String spectrum and dynamics in AdS_3 backgrounds

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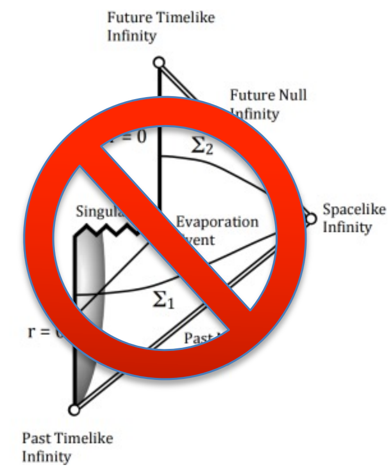
based on arXiv pubs:

1803.08505, 1906.11473, **2005.12344**, **2211.12476,** work in progress
(EJM, Stefano Massai, David Turton)

2303.00234, 2303.17139 (EJM)

still crazy after all these years

- $\sim$50 yrs of the black hole information problem: What's wrong with this picture?



- String theory: First steps toward a resolution via an accounting of near-extremal black hole microstates

- Gauge/gravity duality in principle gives a picture of dynamics, if we can decode the duality map

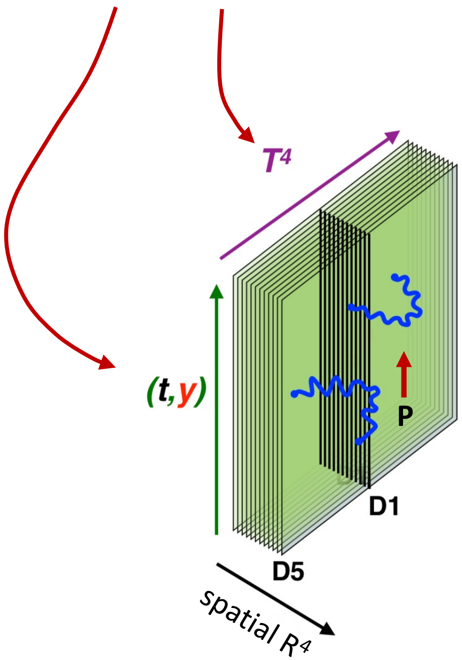
- Theme of this talk: The use of perturbative string theory as a tool to take us beyond supergravity in setting up the holographic dictionary

supersymmetric BH entropy

- A prime example: 3-charge BPS **D1**-**D5**-**P** bound states wrapped on $\mathbb{S}^1_y \times \mathcal{M}$ (where $\mathcal{M}=\mathbb{T}^4$ or K3) become black holes at strong coupling

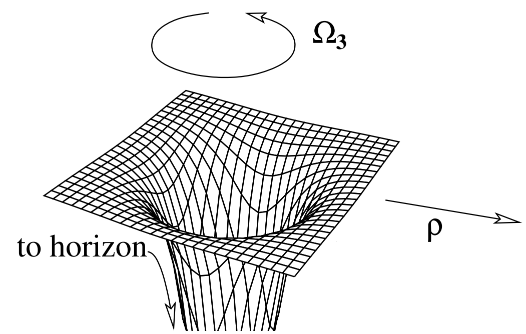
$$S_{BH} = 2\pi\sqrt{n_5 n_1 n_p}$$

(Strominger-Vafa '96)



Weak coupling

Strong coupling

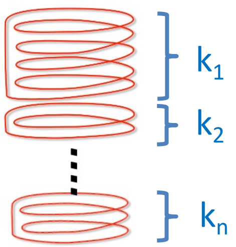


- The weakly coupled CFT is the symmetric product orbifold $(\mathcal{M}^{n_1 n_5})/S_{n_1 n_5}$

BPS states of the symmetric product

- It's useful to focus on BPS states, as these are robust across moduli space

- The general **½-BPS** ground state $|\{N_k^I\}\rangle$ is labelled by a decorated conjugacy class of the symmetric group – a tensor product of N_k cyclic twists sewing together k copies of the CFT on $\mathcal{M}=\mathbb{T}^4$ or K3; each cycle can be in any ½-BPS ground state of $\mathcal{M}$ labeled by I , and $\sum_k kN_k^I = N = n_1 n_5$



- For **¼-BPS** states, apply your favorite collection of fractionated, left-moving oscillator excitations to each cycle

$$\big|\{n_{\ell,a}\},\{m_{j,\beta}\},I\big\rangle_{k\cdot cycle} = \prod_{\ell,a;j,\beta} (\alpha^a_{-\ell/k})^{n_{\ell,a}} (\psi^\beta_{-j/k})^{m_{j,\beta}} \big|I\big\rangle_k$$

connecting branes to geometry

- We would like to tie the intersecting brane picture (weak coupling) to bulk geometry (strong coupling)

- The **F1**-**NS5** duality frame is very useful here – perturbative string theory about the vacuum has a solvable worldsheet description, allowing us to probe beyond the supergravity approximation (Giveon-Kutasov-Seiberg '98)

- The **NS5**-brane tension $\propto 1/g_s^2$ makes it a **solitonic** object in supergravity, therefore much structure will be revealed in semi-classical closed string dynamics, which incorporates the back-reaction of the heavy background

- The worldsheet theory keeps track of what the fivebranes are doing, even in regions of stringy curvature

NS5-F1 ½-BPS ground states

- The general **½-BPS** string condensate on a stack of **NS5**'s has N_k^I excitations of winding number k w/choice of 8B+8F polarizations I (for $\mathbb{T}^4$);

$$|\Psi\rangle = \prod_{\substack{k,\ell,p,q\\ pol's\ I}} \overbrace{\Big(\underbrace{|\alpha\dot\beta\rangle}_{\perp\text{ scalars}}{}_k\Big)^{N_k^{\alpha\dot\beta}} \Big(\underbrace{|AB\rangle}_{\substack{\text{internal}\\ \text{(NS5 gauge multiplet)}}}{}_\ell\Big)^{N_\ell^{AB}}}^{\text{bosons}} \overbrace{\Big(|\alpha B\rangle_p\Big)^{N_p^{\alpha B}} \Big(|A\dot\beta\rangle_q\Big)^{N_q^{A\dot\beta}}}^{\text{fermions}}$$

$\alpha, \dot\beta, A, B = \pm$

$\sum_{k,I} k N_k^I = N = n_1 n_5$

mode number = "*little string*" winding

- Easily understood via T-duality to **NS5**-**P** where this is the Fock space of fractionated BPS momentum modes on a single NS5 wrapping $\mathbb{S}^1_{\tilde{y}}$ n_5 times.

