Gravitational waves from gravitational Chern-Simons term

Atsushi NARUKO

[**C**enter for **G**ravitational **P**hysics, **YITP**]

in collaboration with

R. Namba [McGill] & K. Yonekura [Tohoku]

Relavant action

✓ In the first day, Soda-san has introduced
the “**relevant action**” in this field/conference

$$\mathcal{L} = R - (\nabla\phi)^2 - V(\phi)$$

$\phi F F^*$

$\phi R R^*$

this talk

Contents

primordial

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- see Peter's talk

inflation

✓ inflation is the accelerated expansion…

- see Peter's talk

predictions of inflation

✓ due to the quasi-exponential expansion of the universe..

→ quantum fluctuations generate **tensor perturbations = GWs** whose amplitude is O(H) and hence nearly scale-invariant

$$h_{ij} \sim \mathcal{O}(H)$$

✓ the Friedmann equation relates H with ρ :

$$3M_{\rm pl}^2 H^2 = \rho$$

☞ observing h_{ij} ~ H will determine the energy scale of inflation ??

NO !!!   h > H

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Does the detection of primordial gravitational waves exclude low energy inflation?

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A R T I C L E   I N F O

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A B S T

We show
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see also

- **Peter's talk & work**
- **Namba, Peloso, Shiraishi, Sorbo, Unal [2016]**
- **Dimastrogiovanni, Fasiello, Fujita [2017] …**

Spectator axion-SU(2)

ABSTRACT

We show that a detectable tensor-to-scalar ratio ($r \geq 10^{-3}$) on the CMB scale can be generated even during extremely low energy inflation which saturates the BBN bound $\rho_{\rm inf} \approx (30\ \mathrm{MeV})^4$. The source of the gravitational waves is not quantum fluctuations of graviton but those of $SU(2)$ gauge fields, energetically supported by coupled axion fields. The curvature perturbation, the backreaction effect and the validity of perturbative treatment are carefully checked. Our result indicates that measuring r alone does not immediately fix the inflationary energy scale.

$$+\frac{\lambda}{4f}\chi F^a_{\mu\nu}\tilde{F}^{a\mu\nu} \quad \Rightarrow \quad h_{ij} = \mathcal{O}(H) + h^{\rm source}_{ij}[\mathbf{A}^a]$$

> O(H)

1, VEV at BG

$$\bar{A}^a = \Phi(t)\,\delta^a_i$$

⇔ isotropy of BG

2, chirality in SU(2)

$$A_R \qquad A_L$$

⇔ axion-SU(2) term

3, (sourced) chiral GWs

$$h^{\rm source}_{ij}[\mathbf{A}^a]$$

⇔ GW source at 1st order

more simple model ??

✓ OK.. the result is great..

I don't want to speak ill of
the previous models at all…

but we have to assume

1, SU(2) fields have to **couple with axion** **[parity-violation]**

2, SU(2) fields have specific **BG VEVs**

3, and then **SU(2) fields** source **GWs**

$$+\frac{\chi}{\Lambda} R\tilde{R}$$

parity-violation in **gravity** → (w/o SU(2)) → chiral **GWs**

The model

✓ model : **GR** + **GCS** + **axion** (spectator) + **inflation**

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2} R + \frac{1}{16} \frac{\chi}{\Lambda} R \tilde{R} + P[(\nabla \chi)^2]$$ **+ inflaton**

generic term
w/ shift-symmetry

$$R \tilde{R} = \epsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta}{}^{\mu\nu} R_{\gamma\delta\mu\nu}$$

Sato, Kanno & Soda

✓ Soda-san is great

PHYSICAL REVIEW D **77**, 023526 (2008)

Circular polarization of primordial gravitational waves in string-inspired inflationary cosmology

