Is asymptotically safe inflation eternal?

Recent Progress of Quantum Cosmology

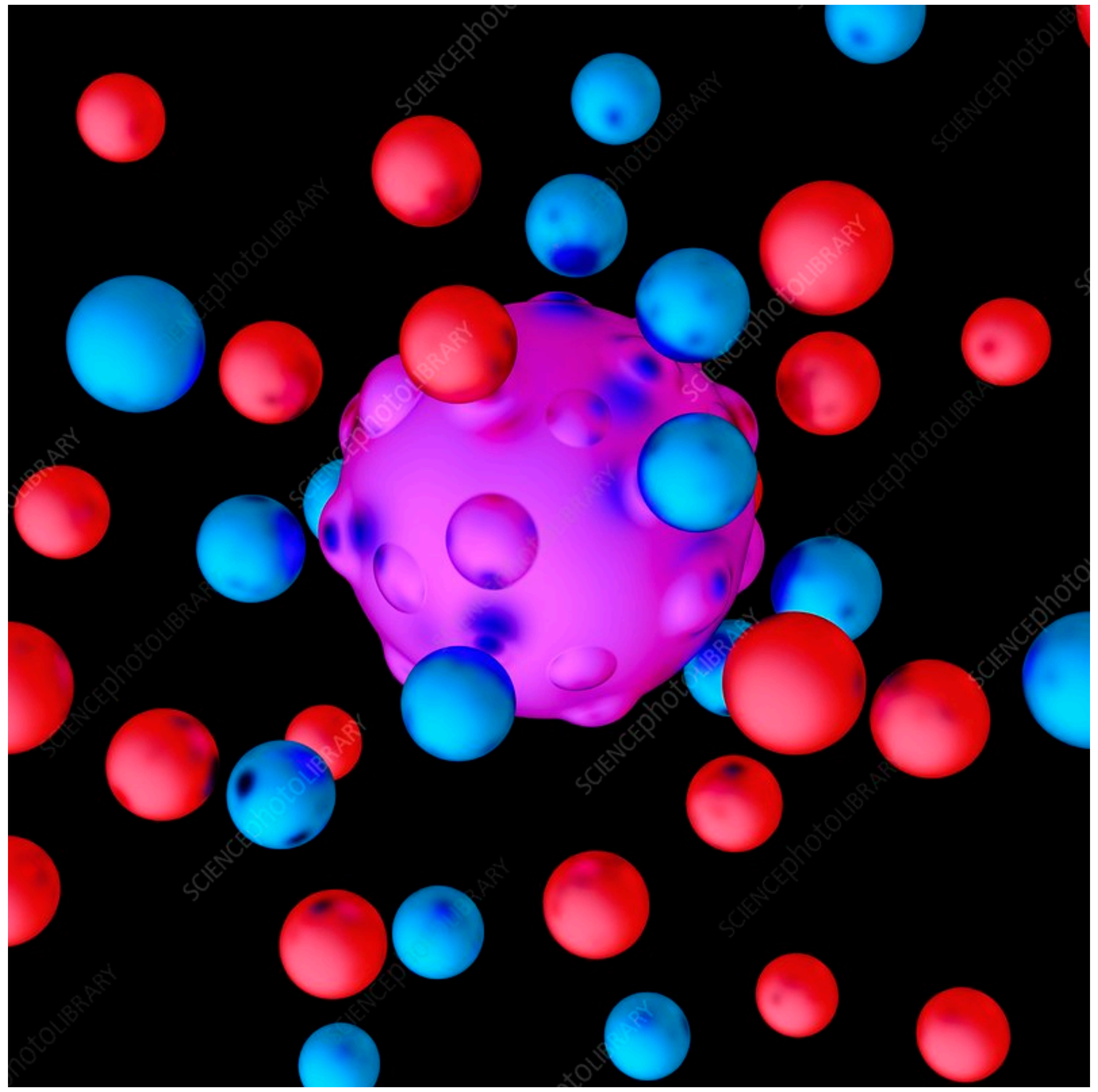
09/11/2021

Jan Kwapisz University of Warsaw, visitor at CP3-Origins

Talk based on 2101.00866, and partly on 2102.13556

Eternal inflation and swampland

Rudelius 2019; Rudelius 2021



$$\frac{V'}{V^{3/2}} < \frac{\sqrt{2}}{2\pi M_P^3}$$

$$M_P \frac{|V'|}{V} > c \sim \mathcal{O}(1)$$

Asymptotic safety

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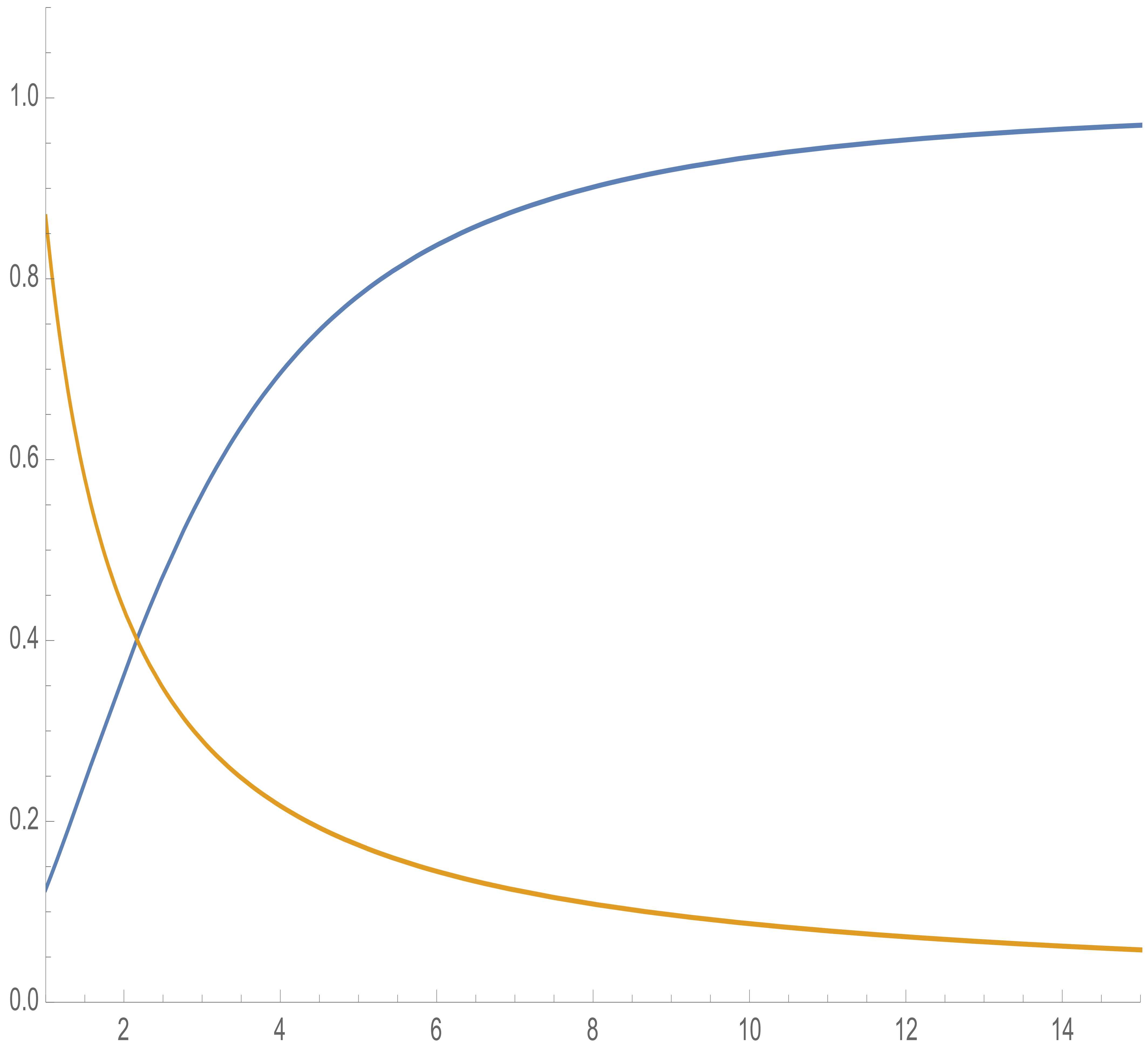

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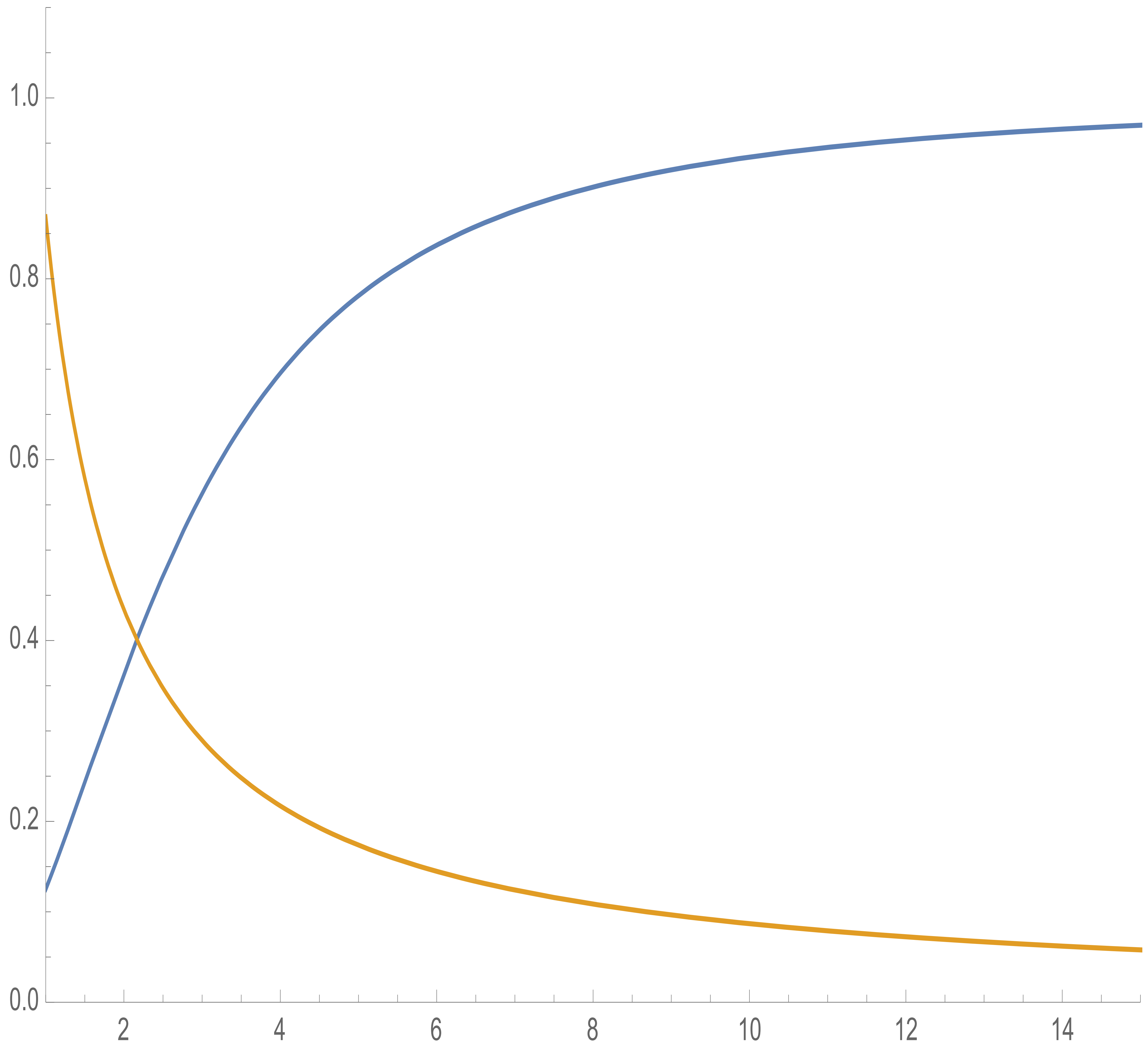