- For K3, there are no fermionic modes, and 16 extra internal bosonic modes

Harvey-Strominger '95

- ***Each of these states corresponds to a particular horizonless geometry***

Lunin-Mathur '01, Taylor '05, Kanitscheider-Skenderis-Taylor '07

NS5-F1 supertube geometry

- The ½-BPS **NS5-F1** supertube geometry

$$ds^2 = -Z_1^{-1}\big(du+\omega\big)\big(dv+\beta\big) + Z_5\, d\mathsf{x}_\perp\!\cdot d\mathsf{x}_\perp + ds^2_{\mathbb{T}^4} \qquad e^{2\Phi} = g_s^2\,\frac{Z_5}{Z_1}$$

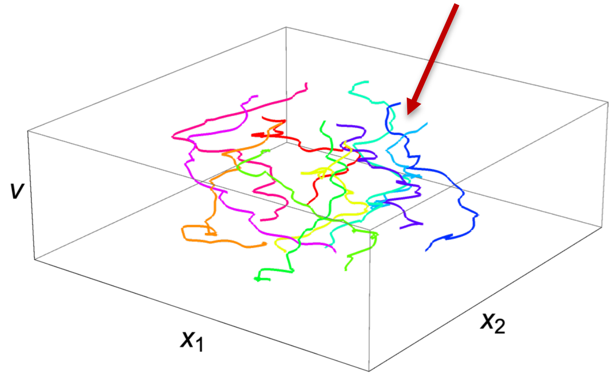
$$B = \tfrac{1}{2} Z_1^{-1}(du+\omega)\wedge(dv+\beta) + b_{ij}\, dx^i \wedge dx^j\ . \qquad db = *_\perp dZ_5$$

is specified by a set of harmonic functions and one-forms sourced by functions $\mathsf{F}^I(v)$ along a contour (Lunin-Mathur '01)

$$Z_5 = \frac{n_5}{2\pi L}\int_0^{2\pi L}\frac{dv}{|\mathbf{x}-\mathsf{F}(v)|^2} \qquad \beta = \mathsf{A}+\mathsf{B}\ , \quad \omega = \mathsf{A}-\mathsf{B}$$

$$\mathsf{A}_i = \frac{n_5}{2\pi L}\int_0^{2\pi L}\frac{dv\,\dot F_i(v)}{|\mathbf{x}-\mathsf{F}(v)|^2}\ , \qquad d\mathsf{B} = *_\perp d\mathsf{A}$$

$$Z_1 = 1+\frac{n_5}{2\pi L}\int_0^{2\pi L}\frac{dv\,\dot{\mathsf{F}}_I\cdot\dot{\mathsf{F}}_I}{|\mathbf{x}-\mathsf{F}(v)|^2}$$

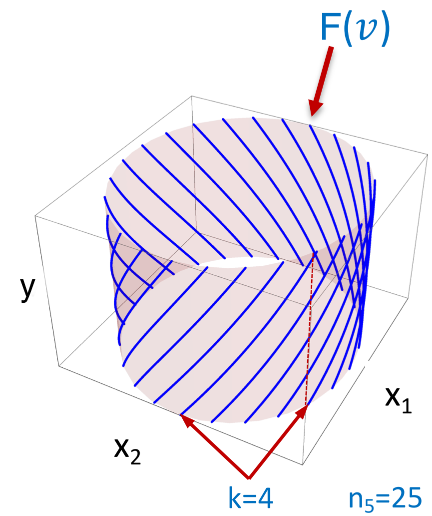


- ***The Fourier mode amplitudes a_k^I of $\mathsf{F}^I(v)$ are coherent state parameters specifying $\langle N_k^I\rangle$***

The Return of the String

- The worldsheet **string** dynamics in the **fivebrane** throat is *exactly solvable* when the n_5 fivebranes are evenly distributed along a circle in a ⊥ plane.

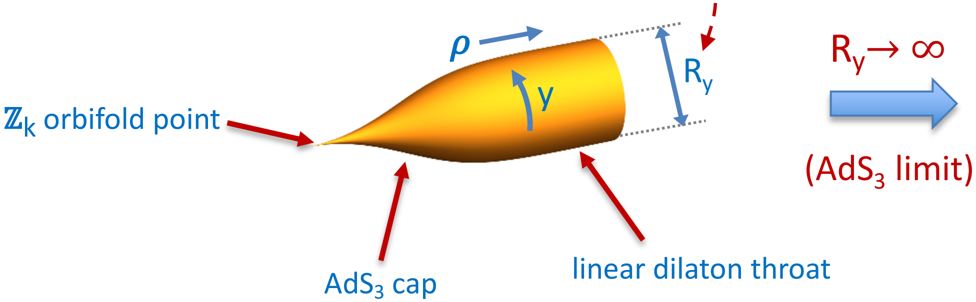
(Giveon-Kutasov '99, EJM-Massai '17)

