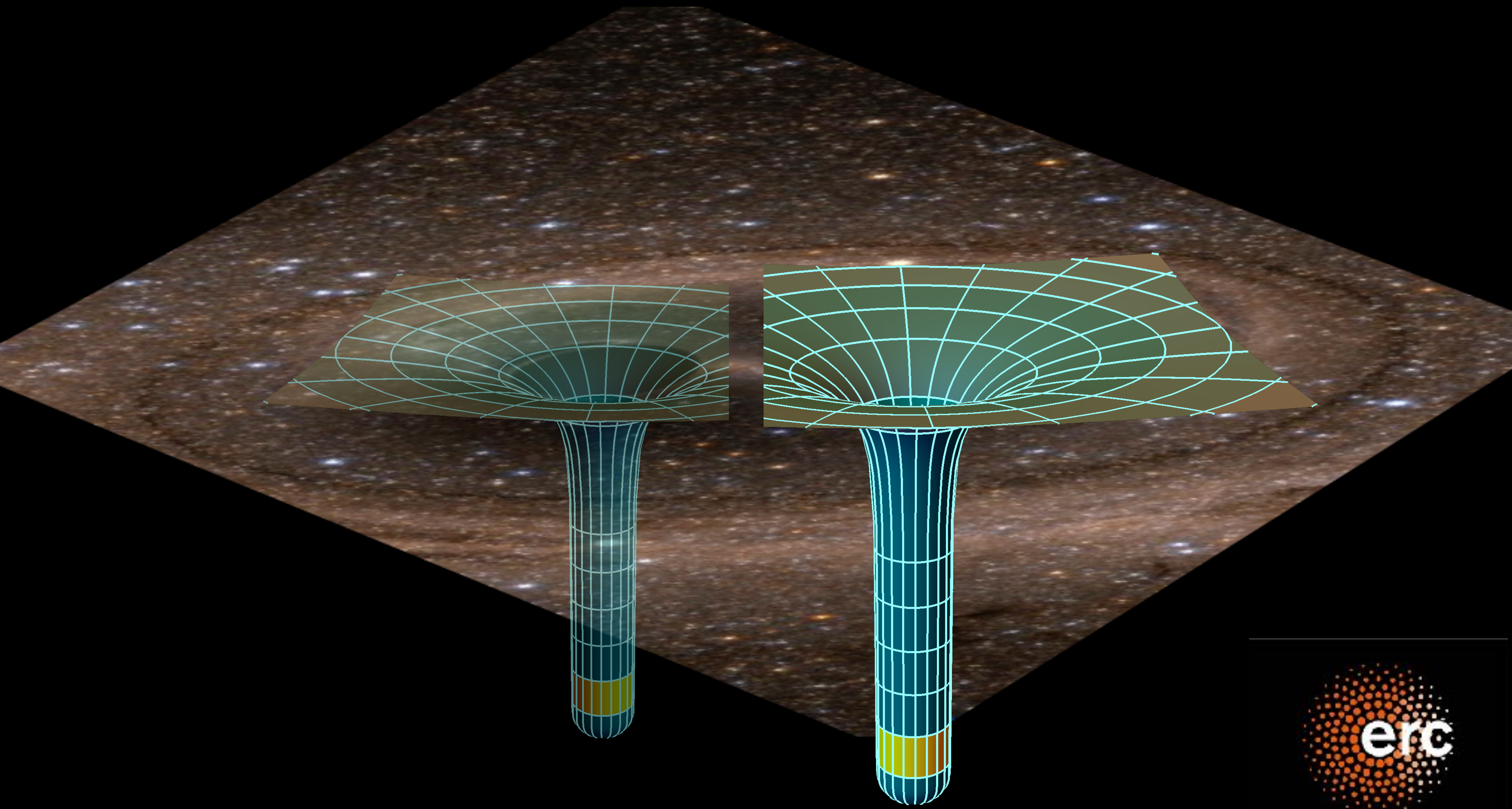
Black-Hole Microstructure in String Theory



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Nick Warner, *April 3, 2023.*

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LIGO/Caltech

An Overview of Microstate Geometries

Outline

- ★ Motivation: The information Problem
- ★ Core ideas for Fuzzballs and Microstate Geometries
- ★ Important supersymmetric examples: The Story of Superstrata
- ★ The holographic dictionary of superstrata
- ★ Replicating Black-Hole-Like Physics in Microstate Geometries
- ★ Non-supersymmetric Microstate Geometries: Microstrata
- ★ Brane fractionation in supergravity
- ★ Final comments

Bena, Martinec, Mathur and Warner,
2203.04981 Snowmass White Paper: Micro- and Macro-Structure of Black Holes
2204.13113 Fuzzballs and Microstate Geometries: Black-Hole Structure in String Theory

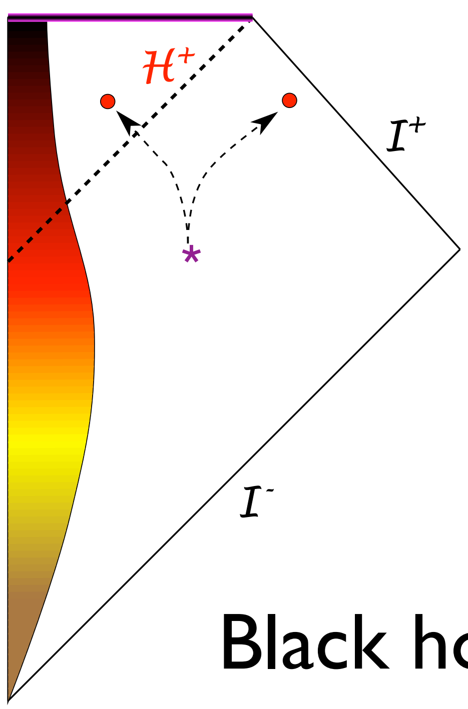
The Black-Hole Information Paradox

Bekenstein-Hawking entropy:

$$S = \frac{k_B c^3}{4\, G\, \hbar} A = \frac{1}{4} \frac{A}{\ell_P^2}$$ *~ k Log(Number of microstates of black hole)*

Number of microstates of Sgr A black hole* **$\sim e^{10^{90}}$**

Hawking radiation



Black holes polarize the vacuum

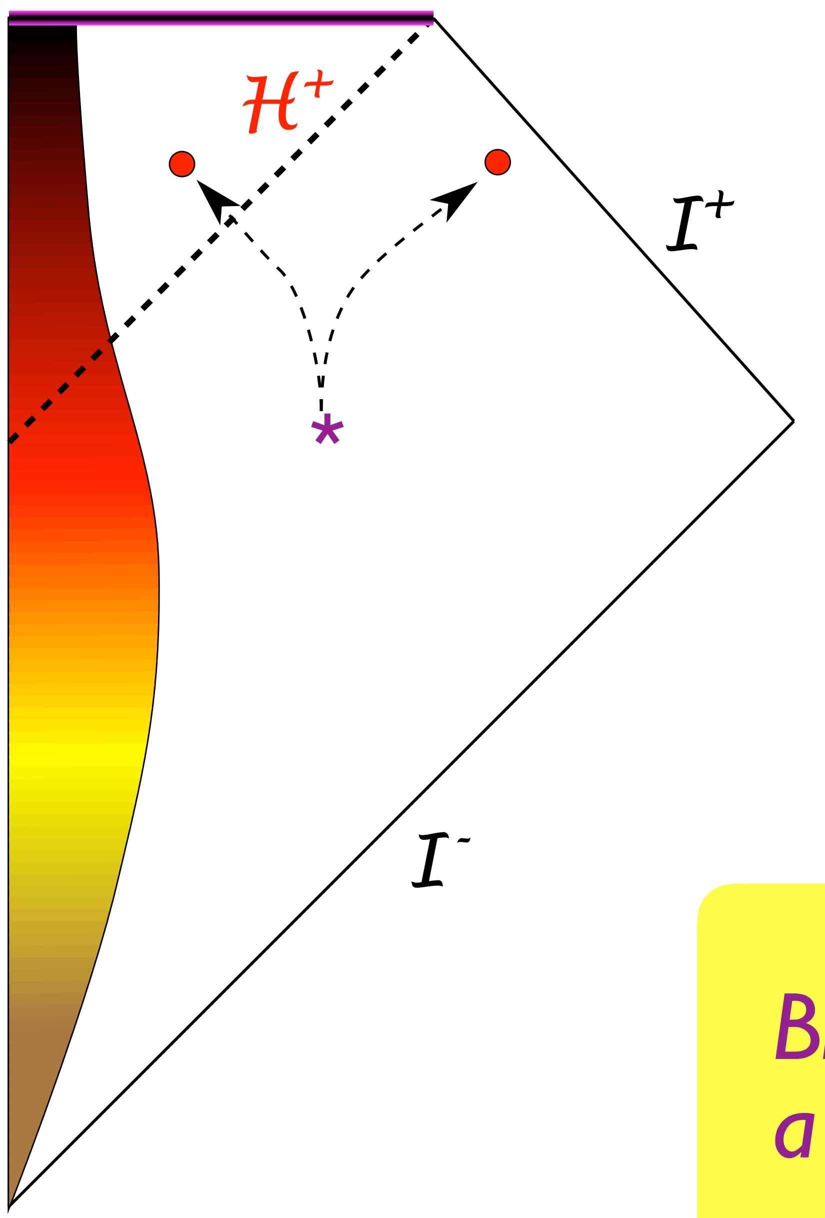
→ Thermal "*Hawking*" radiation at infinity

$$T = \frac{\kappa}{2\,\pi} = \frac{\hbar\, c^3}{8\pi\, G\, k_B\, M}$$

Black holes evaporate into Hawking radiation over vast periods of time

Black-Hole Uniqueness

⇒ *Hawking Radiation* is *almost* featureless: It can encode only the *Bulk State Functions*: mass, angular momentum and charge of the black hole



Black holes, no matter how they form, evaporate into the same (largely featureless) cloud of Hawking Radiation:

$\Rightarrow$ *Impossible to reconstruct the initial state*

Black-hole formation and evaporation results in a vast violation of unitary in quantum mechanics

The Small-Corrections Theorem

Mathur, arXiv:0909.1038

An old conceit: The problem can be fixed through very slow leakage ...

Hawking evaporation is extremely slow:

$$t_{evap} = \frac{5120\,\pi\,G^2\,M_\odot^3}{\hbar\,c^4} \approx 6.6\times 10^{74} s \approx 2.1\times 10^{67} years$$

(for a one solar mass black hole)

Information can leak out very slowly via tiny quantum gravity/string (*(Riemann)^n*) corrections to radiation.

Mathur (2009): ***No!***

Strong sub-additivity of quantum information:

There must be *O(1)* changes to physics at the horizon scale.

One is left with three options as to where the *O(1)* changes must be made:

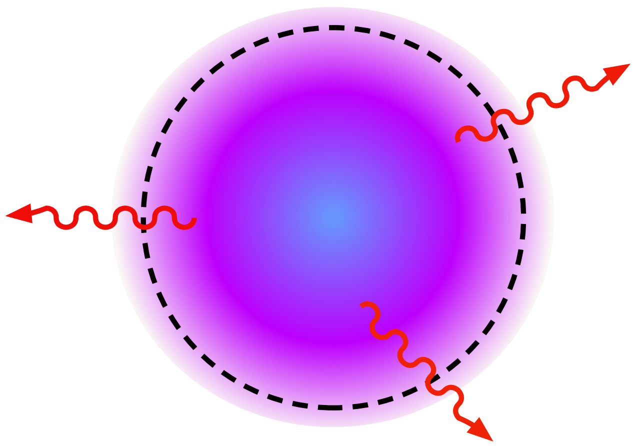
- A black-hole cannot have a smooth geometric horizon as in GR
- Effective field theory must fail at the horizon scale
- There must be vast non-locality of physics on vast scales of time and space

Fuzzballs and Microstate Geometries

The most conservative option …

Replace the black hole of GR by a ***horizonless*** *object that looks like a black hole at large scales, but its structure can be observed and measured by distant observers …*

- *Hawking radiation no more mysterious than the radiation from a compact star or a piece of coal*



- *Challenge: Find new states of matter that can support horizon-scale microstructure and avoid collapse behind a horizon …*
- *Replicate the macroscopic behaviors of the black hole of General Relativity*
- *The greatest challenge: Encode the vast numbers of microstates that went into forming the black hole …* $\left(e^{10^{90}}\right)$

This is impossible in GR coupled to ordinary matter in 3+1 dimensions..

… but all of this is achievable in string theory/higher-dimensional supergravity

⇒ Fuzzballs and Microstate Geometries

Fuzzballs and Microstate Geometries

Philosophy: Broad conceptual ideas

Bena, Martinec, Mathur and Warner,
2203.04981 Snowmass White Paper: Micro- and Macro-Structure of Black Holes
2204.13113 Fuzzballs and Microstate Geometries: Black-Hole Structure in String Theory

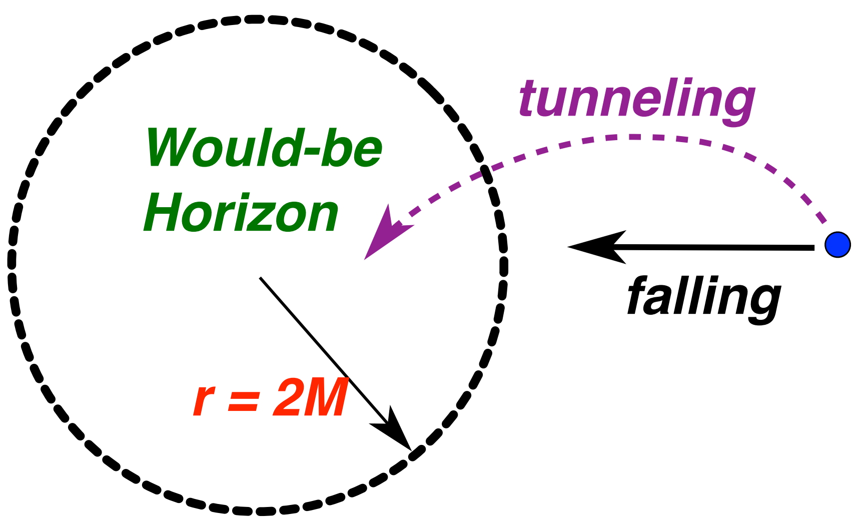
The Invisible Quantum Elephant of Black-Hole Physics

Curvature at horizon: $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}|_{horizon} = \frac{3}{16}\frac{G^2}{M^4} \Rightarrow$ ***Large black hole is classical at horizon scale***

However, because of the extreme density of states, an apparently classical black hole actually *behaves as a quantum object*

Consider a particle falling into a black hole ...

Mathur: 0805.3716; 0905.4483 Mathur and Turton: 1306.5488



Amplitude to tunnel directly into a black hole from nearby ~ $e^{-\alpha M^2/m_P^2}$

α ~ *O(1)*

Number of states inside black hole ~ $e^{+16\pi M^2/m_P^2}$

Fermi Golden Rule: $\mathcal{T}_{i\to f} = \frac{2\pi}{\hbar} < \psi_f | \mathcal{H}_{int} | \psi_i > |^2 \rho$ ← ***density of states***

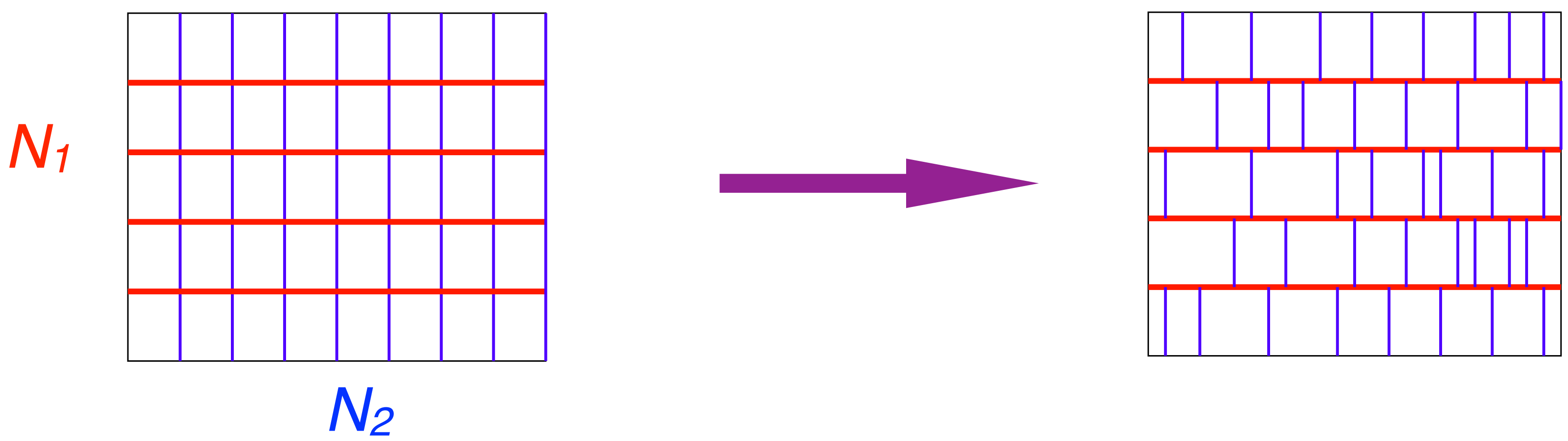
Probability of tunneling during infall time ~ *O(1)!*

Black holes are intrinsically quantum objects whose formation comes about via a quantum (tunneling) phase transition!

Another Variant: Brane Fractionation

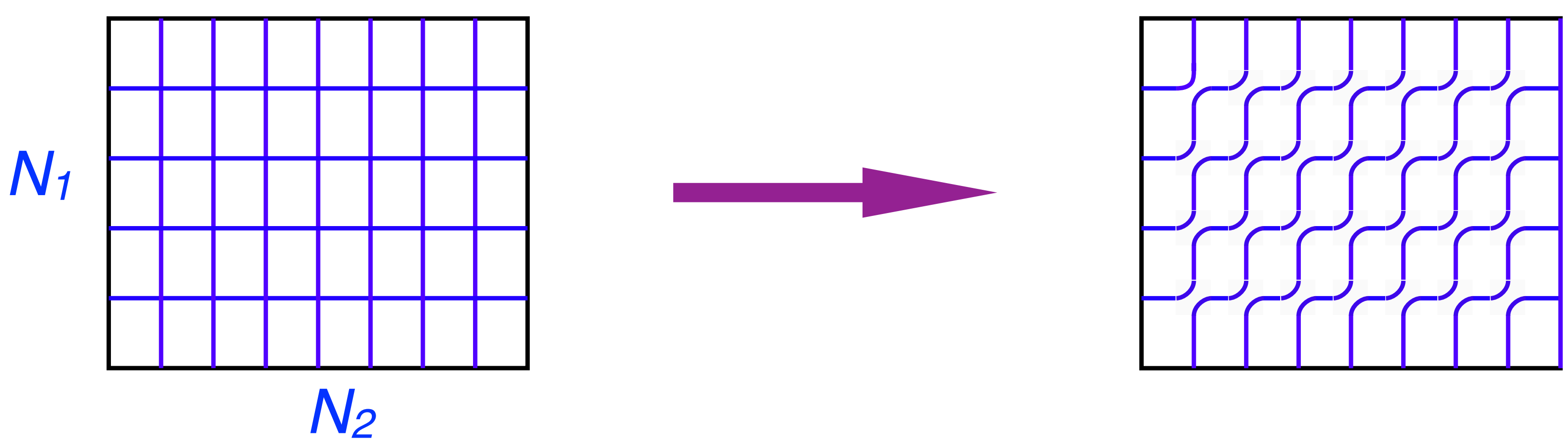
Naively, the scale of quantum gravity effects lead to wave functions of width ℓ_{Planck} or ℓ_{String} ....

However, multiple D-branes wrapping compact manifolds can fractionate:



N_1 N_2 pieces

Or, if they are the same species, they can fractionate into very long branes



One brane of length N_1 N_2 but still with self-intersection N_1 N_2

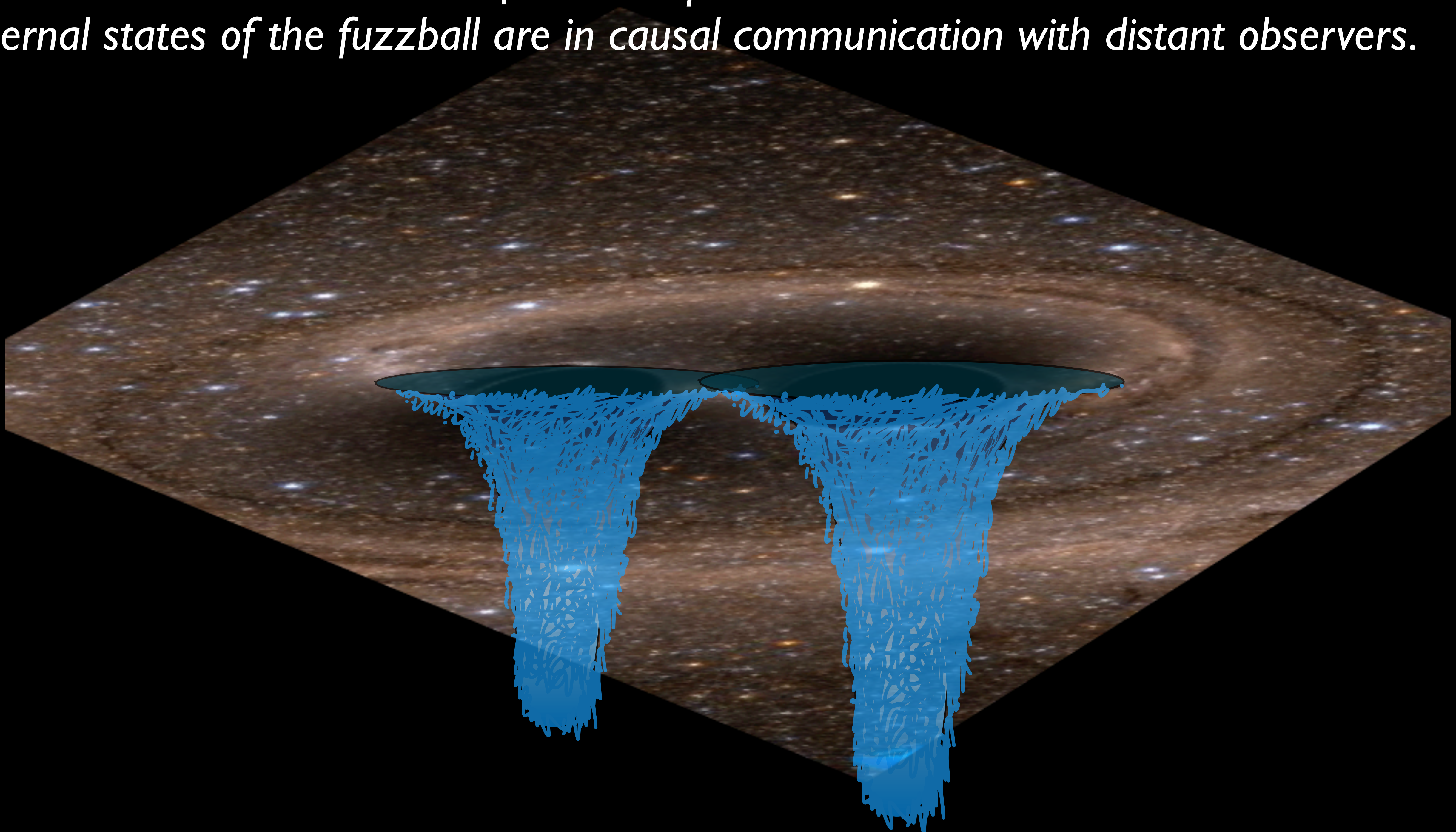
Result of fractionation: *Energy gap decreases by a factor of N^{-1} where $N \equiv N_1 N_2$*

Wave functions of develop a width of $N^{\alpha}\,\ell_{String}$ or $N^{\alpha}\,\ell_{Planck}$

Black holes are really fuzzy branes with horizon-scale wave functions

Fuzzball Paradigm: *Fuzzballs represent a new quantum phase of matter that emerges when it is compressed to black-hole densities, and this new phase prevents the formation of a horizons and singularities*

A fuzzball does not have an information problem because there are no horizons: internal states of the fuzzball are in causal communication with distant observers.



