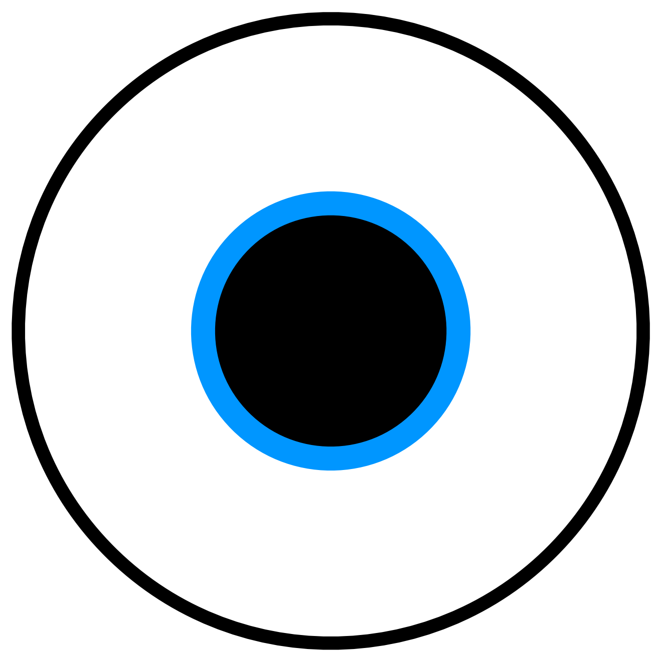
Some candidates of atypical black hole microstates

Kotaro Tamaoka (Nihon U.)

This talk



Microstates of black holes with "end of the world brane"

(Boundary point of view in AdS/CFT with entanglement entropy)

Based on

- Work in progress with

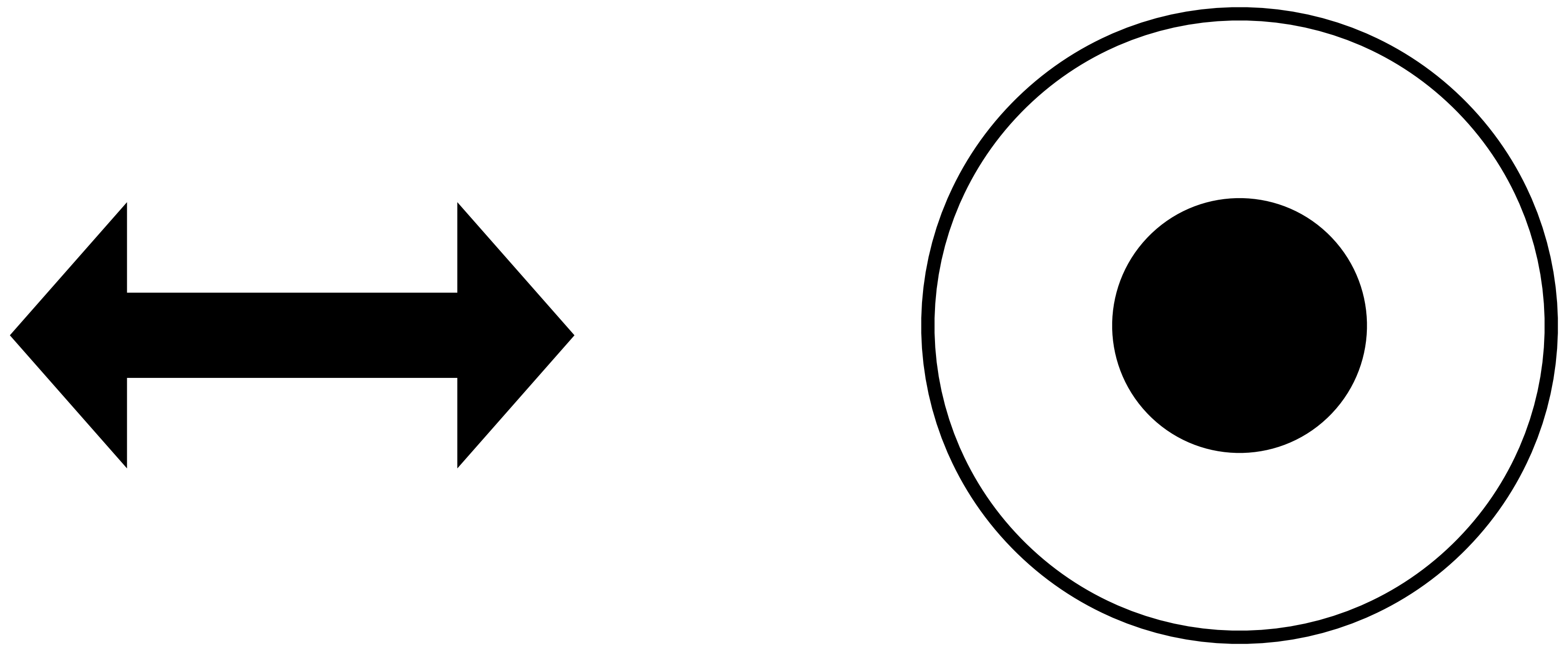
  Yuya Kusuki (Caltech), Yasushi Yoneta (RIKEN), Zixia Wei (RIKEN)

- 1909.06790 with Yuya Kusuki
- See also 2302.03895 by Yoneta-Wei

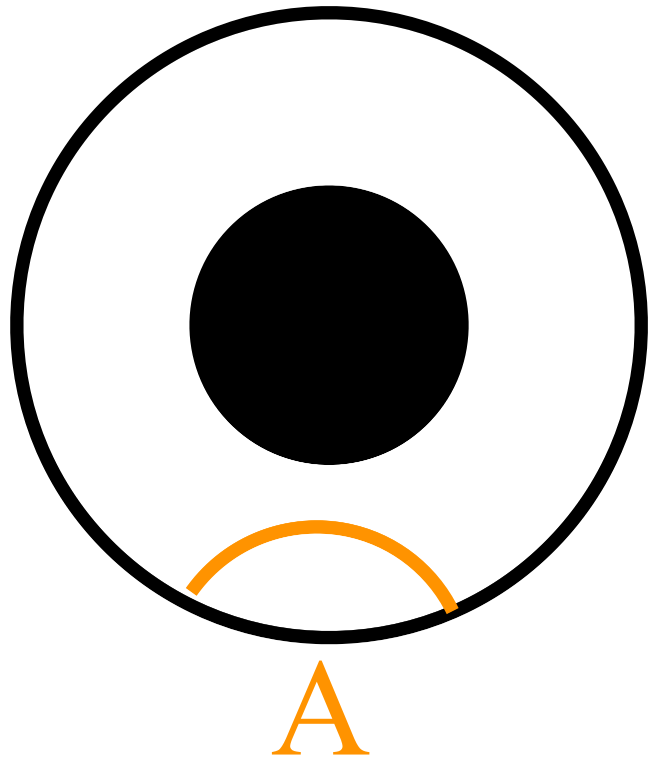
Black holes in AdS/CFT

- Thermal state in CFT with gravity dual (holographic CFT) $\leftrightarrow$ black hole in AdS

$$\rho_\beta = \frac{1}{Z(\beta)} \sum_n e^{-\beta E_n} \left| n \right\rangle \left\langle n \right|$$



- Entropy of the subsystem (entanglement entropy) follows the volume law in CFT



e.g.) static BTZ black holes

$$S(\rho_A) \simeq \frac{\pi c}{3\beta} \ell_A$$

Holographically computed from Ryu-Takayanagi formula

Typical (pure) states

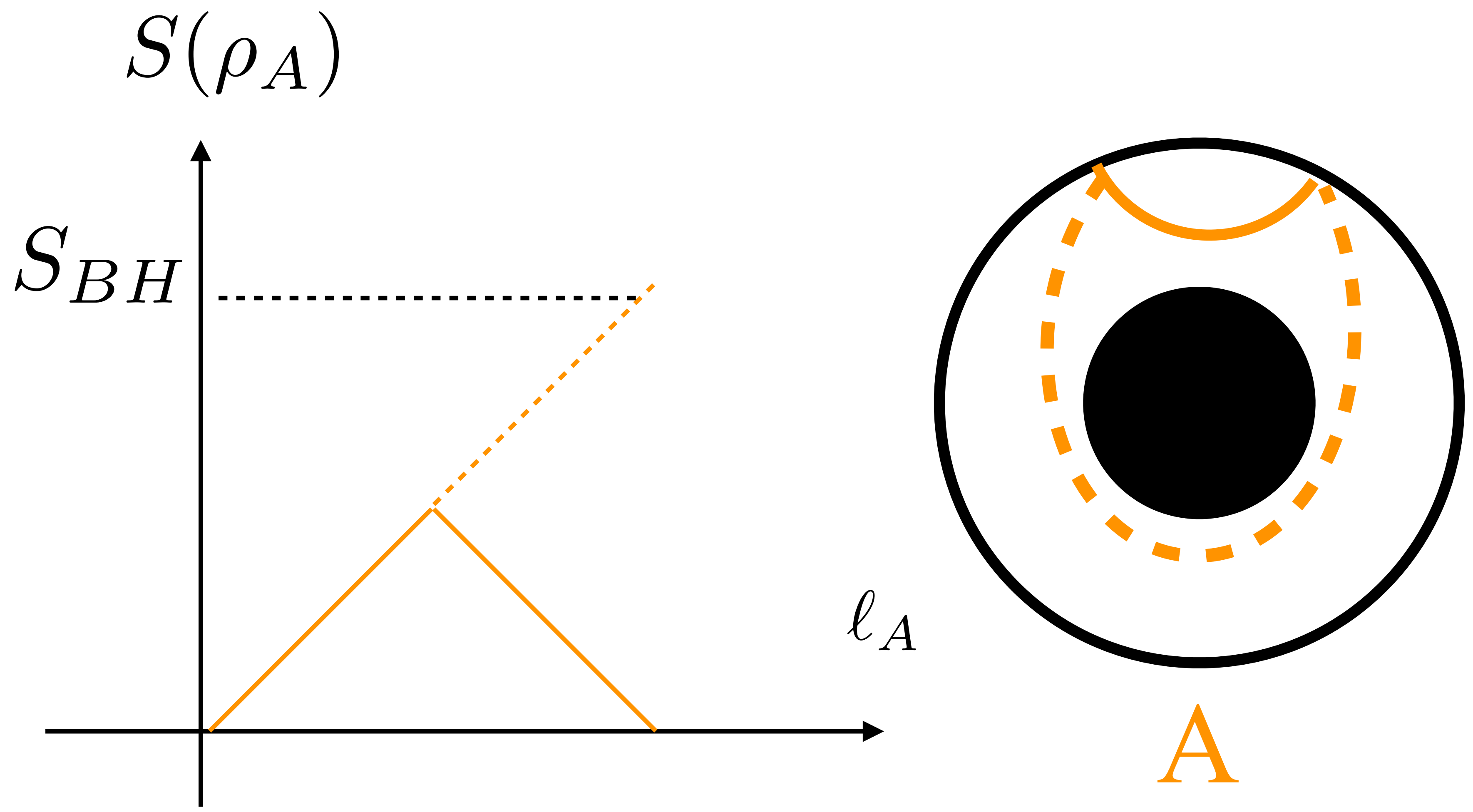
- Let us consider a typical pure state locally looks like the thermal state