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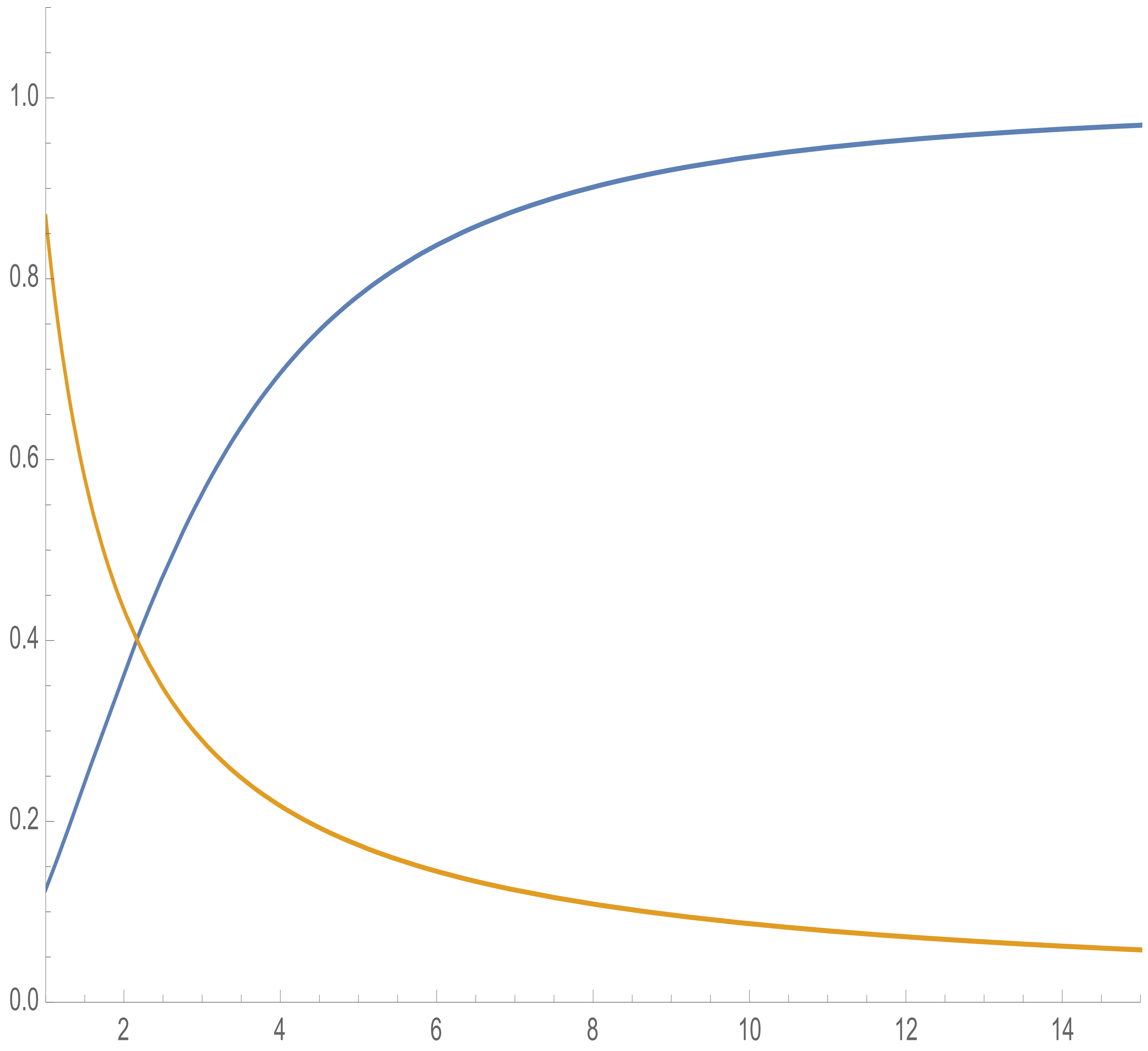
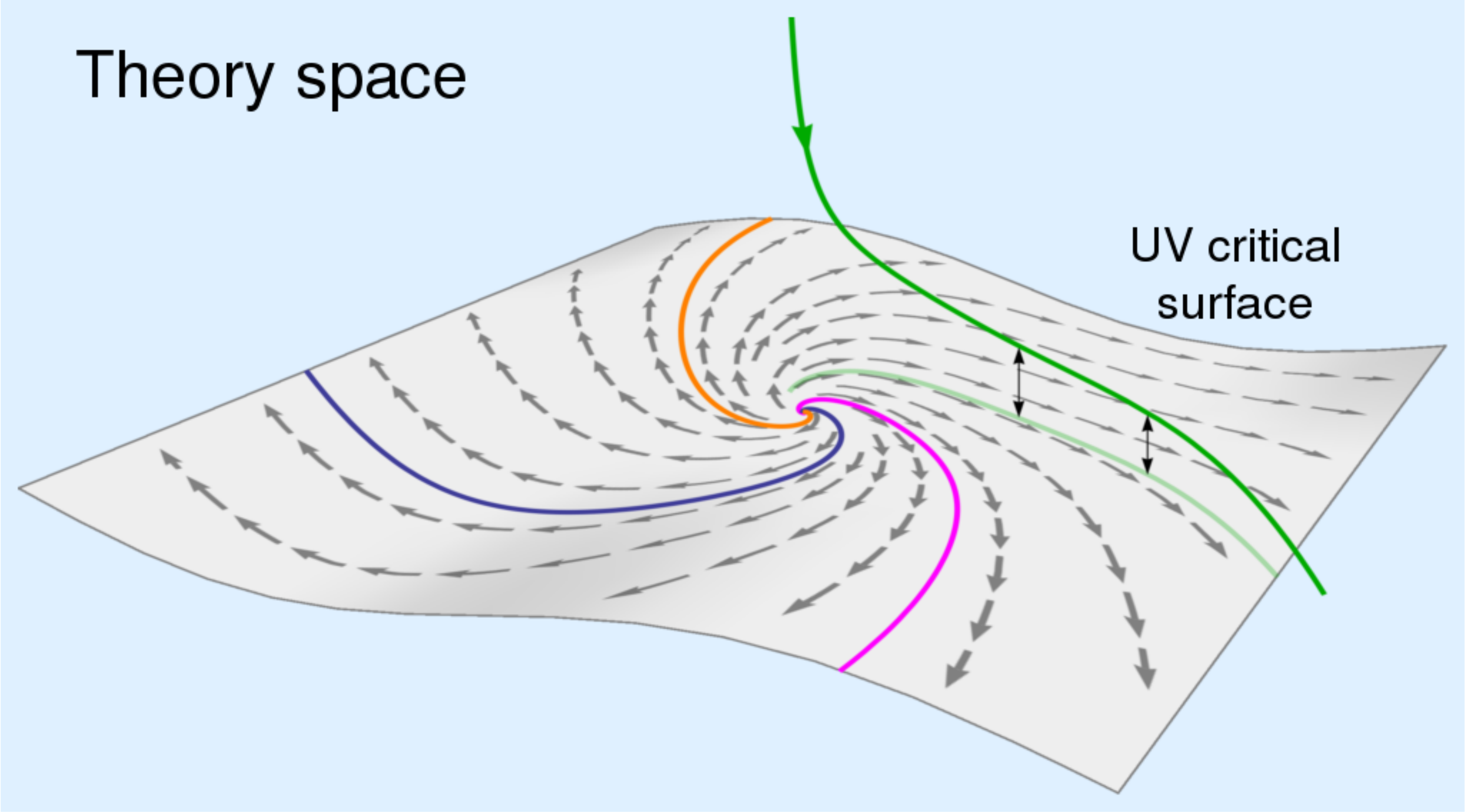
- Constraints on higher-order couplings