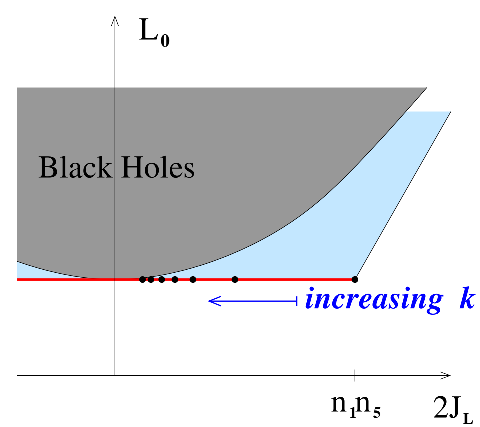


- The worldsheet theory is a gauged WZW model on

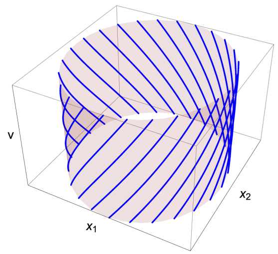
$$\frac{\mathcal{G}}{\mathcal{H}} = \frac{\left(\mathbb{R}_t \times \mathbb{S}^1_y \times \mathbb{T}^4\right)_{||} \times \left(SL(2,\mathbb{R}) \times SU(2)\right)_\perp}{U(1)_L \times U(1)_R}$$

where the gauge orbits under $\mathcal{H}$ are null isometries of $\mathcal{G}$





supertube source perturbations



- This circular source profile corresponds to the fivebrane state with a single transverse scalar mode populated macroscopically

$$|\Psi\rangle = \Big(\underbrace{|{+}{+}\rangle}_{\perp\ \text{scalar}}{}_{k}\Big)^{N/k}$$

- ½-BPS string vertex operators $\mathcal{V}^{\alpha\dot\beta}_{j,w_y}$ (NS·NS) and $\mathcal{S}^{AB}_{j,w_y}$ (R·R) implement transitions

$$\mathcal{V}^{\alpha\dot\beta}_{j,w_y}\ :\ \big(|{+}{+}\rangle_k\big)^{2j+1} \longrightarrow\ |\alpha\dot\beta\rangle_{(2j+1)k+w_y n_5}$$

$$\mathcal{S}^{AB}_{j,w_y}\ :\ \big(|{+}{+}\rangle_k\big)^{2j+1} \longrightarrow\ |AB\rangle_{(2j+1)k+w_y n_5}$$

16 extra of these for K3

- These perturbations take us toward arbitrary ½-BPS backgrounds

$$|\Psi\rangle = \prod_{\substack{k,\ell,p,q\\ pol's\ I}} \overbrace{\Big(\underbrace{|\alpha\dot\beta\rangle_k}_{\perp\ \text{scalars}}\Big)^{N_k^{\alpha\dot\beta}}\Big(\underbrace{|AB\rangle_\ell}_{\text{internal}}\Big)^{N_\ell^{AB}}}^{\text{bosons}}\ \overbrace{\Big(|\alpha B\rangle_p\Big)^{N_p^{\alpha B}}\Big(|A\dot\beta\rangle_q\Big)^{N_q^{A\dot\beta}}}^{\text{fermions (absent for K3)}}$$

½-BPS worldsheet perturbations

- These ½-BPS vertex operators are supergraviton-like, *e.g.*

$$\mathcal{V}^{++}_{j,w_y} = \underbrace{\big(J\bar{J}\,\Phi^{sl}_{j+1}\big)_{j;j,j}}_{\text{Clebsch}}\,\Phi^{su}_{j;j,j}\;\underbrace{e^{iw_yR_y(t+\tilde{y})}}_{\text{winding on } \mathbb{S}^1_y} \qquad + \dots$$

$$\mathcal{V}^{--}_{j,w_y} = \Phi^{sl}_{j+1}\,\overbrace{\big(J\bar{J}\,\Phi^{su}_{j}\big)_{j+1}}\;\overbrace{e^{iw_yR_y(t+\tilde{y})}} \qquad + \dots$$

$$\mathcal{S}^{AB}_{j,w_y} = \big(S^A\bar{S}^B\,\Phi^{sl}_{j+\frac{1}{2}}\;\Phi^{su}_{j-\frac{1}{2}}\big)_{j|j}\;e^{iw_yR_y(t+\tilde{y})} \qquad + \dots$$

BPS condition selects highest weight sates in SL(2,R) and SU(2)

(Kutasov-Larsen-Leigh ’98)

subject to the Virasoro and null gauge constraints.

EJM-Massai-Turton ’18, ’20, ’22
Bufalini-Iguri-Kovensky-Turton ‘22

- There is a 1-1 map of ½-BPS worldsheet vertex operators and ½-BPS deformations. The generic state is obtained by condensing these operators (exponentiating them into the worldsheet action). One sees that indeed the LM backgrounds are coherent string condensates bound to the **NS5**’s.

½-BPS perturbations

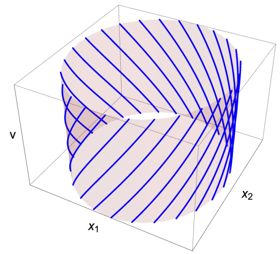
- The center-of-mass wavefunctions have the form

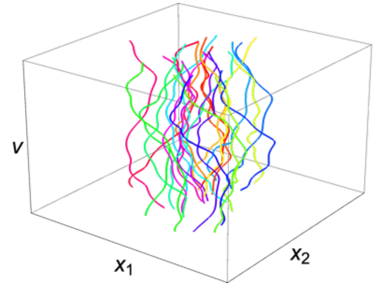
$$\Phi^{sl}_{j,j,j}\,\Phi^{su}_{j,j,j} \sim \left(\frac{a^2\sin^2\theta}{r^2+a^2}\right)^j$$

  and at large j are concentrated along the fivebrane ring source at $r\sim 0$, $\theta\sim\pi/2$

- The vertex operators make deformations that sit along the supertube source in the deepest part of the geometry and make ½-BPS deformations to nearby supertubes. In the supergravity approximation these deformation look like shockwaves; string theory regularizes this and determines back-reaction

- The nonlinear condensation of these excitations leads to general supertube geometries





¼-BPS perturbations

- There are also **¼-BPS** vertex operators describing supergravity perturbations.

