Emergence of space and time in holography

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YIT P workshop

Recent Developments in Quantum Physics of Black Holes 2023

Apr. 5th , 2023

based on recent work with Samuel Leutheusser

arXiv: 2110.05497, 2112.12156 and 2212.13266

Emergence of spacetime in quantum gravity

$G_N \to 0$ limit: Spacetime geometry + QFT in curved spacetime

causal structure, local regions, different notions of times ……

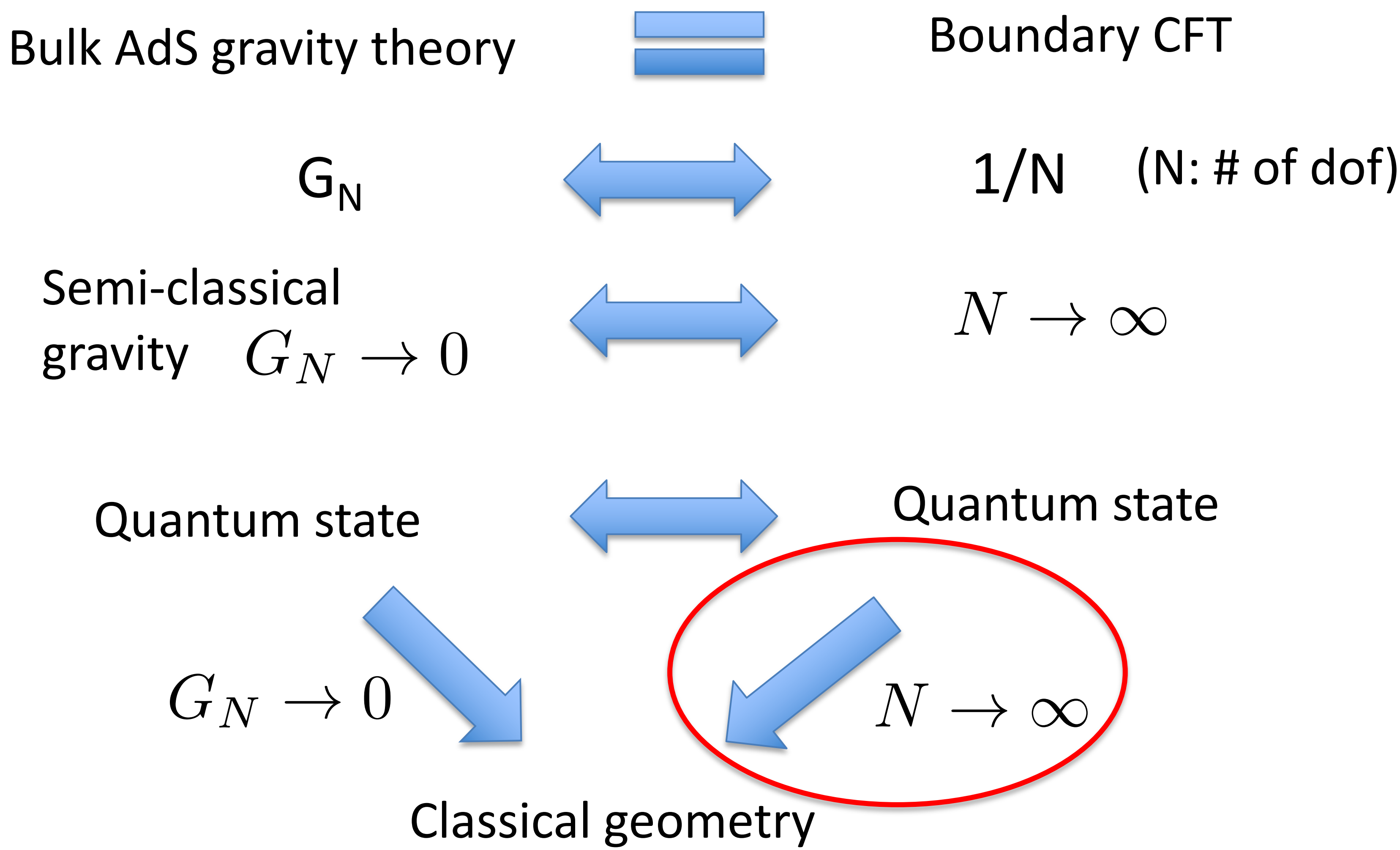
It has long been speculated that such geometric notions are low energy phenomena, emergent in the $G_N \to 0$ limit.

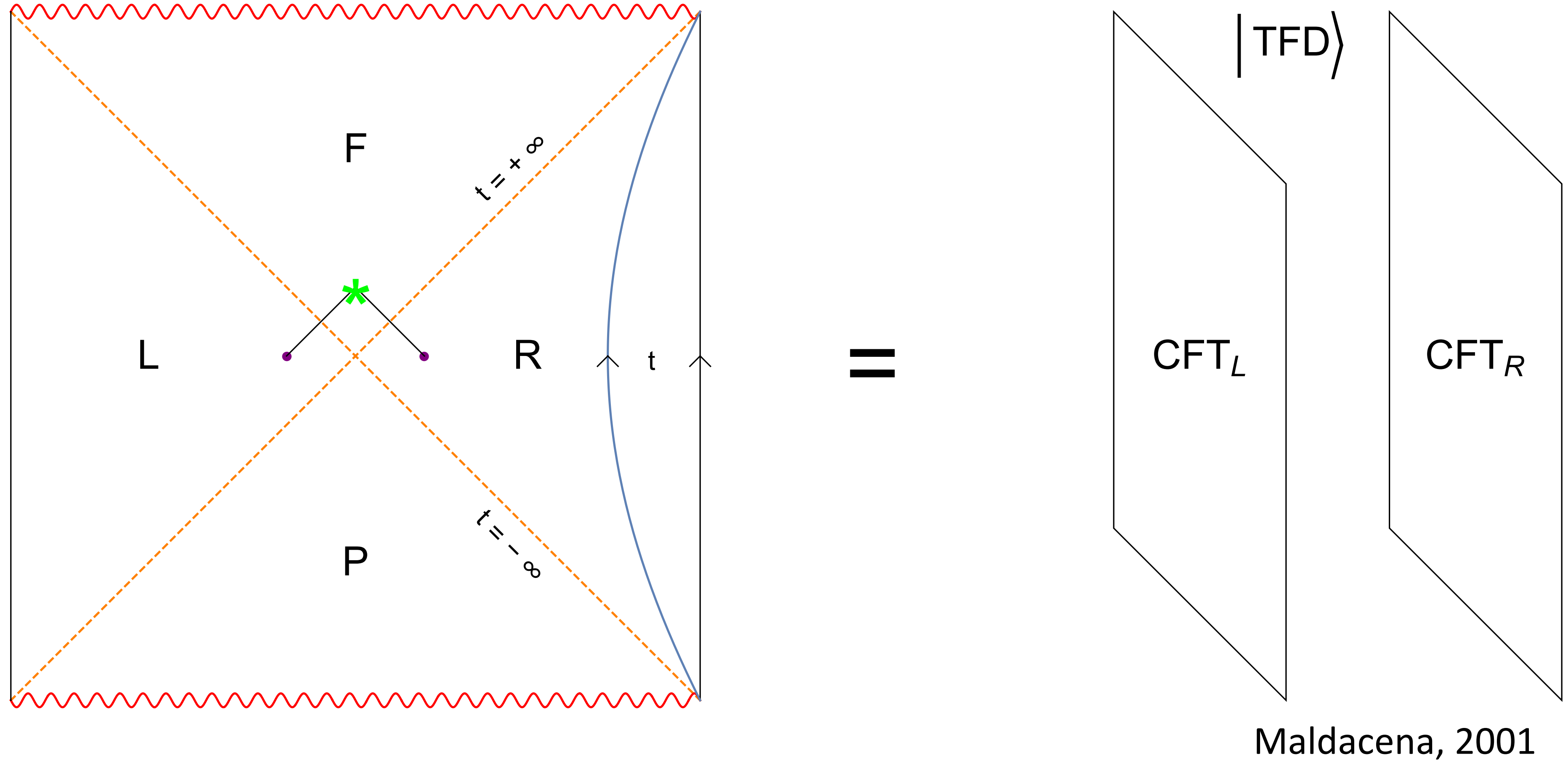
How?

What are the physical and mathematical structures underlying such emergence?

We should be able to answer these questions in the context of the AdS/CFT duality.

AdS/CFT duality





Time-like Killing vector outside the horizon

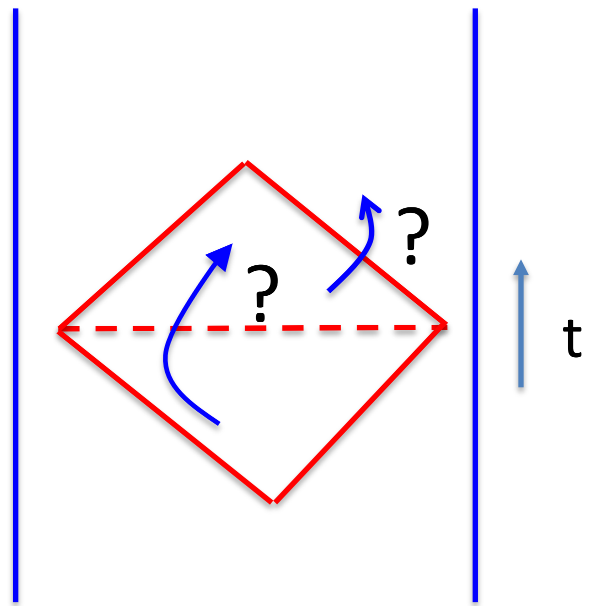
Many mysteries: F and P regions? Kruskal-like time?

horizons and associated causal structure ?

Emergence of space and time in holography

Consider a bulk spacetime, and some causally complete spacetime region in it.

How do we describe such a region in the boundary theory?



"interior" time?

causal structure?

"global" time ? (which can take one outside the region)

Goal of the talk:

Outline a formalism for addressing these questions.

Bulk spacetime locality is a geometrization of emergent boundary type III_1 von Neumann subalgebras

Emergent type III_1 von Neumann subalgebra ⟺ bulk spacetime region

Properties of such emergent type III_1 subalgebras ⟹ Geometric notions such as horizons, times, causal structure, ...

Subalgebra-subregion duality

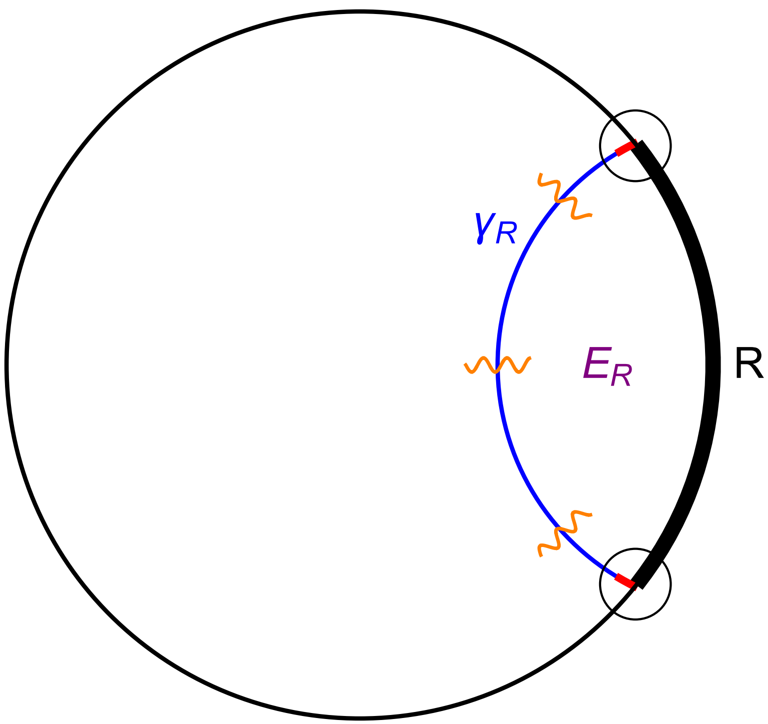
subcase: subregion-subregion duality (entanglement wedge reconstruction)

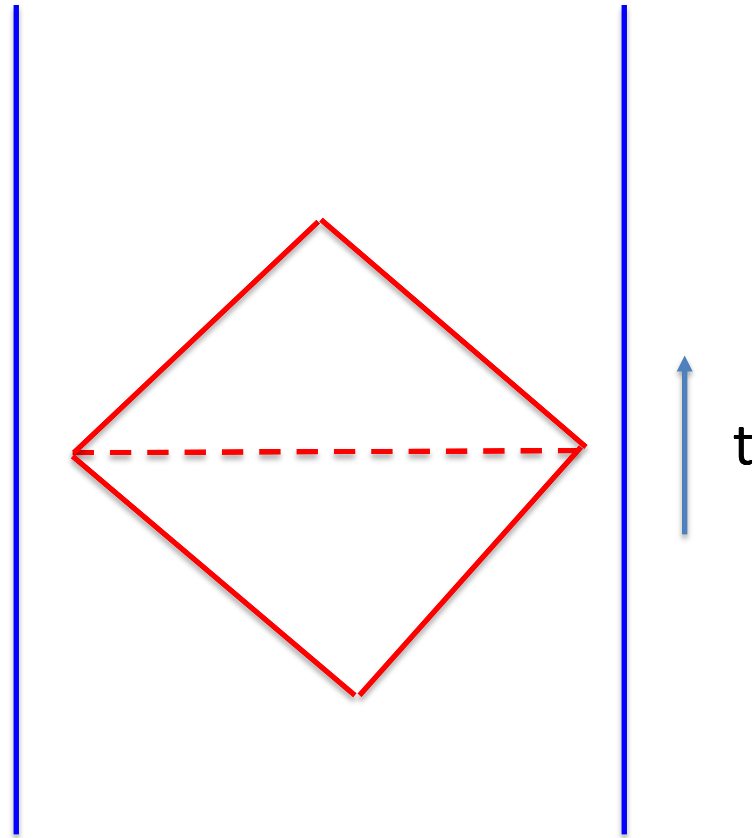
Van Raamsdonk (2009)

Czech, Karczmarek, Nogueira, and Van Raamsdonk (2012)

....