Asymptotic safety

Three dimensional hypersurface: g_N, λ, β



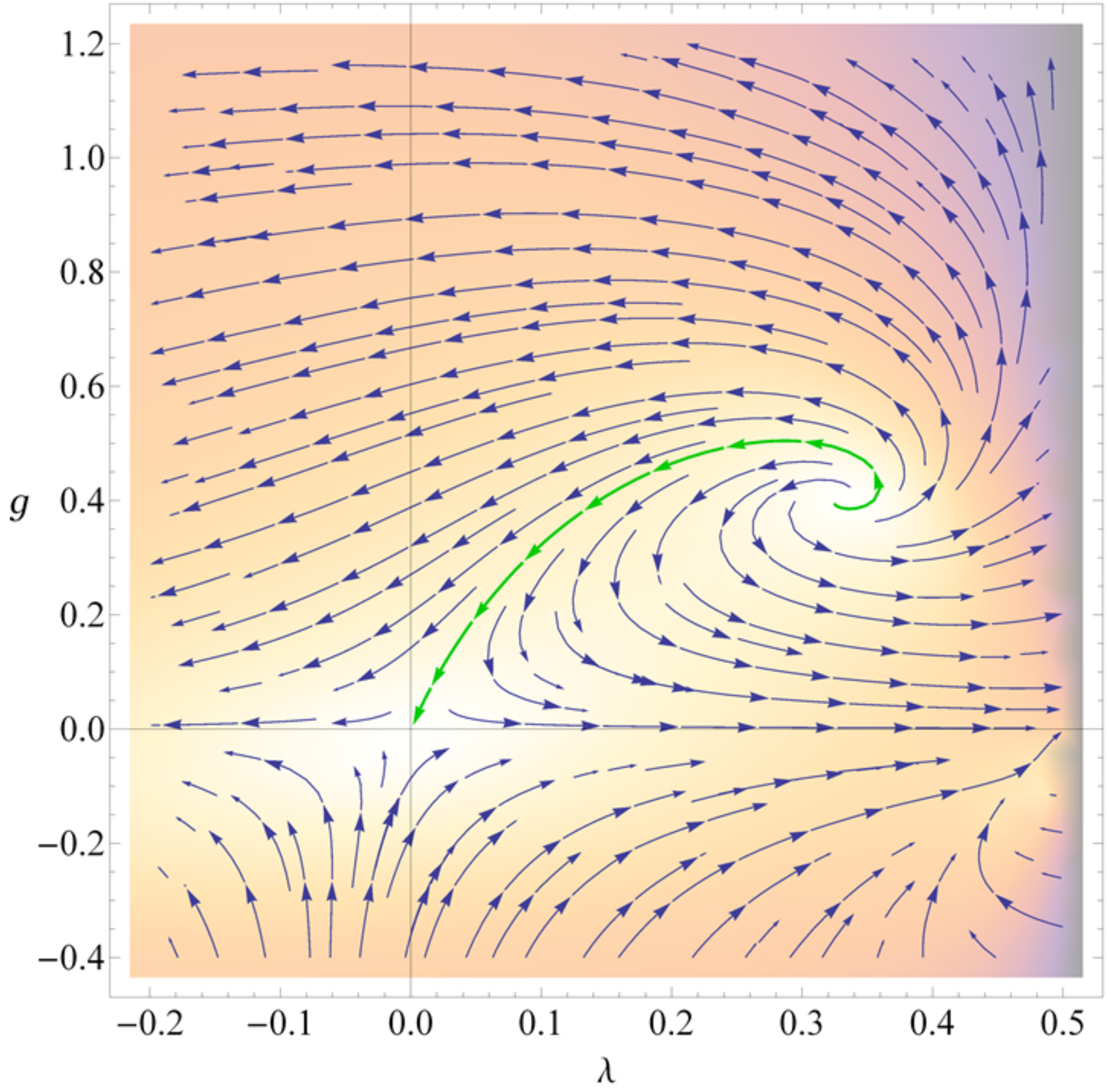
Asymptotic safety in Quantum Gravity

Weinberg's hypothesis

The RG flow for the Einstein-Hilbert action
(see Bonanno et al 2004.06810)

$$\beta_{g_N} = 2g_N - bg_N^2$$

+ Effects on matter: $\beta_i \sim \dfrac{1}{M_P^2(\mu)} g_i$



Inflation

Period of accelerated expansion in the Early Universe

The problems solved by inflationary theory:

- flatness problem
- Homogeneity and isotropy problem
- Small abundance of cosmological defects
- CMB temperature differences of order $\delta T/T \approx 10^{-5}$

Inflation has its own theoretical problems

- Initial conditions
- Eternal inflation (this talk)
- Reheating

Inflation

Slow roll

- The equation of state $\rho \approx -p$ and $\ddot{a} > 0$.

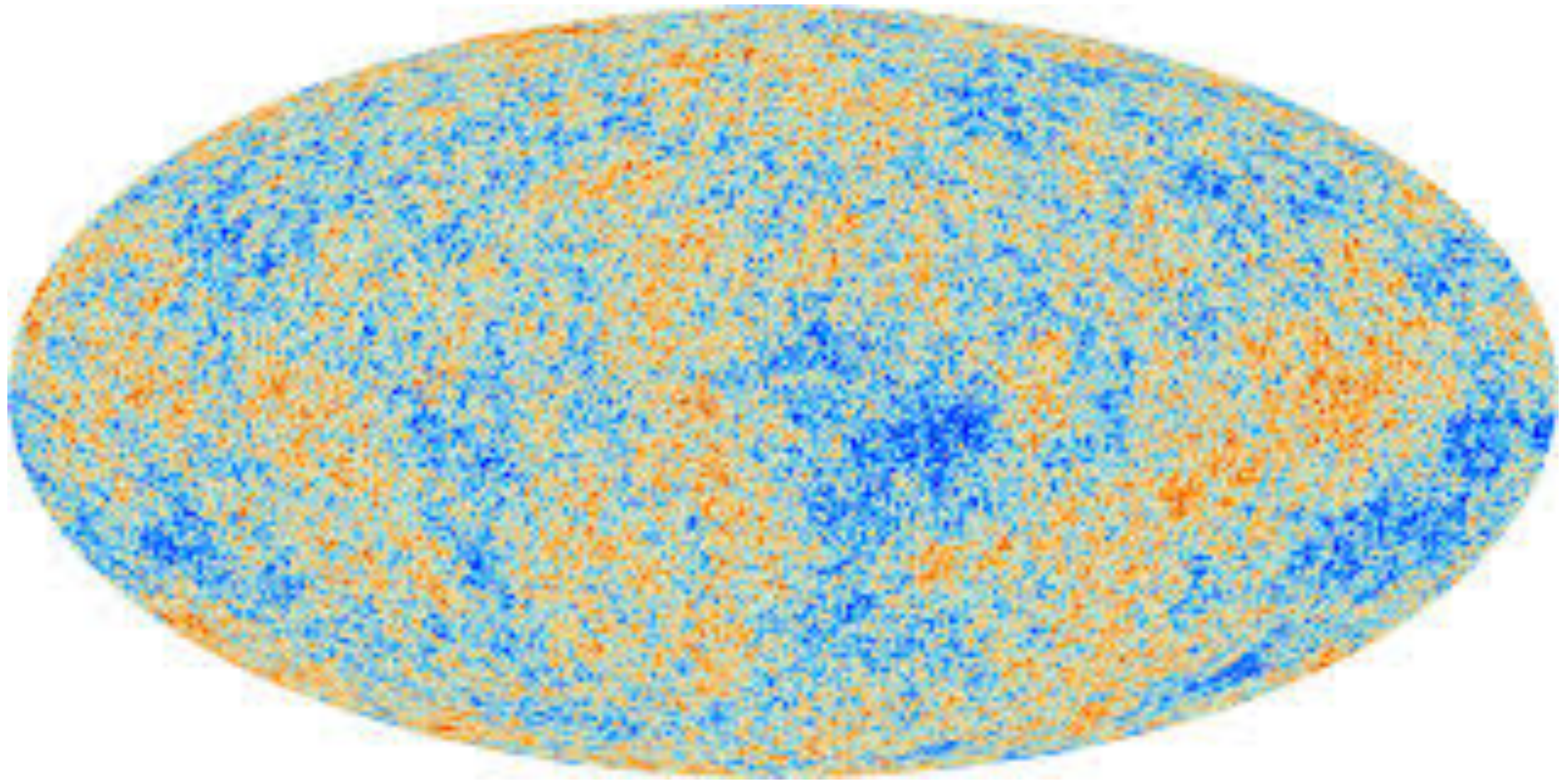
Inflation

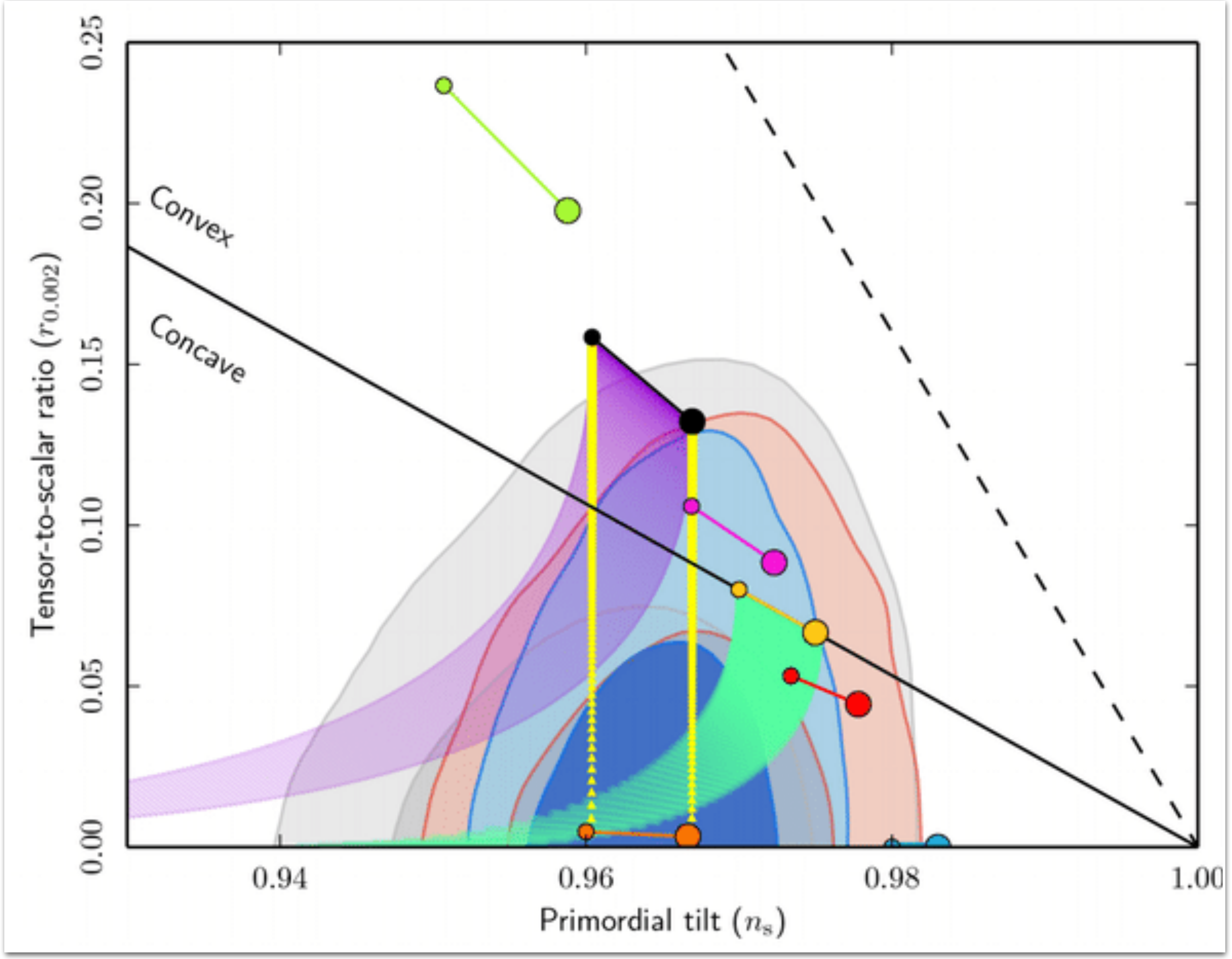
Slow roll

- The equation of state $\rho \approx -p$ and $\ddot{a} > 0$.
- Inflation driven by scalar field ϕ with potential $V(\phi)$.

