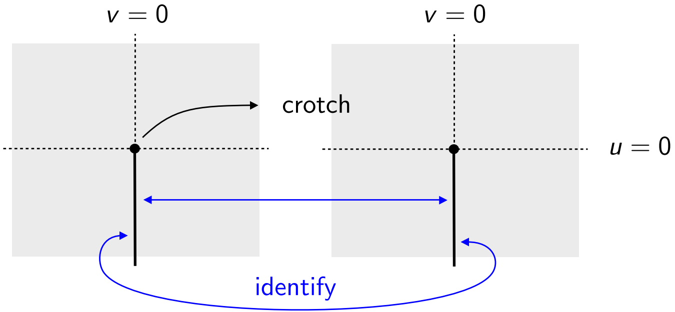
Such **negative mass sources** have the potential to **increase the genus**.

The key point here is that the original spacetime in coordinates (x, y) is a **double cover** of flat space in coordinates (u, v)

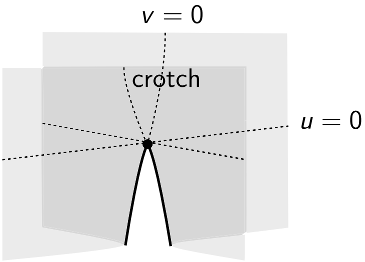
$$u + \mathrm{i}v = re^{\mathrm{i}\theta} \sim \rho^2 e^{2\mathrm{i}\alpha} = (x + \mathrm{i}y)^2$$

so indeed rotating once around the origin in (x, y) circles around the origin *twice* in (u, v) coordinates $\alpha \to \alpha + 2\pi = \theta \to \theta + 4\pi$.

The two covers (or two sheets) are identified along a **branch cut** starting at the singular point $u = v = 0$ and extending out to infinity, just like the complex function $\sqrt{u + \mathrm{i}v}$. This gives the global spacetime



We now see where the name **crotch** for the singular point comes from, it is literally the crotch of a (would-be) pair of pants



To make this into an actual pair of pants one could identify for instance the slice $v = a$ on both sheets and $v = -b$ on both sheets.

The key takeaway is that if we take **two copies of some geometry**, we cut them along two **identical** semi-infinite **lines**, and then **swap identify** the resulting edges, that at the endpoint of the identification we have

$$\sqrt{g}R \subset -4\pi\,\delta(x - x_{\text{crotch}})$$

We now want to mimic this construction in the AdS_2 $R + 2 = 0$ setup. The generalization to generic dimensions will be discussed later.

Consider the metric of the Rindler patches of the TFD in conformal gauge

$$\mathrm{d}s^2 = \frac{-\mathrm{d}u^2 + \mathrm{d}v^2}{\sinh^2 v}$$

Here u is Rindler time and the two asymptotic boundaries are at $v \sim \pm\varepsilon$.

We now want to choose two semi-infinite lines on which we can cut the geometry, and then make swap identifications that implement a crotch singularity

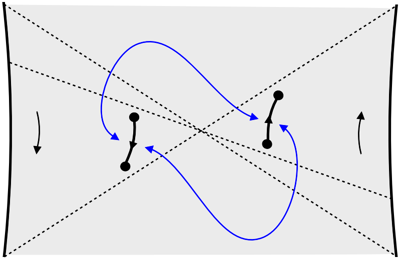
$$\sqrt{g}(R+2) = -4\pi\,\delta(x - x_{\text{crotch}})$$

Since locally any spacetime is flat we are guaranteed that the crotch at the end of the branch cut always has precisely this type of source.

However for general spacetimes we should be careful that the **lines** which we want to identify are actually **compatible**, meaning the glued spacetime is **smooth across the identification**.
In particular this forces the **extrinsic curvature** and length (and dilaton in 2d dilaton gravity) to match on the identified lines.

One natural way to always ensure this is to swap identify **identical** lines in situations where we have **two copies** of the same (patch of) **spacetime**.

In the case of Rindler patches in the TFD for AdS_2 one can for instance take two mirrored half lines at $v = v_0$ and $v = -v_0$ as follows



The arrows are flow of Rindler time u.

Moduli space and gauge fixing time

Before proceeding let me make one comment about this crotch formalism.

As you know, time in quantum gravity is only well-defined on an asymptotic boundary (where gravity is effectively turned off).
Nevertheless the notion of a **time function in the bulk spacetime** is still often extremely useful.
One can think of choosing some time function in the bulk as a particular gauge choice, within which one can do calculations.

The Louko-Sorkin formalism should be thought of as implementing just that.
In some sense (which they make precise) one can think of each singular Lorentzian spacetime as **one to one** related with a **Euclidean spacetime and** a choice of **time slices**.
Having chosen a time slicing, **interactions** take place at specific **time coordinates** t_{crotch} at which time a topological transition occurs.
Integrating over interaction times is gauge equivalent to the integral over Euclidean spacetimes.