Subregion-subregion duality





$$E_R \sim R$$

Equivalently: $\mathcal{M}_{E_R} = \mathcal{X}_R$

more general language

Subalgebra-Subregion duality

Plan

1. Some key elements used

- Von Neunmann algebras
- Large N limit of the AdS/CFT duality

2. Formulation of a duality for a general bulk subregion

Example: boundary emergence of Kruskal-like time and event horizon of an eternal black hole

3. More general examples

4. New insights into subregion-subregion duality

(RT surface without entropy, additivity anomaly, explanation of quantum error corrections)

5. Some future perspectives

Von Neumann (vN) algebras

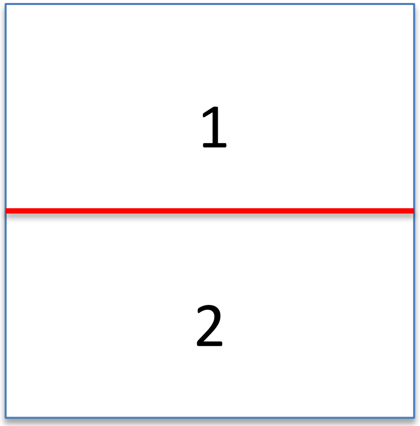
Soon after the development of quantum mechanics and its mathematical foundation using Hilbert space, Von Neumann and Murray initiated the task of classifying operator algebras for all quantum systems.

The von Neumann algebras are classified into: Type I, II, III

Operator subalgebras we normally encounter in QM classes are all type I.

Type II and III are more exotic.

Modern perspective: Classification of entanglement patterns of quantum systems



$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$$

$$\rho_1 = \mathrm{Tr}_2 |\Psi\rangle\langle\Psi|$$

can be equivalently characterized in terms of operator algebra of subsystem 1: type I

Systems with an infinite number of degrees of d.o.f:

$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ may not exist: Infinite amount of entanglement

Type II: A renormalized reduced density operator and entropy may still be defined

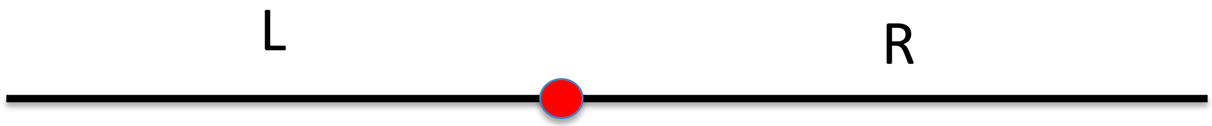
Type III: Even renormalized density operator or entropy does not exist

entanglement is instead characterized by modular flows

Type II and III algebras have since found applications in quantum statistical physics and quantum field theories.

Relativistic QFTs: local operator algebra in any local region R is type III_1

For example:



operator algebra in R is III_1:

- infinite entanglement between R and L (in any state)
- Entanglement structure needed to have sharp causal structure (in any state).

Holography in the large N limit

AdS/CFT duality

Bulk AdS gravity theory = Boundary CFT

G_N ⟷ 1/N (N: # of dof)

Semi-classical gravity $G_N \to 0$ ⟷ $N \to \infty$

Quantum state ⟷ Quantum state

$G_N \to 0$ ↘ ↙ $N \to \infty$

Classical geometry

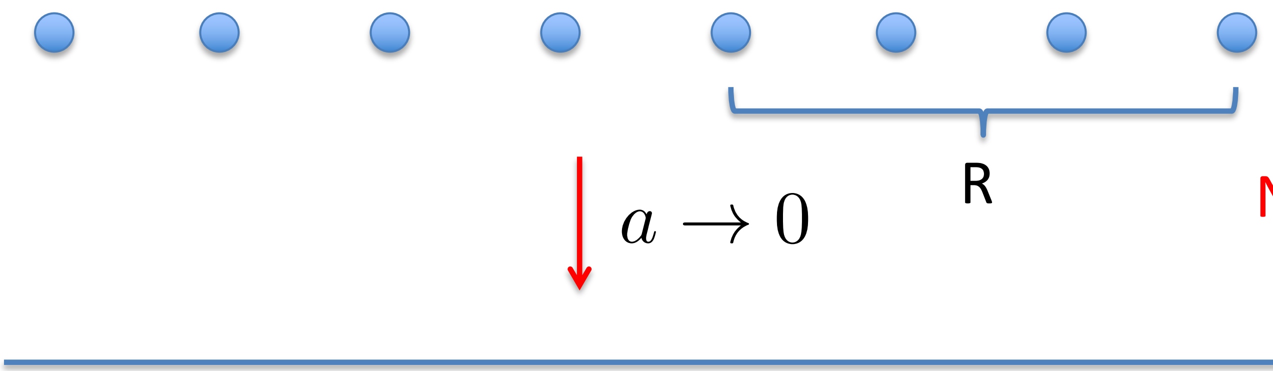
Emergent physical and mathematical structures of the $N \to \infty$ limit

Large N limit of the duality

Consider, e.g. $\mathscr{N}$=4 super-Yang-Mills with gauge group SU(N)

Many states and operators do not have a well-defined large N limit

→ the structures of Hilbert space and operator algebras undergo dramatic changes in the large N limit



$\mathcal{H} = \mathcal{H}_R \otimes \mathcal{H}_L$

$\mathcal{A}_R$: type I

No sharp light-cone

$\mathcal{H}$ not factorizable

$\mathcal{A}_R$: type III_1

Sharp light-cone

Large N limit of operator algebras

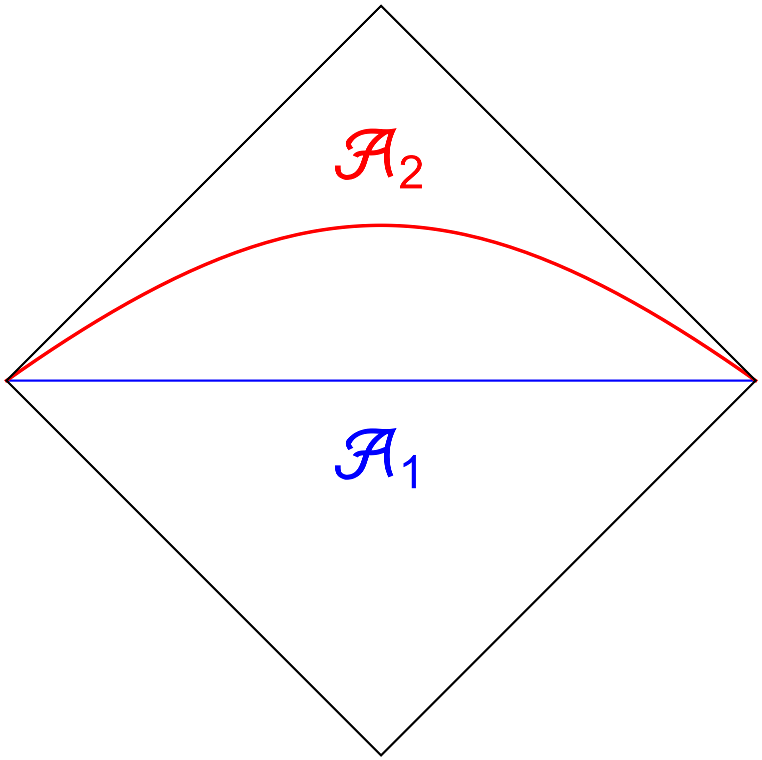
An operator has a sensible large N limit if its vacuum correlation functions have a well-defined $N \to \infty$ limit.

→ finite products of single-trace operators.

form an algebra in the large N limit: single-trace operator algebra

Key features:

- state-dependent
- Single-trace operators at different times are independent



Finite N, $\mathcal{A}_1 = \mathcal{A}_2$ (or in an ordinary QFT)

For algebras of single-trace operators, $\mathcal{A}_1 \neq \mathcal{A}_2$

AdS/CFT duality at large N

geometry $\Longleftrightarrow$ $|\Psi\rangle$

free bulk fields $\Longleftrightarrow$ generalized free fields (single-trace operators)

$$\mathcal{H}^{\mathrm{Fock}} = \mathcal{H}^{\mathrm{GNS}}$$

$$\widetilde{\mathcal{M}} = \mathcal{M}_{\Psi}$$

(operator algebra of $\mathcal{H}^{\mathrm{Fock}}$)

(operator algebra of $\mathcal{H}^{\mathrm{GNS}}_{\Psi}$)

Formulation of duality of a general bulk subregion

Duality for a bulk subregion

Now consider a causally complete bulk spacetime region **b**

Bulk field operator algebra $\tilde{\mathcal{Y}}_{\mathbf{b}}$ in **b** is type III_1

$$\tilde{\mathcal{Y}}_{\mathbf{b}} \subset \widetilde{\mathcal{M}}_{\Psi}$$

So there must be an emergent type III_1 subalgebra

$$\mathcal{Y} \subset \mathcal{M}_{\Psi}$$

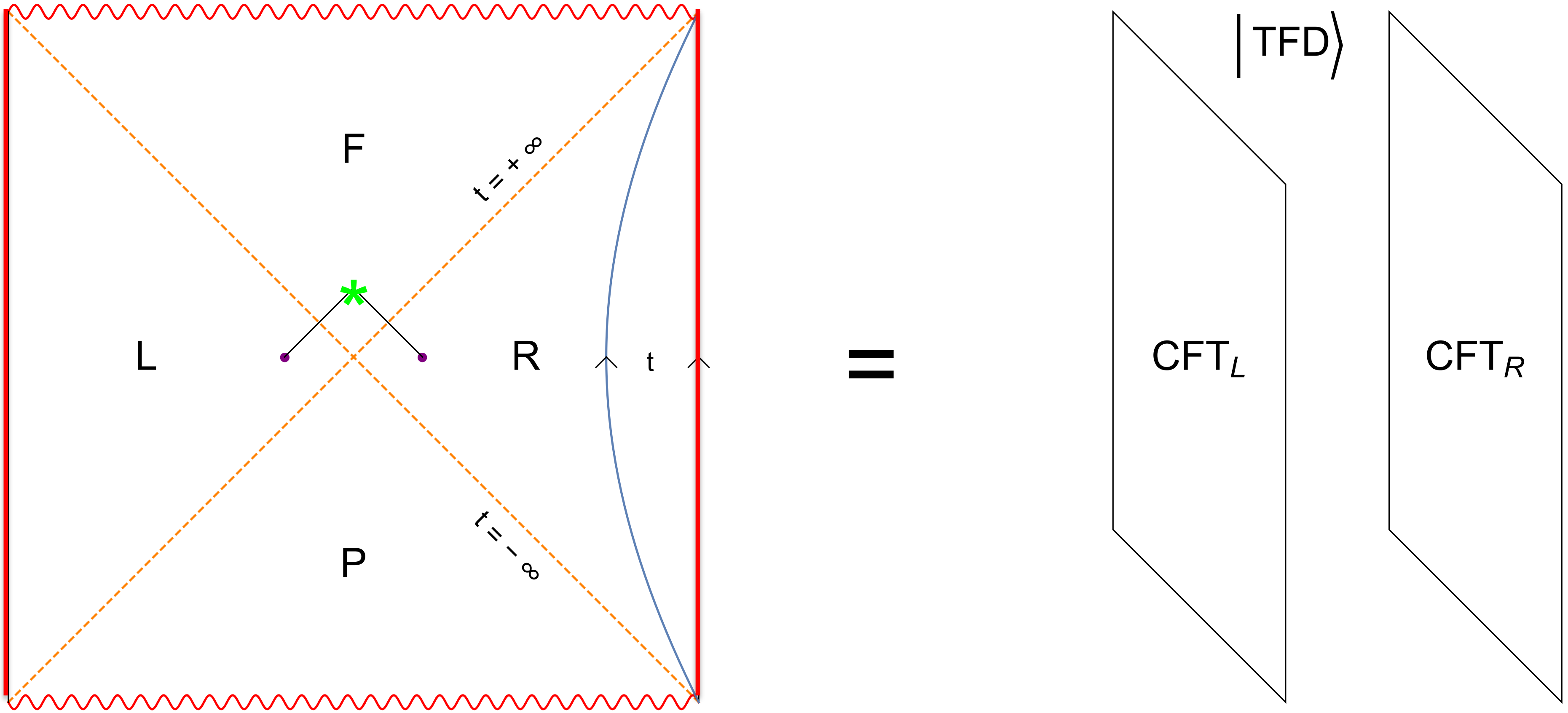
$$\mathcal{Y} = \tilde{\mathcal{Y}}_{\mathbf{b}}$$

Conversely, we would like to conjecture any type III_1 subalgebra $\mathcal{Y} \subset \mathcal{M}_{\Psi}$ there exists some bulk region **b**

$$\mathcal{Y} = \tilde{\mathcal{Y}}_{\mathbf{b}}$$

Example: Emergent times and horizons of an eternal BH

Eternal black hole in AdS



Maldacena, 2001

Boundary description of F and P regions? Kruskal-like time?

