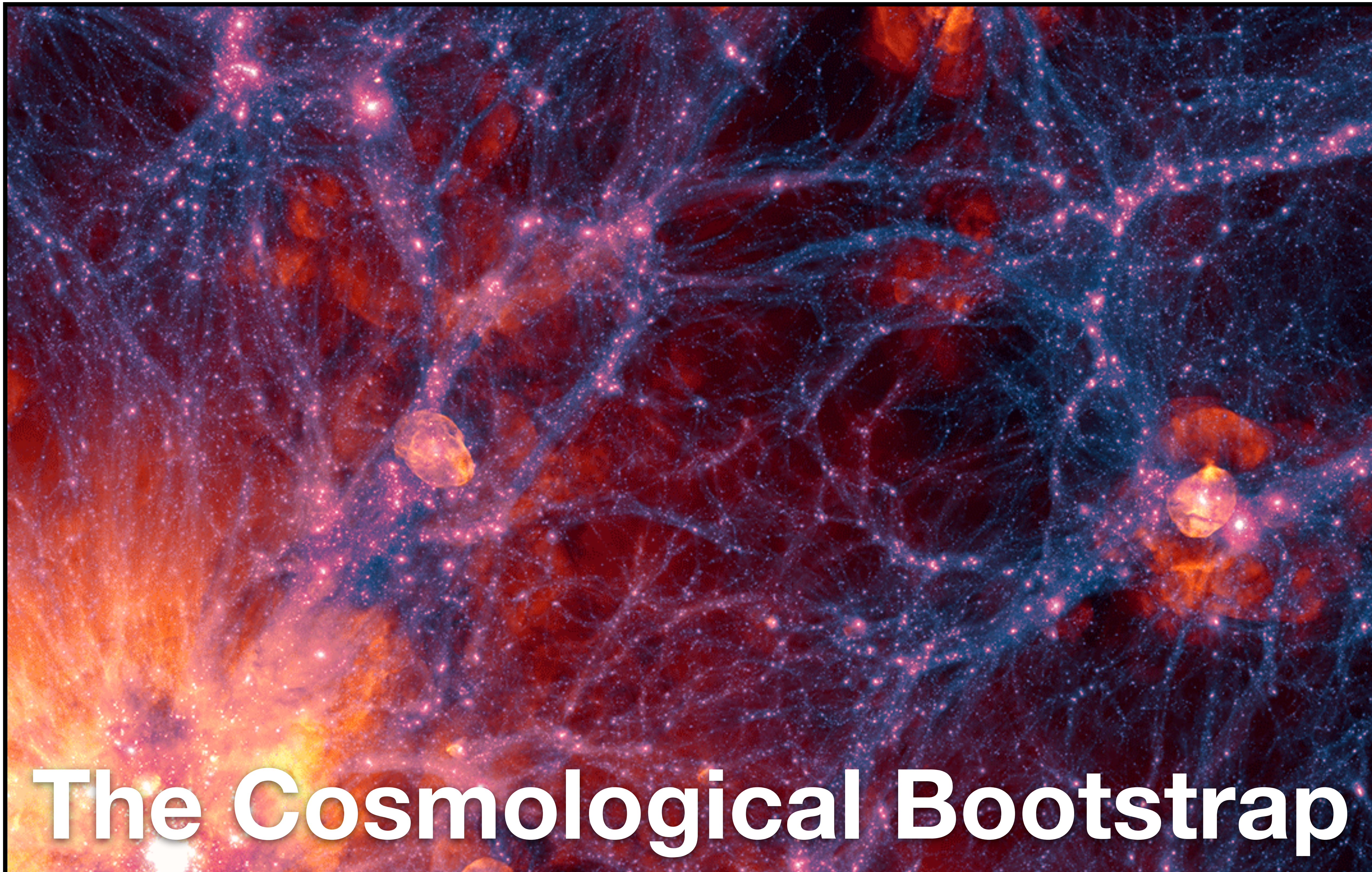

The Cosmological Bootstrap

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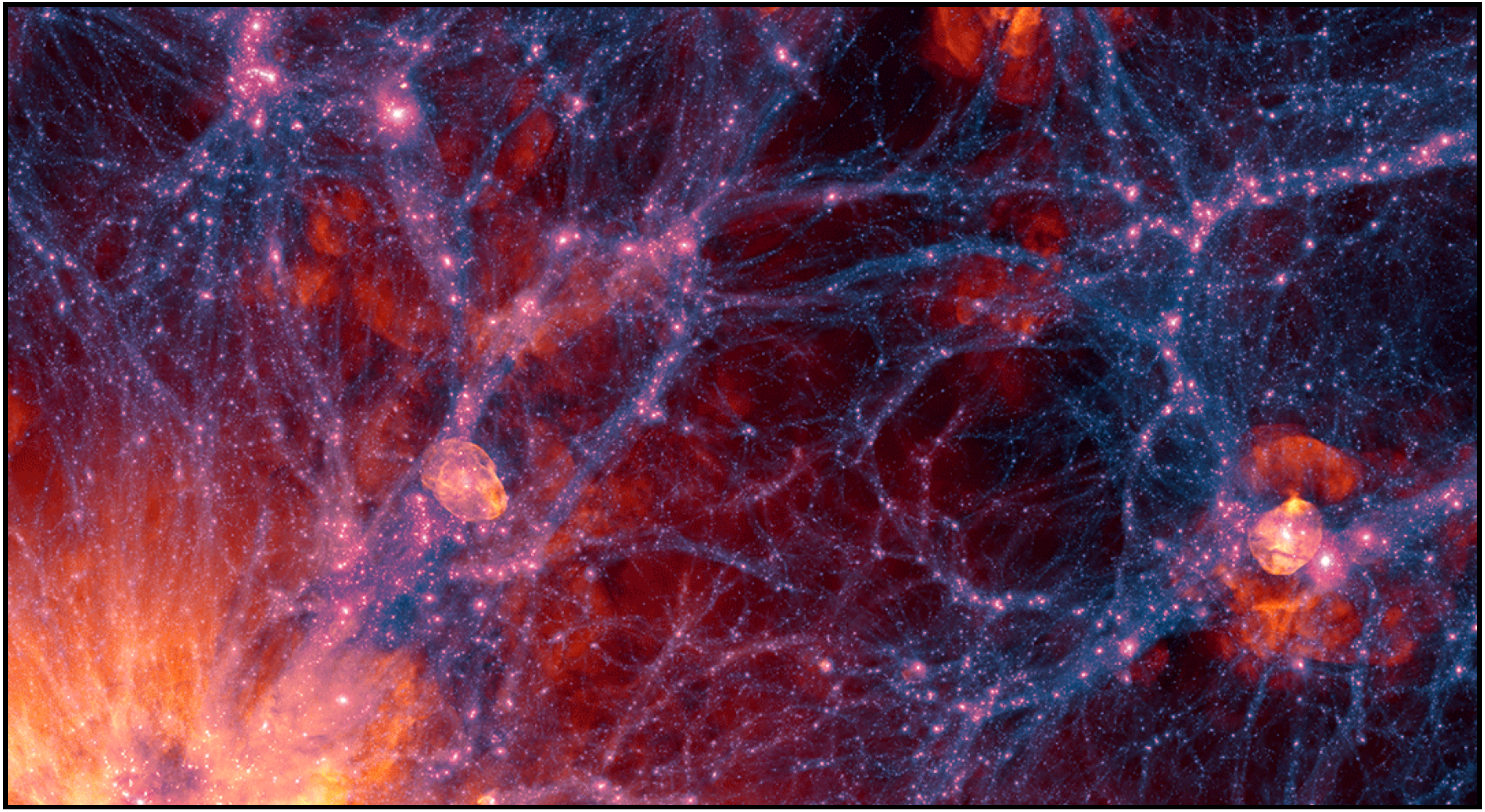
National Taiwan University

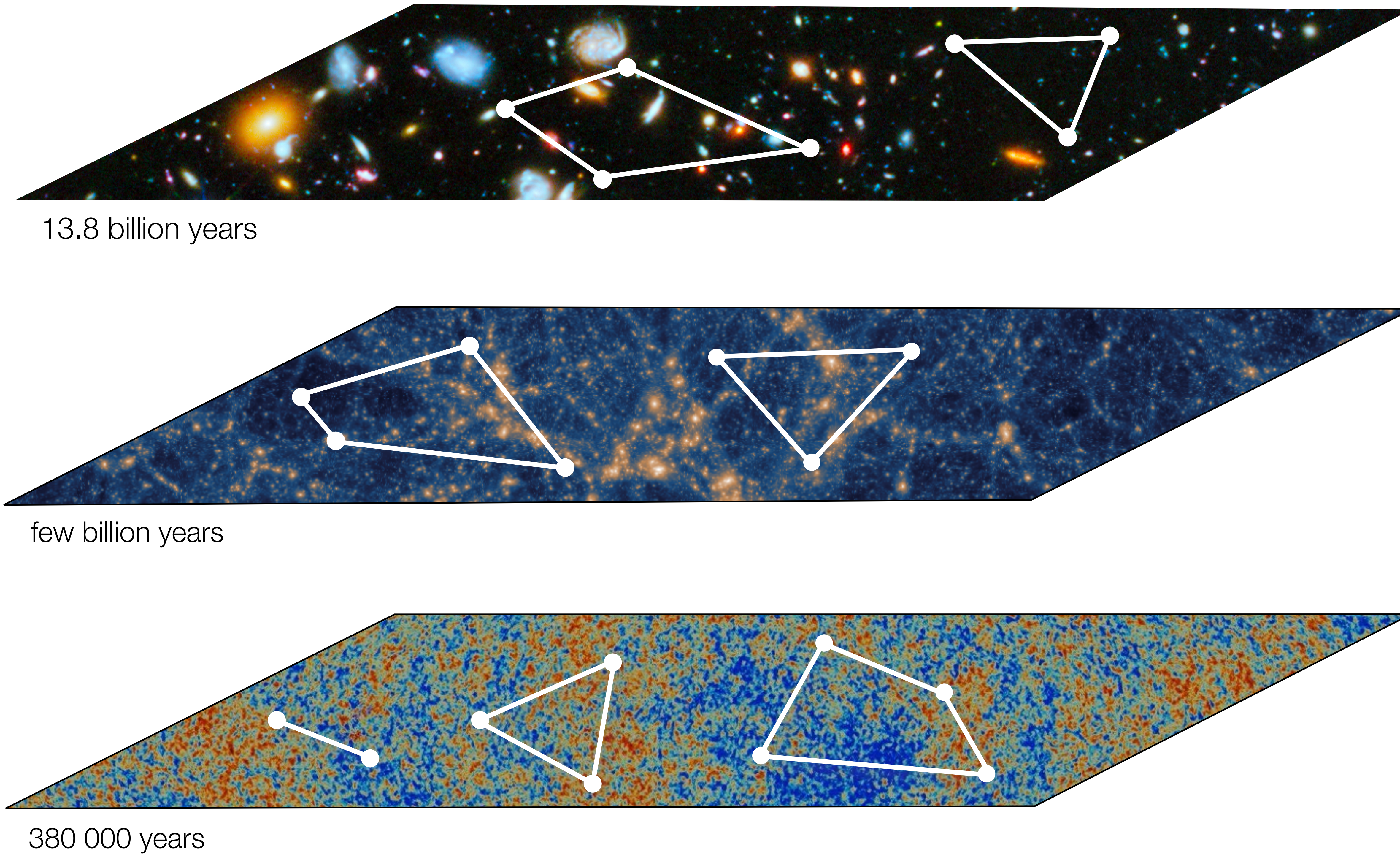
KYOTO
November 2021



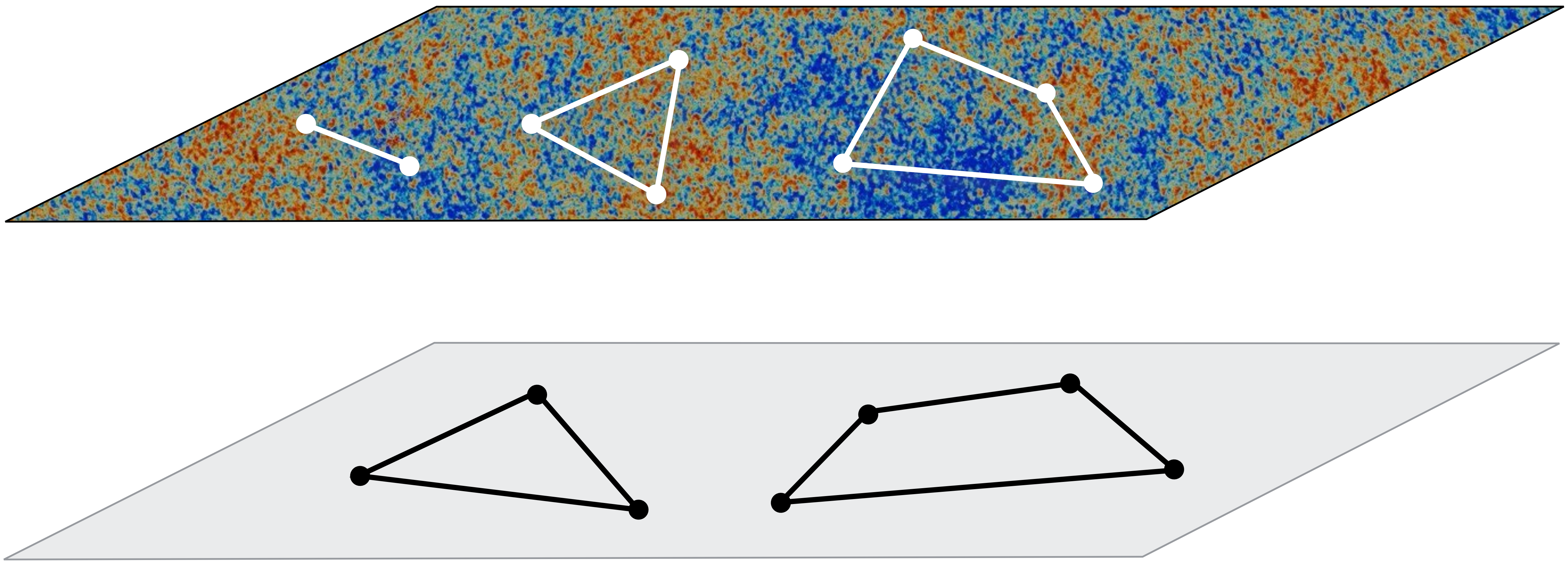
Based on work with

Nima Arkani-Hamed, Wei Ming Chen, Hayden Lee, Guilherme Pimentel,
Carlos Duaso Pueyo and Austin Joyce

