# Search for ultralight scalar dark matter with NANOGrav pulsar timing arrays

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Ryo Kato, Jiro Soda, arXiv:1904.09143.

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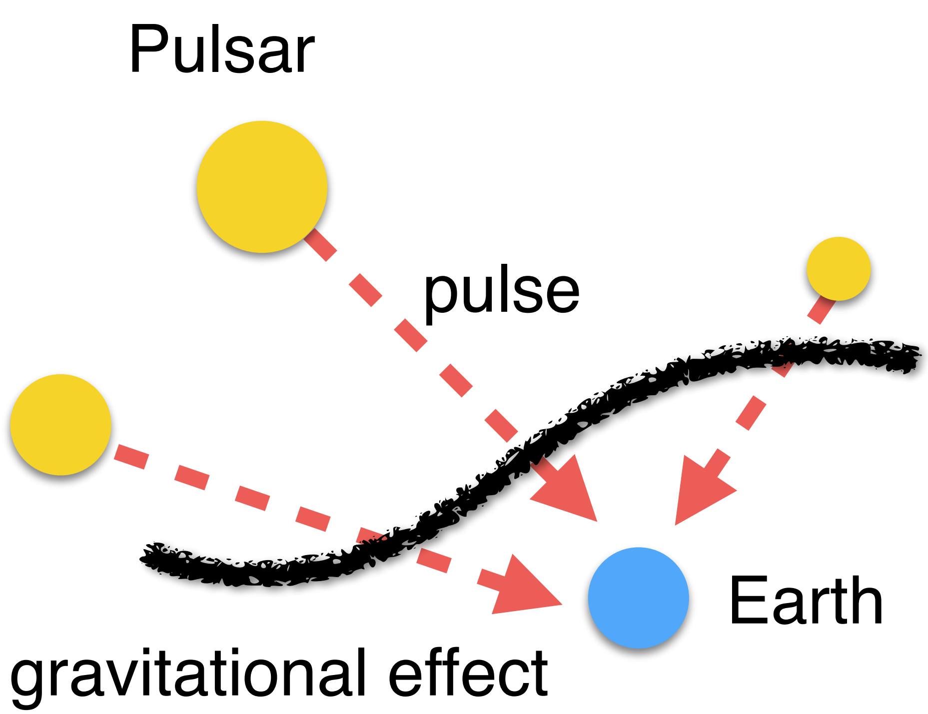
# Pulsar



## Fast spinning neutron star




The gravitational effects can be observed as a delay of pulse arrival times.

Pulsars can be used as a galactic-scale detector,

and sensitive to gravitational effects in a nanohertz frequency band.

# Axion dark matter



## Scalar field predicted by string theory

Feature: The axion behaves as a perfect fluid with an oscillating pressure.

Oscillation of pressure

↓ Einstein's equation

Oscillation of gravitational potential

↓ Geodesic equation

Oscillation of pulse arrival times

If the axion mass is about $10^{-23}\,\mathrm{eV}$ ,

pulse arrival times oscillates with a nanohertz frequency.

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# Feature of the axion



To explain the energy density of the dark matter,
a very large occupation number is required.

$$\frac{\Delta N}{\Delta x^3 \Delta p^3} \sim 10^{96} \left(\frac{\rho}{0.4\mathrm{Gev/cm^3}}\right)\left(\frac{10^{-23}\mathrm{eV}}{m}\right)^4$$

$\rho$ :the energy density of the dark matter

$m$ :the axion mass

Axion can be treated as a classical scalar field.

The solution of the Klein-gordon equation:

$$\phi(t, \vec{x}) = \phi_0(\vec{x}) \cos(mt + \theta(\vec{x}))$$

because

$$k \sim (10\mathrm{kpc})^{-1} \sim 10^5 H_0$$

Galaxy Scale

$$m \sim 10^{-23}\mathrm{eV} \sim 10^{10} H_0$$

$\Rightarrow$

dispersion relation:

$$E_k^2 = k^2 + m^2 \simeq m^2$$

(monochromatic with m)

# Energy momentum tensor 

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2}((\partial\phi)^2 + m^2\phi)$$

Using the equation: $\phi(t, \vec{x}) = \phi_0(\vec{x}) \cos(mt + \theta(\vec{x}))$

with $\partial_i \phi \ll \partial_0 \phi \quad \because k \ll m$

$$T_{00} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 = \frac{1}{2}m^2\phi_0^2$$

$$T_{ij} = \left(\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2\right)\delta_{ij} = \frac{1}{2}m^2\phi_0^2 \cos(2mt + \theta)\delta_{ij}$$

The axion behaves as a perfect fluid with an oscillating pressure.