Boundary description of horizons and associated causal structure ?

Emergent type III_1 vN algebras

At finite N, the (bounded) operator algebra of CFT_R or CFT_L is a type I von Neumann (vN) algebra (not relevant for large N)

In the large N limit,

$\mathcal{M}_R$: algebra generated by single-trace operators of CFT_R

Claim:

$T < T_{\text{Hawking-Page}}$: $\mathcal{M}_R$ is type I vN algebra

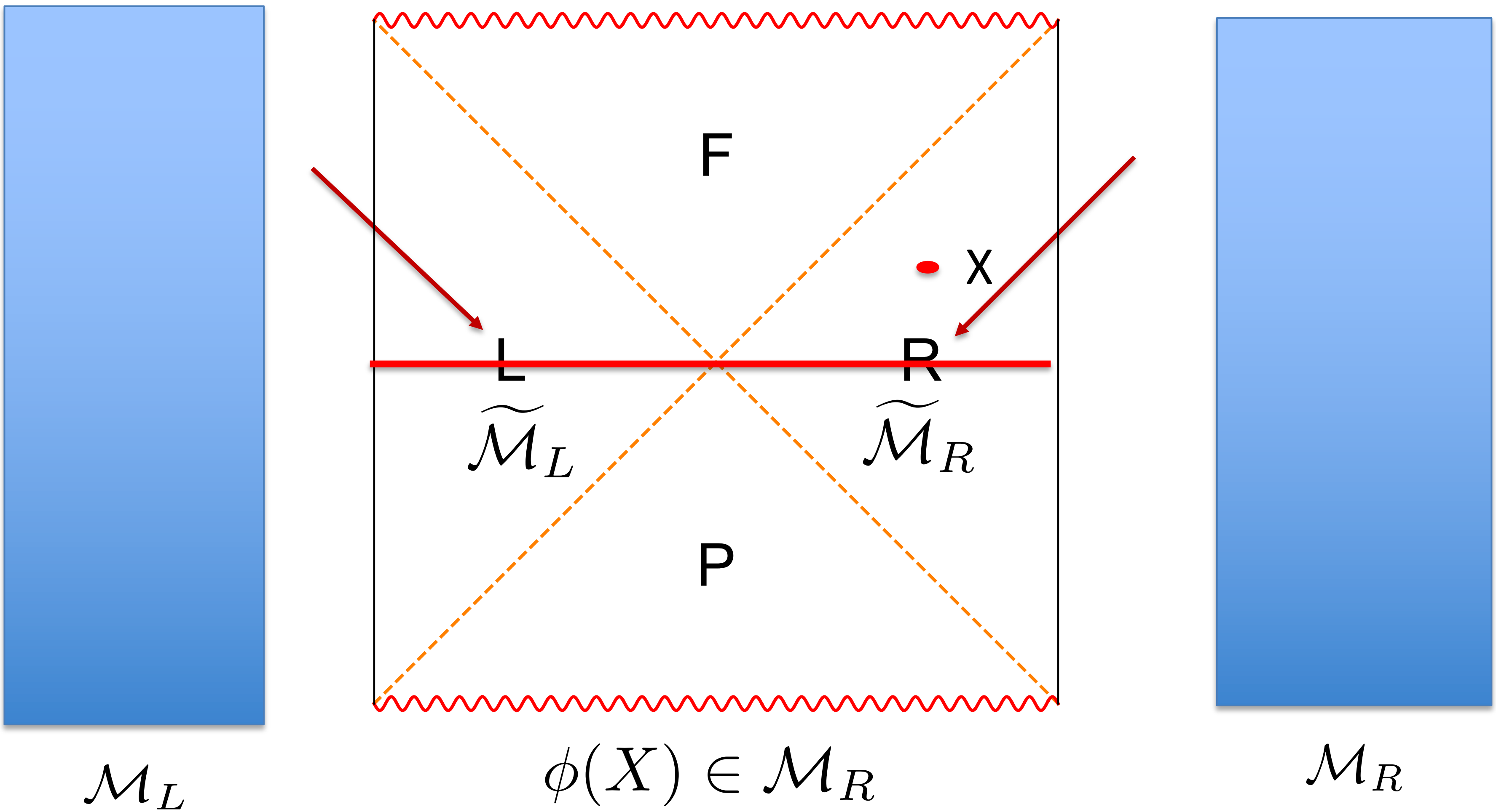
$T > T_{\text{Hawking-Page}}$: $\mathcal{M}_R$ becomes type III_1 vN algebra

Same statements apply to $\mathcal{M}_L$

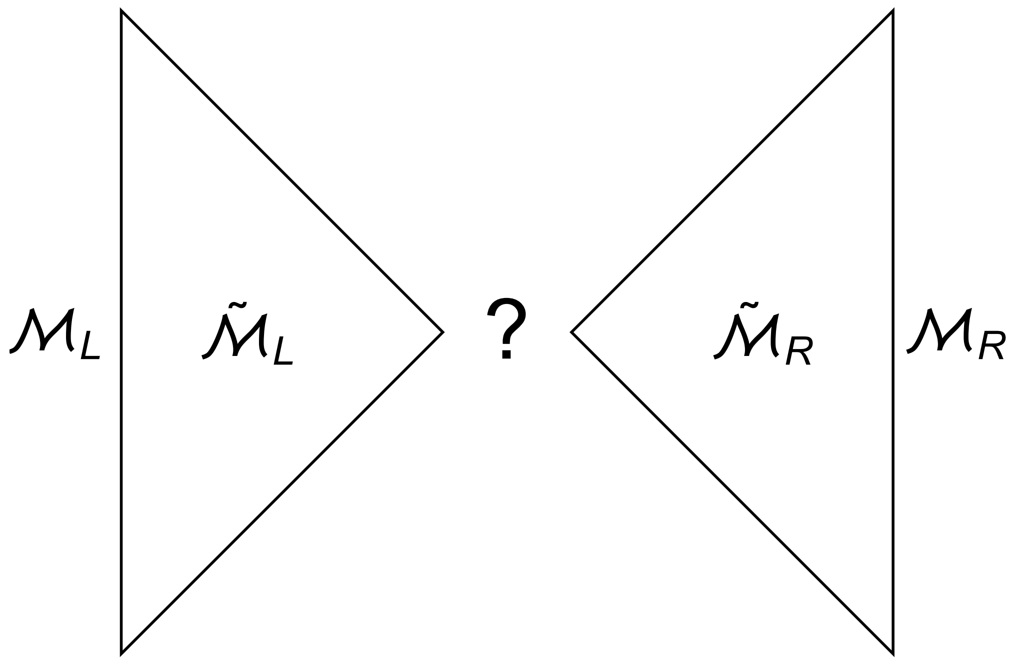
Identification of algebras

$\widetilde{\mathcal{M}}_R, \widetilde{\mathcal{M}}_L$: bulk operator algebras in the R and L regions

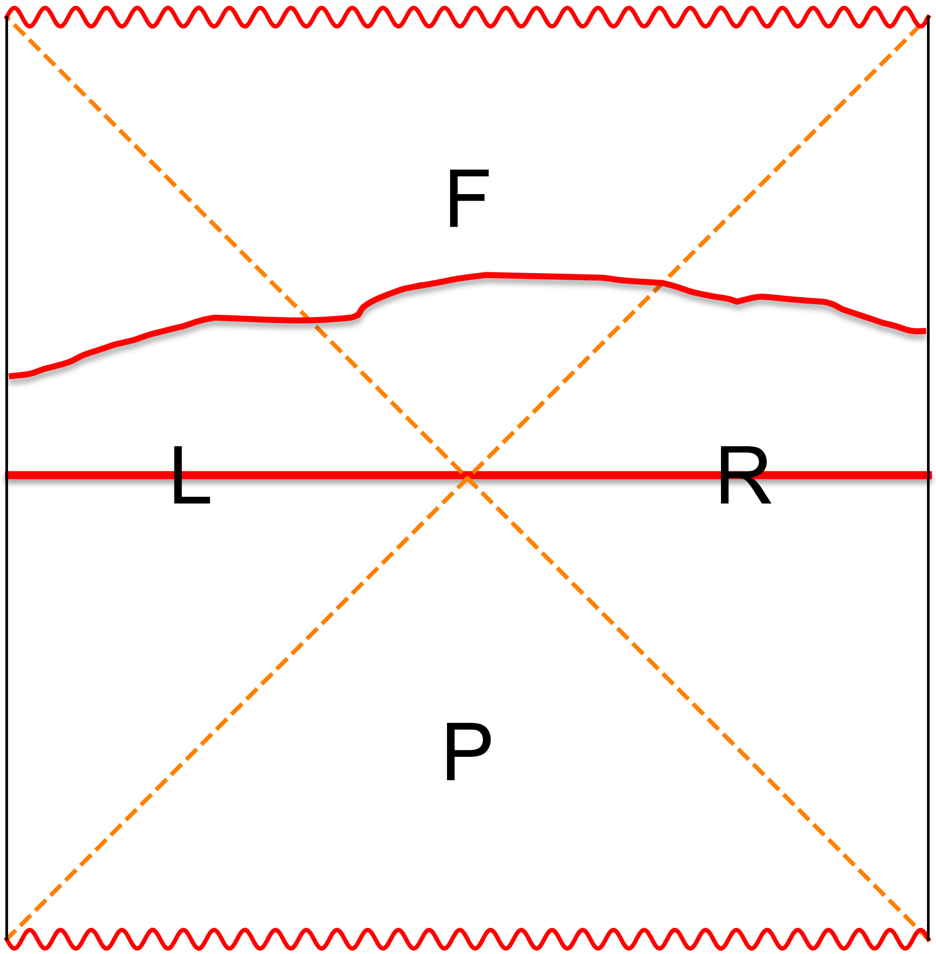
Duality: $\mathcal{M}_R = \widetilde{\mathcal{M}}_R, \quad \mathcal{M}_L = \widetilde{\mathcal{M}}_L$



$$\mathcal{M}_R = \widetilde{\mathcal{M}}_R, \quad \mathcal{M}_L = \widetilde{\mathcal{M}}_L$$



Times in the bulk gravity ?