$$\rho_A = \mathrm{Tr}_{\bar{A}}(|\psi\rangle\langle\psi|) \simeq \mathrm{Tr}_{\bar{A}}\rho_\beta$$

- Mutuality of entanglement entropy

$$S(\rho_A) = -\mathrm{Tr}\rho_A \log \rho_A$$

suggests deviation from the thermal state

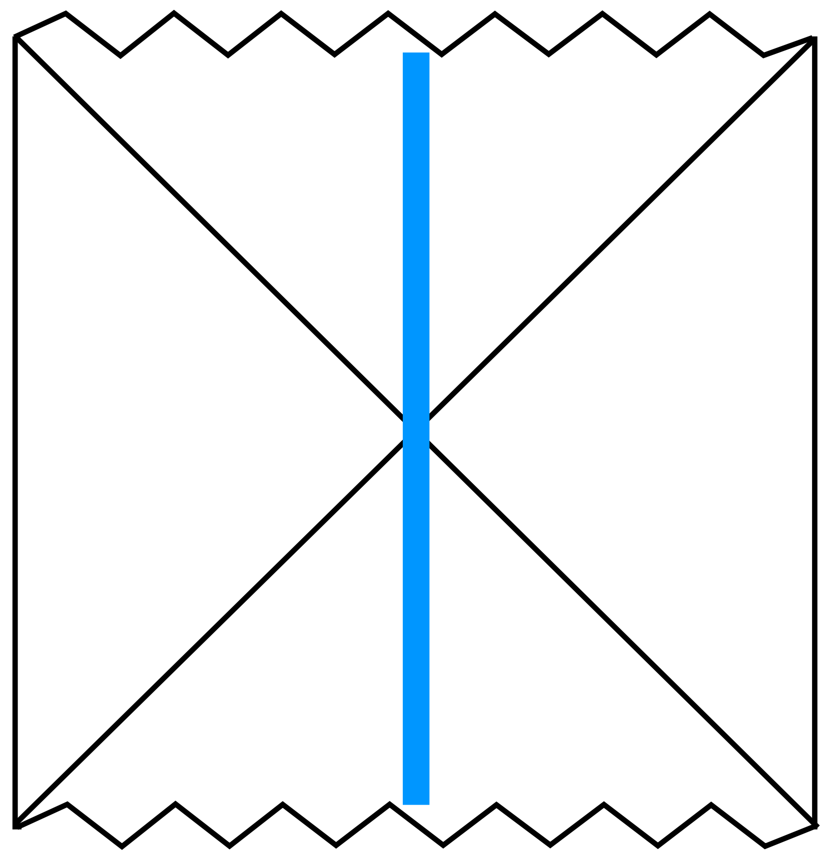


- Still keep the “volume-law” of entanglement

- Enough # to explain BH entropy

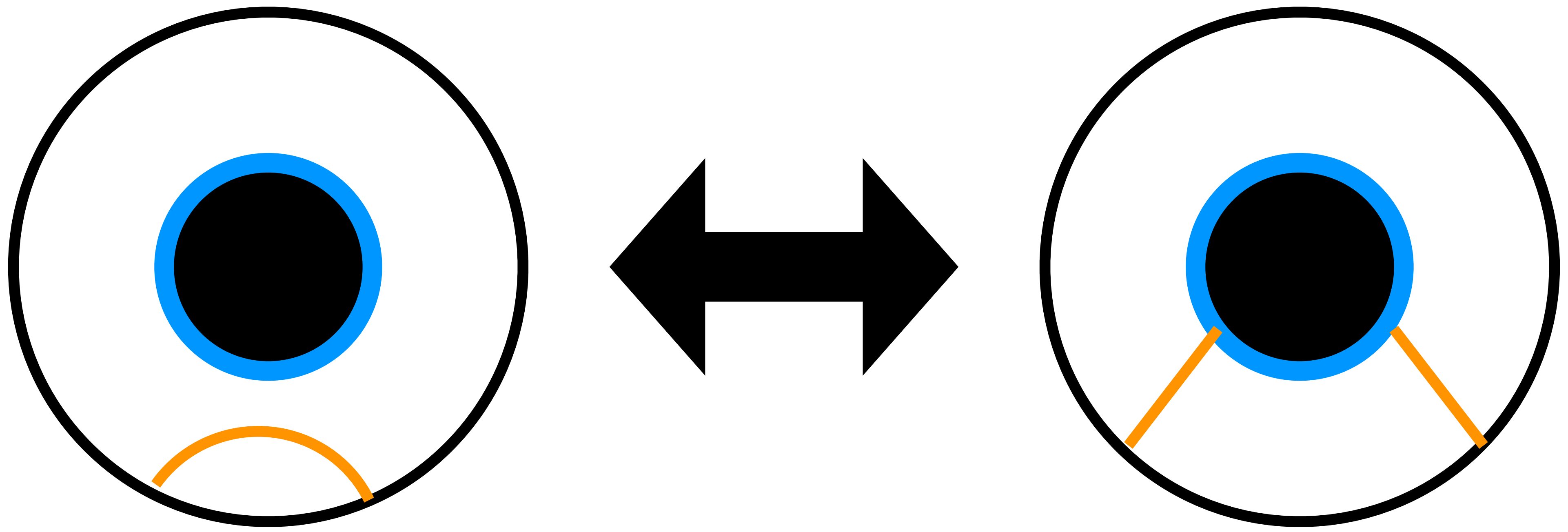
Well-known example in (B)CFT

$$e^{-\frac{\beta}{4}H}|B\rangle$$



Calabrese-Cardy '07

Hartman-Maldacena '13

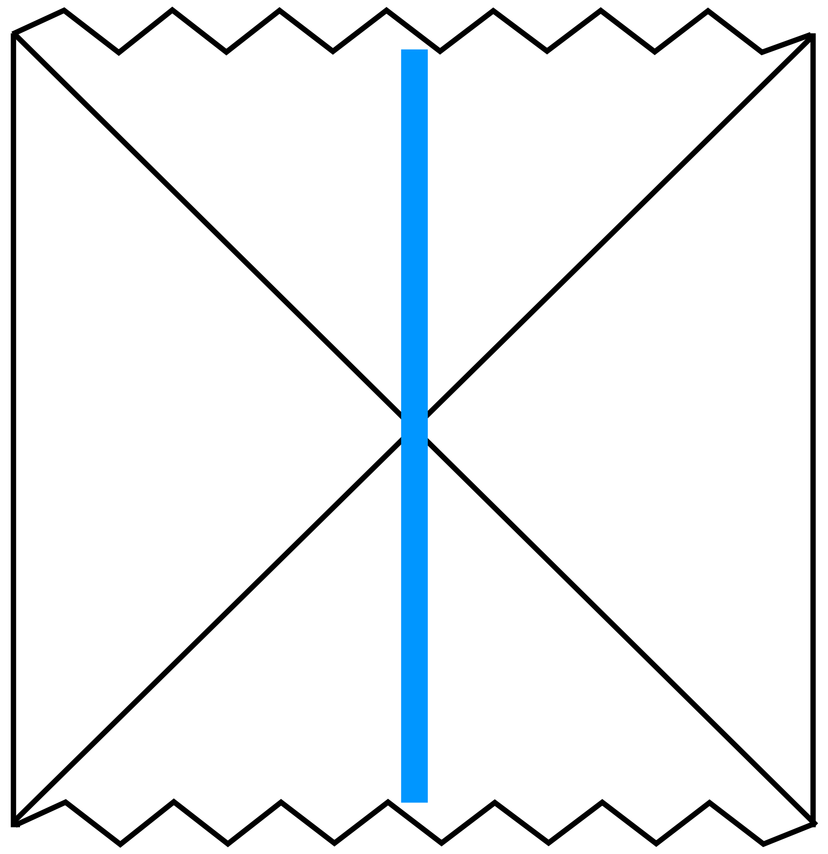


$$S(\rho_A) \simeq \frac{\pi c}{3\beta}\ell_A \qquad S(\rho_A) \simeq \frac{c}{3}\log\beta$$