This is completely analogous to **lightcone string** theory (Mandelstam) for 2d quantum gravity (Usatyuk).
More comments in discussion slide at the end of the talk.
In this sense, in this framework **by definition one** always **knows the spatial geometry for fixed boundary time**, and it is physical to ask questions about its properties, for instance what the spatial volume is.

For higher dimensional gravity is is not obvious that this is an identity covering of a slice of the moduli space of **metrics modulo diffeos**, but we will collect strong evidence in favor of this by reproducing several non-trivial late time predictions (which we discussed earlier).

The spatial geometries are path integrated over of course, this integral one should think of as analogous to not knowing what the bulk spatial geometry is in another gauge choice. In another gauge any one bulk spacetime has different notions of volume for fixed boundary time, one can think of that as choosing different time functions, or **equivalently** as having one fixed time function but integrating over different spatial metrics as function of time. We consider the latter scenario.

Constrained instantons and instanton action

Now we want to become more precise.
Before diving into the examples we want to first understand how these crotch spacetimes are weighted in the gravitational path integral.
In particular we want to know their **instanton action**.

One confusing point in that regard is that in JT gravity the dilaton is a Lagrange multiplier, namely the action is

$$\exp\left(\frac{1}{2}\int \mathrm{d}^2x\sqrt{g}\,\Phi(R+2)\right)$$

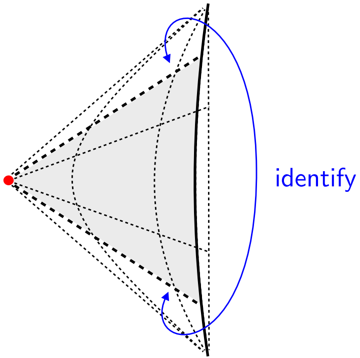
which naively localizes on exactly $\sqrt{g}(R+2)=0$ everywhere.

So naively the path integral has **no saddles** which involve **crotches**, even if one gauge fixes $\mathcal{D}g/$diffeos to be over Lorentzian spacetimes with crotches only.

To resolve this confusion let us take a step back.
Precisely the same situation actually occurs for the **Lorentzian black holes** contributing to $Z(\pm \mathrm{i} T)$. These all have **conical singularities** at the horizon, so naively the Lorentzian JT path integral just vanishes.

conical singularity $\alpha = AT$

$$\sqrt{g}(R+2) = 2(2\pi - \mathrm{i}\alpha)\,\delta(x)$$



The reason there is this singularity with **real-time periodicity** is the same reason that the Euclidean disk has a conical singularity when we force the Hawking temperature of the black hole to differ from the asymptotic length (see also **fixed area states**).

In that case the way to proceed (Marolf) is to **first keep the dilaton at the horizon fixed** and only in the end integrate over this modulus. This is inspired by the **constraint instanton** construction of wormholes by (Stanford) and (Cotler, Jensen) more commonly known as **Lagrange multipliers**.

Very concretely in 2d dilaton gravity we found that one should insert

$$\begin{aligned} 1 &= \int_{-\infty}^{+\infty} \mathrm{d}A \, \delta(A = \Phi(x)) \\ &= \frac{1}{\int \mathrm{d}x \sqrt{g}} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathrm{d}\alpha \int_{-\infty}^{+\infty} \mathrm{d}A \, e^{(2\pi - \mathrm{i}\alpha)A} \, \mathsf{defect}(\alpha) \end{aligned}$$

with the **defects sourcing conical singularities**

$$\mathsf{defect}(\alpha) = \int \mathrm{d}x \sqrt{g} \, e^{-(2\pi - \mathrm{i}\alpha)\Phi(x)}$$

For $\alpha = AT$ we obtain a **solution** to the **sourced** equations of motion

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 - 4A^2 \sinh(\rho)^2 \mathrm{d}t^2 \,, \quad t \sim t + T \,, \quad \Phi = A \cosh(\rho)$$

despite the **original problem** having **no saddles**.

Evaluating the JT action on shell for fixed A and finally integrating over A one recovers the analytical continuation of the Euclidean answer

$$Z(\mathrm{i}T) \sim \int_{-\infty}^{+\infty} \mathrm{d}A\, e^{S_0 + 2\pi A - \mathrm{i}TA^2}$$

Concrete evidence that this trick is **quantitatively accurate**.
The contribution of the singular point is the factor

$$e^{2\pi A}$$

which appeared in the Lagrange multiplier trick.
The takeaway is that one can account for (mildly) singular configurations in the gravitational path integral using **constrained instantons**.