Bulk time evolutions

Boundary automorphisms of

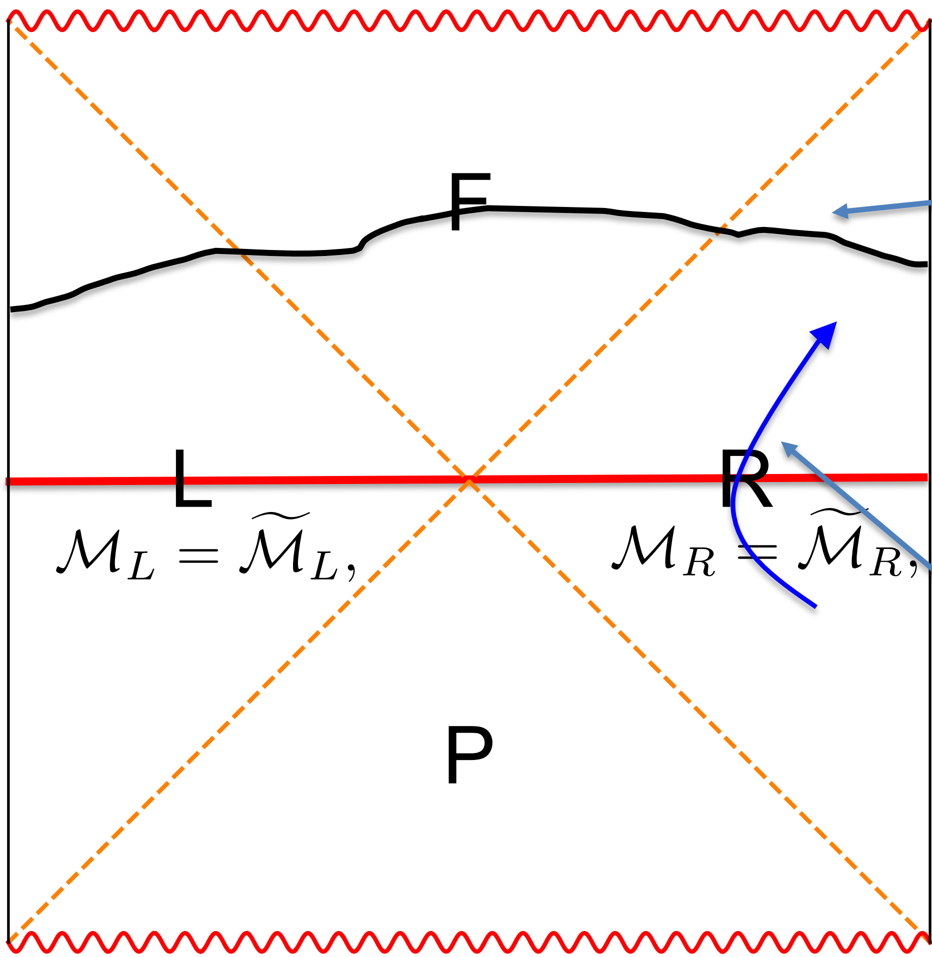
$$\mathcal{M}_R \vee \mathcal{M}_L$$

What kind of automorphism corresponds to bulk time evolution?

Times in the bulk gravity ?

Bulk time evolutions ⟺ Boundary automorphisms of

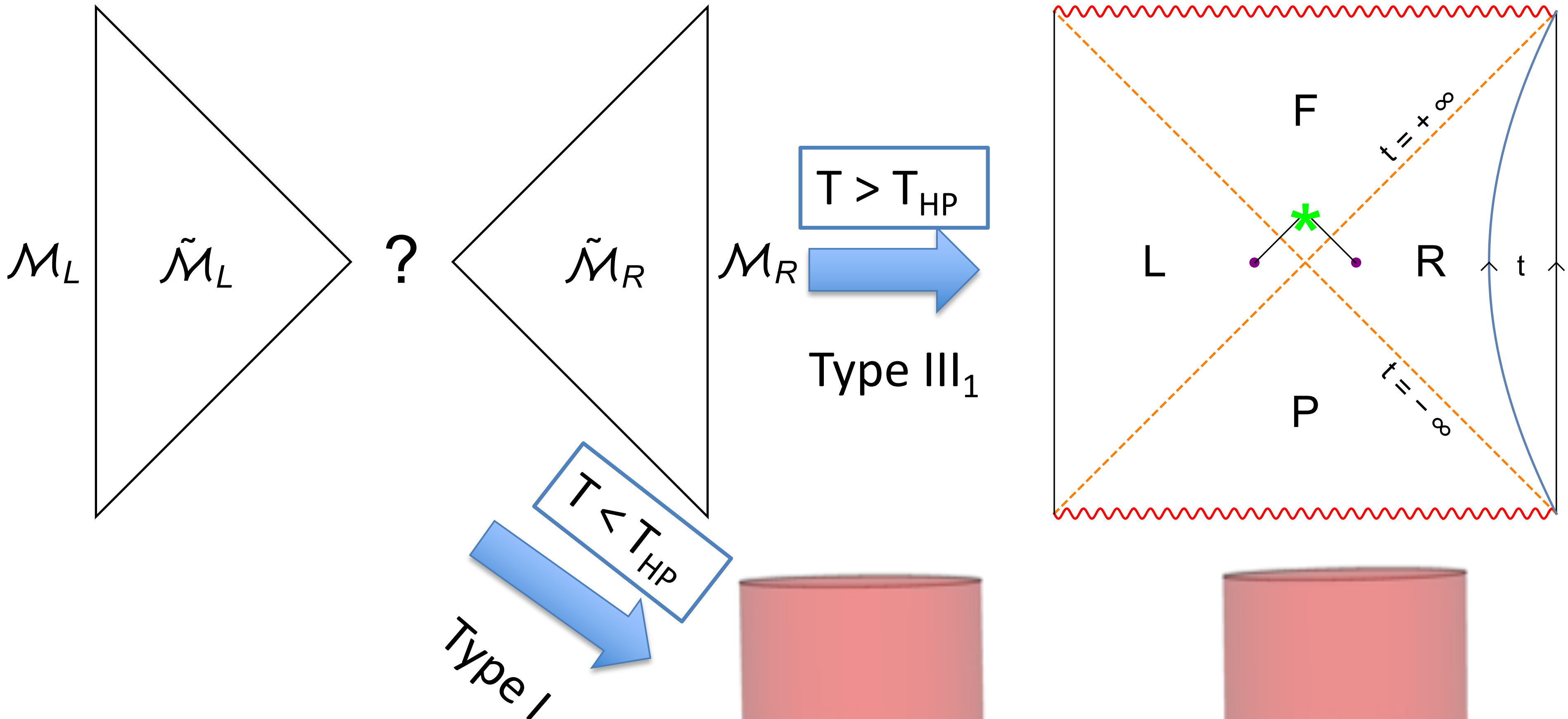
$$\mathcal{M}_R \vee \mathcal{M}_L$$



half-sided modular flows (specific to type III_1)

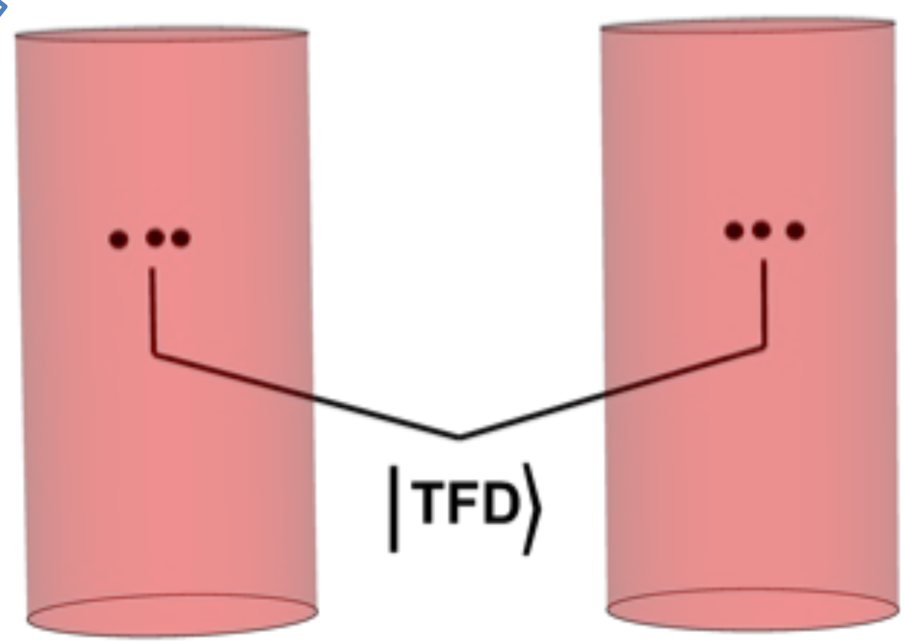
generate F and regions. Sharp boundary signature of horizon: non-analytic behavior under the flow.

Internal time generated by H_R-H_L (modular flow)



(Causal) connectedness from emergent type III_1

(more precise than ER= EPR)



Emergence of Kruskal-like time

Can construct a unitary group U(s) (for example for 2d CFT):

$$U(s) = e^{-iGs}, \qquad G \geq 0$$

$$\Phi(X;s) \equiv U(-s)\phi(X)U(s), \quad X \in R$$

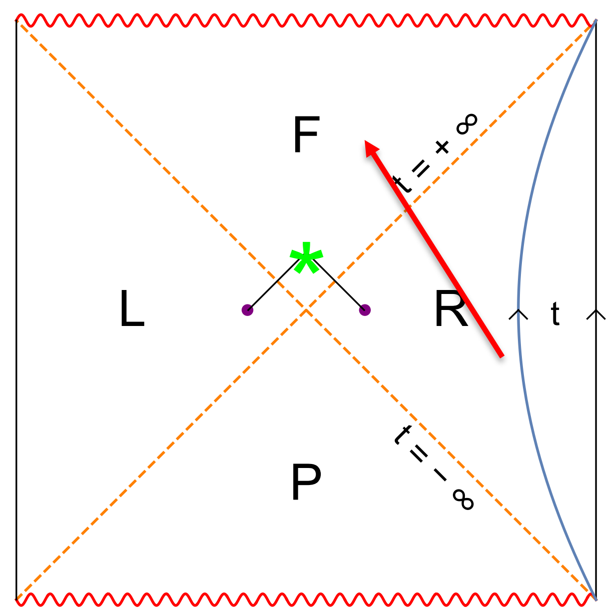
There exists an $s_0 > 0$

$$s < s_0, \quad \Phi(X;s) \in \mathrm{CFT}_R$$

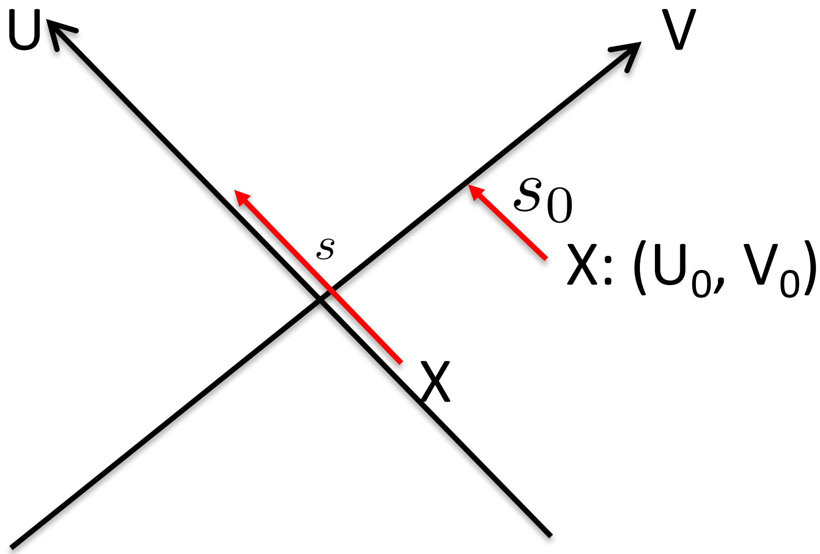
$$s > s_0, \quad \Phi(X;s) \in \mathrm{CFT}_R \otimes \mathrm{CFT}_L$$

signature of a sharp horizon.