# Einstein's equation



$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Newtonian gauge

$$ds^2 = -(1+2\Psi)dt^2 + (1+2\Psi)\delta_{ij}dx^i dx^j$$

00 component: $\Delta\Psi = -4\pi G\rho$

ij component: $\ddot{\Psi} = 4\pi G\rho\cos(2mt+\theta)$

$$\rho \equiv \frac{1}{2}m^2\phi_0^2$$

Separate $\Psi$ into time-independent and time-dependent terms.

$$\Psi = \Psi_0(\vec{x}) + \delta\Psi(t,\vec{x})$$

Then

$$\Delta\Psi_0 = -4\pi G\rho$$

Undetectable

$$\delta\ddot{\Psi} = 4\pi G\rho\cos(2mt+\theta)$$

$$\Rightarrow \delta\Psi = \pi G\rho\cos(2mt+\theta)$$

Oscillation of the gravitational potential $\delta\Psi$
is induced from the pressure oscillation.

# Geodesic equation



To obtain the red shift of the light,
the zero component of the geodetic equation is calculated.

$$\frac{d^2x^0}{d\lambda} + \Gamma^0_{\nu\rho}\frac{dx^\nu}{d\lambda}\frac{dx^\rho}{d\lambda} = 0 \qquad ds^2 = -(1+2\delta\Psi)dt^2 + (1+2\delta\Psi)\delta_{ij}dx^i dx^j$$

$$\Rightarrow \quad \frac{d^2x^0}{d\lambda} = -2\omega_0\frac{d\delta\Psi}{d\lambda}$$

$\omega_0$ :unperturbed frequency

Observed light frequency:

Four-velocity at Earth and pulsar

$$\omega_{\rm obs} = \omega_0 - \omega_0[\delta\Psi(\lambda_{\rm Pulsar}) - \delta\Psi(\lambda_{\rm Earth})] + 2\omega_0\int_{\lambda_{\rm Earth}}^{\lambda_{\rm Pulsar}} d\lambda' \frac{d^2x^0}{d\lambda}$$

$$= \omega_0 + \omega_0[\delta\Psi(\lambda_{\rm Pulsar}) - \delta\Psi(\lambda_{\rm Earth})]$$

# The delay of the light



Red shift:

$$z = \frac{\omega_0 - \omega_{\rm obs}}{\omega_0} = \delta\Psi(\lambda_{\rm Earth}) - \delta\Psi(\lambda_{\rm Pulsar})$$
$$= \pi G[\rho(\vec{x}_{\rm E})\cos(2mt + \theta(\vec{x}_{\rm E})) - \rho(\vec{x}_{\rm P})\cos(2m(t-D) + \theta(\vec{x}_{\rm P}))]$$

The delay of the light:

$$\Delta t = \int_0^t dt\ z$$
$$= \frac{\pi G}{2m}[\rho(\vec{x}_{\rm E})\sin(2mt + \theta(\vec{x}_{\rm E})) - \rho(\vec{x}_{\rm P})\sin(2m(t-D) + \theta(\vec{x}_{\rm P}))]$$

The delay of the light $\Delta t$ can be detected by the pulser timing array as a change in the pulse arrival times.

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# Axion signal



The delay of the pulse arrival time

$$\frac{1}{2\pi f}\left[\Psi\sin(2\pi f t+\alpha)-\Psi\sin(2\pi f t+\alpha_p)\right]$$

$$\Psi\equiv\frac{\pi G\rho}{m^2}\quad,\qquad f\equiv\frac{m}{\pi}$$

$m$ : Axion mass $\qquad$ $\rho$ : Axion energy density

$\alpha, \alpha_p$ : Phases at earth and pulsar

In this study:

- We search for the axion, and if it cannot be detected, we give an upper limit for $\Psi$.
- We investigate whether the axion is necessary for a model which describes the observation data.