This inspires us to **play the same game for the crotches** by inserting

$$1 = \frac{1}{\mathrm{Vol}} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathrm{d}\alpha \int_{-\infty}^{+\infty} \mathrm{d}A\, e^{(-2\pi - \mathrm{i}\alpha)A}\, \mathrm{crotch}(\alpha)$$

$$\mathrm{crotch}(\alpha) = \int \mathrm{d}x_{\mathrm{crotch}} \sqrt{g}\, e^{(-2\pi - \mathrm{i}\alpha)\Phi(x_{\mathrm{crotch}})}$$

Classical **solutions** arise when $\alpha = 0$, they are precisely the geometries with **crotch sources** that we described above.
Notably their contribution to the **on shell action** again can be read off immediately form the A dependent piece of the previous equation

$$\int \mathrm{d}x_{\text{crotch}} \sqrt{g}\, e^{-S_0 - 2\pi A(x_{\text{crotch}})}$$

The S_0 from Einstein Hilbert term on solution R with curvature source.
Where $A(x_{\text{crotch}})$ is the **dilaton at the crotch**, initially kept fixed.

Just like for the black hole, the crotch wants to **extremize this area**, therefore the saddle-point solution one finds upon varying A is that the **crotches accumulate near the horizon**.

$$\frac{d}{dx_{\text{crotch}}} A(x_{\text{crotch}}) = 0 \quad \Leftrightarrow \quad x_{\text{crotch}} = x_{\text{extr}}\,,$$

In 2d dilaton gravity this means we should extremize the dilaton.

The **instanton action** is thus the **entropy** associated with the dominant **extremal surface**.

Example 1. spectral form factor

In the remainder of the talk I will discuss **examples** of the resulting geometries and amplitudes.
I will construct Lorentzian wormholes solutions relevant for several observables, conjecture their moduli space (in analogy with the interacting string picture) and check that this reproduces boundary predictions.

First up is the spectral form factor where we seek a **real time explanation** of the universal equation

$$\sim \sum_{g=0}^{\infty} T^{2g+1} \int_{\dots}^{\infty} \mathrm{d}E \, e^{-2gS(E)} \, e^{-2\beta E} \,, \quad e^{S(E)} = e^{A(E)/4G}$$

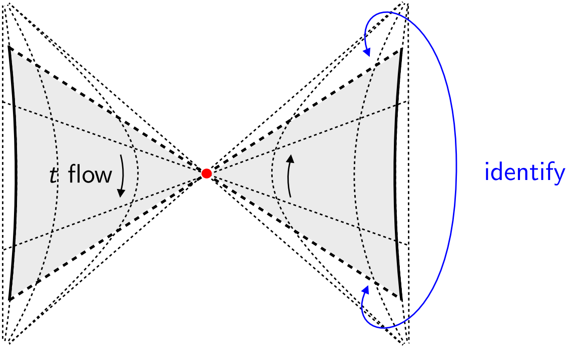
Notice relation with formulas from original baby universe papers (Giddings, Strominger)

$$Z_g(+\mathrm{i}T, -\mathrm{i}T)_{\text{conn}} \sim T^{2g+1} \, e^{-2gS_{\text{inst}}}$$

This suggests **interaction times are zero modes** (approximately).

Let us first construct Lorentzian spacetimes with such crotch singularities in **JT gravity** and explain how they reproduce the boundary prediction.
I will later discuss how this obviously generalizes.
Consider thereto the **double-cone** saddle points (SSS)

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2 - A(E)\,\mathrm{d}t^2}{\sinh(r)^2}\,, \quad \Phi = A(E)\coth(r)$$



This is 2 copies of a real-time **black hole** with horizon area $A(E) = E^{1/2}$.

There is a **moduli space** of double-cone solutions labeled by E with on-shell action $2\beta E$, moreover there is an infamous **twist** zero mode associated with relative time translations between the two boundaries.

Thus the contribution of this bare double-cone geometry is

$$T \int_{\dots}^{\infty} \mathrm{d}E \dots e^{-2\beta E}$$

This is true for **generic gravity models** (SSS, Cotler, Jensen, Stanford). In higher dimensions $E(A)$ where A denotes the transverse area of the horizon of the Lorentzian black hole.

We propose that inserting **crotches on** this moduli space of **double-cone** saddles, one obtains **new saddles** which contribute the predicted answer

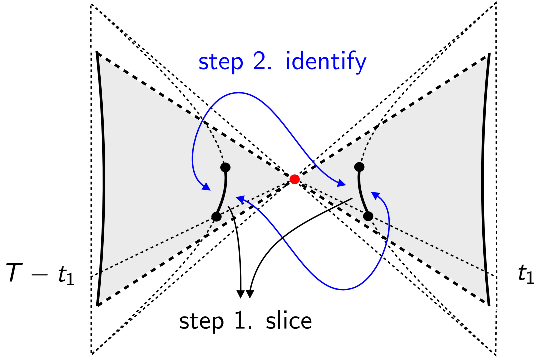
$$\sim \sum_{g=0}^{\infty} T^{2g+1} \int_{\dots}^{\infty} \mathrm{d}E \, e^{-2gS(E)} \, e^{-2\beta E} \,, \quad e^{S(E)} = e^{A(E)/4G}$$

The E integral and one factor T is inherited from the bare double-cone.