Locally looks like the thermal state,
but does not obey the volume law

Well-known example in (B)CFT

$$e^{-\frac{\beta}{4}H}|B\rangle$$



Calabrese-Cardy '07

Hartman-Maldacena '13

- Boundary state is an example of the product state

  Miyaji-Ryu-Takayanagi-Wen '14

- Very useful to understand recent unitary Page curve argument

  AdS/BCFT: Takayanagi '11, … Page curve: Almheiri-Engelhardt-Marolf-Maxfield '19,…

- Not enough number to explain full BH entropy

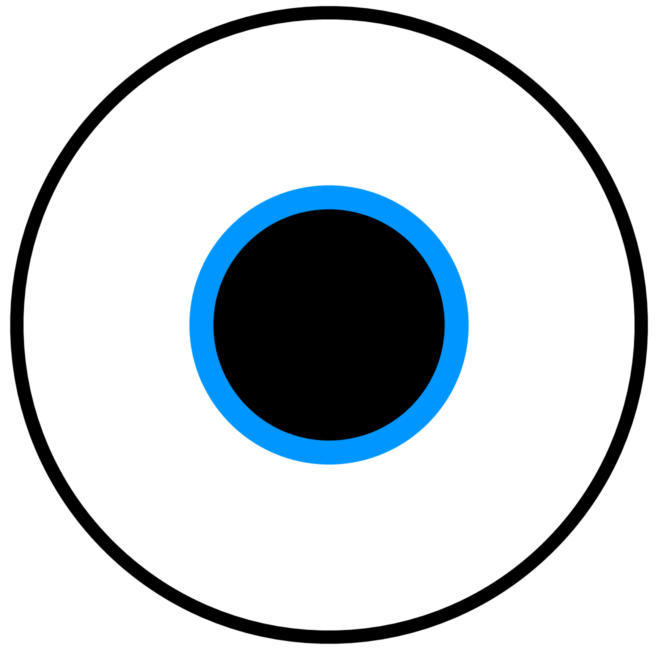
  c.f. Miyaji-Takayanagi -Ugajin '21

- Related to additivity conjecture in QI theory Hayden-Penington '20

  (Called "disentangled states")

Q. Do we have such geometries as many as ~exp[S]?

This talk:



Explicit examples of

black holes microstates with "end of the world brane"

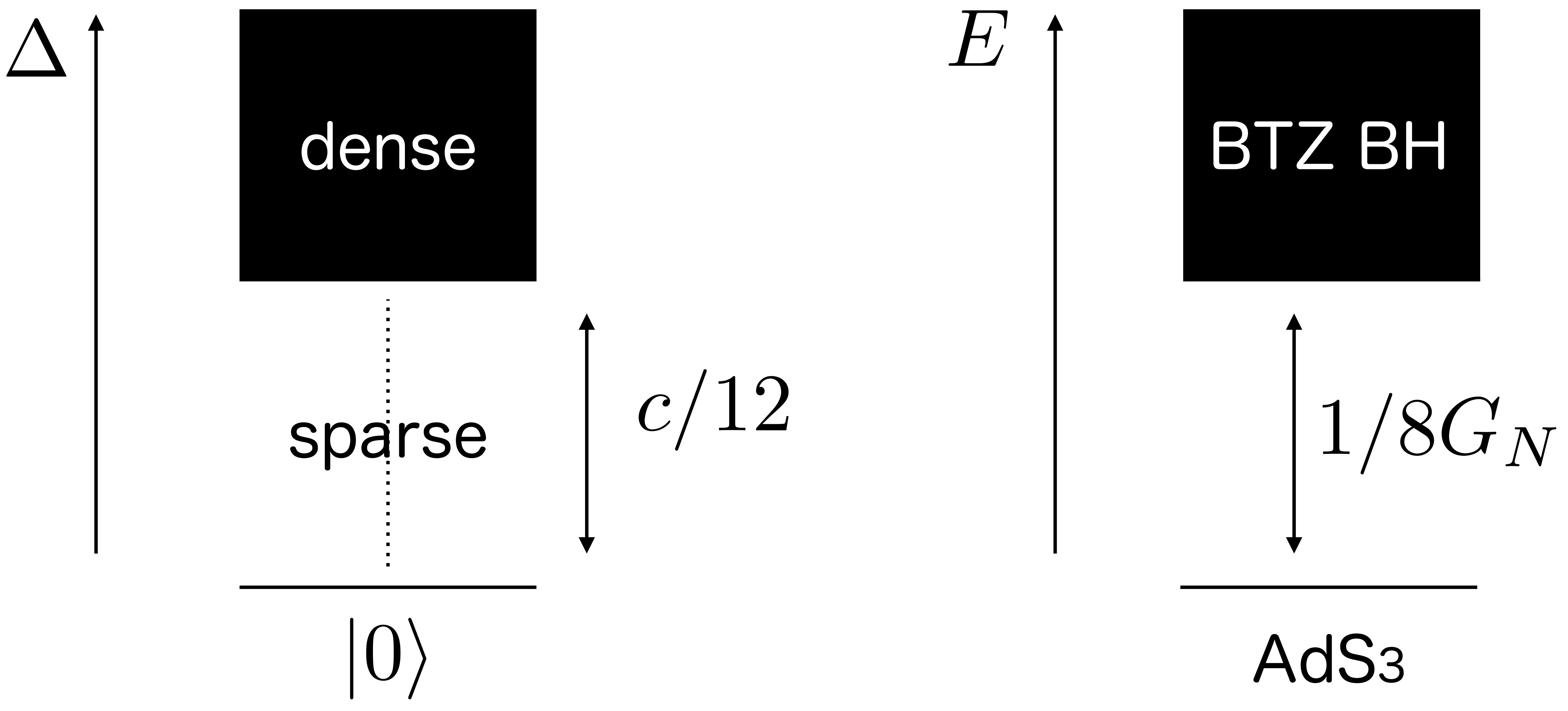
which (approximately) account for the black hole entropy

1. Heavy primary states
2. METTS

Heavy (primary) state in 2D CFT

- 2D holographic CFT
  - large-c limit
  - Sparseness condition

Hartman-Keller-Stoica '14



- Heavy primary state

$$|\mathcal{O}_H\rangle = \mathcal{O}_H(0)\,|0\rangle$$

$$\Delta_H = 2h_H \geq \frac{c}{12}$$

Let us estimate EE and compare it with the bulk results

Computation via Replica trick

Note: the same argument as Asplund-Bernamonti-Galli-Hartman'14 up to conformal block expansion

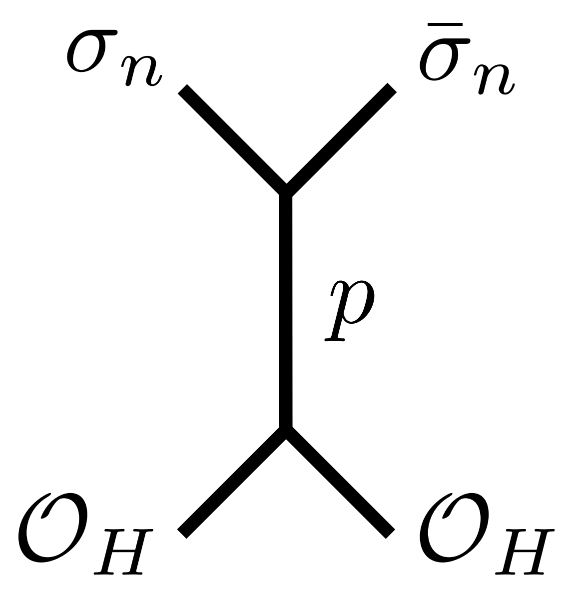
- Compute n-th Renyi entropy (n→1 limit gives entanglement entropy)

$$S^n(\rho_A) = \frac{1}{1-n} \log \mathrm{Tr} \rho_A^n \qquad \text{where} \qquad \rho_A = \mathrm{Tr}_{\bar{A}} \left| \mathcal{O}_H \right\rangle \left\langle \mathcal{O}_H \right| \qquad S(\rho_A) = \lim_{n \to 1} S^n(\rho_A)$$