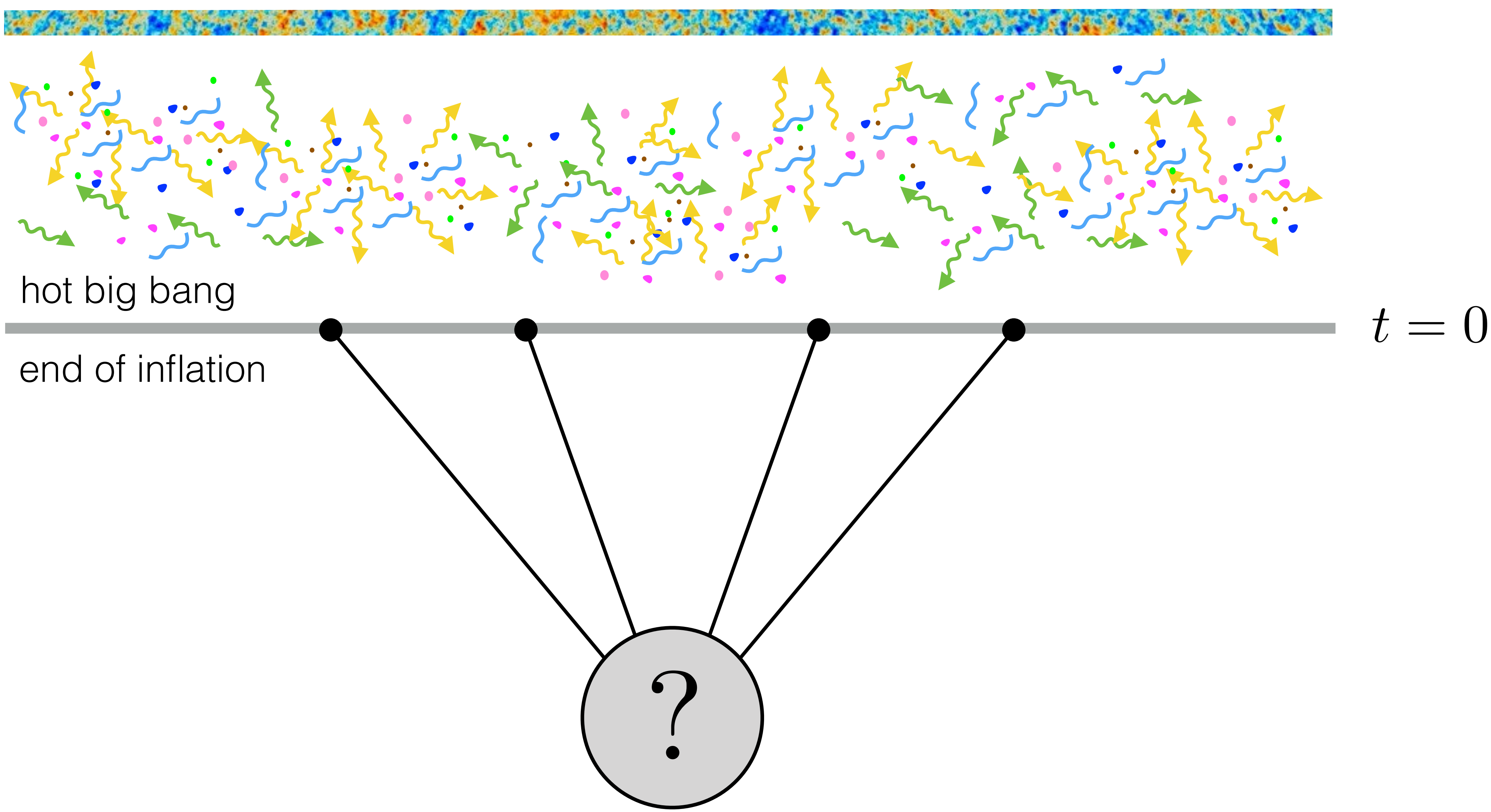
Cosmological structures are not distributed randomly, but display spatial correlations over very large distances:



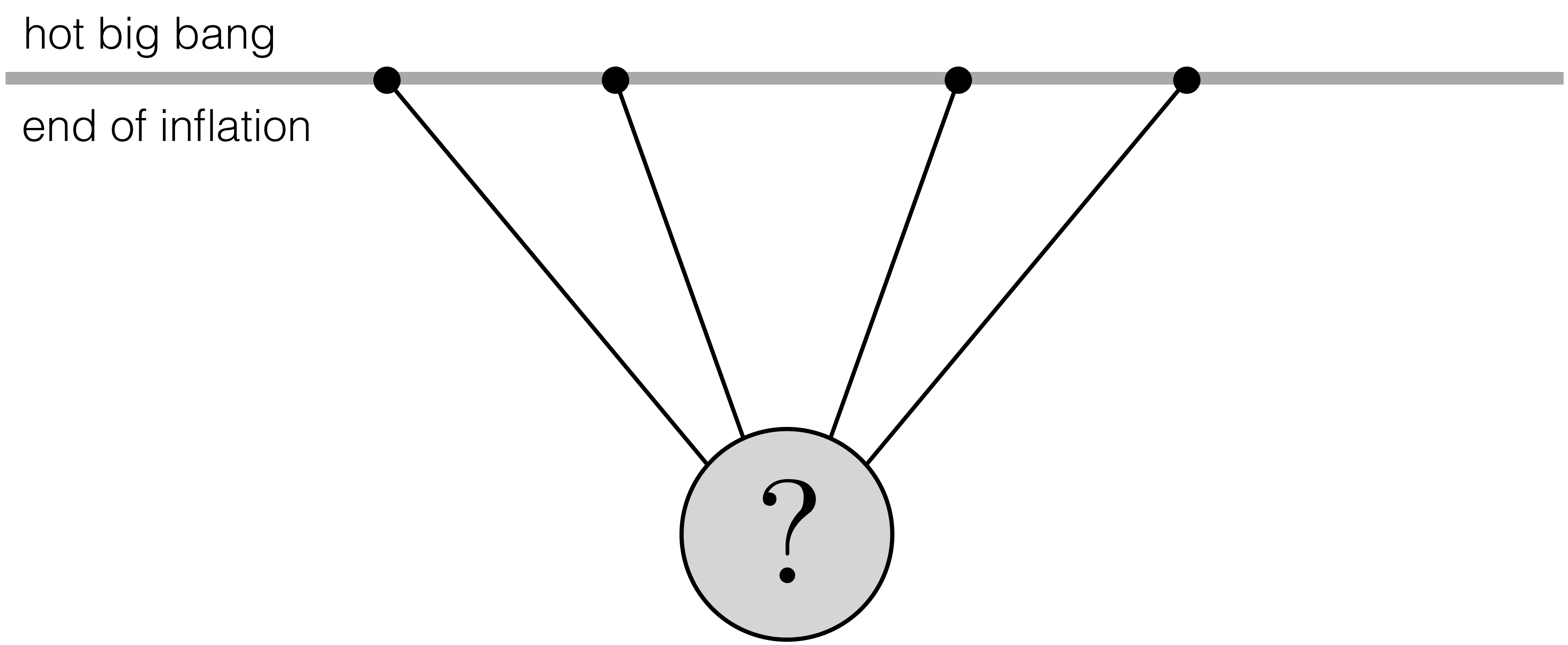
These correlations can be traced back to the beginning of the hot Big Bang:



However, the Big Bang was not the beginning of time, but the end of an earlier high-energy period:

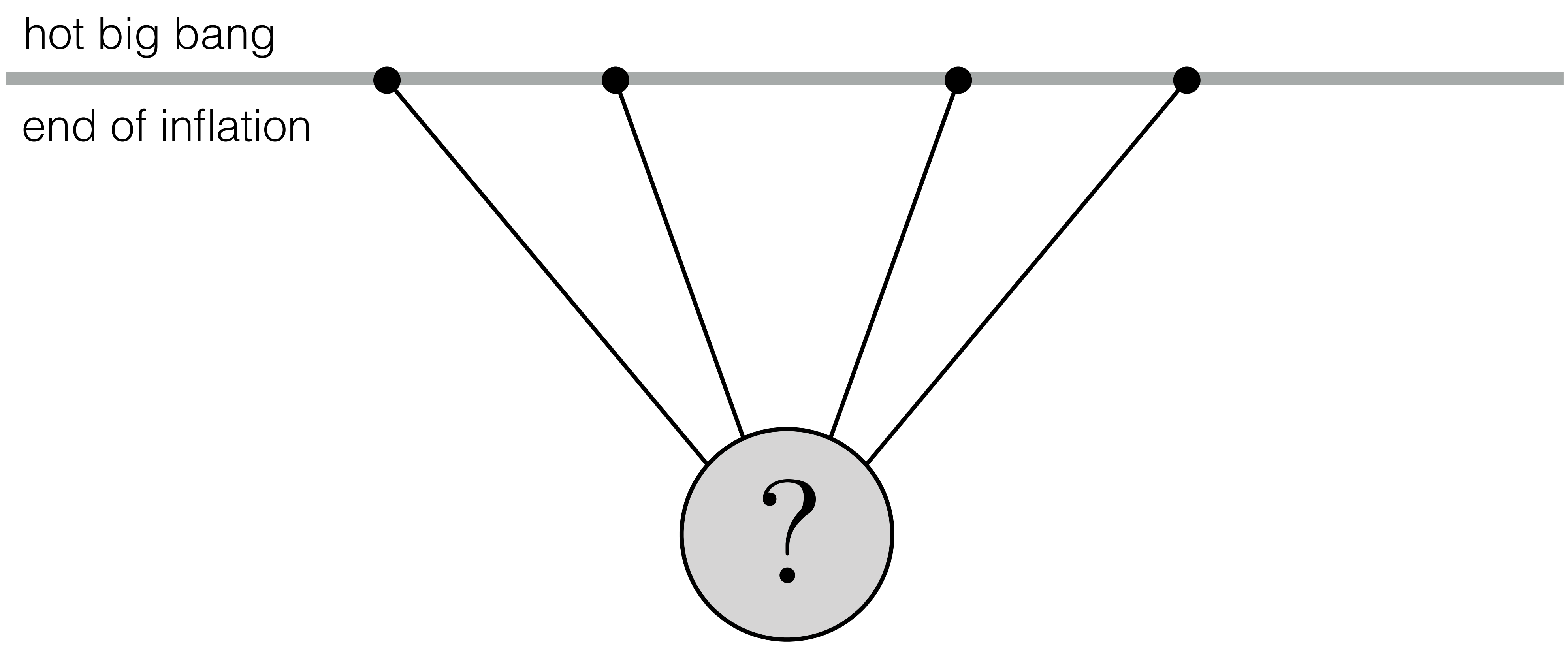


What exactly happened before the hot Big Bang?
How did it create the primordial correlations?



The challenge is to extract this information from the pattern of correlations in the late universe.

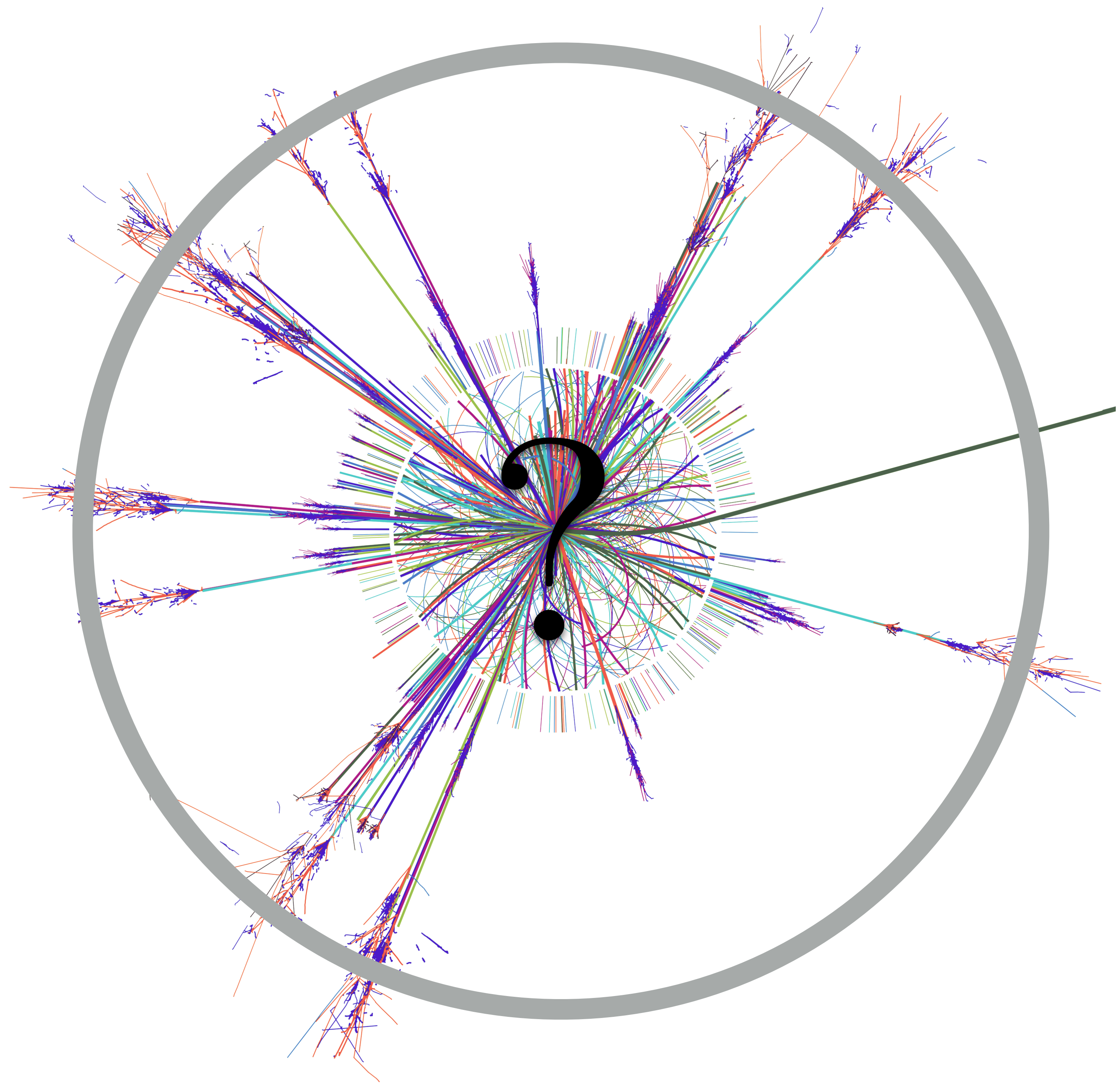
What is the space of consistent correlations?