To obtain double-cone geometries with crotches that solve

$$\sqrt{g}(R+2) = -4\pi \sum_{\text{crotches}} \delta(x - x_c)$$

we can make for instance the following **mirrored identifications**



genus 1 wormhole

The reason that we can **cut and glue** on these lines is because they have matching extrinsic curvature K and length, such that we are assured that the resulting spacetime is smooth and $R = -2$. The dilaton matches too.

The time ordering left and right is imposed by orientability, I can explain this but it takes some time and a blackboard.

Because we are just making identifications on the original double cone, the **metric and dilaton configurations are not affected** (except for the metric singularity at the crotch points)

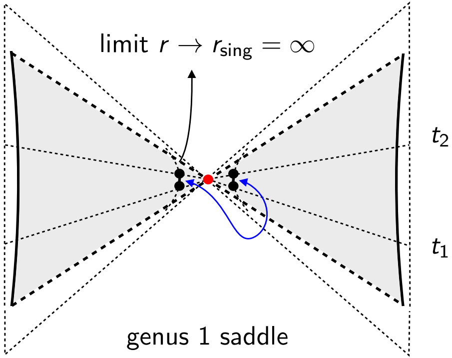
$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2 - A(E)\,\mathrm{d}t^2}{\sinh(r)^2}\,, \quad \Phi = A(E)\coth(r)$$

The question remains what happens at the crotch points. More precisely we want to understand to which degree these configurations are actually **semi-classical solutions**, why these contribute and what their **on-shell action** is.
But we already discussed this. For fixed $A(x_{\text{crotch}})$ they are saddles and each crotch contributes

$$\int \mathrm{d}x_{\text{crotch}}\sqrt{g}\, e^{-S_0 - 2\pi A(x_{\text{crotch}})}$$

In this scenario there is also a **saddle** for the **area integral**.

Namely the area has an extremum (minimum) at the horizon, hence classically the crotches accumulate near the horizon



This means you are only likely to encounter wormholes at the horizon.
This means that each **crotch** contributes the **on-shell action**

$$e^{-S_0 - 2\pi A(E)} = e^{-S(E)} = e^{-S_{\text{inst}}}$$

The **temporal location** of the crotches remains a **zero mode** (unlike the radial location which has a saddle-point). Dilaton only depends on r.

Thus we conclude that each crotch contributes a factor

$$T\, e^{-S(E)}$$

So in the baby-universe language the **instanton action** equals the **black hole entropy**.

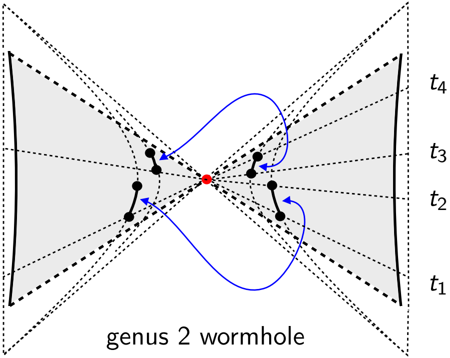
Combining this with the moduli of the parent double-cone we find that these **semi-classical slit geometries** reproduce the full **semi-classical approximation to the plateau**

$$\sim \sum_{g=0}^{\infty} T^{2g+1} \int_{\dots}^{\infty} \mathrm{d}E\, e^{-2gS(E)}\, e^{-2\beta E}\,, \quad e^{S(E)} = e^{A(E)/4G} \quad \square$$

In 2d dilaton gravity one can make this even more precise by describing the theory directly in **lightcone gauge**, for related work see (Usatyuk). There are rigorous statements that integrating over the crotch locations with the flat measure (as we did here) is gauge-equivalent to integrating over the moduli space of Euclidean Riemann surfaces exactly once (Giddings, d'Hoker, Phong, Wolpert).

Part of the bulk diffeo's are gauge-fixed by choosing the bulk slices such that each crotch has identical time coordinates left-and right.

For clarity 2-slit geometries for instance look like



genus 2 wormhole

One crucial point is that this whole construction goes through for **any gravitational theory** (where the double-cone construction works).

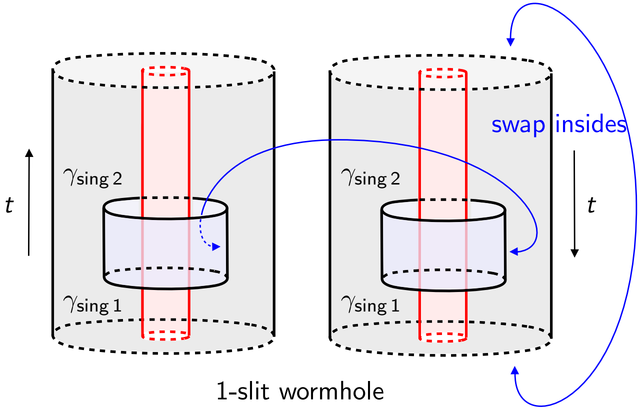
Because the **double-cone** is always exactly **two copies of the black hole** we can always make **identifications on mirroring co-dim one surfaces**, the metric would still be a smooth **solution** to the Einstein equations everywhere except at the end "points" of the identification.