Conversely:

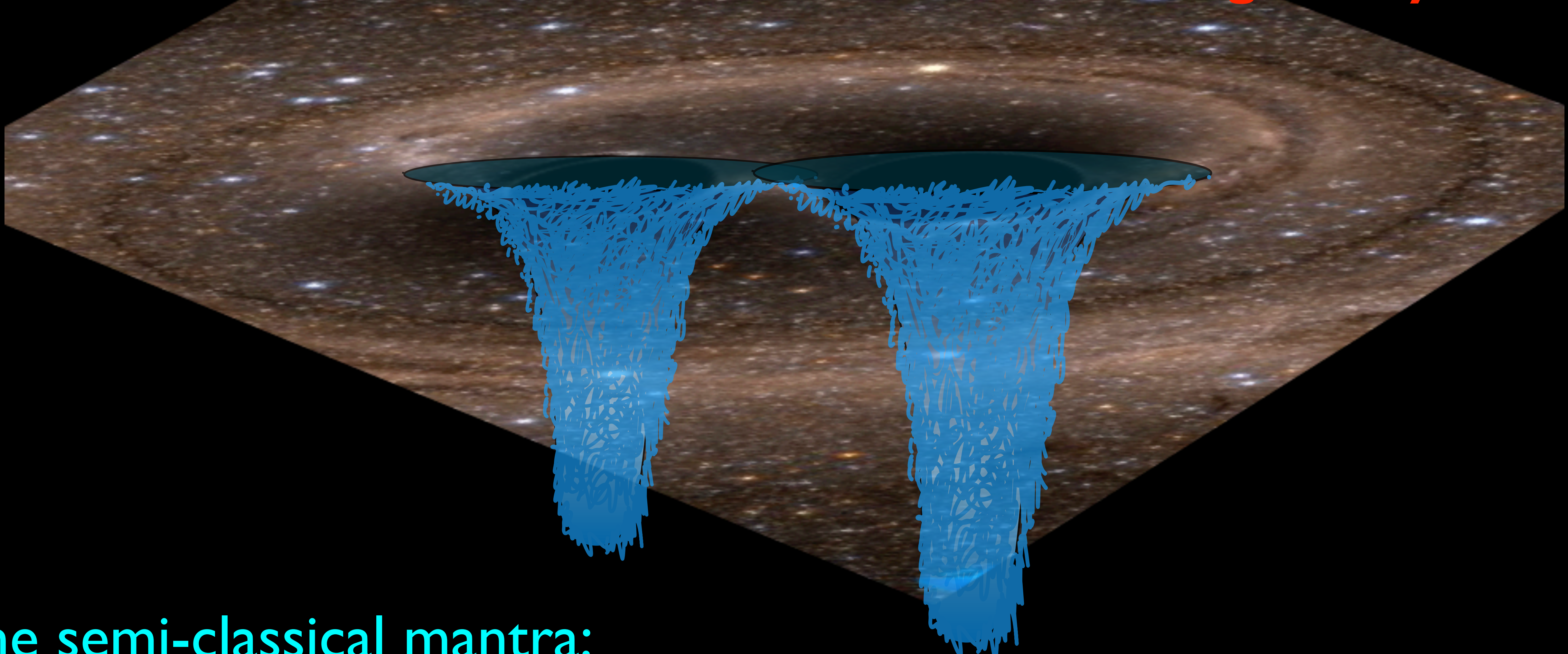
Horizons and singularities only appear if one tries to describe gravity using some "effective" theory (like GR) that has too few degrees of freedom to resolve the physics.

Problem: How do we put computational flesh on the Fuzzball paradigm

… a new quantum phase of matter that emerges when it is compressed to black-hole densities, and this new phase prevents the formation of a horizons and singularities … encoding all the microstructure of a black hole

Requires a UV Completion of General Relativity:

Realize Fuzzballs in ***String theory***



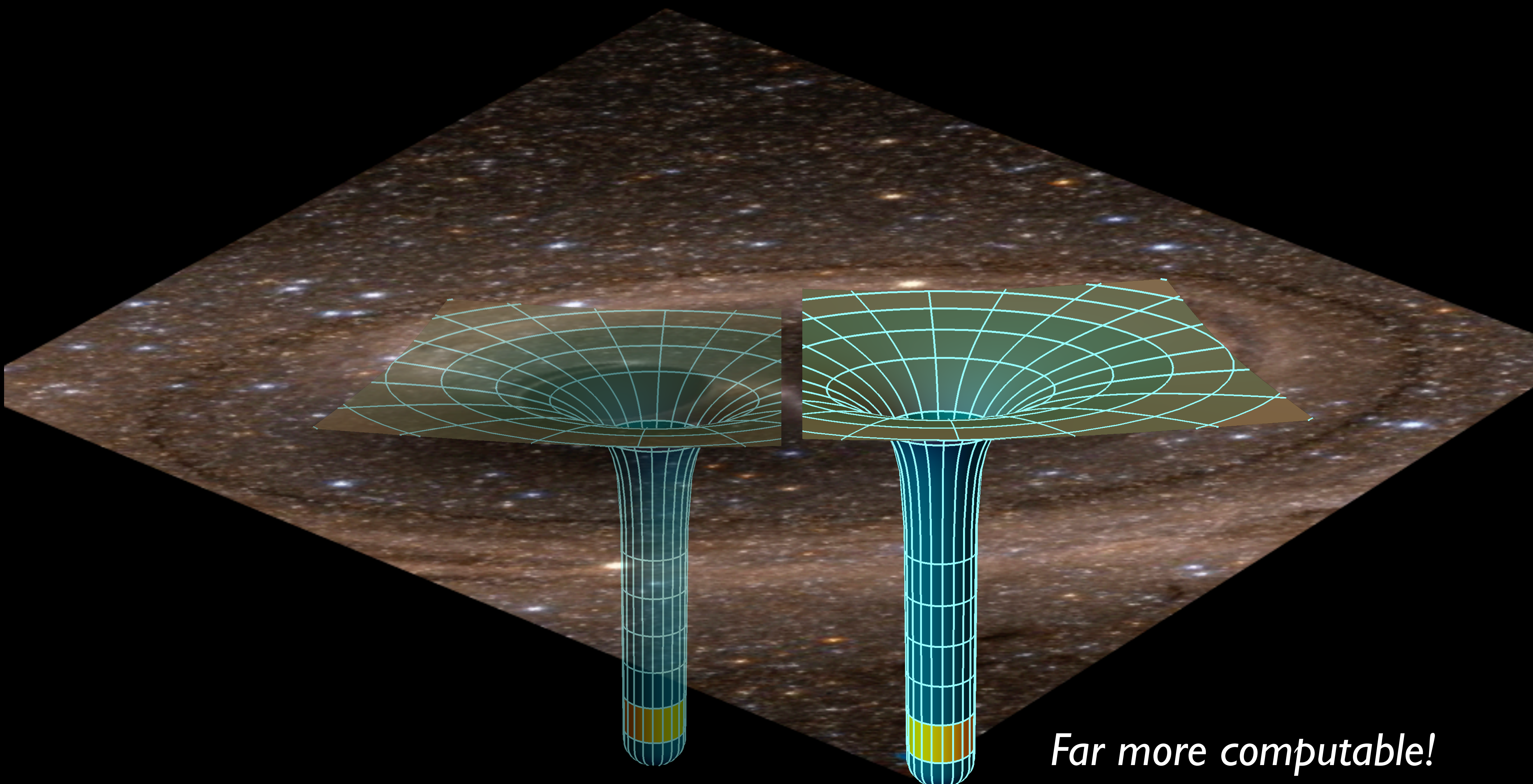
The semi-classical mantra:

Quantum systems have semi-classical limits in terms of coherent states. Fuzzballs with their vast number of microstates should have vast moduli spaces of semi-classical, geometric limits…

<u>Microstate geometries</u>:

<u>Microstate geometries</u> *are the coherent expressions of fuzzballs* within the supergravity limit of string theory.

$\Rightarrow$ ***Smooth***, horizonless "solitonic" solutions to the bosonic sector of supergravity *with the same asymptotic structure as a given black hole*



Far more computable!

Black-hole microstructure in string theory

The most developed example:
The D1-D5 system used by Strominger and Vafa to count microstates

Describing Black-Hole Microstructure in String Theory

Start by simplifying the problem

Look for microstates of **supersymmetric/BPS** black holes "M = Q"

★ Stable and time independent: Hawking Temperature = 0
The *information problem simplifies to the information storage problem.*

★ BPS equations typically first order equations, and sometimes linear.
Much, much simpler than equations of motion.

★ Computationally far simpler. Microstates are all BPS states

★ Microstates "protected by supersymmetry;"
preserved under variation of couplings

★ One can count the microstates using index theory …

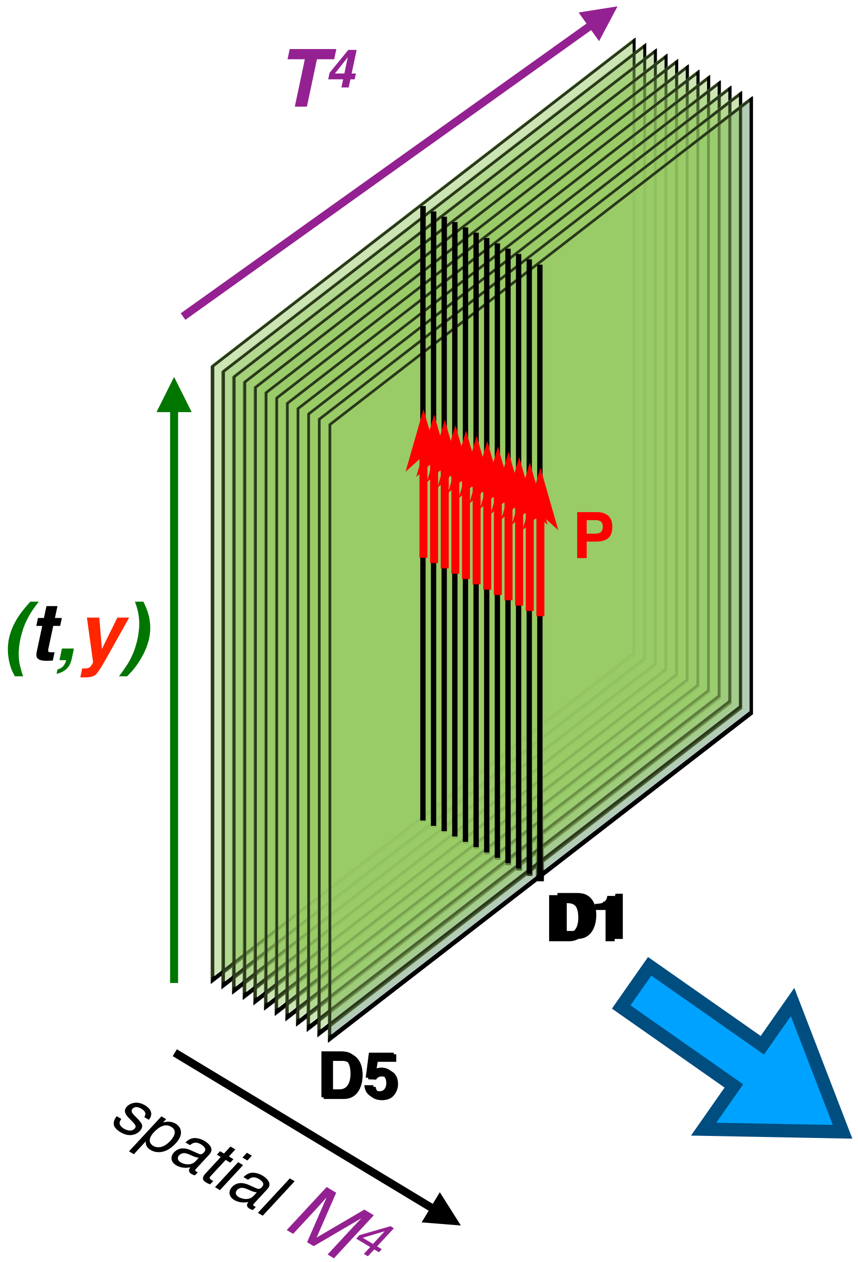
Simplify even further: get rid of gravity

- Vanishing G_{Newton}, g_{String} **Strominger and Vafa: hep-th/9601029**

At $g_{String} = 0$, look for D-brane configurations that become BPS black holes with macroscopic horizon areas at finite $G_{Newton} \sim g^2_{String}$

The D1-D5 system wrapped on $T^4 \times S^1(y)$

Ten dimensional IIB supergravity



D5 branes wrapped on $T^4 \times S^1(y)$

D1 branes wrapped on $S^1(y)$

Common circle: $y \equiv y + 2\pi R_y$

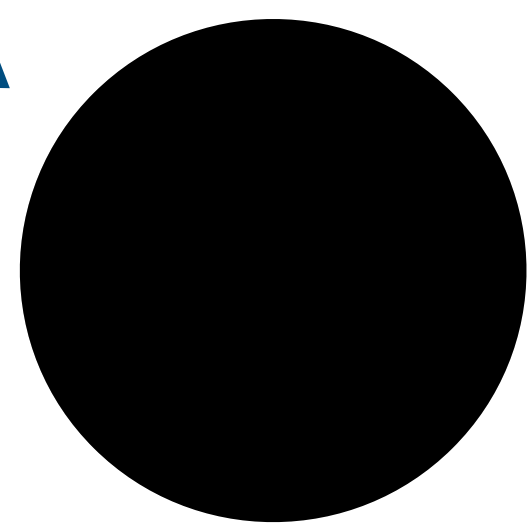
32 supersymmetries

➡ 8 supersymmetries, **¼ BPS**

Add momentum charge**: P**

➡ 4 supersymmetries, **⅛ BPS**

Back-react with finite $G_{Newton} \sim g^2_{String}$ ⇒ Black hole

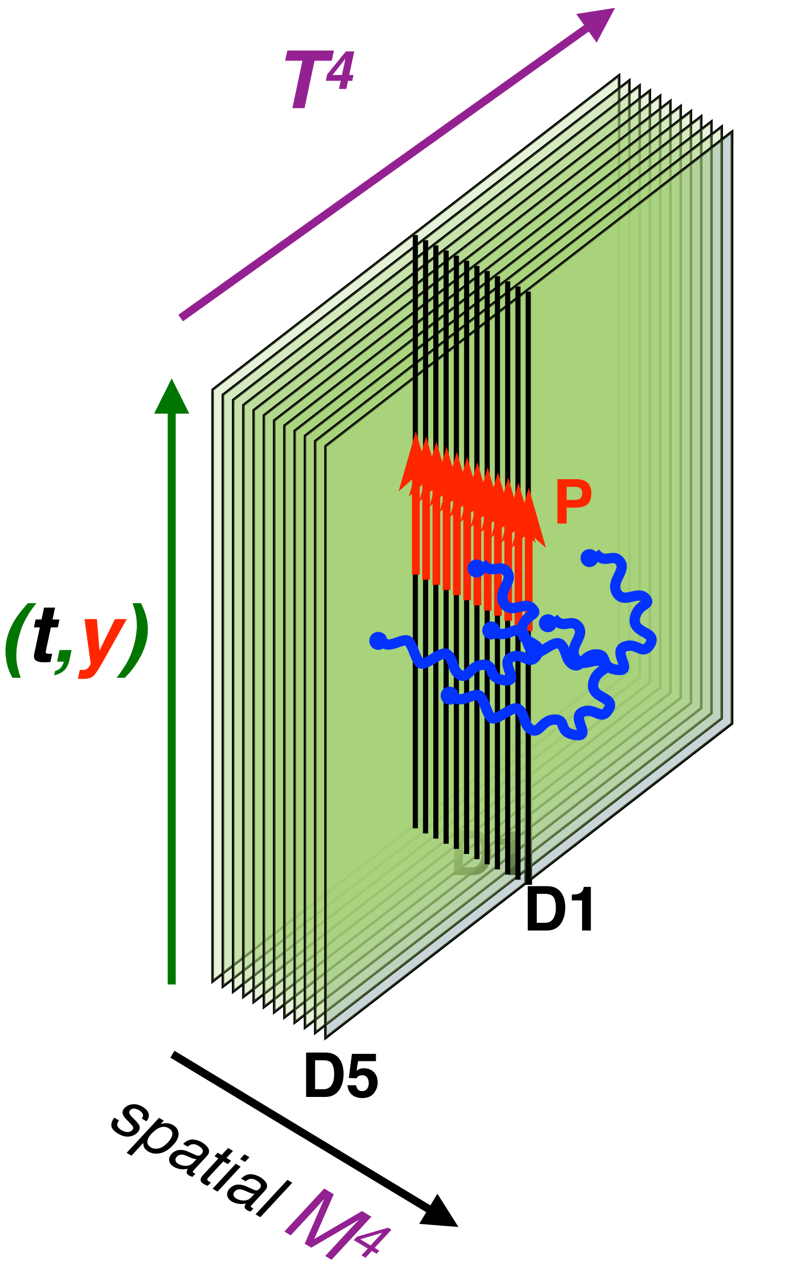


Entropy of the black hole:

$$S = \frac{1}{4} A = 2\pi \sqrt{Q_1 Q_5 Q_P}$$

Now return to $G_{Newton} \sim g^2_{String} = 0$ ….

Microstructure: the D1-D5 system at weak coupling



Momentum carried by massless open superstrings moving in T^4 stretched between D1-D5 branes…

- ✦ $N \equiv N_1 N_5$ Chan-Paton labels: *(4,4) supersymmetric CFT on $S^1(y)$ with* $c = 6\,N = 6\,N_1\,N_5$.
- ✦ The Left + Right moving RR Ground states **¼ BPS**
- ✦ Purely left-moving momentum:

  $Q_P \sim N_P = L_{0,\text{left}} \neq 0$

  Right moving sector: Ramond ground state

  $\Rightarrow$ ⅛ BPS states

Perturbative string states: *Cardy formula:*

$$S \equiv \log\left(\Omega(Q_P)\right) = 2\,\pi\sqrt{\frac{c}{6}\,L_0}$$

$$= 2\,\pi\,\sqrt{N_1\,N_5\,N_P} = 2\,\pi\,\sqrt{Q_1\,Q_5\,Q_P}$$

Strominger, Vafa 1996

Perfect match with black hole! *Declare victory* ….

At vanishing string coupling

Superstrata and Microstrata

Microstate geometries for which we know precise holographic duals

What do D1-D5 microstates become at finite g_{String} ?

The Geometry of the D1-D5 System in IIB Supergravity

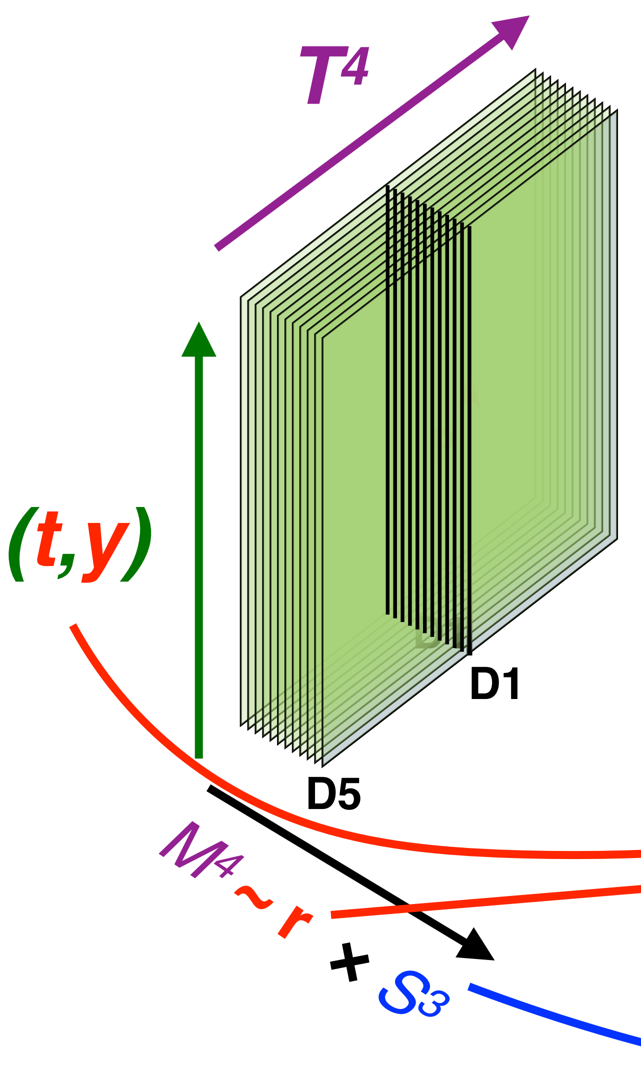
Starting point: the holographic dual of the ¼ BPS ***RR ground states***

Angular momenta: $-\tfrac{1}{2}\mathbf{N} < j_L, j_R < \tfrac{1}{2}\mathbf{N}$ with $\mathbf{N} = N_1 N_5$

Back-reacted geometry ⇔ Gravity dual of D1-D5 CFT: *the D1-D5 supertube*

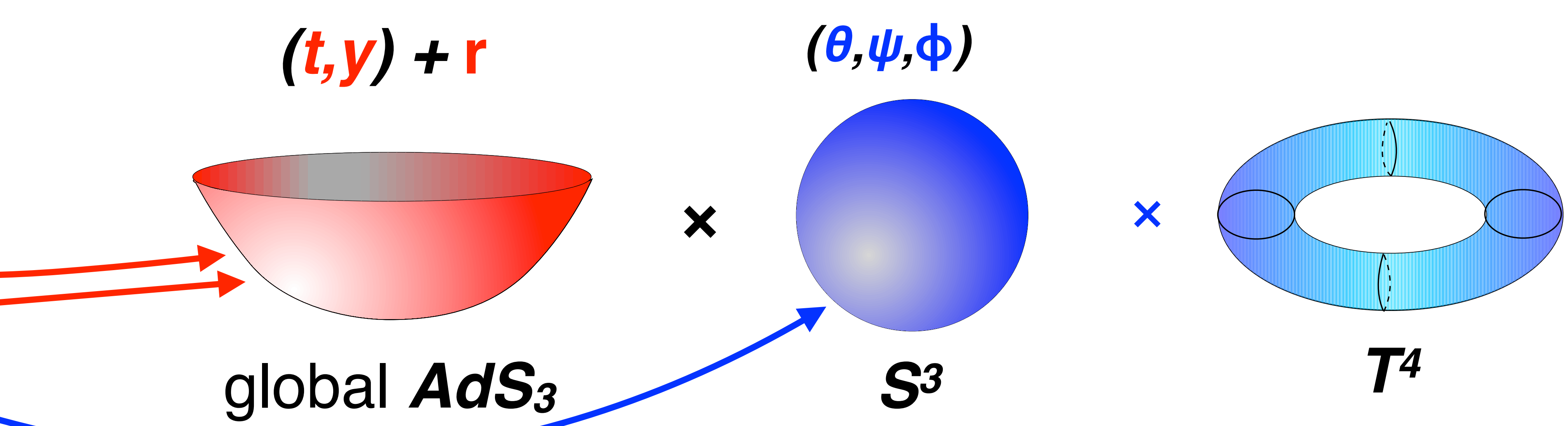
⇔ *Deformations of global* $AdS_3 \times S^3$

Lunin, Mathur, hep-th/0202072; Lunin, Maldacena, Maoz hep-th/0212210
Kanitscheider, Skenderis, and Taylor 0611171 and 0704.0690; Taylor, 0709.1838