locality, causality, unitarity, …

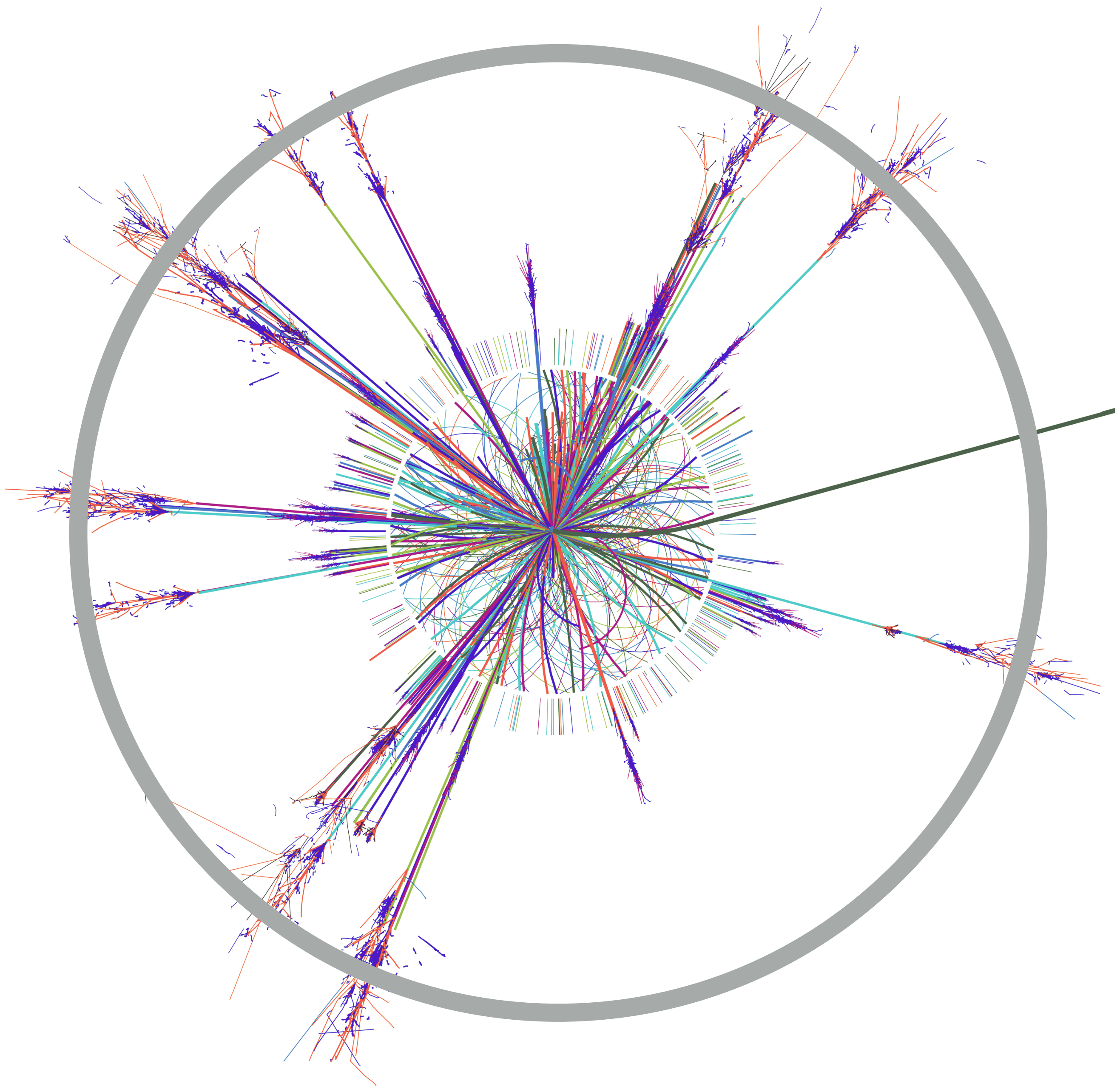
- What correlations are consistent with basic **physical principles**?
- Can these correlations be **bootstrapped** directly?

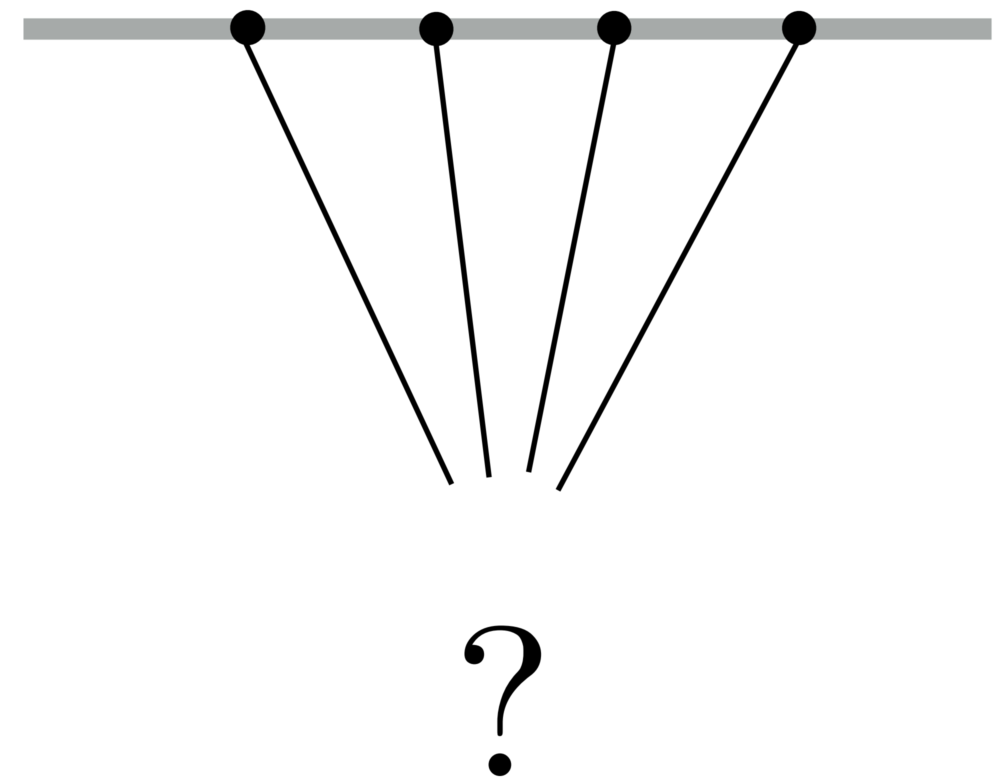
The bootstrap perspective has been very influential for **scattering amplitudes**:



In that case, the rules of **quantum mechanics** and **relativity** are very constraining.

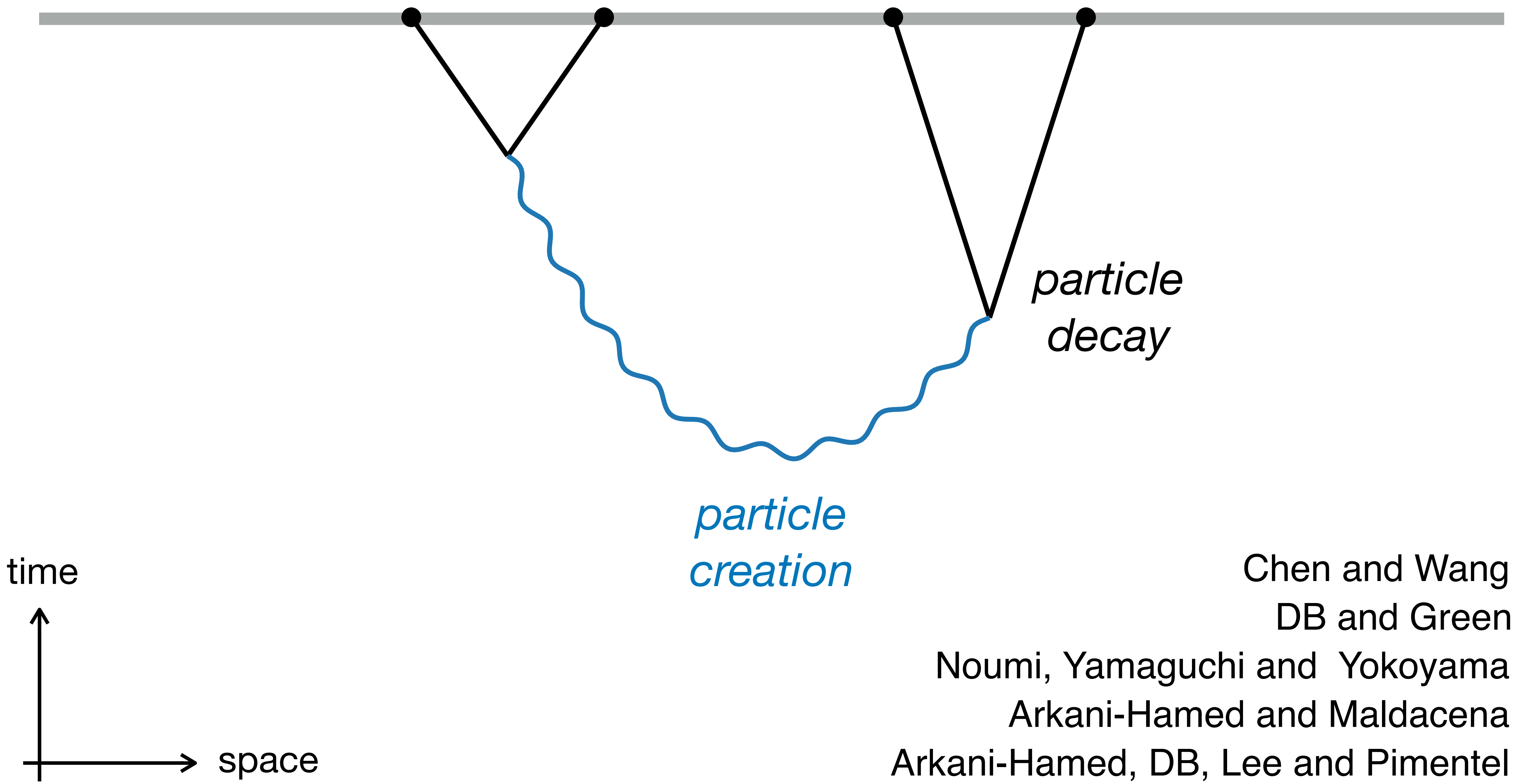
Does a similar **rigidity** exist for cosmological correlators?





Goal: Develop an understanding of cosmological correlators that parallels our understanding of flat-space scattering amplitudes.

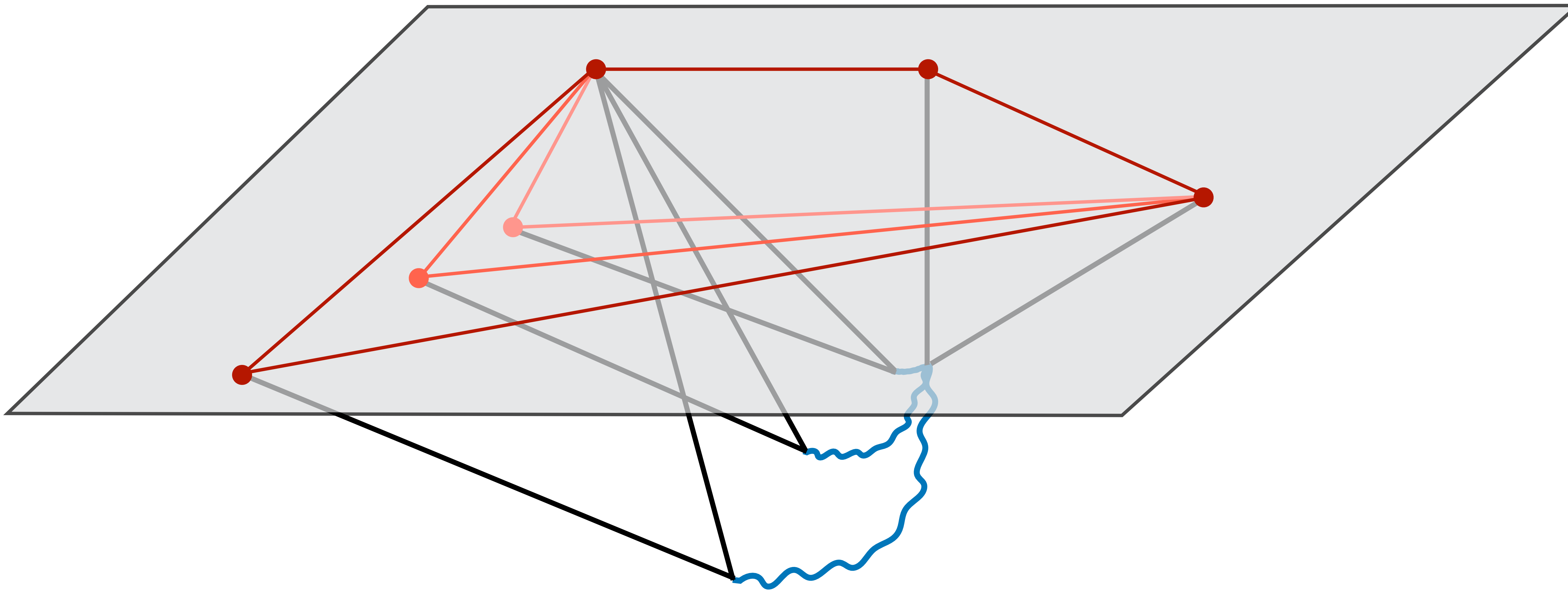
The connection to scattering amplitudes is also relevant because the early universe was like a giant **cosmological collider**:



Chen and Wang [2009]
DB and Green [2011]
Noumi, Yamaguchi and Yokoyama [2013]
Arkani-Hamed and Maldacena [2015]
Arkani-Hamed, DB, Lee and Pimentel [2018]

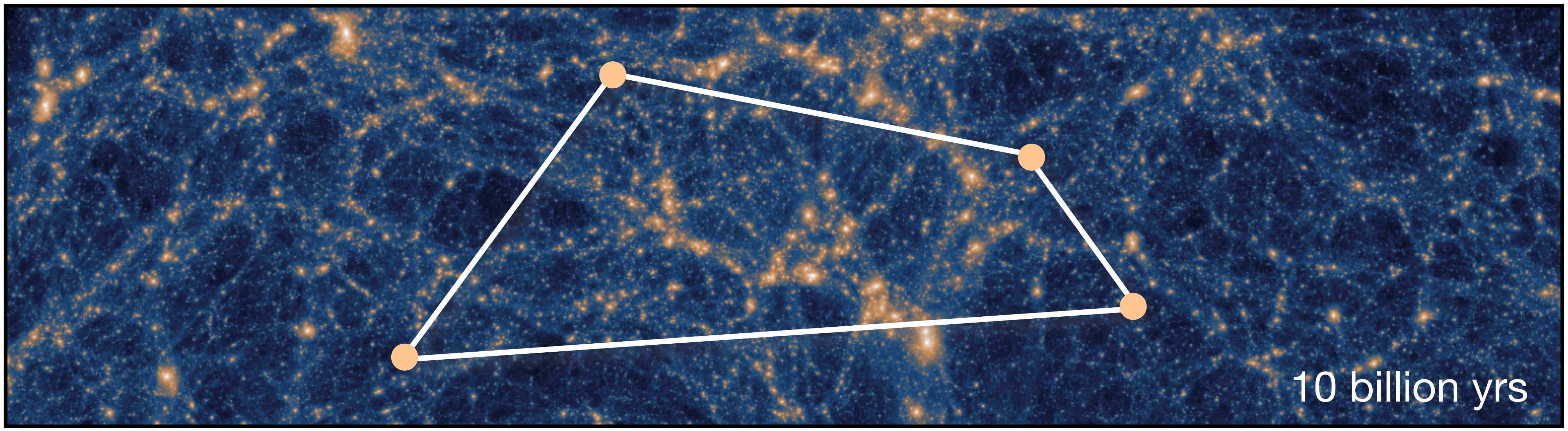
During inflation, the rapid expansion can produce very **massive particles** ($\sim 10^{14}$ GeV) whose decays lead to nontrivial correlations.

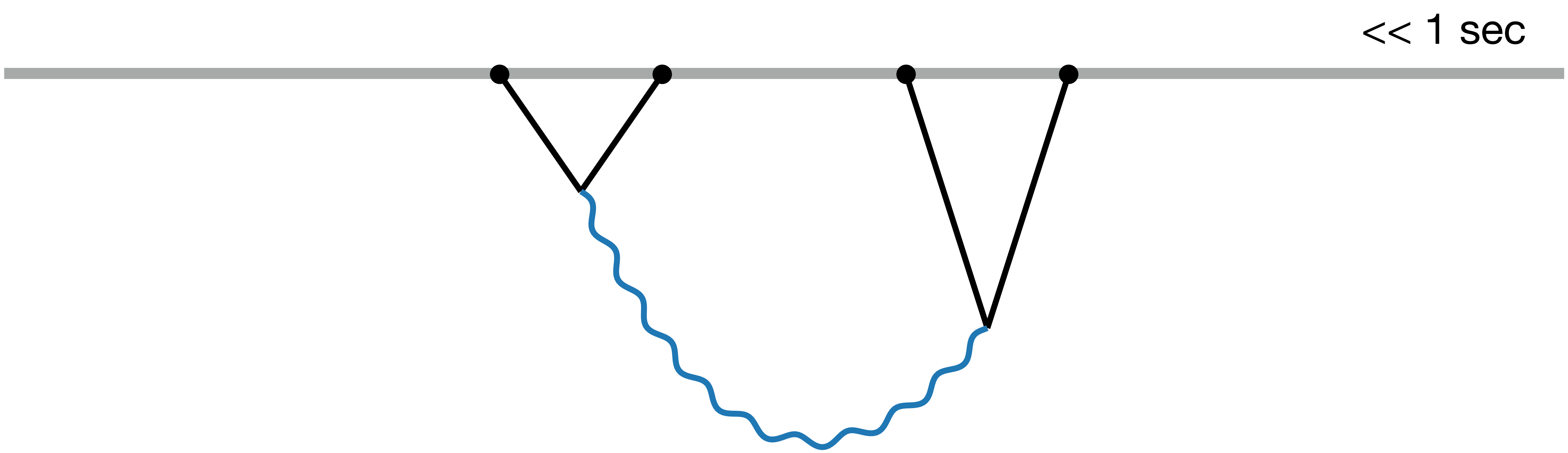
These particles are tracers of the inflationary dynamics:



The pattern of correlations *after* inflation contains a memory of the physics *during* inflation (evolution, symmetries, particle content, etc).

At late times, these correlations leave an imprint in the distribution of galaxies:





Goal: Develop a systematic way to predict these signals.

Outline

I. Basics of the Bootstrap

II. Recent Progress

III. Future Directions

I. Basics of the Bootstrap

The Conventional Approach

How we usually make predictions:

Physical Principles → **Models** → **Observables**

Physical Principles	Models	Observables
locality, causality, unitarity, symmetries	Lagrangians equations of motion spacetime evolution Feynman diagrams	

Works well if we have a well-established theory (Standard Model, GR, …) and many observables.

The Conventional Approach

Although conceptually straightforward, this has some drawbacks:

- It involves unphysical gauge degrees of freedom.
- Relation between Lagrangians and observables is many-to-one.
- Even at tree level, the computation of cosmological correlators involves complicated time integrals:

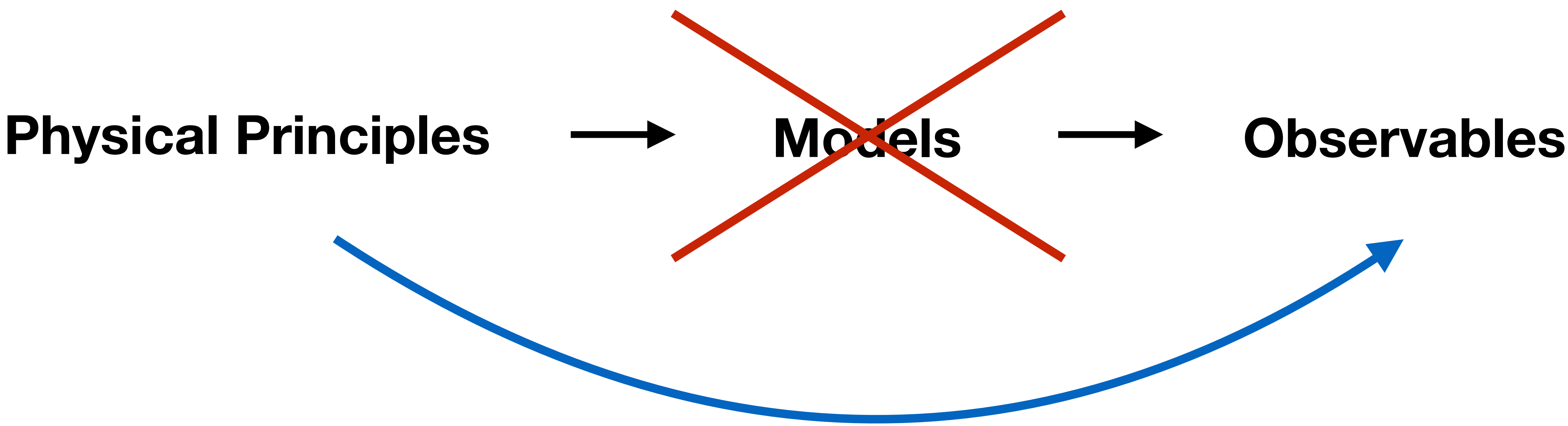
$$\boxed{F_N = \sum_{\text{Diagrams}} \left(\prod_{\text{Vertices}} \int \mathrm{d}t \right) \begin{pmatrix} \text{External} \\ \text{propagators} \end{pmatrix} \begin{pmatrix} \text{Internal} \\ \text{propagators} \end{pmatrix} \begin{pmatrix} \text{Vertex} \\ \text{factors} \end{pmatrix}}$$

Hard to compute!

- Fundamental principles (e.g. locality and unitarity) are obscured.
- To derive nonperturbative correlators and constraints from the UV completion, we need a deeper understanding.

The Bootstrap Method

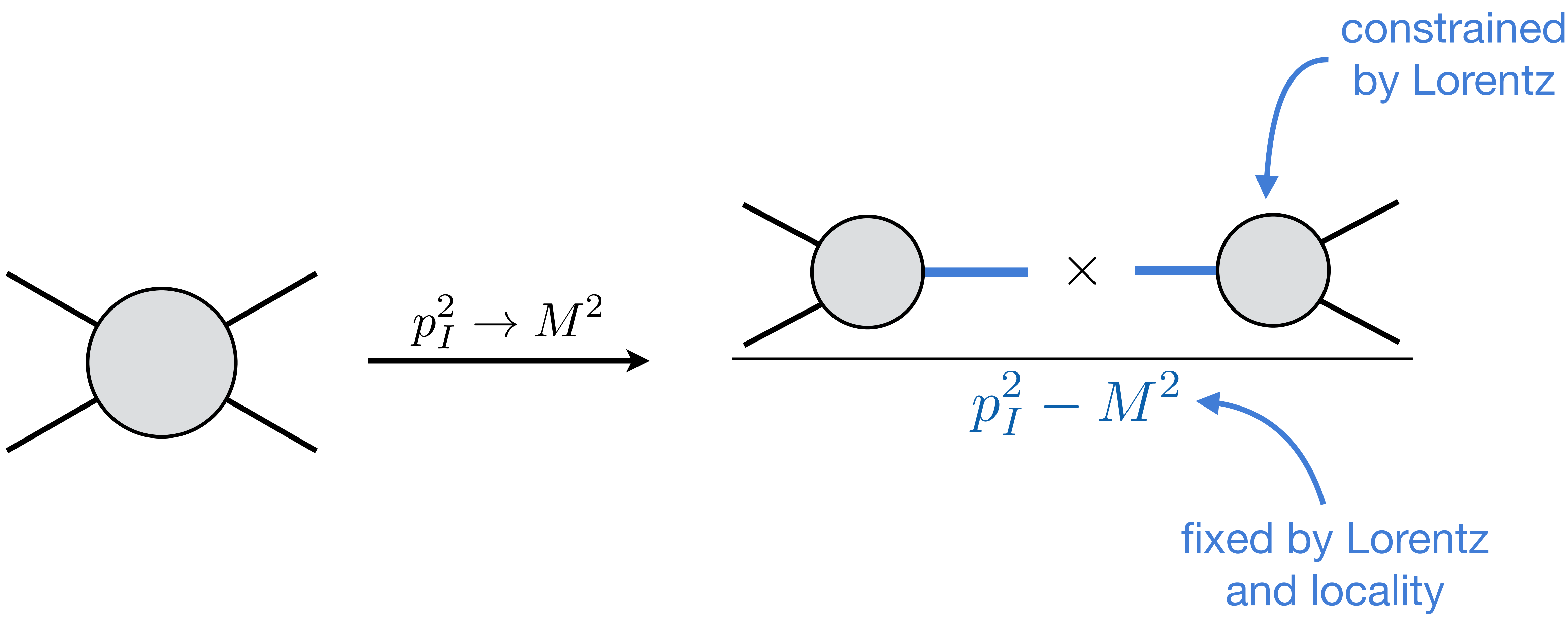
In the bootstrap approach, we cut out the middle man and go directly from fundamental principles to physical observables:



This is particularly relevant when we have many theories (inflation, BSM, …) and few observables.