This introduces **singularities** which are essentially the Louko-Sorkin crotches tensor a transverse space (Marolf) and whose contribution to the **action** is precisely again (Marolf) the **area** of that transverse space

$$e^{-A/4G}$$

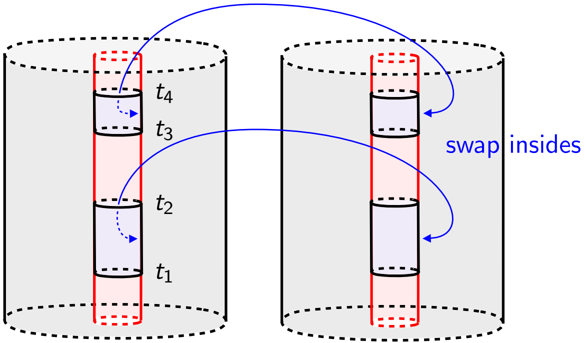
The area is a modulus whose on-shell **extremum** is achieved by letting it coincide with the center of the double-cone, the spatial area of which is precisely the **horizon area** $A(E)$.
The timelike location of this singularity is again a zero mode giving a factor of T and we again **recover the boundary prediction**. $\square$

Before extremizing the area the **co-dimension-1 slit** (on which we swap-identify) stretching between co-dimension-2 crotches may (or may not) **wrap around the horizon**



1-slit wormhole

After extremizing the area to obtain a saddle, these **slits hug the horizon**



2-slit saddle

One suggested physical picture is that even though you think you are falling into your black hole, you might find yourself in the interior of another black hole, you may have been **swapped near the horizon**.
This second black hole could also be in a quantum computer decoding the radiation.
For $T > e^{S(E)}$ there is a phase transition where **slits proliferate** at the horizon and cover significant percentages. Statistics questions?

Example 2. two-point function

As second (physically more interesting) example we reproduce the **semiclassical expansion** of the late time correlator

$$\mathrm{Tr}\Big(\mathcal{O}\, e^{-(\beta/2+\mathrm{i}T)H}\mathcal{O}\, e^{-(\beta/2-\mathrm{i}T)H}\Big) = \big\langle e^{-\Delta\ell}\big\rangle$$
$$\sim \sum_{g=1}^{\infty} T^{2g-1}\int_{\dots}^{\infty} \mathrm{d}E\, e^{-\beta E}\, e^{-\Delta\ell(E)}\, e^{-(2g-1)S(E)}$$

Here $\ell(E)$ is the length of the ER bridge in the TFD at $t=0$

$$\ell(E) = -\log(E)$$

We restrict to 2d dilaton gravity henceforth.

In particular the real-time TFD spacetime has the metric

$$\mathrm{d}s^2 = \frac{\mathrm{d}\sigma^2 - \mathrm{d}X^2}{\sin(\sigma)^2}\,, \quad \Phi = E^{1/2}\frac{\cos(X)}{\sin(\sigma)}$$

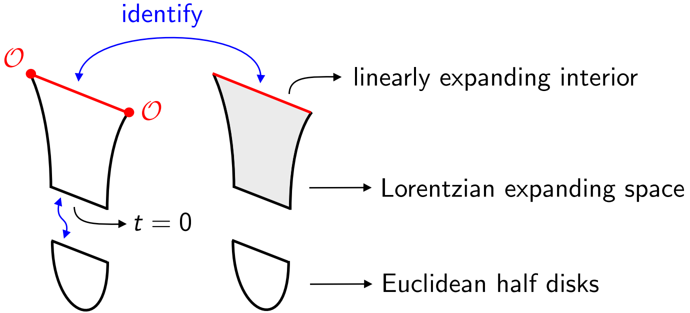
One should think of this spacetime as having boundaries parameterized by (physical) boundary time t

$$\tan(X/2) = \tanh\left(E^{1/2} t\right), \quad \sigma = \pi - \varepsilon \frac{\mathrm{d}X}{\mathrm{d}t}$$

The operators $\mathcal{O}$ are inserted on this boundary trajectory at $t = T/2$ and the shortest geodesic between them is spacelike with **linearly growing length**

$$\ell(E, t) = -\log(E) + 4E^{1/2} t$$

The full spacetime looks like



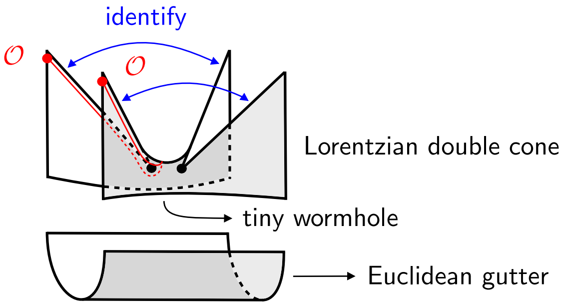
Both bra and ket Lorentzian spacetimes are prepared at $t = 0$ by gluing to half of a Euclidean disk