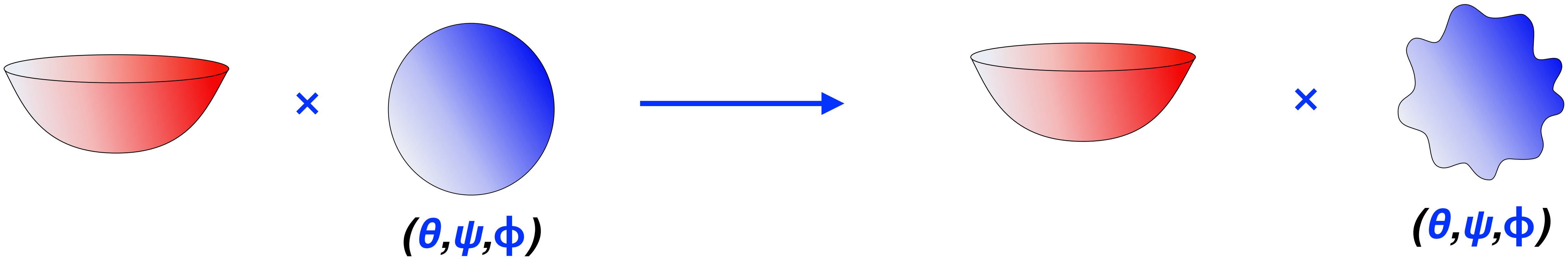
Maximally spinning RR ground state:

$$\left(\left|+\tfrac{1}{2},+\tfrac{1}{2}\right\rangle_1\right)^N \qquad j_L = j_R = \tfrac{1}{2}\mathbf{N} = \tfrac{1}{2} N_1 N_5$$



Smooth geometry …

More general RR ground states: *Harmonic deformation of* S^3



$$\left(|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_1\right)^{\mathbf{N}} \longrightarrow \left(|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_1\right)^{n_{++}} \bigotimes_{k=1}^{N_1 N_5} \left(|\,0,0\,\rangle_k\right)^{n_k}$$

$j_L = j_R = \tfrac{1}{2} N$

$j_L = j_R = \tfrac{1}{2} n_{++}$

$$n_{++} + \sum_{k=1}^{N_1 N_5} k\, n_k = N_1 N_5$$

Generic S^3 phase dependence:

$$\chi_{k_j, m_j} \equiv \tfrac{1}{2}(k_j - 2m_j)\,\psi - \tfrac{1}{2} k_j\, \phi$$

These states have $m_j = 0$

Kanitscheider, Skenderis, and Taylor 0611171 and 0704.0690; Taylor, 0709.1838

Superstrata: *Add momentum excitations … compute the supergravity dual… as a microstate geometry*

This took quite a few years … and a lot of effort …

Bena, de Boer, Shigemori, Warner,
"Double, Double Supertube Bubble," 1107.2650

Giusto, Russo, Turton,
"New D1-D5-P geometries from string amplitudes," 1108.6331

Bena, Giusto, Shigemori, Warner,
"Supersymmetric Solutions in Six Dimensions: A Linear Structure" 1110.2781

Giusto, Russo,
"Perturbative superstrata," 1211.1957

Niehoff, Vasilakis, Warner,
"Multi-Superthreads and Supersheets," 1203.1348

Lunin, Mathur, Turton,
"Adding momentum to supersymmetric geometries," 1208.1770

Vasilakis,
"Corrugated Multi-Supersheets," 1302.1241

Niehoff, Warner,
"Doubly-Fluctuating BPS Solutions in Six Dimensions," 1303.5449

Shigemori,
"Perturbative 3-charge microstate geometries in six dimensions," 1307.3115

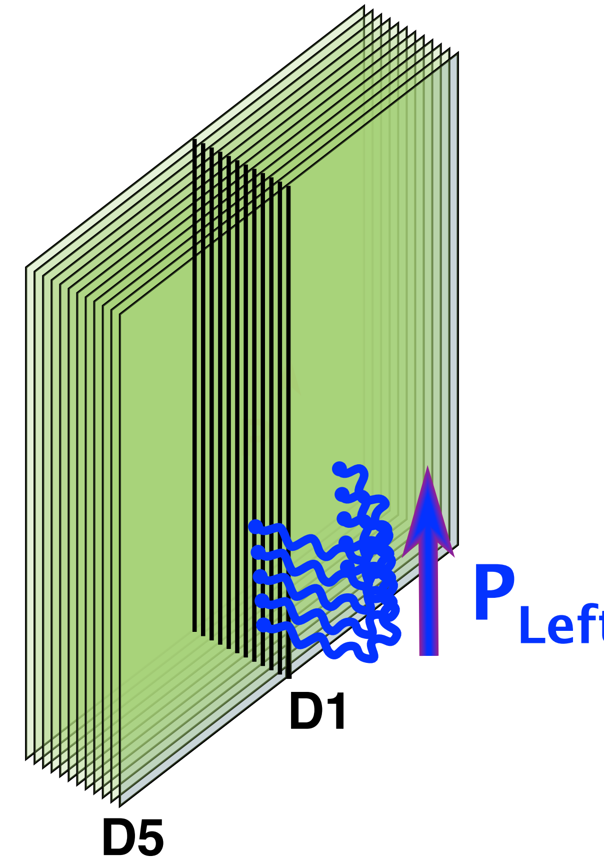
Superstrata

Bena, Giusto, Russo, Shigemori, Warner 1503.01463

Add purely *left-moving* momentum excitations

Right moving sector: *Ramond ground state*

⇒ ⅛ BPS states of the "Supergraviton gas"



Superstratum excitations

Linear superpositions of states of the form

$$\left(|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_1\right)^{n_{++}} \otimes \left(\frac{1}{m!\,n!}(J^+_{-1})^m\,(L_{-1}-J^3_{-1})^n\,|00\rangle_k\right)^{n_{k,m,n}}$$

(cyan) = Particular Ramond ground states (yellow) = Particular Excitations

Degeneracies specified by n_{++}, $n_{k,m,n}$

Angular momenta:

$j_R = \tfrac{1}{2}\, n_{++}$

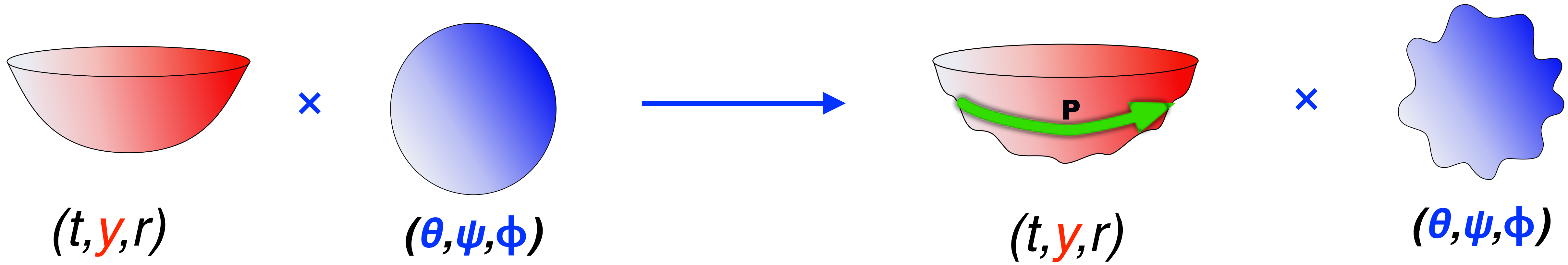
$j_L = \tfrac{1}{2} n_{++} + \sum m\, n_{k,m,n}$

Momentum

$P = L_0 = \sum (m+n)\, n_{k,m,n}$

Fairly rich collection of BPS states and dual BPS geometries …

Momentum excitations: *Harmonic deformation of $AdS_3 \times S^3$*



$$\left(\left|+\tfrac{1}{2},+\tfrac{1}{2}\right\rangle_1\right)^{\mathrm{N}} \longrightarrow \left(\left|+\tfrac{1}{2},+\tfrac{1}{2}\right\rangle_1\right)^{n_{++}} \bigotimes \left(\frac{1}{m!\,n!}\,(J^+_{-1})^m\,(L_{-1}-J^3_{-1})^n\,|00\rangle_k\right)^{n_{k,m,n}}$$

Generic $AdS_3 \times S^3$ phase dependence:

$$\chi_{k_j,m_j,n_j} \;\equiv\; R_y^{-1}\,(m_j+n_j)\,v \;+\; \tfrac{1}{2}\,(k_j-2m_j)\,\psi \;-\; \tfrac{1}{2}\,k_j\,\phi$$

Null coordinate (*left-moving on* AdS_3): $v \;\equiv\; \frac{1}{\sqrt{2}}\,(t\;+\;y)$

Fourier modes (k,m,n) + Fourier coefficient of fields and metric …

⇒ Supergravity solutions ***sourced*** by ***arbitrary functions of three variables***

Heidmann, Warner 1903.07631

Complete solution depends on five variables: $(v, r, \theta, \psi, \phi)$

BPS ⇒ independent of right-moving time: $u \;\equiv\; \frac{1}{\sqrt{2}}\,(t\;-\;y)$

Summary

The CFT data: A class of states in the "supergraviton gas:"

$$\left(|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_1\right)^{n_{++}} \bigotimes \left(\frac{1}{m!\,n!}\,(J^+_{-1})^m\,(L_{-1}-J^3_{-1})^n\,|00\rangle_k\right)^{n_{k,m,n}}$$

Angular momenta:

$j_R = \tfrac{1}{2}\, n_{++}$

$j_L = \tfrac{1}{2} N_{++} + \sum m\, N_{k,m,n}$

Momentum

$P = L_0 = \sum (m+n)\, n_{k,m,n}$

The Geometric Data

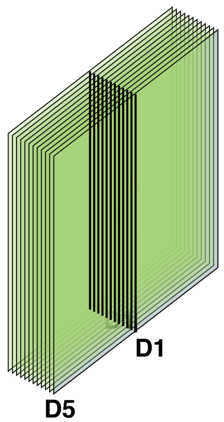
Supergravity: Metric, gauge fields and scalars

n_{++} → Fourier coefficients, a, for angular momentum, j_R

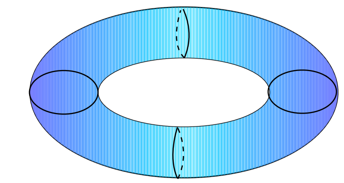
$n_{k,m,n}$ → Fourier coefficients, $b_{k,m,n}$ for momentum modes

$$\chi_{k_j,m_j,n_j} \equiv R_y^{-1}\,(m_j+n_j)\,v \;+\; \tfrac{1}{2}\,(k_j-2m_j)\,\psi \;-\; \tfrac{1}{2}\,k_j\,\phi$$

The supergravity



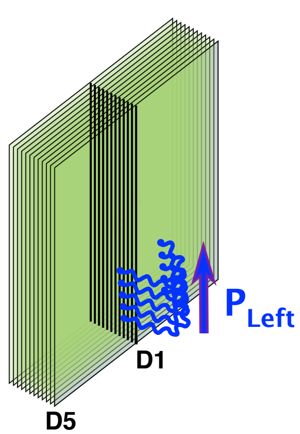
▶ D1-D5 system ⇒ IIB supergravity compactified on T^4



⇒ *Six-dimensional (1,0) supergravity*

- Graviton multiplet: $g_{\mu\nu}$, self-dual tensor gauge field $B^{+}_{\mu\nu}$ + gravitini

▶ Independent D1 + D5 branes: unconstrained $C_{\mu\nu} = B^{+}_{\mu\nu} + B^{(1)-}_{\mu\nu}$



▶ Coherent string excitations

⇒ anti-self-dual NS tensor gauge field $B^{(2)-}_{\mu\nu}$

(anti-self-duality required by supersymmetry)

- Anti-self-dual tensor multiplets: $B^{(i)-}_{\mu\nu}$, scalars, $\phi^{(i)}$ + gauginos

The Simplest six-dimensional (BPS) superstratum metric

Flat M^4 = $\boldsymbol{R^4}$ base transverse to branes with coordinates, y.

$$ds_6^2 = -\frac{2}{\sqrt{\mathcal{P}}}(dv+\beta)\big(du+\omega-\tfrac{1}{2}\mathcal{F}(dv+\beta)\big) + \sqrt{\mathcal{P}}\, d\vec{y}\cdot d\vec{y}$$

Gutowski, Martelli and Reall 0306235

Null coordinates: $u \equiv \frac{1}{\sqrt{2}}(t-y)\,, \qquad v \equiv \frac{1}{\sqrt{2}}(t+y)$

BPS system + Smoothness:

Determines

- *Tensor gauge field fluxes*
- "warp factors," *F*, *P* *and one-forms*, β, *ω*

Miracle ***The BPS equations are linear***

Bena, Giusto, Shigemori, Warner 1110.2781
Giusto, Martucci, Petrini, Russo 1306.1745
Čeplak, Hampton, Warner 2204.07170

Solving BPS equations is an algorithmic process …

One can construct superstratum solutions that correspond to *generic superpositions* of the CFT excitations:

$$\left(|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_1\right)^{n_{++}} \bigotimes \left(\frac{1}{m!\,n!}(J_{-1}^{+})^m\,(L_{-1}-J_{-1}^{3})^n\,|00\rangle_k\right)^{n_{k,m,n}}$$

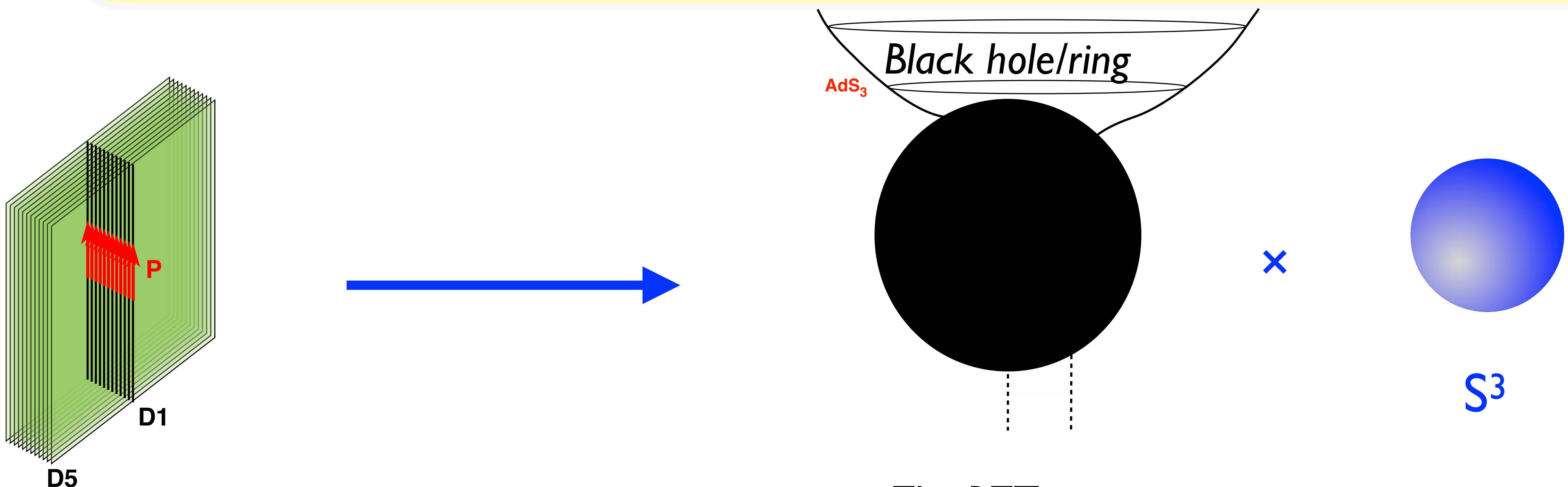
Coarse-grained Back-reaction: Black hole/ring metrics

Vacuum with maximal angular momenta

Back-reacted Geometry

Add pure momentum charge, Q_P, to this state ….

… ignoring details of how the momentum is actually carried by supergravity fields



The BTZ geometry for vanishing j_L, j_R

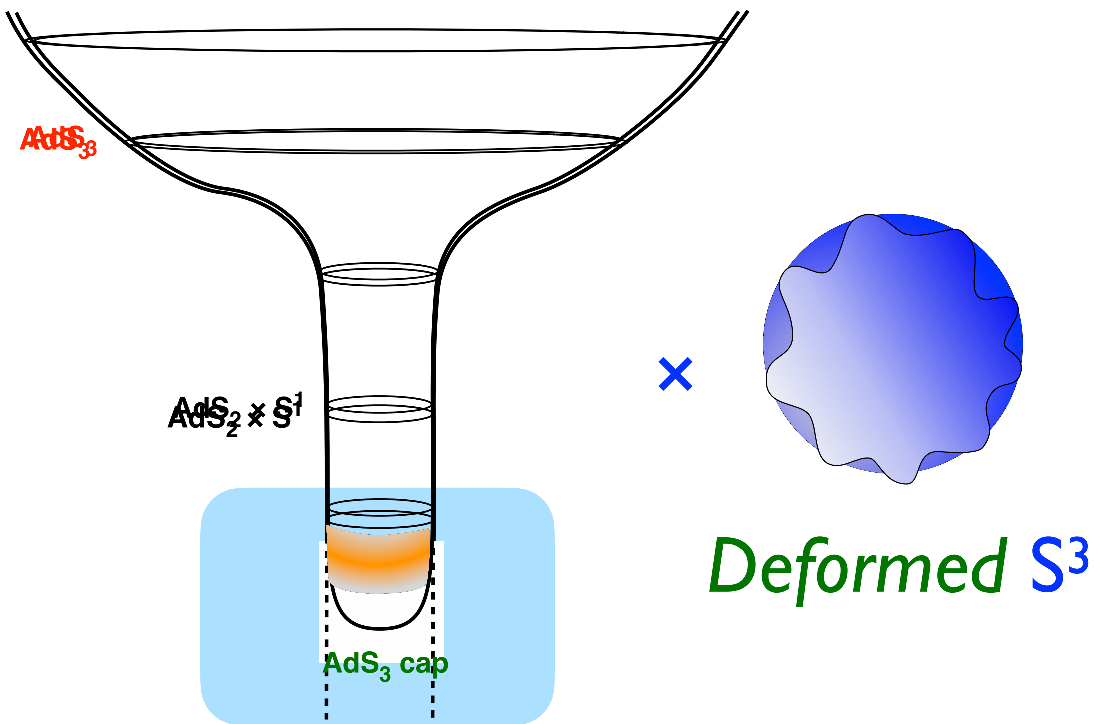
Horizon ⇔ ***Ensemble Averaging*** *over details of momentum charge*

Superstrata:
What does back-reacted microstructure become at ***strong coupling*** ***by developing the precison holography of the microstructure?***

<u>Back-reacted Geometry + Momentum Excitations</u>

The *precision* holographic dictionary relating CFT states to supergravity excitations is well-known and extremely well tested.

And the gravity dual is ***not*** the BTZ Black-hole geometry …



The superstratum:
The correct holographic dual of these black-hole microstates has a Black-hole-like throat but caps off smoothly above the original BTZ horizon …

These geometries are indeed dual to some of the families of supersymmetric microstates in the CFT that were counted by Strominger and Vafa….

Microstate Geometries *capture the true microstate structure …*
(at least for these particular CFT states)

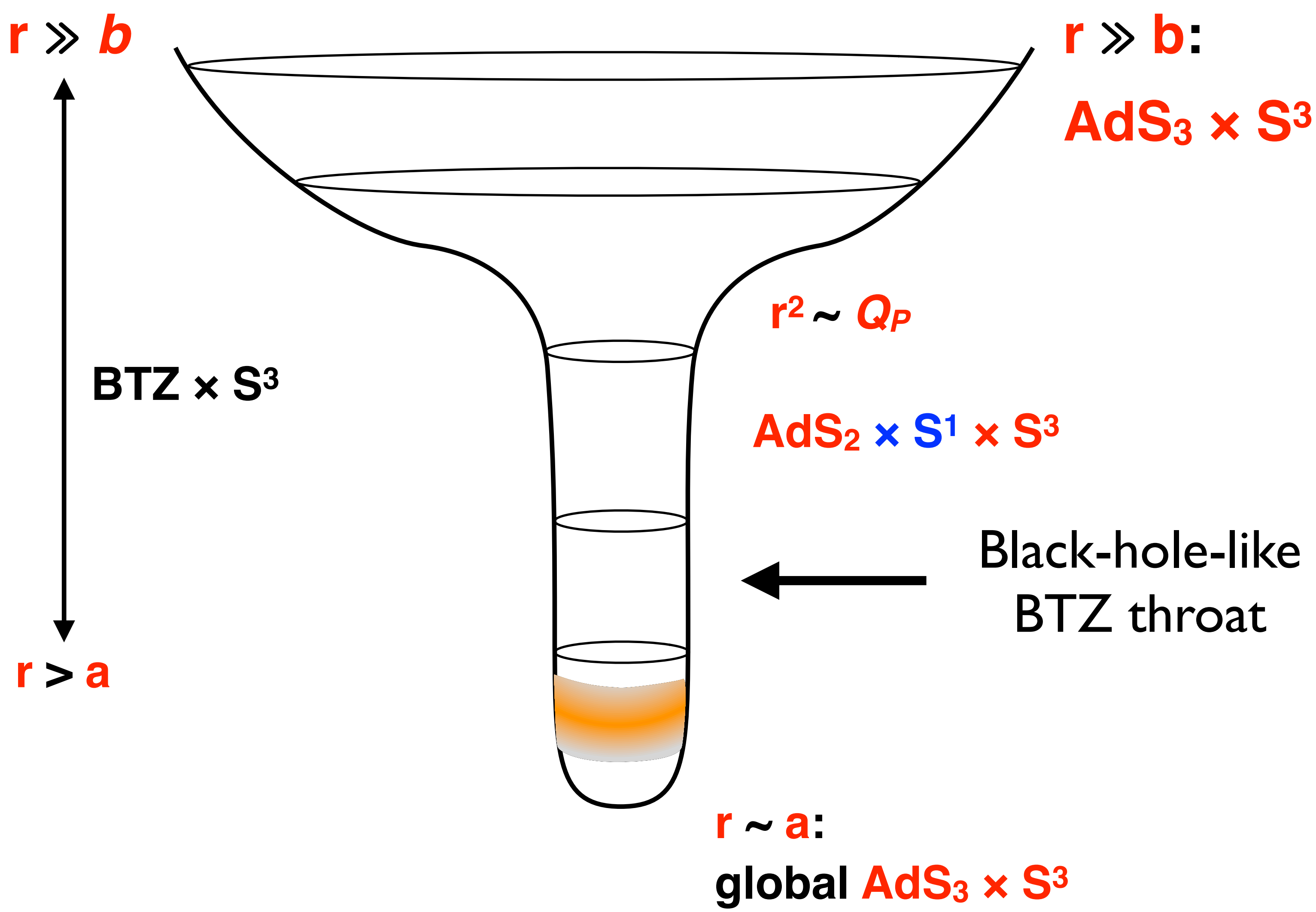
The "Geography" of Simplest Asymptotically AdS Superstrata

Focus on the (2+1)-dimensional base geometry asymptotic to *AdS₃*

D1, D5 charges Q_1, Q_5 set the scale of the underlying *AdS₃*: $R_{AdS} = (Q_1\, Q_5)^{\frac{1}{4}}$

Parameters: *Fourier coefficients of modes:* ***a*** *and* ***b****;* *(Take* ***b*** $\gg$ ***a****, and m = 0)*

Angular momenta: $j_L = j_R \sim a^2 \sim n_{++}$ *Momentum charge* $Q_P \sim b^2 \sim n_{k,m,n}$



AdS₃ in UV of D1-D5 system: **S¹**(y) grows with **r**

Momentum charge, Q_P stablizes **S¹**(y) radius

S¹(y) radius shrinks creating smooth AdS₃ cap

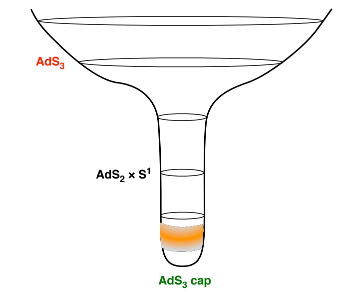
"Capped BTZ" geometries

Bena, Giusto, Martinec, Russo, Shigemori, Warner 1607.03908

An Example: The six-dimensional geometry with (k,m,n) = (1,0,n)