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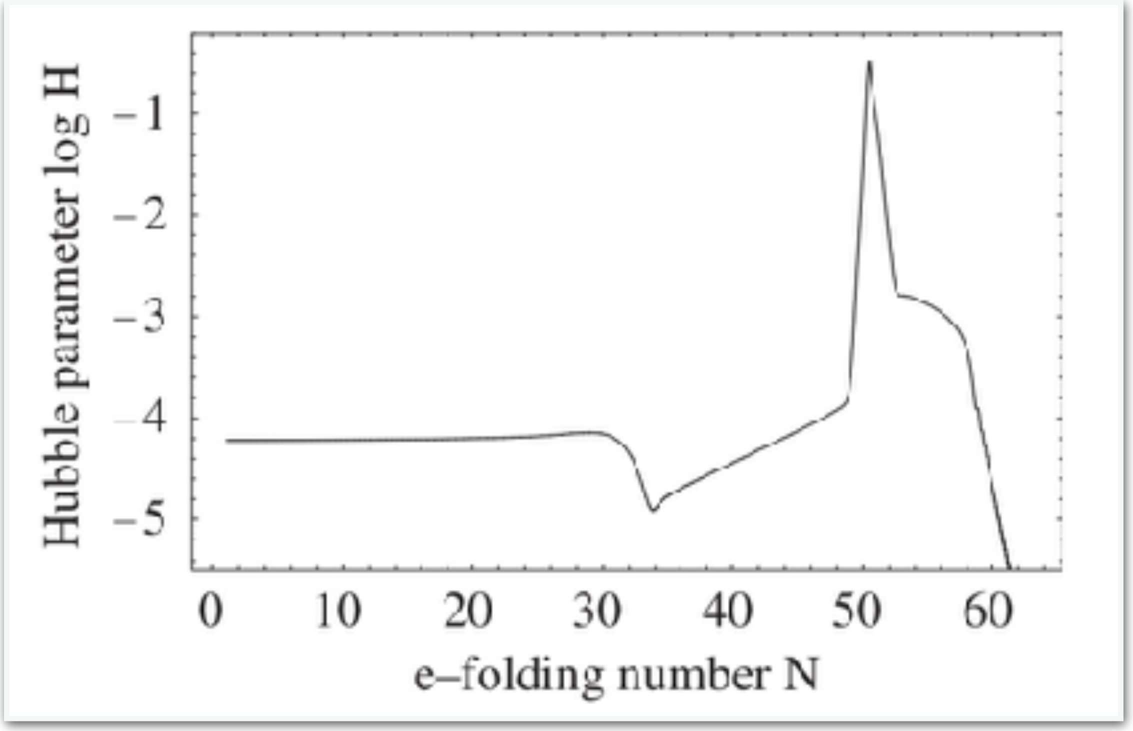
(Received 3 July 2007; published 23 January 2008)

We start with the action motivated from string theory given by [9]

$$S = \frac{1}{2}\int d^4x\sqrt{-g}R - \int d^4x\sqrt{-g}\left[\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi + V(\phi)\right] - \frac{1}{16}\int d^4x\sqrt{-g}\xi(\phi)R^2_{\rm GB} + \frac{1}{16}\int d^4x\sqrt{-g}\omega(\phi)R\tilde{R},$$

R_{GB} affects **BG universe & GWs**

$$R^2_{\rm GB} = R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} - 4R^{\alpha\beta}R_{\alpha\beta} + R^2$$



The model

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generic term
w/ shift-symmetry

$$R\tilde{R} = \epsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta}{}^{\mu\nu} R_{\gamma\delta\mu\nu}$$

✓ BG evolution :

$$3M_{\rm pl}^2 H^2 = \rho_{\rm inf.} + \rho_{\rm axion}$$

GWs from GCS

✓ GWs propagating in z-direction ⇒ helicity decomposition

$$\delta g_{ij} = a^2 \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & h_- & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} 2h_L = h_+ + i h_\times \\ 2h_R = h_+ - i h_\times \end{cases}$$

✓ quadratic action for GWs :

$$S_{GW} = \int \mathrm{d}t\, \mathrm{d}^3x\, a^3 \left(M_{\rm pl}^2 \mathcal{L}_{GR} + \mathcal{L}_{GCS} \right)$$

$$\mathcal{L}_{GR} = \frac{1}{2}\left(|\dot{h}_\pm|^2 - \frac{k^2}{a^2}|h_\pm|^2 \right) \qquad \mathcal{L}_{GCS} = \pm \frac{k}{a}\frac{\dot{\chi}}{\Lambda}\mathcal{L}_{GR}$$

origin of chirality

nature of GWs

✓ action for the canonically normalised GW, $\widetilde{h}$ = $z_\pm$ h :

$$\mathcal{L} = \frac{1}{2} a^3 \left(M_{\rm pl}^2 \pm \frac{k}{a} \frac{\dot{\chi}}{\Lambda} \right) \left(|\dot{h}_\pm|^2 - \frac{k^2}{a^2} |h_\pm|^2 \right)$$

= $z_\pm^2$

$$= \frac{1}{2} \left[|\dot{\widetilde{h}}_\pm|^2 - \left(\frac{k^2}{a^2} - \frac{\ddot{z}_\pm}{z_\pm} \right) |\widetilde{h}_\pm|^2 \right]$$