S-matrix Bootstrap

Much of the physics of scattering amplitudes is controlled by **SINGULARITIES**:

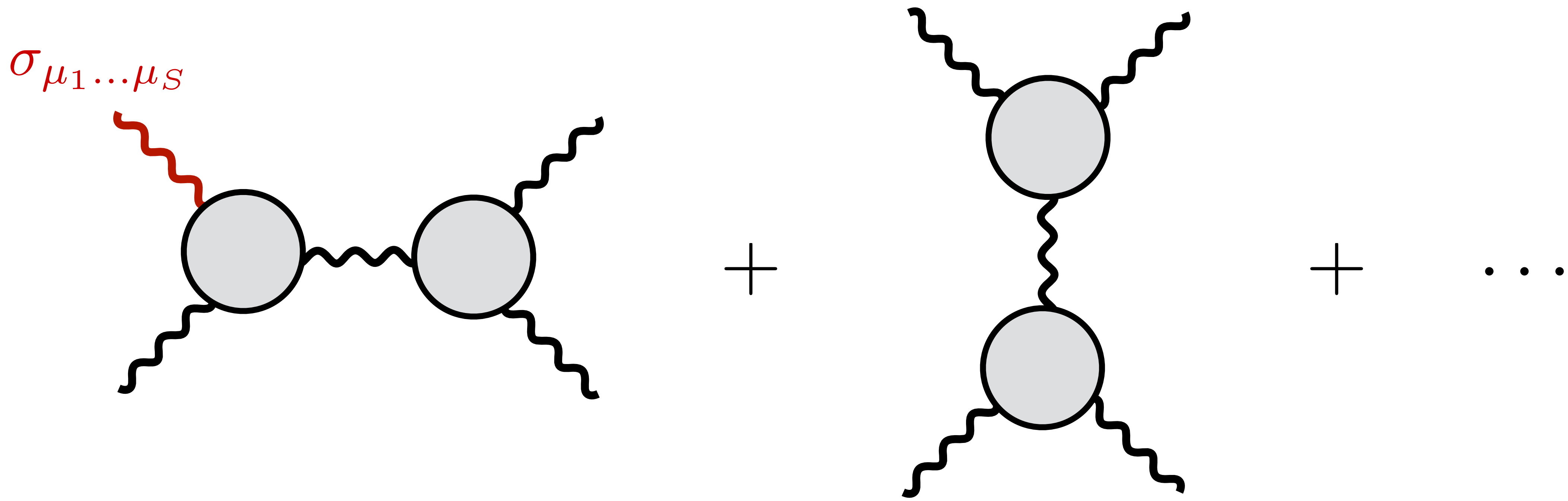


- Amplitudes have **poles** when intermediate particles go on-shell.
- On these poles, the amplitudes **factorize**.

The different singularities are connected by (Lorentz) **SYMMETRY**.

S-matrix Bootstrap

Consistent factorization is very constraining for massless particles:



→ Only consistent for spins $S = \{ 0 \, , \, \frac{1}{2} \, , \, 1 \, , \, \frac{3}{2} \, , \, 2 \, \}$

YM SUSY GR

Benincasa and Cachazo [2007]
McGady and Rodina [2013]

A Success Story

The modern amplitudes program has been very successful:

1. New Computational Techniques

- Recursion relations
- Generalized unitarity
- Soft theorems

2. New Mathematical Structures

- Positive geometries
- Amplituhedrons
- Associahedrons

3. New Relations Between Theories

- Color-kinematics duality
- BCJ double copy

4. New Constraints on QFTs

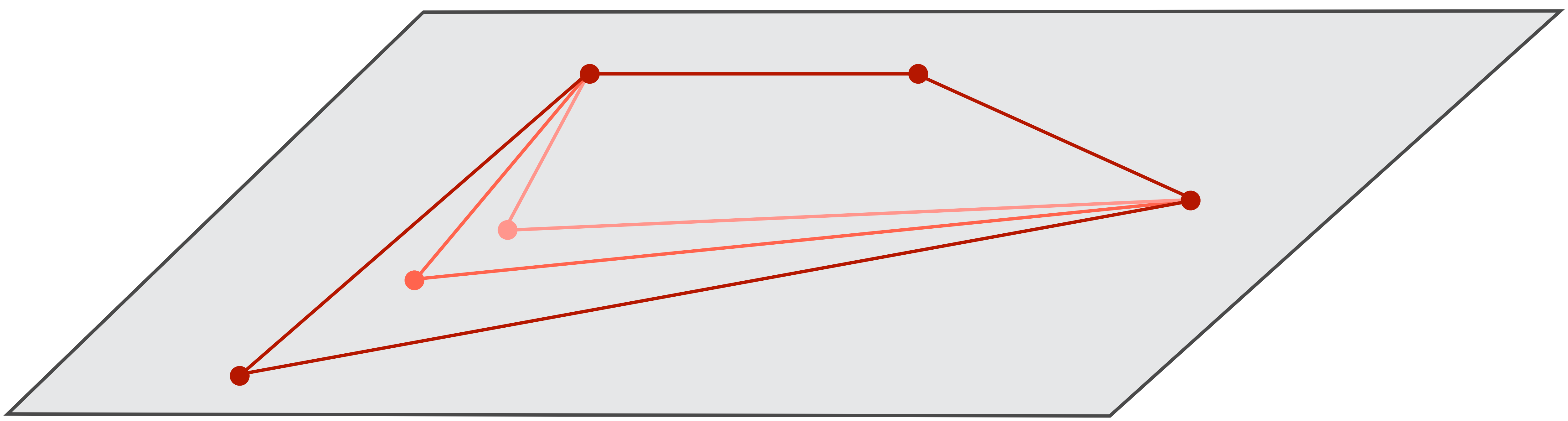
- Positivity bounds
- EFThedron

5. New Applications

- Gravitational wave physics
- Cosmology

Cosmological Bootstrap

If inflation is correct, then all cosmological correlations can be traced back to the future boundary of an approximate **de Sitter spacetime**:



We want to **bootstrap** these boundary correlations directly, without explicit reference to the bulk dynamics.

Cosmological Bootstrap

The logic for bootstrapping these correlators is similar to that for amplitudes:

- **SINGULARITIES:**
  Correlators have characteristic singularities as a function of the external energies.

  ⟶ Analog of resonance singularities for amplitudes.

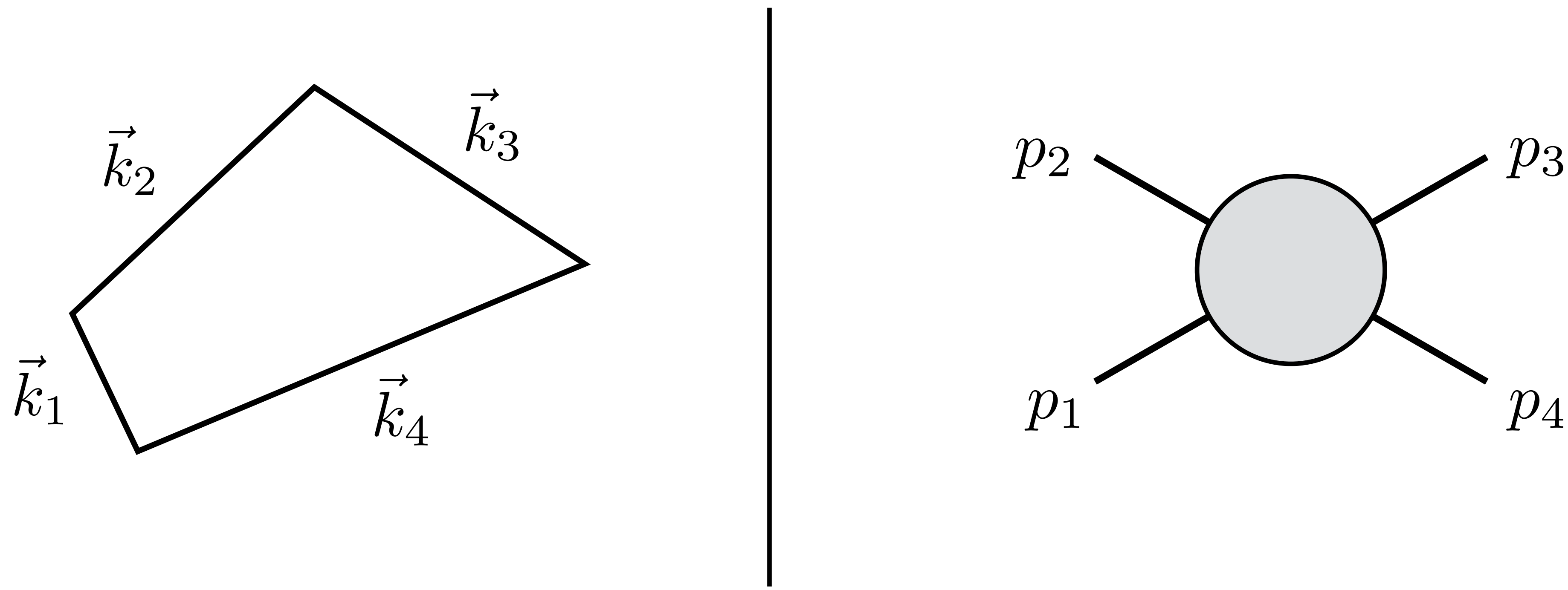
- **SYMMETRIES:**
  These singularities are connected by causal time evolution, which is constrained by the symmetries of the bulk spacetime.

  ⟶ Analog of Lorentz symmetry for amplitudes.

Arkani-Hamed, DB, Lee and Pimentel [2018]
DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]
DB, Chen, Duaso Pueyo, Joyce, Lee and Pimentel [2021]

Singularities

Correlators depend on the same external data as scattering amplitudes:



Two important differences:

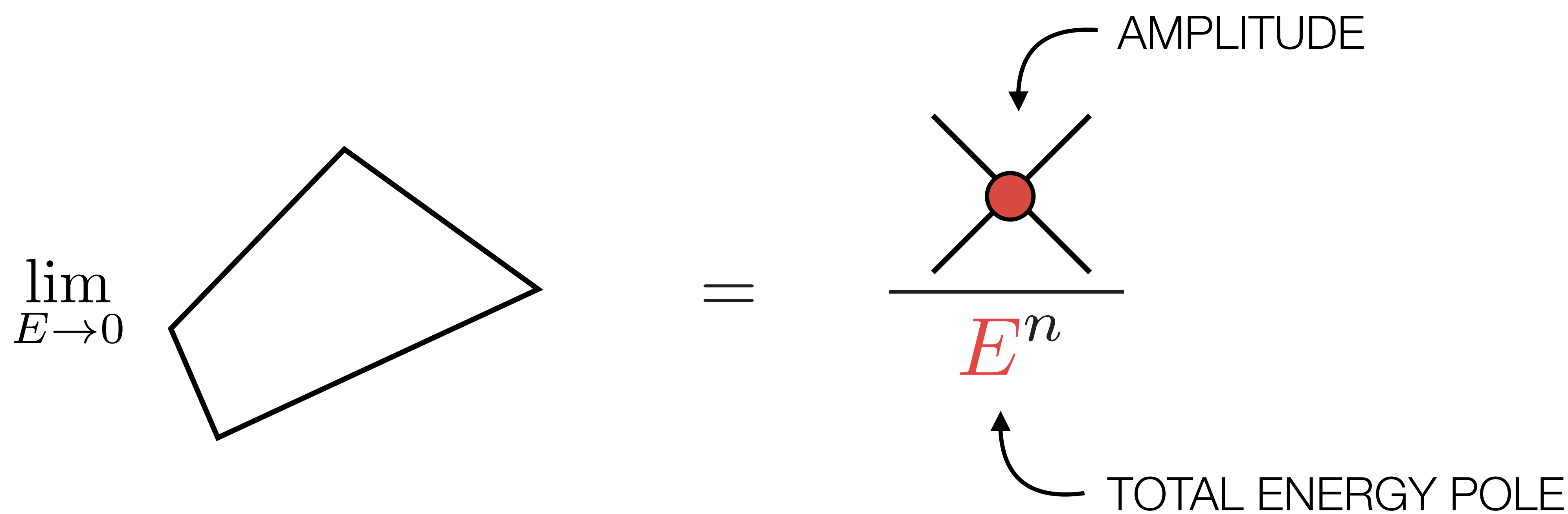
- All energies must be positive: $E_n \equiv |\vec{k}_n| > 0$

- The total energy is **not** conserved in cosmology:

$$E \equiv |\vec{k}_1| + |\vec{k}_2| + |\vec{k}_3| + |\vec{k}_4| \neq 0$$

Singularities

Something interesting happens in the limit of would-be energy conservation:

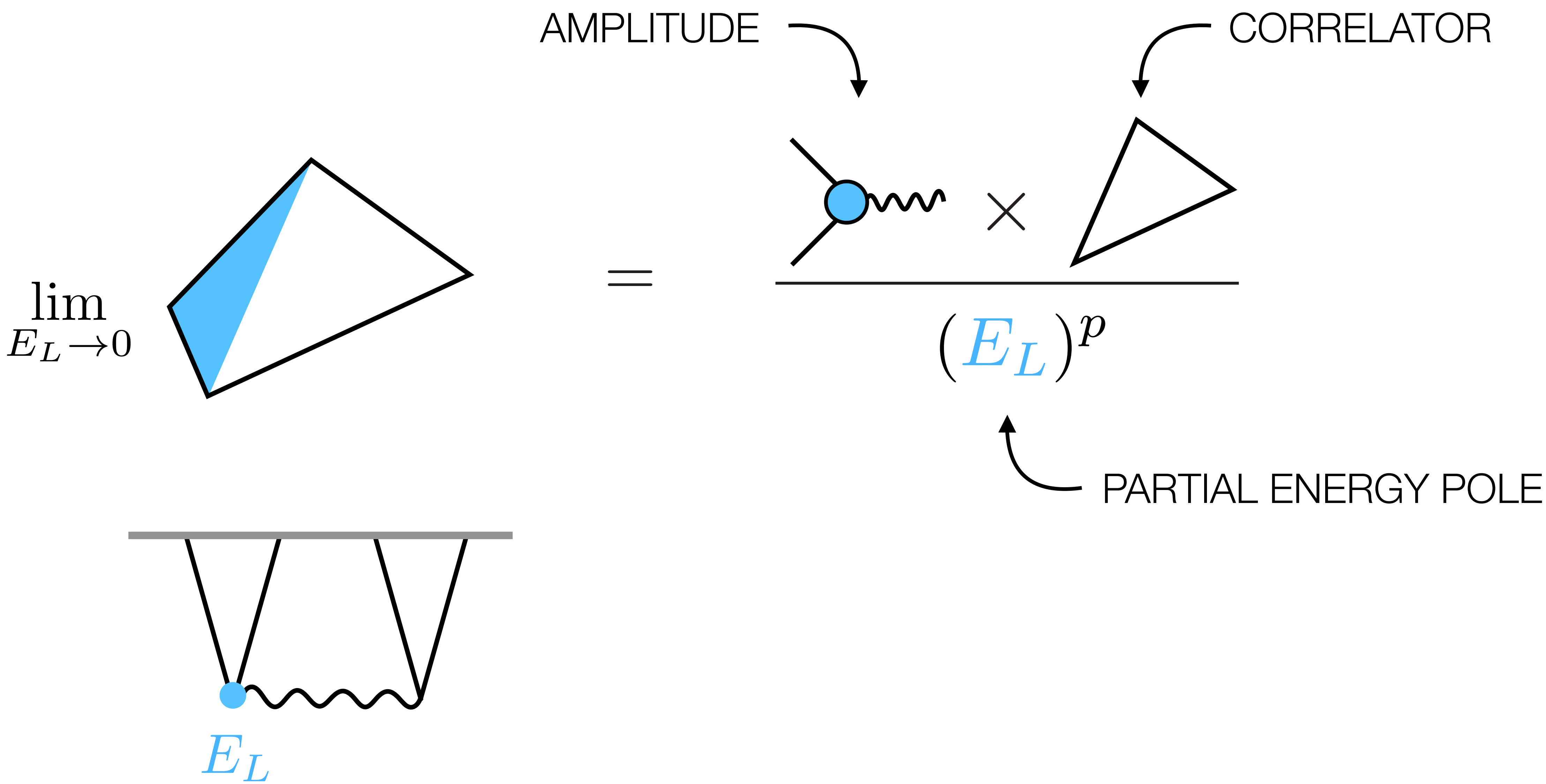


→ Amplitudes live inside correlators.