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + 4E\sinh(\rho)^2\mathrm{d}\tau^2\,, \quad \tau \sim \tau + \beta\,, \quad \Phi = E^{1/2}\cosh(\rho)$$

One checks that the metric and dilaton glue smoothly.
Because the length ℓ grows with time $\left\langle e^{-\Delta\ell}\right\rangle$ **decays exponentially** and so for late time this spacetime does not contribute.

But **wormholes** (we are being told by science fiction) create **shortcuts in spacetime** hence one expects at sufficiently late times shorter geodesics (not growing with time) exist when wormhole come into play. Those would **dominate** at late times.

In particular we are led to consider the following Lorentzian spacetime



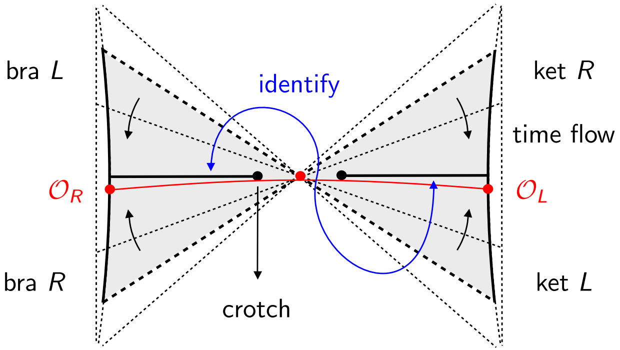
$g = 1$ geometry

The salient feature is that Lorentzian **time evolution** is used to evolve a **double cone** type region instead of the usual TFD bra and ket.
The preparation region in this case is half of a Euclidean double trumpet

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + 4E\cosh(\rho)^2\mathrm{d}\tau^2\,, \quad \tau \sim \tau + \beta\,, \quad \Phi = E^{1/2}\cosh(\rho)$$

One checks that the metric and dilaton glue smoothly.

Indeed this Lorentzian piece can be recognized as a **double cone** region with **one additional identification**



$\sim Te^{-S(E)}$

$g = 1$ geometry

The associated additional **crotch** gives an additional topological suppression factor as compared to the previous calculation

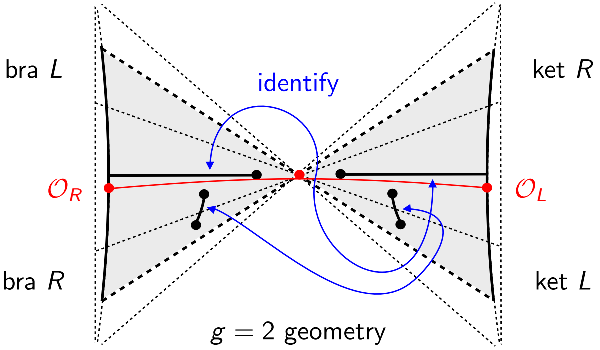
$$e^{-S(E)}$$

Otherwise the on-shell action calculation and zero modes are identical.

For higher genus contributions we can now just mimic our previous construction.

Indeed the **rules of the game** instruct us in general to account for **any number of crotches** (resulting in branch cuts and identifications) near extremal surfaces.

For instance the first subleading correction thus comes from



$\sim T^3 e^{-3S(E)}$

Perhaps physically the most relevant feature of these spacetimes is that the **length** of the final slice does **not grow with time**. In fact the metric and dilaton on this slice are exactly equal to that of the TFD at $t = 0$

$$\mathrm{d}s = \mathrm{d}\rho = \frac{\mathrm{d}\sigma}{\sin(\sigma)}\,, \quad \Phi = E^{1/2}\cosh(\rho) = \frac{E^{1/2}}{\sin(\sigma)}$$

Therefore in each geometry the **correlation does not decay**

$$\left\langle e^{-\Delta\ell}\right\rangle = e^{\Delta \log(E)}$$

as one would expect of wormholes spacetimes.

Combining the elements we recover our **boundary prediction**

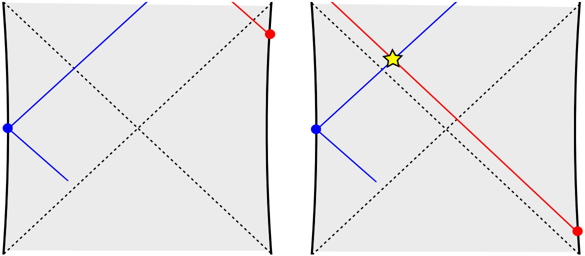
$$\mathrm{Tr}\Big(\mathcal{O}\, e^{-(\beta/2+\mathrm{i}T)H}\mathcal{O}\, e^{-(\beta/2-\mathrm{i}T)H}\Big) = \left\langle e^{-\Delta\ell}\right\rangle$$
$$\sim \sum_{g=1}^{\infty} T^{2g-1}\int_{\dots}^{\infty}\mathrm{d}E\, e^{-\beta E}\, e^{\Delta\log(E)}\, e^{-(2g-1)S(E)} \quad \square$$

This prediction is quite difficult to get out of a Euclidean calculation so this is a rather **nontrivial match**.