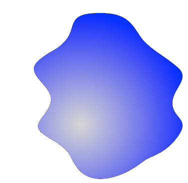
Bena, Turton, Walker, Warner 1709.01107

$$ds_6^2 = \sqrt{Q_1 Q_5}\left[\Lambda\, \widehat{ds}_3(r) + \widetilde{ds}_3(r,\theta)\right]$$



$$\widehat{ds}_3^{\,2} = \frac{dr^2}{r^2+a^2} + \frac{2\,r^2(r^2+a^2)}{R_y^2\,a^4}\,dv^2 - \frac{1}{2\,R_y^2}\,\frac{1}{A^4 G^2}\Big(\underbrace{du+dv}_{dt} + \frac{2A^2 r^2}{a^2}\,dv\Big)^2$$

$$\widetilde{ds}_3^{\,2} = \Lambda\,d\theta^2 + \frac{1}{\Lambda}\sin^2\theta\left(d\varphi_1 - \frac{1}{\sqrt{2}\,R_y A^2}(du+dv)\right)^2 + \frac{G}{\Lambda}\cos^2\theta\left(d\varphi_2 + \frac{1}{\sqrt{2}\,R_y\,a^2 A^2\,G}\left(a^2(du-dv) - b^2 F\,dv\right)\right)^2$$

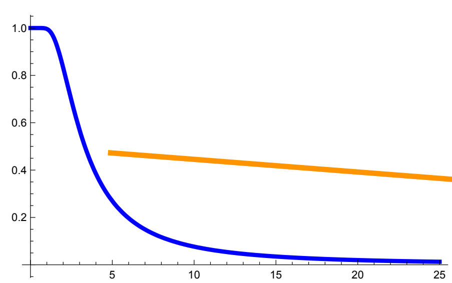


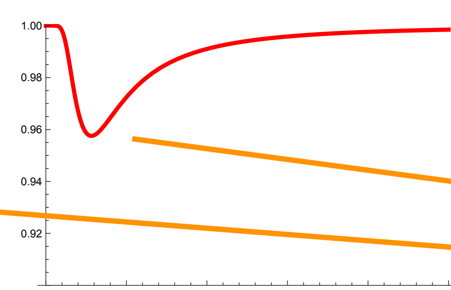
Bump functions and parameters:

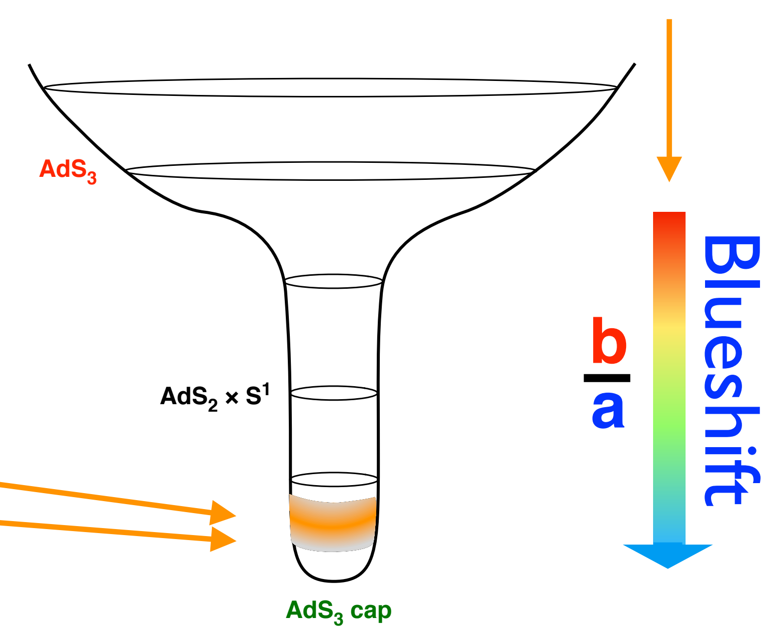
Warp factor: $\Lambda \equiv \sqrt{1 - \frac{a^2\,b^2}{(2a^2+b^2)}\,\frac{r^{2n}}{(r^2+a^2)^{n+1}}\,\sin^2\theta}$

$$A \equiv \sqrt{1+\frac{b^2}{2a^2}} \sim \frac{b}{\sqrt{2}\,a}$$

$$F \equiv 1 - \frac{r^{2n}}{(r^2+a^2)^n}\,, \qquad G \equiv 1 - \frac{a^2\,b^2}{2a^2+b^2}\,\frac{r^{2n}}{(r^2+a^2)^{n+1}}$$







Asymptotically Flat Superstrata

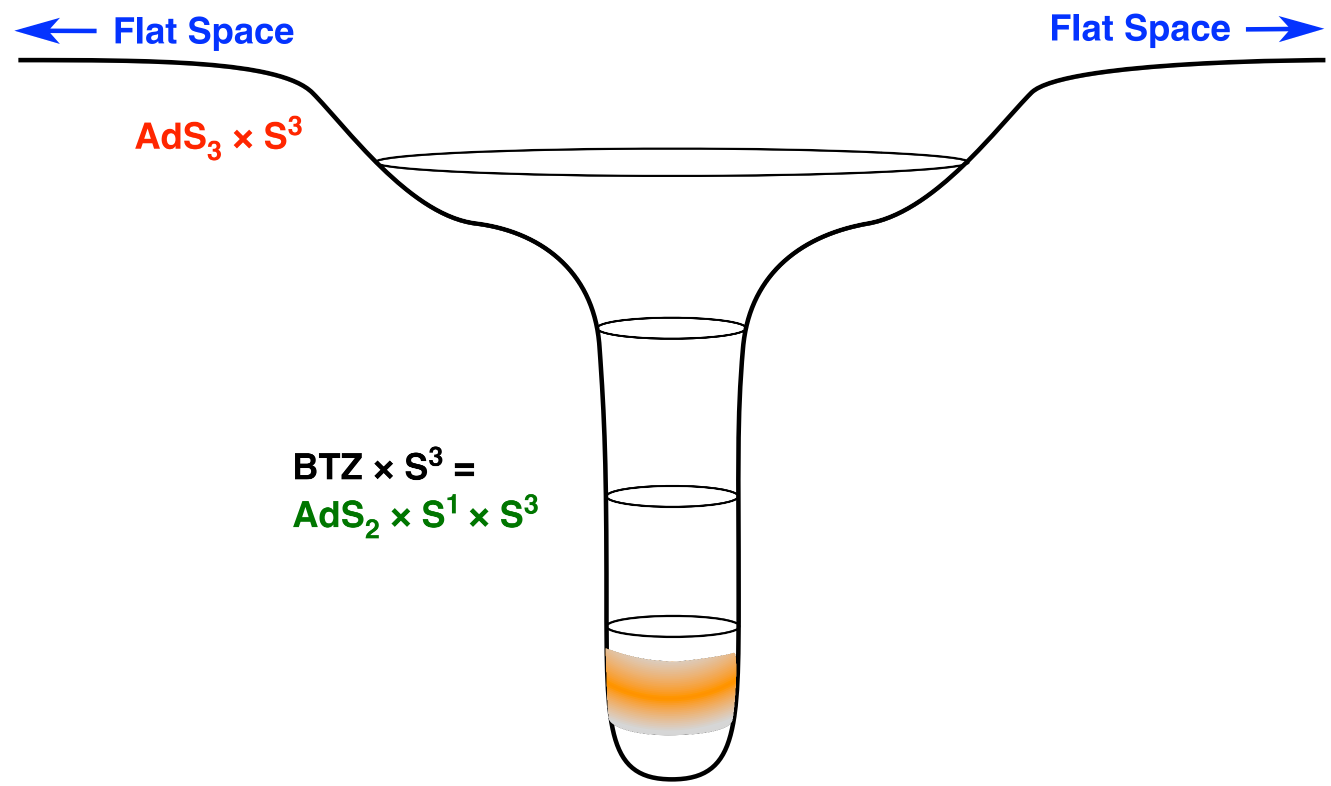
Bena, Giusto, Martinec, Russo, Shigemori, Warner 1711.10474

Embed these asymptotically AdS_3 microstate geometries into asymptotically-flat space-times

Algorithmic process:

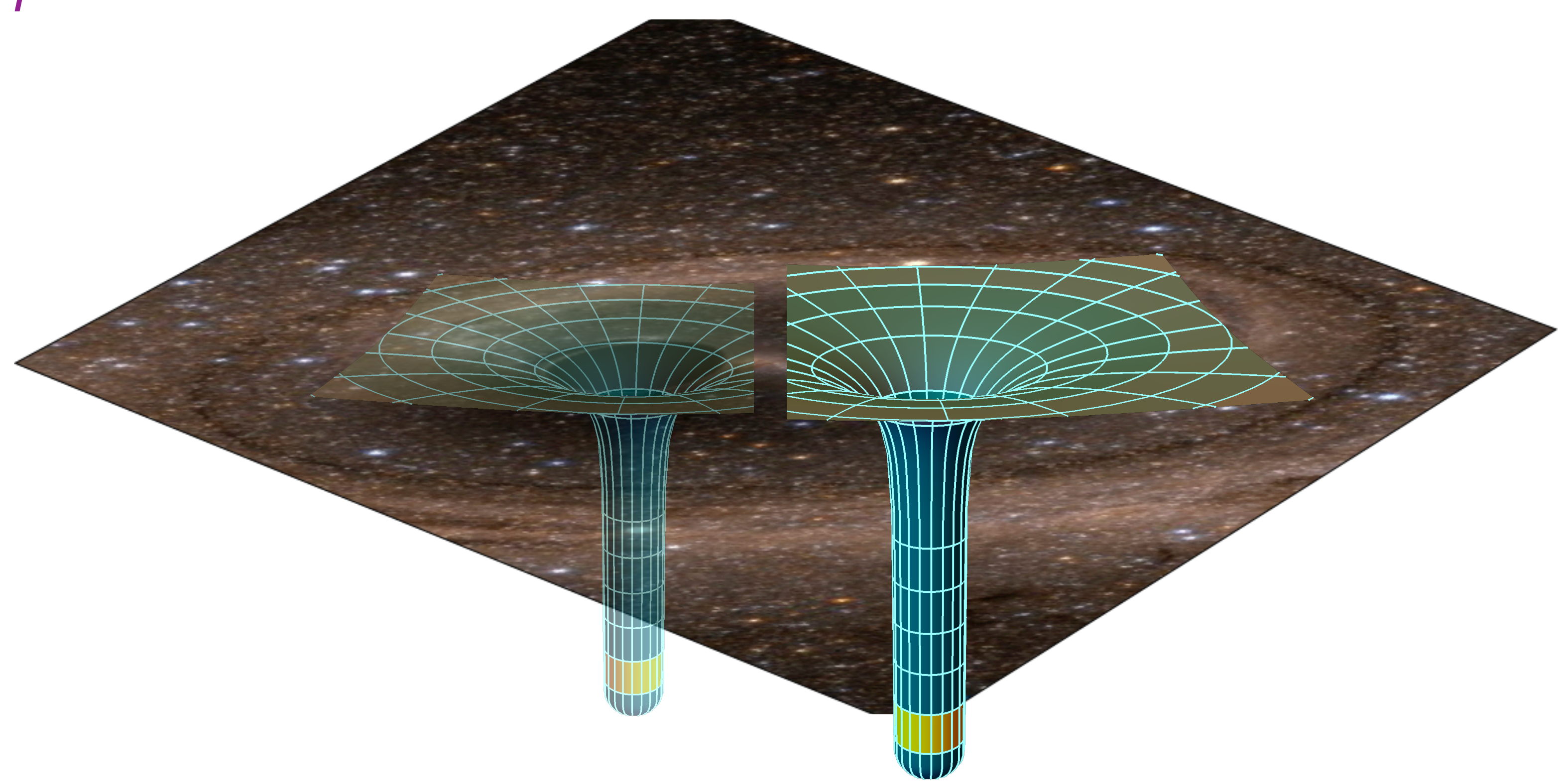
Add parameters (the "1's") to metric warp factors …

BPS equations still linear



At large r, the $S^1(y)$ limits to a fixed radius, R_y, and the S^3 combines with the radial coordinate to make $\mathbb{R}^{4,1}$

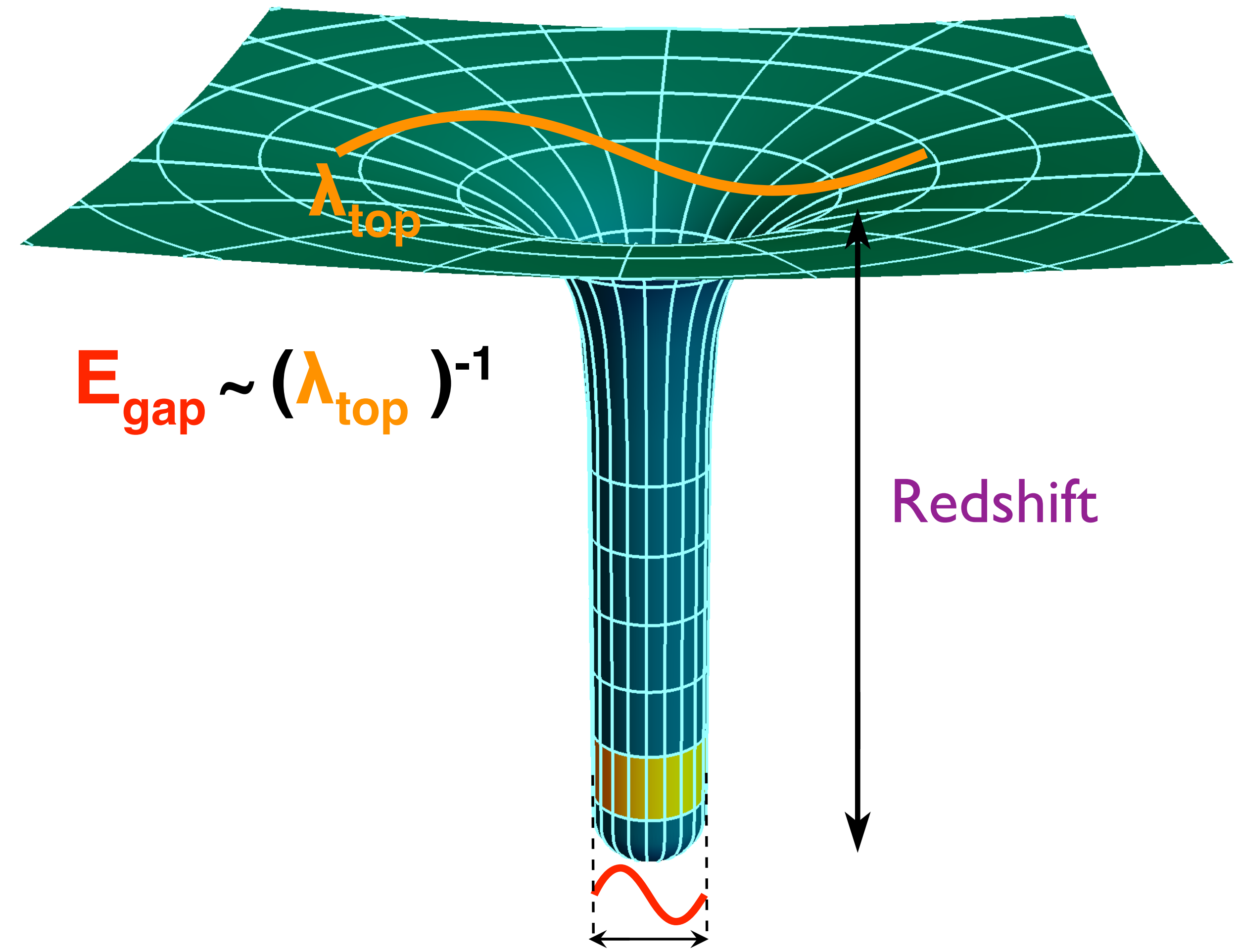
The space-time is asymptotic to $\mathbb{R}^{4,1} \times S^1$



Features of Superstrata:
Laboratories for new near-horizon physics

The Energy Gap

- ★ Find the longest wavelength, λ_0, excitation that can be localized at the bottom of the throat.
- ★ Compute the redshift factor from the bottom to top of the throat: λ_{top} = Redshift × λ_0
- ★ $E_{gap} \sim (\lambda_{top})^{-1}$



Deep throat $\frac{b^2}{a^2} \gg 1 \quad \Rightarrow$

$$E_{gap} = \frac{a^2}{b^2}\mu \approx \frac{j_L\,\mu}{N_1 N_5} \sim \frac{1}{C_{CFT}}$$

The deepest possible throats have $j_L = ½$; μ is a number of order 1

Tyukov, Walker, Warner 1710.09006 Bena, Heidmann, Turton 1806.02834

Deep, scaling geometries are dual to states in the maximally-twisted/ most-highly-fractionated sector of the underlying D1-D5 CFT

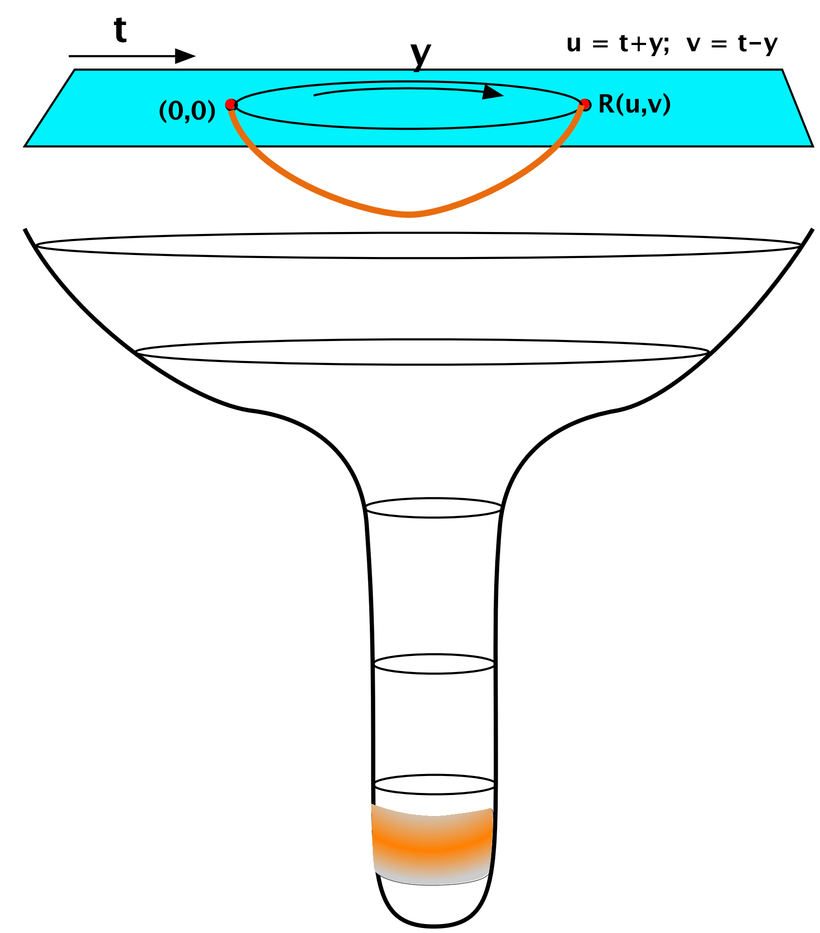
Probing with waves: Green functions

I. Bena, P. Heidmann, R. Monten, N.P. Warner 1905.05194

In some superstrata the six-dimensional massless wave equation is separable

Bena, Turton, Walker and Warner 1709.01107

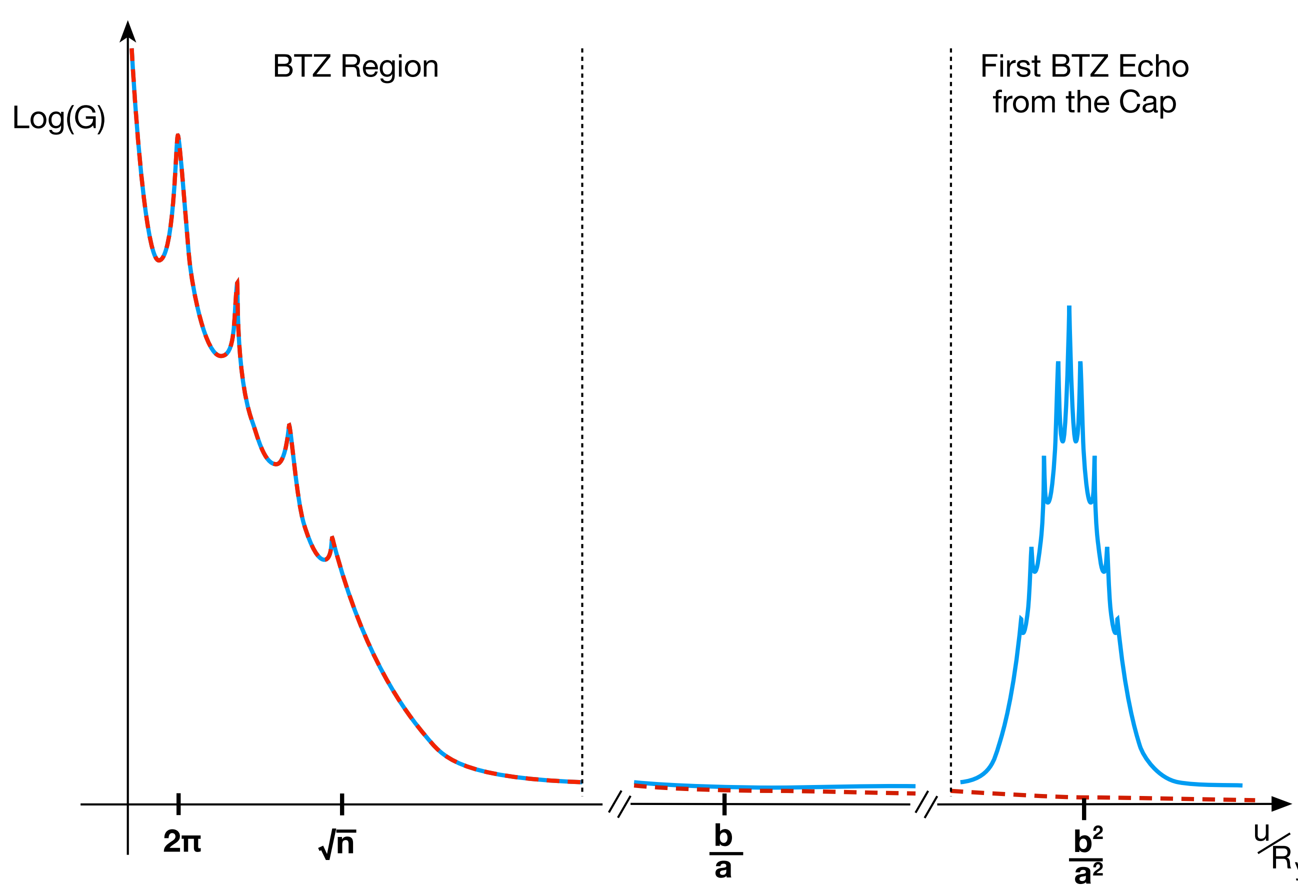
$$\Phi(y,t;r) \;=\; \beta(y,t)\, r^{\Delta-d}\,(1+\mathcal{O}(r^{-2})) + \alpha(y,t)\, r^{-\Delta}\,(1+\mathcal{O}(r^{-2}))$$



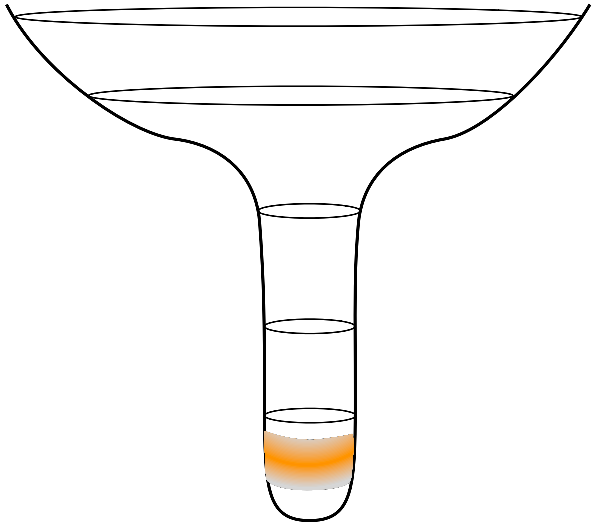
Isolate normalizable $\alpha(y,t)$ and non-normalizable modes $\beta(y,t)$….

