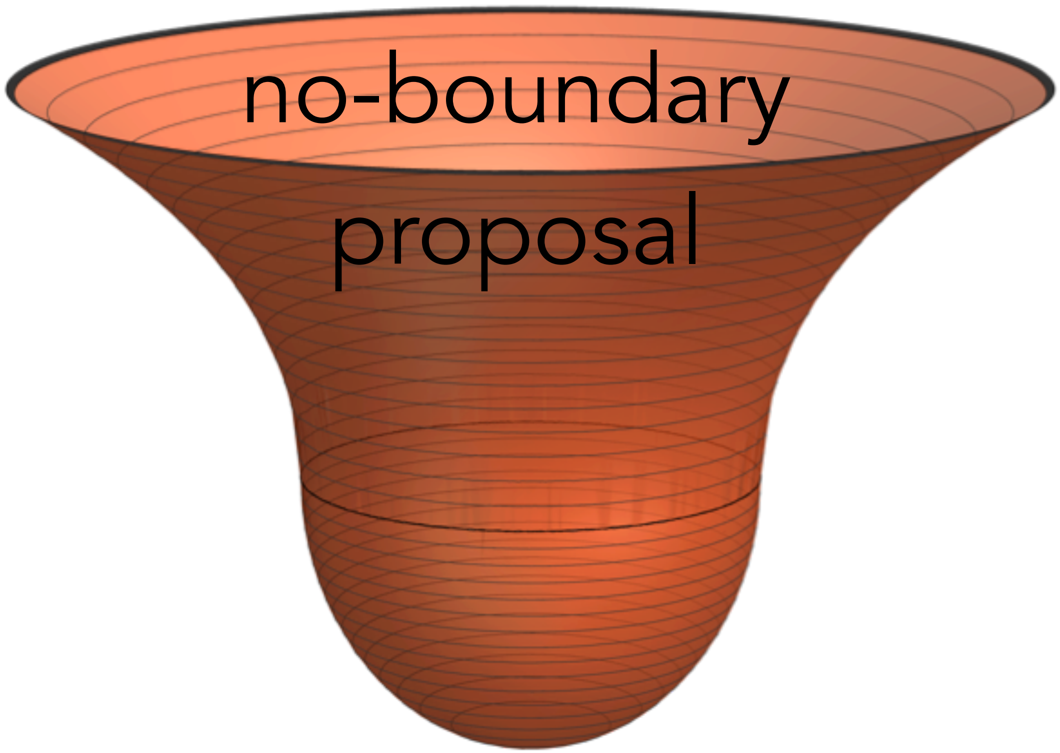
Recent developments in the minisuperspace no-boundary proposal



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European Research Council
Established by the European Commission
Supporting top researchers from anywhere in the world

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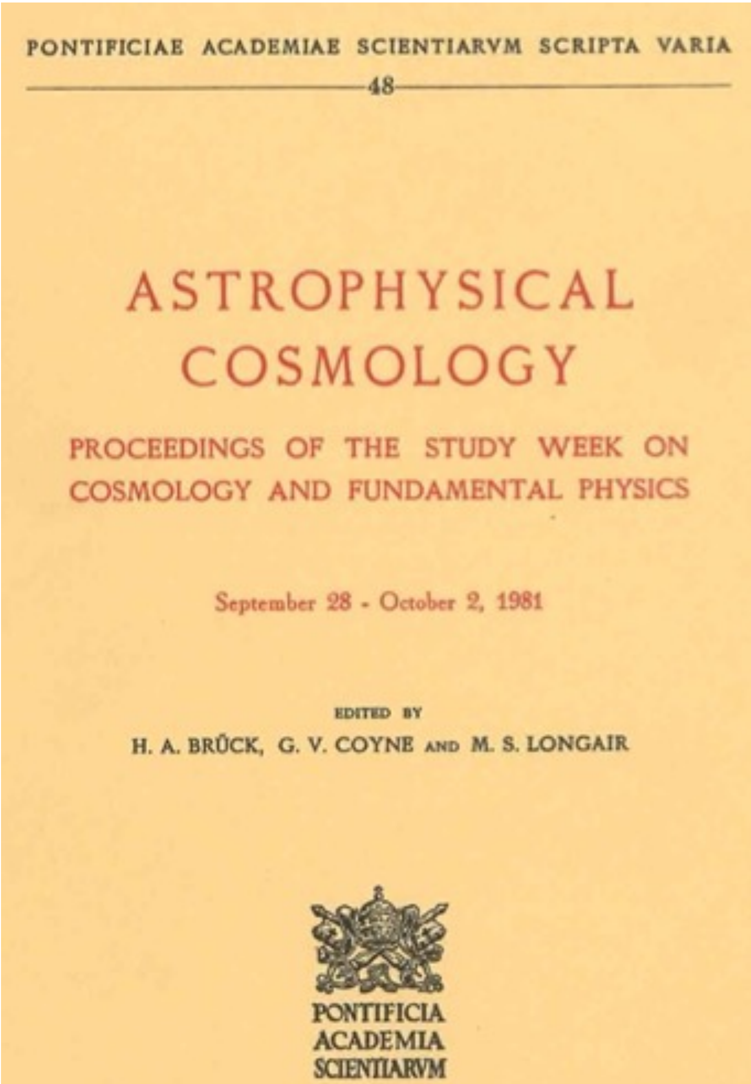
Plan

- 40 years of the no-boundary proposal (from 1981 onwards)
- Recent developments (last few years)
- Complex metrics (à la Kontsevich-Segal)

First appearance – September 1981



Allocution of Pope John Paul II

Toute hypothèse scientifique sur l'origine du monde, comme celle d'un atome primitif d'où dériverait l'ensemble de l'univers physique, laisse ouvert le problème concernant le commencement de l'univers. La science ne peut par elle-même résoudre une telle question: il y faut ce savoir de l'homme qui s'élève au-dessus de la physique et de l'astrophysique et que l'on appelle la métaphysique; il y faut surtout le savoir qui vient de la révélation de Dieu.

"Science by itself cannot resolve this question [of the beginning of the universe]… it requires the knowledge that comes from the revelation of God"

First appearance – September 1981

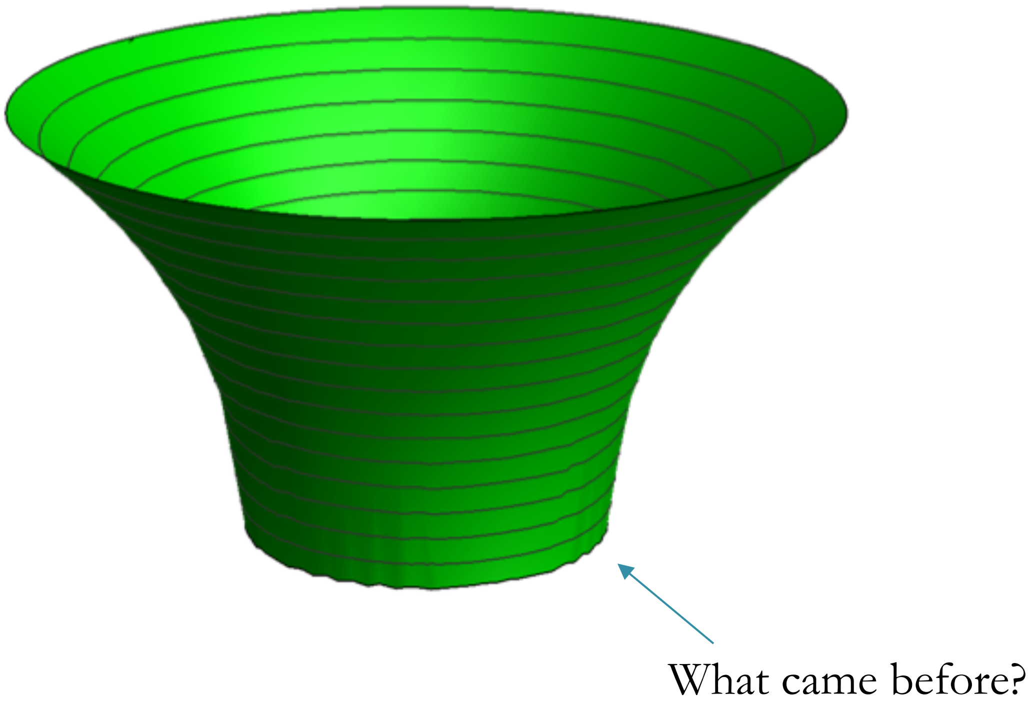
THE BOUNDARY CONDITIONS OF THE UNIVERSE

S.W. HAWKING
University of Cambridge, Cambridge

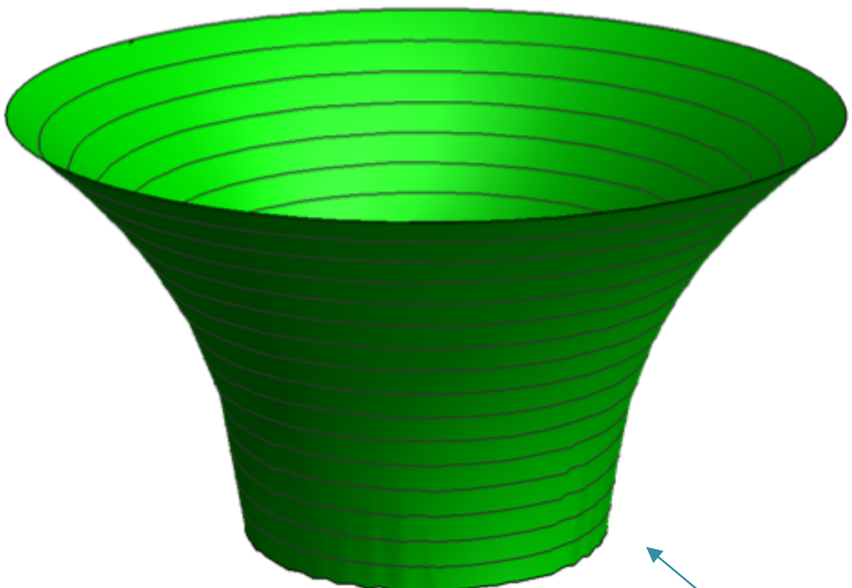
This paper considers the questions of what are the boundary conditions of the universe and where should they be imposed. It is difficult to define boundary conditions at the initial singularity and, even if one could, they would be insufficient to determine the evolution of the universe. In order to overcome this problem it is suggested that one should adopt the Euclidean approach and evaluate the path integral for quantum gravity over positive definite metrics. If one took these metrics to be compact, one would avoid the need to specify any boundary conditions for the universe. This approach might explain why the apparent cosmological constant is zero, why the universe is spatially flat, and why it was in thermal equilibrium at early times.