# Bayesian statistics



- Parameter estimation

Bayes' theorem:

$$p(\vec{\theta}|d) \propto p(d|\vec{\theta})p(\vec{\theta})$$

$\vec{\theta}$ : parameter

$d$ : observation data

The goal of the Bayesian estimation is to update the prior knowledge of the parameter $p(\vec{\theta})$ to the posterior knowledge $p(\vec{\theta}|d)$ using the data $d$ .

- Bayesian hypothesis testing

Bayes factor:

$$B_{10} \equiv \frac{p(d|M_1)}{p(d|M_0)}$$

$M_0, \ M_1$ : hypothesis

The goal of the Bayesian hypothesis testing is to compute the Bayes factor $B_{10}$.

Interpretation of the Bayes factor (Jeffreys' scale)

| B10 | Evidence in favor of M0 |
|---|---|
| 1-3 | Week |
| 3-20 | Positive |
| 20-150 | Strong |

# Data analysis



Pulsar observation data: The NANOGrav Eleven-Year Data Set

https://data.nanograv.org

Program: Based on PAL2 (Bayesian inference package for pulsar timing data)

https://github.com/jellis18/PAL2

## First Analysis

Model: $s + n_{\rm red}$

$s$ : Axion signal

$n_{\rm red}$ : Red noise

Red noise: The power spectral density of the noise has most of their power at low frequencies in a given data set.

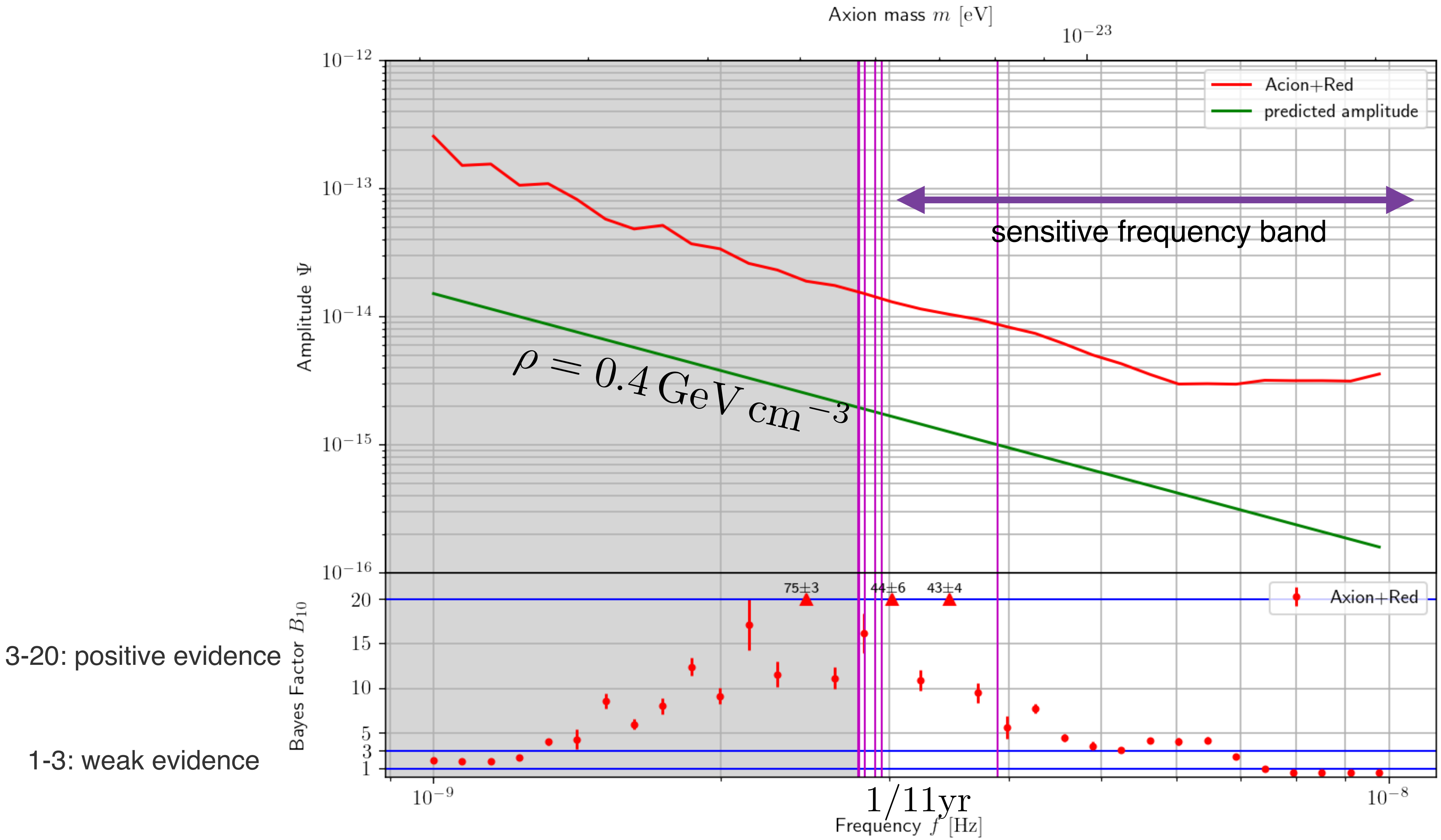
Source:
- Fluctuations of the rotation speed of the pulsar
- Diffractive and refractive interstellar effects