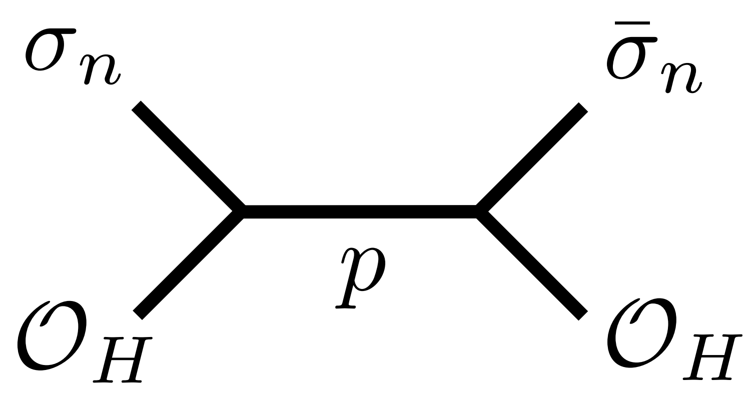
- Replica partition function can be computed from the correlation function of twist operators

$$\mathrm{Tr} \rho_A^n = \langle 0 | \mathcal{O}_H^{\otimes n}(\infty) \sigma_n(z_1) \bar{\sigma}_n(z_2) \mathcal{O}_H^{\otimes n}(0) | 0 \rangle_{\mathrm{CFT}^{\otimes n}/Z_n} \qquad \Delta_n = 2h_n = \frac{c}{12}\left(n - \frac{1}{n}\right)$$

$$= \sum_p (\text{OPE coefficients})^2$$



$$= \sum_p (\text{OPE coefficients})^2$$

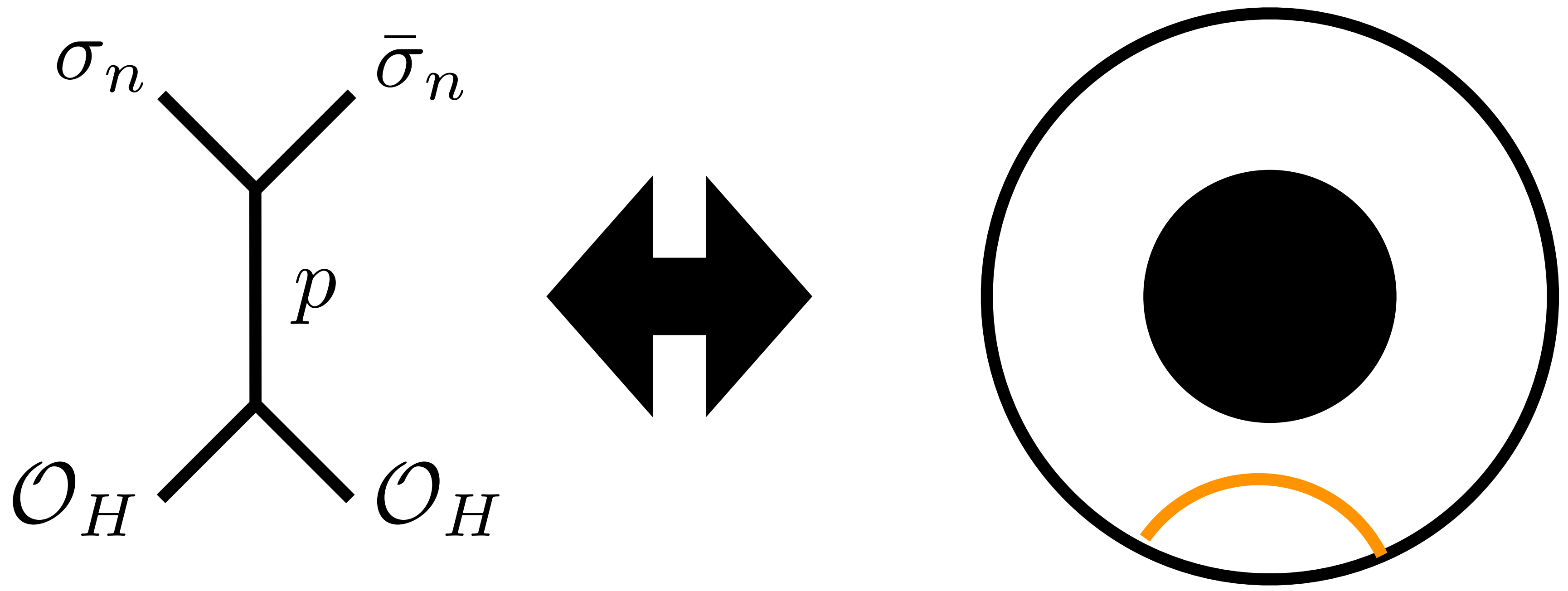


Conformal block approximation

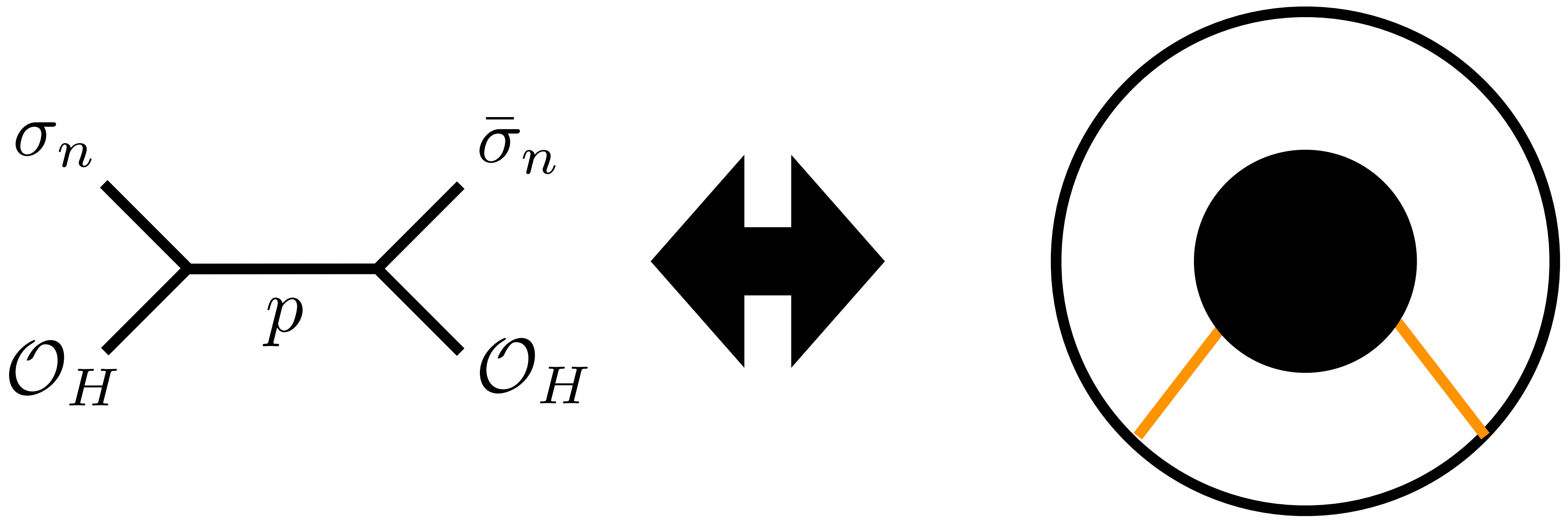
- Dominant contribution @ large-c limit

  = **the lowest conformal dimension for appropriate channel**

- Conformal block @ large-c limit is well-studied in literature



Asplund-Bernamonti-Galli-Hartman'14



Banerjee-Datta-Sinha '16

Note: OPE coefficients for s-channel can be estimated from modular bootstrap (no suppression @ large-c)

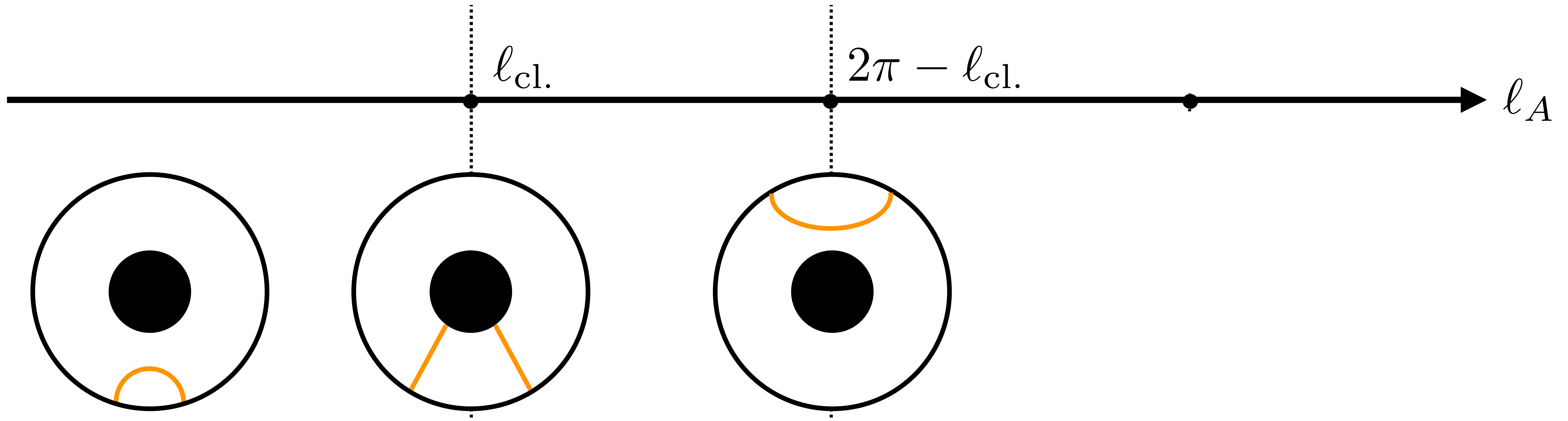
Intermediate phase requires “brane”