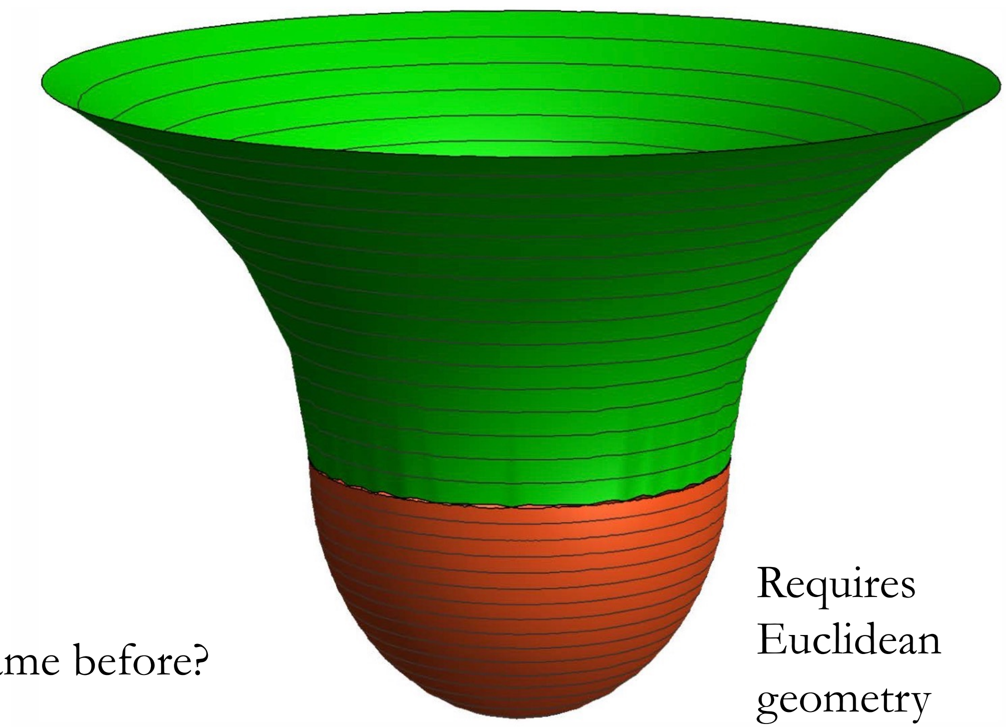
[*Pontif.Acad.Sci.Scr.Varia* 48 (1982) 563-574]

- Going back in time to an earlier phase, one can always ask what came before
- This leads to an infinite regression…



- If the geometry is smoothly rounded off, then
  - The infinite regression is avoided
  - The initial singularity is avoided
  - We may not need to specify any boundary conditions





Expectations

- Ideas spelled out in great detail by Hartle & Hawking
  [*Phys.Rev.D* 28 (1983) 2960-2975]

- The wave function of the universe should be calculated via a path integral defined by a sum over compact, regular, Euclidean metrics

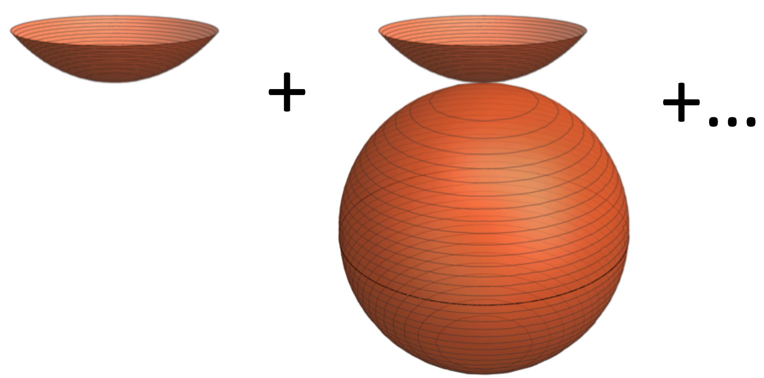
$$\Psi(final) = \int_{no-boundary}^{final} e^{-S_E/\hbar}$$

Expectations

- The w.f. should be real (because defined over real Euclidean metrics)
- The w.f. is expected/hoped to be unique
- The w.f. represents the ground state of the universe

$$e^{iEt} \overset{t=-i\tau}{\longrightarrow} e^{E\tau}$$ lowest E survives as $\tau \to -\infty$

- The w.f. gives different probabilities for different initial conditions
- Topology change can be incorporated: $\Psi(a=0) \neq 0$



Issues

- Conflict with uncertainty principle
- Friedmann equation (with k=+1):

$$\dot{a}^2 + 1 = a^2 \left(\frac{1}{6} \dot{\phi}^2 + \cdots \right)$$

- *Compactness* requires that at origin a=0
- *Regularity* requires a condition on expansion rate,

$$\boxed{\dot{a}^2 + 1 = 0} \quad \text{at} \quad a = 0 \qquad (\text{i.e.} \quad a_{,\tau} = \pm 1)$$

- Hence one is trying to fix both the field and the momentum

Issues

- Conformal mode problem

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + a(t)^2\mathrm{d}\mathbf{x}^2$$

$$S = \int \mathrm{d}t N \left(-3a\frac{\dot{a}^2}{N^2} + \frac{1}{2}a^3\frac{\dot{\phi}^2}{N^2} + \cdots \right)$$

Opposite signs

- Action unbounded above and below, so how should the Euclidean path integral $\int e^{-S_E/\hbar}$ be defined?

Issues

- Uniqueness?

- Early studies by Halliwell and Louko showed that a wide variety of boundary conditions and integration contours (especially for the lapse N) seemed possible in minisuperspace models

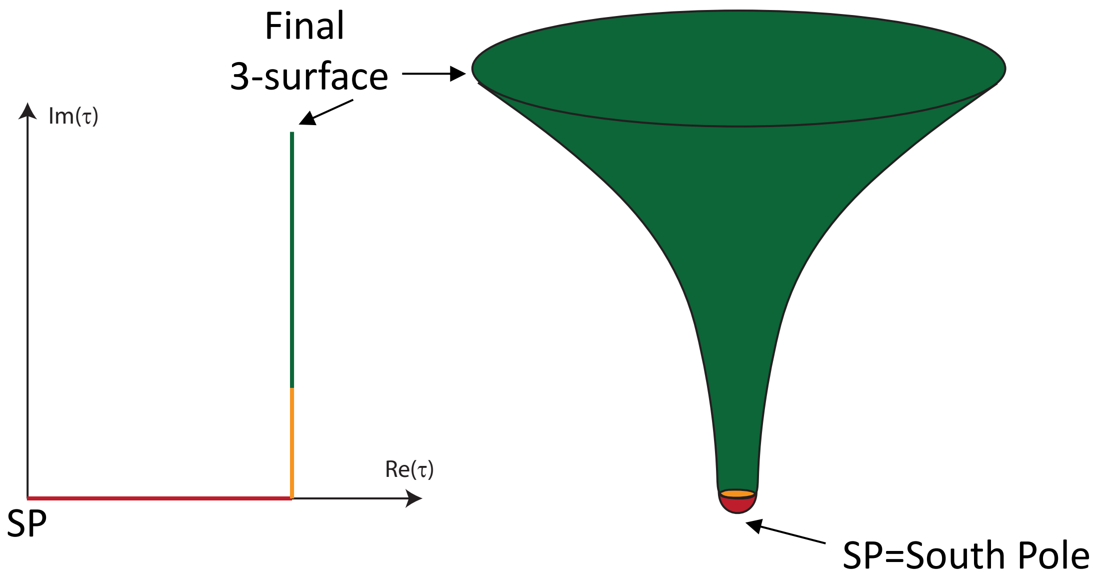
Early works

- Pragmatic approach:
  Focus on saddle points (= possibly complex solutions of classical equations of motion) of path integral, since these provide dominant contribution to path integral