Inflation

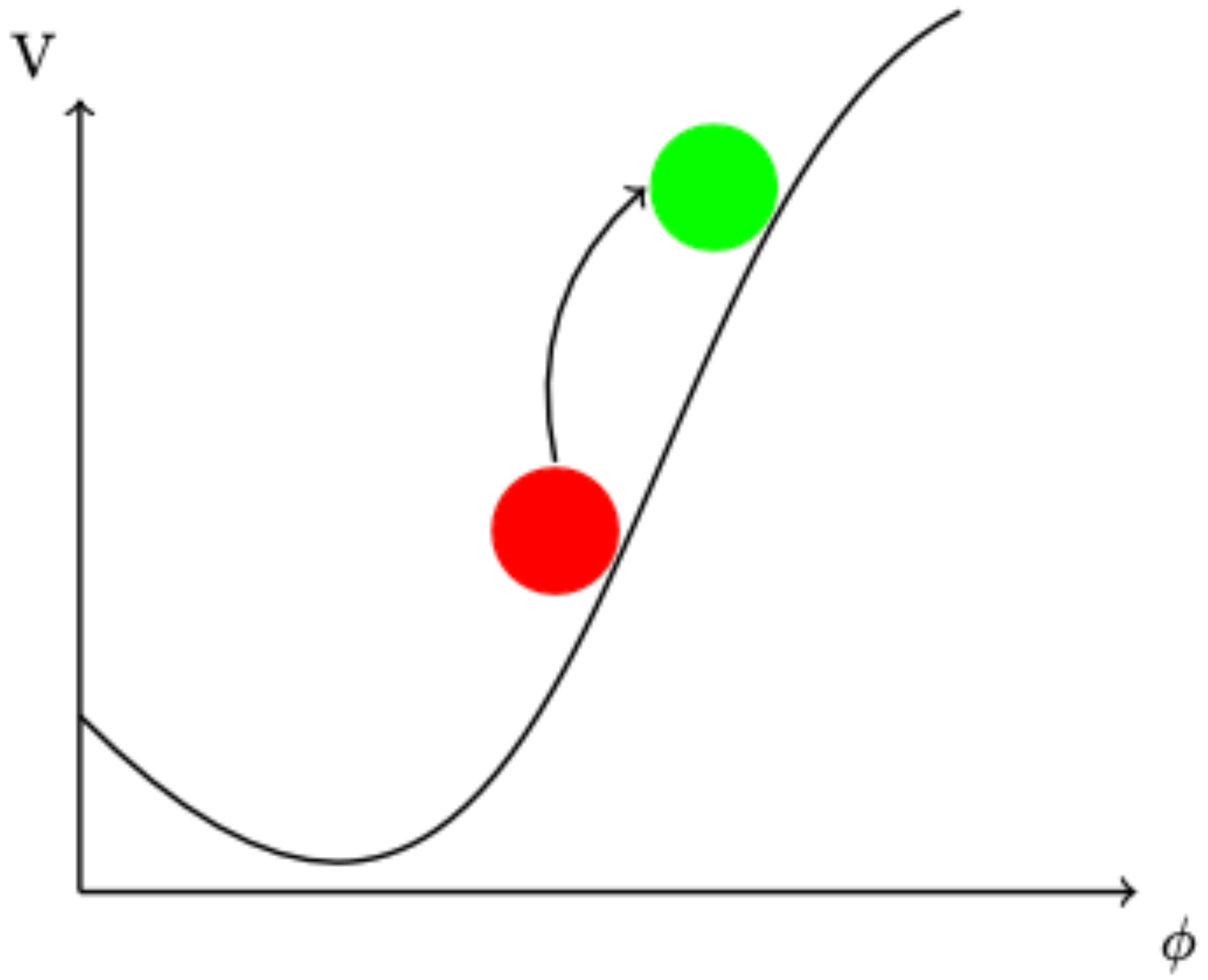
Quantum fluctuations





Inflation

Quantum “jumps” and tunnelling



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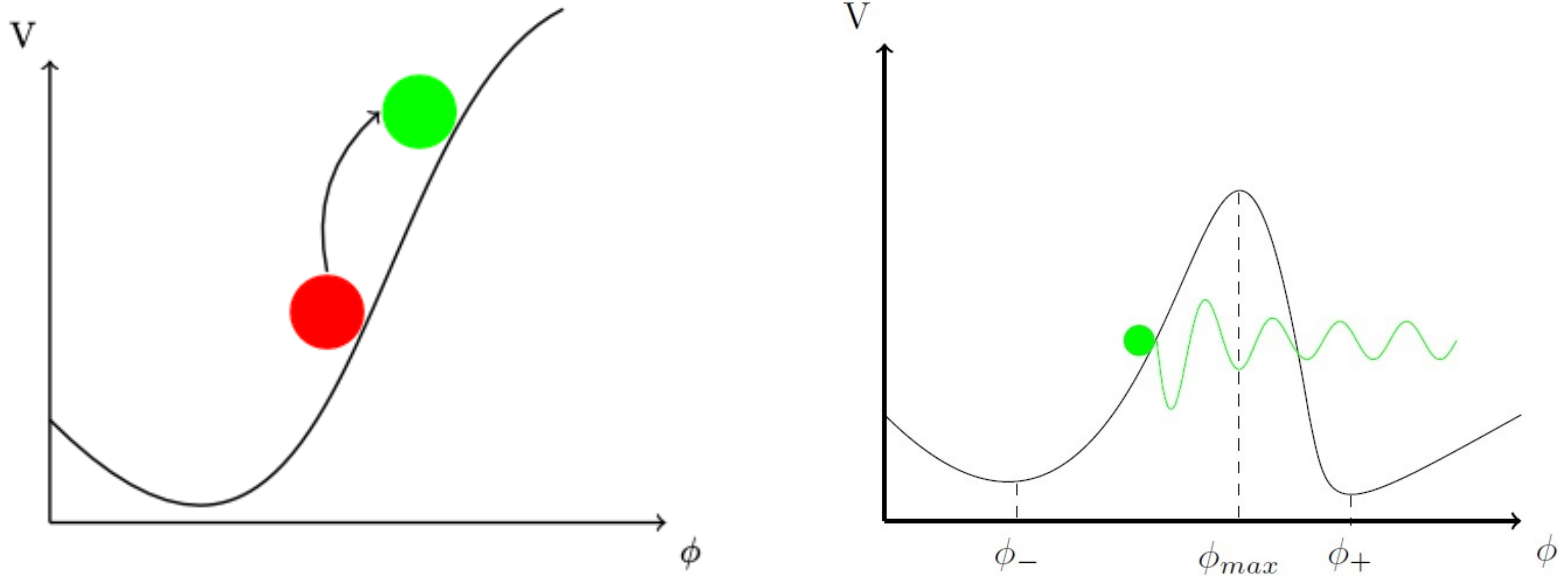
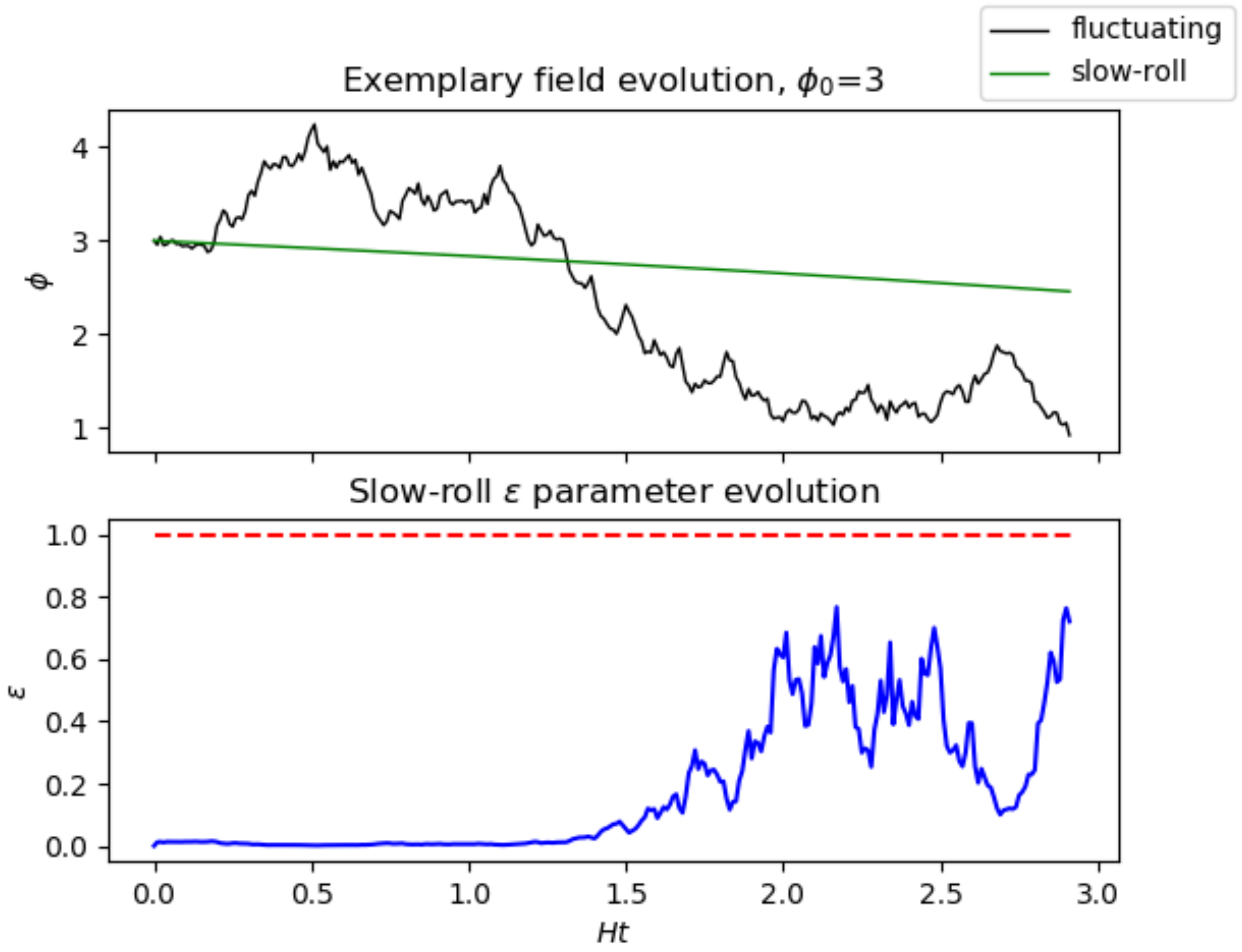