$$\langle n_{red} n_{red}^T \rangle = \int_{-\infty}^{\infty} df \, \mathrm{e}^{-2\pi i f t} P(f) \qquad P(f) \equiv A f^{-\gamma}$$

# First analysis



Upper limit on the axion amplitude $\Psi$ as a function of the frequency




The upper limit of the amplitude is still one order of magnitude larger than the expected amplitude.

The Bayes factor exceeds 20, and so the existence of the axion can not be denied.

# Uncertainty of earth orbit



## In the case of gravitational waves

Uncertainties of the earth orbit affects
upper limits and Bayes factors.

Z. Arzoumanian et al. The NANOGrav 11-year Data Set: Pulsar-timing Constraints On The Stochastic Gravitational-wave Background. *Astrophys. J.*, 859(1):47, 2018.

## Second Analysis

Model: $s + n_{\mathrm{red}} + n_{\mathrm{orbit}}$

$s$ : Axion signal

$n_{\mathrm{red}}$ : Red noise

$n_{\mathrm{orbit}}$ : Uncertainties of earth orbit

Sources of $n_{\mathrm{orbit}}$:

- Error of ecliptic longitude
- Mass errors of Jupiter, Saturn, Uranus, and Neptune
- Orbit error of Jupiter

The Jupiter has a orbital period of 11.86 years and a large mass.