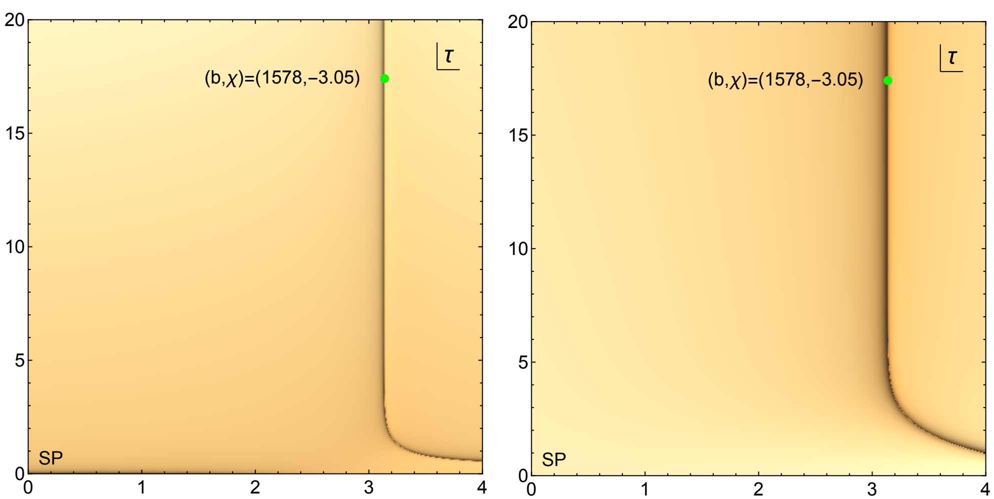
  [*recent Refs.:* De Alwis; Halliwell, Hartle & Hertog]

- Advantages:
  - Can specify both field values and momenta at nucleation
  - Do not need to worry about conformal mode problem
  - Uniqueness is only a problem if there exist multiple inequivalent saddle points

Early works – results



- Appropriate saddle points exist!
- When a scalar field with inflationary potential is included, the geometry is complex – *fuzzy instantons*



Scale factor   Scalar field

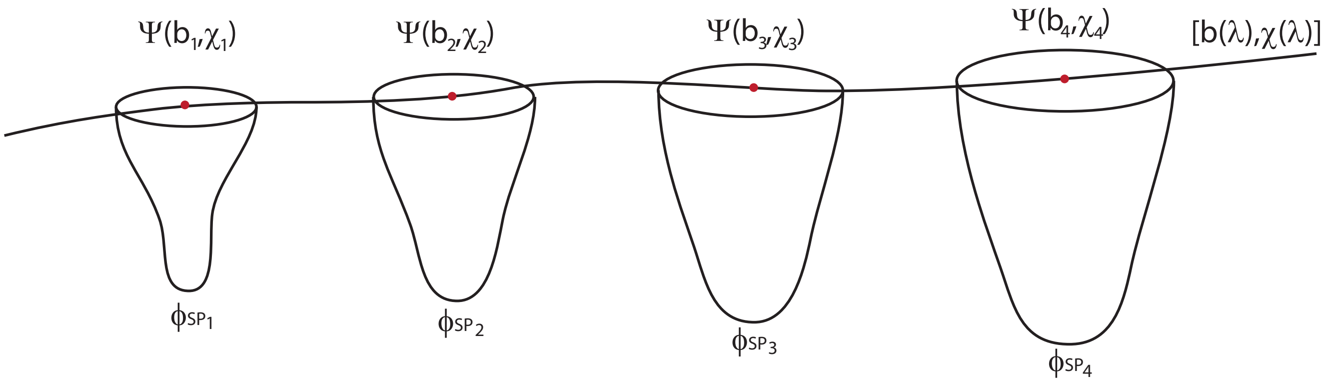
Dark lines
=
Real field values

b, final value of a
χ, final value of ϕ

[Lyons]

Early works – results

- A sequence of instantons with changing boundary data shows how the wave function evolves



- If the wave function is of WKB form, i.e. slowly varying amplitude, fast varying phase (as the universe expands),

$$\Psi \approx A e^{iP} \quad \text{with} \quad \partial P \gg \partial A$$

then one can assign an approximately conserved probability $|\Psi|^2 = A^2$ to the associated history

[Hartle, Hawking & Hertog; Vilenkin]

Early works – results

- Does the wave function automatically reach WKB form?
- No! need attractor mechanism

- Best understood example: inflation
- E.g. $m^2\phi^2$ potential

$$\Psi(a_f, \phi_f, \phi_{SP}) \approx e^{\frac{i}{\hbar} S_{on-shell}}$$

$$\approx e^{\frac{12\pi^2}{\hbar V(\phi_{SP})} + i \frac{2\pi^2}{\hbar} \sqrt{\frac{2}{3}} m \phi_f a_f^3}$$

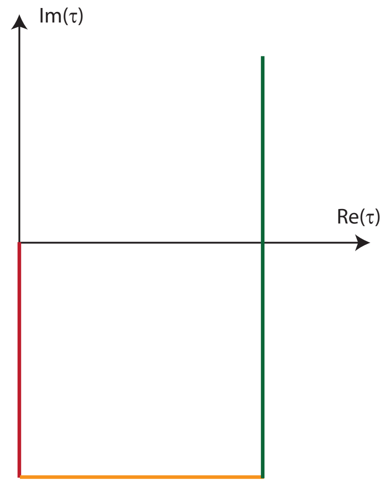
Weighting almost constant for slow-roll

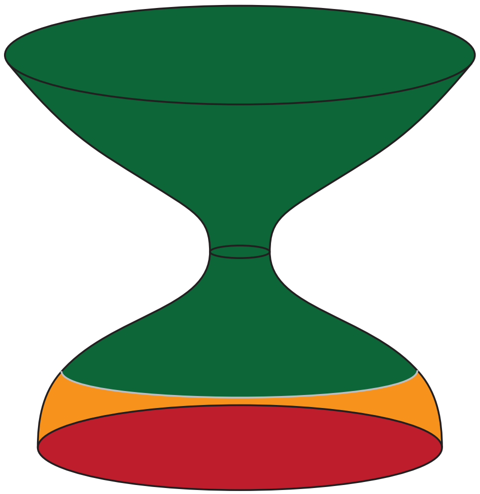
Phase grows as volme

[Lyons]

Early works – results

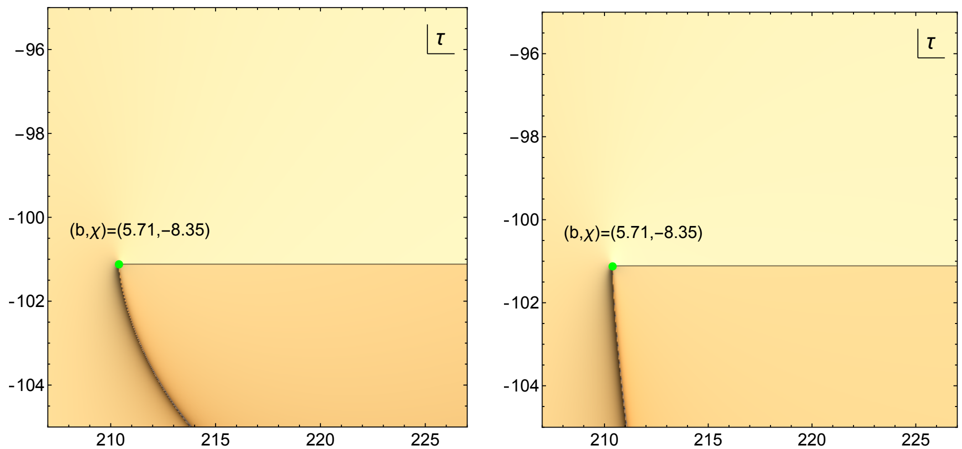
- Only other mechanism known: ekpyrosis





- WKB reached as the universe contracts
- Weighting

$$\Psi \approx e^{\frac{1}{|V(\phi_{SP})|}}$$



Scale factor   Scalar field

- Connection to present day requires an understanding of the bounce phase

[Battarra & JLL]

Early works – results

- The no-boundary wave function explains the origin of classical realms, but *only* in regions of the potential allowing for inflation (positive, flat) or ekpyrosis (negative, steep)

- Low values of potential come out as preferred, i.e. short inflationary phases or long ekpyrotic phases

- This has led to speculations that one needs to use anthropic reasoning to argue for a long inflationary phase, and possibly eternal inflation

- In my view this is premature, as a rigorous understanding of probabilities, and of the possible scalar potentials in fundamental theory, is still lacking

[Hartle, Hawking & Hertog]

Early works – results

- Consider perturbations around saddle geometry, e.g. tensor perturbations h (for spherical harmonic k)

$$\ddot{h} + 3\frac{\dot{a}}{a}\dot{h} + \frac{k(k+2)}{a^2}h = 0$$

- 2 solutions: one is regular at a=0, the other blows up
- Regular solution corresponds to Bunch-Davies vacuum (for de Sitter case)