Not possible for a type I vN algebra.



$$\Phi(X;s) \equiv U(-s)\phi(X)U(s), \quad X \in R$$



U, V: Kruskal null coordinates

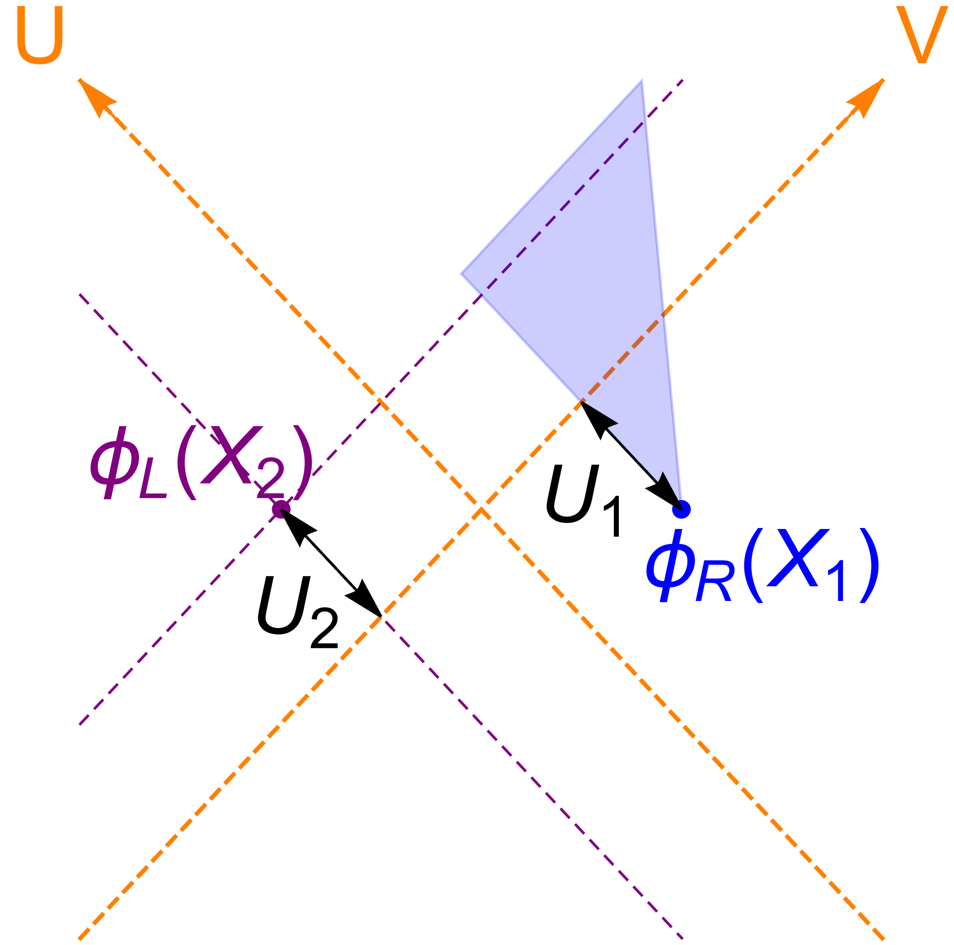
$$s_0 = -U_0$$

X near the horizon, local transformation: Kruskal null translation

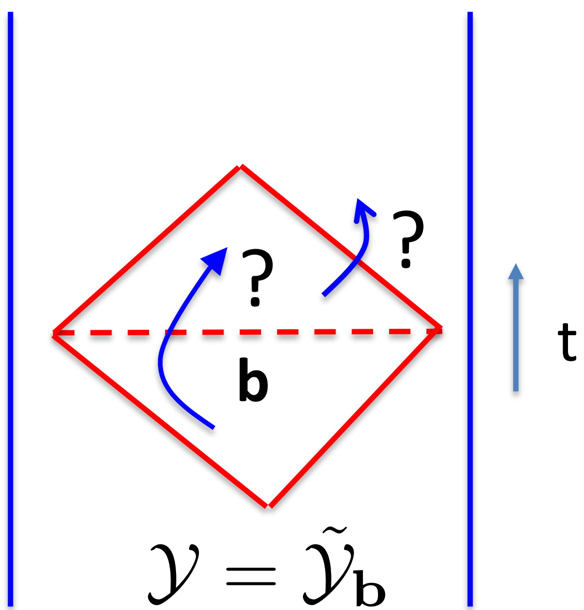
General X: transformation is nonlocal, but respects the casual structure

Causal structure

$$[U^\dagger(s)\phi_R(X_1)U(s), \phi_L(X_2)] = \begin{cases} 0 & s < |U_1| + U_2 \\ \neq 0 & s > |U_1| + U_2 \end{cases}$$



Emergent geometric properties



1 “interior” time of **b**

described by modular flow of $\mathcal{Y}$

2. “Global” time flows taking one outside **b**

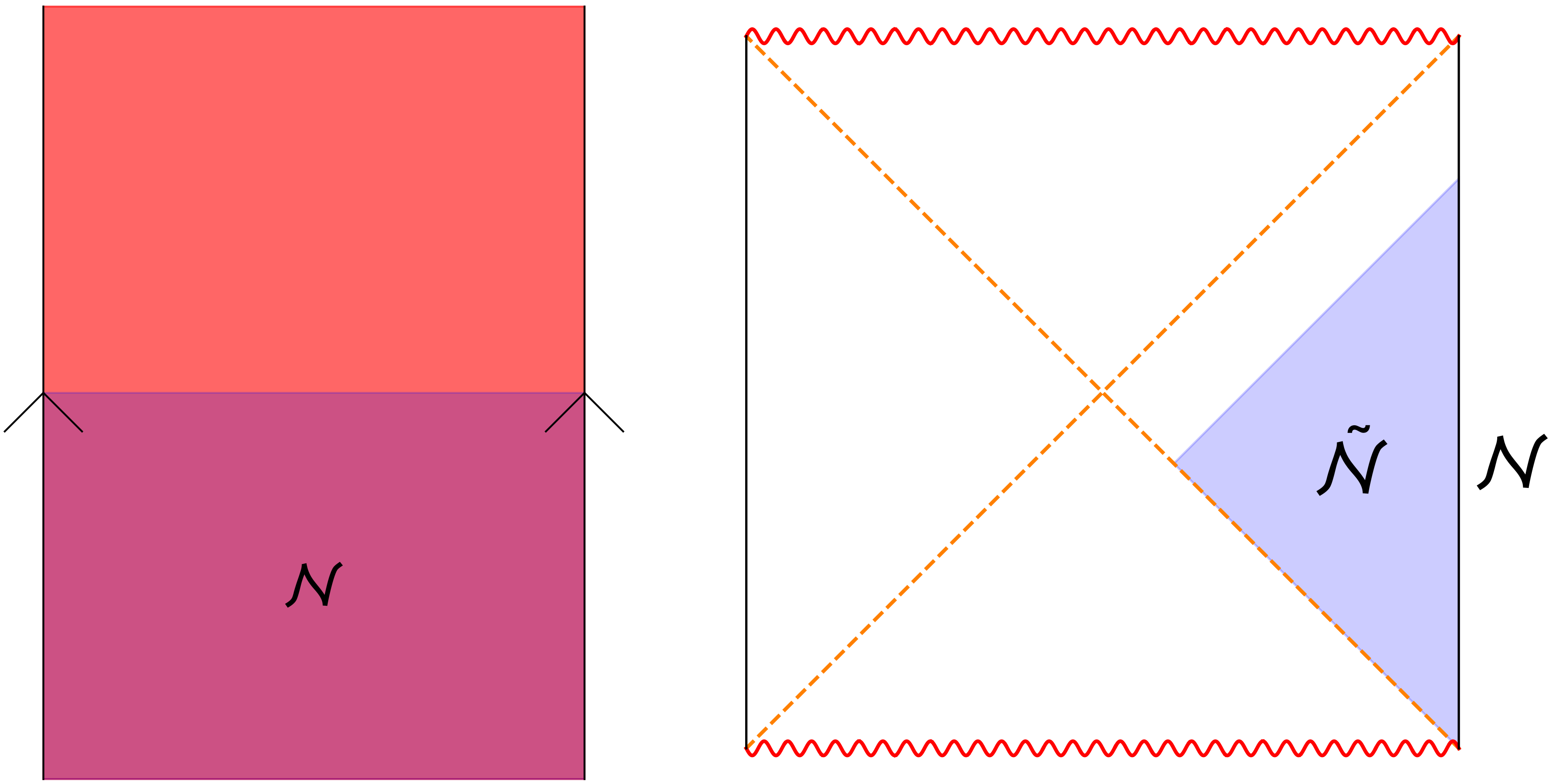
described by half-sided modular flow

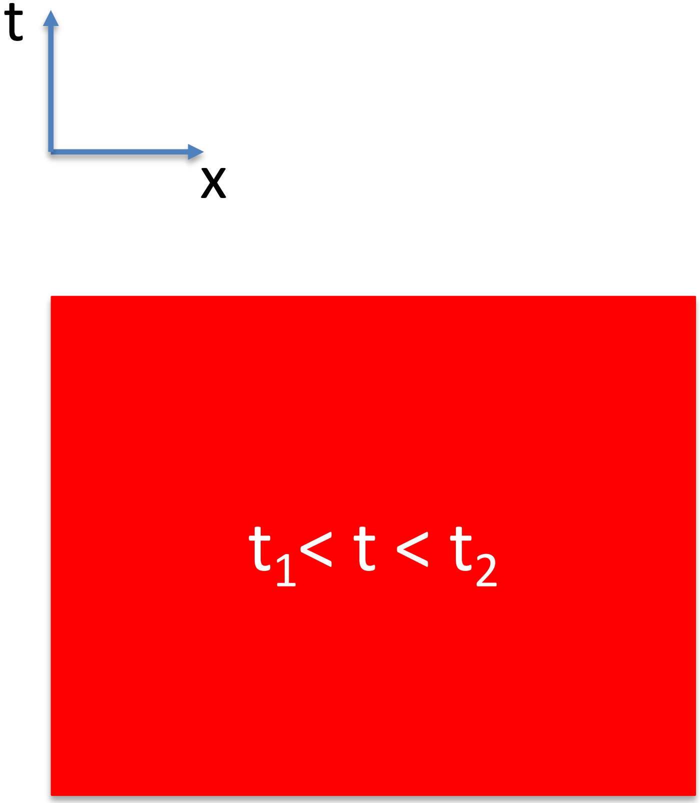
3. Causal structure from non-analytic behavior under half-sided modular flows