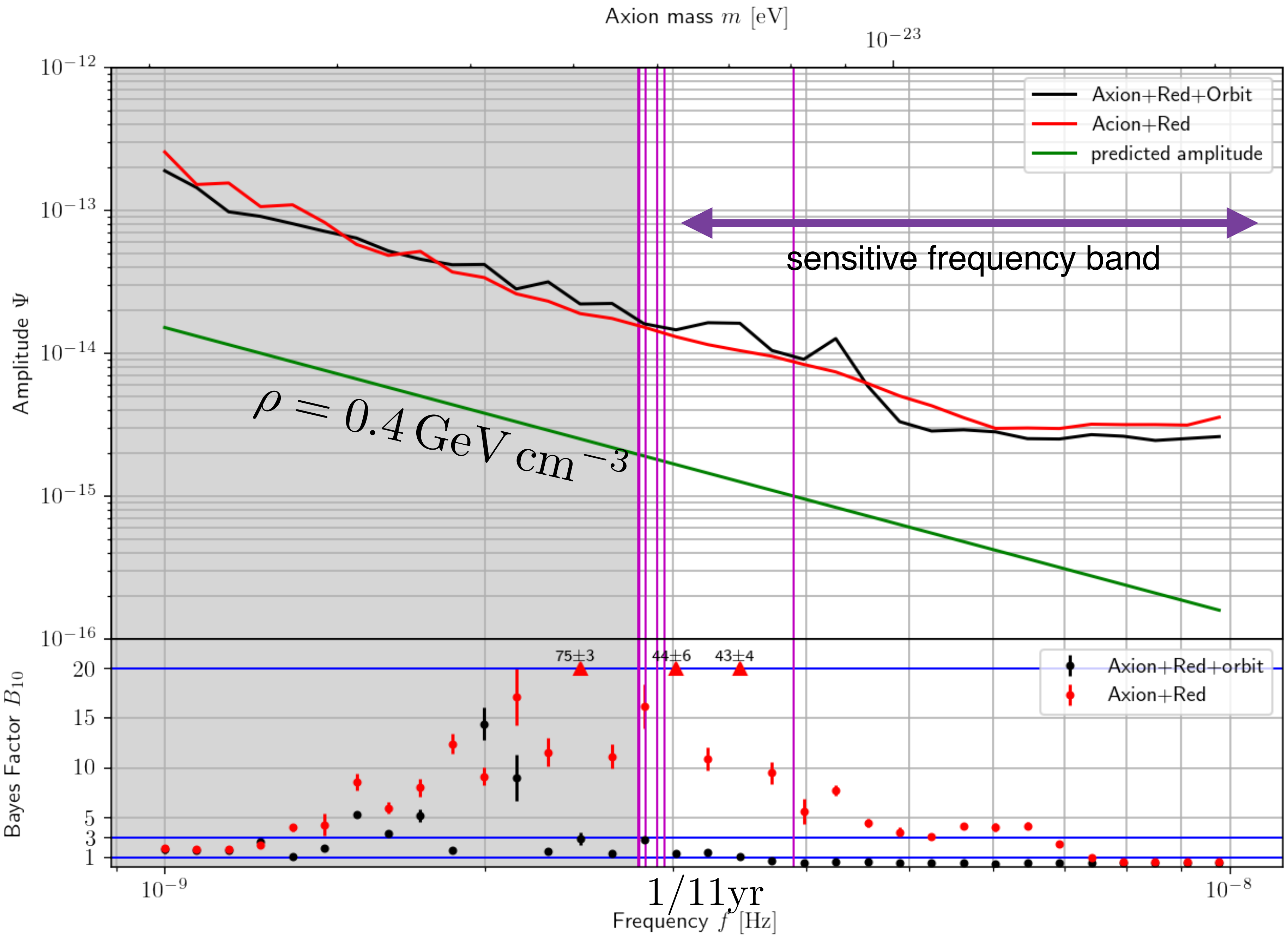
→

The Jupiter produces a large noise in the low frequency region of pulser timing arrays.