due to homology constraints of Ryu-Takayanagi formula

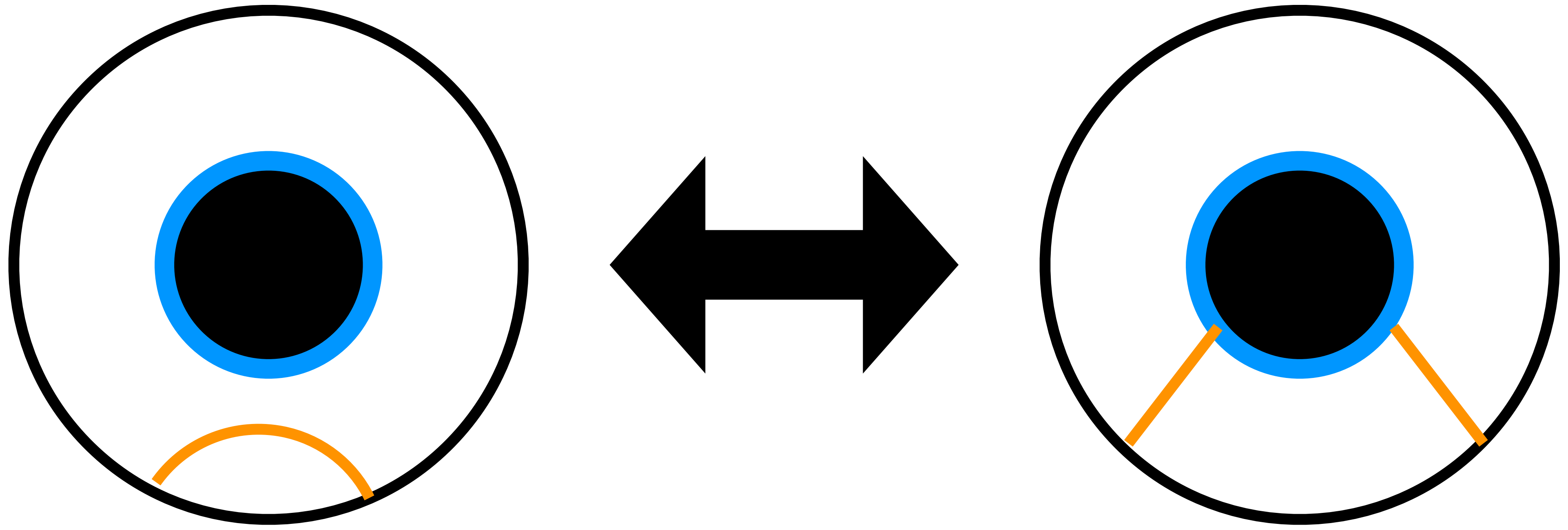
Kusuki-KT

$$S(\rho_A) = \begin{cases} \frac{c}{3}\log\frac{\beta_H}{\pi\epsilon}\sinh\left(\frac{\pi\ell_A}{\beta_H}\right) & (0<\ell_A<\ell_{\rm cl.}), \\ \frac{c}{3}\log\frac{\beta_H}{\pi\epsilon} & (\ell_{\rm cl.}<\ell_A<2\pi-\ell_{\rm cl.}), \\ \frac{c}{3}\log\frac{\beta_H}{\pi\epsilon}\sinh\left(\frac{\pi(2\pi-\ell_A)}{\beta_H}\right) & (2\pi-\ell_{\rm cl.}<\ell_A<2\pi), \end{cases}$$

$$\beta_H = \frac{2\pi}{\sqrt{\frac{24}{c}h_H - 1}} \qquad \sinh\left(\frac{\pi\ell_{\rm cl.}}{\beta_H}\right) = 1 \ \left(\Leftrightarrow \ell_{\rm cl.} = \frac{\beta_H}{\pi}\log(1+\sqrt{2})\right)$$



Summary so far



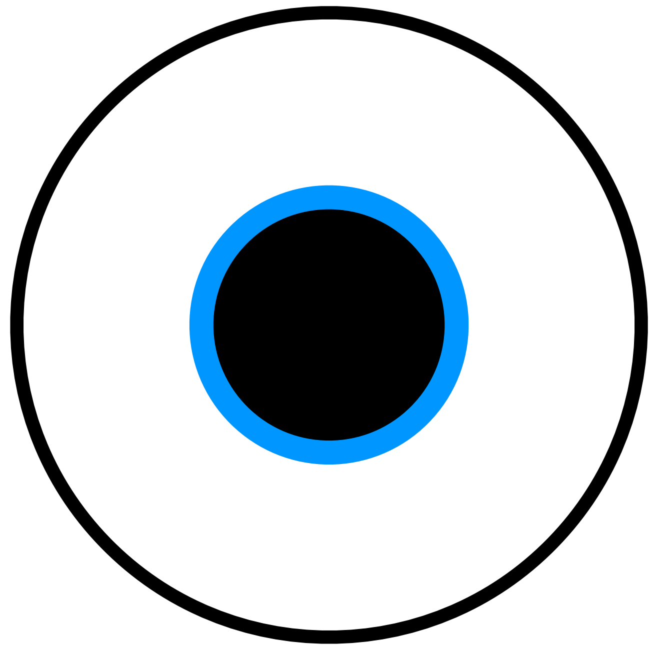
- Assuming we can apply Ryu-Takayanagi formula

  → end of the world brane on the horizon

- Enough # of states to explain BH entropy @ large-c

$$\rho(h) \sim \exp\left[2\pi\sqrt{\frac{c-1}{6}\left(h-\frac{c-1}{24}\right)}\right]$$

This talk:



Explicit examples of

black holes microstates with "end of the world brane"

which (approximately) account for the black hole entropy

1. Heavy primary states
2. METTS

Minimally Entangled Typical Thermal State

White

Thermal state can be decomposed into sum of METTS:

$$\rho = \frac{1}{Z(\beta)} e^{-\beta H} = \frac{1}{Z(\beta)} e^{-\frac{\beta}{2}H} \hat{1} e^{-\frac{\beta}{2}H}$$

$$\hat{1} = \sum_i |P_i\rangle\langle P_i|$$

$$= Z(\beta)^{-1} \sum_i e^{-\frac{\beta}{2}H} |P_i\rangle \langle P_i| e^{-\frac{\beta}{2}H}$$

P_i : product state in a particular basis

e.g.) $|P\rangle = |\uparrow\downarrow\uparrow\downarrow \cdots\rangle$

$$\rho = \sum_i p(\tau, P_i) |\mu(\tau, P_i)\rangle \langle \mu(\tau, P_i)| \quad \text{where} \quad |\mu(\tau, P_i)\rangle = \mathcal{N} e^{-\tau H} |P_i\rangle$$

Minimally Entangled Typical Thermal State

$$|\mu(\tau, P_i)\rangle = \mathcal{N} e^{-\tau H} |P_i\rangle$$

P_i : product state in a particular basis

e.g.) $|P\rangle = |\uparrow\downarrow\uparrow\downarrow \cdots\rangle$

- A natural generalization of conformal boundary state
- Efficient decomposition for numerical simulation,

but not explored in CFT (critical systems) so far

(Not scale with $\ell_A \rightarrow$ efficient even for critical system!)

Averaged Entanglement Entropy

Let us estimate averaged value of EE for METTS:

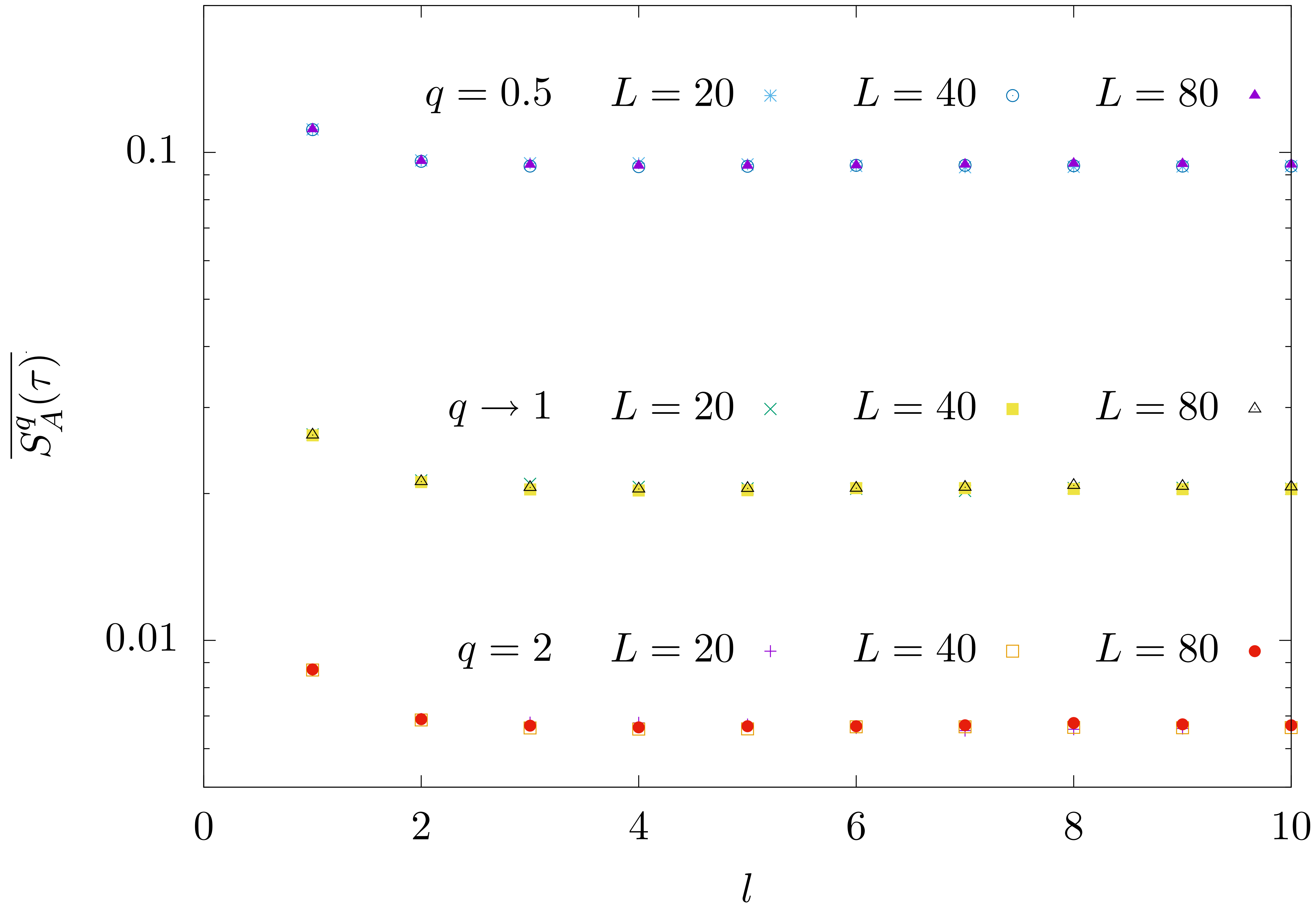
$$\overline{S_A^q} \equiv \sum_i p(\tau, P_i) S_A^q(|\mu(\tau, P_i)\rangle)$$