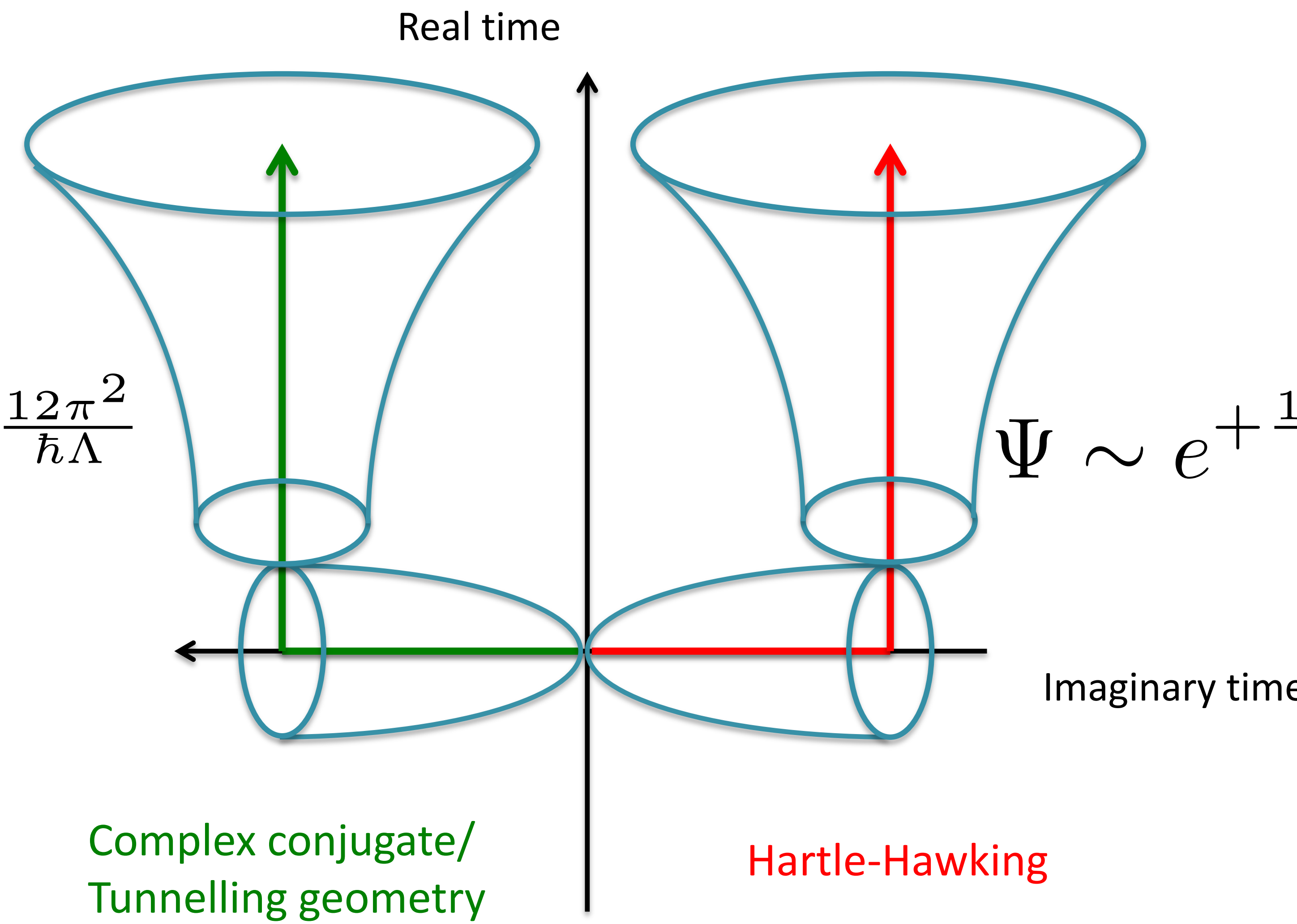
- Thus the no-boundary proposal *explains* the initial state of perturbations

[Halliwell & Hawking]

A subtlety

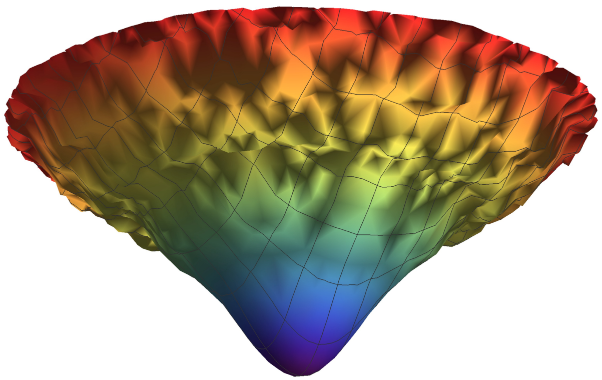
$$\Psi \sim e^{-\frac{12\pi^2}{\hbar\Lambda}}$$

$$\Psi \sim e^{+\frac{12\pi^2}{\hbar\Lambda}}$$



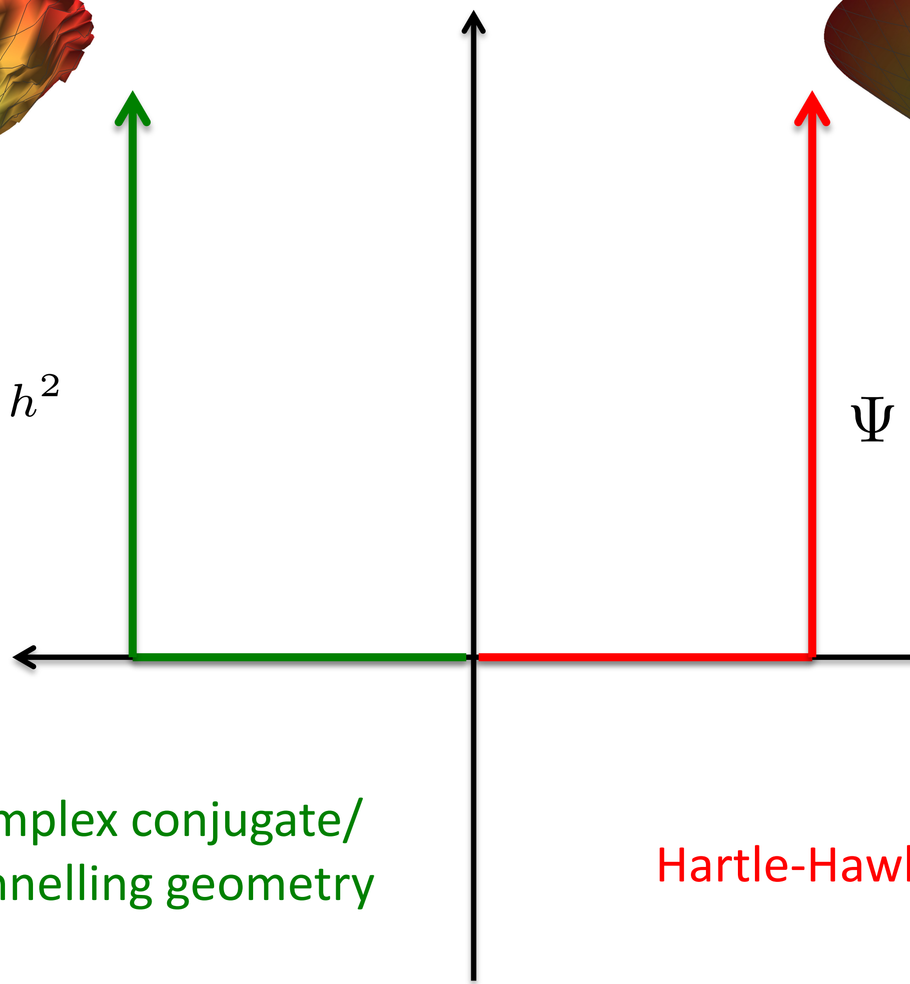
Effectively opposite Wick rotations

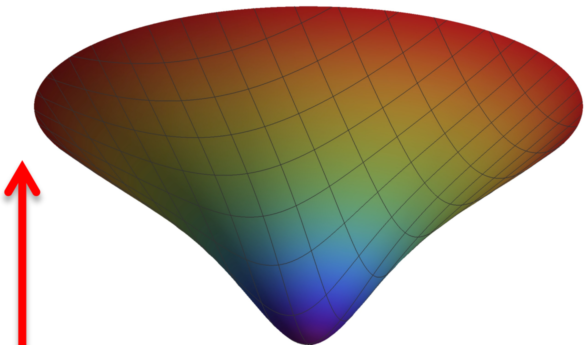
A subtlety



$$\Psi \approx e^{-\frac{12\pi^2}{\hbar\Lambda} + \frac{3k^3}{2\hbar\Lambda}h^2}$$

Unstable





$$\Psi \approx e^{+\frac{12\pi^2}{\hbar\Lambda} - \frac{3k^3}{2\hbar\Lambda}h^2}$$

Stable

No-Boundary Proposal – Implementation?