Eternal Inflation

Quantum fluctuations

- Split into background and fluctuations
  $\phi(t, \overrightarrow{x}) = \phi_{cl}(t, \overrightarrow{x}) + \delta\phi(t, \overrightarrow{x})$



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  $$\phi(t,\vec{x}) = \phi_{cl}(t,\vec{x}) + \delta\phi(t,\vec{x})$$
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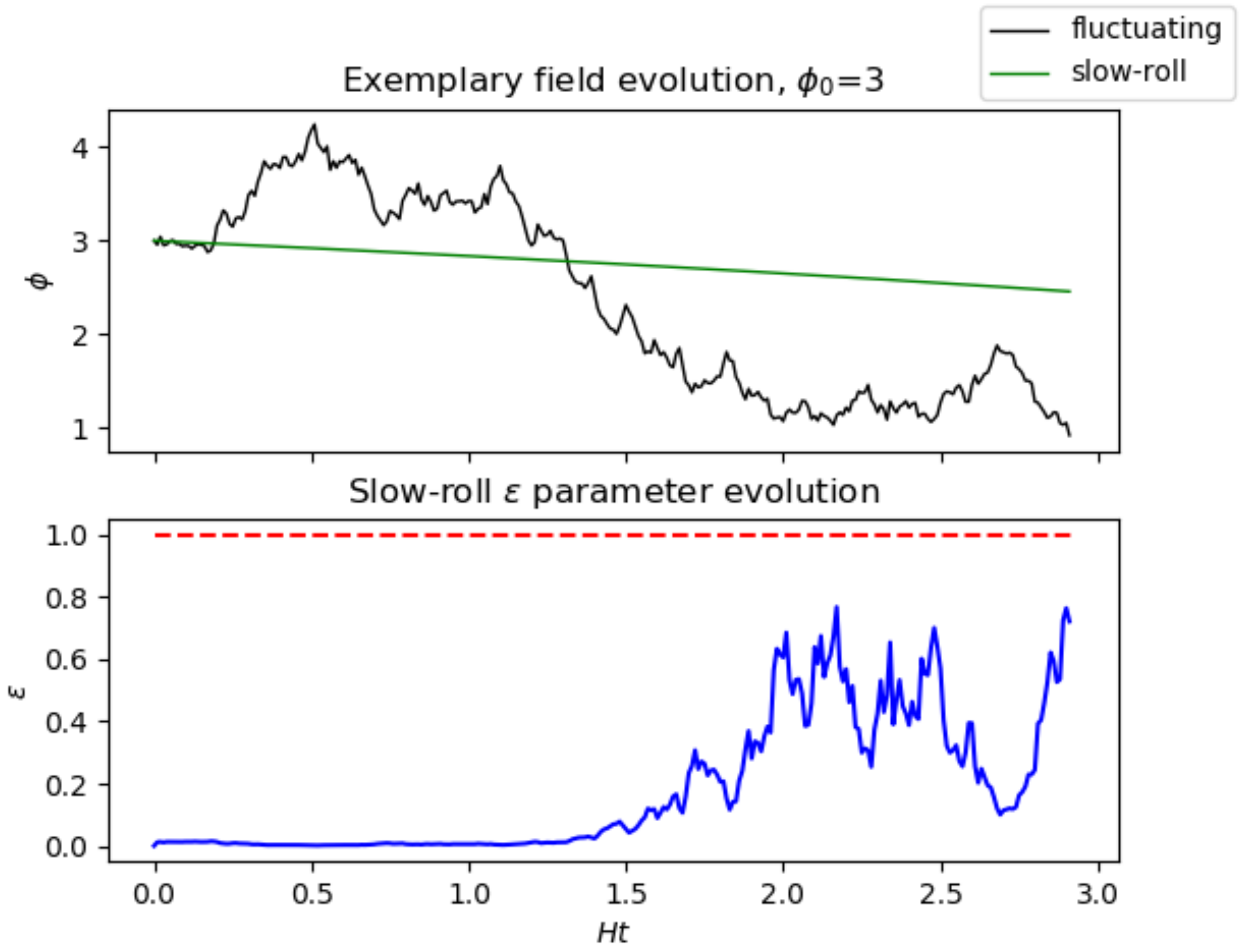
  $$3H\dot{\phi} + \frac{\partial V}{\partial \phi} = \mathcal{N}(t).$$



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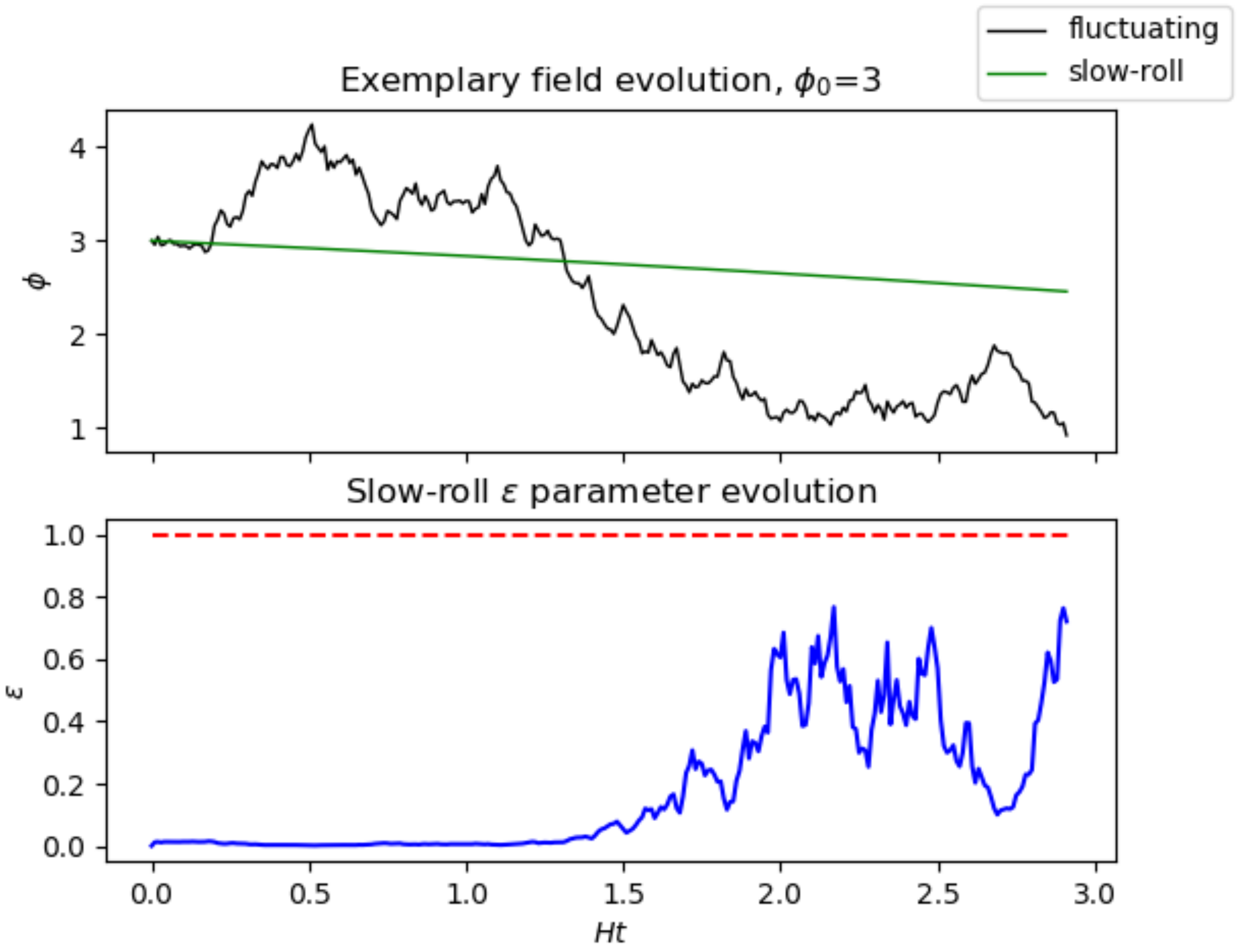
- The $\mathcal{N}(t)$ is Gaussian with mean 0 and variance $\sigma = H^3 t/(4\pi^2)$.