4. Given $\mathcal{Y}$, region **b**, including its light-cone boundary can in principle be determined.

More general examples

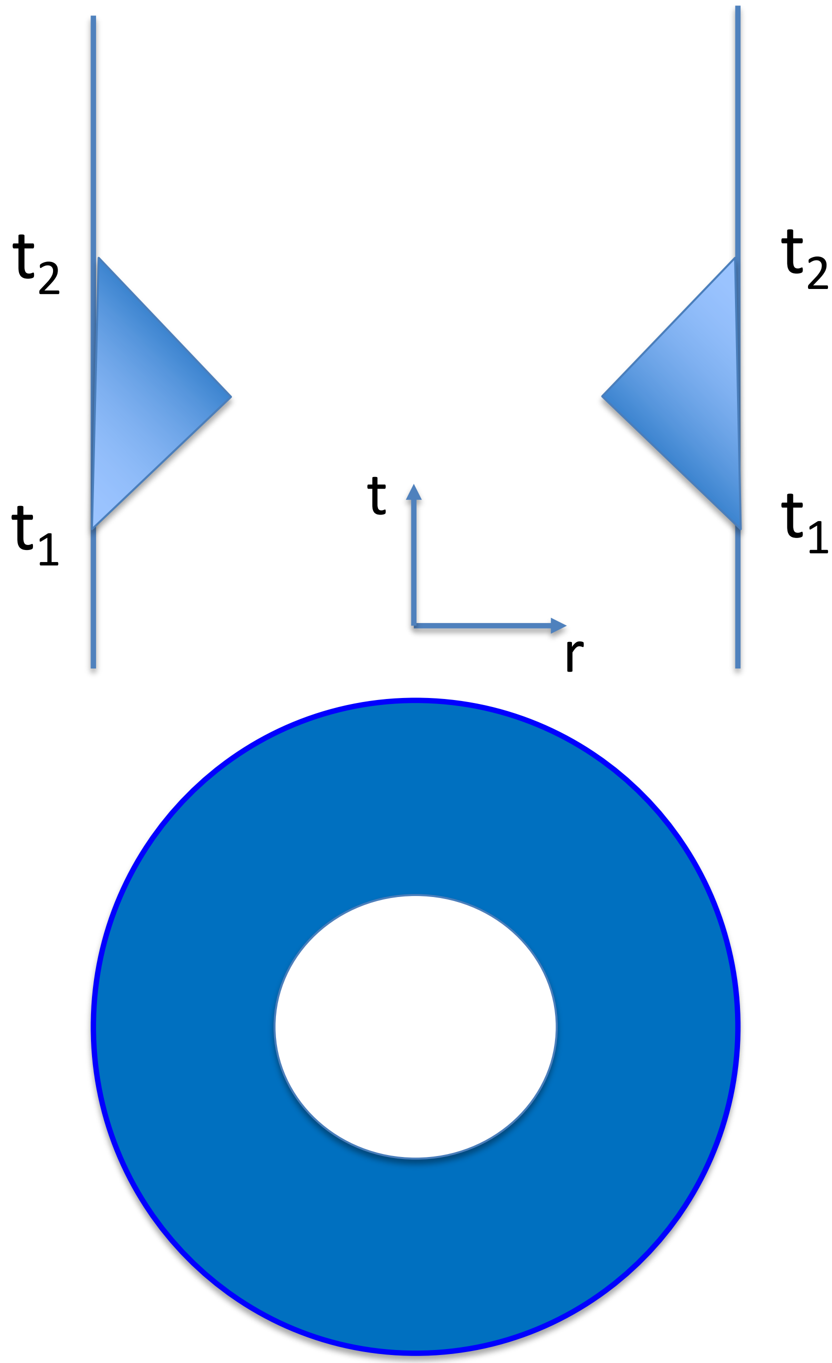
Examples

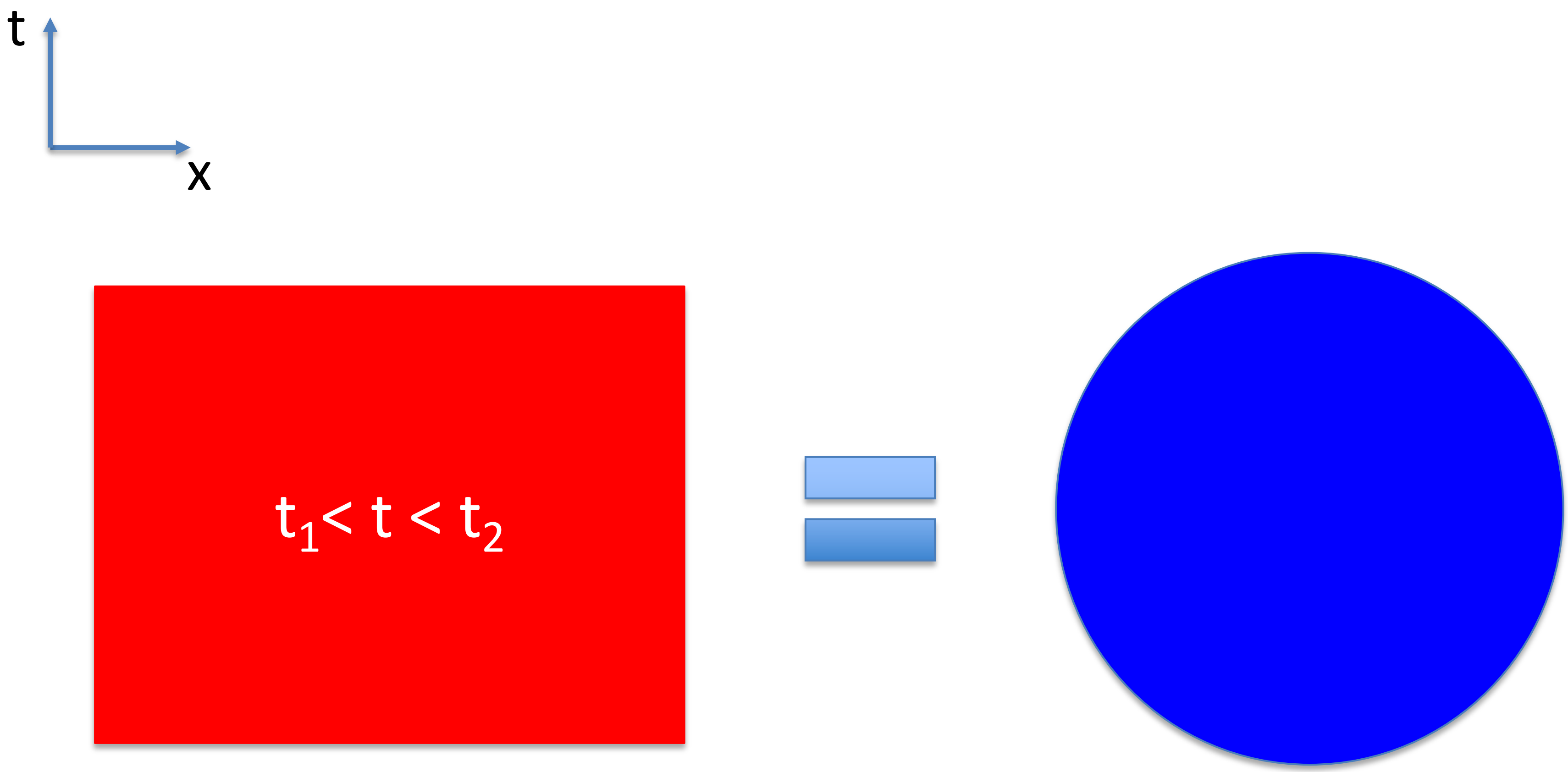




CFT$_2$ in the vacuum

$$t_2 - t_1 < \pi R$$

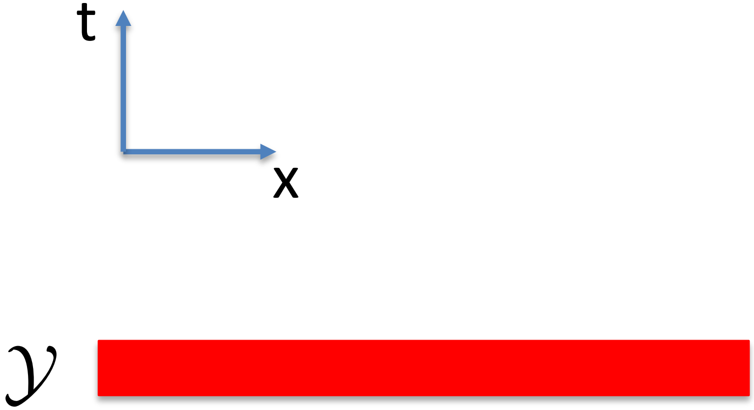




CFT_2 in the vacuum

Full bulk operator algebra

$$t_2 - t_1 \geq \pi R$$

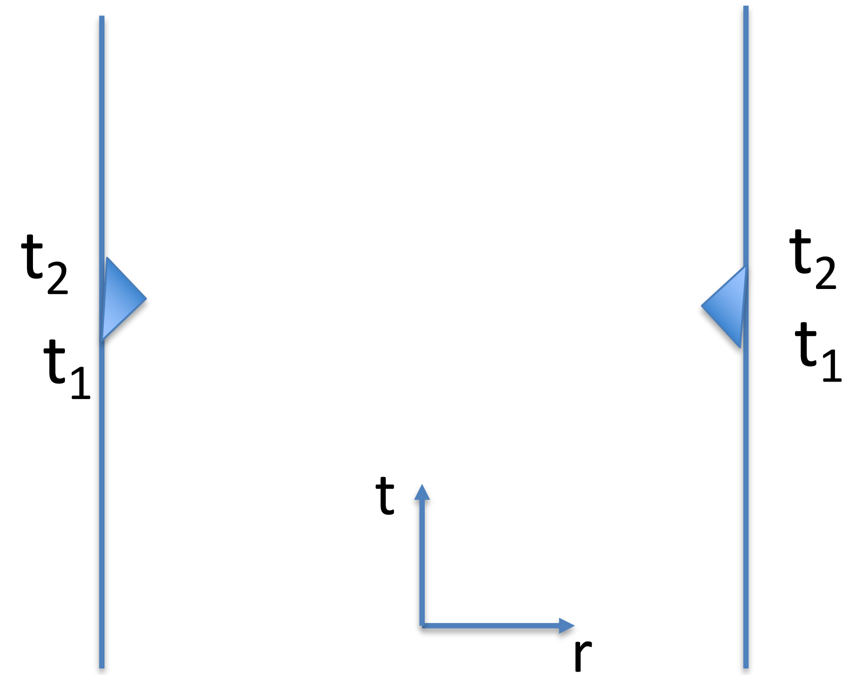


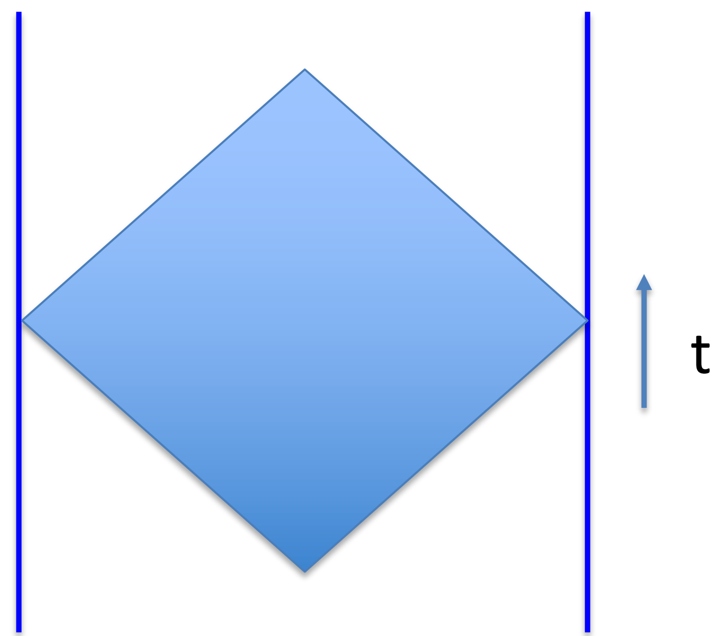
$t_1 < t < t_2$

$$t_2 \to t_1$$

$$\mathcal{Y}'$$

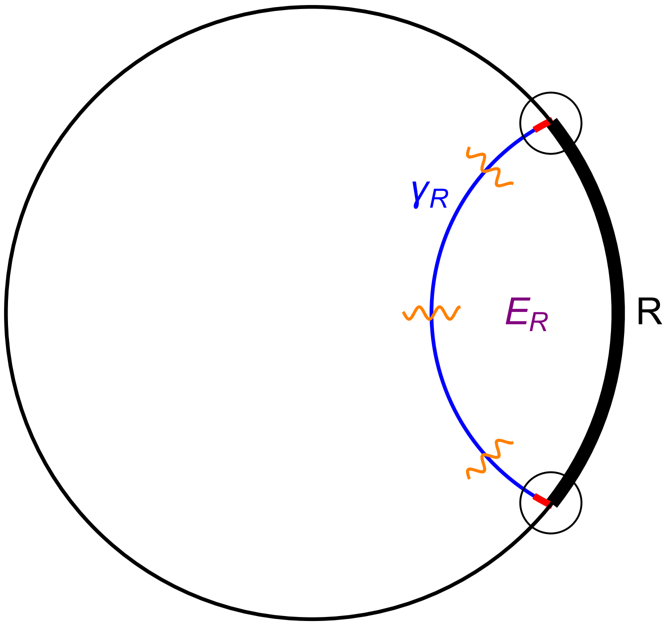
(no obvious boundary geometric description)





Insights into subregion-subregion duality

Emergent type III_1 algebra in a local region



Consider a local region R in the boundary theory.

At finite N, $\mathcal{B}_R$ is type III_1

We can introduce a short-distance cutoff to turn it into type I.

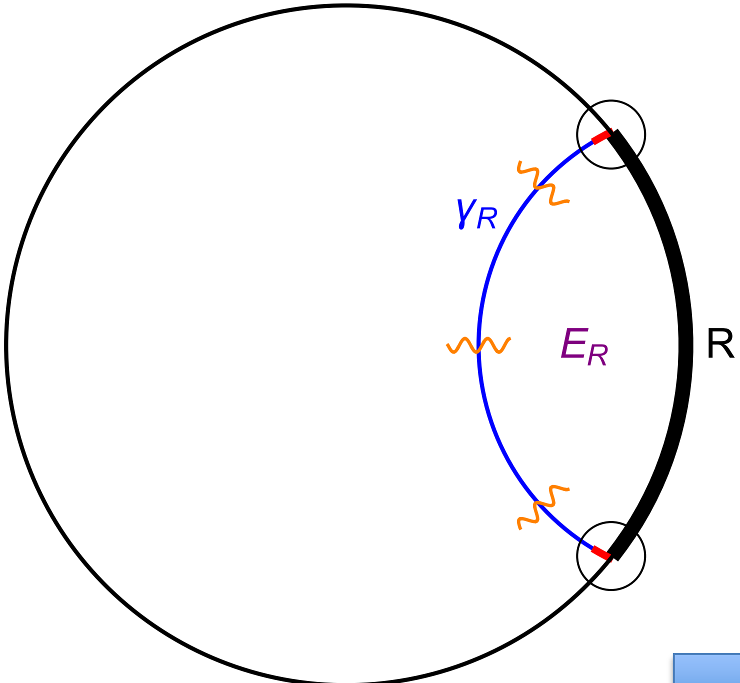
In the large N limit, there is an emergent type III_1 algebra:

$$\mathcal{X}_R = \lim_{N\to\infty, |\Psi\rangle} \mathcal{B}_R$$

Entanglement wedge reconstruction: $\mathcal{X}_R = \mathcal{M}_{E_R}$

$\mathcal{M}_{E_R}$: bulk operator algebra in the entanglement wedge

Entanglement wedge without entropy



Given boundary algebra $\mathcal{X}_R$

We can try to find the bulk region whose operator algebra is equivalent to $\mathcal{X}_R$

→ RT surface without using entropy

We have worked out some simple examples, but how the minimal surface prescription arises in this language is still missing.