- We will consider the simple system

$$\Psi = \int_{\mathcal{C}} \delta N\, \delta q\, e^{iS(N,q)/\hbar}$$

Can add ghosts and choose constant N gauge in a integral – see e.g. [Teitelboim]

with

$$S = \frac{1}{2} \int \mathrm{d}^4x \sqrt{-g}\,(R - 2\Lambda)$$

Scale factor squared

- With the metric $\mathrm{d}s^2 = -\dfrac{N^2}{q(t)}\mathrm{d}t^2 + q(t)\mathrm{d}\Omega_3^2$

[Halliwell & Louko]

the action becomes quadratic

$$S = 2\pi^2 \int \mathrm{d}t \left(-\frac{3}{4N}\dot{q}^2 + 3N - N\Lambda q \right)$$

No-Boundary Proposal

- Implementation?
- First impose boundary condition that universe starts at zero size $q(t=0)=0$
- This implements the idea that we are summing over compact metrics

Ordinary integral for lapse

- We are left with an ordinary integral over the lapse function

$$\Psi = \int \frac{\mathrm{d}N}{N^{1/2}} e^{\frac{i}{\hbar}\left[N^3 \frac{\Lambda^2}{36} + N\left(3 - \frac{\Lambda}{2} q_1\right) - \frac{3}{4N} q_1^2\right]}$$

Integrate over real N

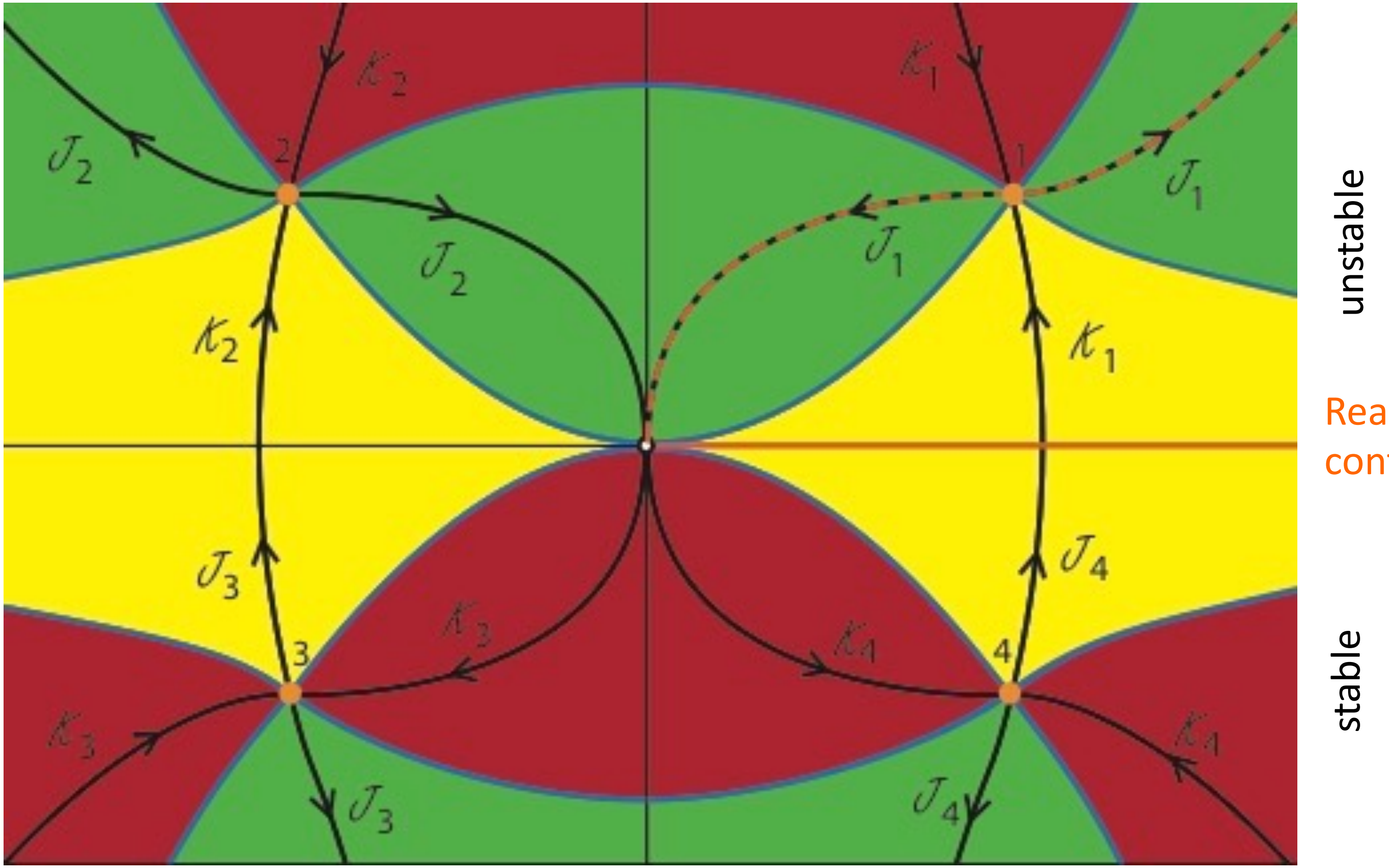
- There are 4 saddle points

$$N_s = \frac{3}{\Lambda}\left[\pm(\frac{\Lambda}{3} q_1 - 1)^{1/2} \pm i\right]$$

- The saddle points are complex
- Now we can apply Picard-Lefschetz theory (framework for systematic saddle point approximation)

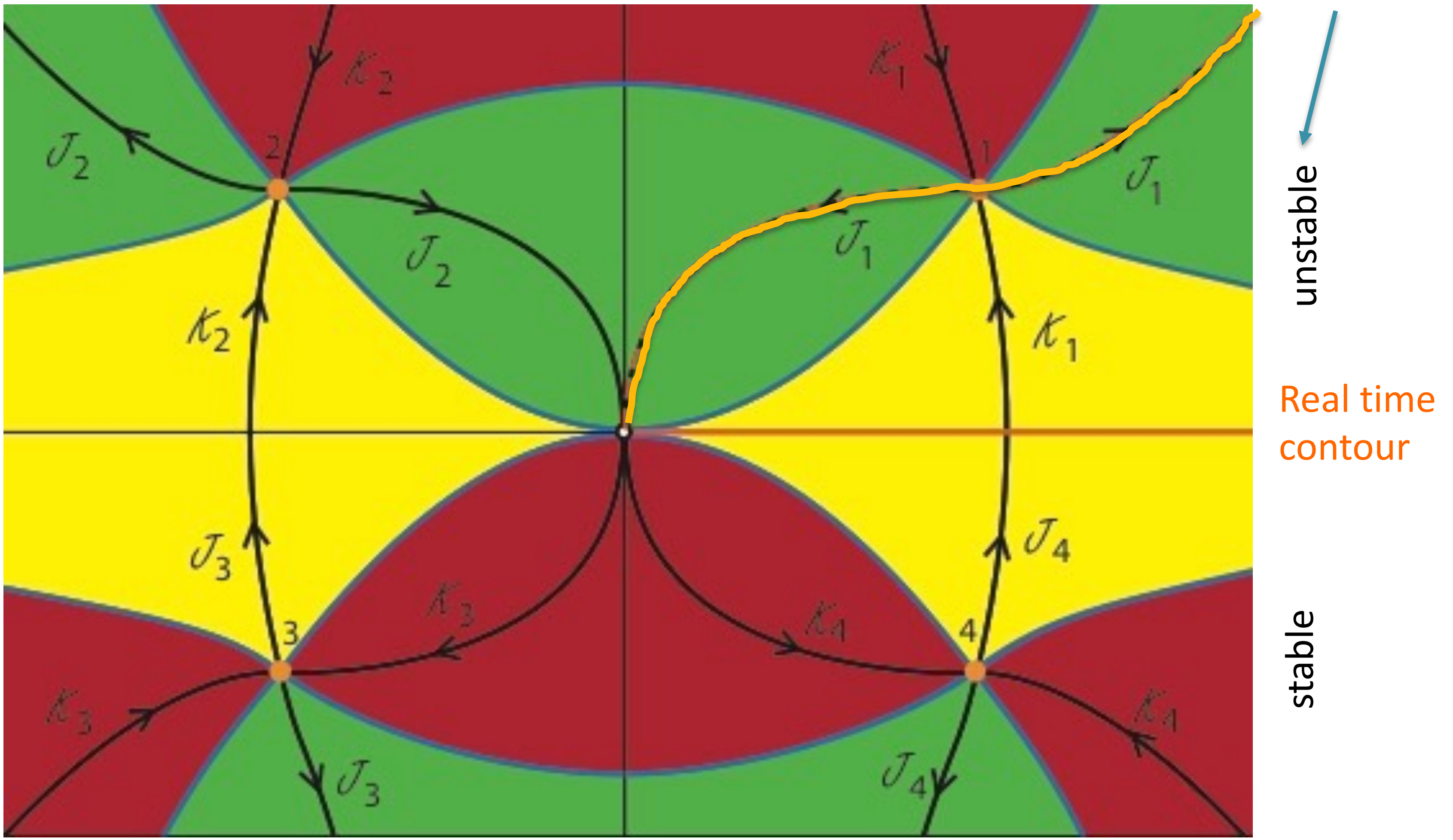
Steepest descent of lapse integral

- Arrows: downwards flow



[Feldbrugge, JLL & Turok]