Raju [2012]
Maldacena and Pimentel [2011]

Singularities

Additional singularities arise when the energy of a subgraph is conserved:



Arkani-Hamed, Benincasa and Postnikov [2017]
Arkani-Hamed, DB, Lee and Pimentel [2018]
DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

Singularities

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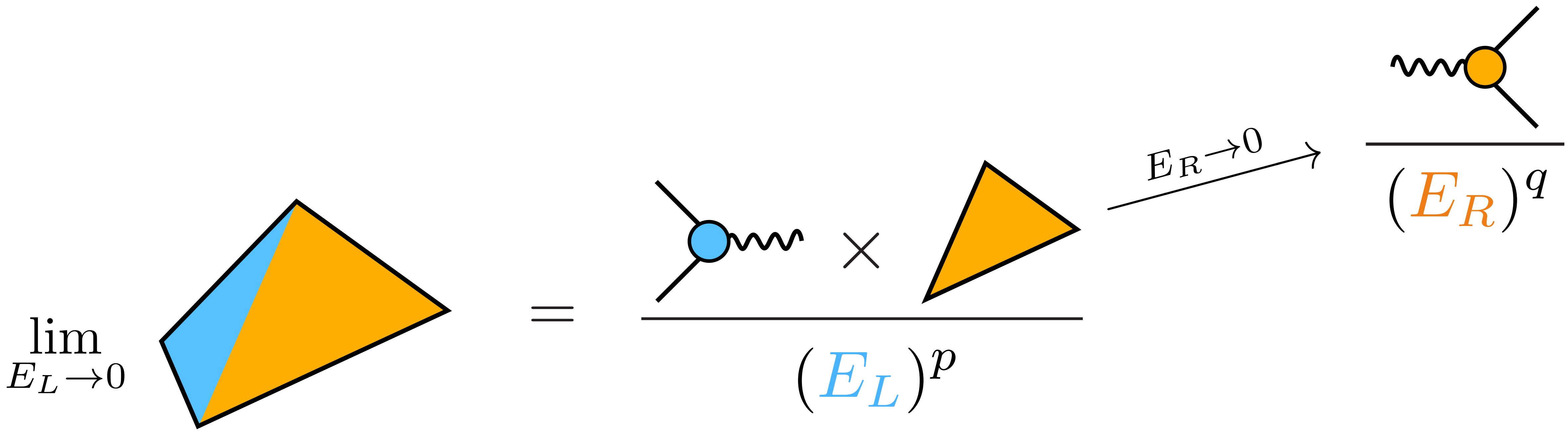


→ Amplitudes are the building blocks of correlators.

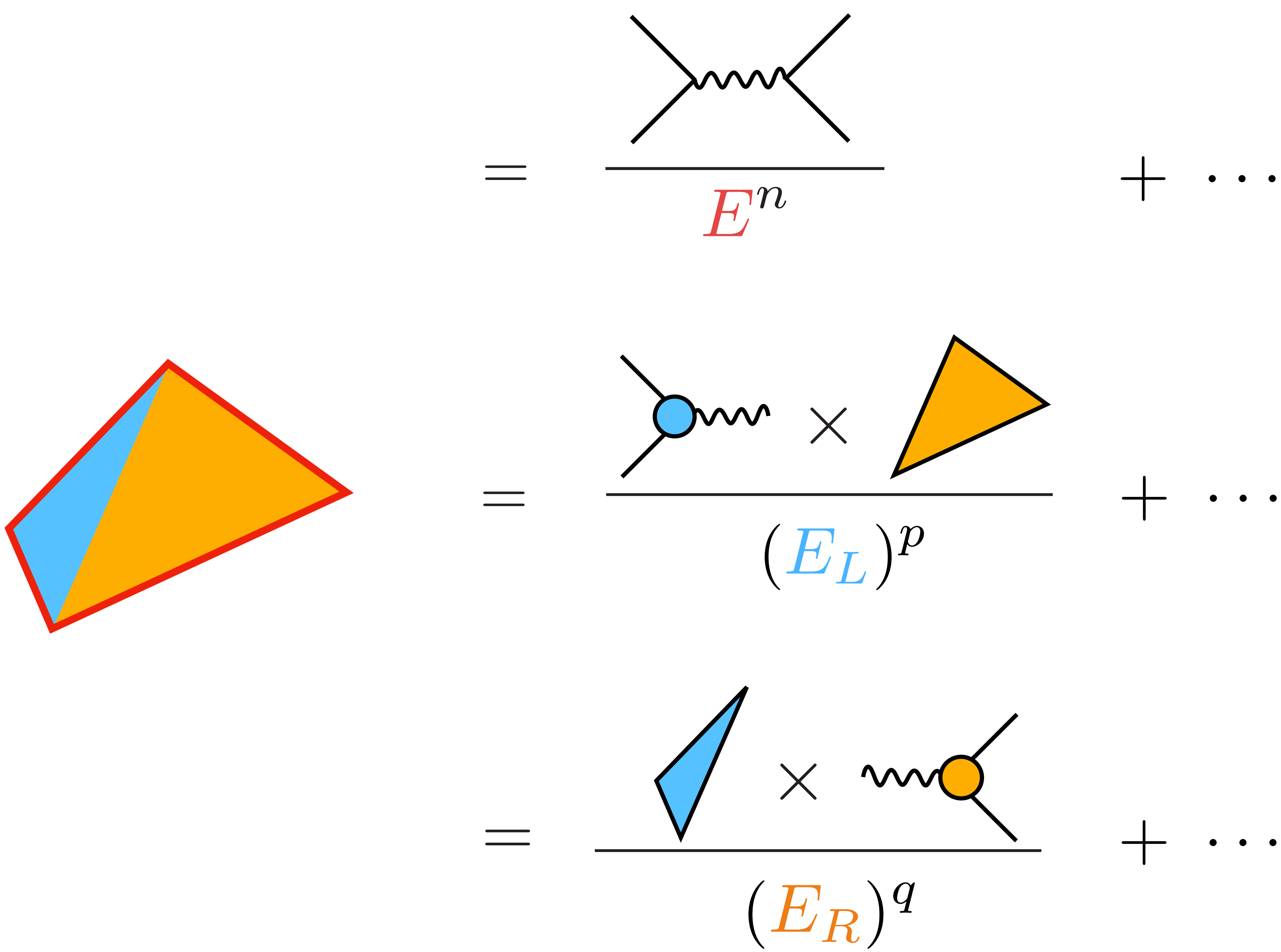
Arkani-Hamed, Benincasa and Postnikov [2017]

Arkani-Hamed, DB, Lee and Pimentel [2018]

DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

Singularities

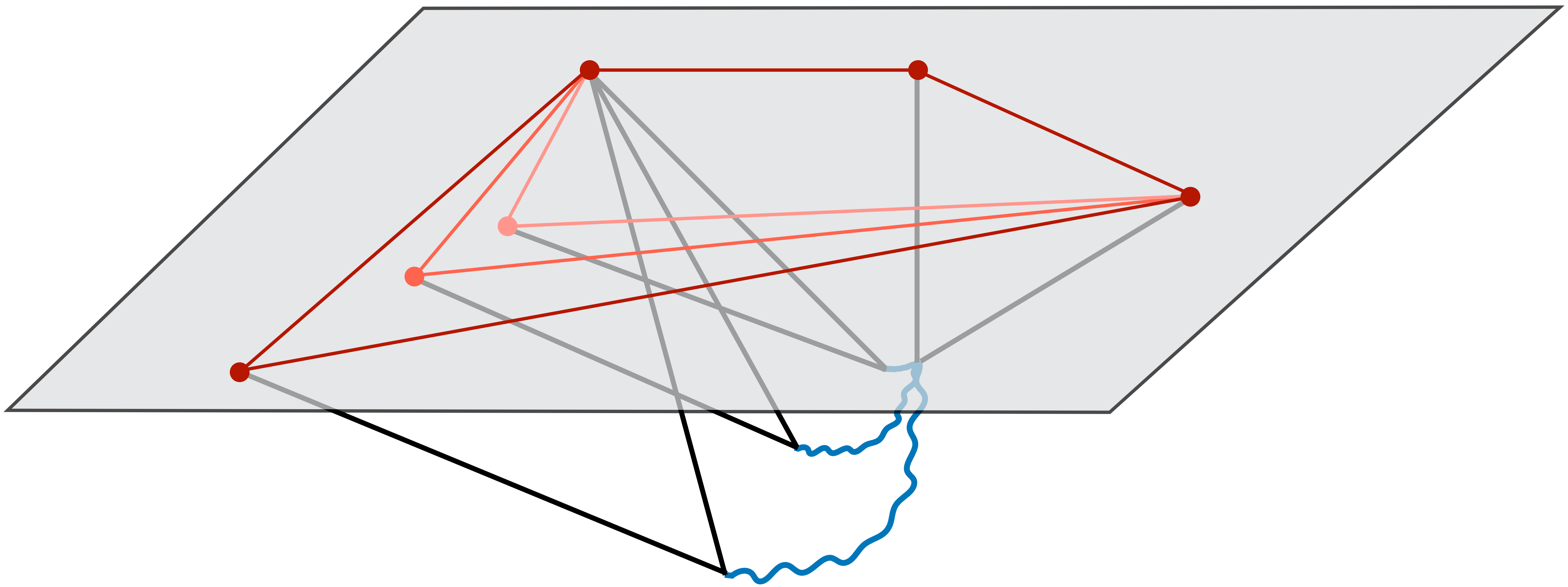
At tree level, these are the only singularities of the correlator:



To determine the full correlator, we must connect these singular limits.

Time Without Time

Time evolution in the bulk = momentum scaling on the boundary:



The allowed deformations are constrained by **symmetries**. For example:
In dS, the boundary correlators must satisfy conformal Ward identities.

Time Without Time

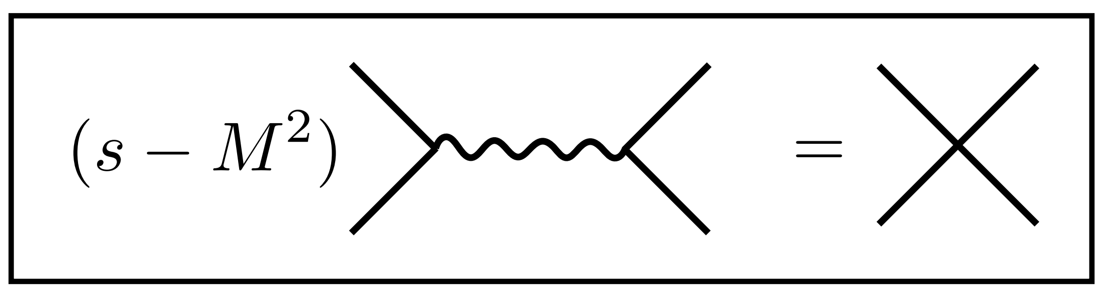
Tree-exchange correlators satisfy:

CONTACT SOLUTION

$$\left(\partial_{E_L}^2 + M^2\right) F_E = F_C$$

FIXED BY SYMMETRY

EXCHANGE SOLUTION



Given the known singularities, the solutions can be classified.