where $$S_A^q(|\mu(\tau, P_i)\rangle) \equiv \frac{1}{1-q} \log \mathrm{Tr}\left[\left(\rho_A^{|\mu(\tau,P_i)\rangle}\right)^q\right]$$

$$\rho_{\beta=2\tau} = \sum_i p(\tau, P_i) \left|\mu(\tau, P_i)\right\rangle \left\langle \mu(\tau, P_i)\right|$$

$$|\mu(\tau, P_i)\rangle = \mathcal{N} e^{-\tau H} |P_i\rangle$$

Averaged EE for half line

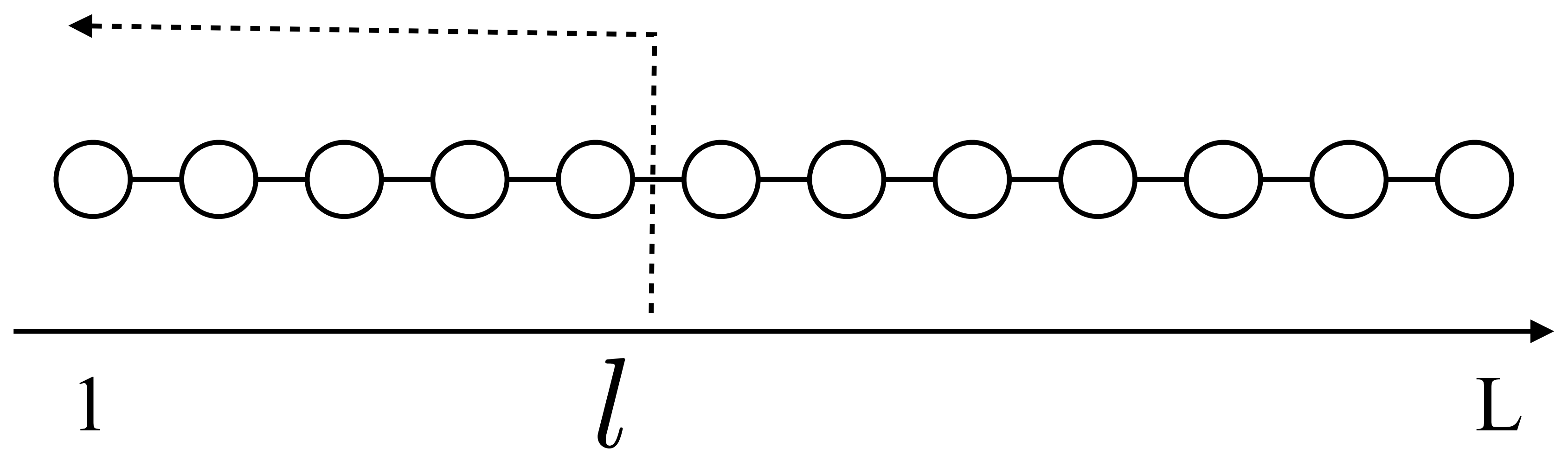


$\mu = z, \tau = 2 \qquad \gamma = 1/2$ (critical)

- Transverse Ising model

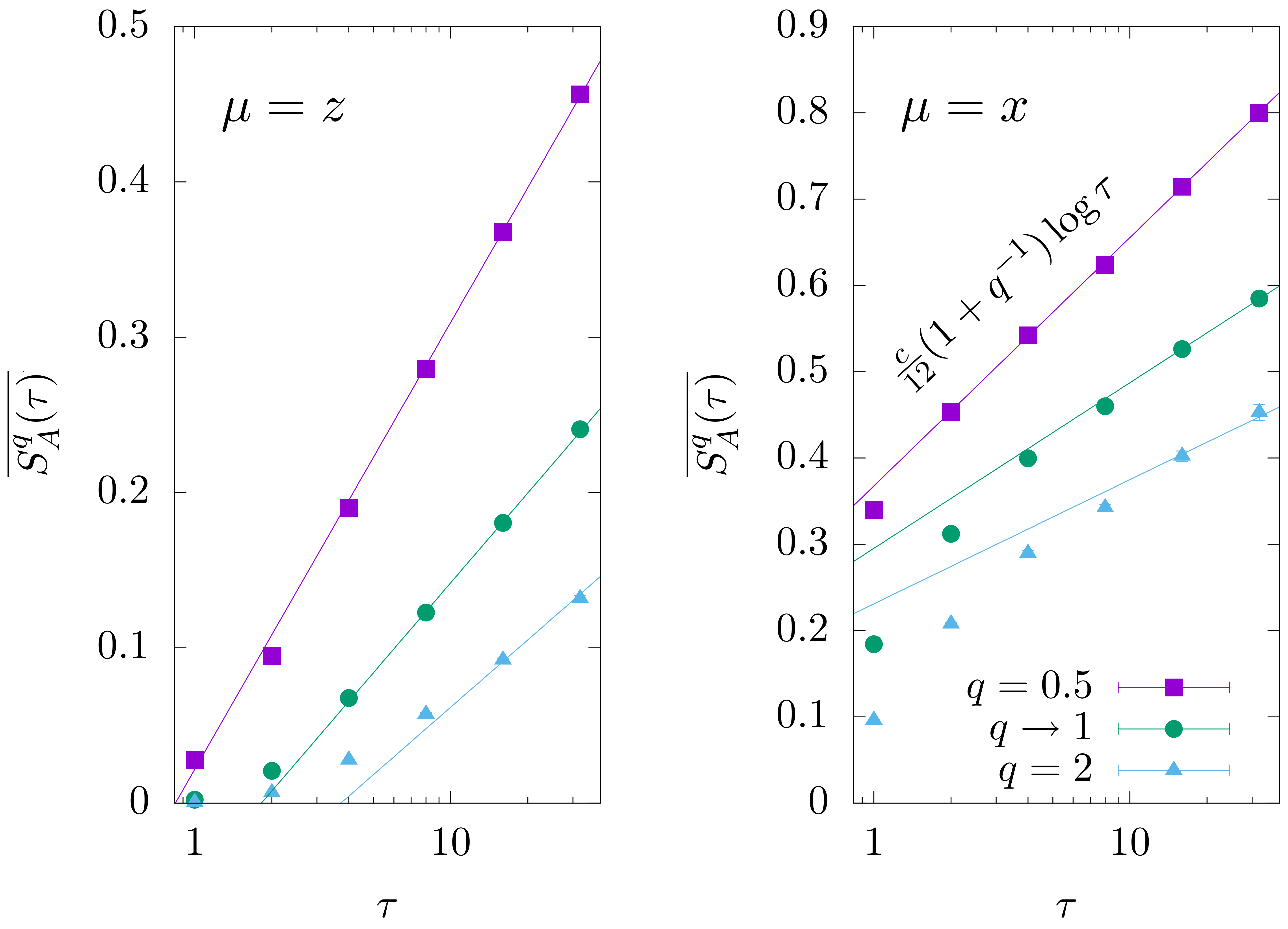
$$H = -\sum_{i=1}^{L-1} \hat{S}_i^z \hat{S}_{i+1}^z - \gamma \sum_{i=1}^{L} \hat{S}_i^x$$