Additivity anomaly (I)

$\mathcal{B}_R$ obeys additivity:

$$\mathcal{B}_{R_1} \vee \mathcal{B}_{R_2} = \mathcal{B}_{R_1 \cup R_2}$$

$$\mathcal{B}_{R_1} \cap \mathcal{B}_{R_2} = \mathcal{B}_{R_1 \cap R_2}$$

$\mathcal{X}_R$ in general does not:

$$\mathcal{X}_{R_1} \vee \mathcal{X}_{R_2} \subseteq \mathcal{X}_{R_1 \cup R_2}$$

$$\mathcal{X}_{R_1 \cap R_2} \subseteq \mathcal{X}_{R_1} \cap \mathcal{X}_{R_2}$$

The inequality arises from taking the large N limit

Additivity anomaly (II)

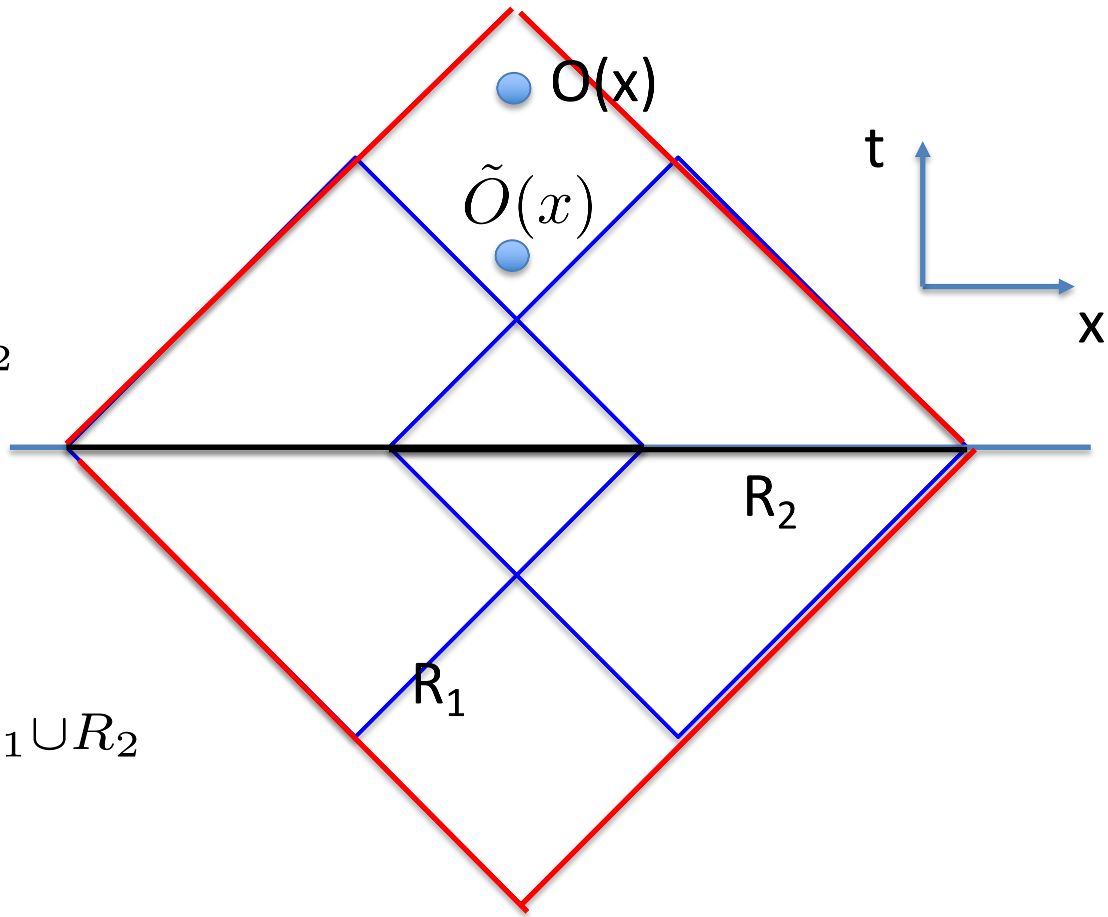
Simplest example:

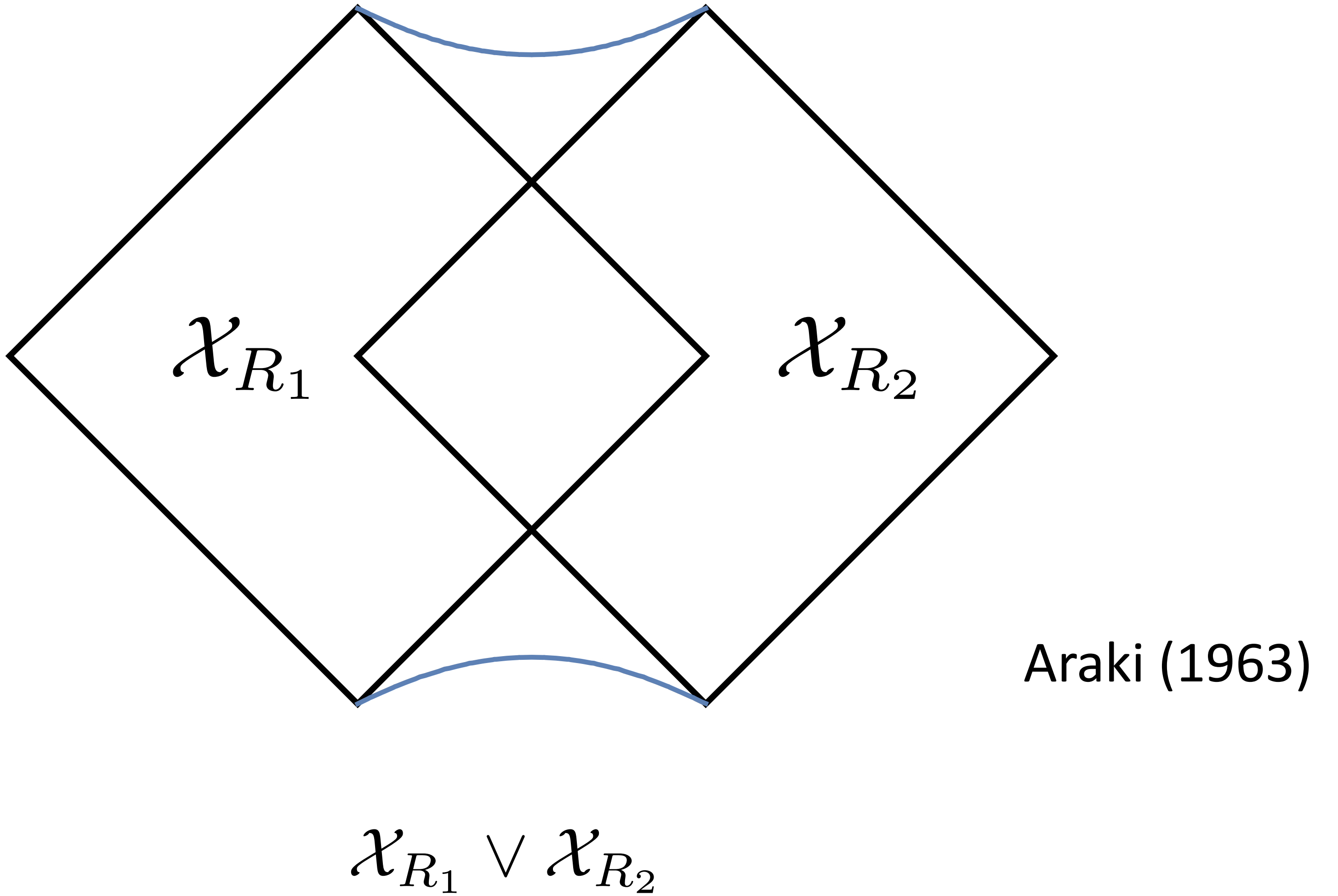
$$O(x) \in \mathcal{X}_{R_1 \cup R_2}$$

$$O(x) \notin \mathcal{X}_{R_1} \vee \mathcal{X}_{R_2}$$

$$\tilde{O}(x) \in \mathcal{X}_{R_1} \vee \mathcal{X}_{R_2}$$

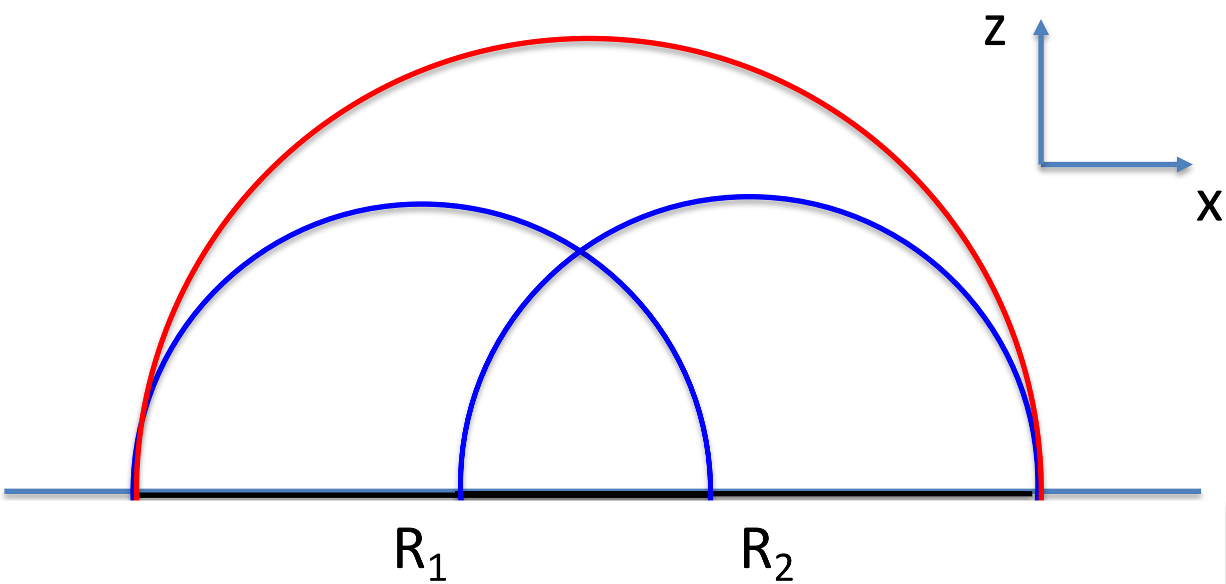
$$\mathcal{X}_{R_1} \vee \mathcal{X}_{R_2} \subset \mathcal{X}_{R_1 \cup R_2}$$




$\mathcal{X}_{R_1}$
$\mathcal{X}_{R_2}$
Araki (1963)
$\mathcal{X}_{R_1} \vee \mathcal{X}_{R_2}$

Implications of additivity anomaly (I)

Additivity anomaly underlies many properties we observe on the gravity side including :



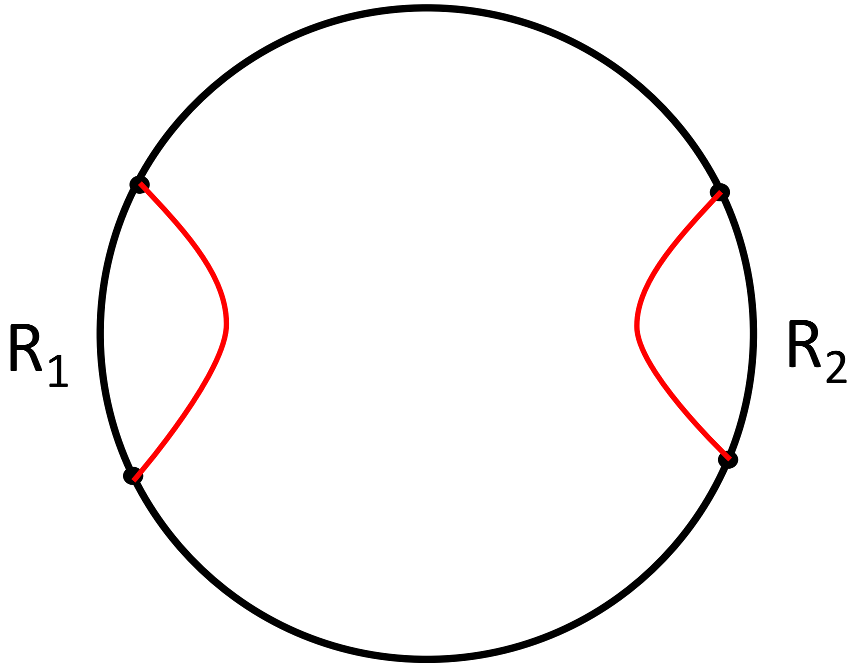
$$\mathcal{X}_{R_1} \vee \mathcal{X}_{R_2} \subset \mathcal{X}_{R_1 \cup R_2}$$