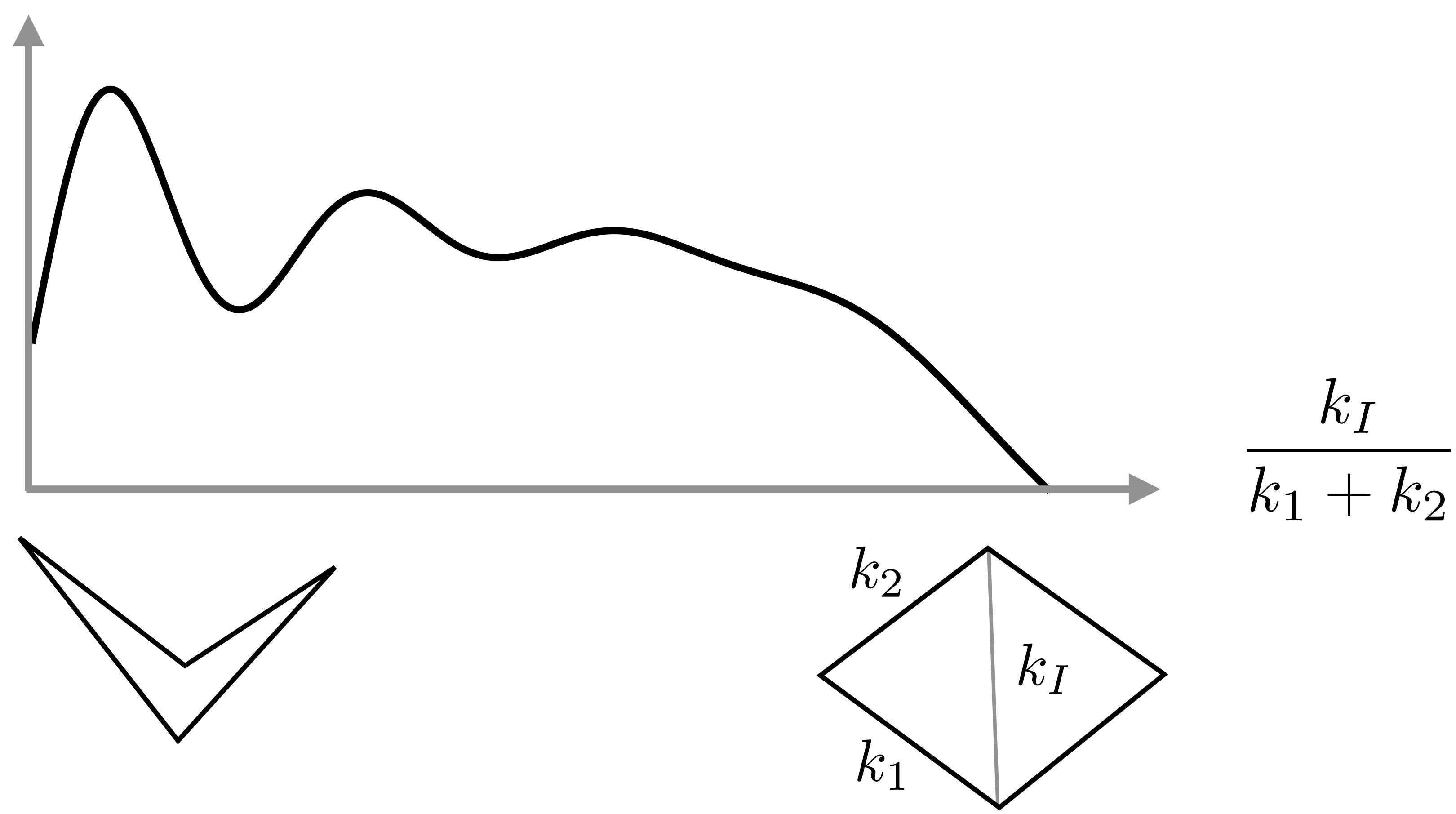
Arkani-Hamed, DB, Lee and Pimentel [2018]

Particle Production

In the limit of small internal momentum, the equation becomes

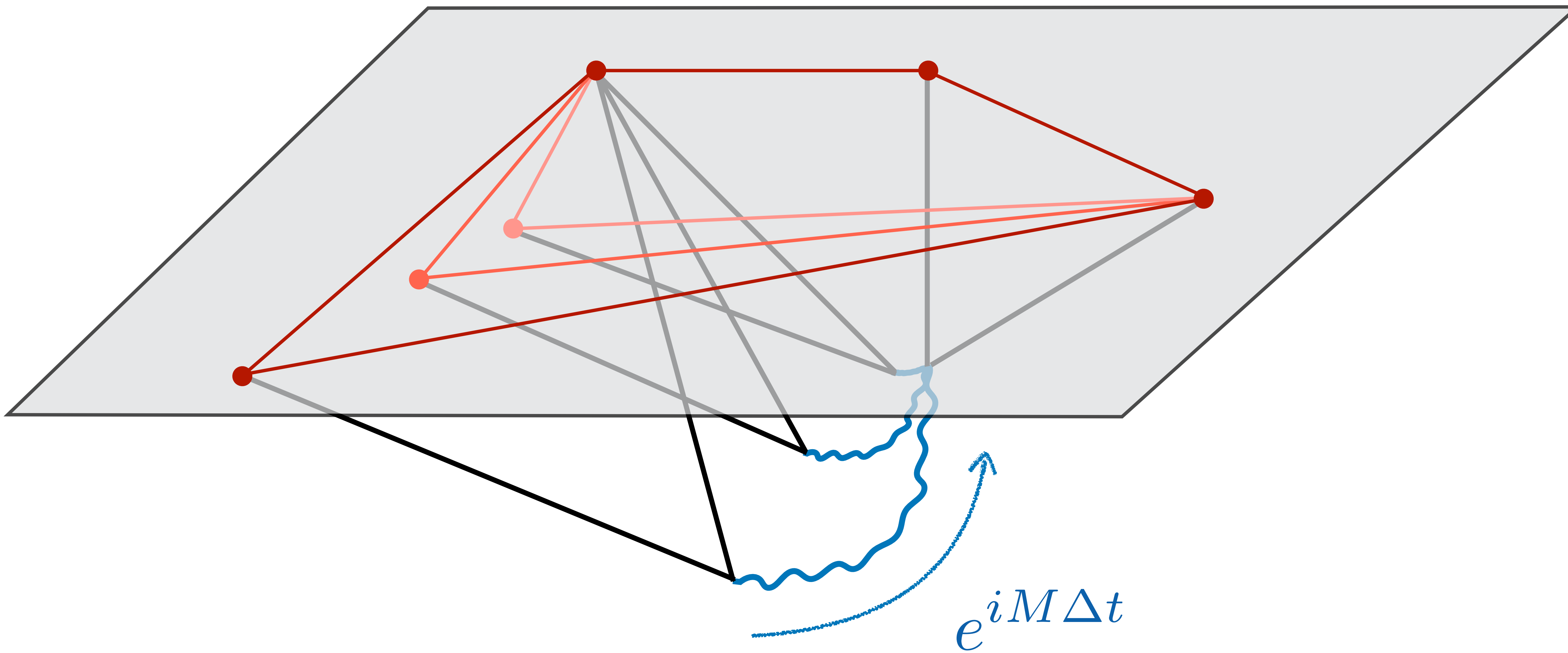
$$\boxed{\left[\frac{d^2}{dt^2} + M^2\right] F_E = \frac{1}{\cosh t}} \qquad t = \ln(E_L/E_R)$$

and the solution has an oscillatory feature



Particle Production

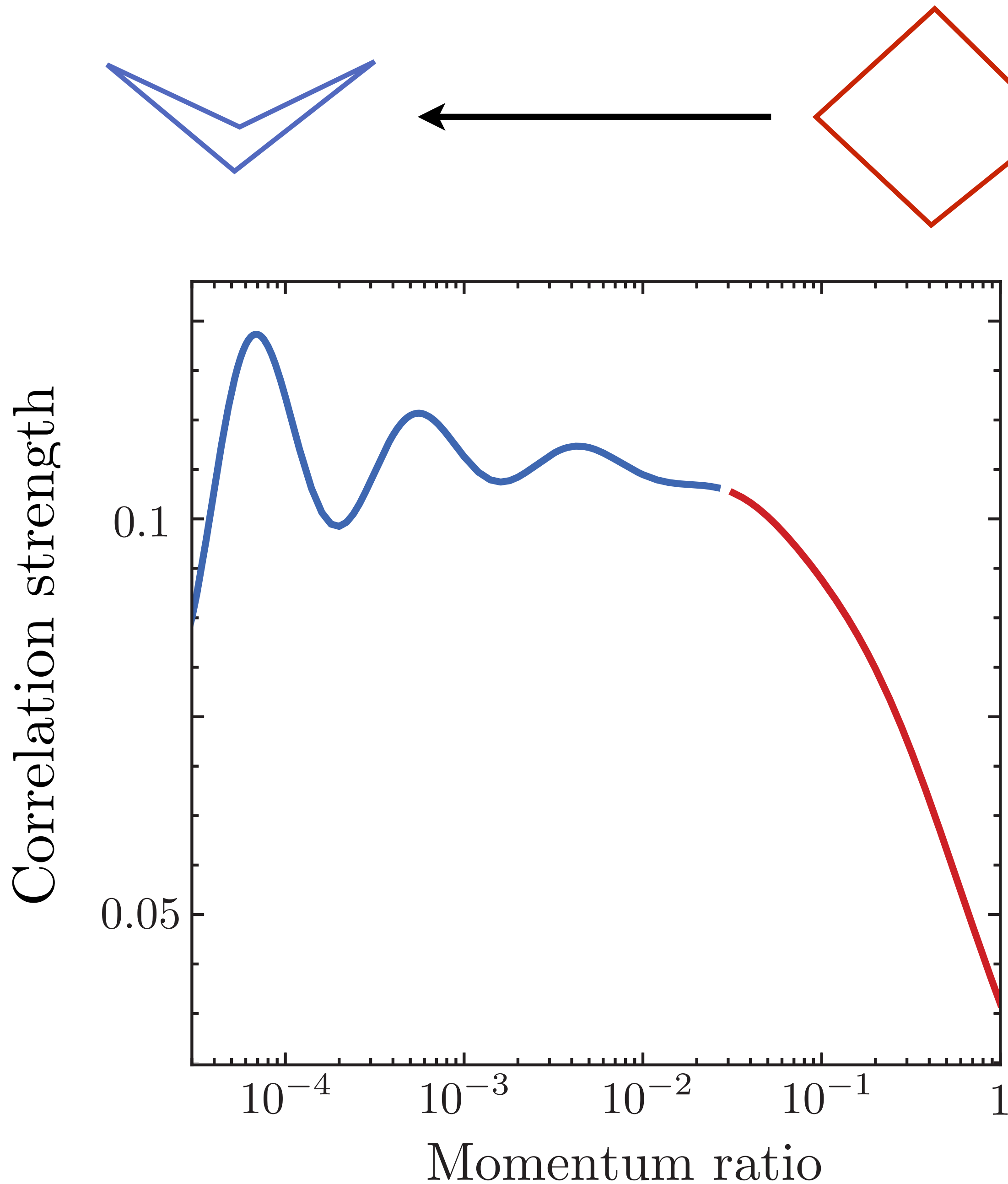
These oscillations reflect the evolution of the massive particles during inflation:

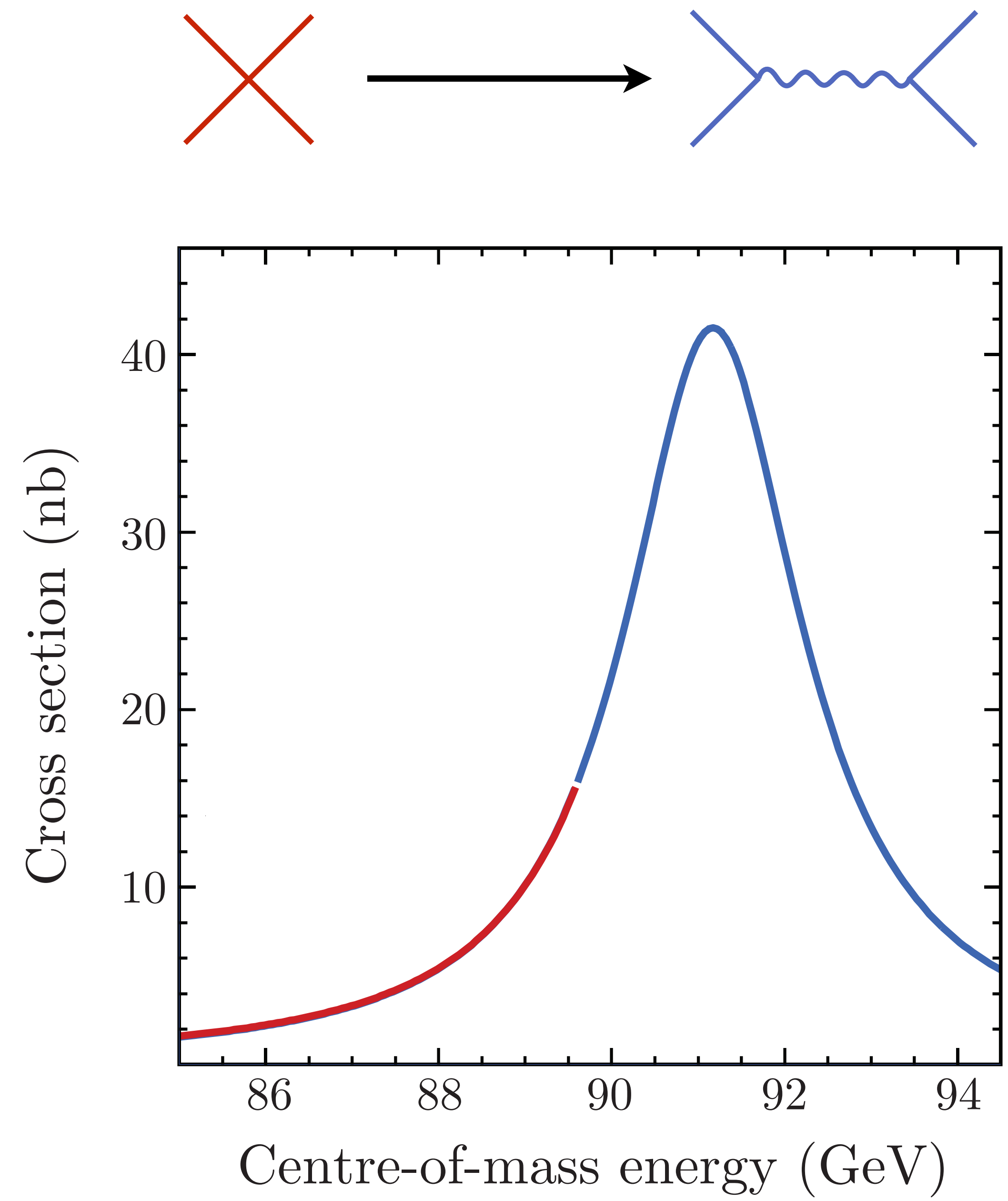


Time-dependent effects emerge in the solution of the time-independent bootstrap constraints.

Cosmological Collider Physics

The oscillatory feature is the analog of a **resonance** in collider physics:





II. Recent Progress

So far, our analysis was limited in two ways:

1. We have assumed an approximate de Sitter symmetry.

$\longrightarrow$ We would like to allow for **symmetry breaking**.