Example 3. firewall probability

As final example we consider the typical state firewall setup which was recently reconsidered by (Stanford, Yang).
The following has not appeared yet.

Consider a perturbed two-sided black hole.
If you jump in at $t \gg 0$ then you do not encounter this perturbation.
We call any $t \gg 0$ spatial slice **expanding** because the size grows.



If you jump in at $t \ll 0$ you encounter a highly **blueshifted** perturbation and you die honorably, effectively you encountered some **firewall**.
We call any $t \ll 0$ spatial slice **contracting** because the size shrinks.

Boundary arguments (Susskind ...) suggest that **surprises** may occur. Even though you might jump in at $t \gg 0$, the spatial slice of the geometry which you encounter could actually be a $t \ll 0$ slice of the two-sided black hole.

The gravitational origin of these surprises is **wormhole** physics. Indeed in the previous example we already saw that wormhole can cause the spatial slice at boundary time $t \gg 0$ to become the $t = 0$ slice of the eternal black hole.

We now generalize this and ask what the **probability** is that the spatial slice at $t \gg 0$ is expanding (you **survive**) or contracting (you **die**). One can relate the age T_ℓ of the bulk slice with its length

$$\ell = -\log(E) + 4E^{1/2}|T_\ell|$$

therefore we should be interested in the **length distribution** $P(\ell)$ of the **bulk spatial slice** which appears in observables as

$$\langle \mathcal{O} \rangle = \int_{-\infty}^{+\infty} \mathrm{d}\ell \, P(\ell) \, \mathcal{O}(\ell)$$

As emphasized earlier, our **Lorentzian path integral** construction has the benifit that we have essentially gauge fixed the metrics such that it is obvious for any geometry what this spatial length is.
Essentially we gauge fixed bulk time.
In contrast this is more subtle in the Euclidean setup, leading to computational difficulties for (Stanford, Yang) for $g > 1$.

Omitting the derivation (based on random matrices) for time constraints one finds the **boundary prediction** for the probabilities

$$P_{\text{exp}}(T_\ell) = \frac{\theta(T_\ell)}{2\pi\rho(E)} \int_{-\infty}^{+\infty} \mathrm{d}\omega\, e^{-\mathrm{i}\omega(T-T_\ell)} \left(\rho(E)^2 + \delta(\omega)\rho(E) - \frac{\sin(\pi\rho(E)\omega)^2}{\pi\omega^2}\right)$$

$$P_{\text{con}}(T_\ell) = \frac{\theta(T_\ell)}{2\pi\rho(E)} \int_{-\infty}^{+\infty} \mathrm{d}\omega\, e^{-\mathrm{i}\omega(T+T_\ell)} \left(\rho(E)^2 + \delta(\omega)\rho(E) - \frac{\sin(\pi\rho(E)\omega)^2}{\pi\omega^2}\right)$$

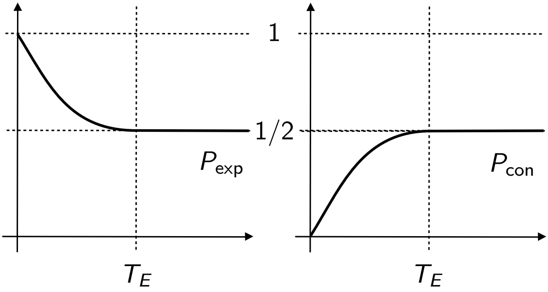
This formula was not known but you should believe me that it is accurate.
The Lorentzian geometries will clarify why one is expanding and the other is contracting.
You should not remember this formula for what follows.

Your **survival probability** is obtained by integrating over ℓ

$$P_{\text{exp}} = 1 - \min\left(\frac{T}{T_E} - \frac{1}{2}\left(\frac{T}{T_E}\right)^2, \frac{1}{2}\right)$$

$$P_{\text{con}} = \min\left(\frac{T}{T_E} - \frac{1}{2}\left(\frac{T}{T_E}\right)^2, \frac{1}{2}\right)$$

Subtracted infinite equal constants imposing that we know the spatial geometry at $T = 0$, furthermore $T_E = 2\pi\rho(E)$



Notice

$$P_{\text{exp}} + P_{\text{con}} = 1$$

and both assymptote to exactly $1/2$ for $T \to \infty$.
Therefore jumping in at very late times is essentially a **coin flip**. This was expected on general grounds (Susskind . . .).
The quadratic in T piece matches the calculation of (Stanford, Yang).
The 1 is the answer without wormholes.