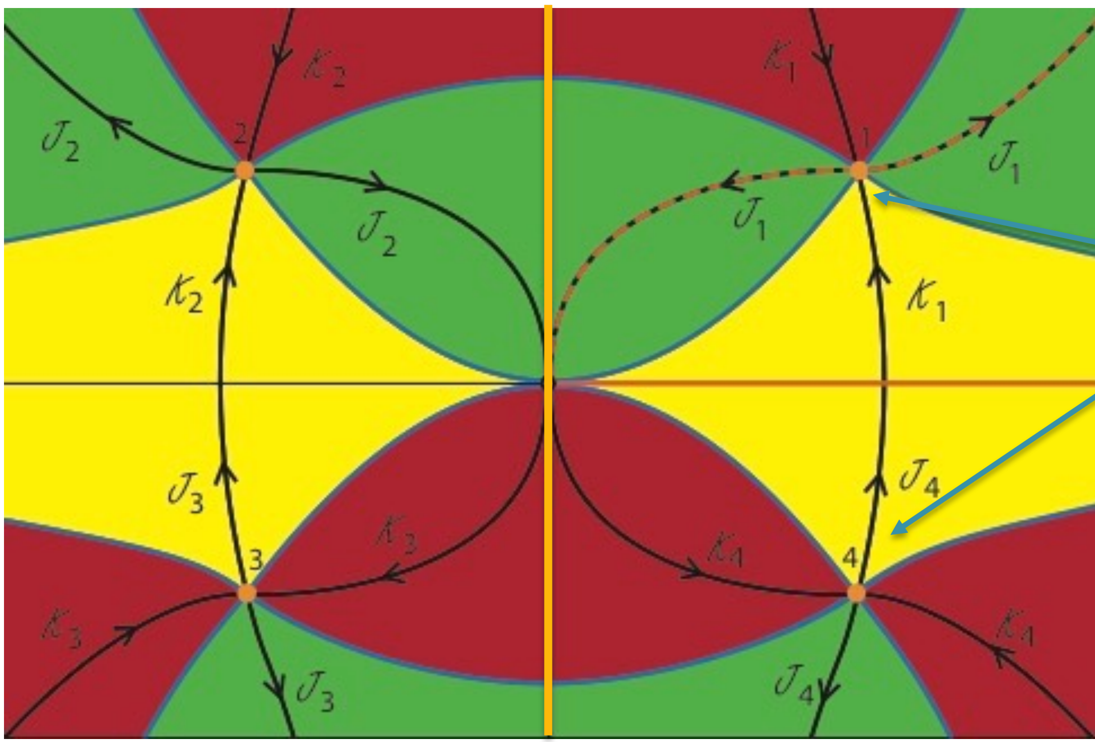
Steepest descent of lapse integral



[Feldbrugge, JLL & Turok]

Remarks

Euclidean contour diverges,
hence sum over Euclidean metrics *does not exist*



unstable

stable

Even if a stable saddle point is included, the steepest descent line flows to the unstable saddle point, hence instabilities will be present

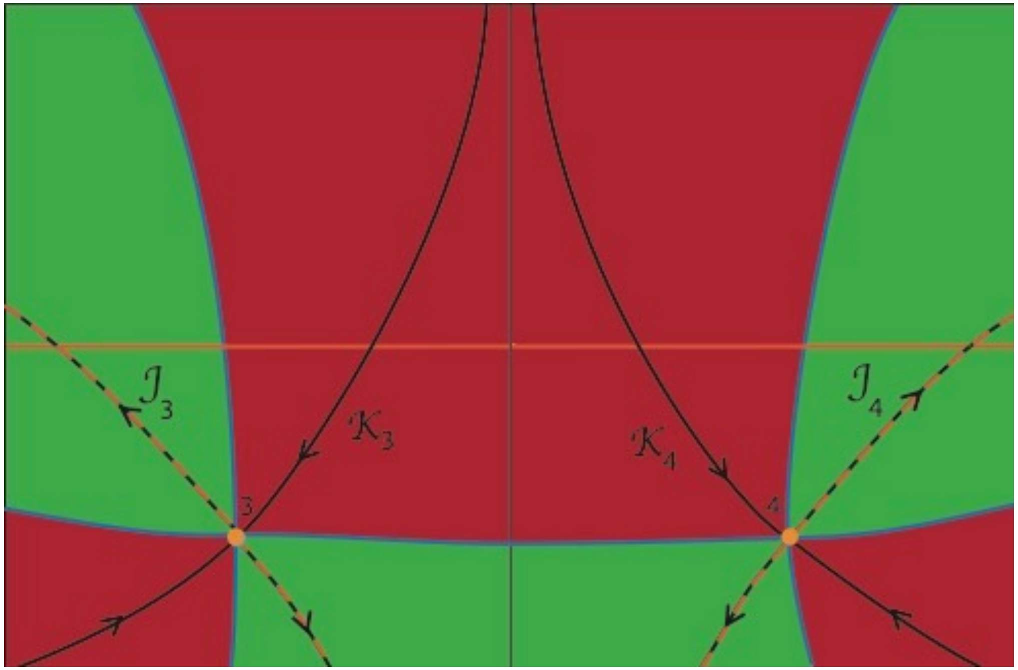
Debate: [Diaz-Dorronsoro, Halliwell, Hartle, Hertog & Janssen; Feldbrugge, JLL & Turok]

New definition

- Can impose regularity, rather than compactness, as a boundary condition on "initial" hypersurface

$$\frac{\dot{q}}{2N} = i$$

- This effectively fixes Wick rotation, and eliminates unstable saddle points

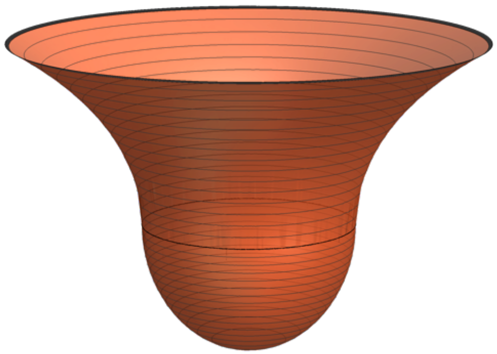


Allows for a Lorentzian contour of integration

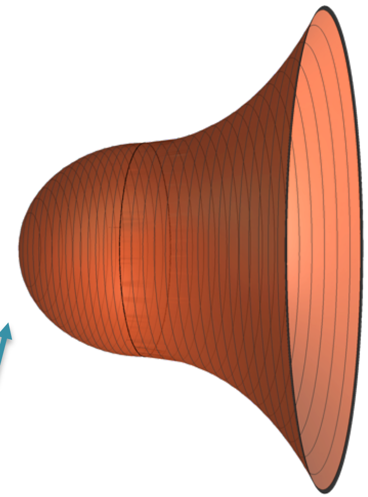
At the saddle points compactness is nevertheless achieved: a(0)=0

[Louko;
Di Tucci, JLL & Sberna]

Correspondence with AdS



De Sitter:
Boundary at a fixed time



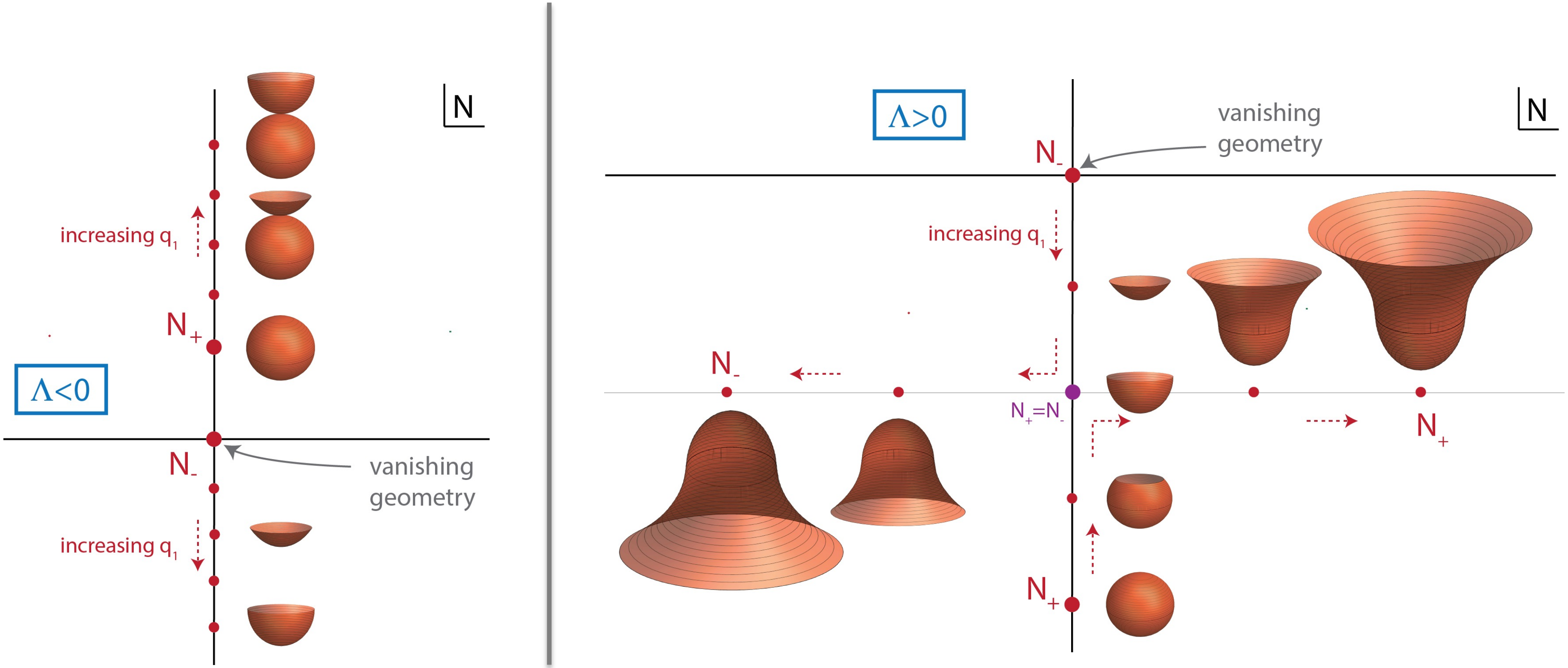
Anti De Sitter:
Boundary at a fixed spatial location (→ ∞)

smooth/regular boundary condition

[Di Tucci, Heller & JLL]