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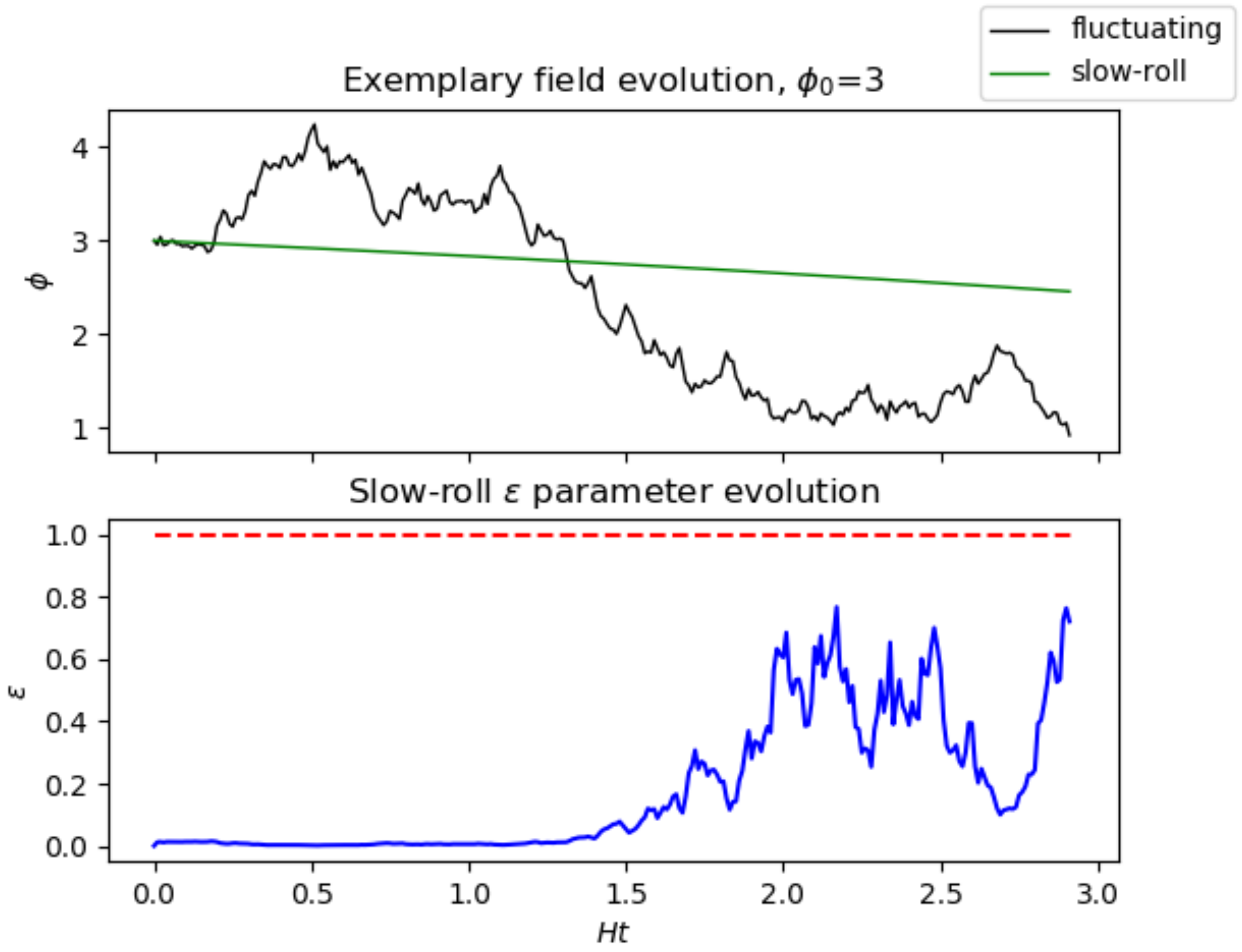
- To find $P[\phi, t]$ we histogram.



Eternal Inflation

How inflation becomes eternal

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$$\text{Pr}[\phi > \phi_{end}, t] \sim \exp(-At), \qquad \lim_{t\to\infty} \text{Pr}[\phi > \phi_{end}, t] = 0,$$

- Take into account the exponential expansion of the Universe
  $U(\phi > \phi_{end}, t) = \text{Pr}[\phi > \phi_{end}, t] \times U_0 e^{3Ht}$

Eternal Inflation

Starobinsky inflation

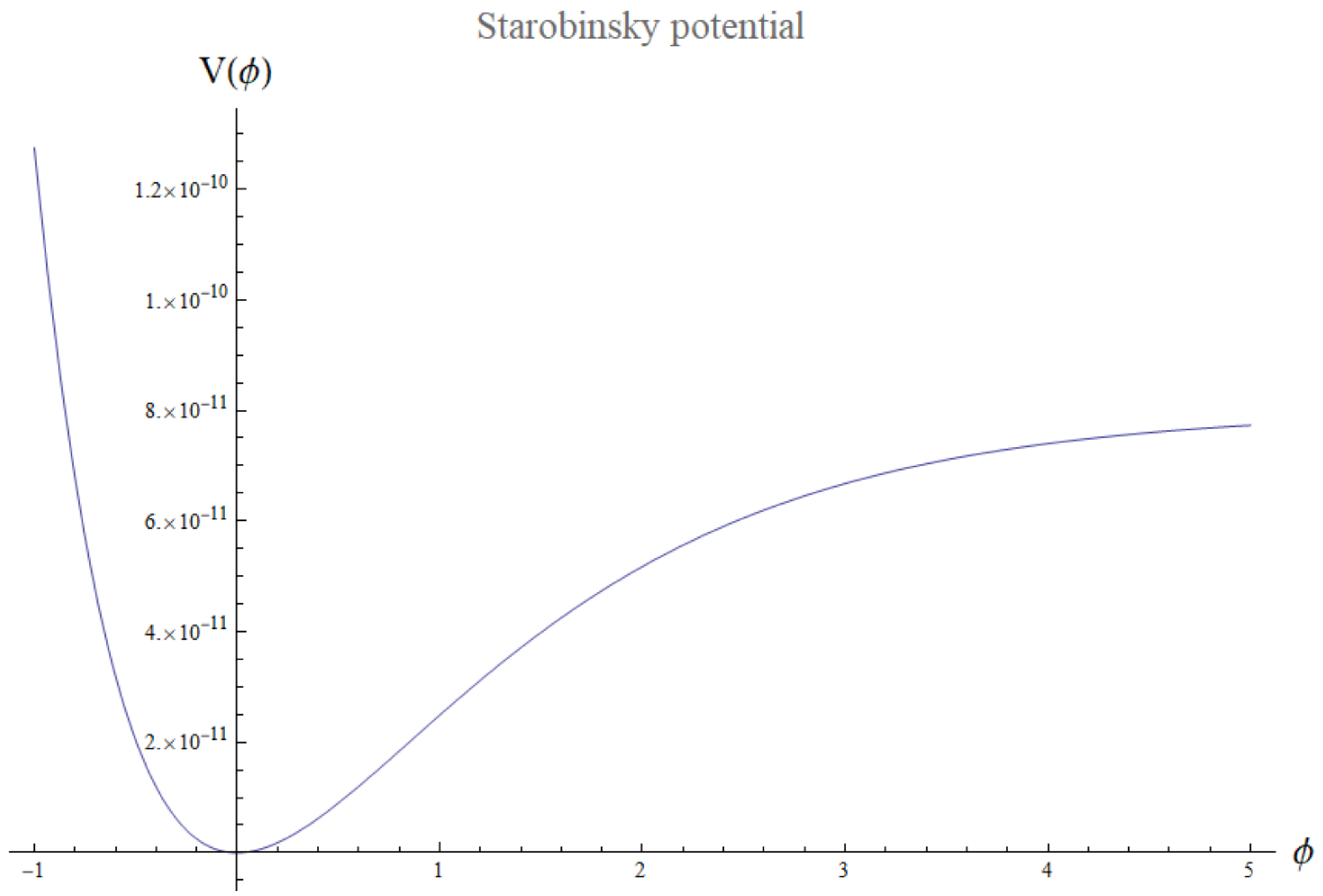
- Starobinsky action:

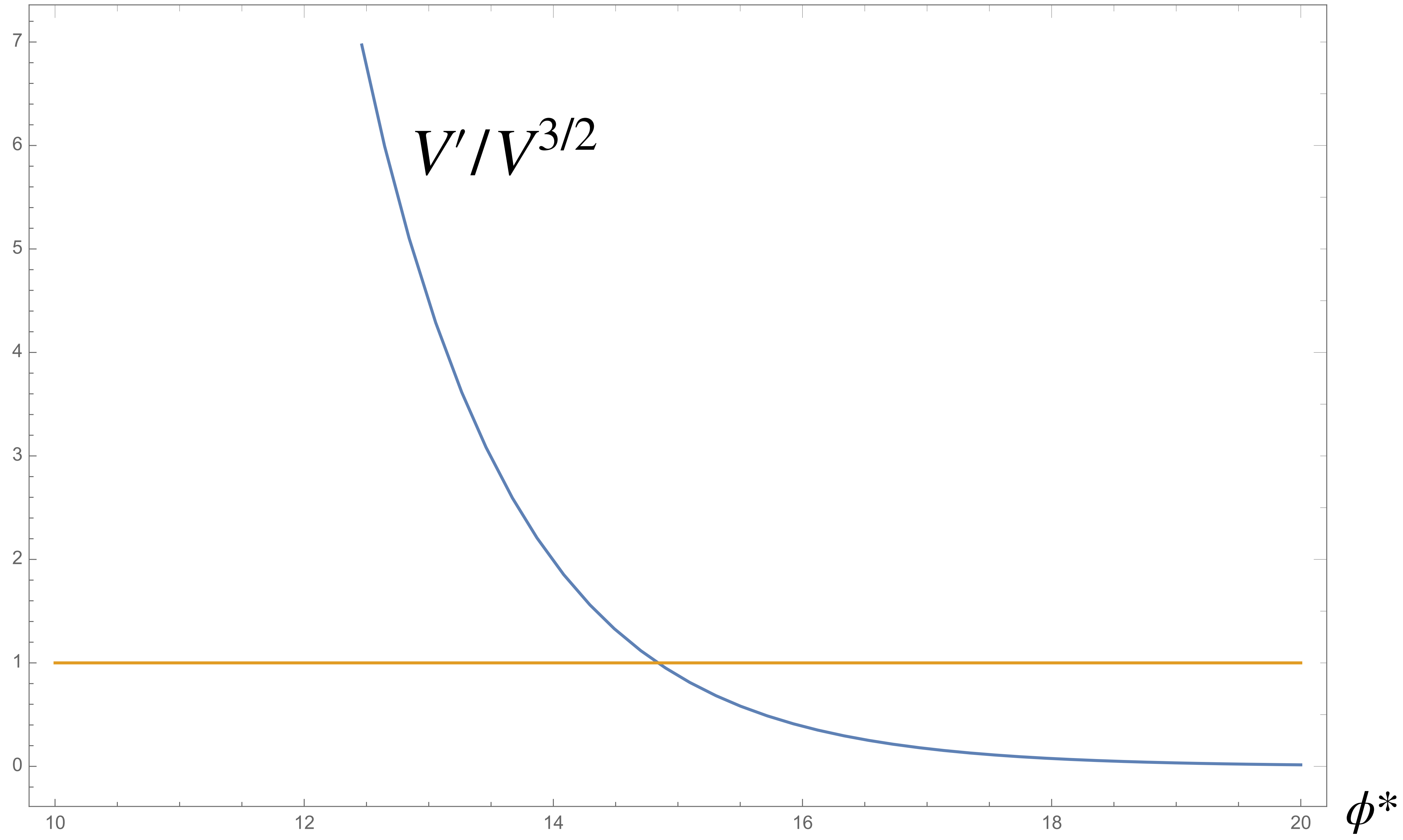
$$S = \frac{1}{2}\int \sqrt{g} d^4x \left(M_{Pl}^2 R + 1\frac{1}{6M^2} R^2 \right).$$

effective Starobinsky potential

$$V(\phi) = V_0 \left(1 - \exp\left(-\sqrt{\frac{2}{3}} \frac{\phi}{M_P} \right) \right)^2,$$

with $V_0 \approx 8 \times 10^{-11} M_{Pl}$.