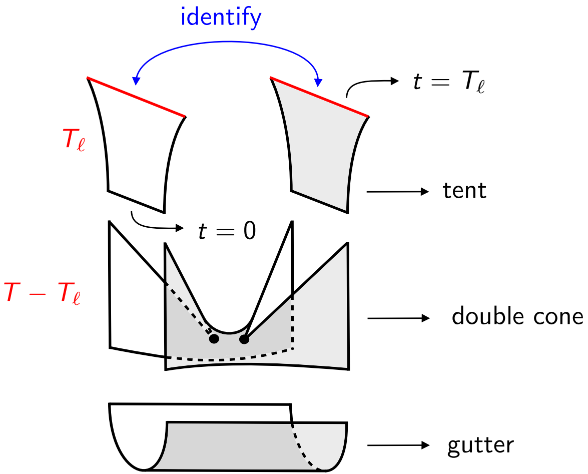
For comparison with Lorentzian wormholes we note that one can associate a **semiclassical genus expansion** with this exact answer

$$P_{\text{exp}}(T_\ell) \sim \theta(T_\ell)\, |T - T_\ell|^{2g+1}\, e^{-(1+2g)S(E)}$$
$$P_{\text{con}}(T_\ell) \sim \theta(T_\ell)\, |T + T_\ell|^{2g+1}\, e^{-(1+2g)S(E)}$$

This takes some work, essentially the techniques are the same contour deformation used for the previous observables.

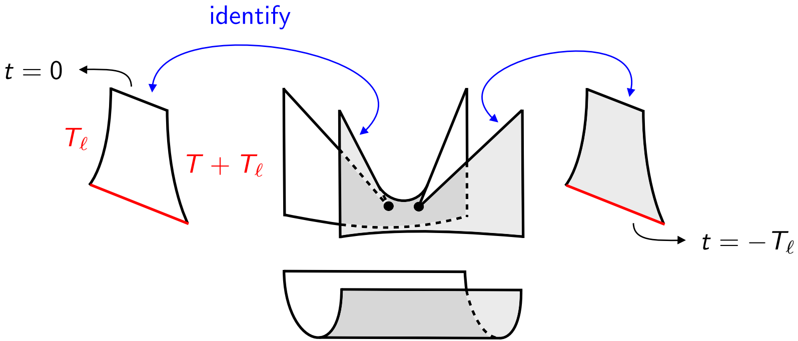
What I want to highlight is that this expansion, which is quite difficult to get even out of the Euclidean calculation in 2d (where only the $g = 1$ result was computed by (Stanford, Yang)) is quite **elementary** to find by **counting Lorentzian spacetimes**.

In particular the **expanding** spacetimes are



The topological suppression works out and the zero modes from temporal crotch locations give powers of $|T - T_\ell|$.
Crotches are located close to the horizon in the double cone region but not shown to avoid cluttering

Similarly the **contracting** spacetimes are



The zero modes from temporal crotch locations give powers of $|T + T_\ell|$.

We see that **wormholes** can **rejuvenate** the two sided black hole to **any bulk age** and that accounting for this one reproduces the expected $P_{\text{con}} = P_{\text{exp}} = 1/2$ at infinite boundary time.
Expect for $T_\ell = 0$ none of these contribute to the two point function because the lengths are exponentially large and this is exponentially suppressed in any correlator $\langle e^{-\Delta \ell} \rangle$.

Discussion

To reiterate the main message.
Instead of gauge fixing $\mathcal{D}g/$diffeos to **smooth complex** spacetimes one can gauge fix $\mathcal{D}g/$diffeos to **real-time** spacetimes with crotches. The two **gauge slices** are related by singular diffeomorphisms.

In setups without holographic boundaries one can proof that the covering is one-to-one, this is why **lightcone strings** works (D'Hoker, Giddings, Phong, Wolpert, Usatyuk). In our case we have argued (in the paper) the map is also one-to-one, and the fact that are calculations recover the Euclidean answers are evidence in favor of that.
The Jacobian from the gauge-fixing in general could be important, but for our late-time observables can be shown to be irrelevant.

The resulting Lorentzian spacetime wormholes one finds from this are very efficient in reproducing complex results from analytical continuation of Euclidean calculations in two dimensions and give nontrivial predictions in higher dimensions, matching with boundary predictions.

These real-time wormholes seem like a relevant new ingredient that should help us understand **real time black hole physics**.
If you fall into some black hole, you will not experience some Euclidean path integral nor do you escape the interior via some Euclidean wormhole. Then what does happen to you?
Calculations suggest that our **slit geometries** should be relevant for your experiences.
The final example involving the **firewall problem** is a first (and not yet completely understood) example of that.
I could also explain what comes of replica wormholes and how islands appear.

Thanks.