effective mass

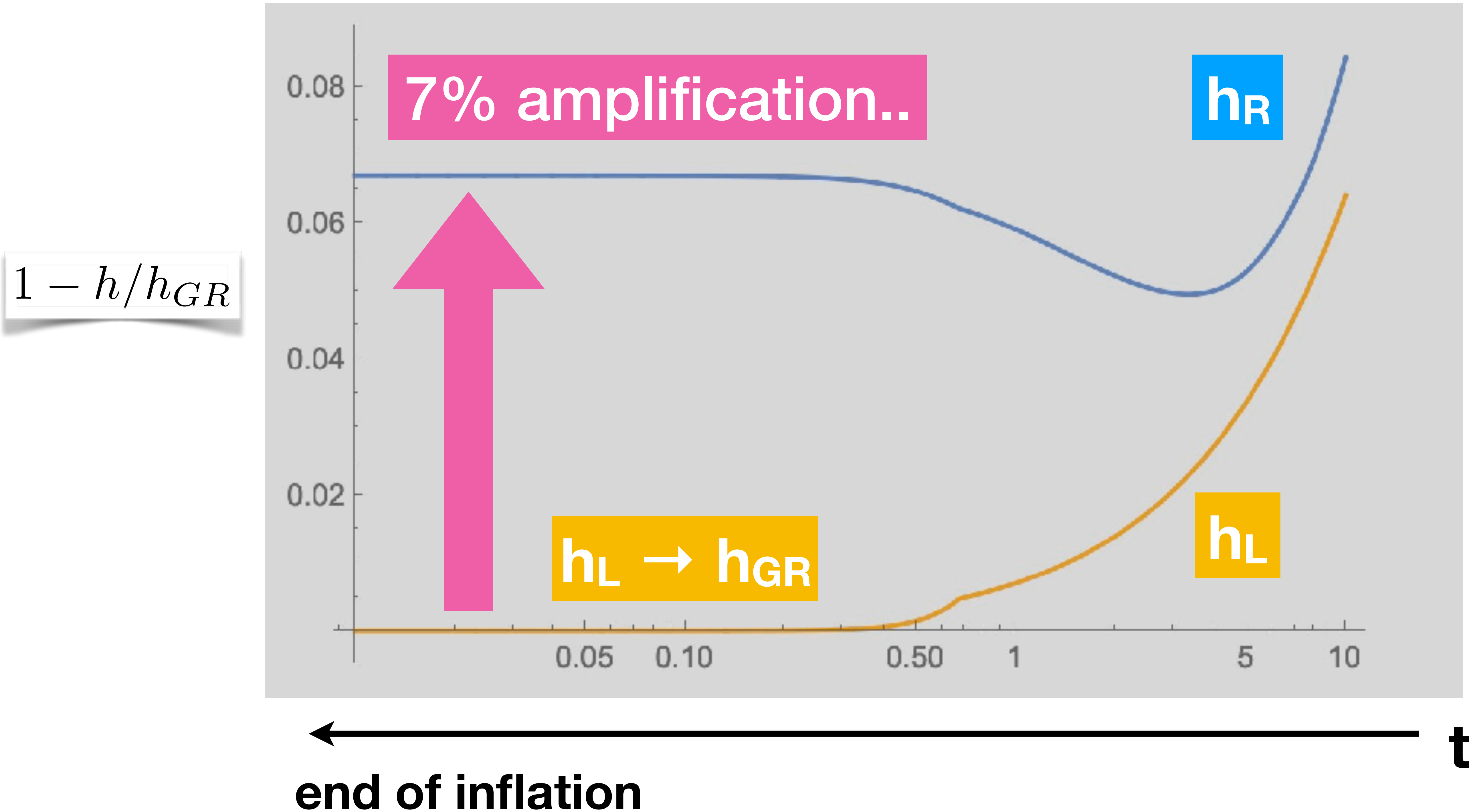
$$\ddot{\phi} + 3\, c_s^2\, H \dot{\phi} = 0$$

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2 X P_{,XX}}$$

$$X = -(\nabla\chi)^2/2 = \dot{\chi}^2/2$$

$$\frac{\ddot{z}_\pm}{z_\pm} \simeq H^2 + \mathcal{O}(\varepsilon_H\,, \eta_H) \pm \frac{k}{a M_{\rm pl}^2} \frac{\dot{\chi}}{\Lambda} \left[H \partial_t (c_s^2) + (1 + c_s^2) H^2 \right]$$

preliminary result



theoretical subtleties

✓ UV-behaviour of GWs :

→ since the GCS includes more spatial derivatives,

it can **dominate on small scales**..

$$\mathcal{L} = M_{\rm pl}^2 R + \frac{\chi}{\Lambda} R \widetilde{R} \;\rightarrow \dot{h}^2 - \frac{k^2}{a^2} h^2 + \frac{k}{a} \frac{\dot{\chi}}{\Lambda} \dot{h}^2 + \left(\frac{k}{a}\right)^3 \frac{\dot{\chi}}{\Lambda} h^2$$

✓ By tuning the functional form of P [(∇χ)^2], GWs can be amplified (as much as you want).

→ but… **instabilities in scalar perturbations** can show up

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2X P_{,XX}} \quad \mathbf{\geq 0}$$

conclusion

- ✓ We have considered a model with the gravitational Chern-Simons term keeping the shift-symmetry of axion.

- ✓ With the nice form of axion's kinetic term, the amplitude of sourced GWs can be larger than that from vacuum fluct.

- ✓ Despite this interesting results, there are several theoretical subtleties which should be overcome..
  - -- UV behaviour of GWs
  - -- stability of axion

Thank you very much