Eternal Inflation

RG improved Starobinsky inflation

- Transplanckian values of the fields requires taking into account quantum gravity effects

- Quadratic gravity Lagrangian can be RG-improved

$$L_k = \frac{k^2}{16\pi g_k}(R - 2\lambda_k k^2) - \beta_k R^2$$

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- $\beta = \beta^* + b_0 \left(\frac{k^2}{\mu^2}\right)^{-1/2}$, $k^2 \to \xi R$ and $\alpha = -2\mu^{\theta_3} b_0 / M_{PL}^2$.

Eternal Inflation

RG improved potential

- In the Einstein frame one obtains the following potential

$$
\begin{aligned}
V_{\pm}(\phi) = \frac{m^2 e^{-2\sqrt{\frac{2}{3}}\kappa\phi}}{256\kappa} \Bigg[& 192(e^{\sqrt{\frac{2}{3}}\kappa\phi} - 1)^2 - 3\alpha^4 + 128 \\
& -\sqrt{32}\alpha[(\alpha^2 + 8e^{\sqrt{\frac{2}{3}}\kappa\phi} - 8) \pm \alpha\sqrt{\alpha^2 + 16e^{\sqrt{\frac{2}{3}}\kappa\phi} - 16}] \\
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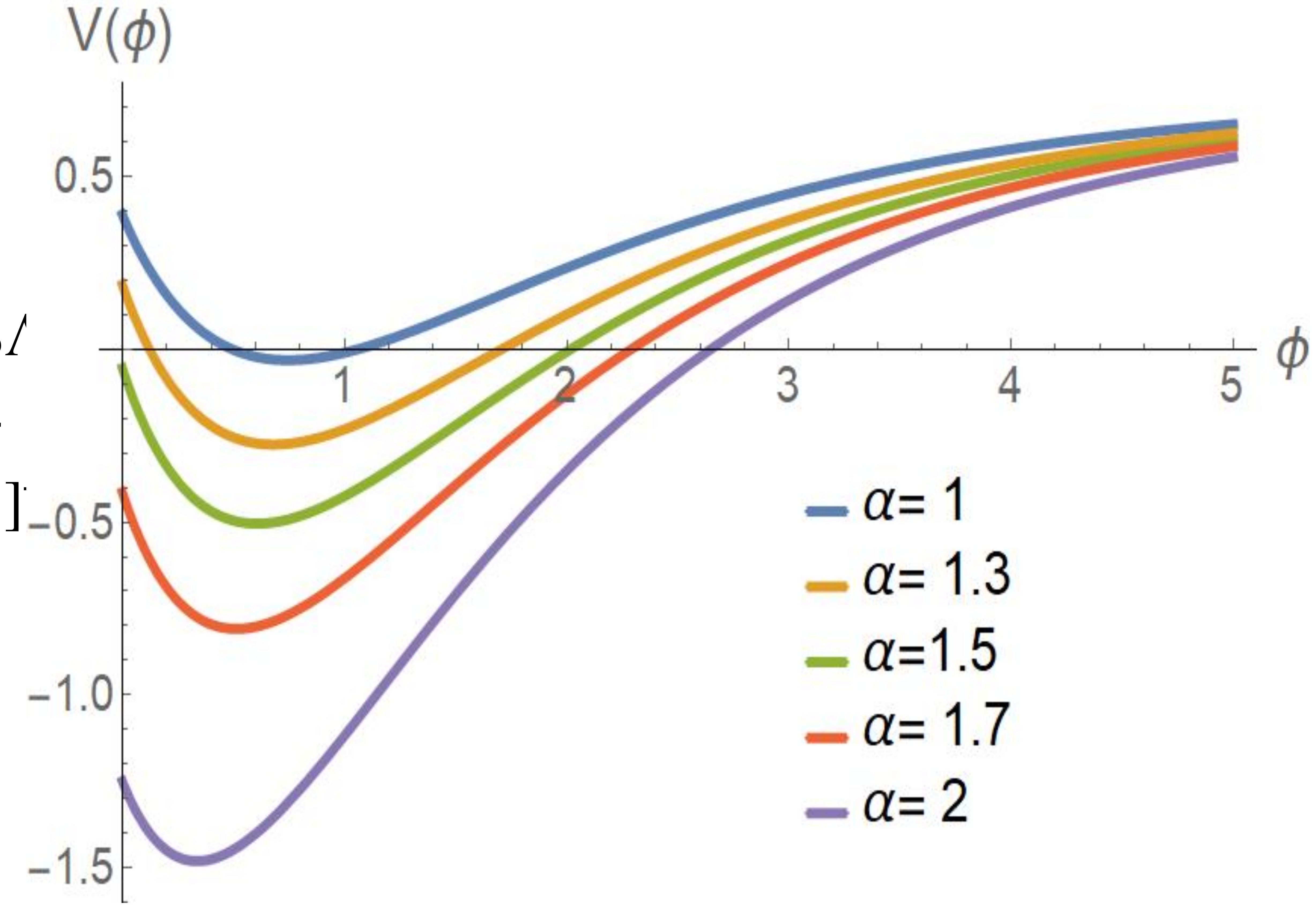

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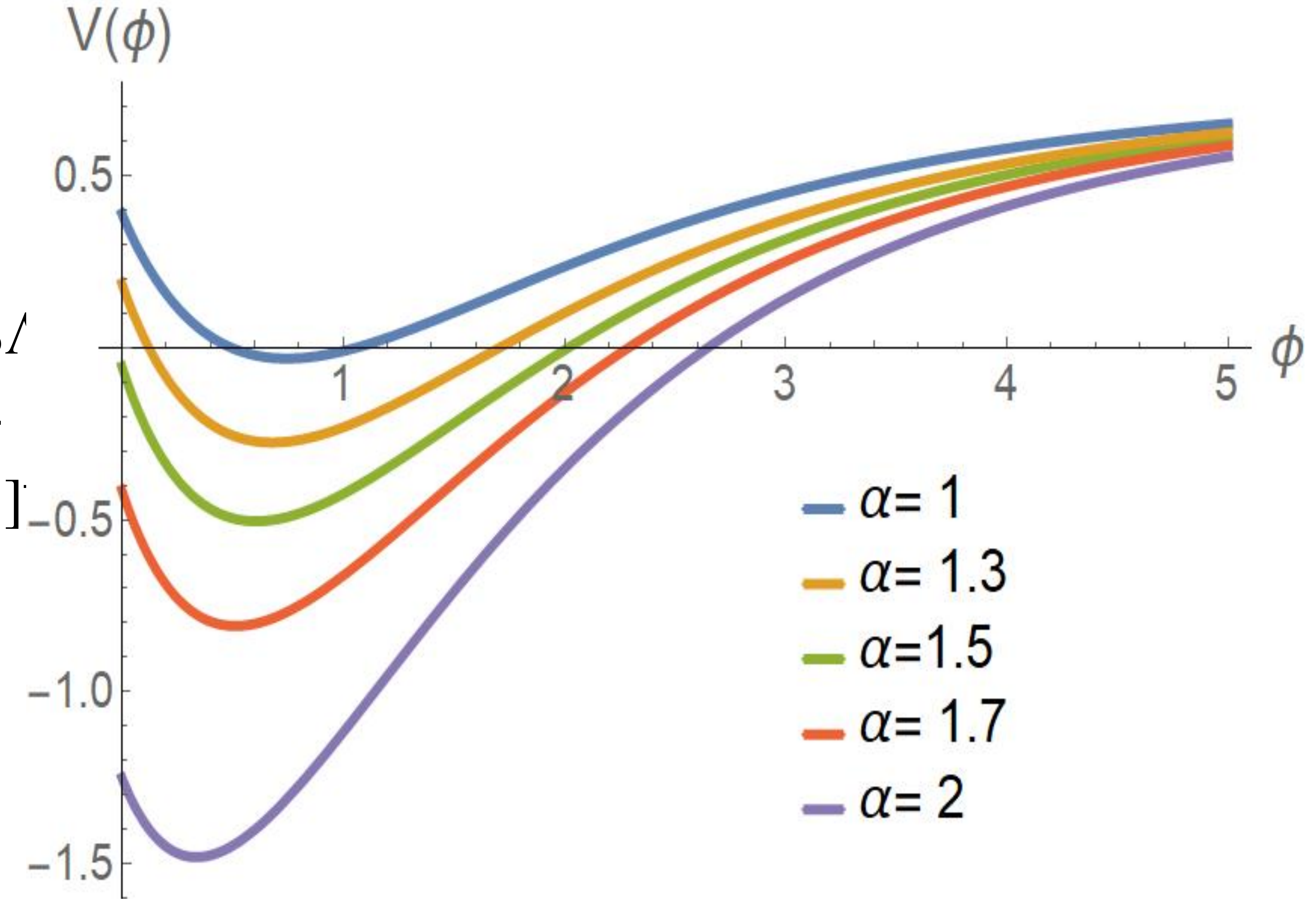
- The potential has a minimum for $V_+(\phi)$ for $\alpha \in [1,3]$ and $\Lambda \in [0,1.5]$.