- Gravitational path integrals in AdS require the identical boundary condition in interior
- AdS/CFT implies $\Psi = Bi\left[\left(\frac{18\pi^2}{-\hbar\Lambda}\right)^{2/3}\left(1-\frac{\Lambda}{3}q_f\right)\right]$

Correspondence with AdS



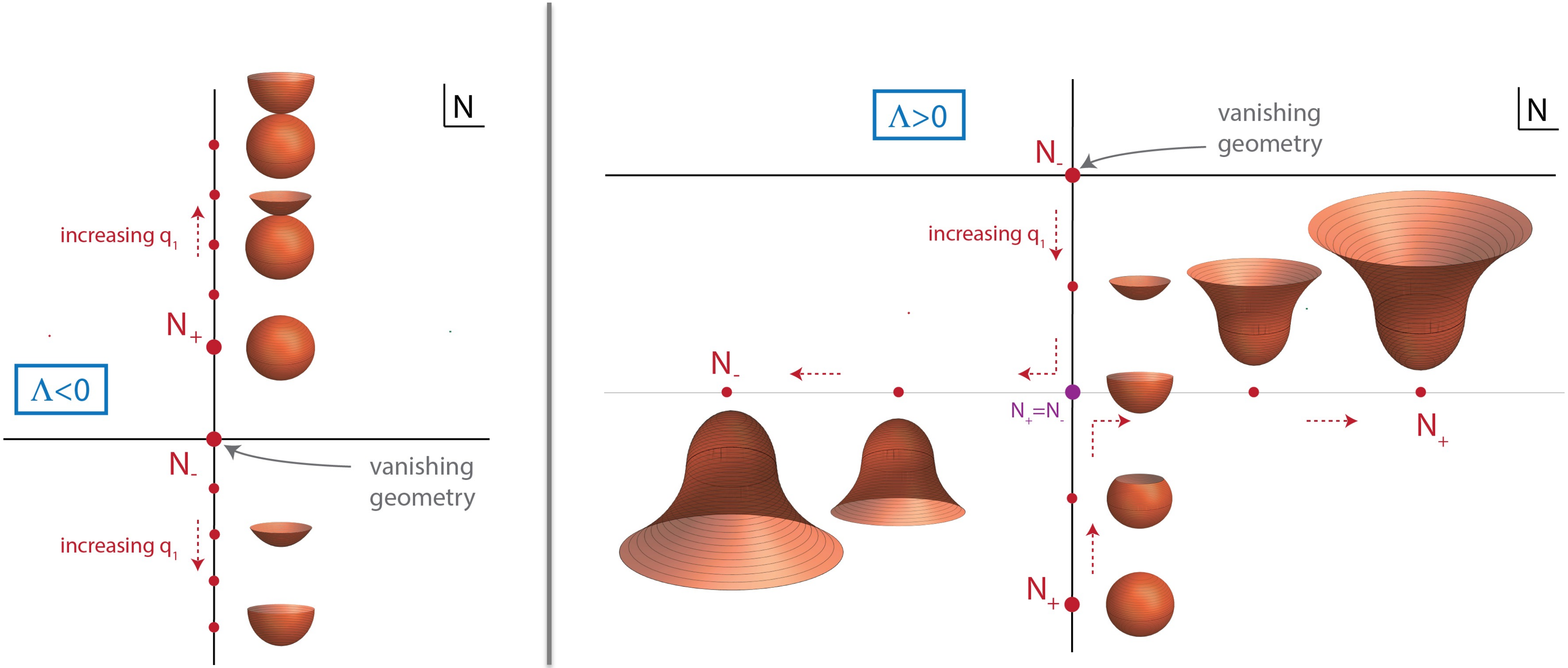
For AdS: relevant saddle point is N_-

Analytic continuation: $$Bi\left[\left(\frac{18\pi^2}{-\hbar\Lambda}\right)^{2/3}\left(1-\frac{\Lambda}{3}q_f\right)\right] = \sqrt{3}\,Ai\left[\left(\frac{18\pi^2}{\hbar\Lambda}\right)^{2/3}\left(1-\frac{\Lambda}{3}q_f\right)\right]$$

For dS: relevant saddle is also N_-

[JLL]

Correspondence with AdS



Implications:

- no contribution from topology changing (Hawking-Moss type) geometries at nucleation of the universe

- Stokes phenomenon describes emergence of time

[JLL]

Simple no-boundary condition

- At the nucleation of the universe, wave function satisfies Wheeler-DeWitt equation in momentum space $q \to i\frac{\partial}{\partial p}$

$$\hat{H}_{(p)}\Psi = 0 \to (p^2 + 36\pi^4)\Psi + 12\pi^4\Lambda i\frac{\partial \Psi}{\partial p} = 0$$

- Regularity requires, $p_0 = -6\pi^2 i$ so that we find that at the no-boundary point we have

$$i\frac{\partial}{\partial p}\Psi = \hat{q}\,\Psi = 0 \qquad \text{no-boundary condition}$$

- Two interpretations:
  - No momentum flow into/out of universe at nucleation
  - Zero size condition imposed in momentum, rather than real, space

[JLL]

Some open questions

- Rigorous definition of probabilities
- Testable predictions?
- Going beyond minisuperspace
- Effective 4d vs full string theory
  - Robust to inclusion of Riemann^n terms [Jonas & JLL]

Complex Metrics

- Complex/Euclidean metrics are useful: used to derive black hole thermodynamics, used to define gravitatioal path integrals over thimbles, used for topology change, tunneling, etc…

- *Which complex metrics should be allowed?*

- Early work by Louko-Sorkin (gr-qc/9511023)
- Recent work by Kontsevich-Segal (2105.10161)
- Discussed by Witten (talk at *Damour Fest*)

Kontsevich-Segal criterion

- Scalars and gauge fields have local covariant stress-energy tensor [Weinberg-Witten theorem]
- More generally require p-form actions to be well defined on complex background (path integral should converge), then can define QFT on these backgrounds

$$|e^{\frac{i}{\hbar}S}| < 1 \to Re\left[i\int d^4x\sqrt{-g}g^{i_1j_1}\cdots g^{i_{p+1}j_{p+1}}F_{i_1\cdots i_{p+1}}F_{j_1\cdots j_{p+1}}\right] < 0$$

- Locally write metric in diagonal form $g_{\mu\nu} = \delta_{\mu\nu}\lambda_\nu$
- This implies

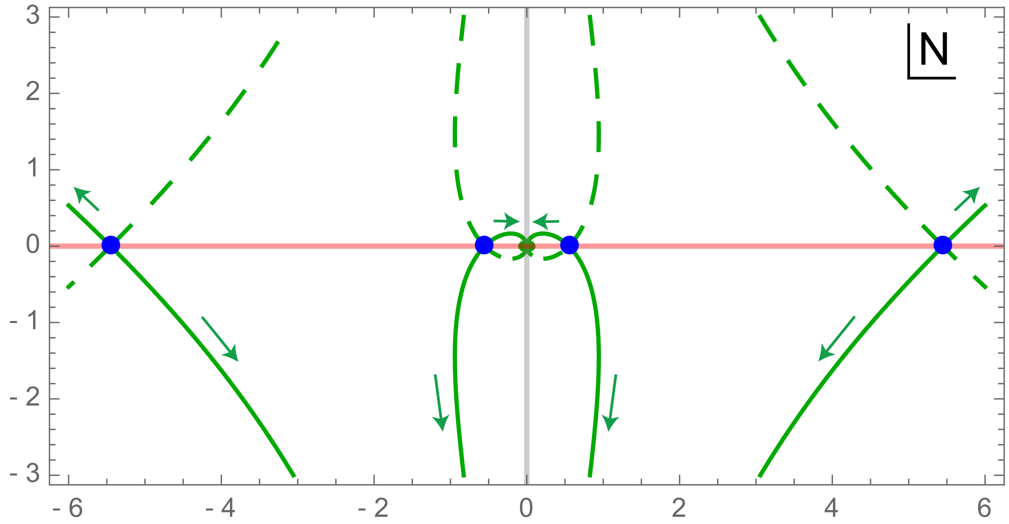
$$\Sigma \equiv \sum_\mu |Arg(\lambda_\mu)| < \pi$$

Lorentzian metrics at boundary of allowable domain

- For Lorentzian metrics (-+++), we have $\Sigma = \pi$, i.e. they are on the boundary of the allowable domain!

[Kontsevich-Segal]

- Classical boundary conditions:



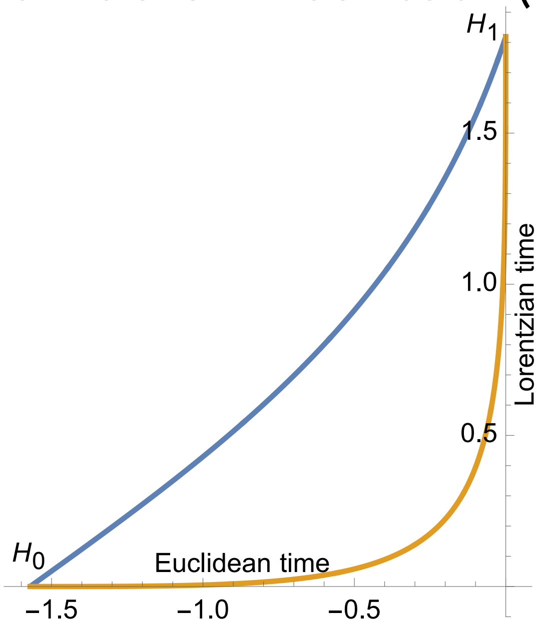
- Thimbles are cut in half, at the location of the saddles!