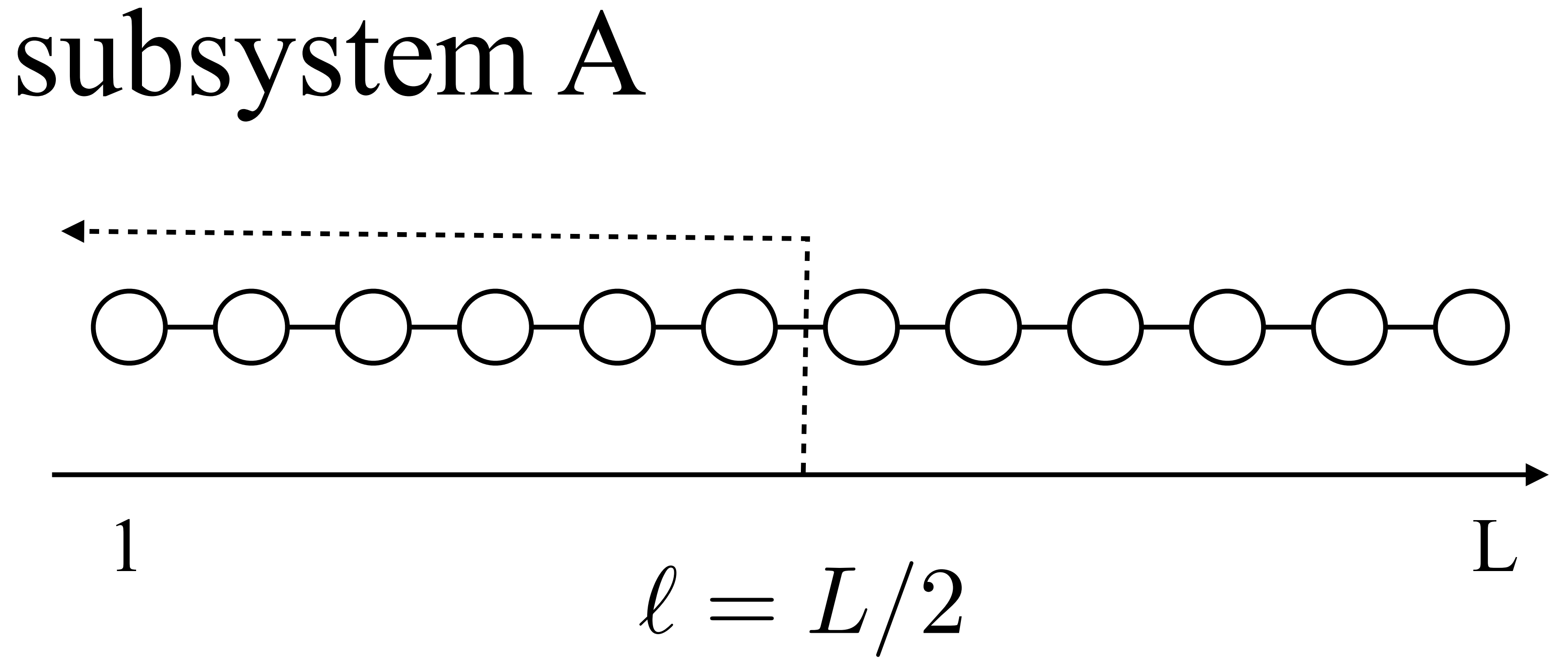
subsystem A



- EE does not obey volume law

Inverse temperature τ-dependence agrees with CFT results





Time evolution (early time)

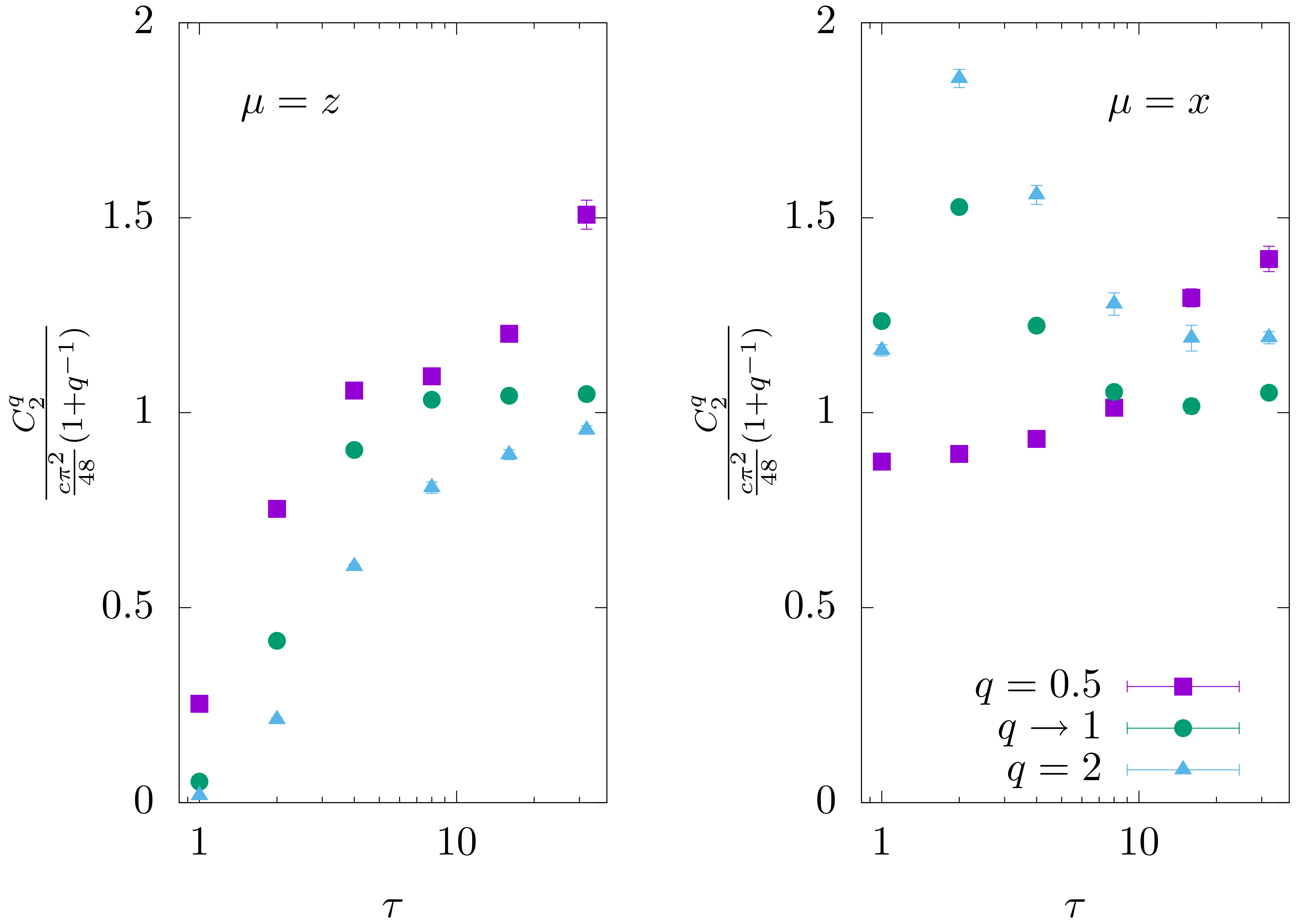
- Consider real time (quench) dynamics via τ → τ+it

$$\overline{S_A^q} = \frac{c}{12}(1+q^{-1})\log\tau + C_2^q\left(\frac{t}{\tau}\right)^2 + \cdots$$

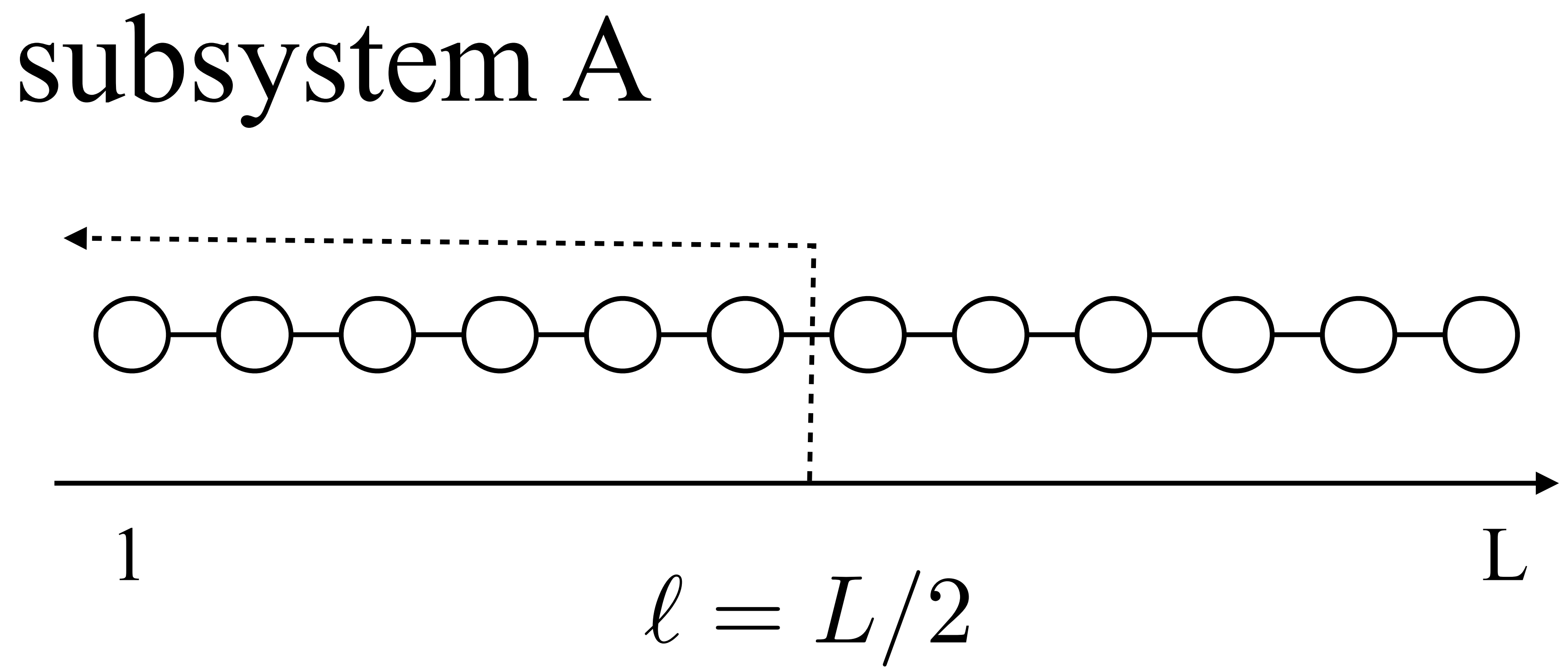
- Compare with the Hartman-Maldacena boundary state

$$S_A^q = \frac{c}{12}(1+q^{-1})\log\tau + \frac{c\pi^2}{48}(1+q^{-1})\left(\frac{t}{\tau}\right)^2 + \cdots$$

Results at large τ agree with HM state



$L = 640$



Averaged METTS ~ HM state?