Boundary-to-Boundary Response function:

$$G(u,v) \;=\; \frac{\delta\alpha}{\delta\beta}$$



The “Response Function” of superstrata



- Exponential/Thermal decay of correlators determined by

$$T_L = \frac{1}{2\pi}\sqrt{\frac{N_P}{N_1 N_5}} \approx \frac{\sqrt{n}}{2\pi\, R_y}$$

⇒ Black-hole like behavior for times ≪ $N_1 N_5 R$

- No quasi-normal modes: states do not decay through a horizon
- Echoes, time-scale set by $(E_{gap})^{-1} \sim c_{CFT}\, R \sim N_1 N_5 R$

⇒ Information recovery

- The cap looks like a highly red-shifted global AdS_3 global
- Sharp, very coherent echoes

⇔ This superstratum is a highly coherent, specialized state: ***Far from typical***

Tidal Forces

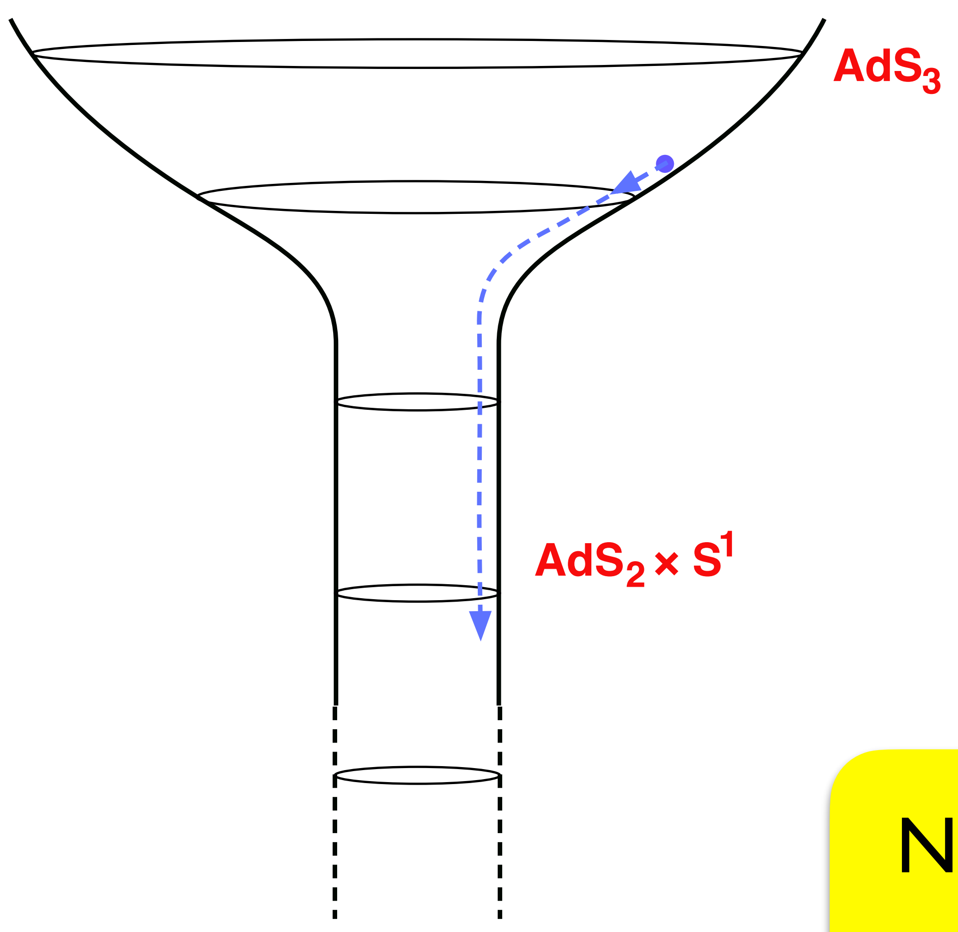
Drop in a probe particle from high above throat:

it reaches ultra-relativistic speeds as it falls

Geodesic Deviation: The *Tidal Tensor* determines the tidal stress in an extended object whose center of mass follows a geodesic with proper velocity, V^μ:

$$\mathcal{A}^\mu{}_\nu \equiv R^\mu{}_{\rho\nu\sigma} V^\rho V^\sigma$$

BTZ metric



BTZ has *locally* same curvature as AdS_3

$$R_{AdS} = (Q_1 Q_5)^{\frac{1}{4}}$$

Tidal tensor magnitude along radial infall:

$$|\mathcal{A}| \sim \frac{1}{\sqrt{Q_1 Q_5}} \sim \frac{1}{\sqrt{N_1 N_5}}$$

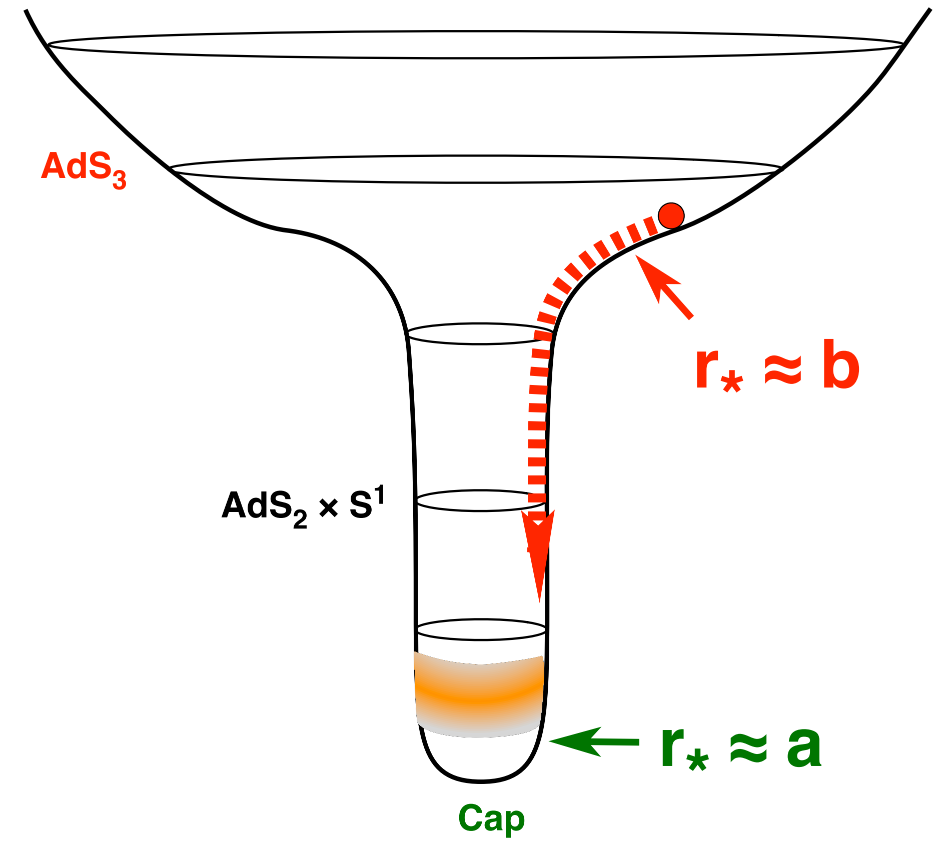
Vanishes for large $\boldsymbol{N} \equiv N_1 N_5$

No "drama at the horizon"

… as with any suitably macroscopic black hole

Tidal Forces in Microstate Geometries

Tyukov, Walker and Warner 1710.09006
Bena, Martinec, Walker and Warner 1812.05110



Tidal tensor also has higher multipole moments:
"Small deviations" from constant curvature BTZ
amplified by ultra-relativistic speeds of the probe …
infalling matter encounters string-scale tidal forces …

For simplest microstate geometries

Tidal forces hit the string scale at $r \sim \sqrt{a\,b}$

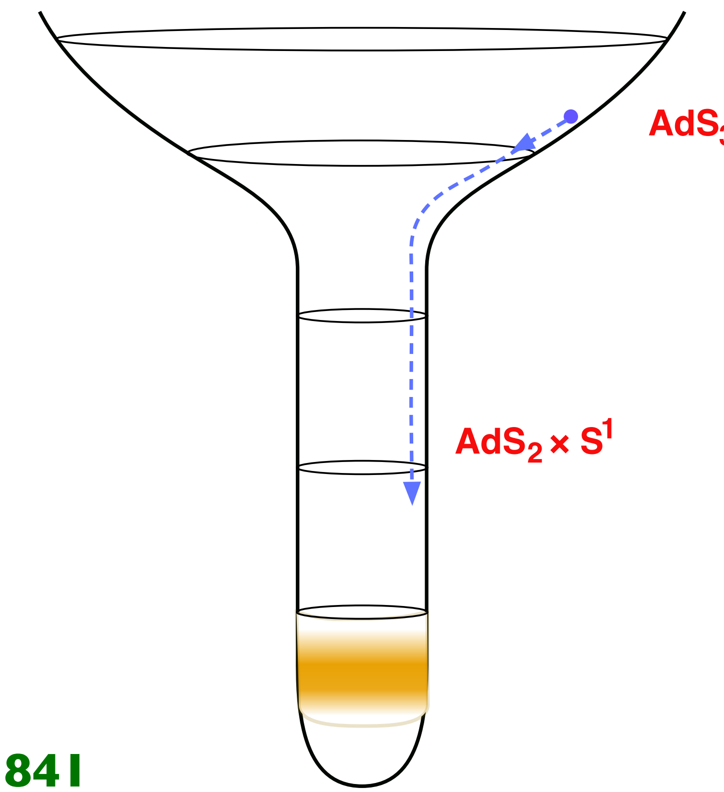
With some fine-tuning one can delay onset …

Bena, Houppe and Warner: 2006.13939

… but the tidal forces reach string scale before the probe reaches the cap

BUT infalling matter is really a string … so what happens to it?

Tidal Trapping in Superstrata



The infalling probe is made of strings:
They becomes excited into massive modes as a result of the tidal forces

- ✦ *The ultra-relativistic speed of probe in throat:* *Compute string excitations in Penrose limit*
- ✦ *Probe passes through the cap extremely fast*
  $\Rightarrow$ *The string excitations are limited …*

Martinec and Warner 2009.07847 **Ceplak, Hampton and Li 2106.03841**

Suppose the probe is massless/low-mass state of energy *α'E.*

Expected string oscillation number:

$$\langle \mathcal{N}_{\rm osc} \rangle \approx \left(\frac{b\,(\alpha' E)}{n\,a^2} \right)^2$$

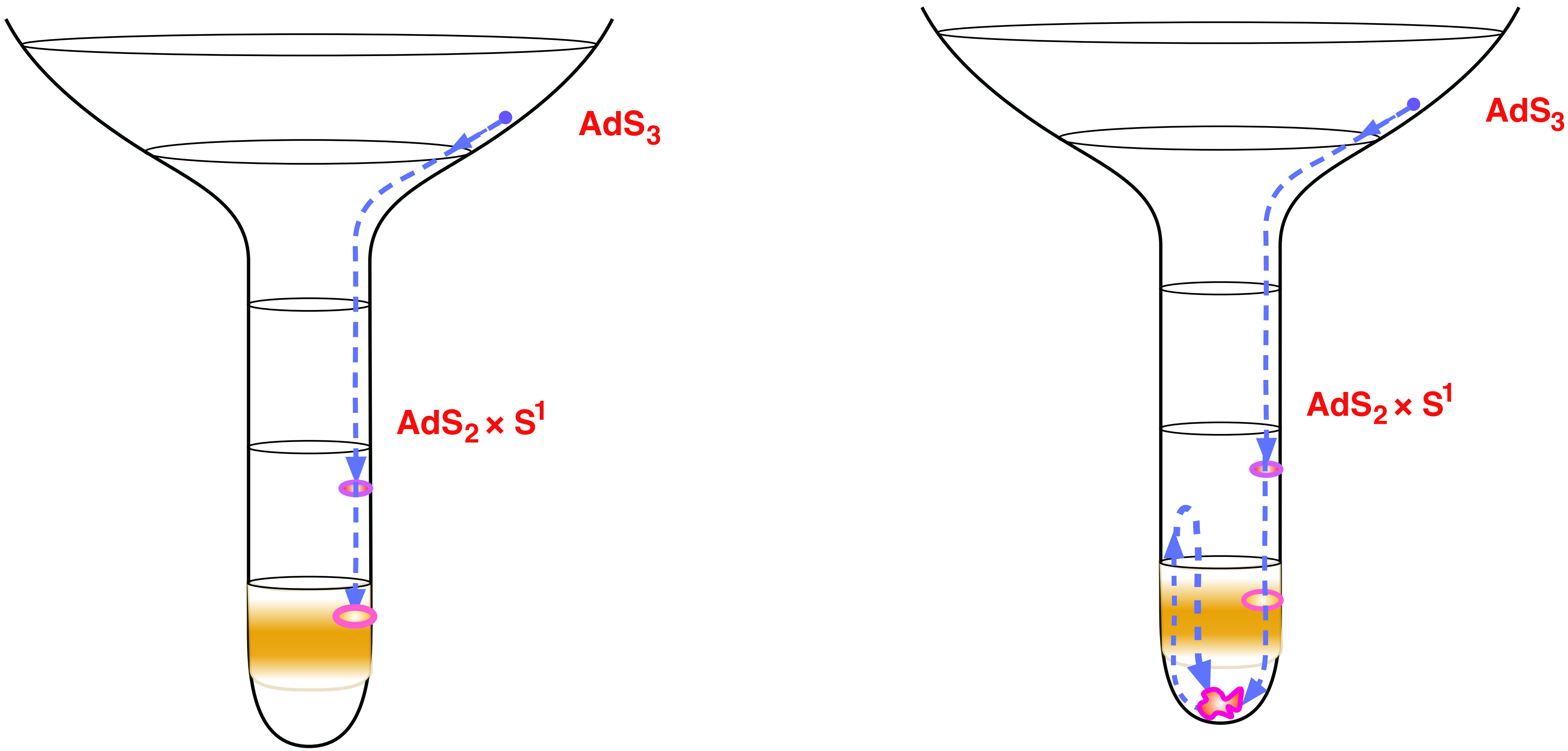
$\Rightarrow$ *The string exits the cap as a much more massive string state.*

As with all tidal phenomena, the energy for the excitations comes from the kinetic energy of the particle being influenced by the tide …

The probe is *trapped* by the geometry …

and *scrambled* into an intrinsically stringy state

Each subsequent pass excites the string further, trapping it more deeply …

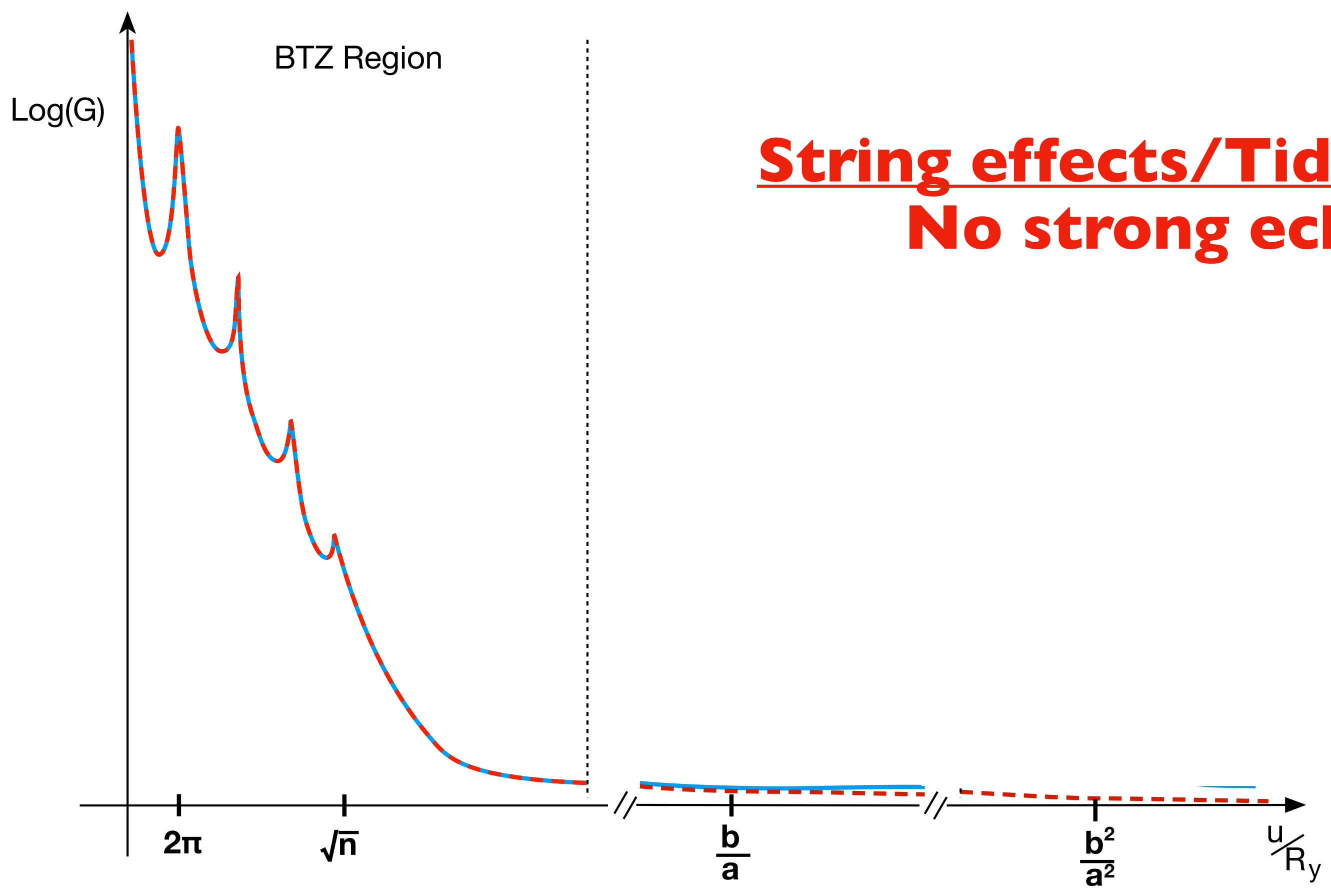


<u>Another black-hole behavior:</u>

Trapping and scrambling of infalling matter - No sharp echoes

***BUT No Horizons**: Microstructure can be seen, observed and measured by distant observers. Information is ultimately recovered*

Boundary-to-Boundary Response function:

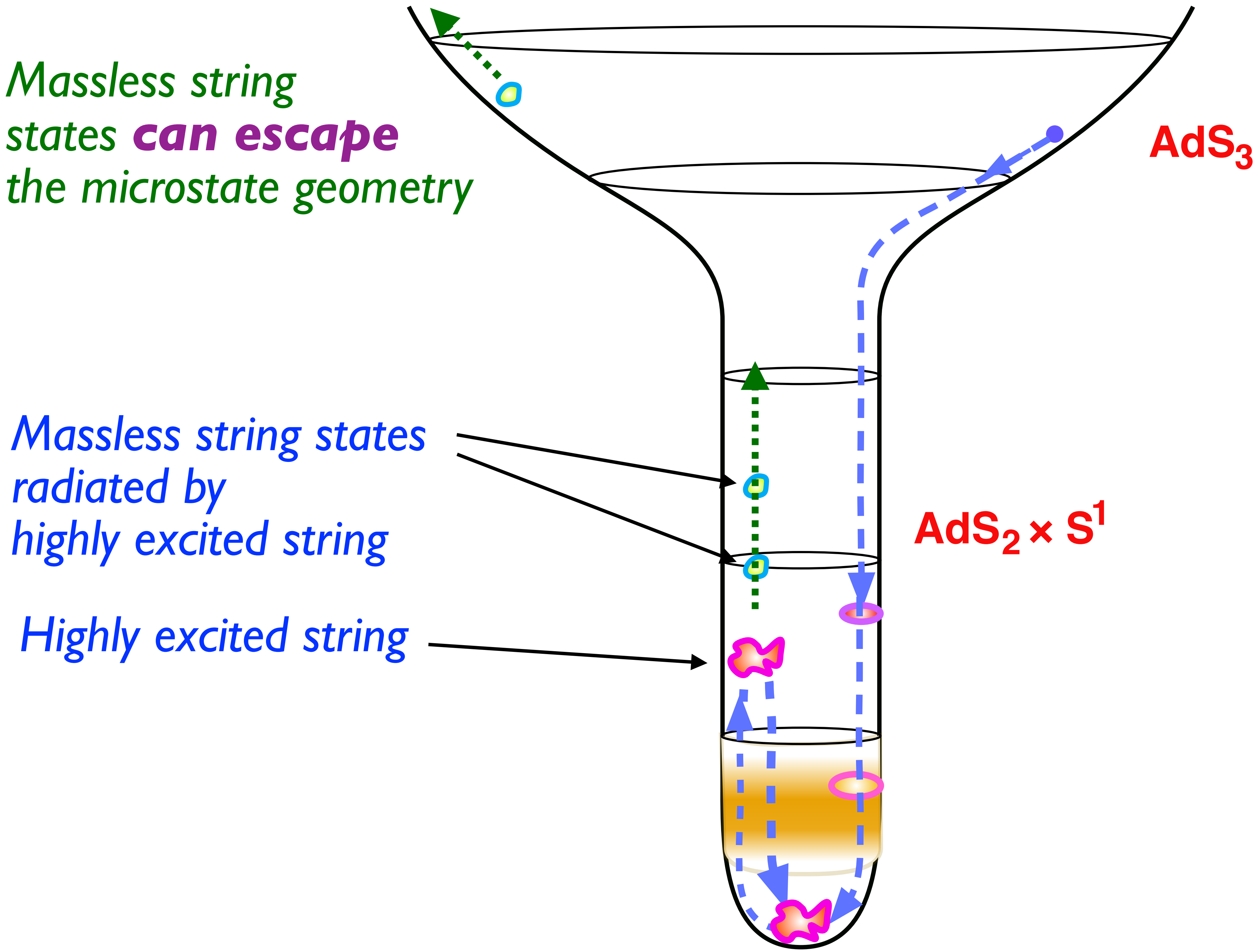


String effects/Tidal Trapping
No strong echoes …

Even supersymmetric, smooth microstate geometries exhibit complete absorption of matter …

Full string answer to Boundary-Boundary correlators:
Thermal decay and no sharp echoes

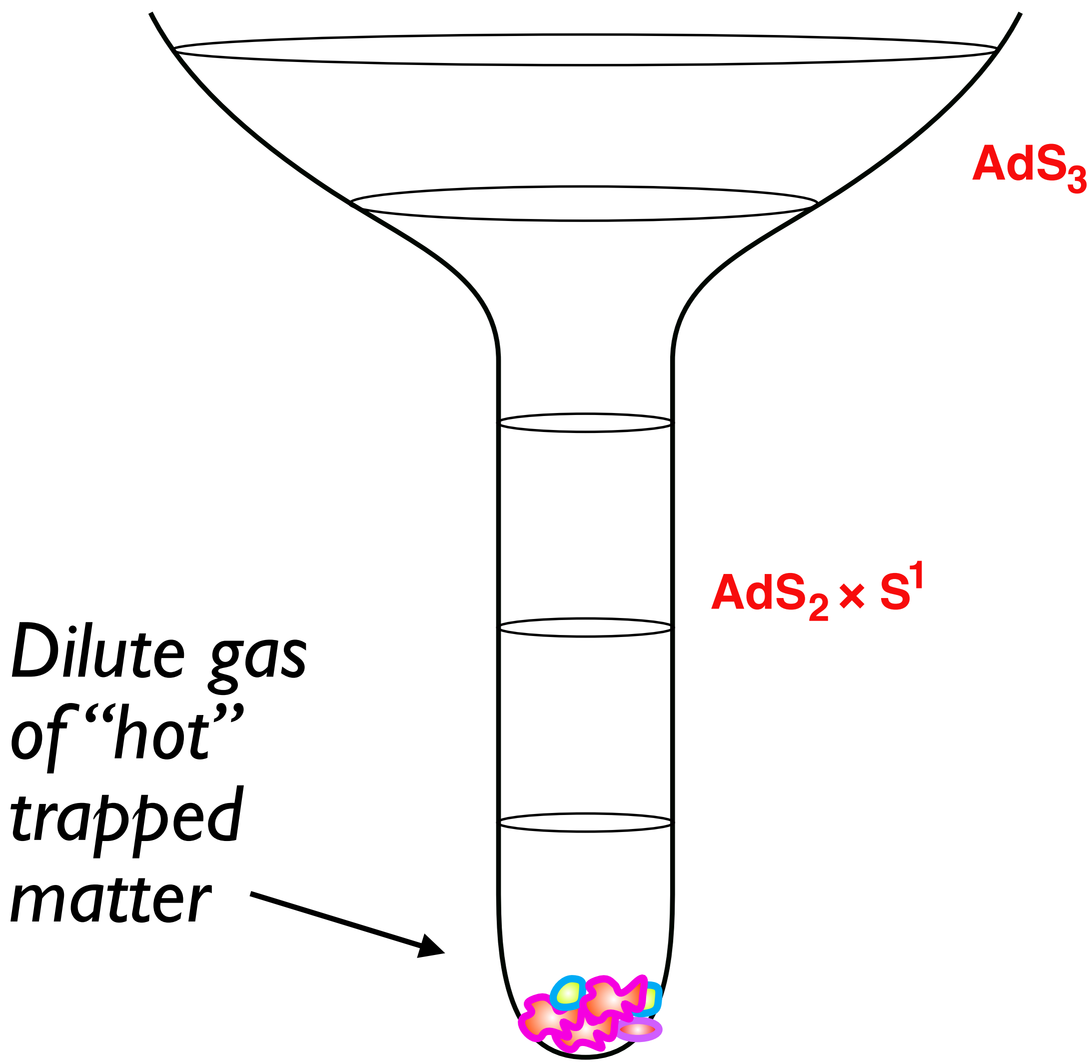
Very Weak Standard Model Echoes?