½-BPS $\mathcal{V}^{++}_{j,w_y} = \left(J\bar{J}\,\Phi^{sl}_{j+1}\right)_{j;j,j}\,\Phi^{su}_{j;j,j}\;e^{i\underbrace{w_y}R_y(t+\tilde{y})} \quad + \ldots$

carrying winding on $\mathbb{S}^1_y$

¼-BPS *sugra* $\mathcal{V}^{++}_{j,n,m} = \left(J\bar{J}\Phi^{sl}_{j+1}\right)_{j;\underbrace{j+n},j}\,\Phi^{su}_{j;\underbrace{j-m},j}\;\exp\left[i\frac{n_y}{\underbrace{R_y}}(t+y)\right] \quad + \ldots$

not highest wt ⇒ not BPS on left

carrying momentum on $\mathbb{S}^1_y$

gauge constraints set $kn_y{=}m{+}n$

- The excitation under global SL(2,R)xSU(2) corresponds to a CFT deformation

$$\left(|{+}{+}\rangle_k\right)^{2j+1} \;\longrightarrow\; \left(J^+_{-\frac{1}{k}}\right)^m \left(L_{-\frac{1}{k}} - \tfrac{1}{k}J^3_{-\frac{1}{k}}\right)^n |{+}{+}\rangle_{(2j+1)k}$$

- Changing the polarization state of the vertex operator allows us to get any ground state polarization $|I\rangle_{(2j+1)k}$ including "supercharged" and 6d vector modes. All 8B+8F left supergraviton polarizations are possible; BPS on the right limits us to 2B+2F polarizations.

¼-BPS perturbations

- The center-of-mass wavefunctions have the form

$$\Phi^{sl}_{j,j+n,j}\,\Phi^{su}_{j,j-m,j} \sim r^n\left(\frac{a^2}{r^2+a^2}\right)^{j+n/2}(\sin\theta)^{2j-m}\,(\cos\theta)^m$$

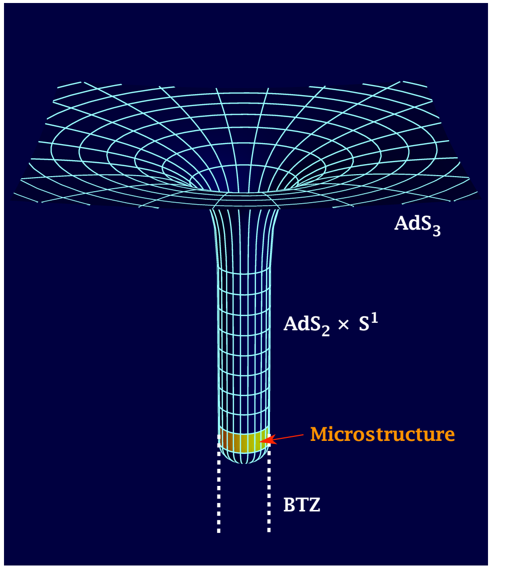
  and again at large j and $m=n=0$ are concentrated along the fivebrane ring source at $r\sim 0$, $\theta\sim\pi/2$. As m, n increase the excitation is pushed away from the ring

- These vertex operators make deformations that sit in the deepest part of the geometry and make ¼-BPS momentum excitations

- The nonlinear condensation of these excitations leads to *superstratum* geometries

nonlinear ¼-BPS deformations

- *superstrata* are the *exact* supergravity solutions obtained by condensing the ¼-BPS *supergravity* vertex ops into the WS action
  Bena, Giusto, Russo, Shigemori, Warner '15
- Account for "supergravity elliptic genus" states
  de Boer '98
- The *superstratum* excitations are in 1-1 correspondence with ¼-BPS *supergravity* vertex ops in the worldsheet construction



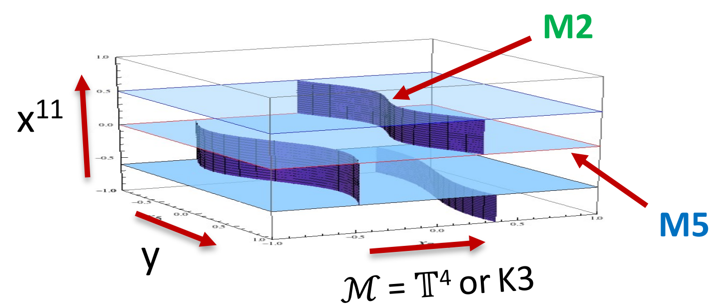
- *The finest modings are absent* – c.o.m. momentum excitations have moding $\propto 1/k$, string oscillators contribute momentum $\propto 1/w_y$, while the cycle length created by the vertex operator is $(2j+1)k+w_y n_5$
- It's not clear that such fine excitations exist in the supergravity regime – they could be lifted from the BPS bound as we traverse the moduli space

the importance of fractionation

- A key feature of the BPS black hole entropy formula $S_{BH} = 2\pi\sqrt{n_5 n_1 n_p}$ is that the entropy of a given constituent is *multiplicatively enhanced* by the presence of other types of constituent in the bound state

- This entropy enhancement is a result of *charge/tension fractionation:*
  - String winding fractionates momentum $\delta\mathrm{P} \propto \frac{n_p}{n_1}$
  - Fivebranes fractionate strings into n_5 constituent *little strings* whose tension is also fractionated $(\alpha')_{little} = n_5\alpha'$ (Dijkgraaf-Verlinde[2] '96,'97, Seiberg '97)

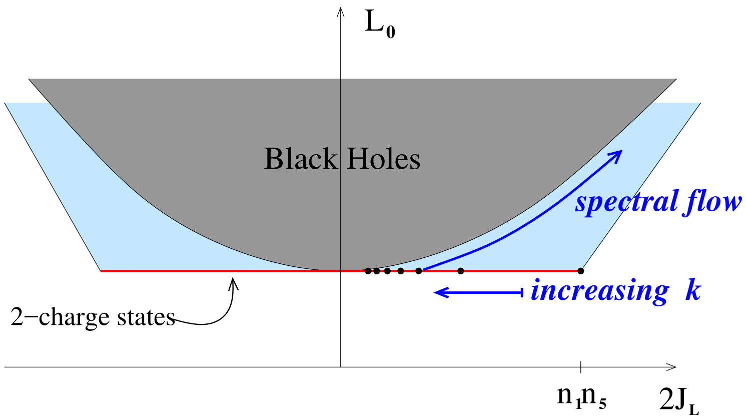
- Cartoon: in the M-theory lift of IIA: **M2**-branes (the lift of IIA **F1**'s) break apart on the **M5**'s (the lift of IIA **NS5**'s)