# Second analysis



Upper limit on the axion amplitude $\Psi$ as a function of the frequency




3-20: positive evidence

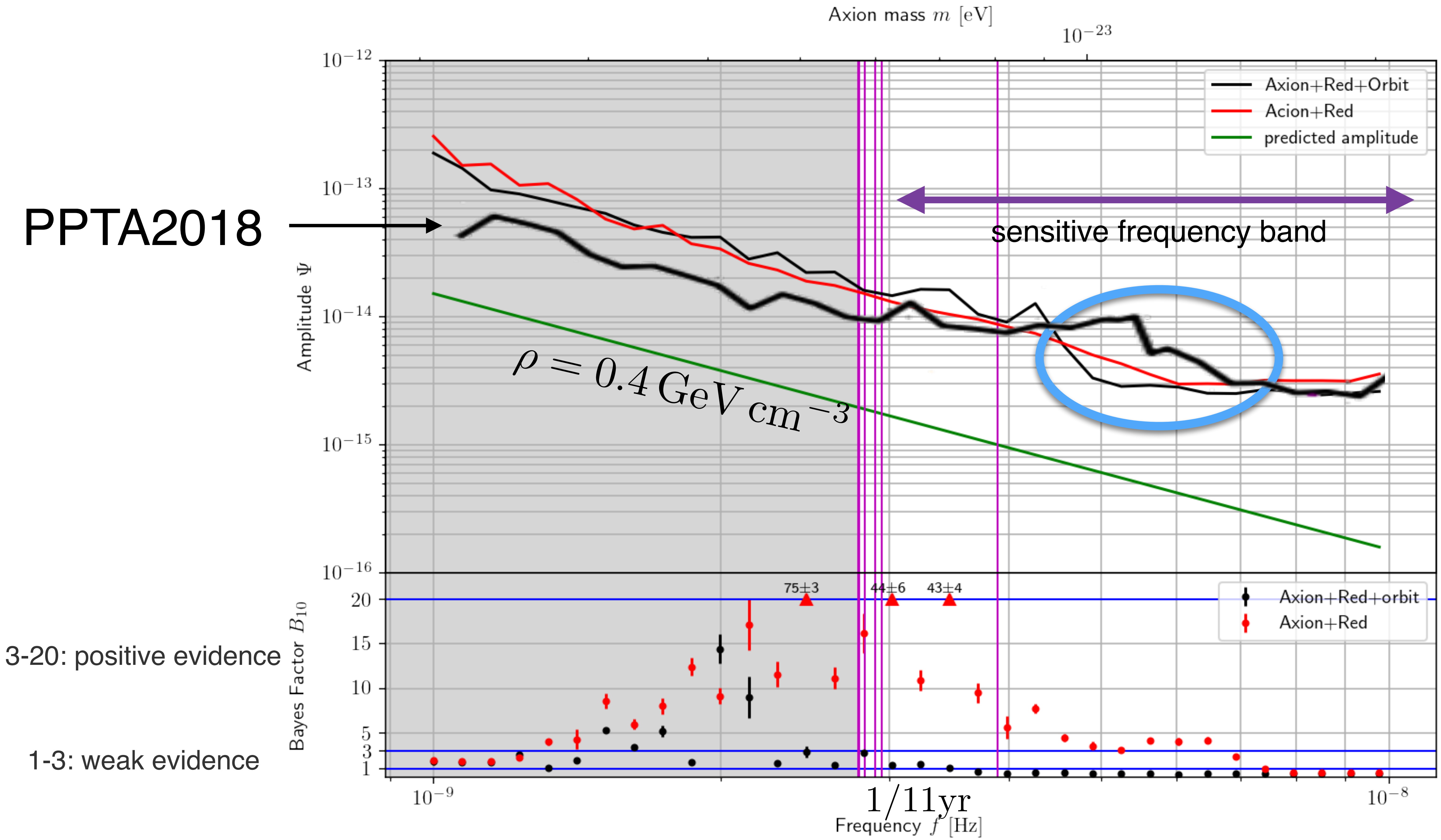
1-3: weak evidence

Almost the same upper limit as in the first analysis is obtained,
but most of the Bayes factors are less than 3, and so no axion is detected.

# Comparison with previous study



Upper limit on the axion amplitude $\Psi$ as a function of the frequency




Previous study: Parkes Pulsar Timing Array Twelve-Year Data Set (PPTA2018)

N. K. Porayko *et al.*, Phys. Rev. D **98** (2018) no.10, 102002

Up to three times stronger upper limit is obtained in the blue circle.

# Fixed noise analysis



Bayes factor is less than 3 $\Leftrightarrow$ Probability that axion is not required is more than 75%

We would like to know whether the noise is sufficient to absorb the signal of the axion.