- Entanglement entropy suggests that the averaged behavior of METTS (at large scale) can be well-approximated by Hartman-Maldacena boundary states

$$|\mu(\tau, P_i)\rangle = \mathcal{N} e^{-\tau H} |P_i\rangle \quad \sim \quad \mathcal{N} e^{-\tau H} |B\rangle$$

→ Near-horizon structure will be similar to the HM boundary state

- However, local correlation functions "feel" different temperature (next slide)

Correlation function

- METTS

$$\sum_i p_i \langle \mu(\tau, P_i)|\mathcal{O}_1 \cdots |\mu(\tau, P_i)\rangle = Z(2\tau)^{-1}\mathrm{Tr}(e^{-2\tau H}\mathcal{O}_1 \cdots)$$

- Conformal boundary state

$e^{-\tau H}|B\rangle$ locally thermal state with $\beta = 4\tau$

(for sufficiently close operators)

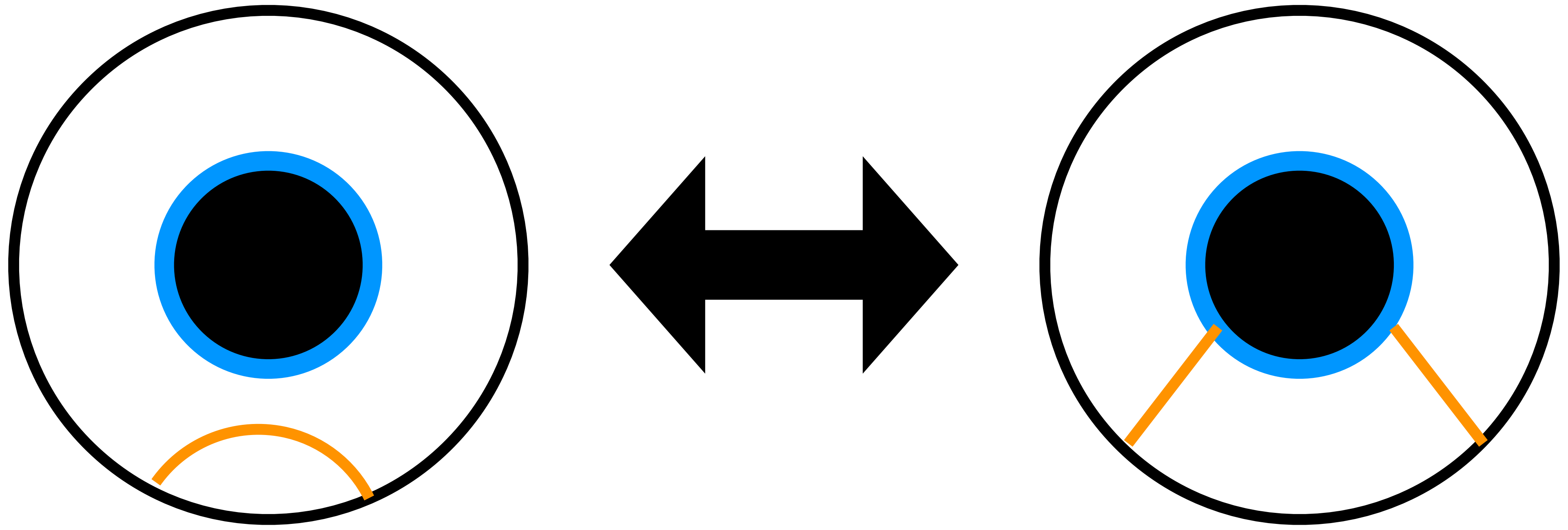
- UV structure should be different (METTS has no conformal symmetry in general)

Summary

- We studied heavy primary states and METTS in 2d CFT

  → Atypical in the sense that they do not follow volume-law of entanglement

- Gravity dual maybe identified with black hole with the end of the world brane



$$S(\rho_A) \simeq \frac{\pi c}{3\beta}\ell_A \qquad\qquad S(\rho_A) \simeq \frac{c}{3}\log\beta$$

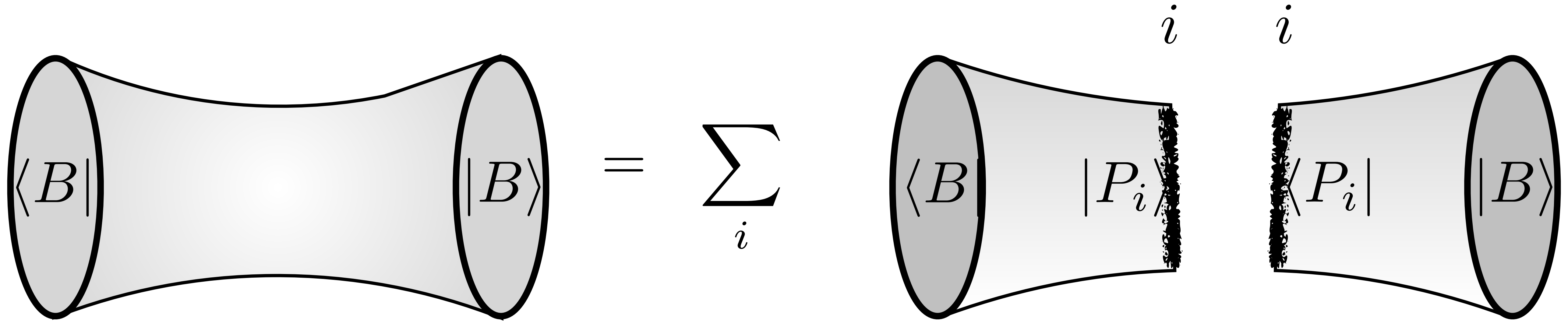
Discussion

- Each METTS can be identified with a unique ground state of local Hamiltonian

c.f. Perez-Garcia—Verstraete — Wolf — Cirac '06

→ One might interpret METTS decomposition
as "average over theories" instead of "average over states"

Analogy with half-wormholes?



c.f. García-García — Godet '21 (2d CFT on JT gravity, METTS average corresponds to B.C. of matter fields)

Let us thank the organizers !

Organizers

Local organizers

Norihiro Iizuka (Osaka University), Masaki Shigemori (Nagoya University), and Tadashi Takayanagi (YITP, Kyoto University)

International advisors

Emil Martinec (Chicago U) and Nick Warner (CEA Saclay & USC)

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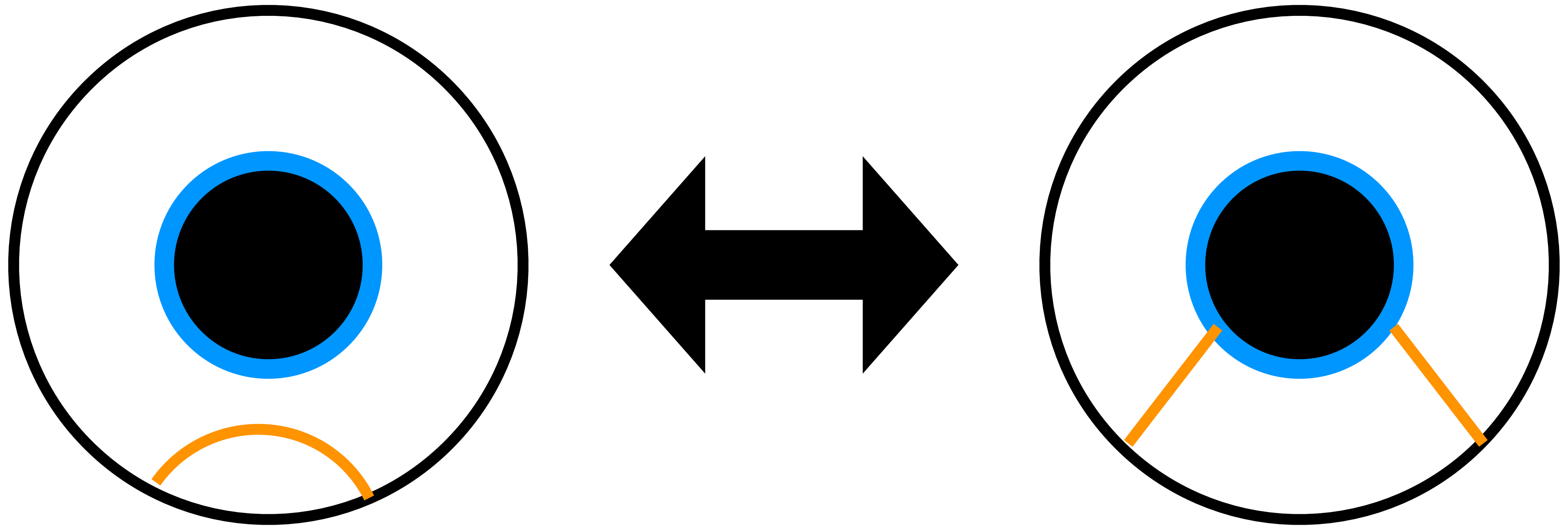

$S(\rho_A) \simeq \frac{\pi c}{3\beta}\ell_A$

$S(\rho_A) \simeq \frac{c}{3}\log\beta$

Thanks!