Massless string spectrum → Standard Model Physics

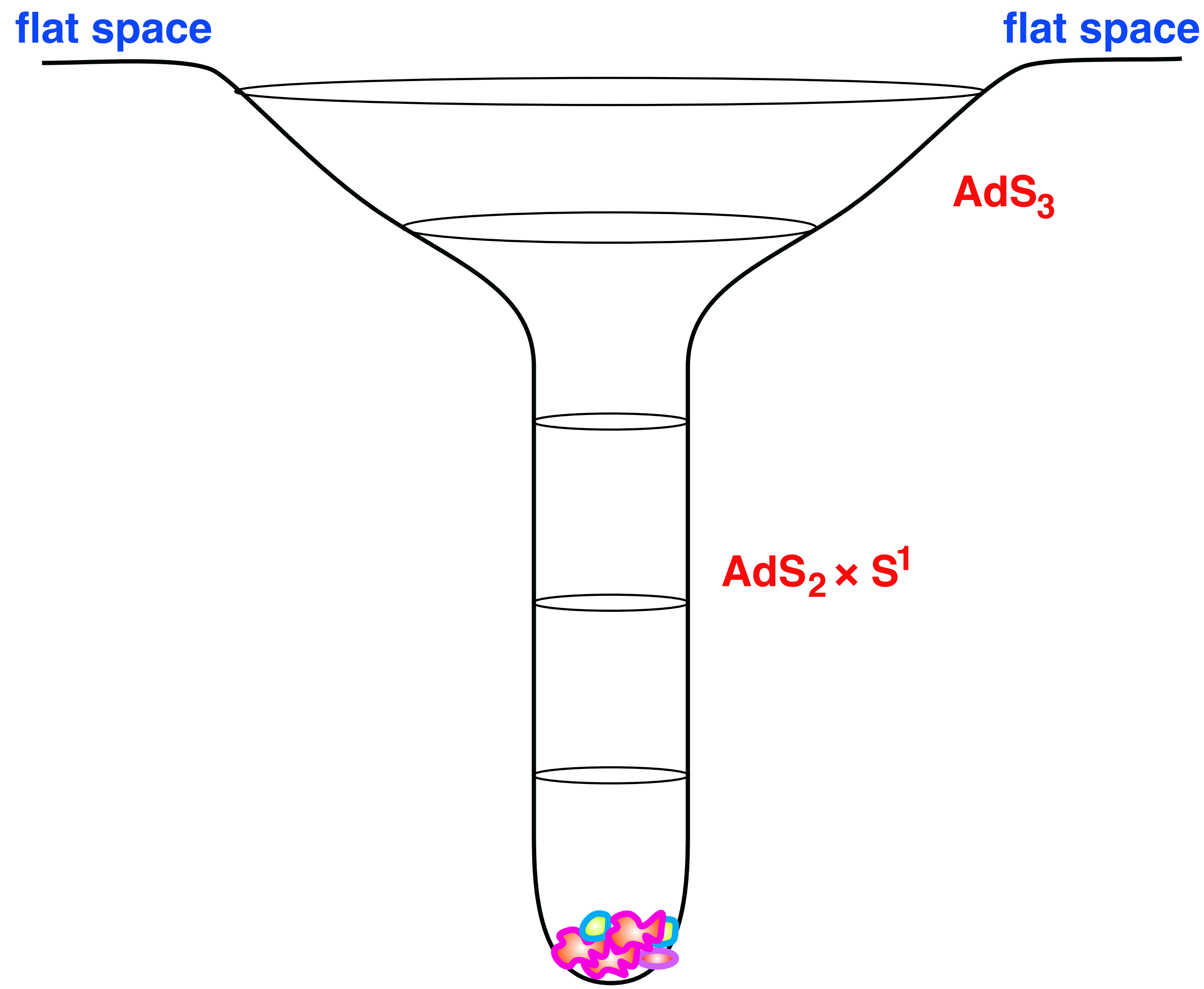
Warming up Microstate Geometries

Very Near BPS



Matter cannot "escape to infinity" in AdS

Connect the superstratum to flat space

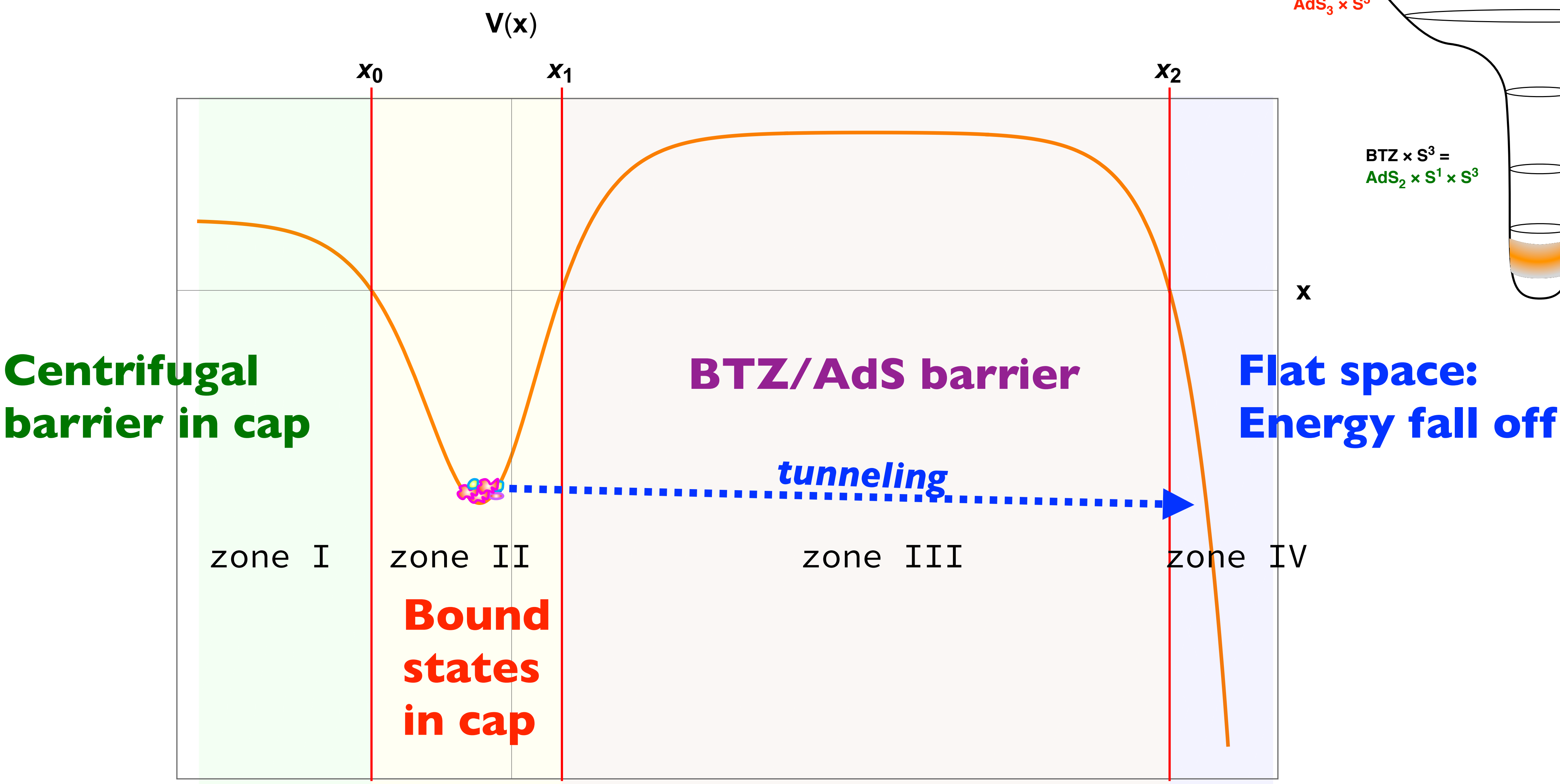


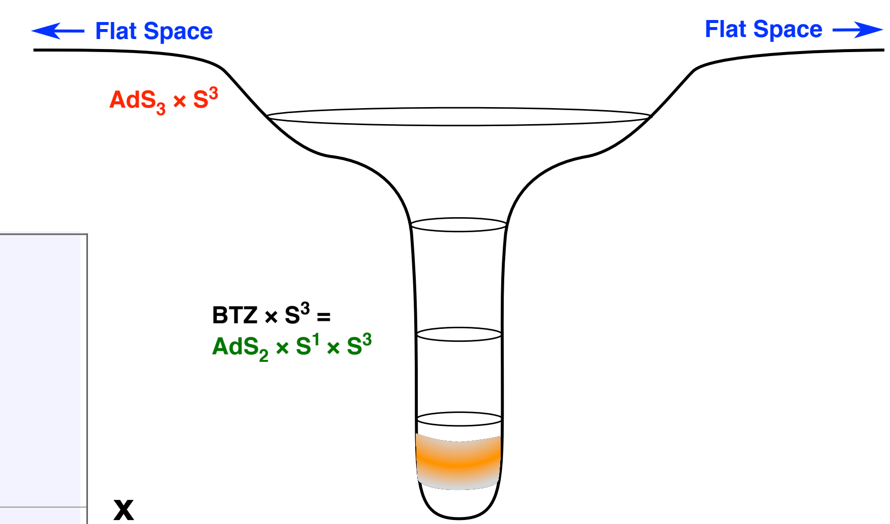
Matter can "escape to infinity" here

This should have an infinitesimal Hawking temperature

Effective WKB potential for tunneling out of superstrata

Bena, Eperon, Heidmann and Warner: 2005.11323





Compute amplitudes and tunneling decay rates of bound states in the cap:

$$t_{\rm decay} \sim \left(E_{\rm gap}\right)^{-2\,\Delta+1} \sim \left(c_{\rm CFT}\right)^{2\,\Delta-1} \qquad \Delta \gg 1$$

Mass of trapped particle $\sim \Delta(\Delta-2)$

The State of the Art for BPS/Supersymmetric Solutions

- ★ Diverse approaches to constructing such geometries based on different charge carriers in various duality frames. *Superstrata are part of a "zoo."*

- ★ Precision holography of microstate geometries: Superstrata mapped onto "Supergraviton gas" of D1-D5 system

- ★ Superstrata are sampling the "typical sector" of the D1-D5 CFT. $E_{gap} \sim \frac{1}{\mathbf{c}_{\mathbf{CFT}}}$

- ★ Tidal trapping; slow decay into flat space: *Hawking radiation?*

- ★ Large numbers of such geometries that approximate black-hole geometries arbitrarily closely $\Rightarrow$ *Extensive (?) sampling of black-hole phase space*

- ★ Entropy of states captured by known superstrata:

$$S_{Superstrata} \sim \sqrt{N_1 \, N_5} \, N_P{}^{1/4} < \sqrt{N_1 \, N_5 N_P} \sim S_{Black\,hole}$$

Shigemori 1907.03878; Mayerson, Shigemori, 2010.04172

Entropy of black hole ~ Entropy of string states around superstrata?

Two issues:

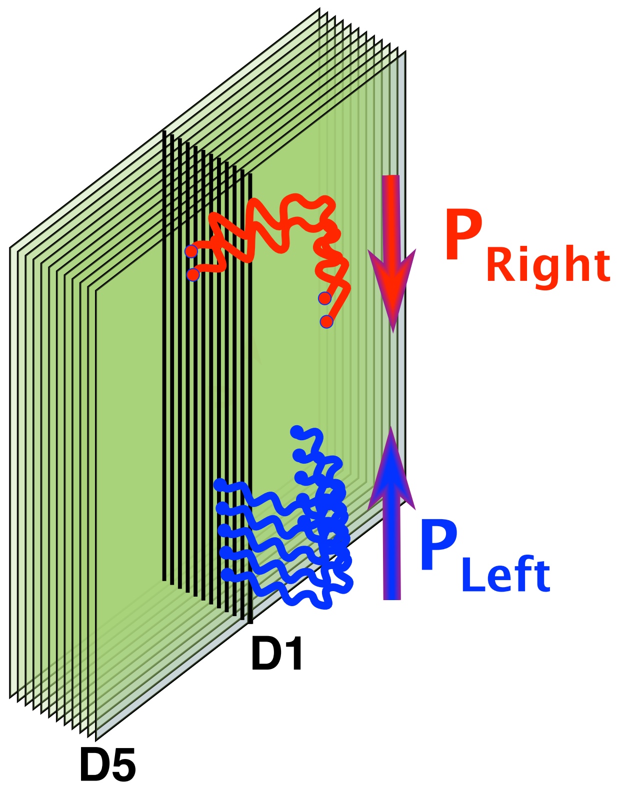
- ★ ***Non-extremal (non-susy) microstate geometries***
- ★ ***Can one find more microstate geometries?***

$$S_{Superstrata} \sim \sqrt{N_1 \, N_5} \, N_P{}^{1/4} < \sqrt{N_1 \, N_5 N_P} \sim S_{Black\, hole}$$

The semi-classical mantra:

Fuzzballs with their vast number of microstates should have vast moduli spaces of semi-classical, geometric limits…

Microstrata:
Non-extremal (non-susy) microstate geometries



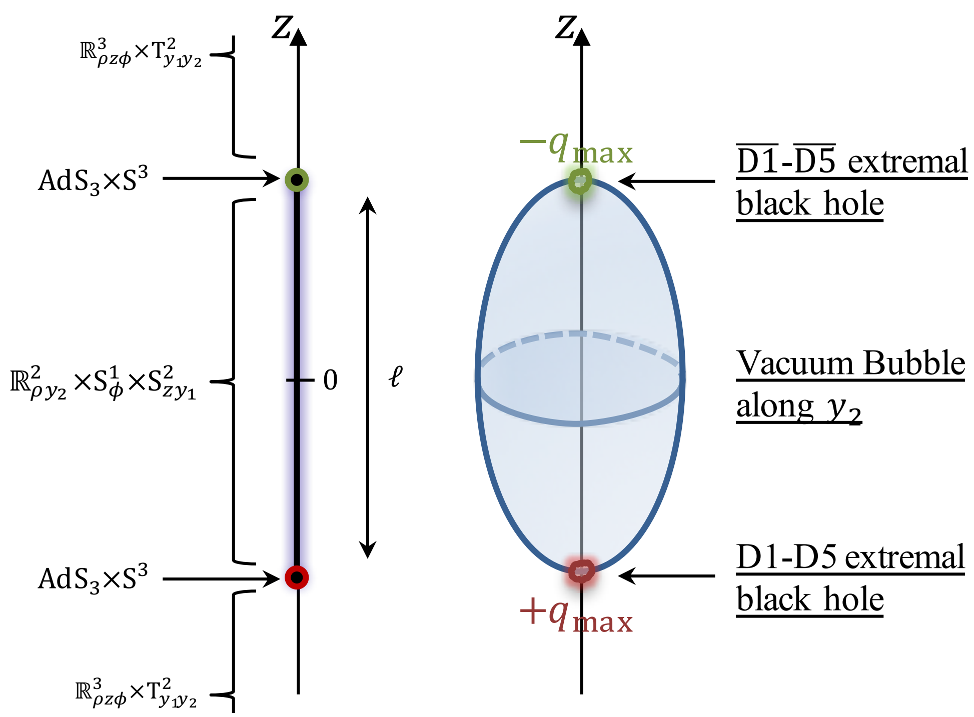
Supersymmetry breaking in Bubbled Microstate Geometries

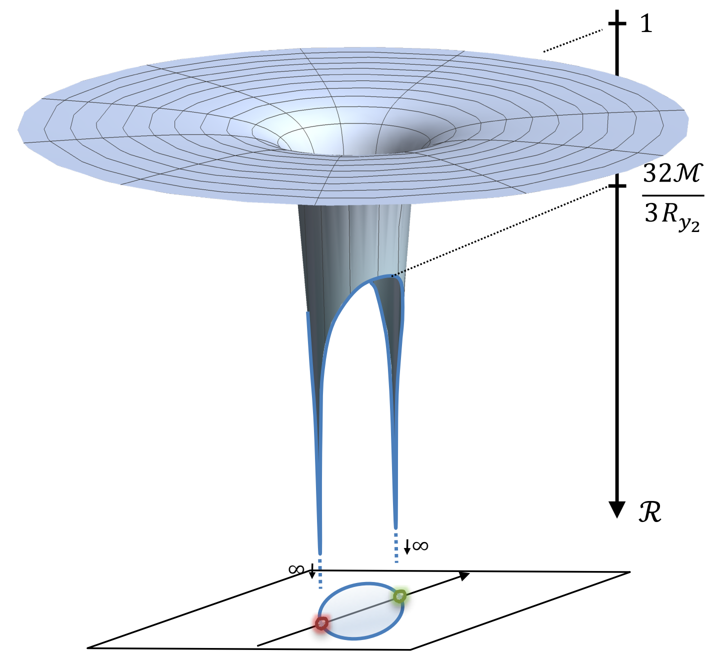
P. Heidmann Non-BPS Floating Branes and Bubbling Geometries (2112.03279)

I Bah and P. Heidmann Non-BPS Bubbling Geometries in AdS₃ (2210.06483) **Heidmann's talk**

P. Heidmann and A. Houppe Solitonic Excitations in AdS₂ (2212.05065)

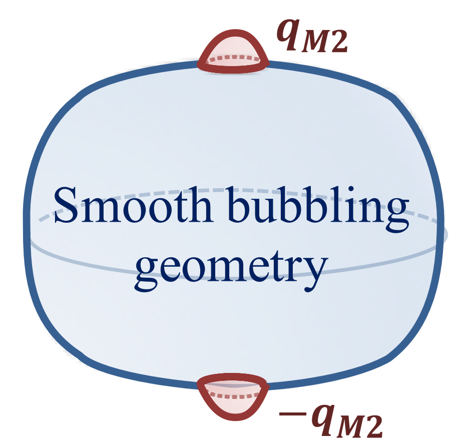
I. Bah, P. Heidmann and P. Weck Schwarzschild-like Topological Solitons (2203.12625)





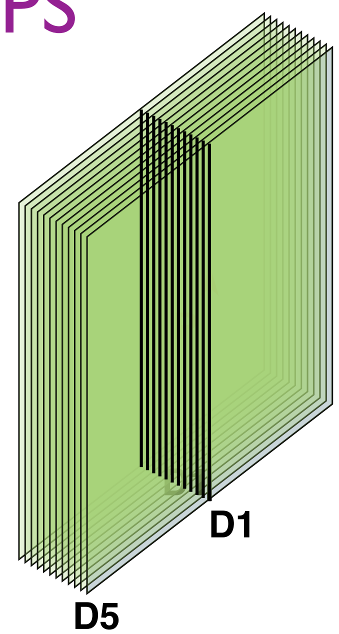
I Bah and P. Heidmann Geometric Resolution of Schwarzschild Horizon (2303.10186)

Fully back-reacted, exact, horizonless, *smooth* non-extremal microstate geometries: *Schwarzschild-like*



Black-Hole Microstructure: Momentum Excitations

¼ BPS

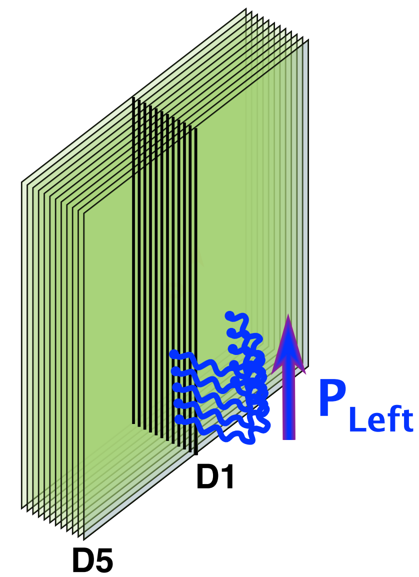


Vacuum for Left/Right Movers

(4,4) supersymmetry

Supertubes

⅛ BPS

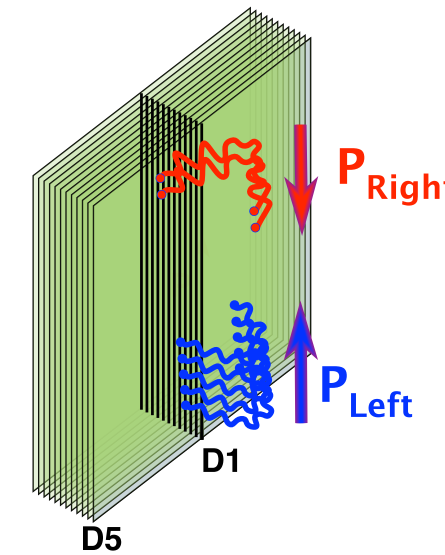


Vacuum for Right Movers only

(0,4) supersymmetry

Superstrata

Non-BPS



Left/Right excited states

(0,0) supersymmetry

Microstrata

Supersysmmetric solutions are much simpler

- ★ BPS Equations are *first order* and, for superstrata, *linear*
- ★ Supersymmetry ⇒ time independent; $\mathbf{T_{Hawking}} = 0$