- **F1** *strings lack a factor of* n_5 *in fractionation*

¼-BPS supertube backgrounds

- One can add momentum and angular momentum to the bkgd via *"fractional spectral flow"* of the CFT state, coherently exciting left-moving SU(2) currents on each cycle of the CFT (w/integer momentum on each cycle)



$$|\Psi\rangle = \Big(J^+_{-2/k} J^+_{-4/k} \cdots J^+_{-2s/k} |{+}{+}\rangle_k \Big)^{N/k} \equiv \Big(\mathcal{O}_{s/k} |{+}{+}\rangle_k \Big)^{N/k}$$

NB: need $\frac{s(s+1)}{k} \in \mathbb{Z}$

- The dual geometry is known; one of its interesting features is an ergoregion. Remarkably, a modification of the worldsheet null-gauged WZW model

$$\frac{\mathcal{G}}{\mathcal{H}} = \frac{\big(\mathbb{R}_t \times \mathbb{S}^1_y \times \mathbb{T}^4\big)_{||} \times \big(SL(2,\mathbb{R}) \times SU(2)\big)_\perp}{U(1)_L \times U(1)_R}$$

again describes these backgrounds, by changing the embedding $\mathcal{H} \subset \mathcal{G}$

¼-BPS perturbations of ¼-BPS supertubes

- There are again **¼-BPS** vertex operators describing supergravity excitations

¼-BPS
sugra

$$\mathcal{V}^{++}_{j,n,m} = \underbrace{\big(J\bar{J}\Phi^{sl}_{j+1}\big)_{j;j+n,j}}_{\text{not highest wt}} \; \underbrace{\Phi^{su}_{j;j-m,j}}_{\text{not BPS on left}} \; \exp\Big[i\underbrace{\frac{n_y}{R_y}}_{\text{carrying momentum on } \mathbb{S}^1_y}(t+y)\Big] \quad + \ldots$$

not highest wt ⇒ not BPS on left carrying momentum on $\mathbb{S}^1_y$

- The worldsheet gauge constraints impose

$$k\, n_y = n + (2s+1)m - 2sj$$

- The vertex operators implement corresponding transitions in the CFT state:

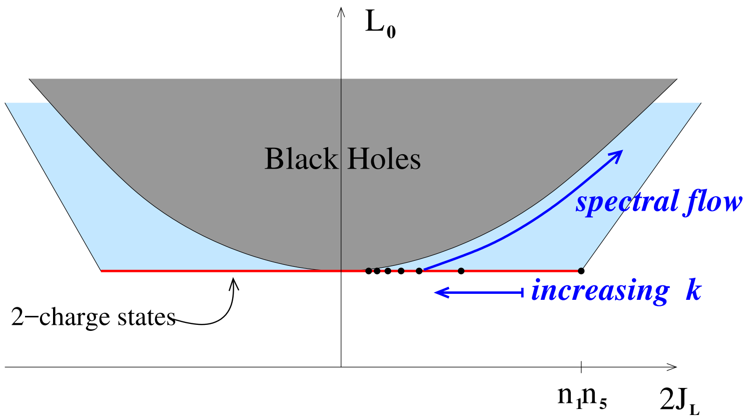
$$\Big(\mathcal{O}_{s/k}|{+}{+}\rangle_k\Big)^{2j+1} \longrightarrow \; \big(L_{-1/k}\big)^n \big(J^+_{-(2s+1)/k}\big)^m \mathcal{O}_{s/k}|{+}{+}\rangle_{(2j+1)k}$$

- Note that some of these strings have $n_y < 0$, and because they are BPS, also have $E < 0$; these live in the ergoregion. The superstrata arising from coherent condensates of these modes have not been worked out.

non-susy coherent states

- One can also make exact non-susy backgrounds, via fractional spectral flow on both left and right

$$\big|\Psi\big\rangle = \Big(\mathcal{O}_{s/k}\,\overline{\mathcal{O}}_{\bar{s}/k}\big|{+}{+}\big\rangle_k\Big)^{N/k}$$



- The dual background is again known (the "JMaRT" geometry); again it has an ergoregion, and in spite of being non-supersymmetric, is stable in the fivebrane decoupling limit. Again it has a description as a gauged WZW model

$$\frac{\mathcal{G}}{\mathcal{H}} = \frac{\big(\mathbb{R}_t \times \mathbb{S}^1_y \times \mathbb{T}^4\big)_{||} \times \big(SL(2,\mathbb{R}) \times SU(2)\big)_\perp}{U(1)_L \times U(1)_R}$$

via a modification of the embedding $\mathcal{H} \subset \mathcal{G}$

perturbations of non-susy backgrounds

- There are again **¼-BPS** vertex operators describing supergravity excitations

¼-BPS
sugra

$$\mathcal{V}^{++}_{j,n,m} = \big(J\bar{J}\Phi^{sl}_{j+1}\big)_{j;\underbrace{j+n,j+\overline{n}}} \Phi^{su}_{j;\underbrace{j-m,j-\overline{m}}} \exp\Big[i\frac{n_y}{\underbrace{R_y}}(t+y)\Big] + \dots$$

not highest wt ⇒ not BPS both L&R   carrying momentum on $\mathbb{S}^1_y$

- The worldsheet gauge constraints impose

$$k\,n_y = n - \overline{n} + (2s+1)m - (2\overline{s}+1)\overline{m} - 2(s-\overline{s})j$$

- The vertex operators implement corresponding transitions in the CFT state:

$$\Big(\mathcal{O}_{s/k}\overline{\mathcal{O}}_{\overline{s}/k}|{+}{+}\rangle_k\Big)^{2j+1} \longrightarrow \big(L_{-\frac{1}{k}}\big)^n \big(J^+_{-\frac{2s+1}{k}}\big)^m \big(\overline{L}_{-\frac{1}{k}}\big)^{\overline{n}} \big(\overline{J}^+_{-\frac{2\overline{s}+1}{k}}\big)^{\overline{m}} \mathcal{O}_{\frac{s}{k}}\overline{\mathcal{O}}_{\frac{\overline{s}}{k}}|{+}{+}\rangle_{(2j+1)k}$$