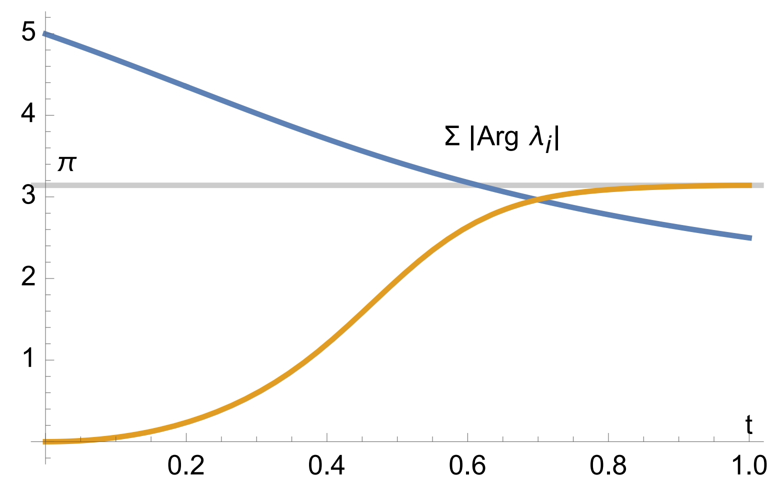
[JLL]

No-boundary

- Constant lapse saddle points (blue) naively violate K-S criterion
- Can deform contour (orange), then K-S is satisfied

[Witten]



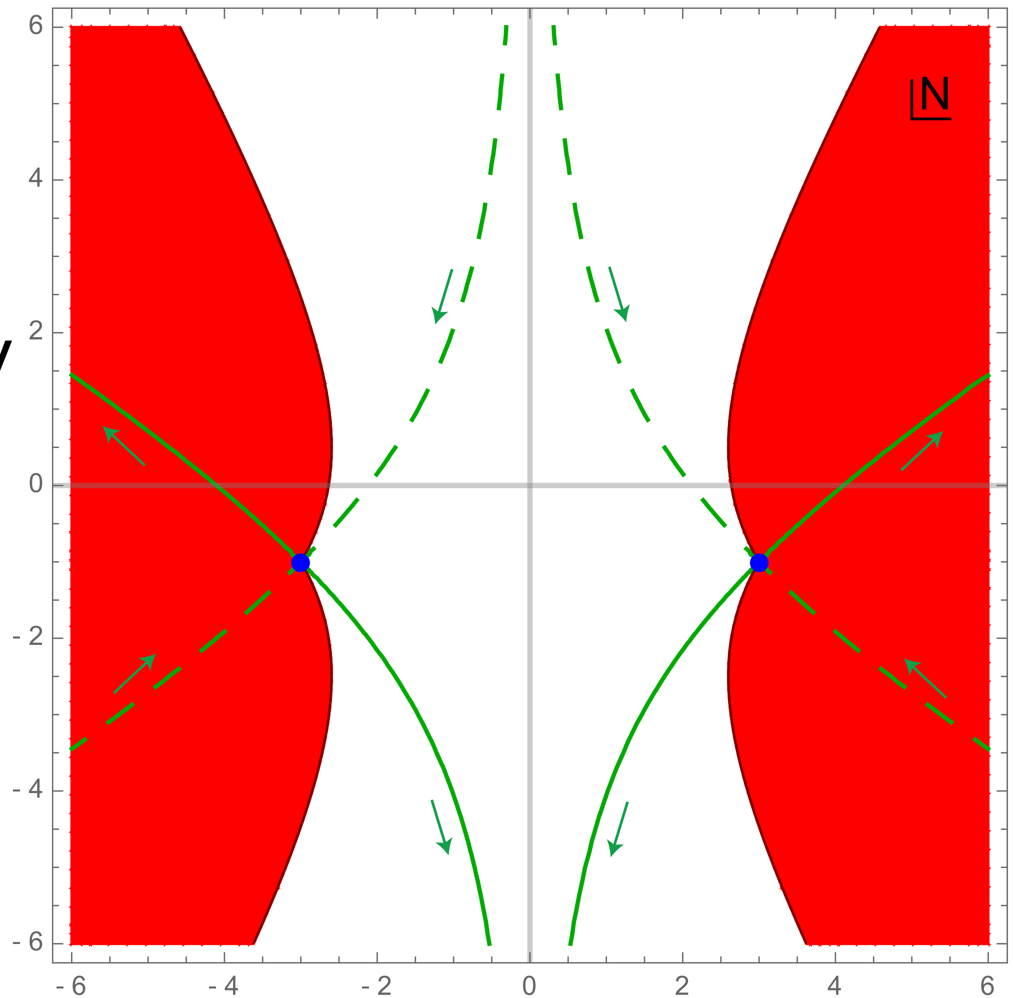


- Will treat such metrics as equivalent, hence a metric is retained if it can be deformed to one satisfying K-S
- For large anisotropies, there may be an obstruction to deforming the contour

[Bramberger, Farnsworth & JLL]

No-boundary $\Lambda>0$

- Vast domain ist not allowable
- Saddles at edge, even though they are complex!
- Thimbles again cut off at saddles
- Asymptotic near-Lorentzian region not allowed



[JLL]

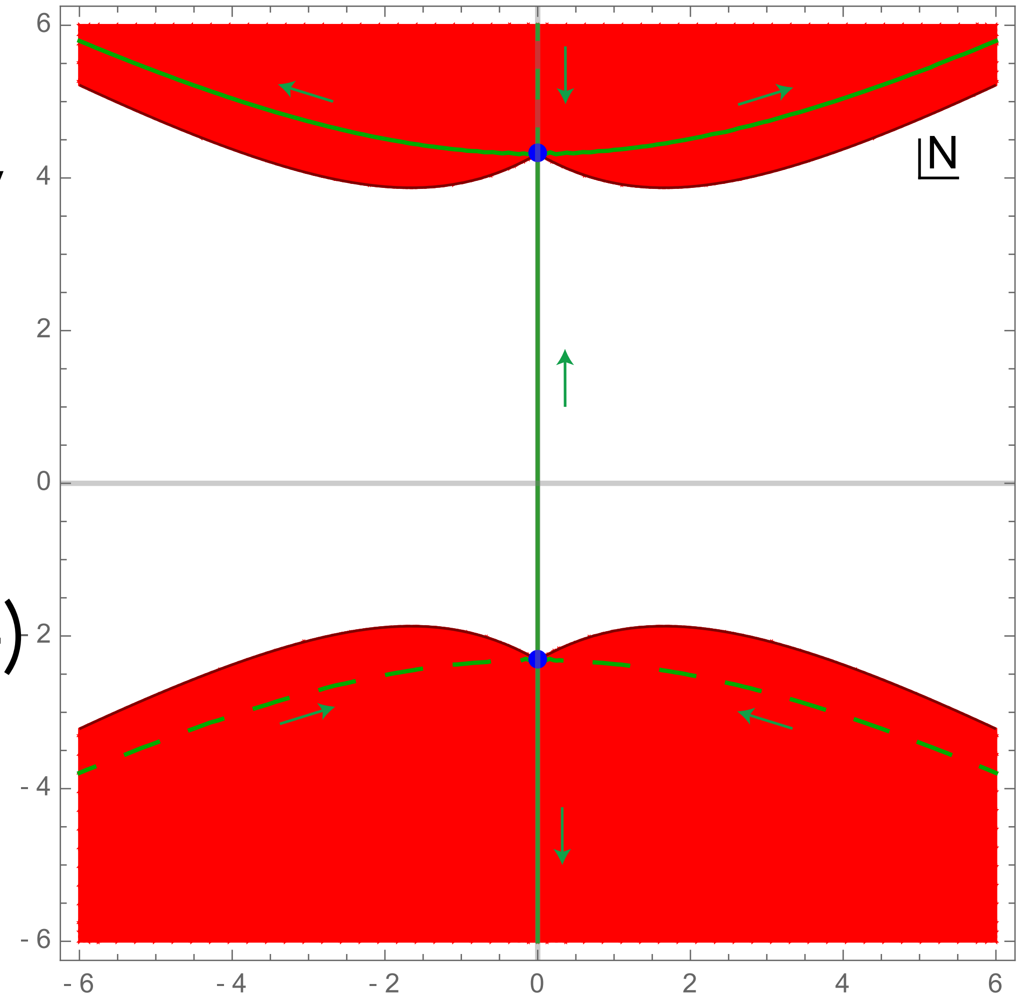
Including a scalar

- No-boundary instantons typically involve complex scalars, *except* at extrema of the potential
- *If one insists* on having a real scalar, then this might help explain why the evolution of an inflaton started at a local maximum
- At the same time, other scalars would preferentially be at local minima, which might explain why physical constants do not vary in our universe

[JLL]

AdS, Λ<0

- Saddles at edge, even though they are Euclidean!
- Thimbles again cut off at saddles
- Asymptotic (near-) Euclidean region not allowed



[JLL]

Outlook

- The K-S criterion implies a tension between allowable metrics and Lefschetz thimbles
- But the saddle points seem to remain at the edge of the allowable domain in physically interesting settings

- Much remains to explore…