Non-supersysmmetric solutions: very hard

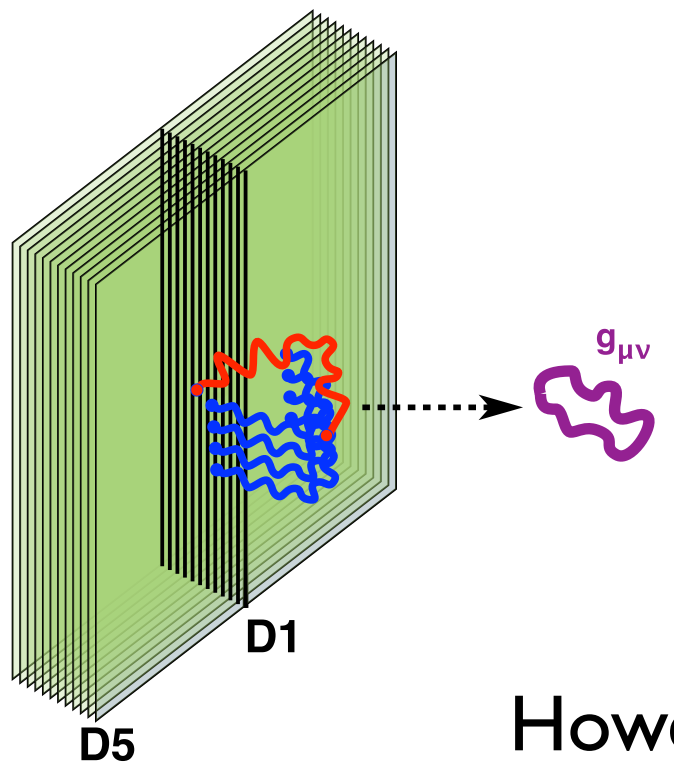
- ★ Second order, fully non-linear equations of motion; typically time-dependent

However:

Superstrata + holographic dictionary ⇒ explicit construction of some microstrata

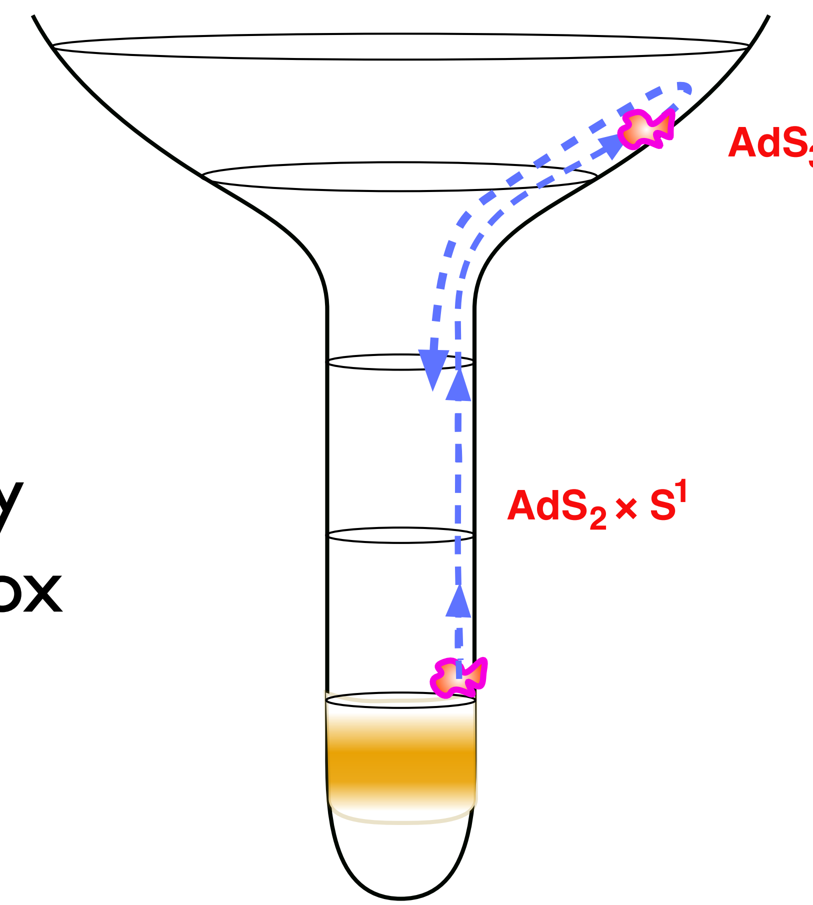
Another challenge:

Microstrata will decay into graviton multiplet excitations



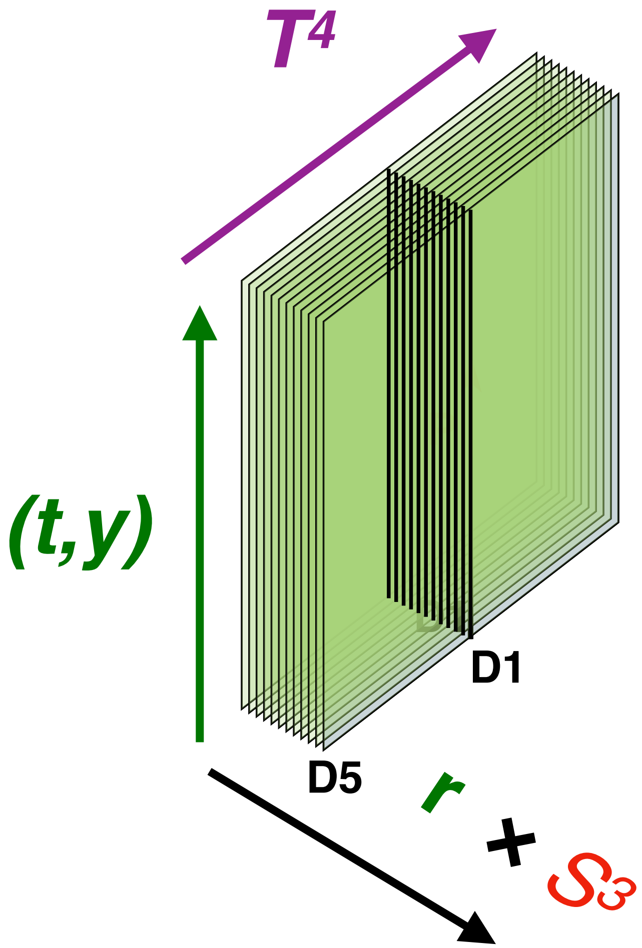
$\Rightarrow$ Solutions are necessarily time dependent?

However: Imposing AdS_3 boundary conditions effectively puts it in a box and stabilizes against decay …



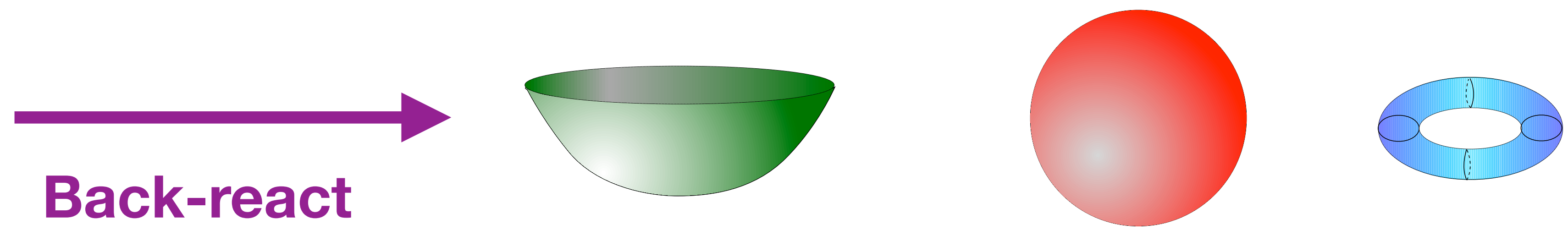
Huge simplification: Asymptotically AdS_3 microstrata can be made time-independent: *Non-extremal microstates in equilibrium with their "Hawking radiation"*

Another huge simplification:



Gauged Supergravity in three dimensions

The maximally supersymmetric supergravity ground state



(t,y,r) (θ,ψ,ϕ)

$AdS_3 \times S^3 \times T^4$

Reduce on the T^4

⇒ *Six-dimensional (1,0) supergravity coupled to tensor multiplets.*

Second Compactification:

Reduce *using special, very restricted modes* on the S^3

⇒ *Three-dimensional SO(4) gauged N= 4 supergravity coupled to hypermultiplets*

We have to solve the *equations of motion* numerically/perturbatively:
Much easier in (t,y,r) than in (t,y,r,θ,ψ,ϕ) *all together*

The Three-Dimensional Action

Mayerson, Walker and Warner, 2004.13031

Scalar fields, m_{AB} (inverse m^{AB}) and χ_A, coupled to gravity and SO(4) KK Maxwell fields, $A^{AB} = -A^{BA}$, from the S^3 fibration

$$\mathcal{L} = \tfrac{1}{4} R - \tfrac{1}{16} \mathrm{Tr}\big[(\mathcal{D}_\mu m) m^{-1} (\mathcal{D}^\mu m) m^{-1}\big] - \tfrac{1}{8} m^{AB} (\mathcal{D}_\mu \chi_A)(\mathcal{D}^\mu \chi_B) - V$$
$$- \tfrac{1}{8} m_{AC} m_{BD} F^{AB}_{\mu\nu} F^{\mu\nu CD} - \tfrac{1}{2} g_0 \varepsilon^{\mu\nu\rho} \big(A_\mu{}^{AB} \partial_\nu \widetilde{A}_\rho{}^{BA} - \tfrac{4}{3} g_0 A_\mu{}^{AB} A_\nu{}^{BC} A_\rho{}^{CA}\big)$$
$$+ \tfrac{1}{16} \varepsilon^{\mu\nu\rho} Y_{\mu AB} F^{AB}_{\nu\rho}$$

$$Y_{\mu AB} \equiv \chi_B \mathcal{D}_\mu \chi_A - \chi_A \mathcal{D}_\mu \chi_B$$

$$V = \tfrac{1}{4} g_0^2 \det(m^{AB}) \Big[2\big(1 - \tfrac{1}{4}(\chi_A \chi_A)\big)^2 + \big(m_{AB}\big(m_{AB} + \tfrac{1}{2} \chi_A \chi_B\big) - \tfrac{1}{2} m_{AA} m_{BB}\big)\Big]$$

Maximally supersymmetric vacuum: $\chi_A = 0$, $m_{AB} = \delta_{AB}$

Useful to define *scale-free* coordinates: $\xi = \dfrac{r}{\sqrt{r^2 + a^2}}, \quad \tau = \dfrac{t}{R_y}, \quad \psi = \dfrac{\sqrt{2}\, v}{R_y}$

Simplify the problem even further using the "Q-ball/Coiffuring trick"

Only scalars are time and angle dependent: e.g. $\chi_1 + i\chi_2 = \nu(\xi)\, e^{i(\omega\tau + n\psi)}$

Phases cancel in energy-momentum tensor and in currents
⇒ *Metric and Maxwell fields can be restricted to* ***functions of ξ(r) alone***

→ Many new non-extremal/non-BPS solutions

The Simplest Family of Solutions

Ganchev, Houppe and Warner, 2107.09677
Ganchev, Giusto, Houppe and Russo, 2112.03287

- The hypermultiplets: $\chi_1 + i\chi_2 = \nu(\xi)\, e^{\, i\,(\omega\,\tau + n\,\psi)}$

- The scalar shape modes: $$m_{AB} = \begin{pmatrix} e^{2\,\mu_1}\,\mathcal{M}_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 2} & e^{2\,\mu_2}\,\mathbb{1}_{2\times 2} \end{pmatrix}$$

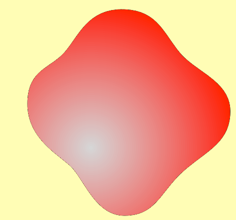
$$(\mathcal{M}_{11} - \mathcal{M}_{22}) \; + \; 2i\mathcal{M}_{12} \; = \; e^{2\,\mu_0}\, e^{2i(\omega\tau + n\psi)}$$

- Maxwell fields

$$\tilde{A}^{12} \; = \; \frac{1}{g_0}\left[\,\Phi_1(\xi)\, d\tau \; + \; \Psi_1(\xi)\, d\psi\,\right]$$
$$\tilde{A}^{34} \; = \; \frac{1}{g_0}\left[\,\Phi_2(\xi)\, d\tau \; + \; \Psi_2(\xi)\, d\psi\,\right]$$

Six-dimensions

Tensor Gauge Fields

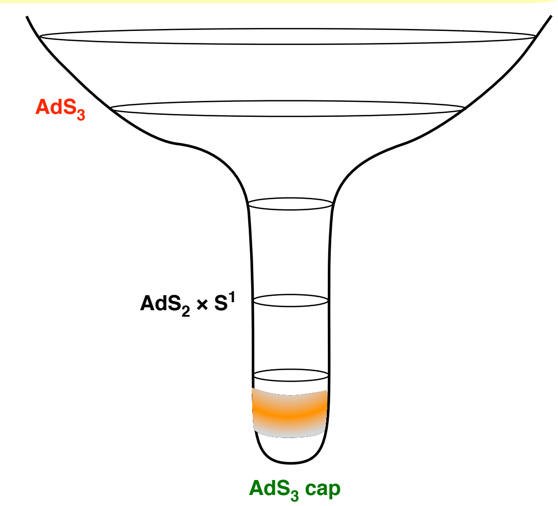


S^3 Shape

Fibering of the S^3 over the space-time

- The Space-Time

$$ds_3^2 \; = \; g_0^{-2}\left[\, -\,{\Omega_1}^2\left(d\tau + \frac{k}{(1-\xi^2)}\, d\psi\right)^2 \; + \; \frac{{\Omega_0}^2}{(1-\xi^2)^2}\left(d\xi^2 \; + \; \xi^2\, d\psi^2\right)\right]$$

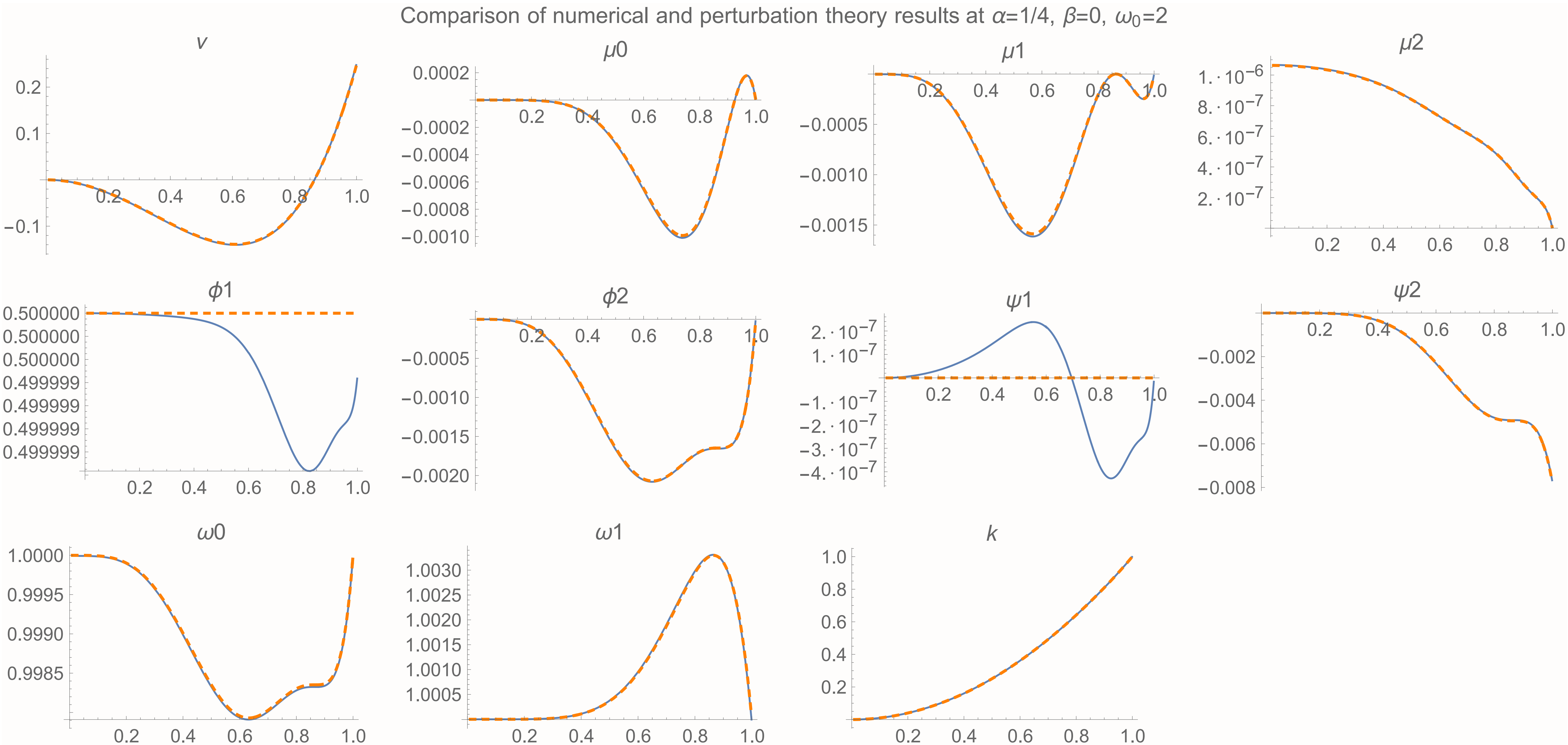


The Ansatz: Eleven functions of one variable, ξ:

ν, μ_0, μ_1, μ_2 , Ω_0, Ω_1, k, $\Phi_1, \Psi_1, \Phi_2, \Psi_2$

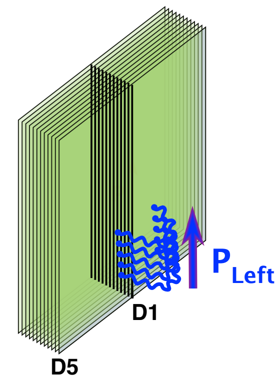
Scalars *3D Geometry* *Electromagnetic KK Fields*

Solve: Perturbation theory and Numerics



The Current State of the Art in Microstrata

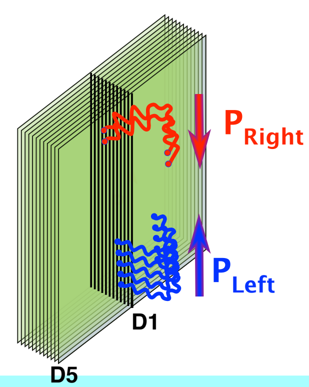
Superstratum states



$$\left(|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_1\right)^{N_{++}} \otimes \left(\frac{1}{m!\,n!}\,(J^+_{-1})^m\,(L_{-1}-J^3_{-1})^n\,|00\rangle_k\right)^{N_{k,m,n}}$$

In Principle:

Microstratum states



$$\left(|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_1\right)^{N_{++}} \bigotimes \left(\frac{1}{m!\,n!\tilde{m}!\tilde{n}!}\,(J^+_{-1})^m\,(L_{-1}-J^3_{-1})^n(\tilde{J}^+_{-1})^{\tilde{m}}\,(\tilde{L}_{-1}-\tilde{J}^3_{-1})^{\tilde{n}}\,|00\rangle_{k=1}\right)^{N_{k=1,m,n,\tilde{m},\tilde{n}}}$$

Perturbations can involve any linear superpositions of these states

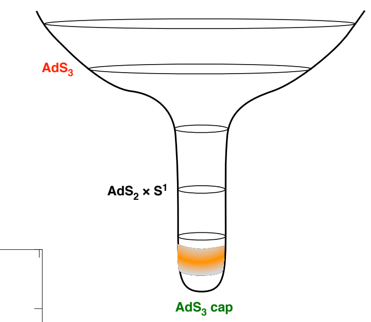
In Practice

High orders perturbation theory limited to one of two microstratum modes

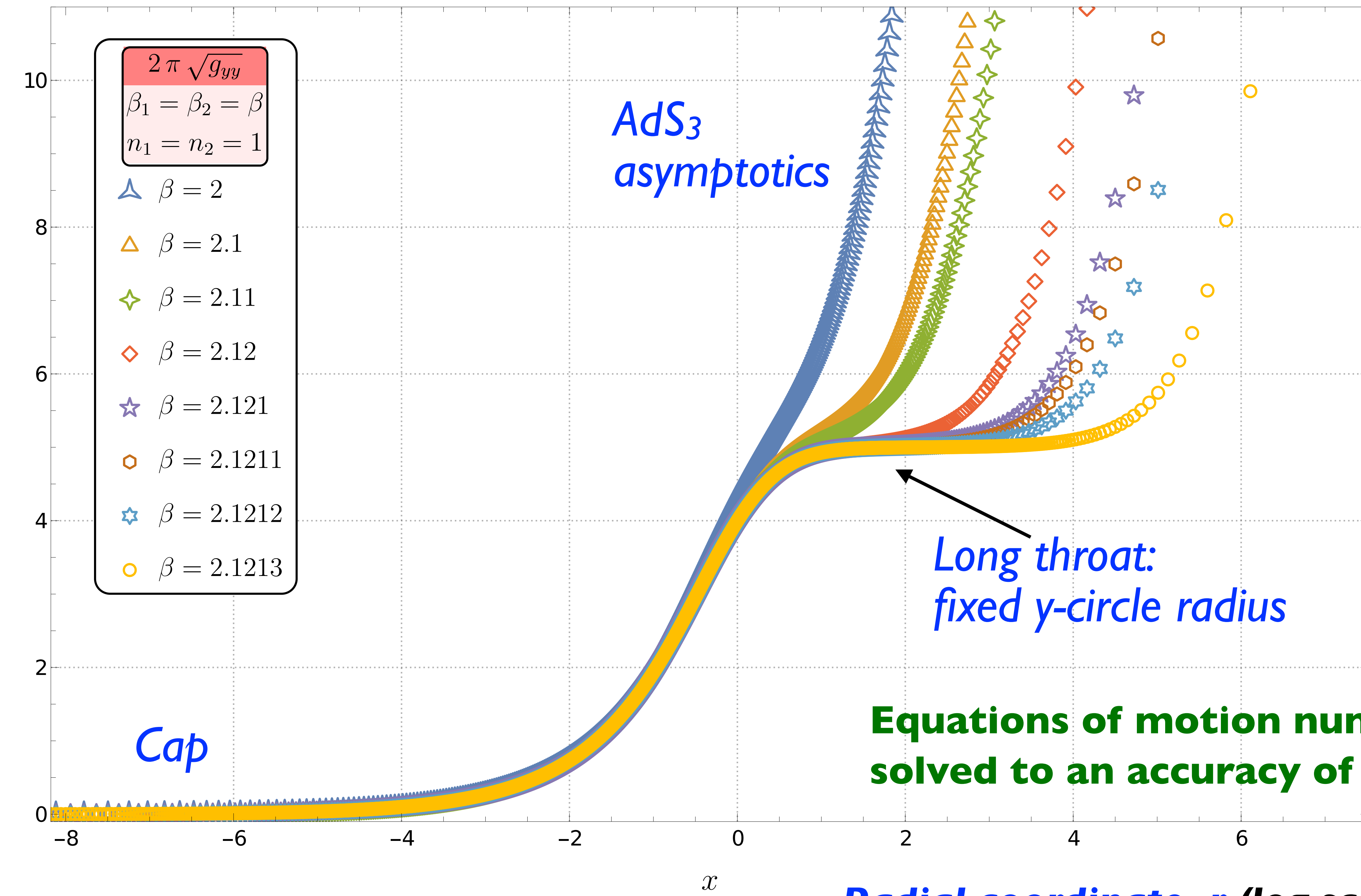
Numerics: One can explore many more modes and their interactions ..