- Again some of these live in the ergoregion, but at the linearized level there is no instability. The *microstrata* arising from coherent condensates of these modes have not been worked out.

perturbations of non-susy backgrounds

- There are again **¼-BPS** vertex operators describing supergravity excitations

¼-BPS
sugra

$$\mathcal{V}^{++}_{j,n,m} = \left(J\bar{J}\Phi^{sl}_{j+1}\right)_{j;\underbrace{j+n,j+\overline{n}}}\,\Phi^{su}_{j;\underbrace{j-m,j-\overline{m}}}\,\exp\left[i\frac{n_y}{\underbrace{R_y}}(t+y)\right] + \dots$$

not highest wt ⇒ not BPS both L&R   carrying momentum on $\mathbb{S}^1_y$

- While these modes are stable in the decoupled fivebrane theory, the story changes when the asymptotically flat region is reintroduced

- Matching the throat modes onto outgoing spherical waves leads to an imaginary part to the frequency of ergoregion modes. Half the modes are exponentially growing instabilities.

Chowdhury-Mathur '07 – '09
Chakrabarty-Turton-Virmani '15

- The worldsheet tells us the how the instabilities change the background:

$$\left(\mathcal{O}_{s/k}\overline{\mathcal{O}}_{\bar{s}/k}|{+}{+}\rangle_k\right)^{2j+1} \longrightarrow \left(L_{-\frac{1}{k}}\right)^n\left(J^+_{-\frac{2s+1}{k}}\right)^m\left(\overline{L}_{-\frac{1}{k}}\right)^{\overline{n}}\left(\overline{J}^+_{-\frac{2\bar{s}+1}{k}}\right)^{\overline{m}}\mathcal{O}_{\frac{s}{k}}\overline{\mathcal{O}}_{\frac{\bar{s}}{k}}|{+}{+}\rangle_{(2j+1)k}$$

which suggests the throat deepens and gains a gas of excitations

¼-BPS (& Stringy) perturbations

- There are also Stringy **¼-BPS** perturbations, *e.g.*

½-BPS

$$\mathcal{V}^{++}_{j,w_y} = \left(J\bar{J}\,\Phi^{sl}_{j+1}\right)_{j;j,j}\,\Phi^{su}_{j;j,j}\;e^{\underbrace{iw_y}_{\text{carrying winding on } \mathbb{S}^1_y} R_y(t+\tilde{y})} \quad + \ldots$$

¼-BPS
sugra

$$\mathcal{V}^{++}_{j,n,m} = \left(J\bar{J}\Phi^{sl}_{j+1}\right)_{j;\underbrace{j+n}_{},j}\,\Phi^{su}_{j;\underbrace{j-m}_{},j}\;\exp\left[i\underbrace{\frac{n_y}{R_y}}_{\text{carrying momentum on } \mathbb{S}^1_y}(t+y)\right] \quad + \ldots$$

not highest wt ⇒ not BPS on left

¼-BPS
Stringy

$$\underline{\mathcal{P}(L\ osc.)}\left(\bar{J}\Phi^{sl}_{j+1}\right)_{j,\underline{j+n},j}\,\Phi^{su}_{j,\underline{j-m},j}\;\exp\left[\underline{iw_yR_y}(t+\tilde{y}) + i\underline{\frac{n_y}{R_y}}(t+y)\right] \quad + \ldots$$

L stringy excitations

not L highest wt
⇒ not BPS on L

carrying momentum
<u>**and**</u> winding on $\mathbb{S}^1_y$ ⇒ $\underbrace{n+m = kn_y = \frac{k}{w_y}N_L}_{\text{level matching}}$

oscillator excitations required by world sheet constraints

- **There are *many more perturbative string than supergravity deformations, yet still falling short of the BTZ entropy. Holographic map is not understood***

Summary, part I

- Worldsheet string theory in AdS_3 reveals a remarkable amount of structure. Gauged WZW models provide a perturbative construction of excitations around well-studied backgrounds (conical defect supertubes and their spectral flows)

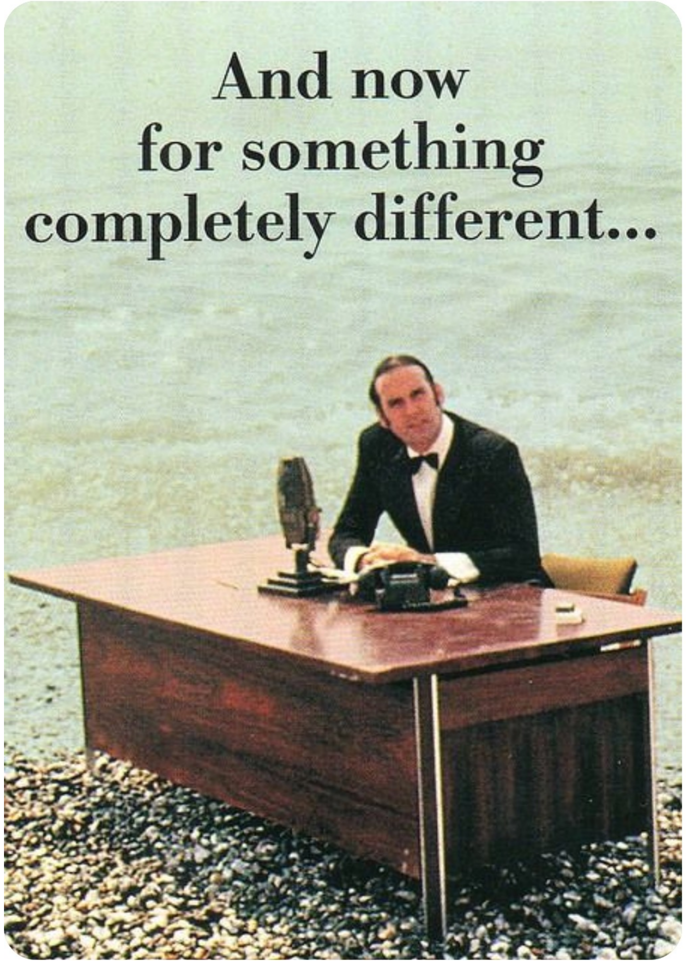
- In particular, one sees all possible excitations whose coherent condensates are superstrata. Vertex operators facilitate our understanding of the duality map, specifically the transitions among states that they mediate