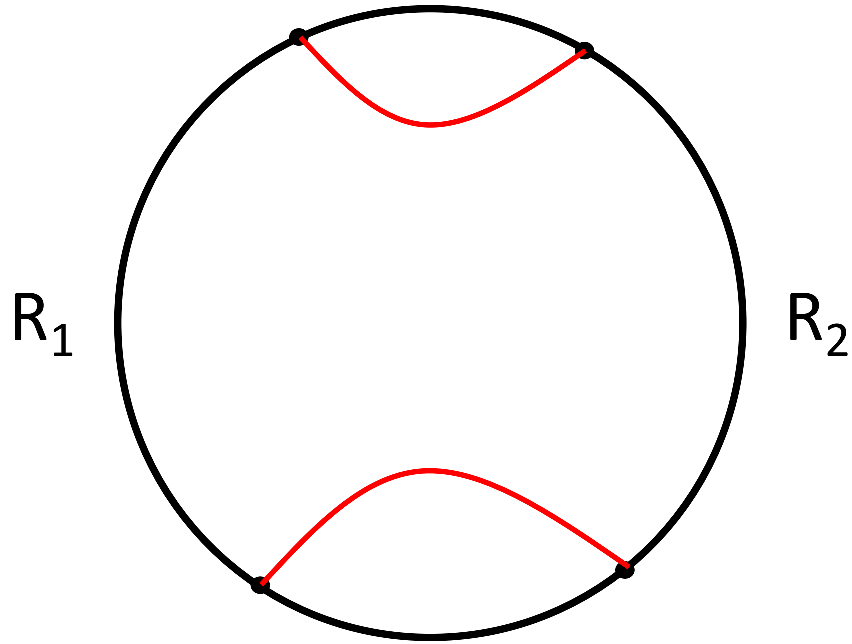
This has been interpreted in terms of quantum error corrections

Almheiri, Dong, and Harlow

Here we give its physical origin.

Implications of additivity anomaly (II)





$$\mathcal{X}_{R_1} \vee \mathcal{X}_{R_2} = \mathcal{X}_{R_1 \cup R_2}$$

$$\mathcal{X}_{R_1} \vee \mathcal{X}_{R_2} \subset \mathcal{X}_{R_1 \cup R_2}$$

New Insights into subregion-subregion duality

1. Give a precise mathematical definition

2. Entanglement wedge without entropy.

RT surface without
using entropy

3. Additivity anomaly and implications

Underlies many bulk properties, including quantum error correction properties

Future perspectives

- Including 1/N corrections, the role of conserved charges
  (boundary manifestation of Gauss law and bulk nonlocality)

Bahiru, Belin, Papadodimas, Sarosi, Vardian
arXiv: 2209.06845

- connections with finite N

Type II

Witten (arXiv:2112:12828)
Chandrasekaran, Penington, Witten, 2209.10454

Type I: (finite N)

- Implications for holography in flat and cosmological spacetimes

Chandrasekaran, Longo, Penington, Witten, 2206.10780

- New perspectives on single-sided or evaporating BHs,
  Derivation of "island''

- Entropy associated with general bulk surfaces

……

Thank you!

Large N limit of the Hilbert space

Many states do not have a well-defined large N limit.

A state has a well-defined large N limit, if correlation functions of single-trace operators (with expectation value subtracted) in it have well-defined $N \to \infty$ limits.

We refer to a state with well-defined large N limit and factorization property as a semi-classical state.

Examples: vacuum , thermal density operator, thermal field double

....

For a semi-classical state $|\Psi\rangle$, we can build a Hilbert space around it by acting finite products of single-trace operators on it. (GNS Hilbert space $\mathcal{H}_{\Psi}^{GNS}$)

In the large N limit, only semi-classical states and states around them survive.

The full state space splits into disconnected GNS Hilbert spaces around semi-classical states.

Different Ψ lead to different single-trace operator algebra with possibly very different mathematical and physical properties

The same structure appears on the gravity side.

We quantize gravity fields around a geometry to obtain the Fock space around that geometry.

Half-sided modular translation

There is a very special structure associated with type III_1 vN algebra.

Suppose $\mathcal{M}$ is a von Neumann algebra and the vector $|\Omega\rangle$ is cyclic and separating for $\mathcal{M}$

Suppose there exists a von Neumann subalgebra $\mathcal{N}$ of $\mathcal{M}$ with the properties:

$|\Omega\rangle$ is cyclic for $\mathcal{N}$

$$\Delta_{\mathcal{M}}^{-it}\mathcal{N}\Delta_{\mathcal{M}}^{it} \subset \mathcal{N}, \quad t \leq 0$$

Then there exists a unitary group U(s), with the following properties:

Borchers, Wiesbrock

$$U(s) = e^{-iGs}, \qquad G \geq 0$$

$$U(s)\Omega = \Omega, \quad \forall s \in \mathbb{R}$$

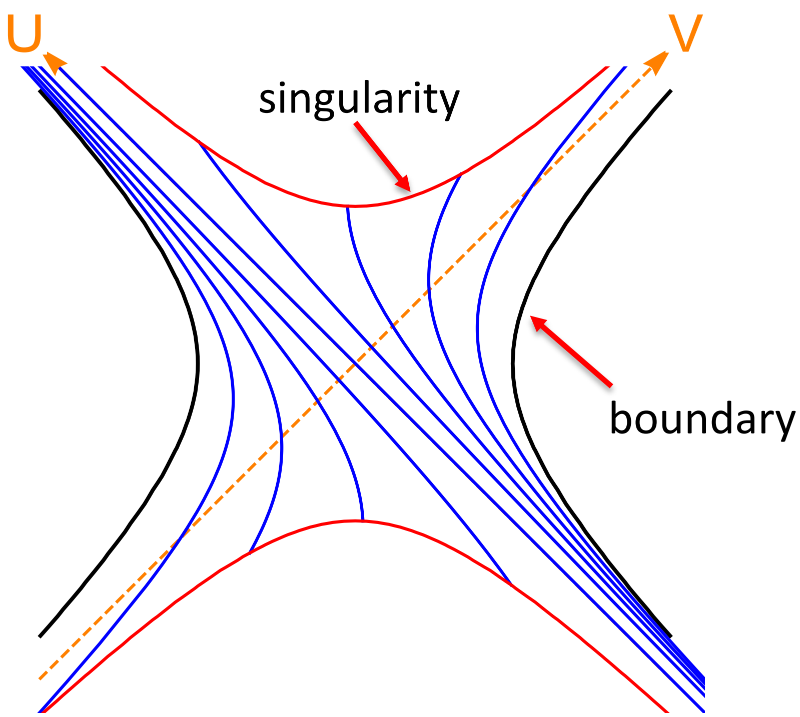
This can be used to generate "new" times!

Flow pattern in the large mass limit

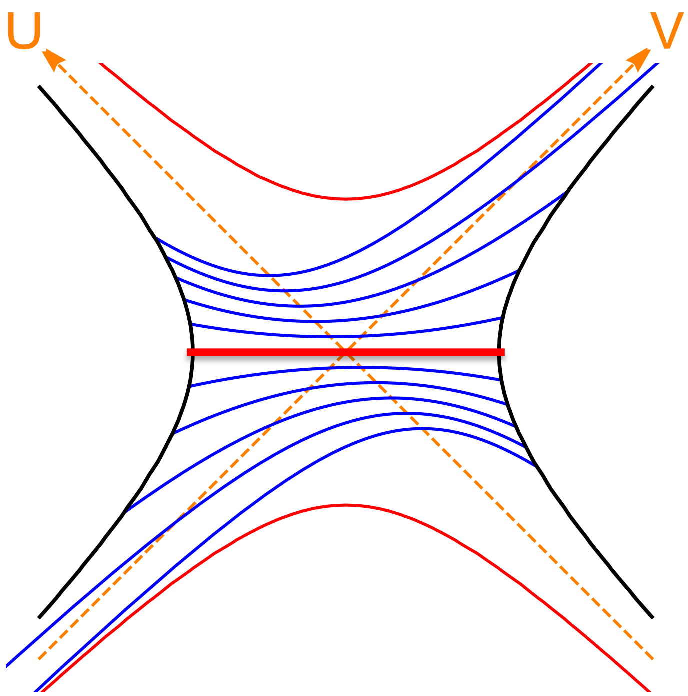
$$\Phi(X;s) \equiv U(-s)\phi(X)U(s) \propto \phi(X_s)$$

$$U_s = U_0 + s, \quad V_s = \frac{V_0}{1 - sV_0}$$

average over boundary spatial directions

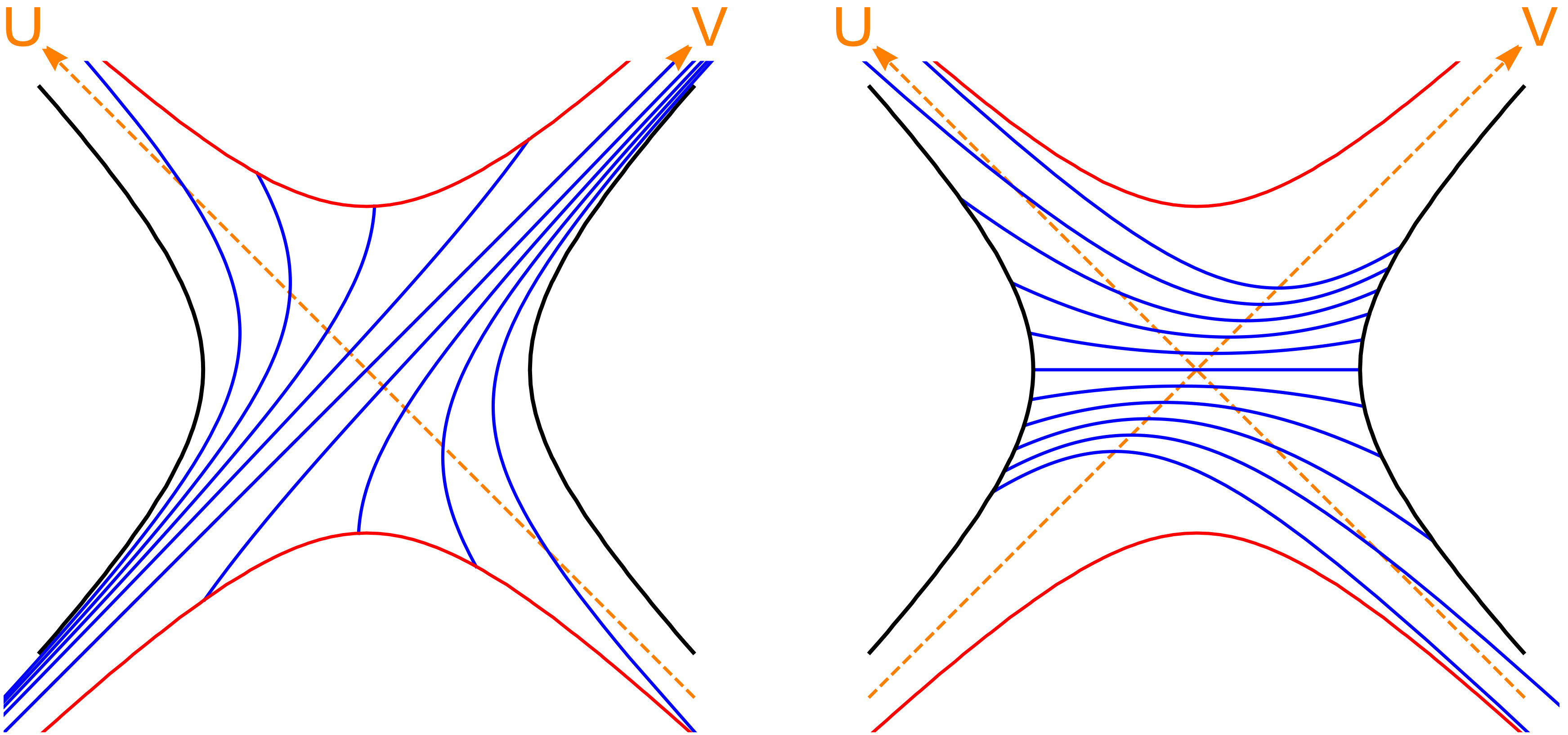


Family of trajectories



Constant-s slices

V-type Flows



We can also consider compositions of such Kruskal-like U-type and V-type flows.

There are an infinite number of such emergent times
(by definition diffeomorphism invariant)