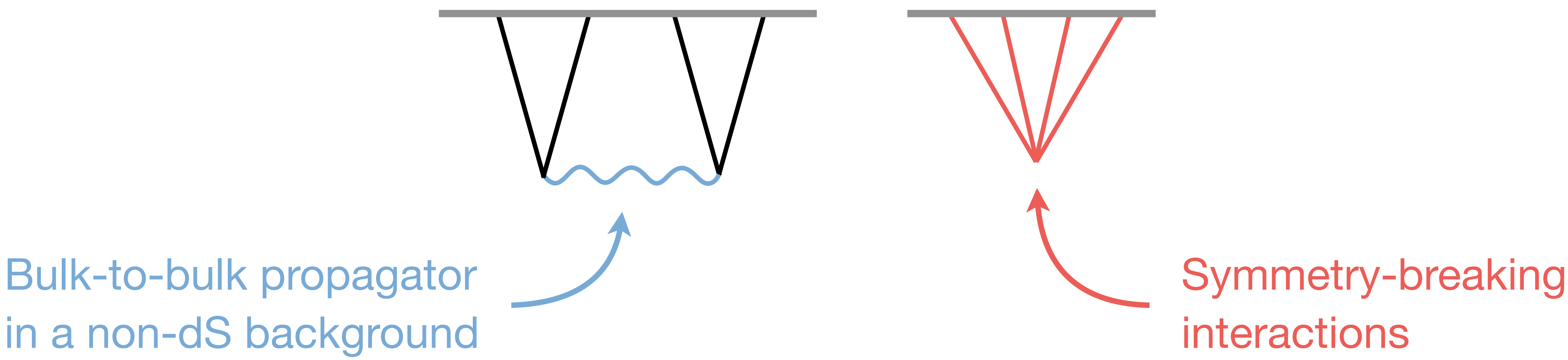
2. We have studied only scalar correlators.

$\longrightarrow$ We would like to allow for **particles with spin**.

Symmetry Breaking

The symmetries can be broken at the level of the **background** and the **interactions**. This can be incorporated in the boundary differential equation:

$$\boxed{\left(\partial_{E_L}^2 + M^2\right)} F_E = \boxed{F_C}$$



A systematic classification of these effects is still an open problem.

Symmetry Breaking

Another way to account for symmetry breaking is in the **transmutation** of flat space correlators:

$$F^{(\mathrm{FRW})} = \mathcal{T}\left(F^{(\mathrm{flat})}\right)$$

Flat space correlator:
easier to bootstrap or compute

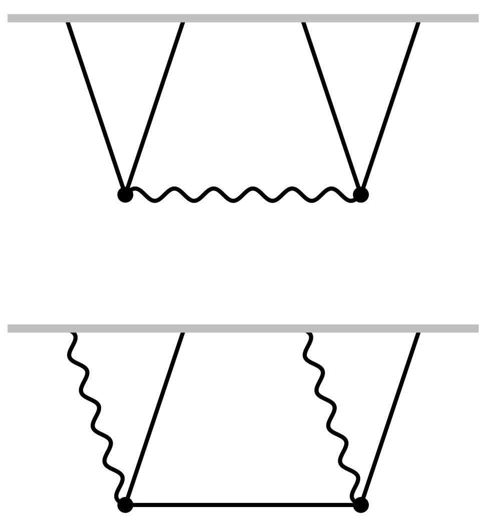
- So far, the relevant transmutation operators are found "experimentally".
- A complete theory of the transmutation is still an open problem.

DB, Chen, Duaso Pueyo, Joyce, Lee and Pimentel [2021]

Particles with Spin

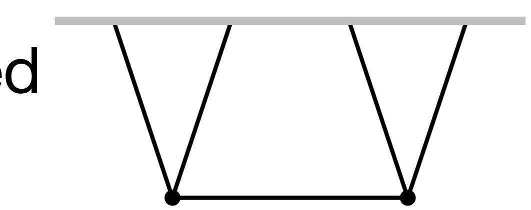
Spin-raising operators can create correlators with spinning particles (internal and external):

$$F_S = \mathcal{S}(F_E)$$



Spinning correlator

Scalar seed correlator



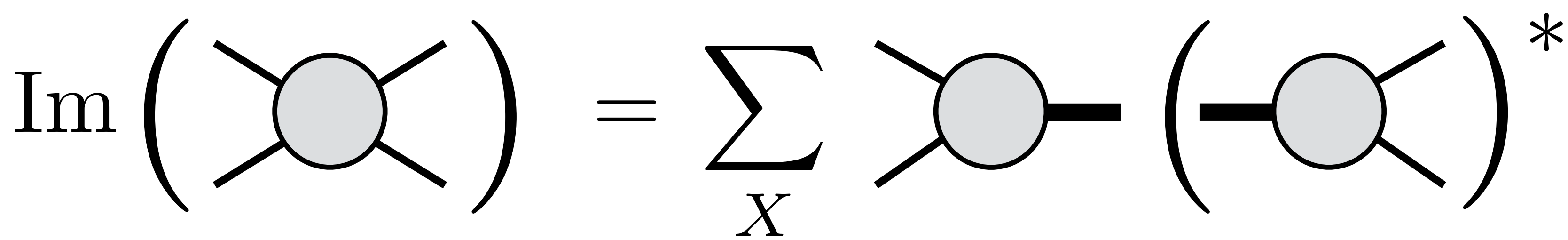
- These spin-raising operators have been classified in conformal field theory.

Arkani-Hamed, DB, Lee and Pimentel [2018]
DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]
DB, Chen, Duaso Pueyo, Joyce, Lee and Pimentel [2021]

Unitarity

For massless spinning particles, the correlators can be fixed by **unitarity**.

- For scattering amplitudes, unitarity implies the **optical theorem**



and the Cutkosky **cutting rules**.

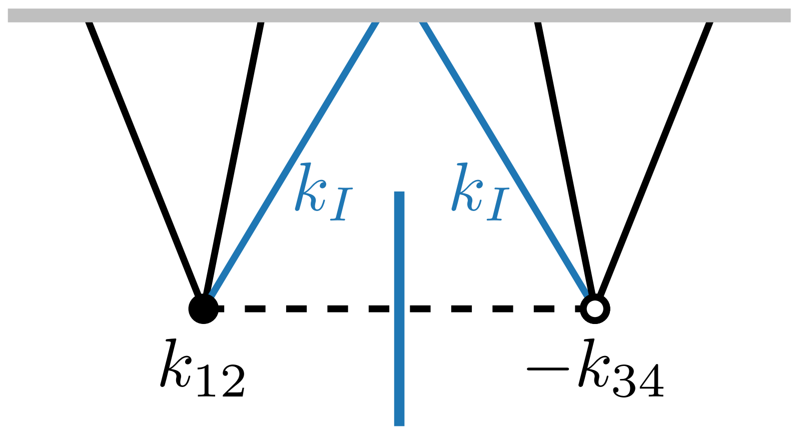
- What are the constraints of unitarity for cosmological correlators?

Until recently this was not know.

Unitarity

Goodhew, Jazayeri and Pajer derived the **cosmological optical theorem**

$$\boxed{F_4(k_n) + F_4^*(-k_n) = \tilde{F}_3^L \otimes \tilde{F}_3^R}$$



and associated **cutting rules**:

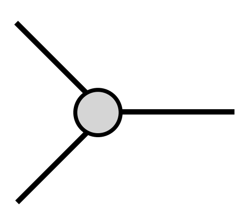
$$\boxed{F_N(k_n) + F_N^*(-k_n) = -\sum_{\text{cuts}} F_N}$$

- Unitarity enforces consistent factorization away from the energy poles.
- In some cases, this fixes the correlator without using de Sitter symmetry.

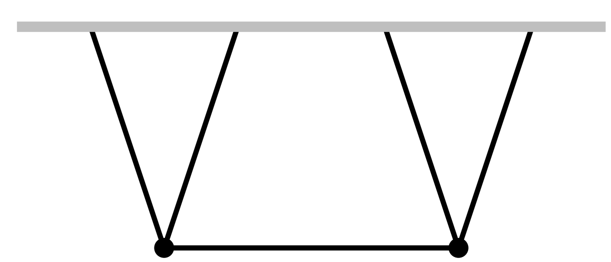
Goodhew, Jazayeri and Pajer [2020]
Jazayeri, Pajer and Stefanyszyn [2021]
DB, Chen, Duaso Pueyo, Joyce, Lee and Pimentel [2021]

The Space of Cosmological Correlators

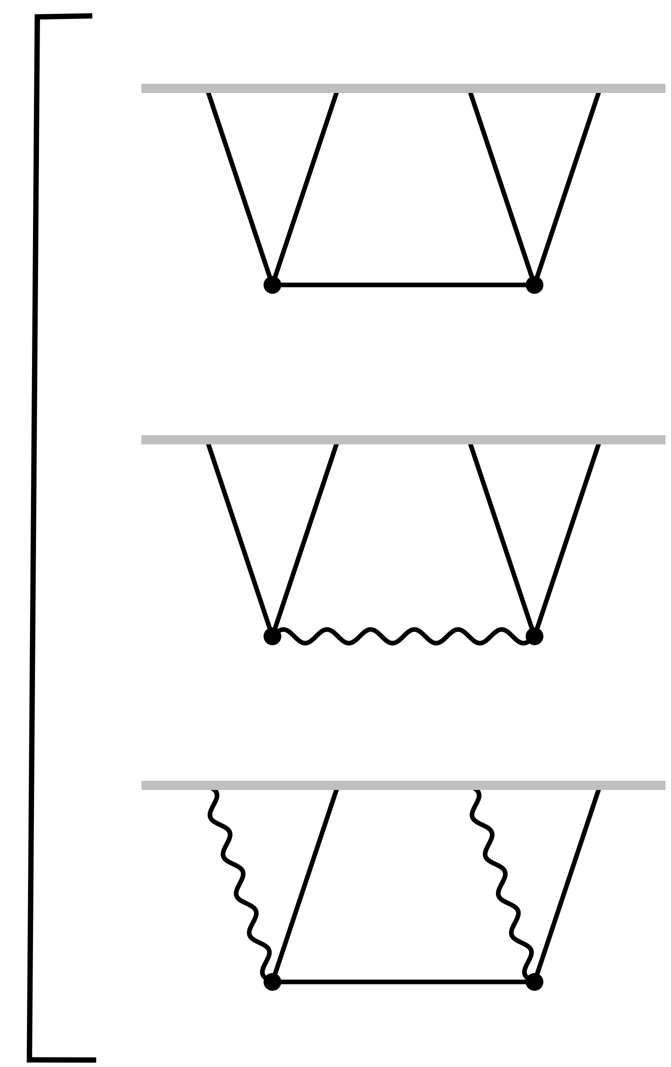
This has allowed us to bootstrap a large number of cosmological correlators (using scattering amplitudes and flat-space correlators as input):



Scattering amplitudes



Flat-space correlators



Cosmological correlators

DB, Chen, Duaso Pueyo, Joyce, Lee and Pimentel [2021]

III. Future Directions

Cosmological correlation functions have a rich structure:

- Much of this structure is controlled by **singularities**.
- Behavior away from singularities captures **local bulk dynamics**.

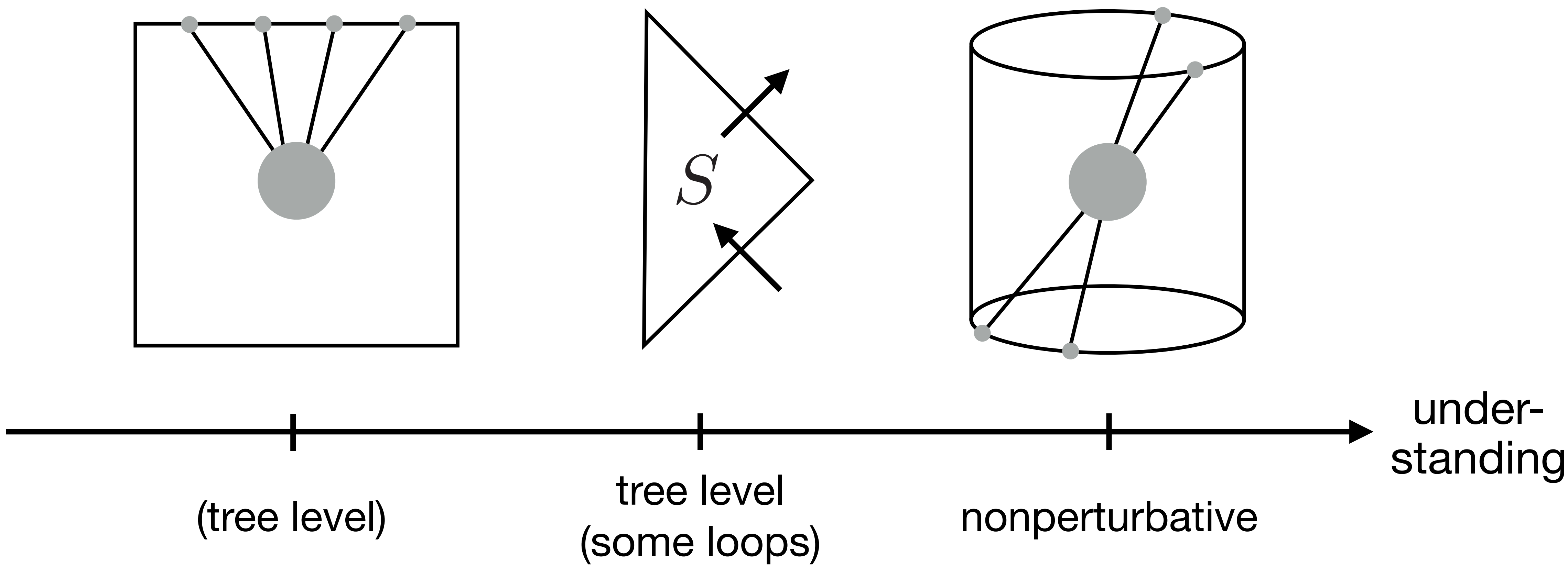
The bootstrap approach has led to new **conceptual insights**:

- Complex correlators can be derived from simpler seed functions:

It also has **practical applications**:

- Signatures of massive particles during inflation can be classified.

Only the beginning of a systematic exploration of cosmological correlators:



Important open questions are

- Are there **hidden structures** to be discovered?
- Can we go **beyond Feynman diagrams**?
- What is the analytic structure **beyond tree level**?
- What is the space of **UV-complete correlators**?
- What are its **observational signatures**?

Thank you for your attention!