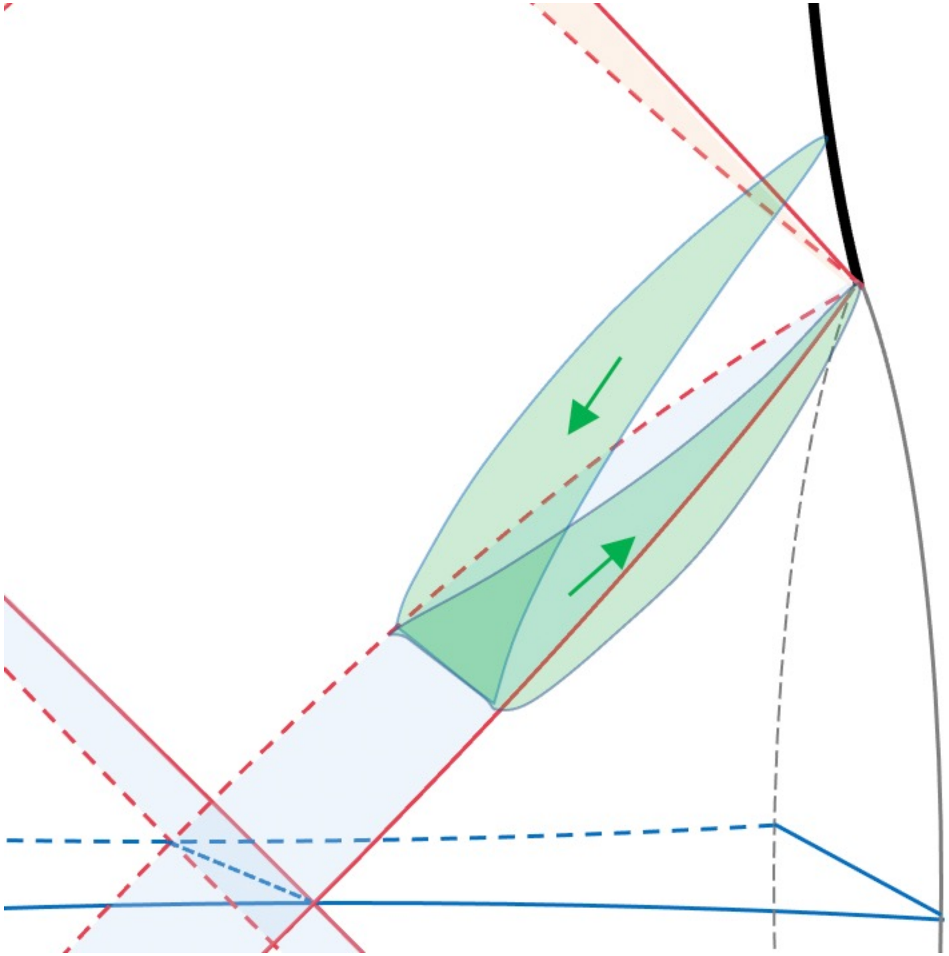
- The most fractionated excitations are absent from the perturbative string spectrum; where are they? The answer may lie in strongy-coupled *little string theory*

- There may exist perturbatively stringy extensions of superstrata in which one allows singularities corresponding to perturbative string sources. Supertube shockwaves are a simple example in the ½-BPS setting known to be regularized by stringy effects

(Lunin-Mathur-Saxena '02, Chakrabarty-Rawash-Turton '21)

charge emission from black holes

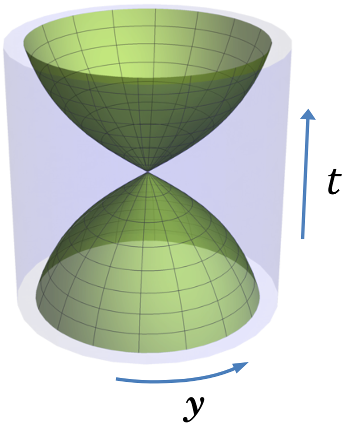




string emission from BTZ black holes

- The **F1**-**NS5**-**P** (BTZ) black hole is a useful laboratory for the study of black holes in string theory. The BTZ solution is the SL(2,R)/$\mathbb{Z}$ WZW model

- In AdS_3. there is a continuum of "long string" excitations which are the vestige of the Coulomb branch of **F1**'s wrapping $\mathbb{S}^1_y$ (for BTZ, twisted sectors of the $\mathbb{Z}$ orbifold). Field theory observables in AdS_3 are boundary correlators; but long strings have an **S-matrix.** BTZ black holes can decay by emitting long strings, even in the decoupled AdS_3 theory



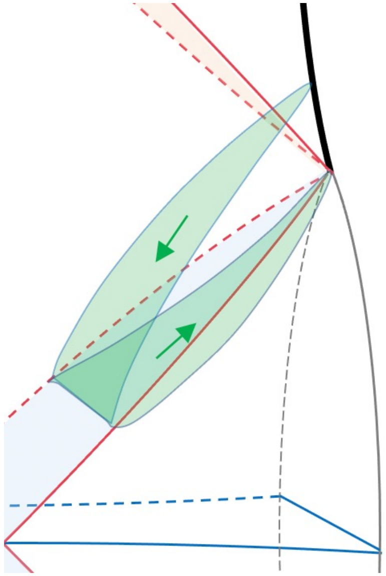
- The information paradox in this instance is the question of whether radiated strings are just scrambled versions of the ones that collapsed to make the black hole (*i.e.* unitarity is preserved), or whether they instead arise from vacuum fluctuations (as predicted by the Hawking process), and thus have no relation to the strings that made the black hole

charge emission from black holes

- One can use the first-quantized formalism to calculate the probability $\boldsymbol{\Gamma}$ for a black hole (BTZ in particular) to emit quanta carrying the various conserved charges E, n_p, J, n_1, *etc.*

- Particles/strings/branes travel a non-classical trajectory near the horizon; they "tunnel across"