Last Analysis

Model: $n_{\rm red} + n_{\rm orbit}$ then Model: $s$

$s$ : Axion signal $n_{\rm red}$ : Red noise

$n_{\rm orbit}$ : Uncertainty of earth's orbit

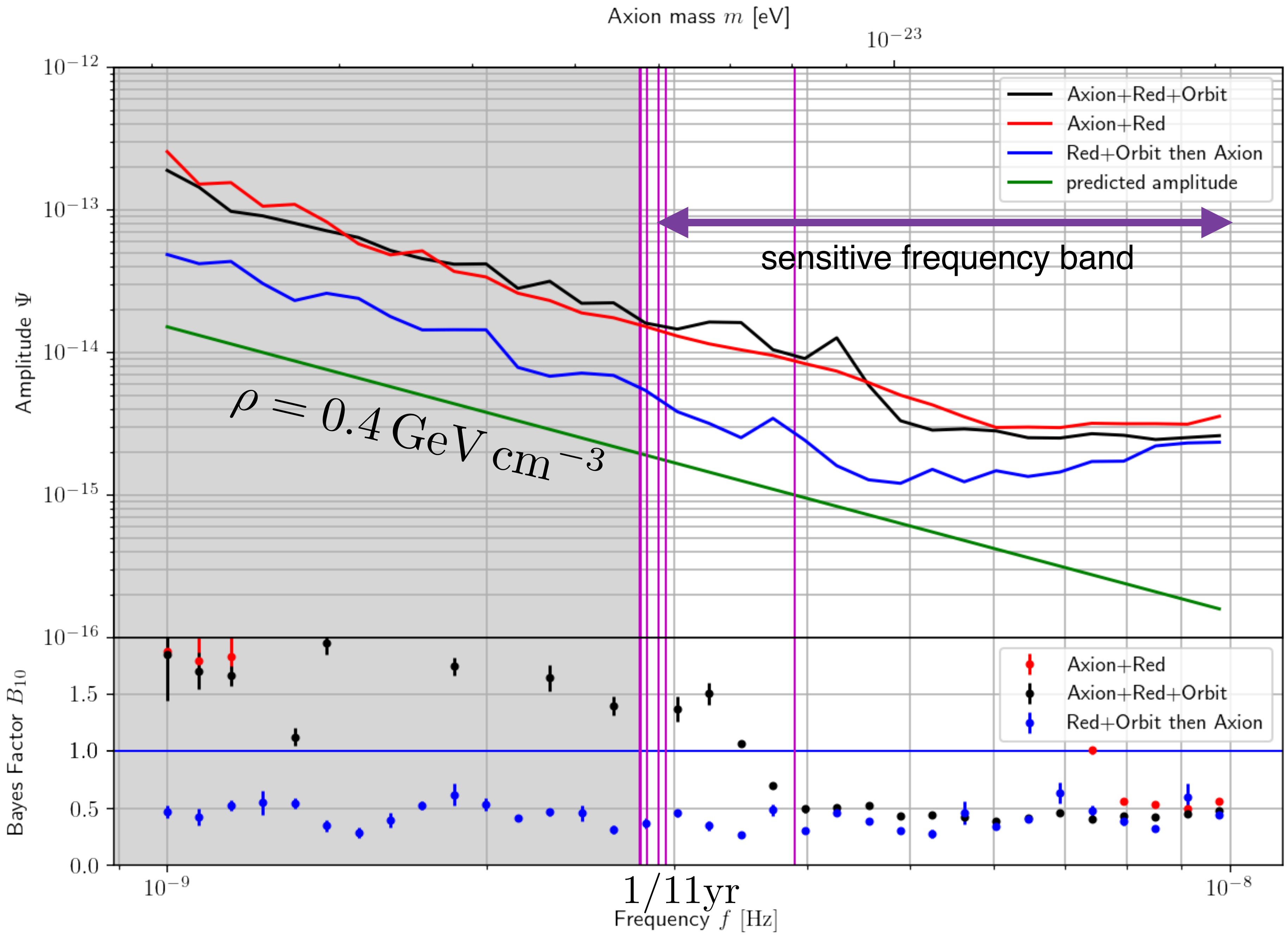
Note that,

It is difficult to distinguish the signal and the noise. Therefore, this analysis cannot be regarded as giving upper limits to the axion.

# Last analysis



Upper limit on the axion amplitude $\Psi$ as a function of the frequency




<1: no evidence

The model of only $n_{\rm red}$ and $n_{\rm orbit}$ absorbs the axion signal sufficiently.

It suggests that the axion is unnecessary for the model of the observation data.

# Summary



We set the upper limit of the axion amplitude
using the recent pulsar observation data.

- By considering the uncertainty of the earth’s orbit, we revealed that the axion is not detected.
- In comparison with the previous study (Porayko et.al.2018), we obtain up to three times stronger upper limit.
- We clarified that the only the red noise and the uncertainty of the earth’s orbit are sufficient for the model of observation data.