Numerical construction of strongly non-BPS microstrata

Ganchev, Giusto, Houppe, Russo and Warner, to appear



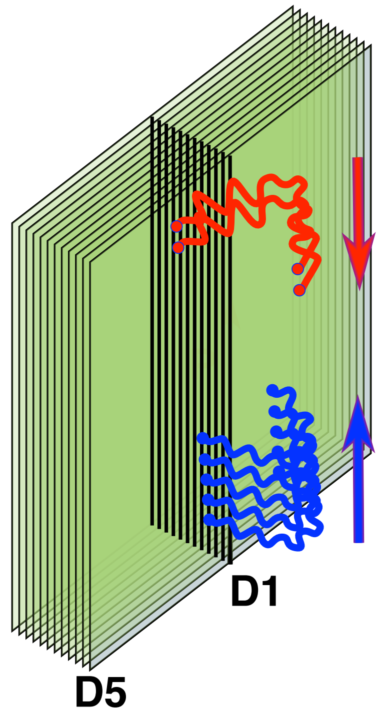
Circumference of the y-circle



Radial coordinate, r (log scale)

Equations of motion numerically solved to an accuracy of 10^{-50}.

Important results



- Non-extremal microstate geometries **exist**/can be constructed as stable gravitational solitons; many examples

- Precision holography maps microstrata onto non-BPS combinations of left + right-moving momentum states

- Normal modes of oscillation of microstrata have frequencies that depend non-linearly on the amplitudes of the states

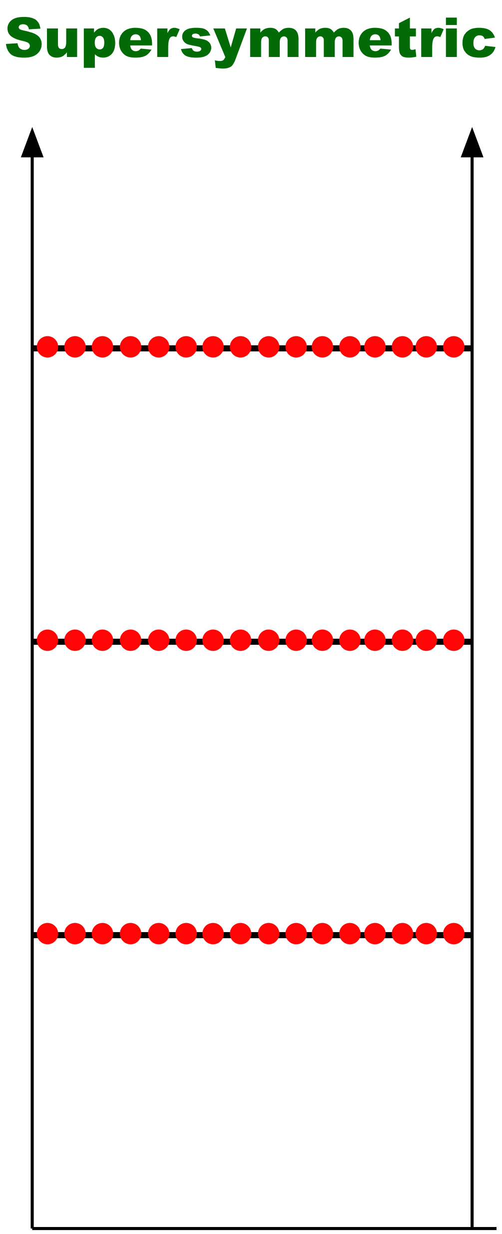
$$\omega_{non-BPS} = \omega_{Semi-classical} + \omega_{Anomalous}$$

$$\omega_{Anomalous} \sim -(Amplitude)^2 + \ldots$$

❖ Anomalous dimensions negative
$\Rightarrow$ Energies of microstrata decrease monotonically below semi-classical: Binding energy increases as supersymmetry breaking becomes larger …

❖ Transition to chaotic spectra

Spectrum of Supestrata vs Microstrata



Energy Gap ~ $\frac{1}{N}$

Degeneracies » 1

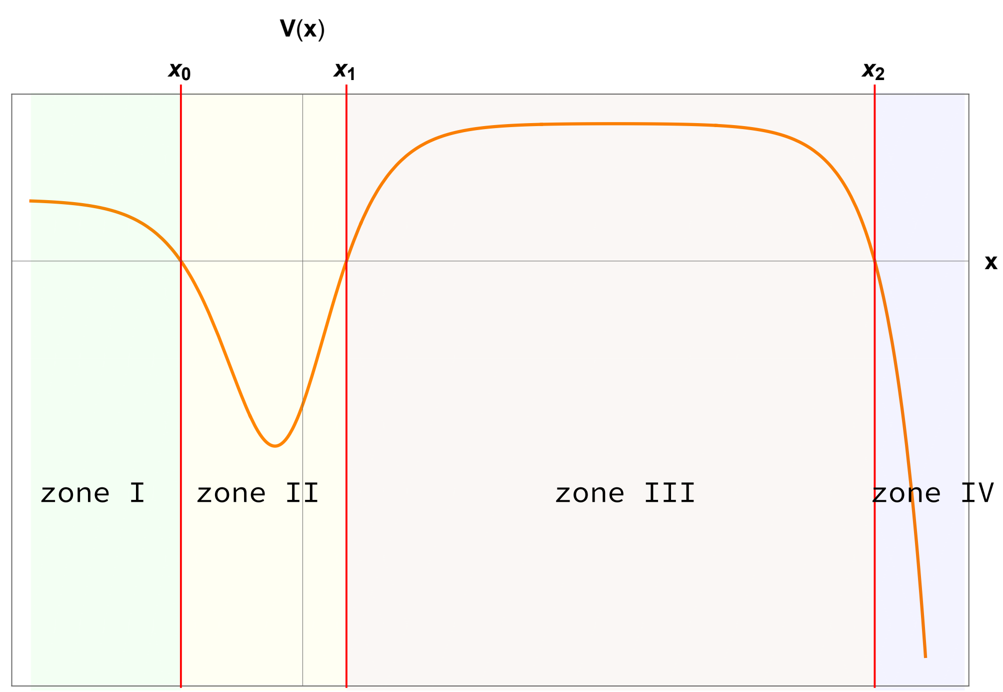
Superstrata

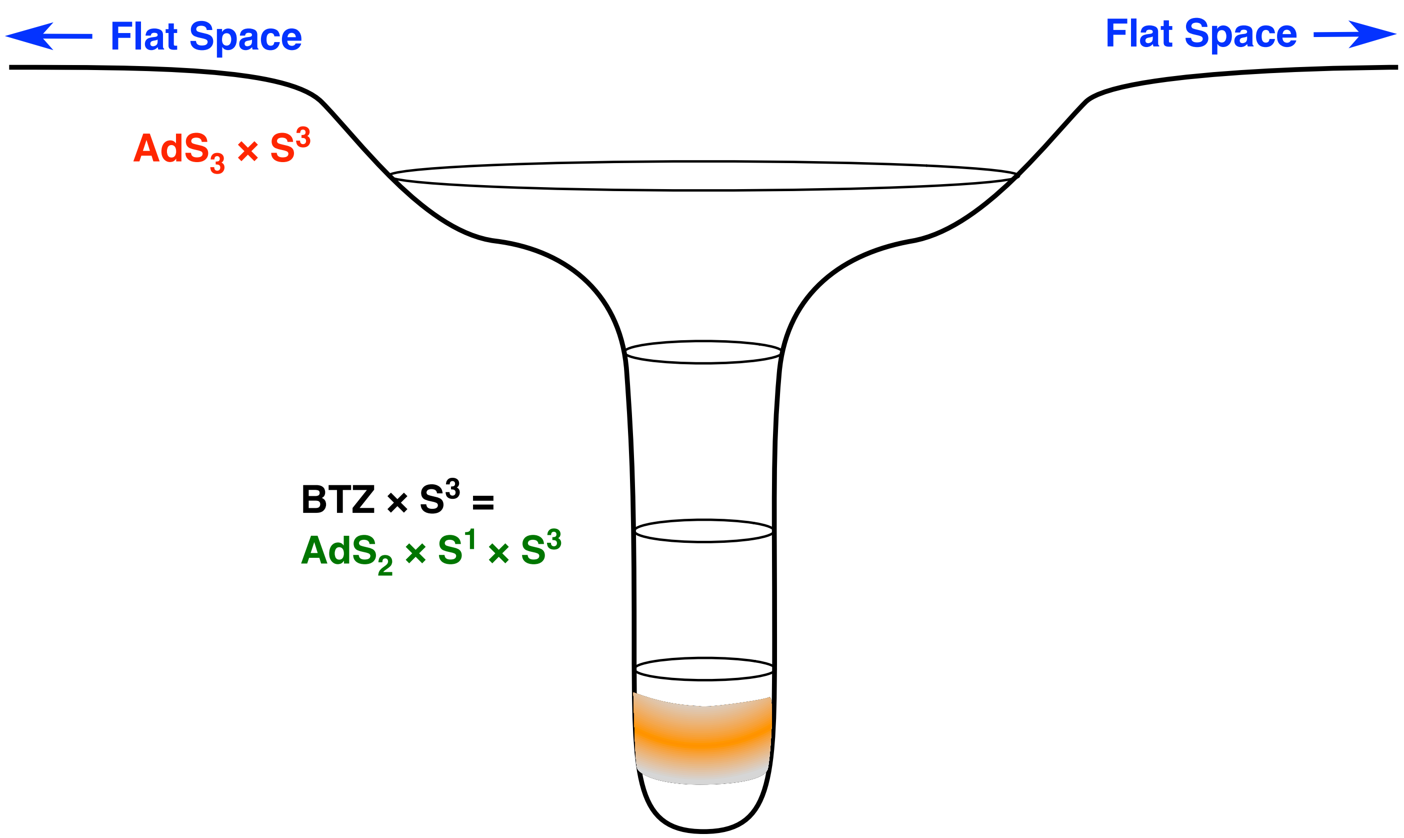
Driven by non-linear effects: $\omega_{Anomalous} \sim -(Amplitude)^2 + \ldots$

Next Steps …

- More complicated multi-mode states: transition to chaos in detail

- Couple to flat space …





and compute Decay/"Hawking radiation" as a tunneling process …

Now including back-reaction …

Generalizing Superstrata: Super-mazes and Themelia

Fractionated sectors of brane systems

More microstate geometries …

★ Supersymmetric Black-hole entropy

$$S \;=\; \tfrac{1}{4}\,A \;=\; 2\,\pi\,\sqrt{Q_1\,Q_5\,Q_P}$$

★ Entropy of states captured by known superstrata:

$$S_{Superstrata} \;\sim\; \sqrt{N_1\,N_5}\,{N_P}^{1/4} \;<\; \sqrt{N_1\,N_5 N_P} \;\sim\; S_{Black\,hole}$$

Shigemori 1907.03878; Mayerson, Shigemori, 2010.04172

★ The semi-classical mantra: *Quantum systems have semi-classical limits in terms of coherent states. Fuzzballs with their vast number of microstates should have vast moduli spaces of semi-classical, geometric limits…*

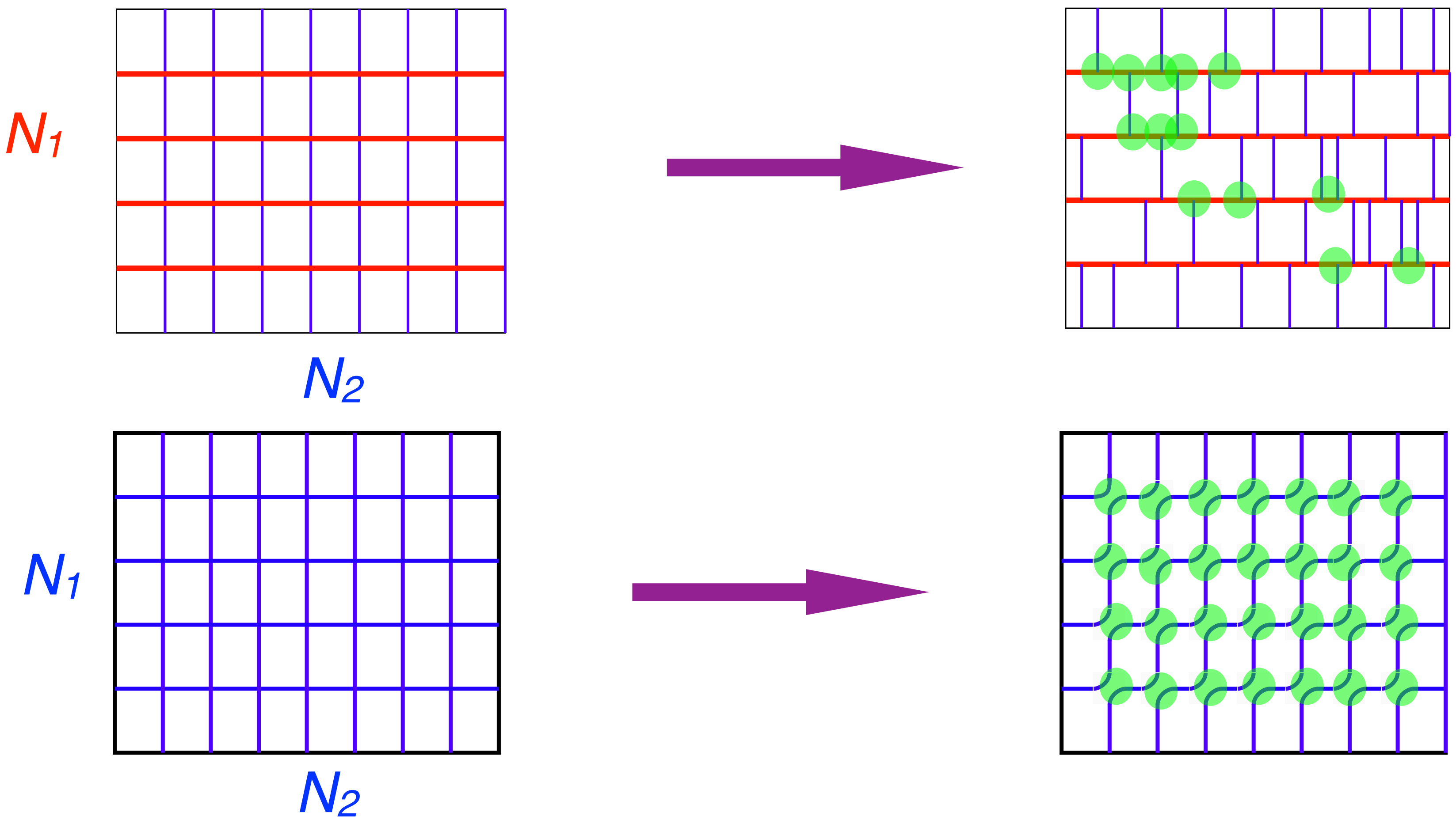
This should also be true of the highly fractionated sectors …

⇒ The phase space of black-holes/fuzzballs must contain vastly more microstate geometries …

What are we missing?

Fractionation

Martinec + Martinec, Massai, Turton



Fractionation:

Huge increase in number of degrees of freedom: moduli of brane intersections

$\Rightarrow$ *Central charge* $\sim N_1 N_2 \Rightarrow (E_{gap})^{-1} \sim c_{CFT} \sim N_1 N_2 R$

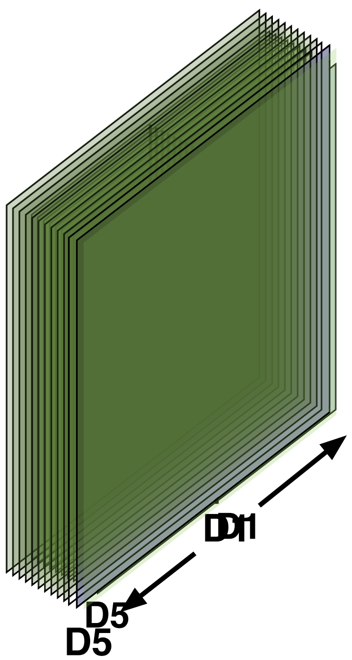
Each brane intersection ● *corresponds to a possible momentum carrier*

$\Rightarrow S \sim \sqrt{N_1 N_2 N_P}$

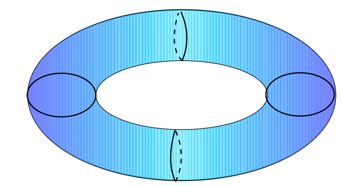
Soft modes: very long D-branes/D-brane effective tension $\sim (N_1)^{-1}$

How can you see coherent avatars of all this in supergravity? **Bena talk**

Going beyond superstrata



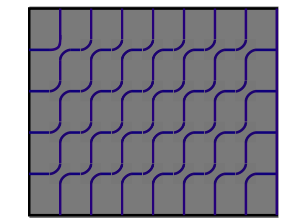
D1-D5 system ⇒ IIB supergravity compactified on T^4



Superstrata

⇒ *Six-dimensional supergravity*

But only if you smear the D1's over the D5's …



⇒ *Details of fractionation lost*

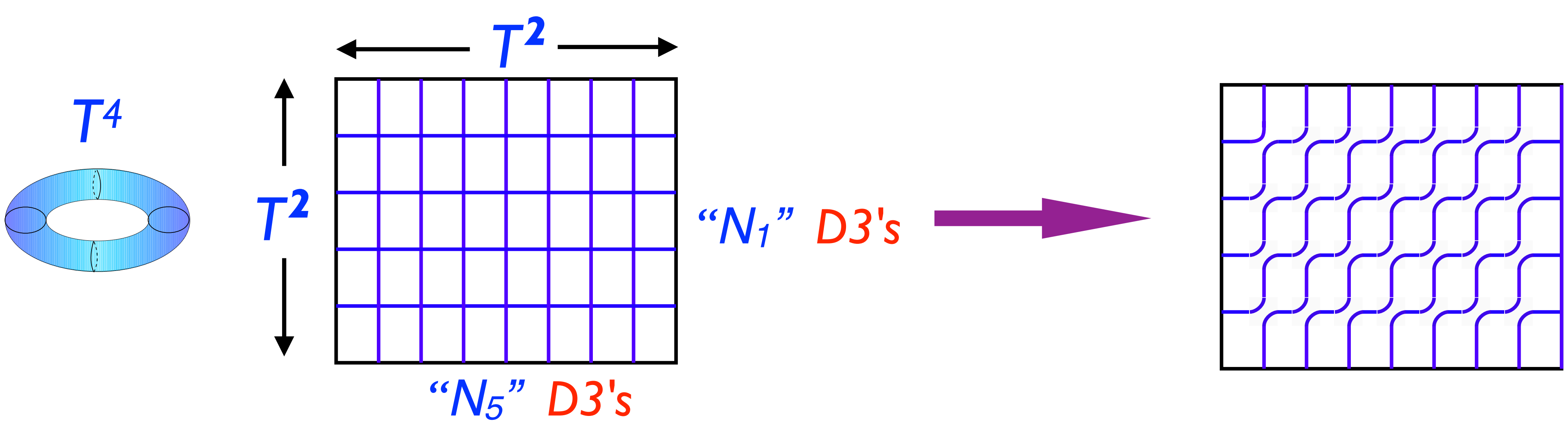
Worse: *Details of fractionation are averaged:*
Ignoring how momentum is encoded ⇒ *Horizons*

➞ *Degenerate corners of superstratum moduli space*

Bena, Ceplak, Hampton, Li, Toulikas, Warner: 2202.08844

To see fractionation in supergravity one must allow *local excitations* on the T^4

Simple formulation: T-dualize D1-D5 system twice ➞ D3-D3



Single D3 brane of length $N_1 N_5$
Whose shape has $N_1 N_5$ moduli

There are similar formulations for fractionated D2-D4 or M2-M5

Look for solutions in the full IIB/IIA/M-theory

First steps to solving this problem:

Very similar to the pre-history of superstrata

Resolving black-hole microstructure with new momentum carriers,
Bena, Ceplak, Hampton, Li, Toulikas, Warner: 2202.08844

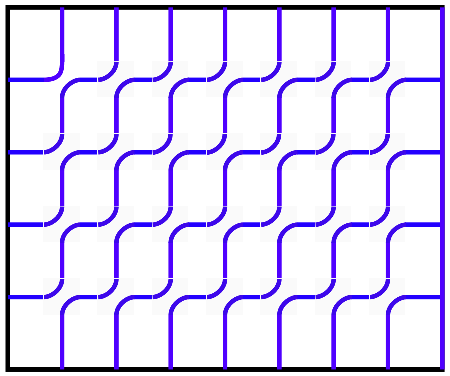
Linearizing the BPS equations with vector and tensor multiplets
Ceplak, Hampton, Warner: 2204.07170

The (amazing) Super-Maze
Bena, Ceplak, Hampton, Li, Toulikas: 2211.14326

Themelia: the irreducible microstructure of black holes
Bena, Ceplak, Hampton, Houppe, Toulikas, Warner: 2212.06158

Vector Superstrata
Ceplak: 2212.06947

Comments/Challenges

T^4 

Resolve $N_1 N_2$ intersection points

Average separation between intersections:

$$\ell_{detail} \sim (N_1 N_2)^{-\frac{1}{4}} \, \ell_{T^4}$$

This is generically going to be sub-Planckian: outside the supergravity approximation ..

… but maybe not after back-reacted momentum excitations?

The semi-classical mantra:

Fuzzballs with their vast number of microstates should have vast moduli spaces of semi-classical, geometric limits…

Obvious semi-classical limit:

Break branes into groups: $N_1 = p_1 M_1$, $N_2 = p_2 M_2$ and seek supergravity configurations of length $M_1 M_2$ with $p_1 p_2$ branes in a strand.

★ Take $p_1 p_2$ large enough for coherent states with significant gravity

★ Take T^4 to be large enough and $M_1 M_2$ small enough and so that: $\ell_{detail} \sim (M_1 M_2)^{-\frac{1}{4}} \, \ell_{T^4} \gg \ell_{Planck}$

$\Rightarrow$ *Semi-classical supergravity limit of fractionated states*

Interesting AdS-CFT issues: Is this 1+1 CFT or 3+1 dimensional QFT? Are twisted sector states in 1+1 dimensions a limit of states in 3+1 dimensional Yang-Mills?

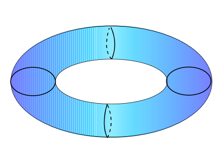
Final comments: Superstrata and Microstrata

- ★ Backed by parallel developments in high-precision holography
  - ❖ Deep, scaling superstrata *accessing the typical sector* of the CFT
    $E_{gap} \sim (c_{CFT})^{-1}$
  - ❖ Dictionary of microstate structure captured by gravity ..
- ★ Black-hole-like behavior
  - ❖ *Geometry closely approximates that of black holes*
  - ❖ *Tidal scrambling and Tidal trapping*
  - ❖ *Bound states and tunneling from superstrata: Hawking radiation*
- ★ Whole new universe of non-extremal microstrata …
  - ❖ *Existence!* ❖ *Spectrum* ❖ *Transition to chaos*
- ★ Entropy of states captured by known superstrata:

$$S_{Superstrata} \sim \sqrt{N_1 N_5} N_P{}^{1/4} < \sqrt{N_1 N_5 N_P} \sim S_{Black\,hole}$$

- ★ New ideas to extend superstrata/ microstrata so that

$$S_{Superstrata} \sim \sqrt{N_1 N_5 N_P} \sim S_{Black\,hole}$$

Replace the T^4  by much more complicated string-web topologies

Final comments on *Microstate Geometries*

★ *The semi-classical mantra*: There will always be coherent expressions of the *fuzzball phase-space* that can be captured by supergravity

★ The only precise, well-defined backgrounds for supporting and doing the analysis of horizon-scale microstructure. **Gibbons and Warner, arXiv:1305.0957**

★ The practical: Generic fuzzballs are still impossible to construct; microstate geometries provide a precise starting point for exploring different phases of black-hole physics and studying horizon-scale microstructure

★ Supergravity can also describe large-scale collective effects of strongly-coupled quantum systems: effective geometries and effective hydrodynamics of fuzzballs …

Bena, Martinec, Mathur and Warner,
2203.04981 Snowmass White Paper: Micro- and Macro-Structure of Black Holes
2204.13113 Fuzzballs and Microstate Geometries: Black-Hole Structure in String Theory