(Kraus-Wilczek '94, Parikh-Wilczek '98)



- The tunneling amplitude is the imaginary part of the classical action for the string center of mass dynamics

$$\mathcal{S}_{worldsheet} = \int \left(\frac{1}{\gamma} G_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - \gamma m^2 - A_\mu \dot{x}^\mu \right)$$

- Passing to the Hamiltonian form of the action

$$\mathcal{S}_{worldsheet} = \int \left(p_\mu dx^\mu - \gamma \mathcal{H} \right)$$

- The Hamiltonian vanishes, leaving the reduced action $\int p\, dx$

charge emission from black holes

- Solving the constraint $\mathcal{H}=0$ for p_r, the vanishing of g_{tt} near the horizon implies that p_r has a pole (for outgoing strings) whose residue depends only on geometric properties of the horizon (surface gravity *etc.*)

$$p_r \sim \frac{E - \Omega j - \Phi q}{\kappa_+ \, (r - r_+)} \qquad (E = p_t \, , \, j = p_y \ etc.)$$

- Deforming the integration contour into the complex $\boldsymbol{r}$ plane to avoid the pole, and using the principal value prescription, the integral of the trajectory across the horizon yields (using $T = \kappa_+/2\pi$)

$$Im\, \mathcal{S} = \frac{E - \Omega j - \Phi q}{2T} = -\frac{dM - \Omega\, dJ - \Phi\, dQ}{2T} = -\frac{1}{2}\, dS_{BH}$$

  where in the last equality we have used the first law

- Particles/strings/branes thus travel a non-classical trajectory near the horizon; they "tunnel out", and the tunneling probability is simply the change in the black hole entropy (Kraus-Wilczek '94, Parikh-Wilczek '98)

string emission from BTZ black holes

- The BTZ black hole entropy formula matches that of the dual CFT

$$S_{BH} = 2\pi\Big(\sqrt{n_1 n_5 (E+n_p)/2} + \sqrt{n_1 n_5 (E-n_p)/2}\,\Big)$$

- The BTZ energy $\boldsymbol{E}$ and angular momentum $\boldsymbol{n_p}$ are related to the inner and outer horizon radii of the BTZ geometry ($N = n_1 n_5 = c/6$)

$$E = \tfrac{1}{2} N (r_+^2 + r_-^2) \quad , \quad J = n_p = N r_+ r_- \quad \Longrightarrow \quad S_{BH} = 2\pi N r_+$$

- The change in entropy resulting from winding string emission is then

$$\Delta S_{BH} = -\frac{r_+\,\delta E_{\rm str} - r_-\,\delta J_{\rm str}}{r_+^2 - r_-^2} - \pi n_5 r_+\,\delta n_1$$

- Thus for large black holes, winding string emission is very rare.
  The BTZ entropy formula is that of the *little string* that governs **NS5** brane dynamics; removing winding subtracts from the length of the *little string*, and in proportion its entropy

charge emission from black holes

- The tunneling probability is a universal result for **all** black holes:

$$\boxed{\boldsymbol{\Gamma \sim \mathrm{e}^{\Delta S_{\mathrm{BH}}} = \mathrm{e}^{S_f - S_i}}}$$

where S_{BH} is the black hole entropy as a function of the various conserved charges $E, J \propto n_p, \mathrm{Q} \propto n_1$, *etc.*

- The horizon here is not a causal barrier but rather an accounting tool – black holes are black simply because their enormous internal phase space means that an object falling in has a hard time finding its way out. The emission probability is purely a matter of *phase space*

- Think of this as an inclusive decay process, where we sum over all possible final states:

$$\Gamma = \sum_f \left| \langle \psi_f | \psi_i \rangle \right|^2$$

i.e. a sum over $\mathbf{e}^{S_f}$ final states, w/transition matrix elements of order $\boldsymbol{e^{-S_i/2}}$

It’s all phase space

- This a variant of the Eigenstate Thermalization Hypothesis, which says that for typical “simple” observables $\mathcal{O}$ in highly excited states of a chaotic system

$$\langle E_a | \mathcal{O} | E_b \rangle \approx \mathcal{F}_{\mathcal{O}}(\bar{E})\, \delta_{ab} + \underline{e^{-S(\bar{E})/2}}\, \mathcal{G}_{\mathcal{O}}(\bar{E}, \omega)\, \mathcal{R}_{ab}$$

where $\bar{E} = \frac{1}{2}(E_a + E_b)$, $\omega = E_b - E_a$, $\mathcal{F}$ and $\mathcal{G}$ are smooth functions, and $\mathcal{R}$ is a random matrix of unit variance.

- We have computed a transition matrix element of the interaction Hamiltonian between the black hole and its environment, that emits a Hawking quantum; we found a result that can be interpreted as

$$\langle \psi_f | \mathcal{H}_{\rm int} | \psi_i \rangle \approx e^{-S(E_i)/2}\, \mathcal{G}(\bar{E}, \omega)\, \mathcal{R}_{\rm fi}$$

- Note that the adjoint (absorption) process has a matrix element $\approx \boldsymbol{e^{-S_f/2}}$ which cancels against the sum over final states; the absorption is order one (in the WKB analysis, the ingoing trajectory has no tunneling segment).

Summary, part II

- The Hawking emission calculation can be extended to rare decay processes, such as the emission of background branes. The emission probability is purely ***phase space kinematics***, and can be interpreted as the probability of excitations finding their way out of the enormous internal phase space of the black hole. The role of the horizon is to account for this phase space, rather than to act as a causal barrier

- The result is universal, applying *e.g.* to any collection of intersecting branes in toroidally compactified string theory (such as ***Dp***-branes on $\mathbb{T}^p$ including $p=0$)

- For emissions carrying the background charge(s), unitarity implies that the emitted branes are the ones that formed the black hole to begin with, rather than resulting from some vacuum fluctuation at the horizon

- But then the Gauss law and near-horizon locality imply that the branes emerging from the horizon were at the horizon just beforehand, and so the wavefunction of the background branes has support out to the horizon scale – the interior is not the vacuum, and the black hole is a fuzzball