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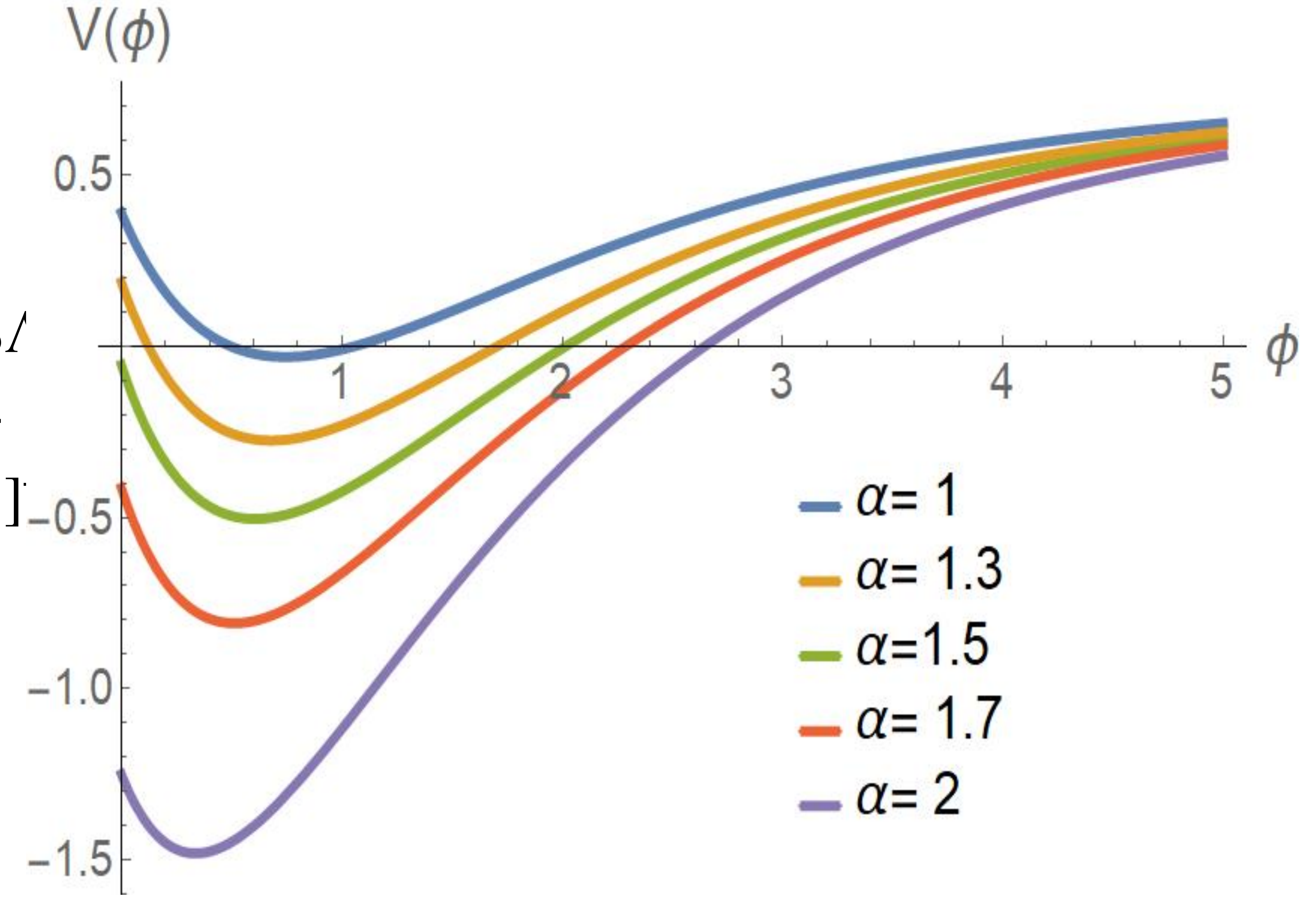

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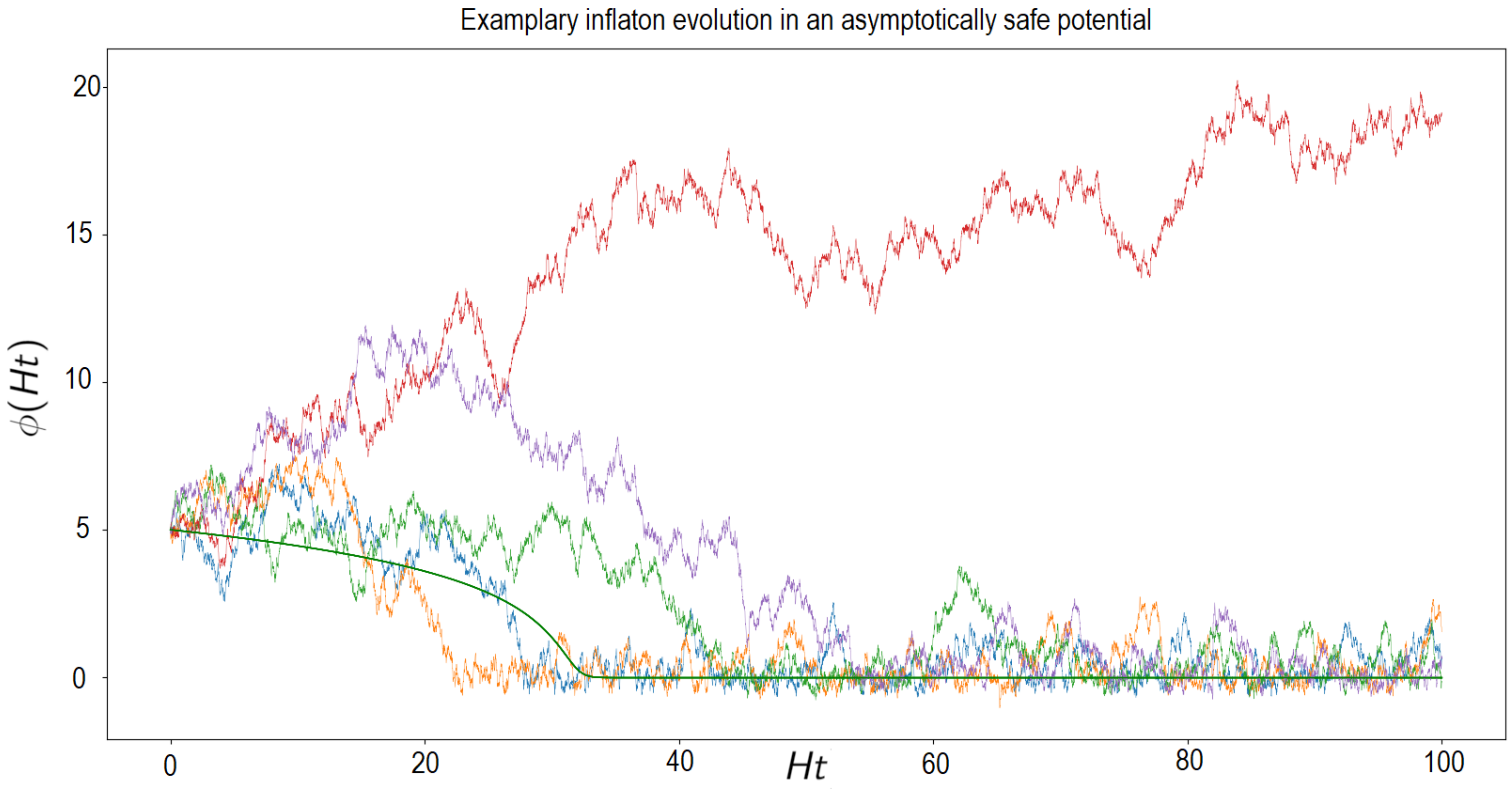
- $\alpha \to 0$, $\Lambda \to 0$ Starobinsky.

Eternal inflation

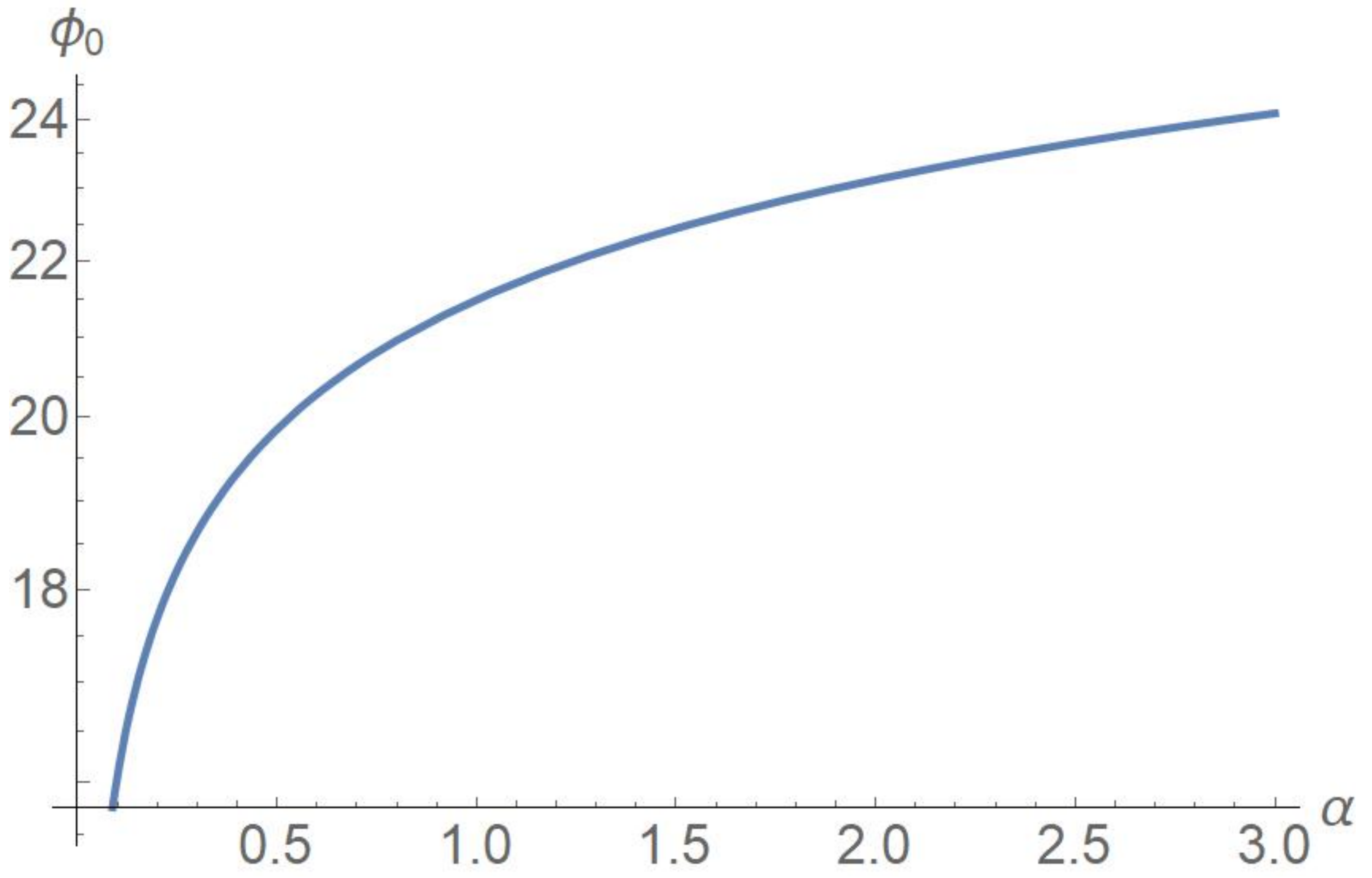
Stochastic evolution



Eternal inflation

Asymptotic safety

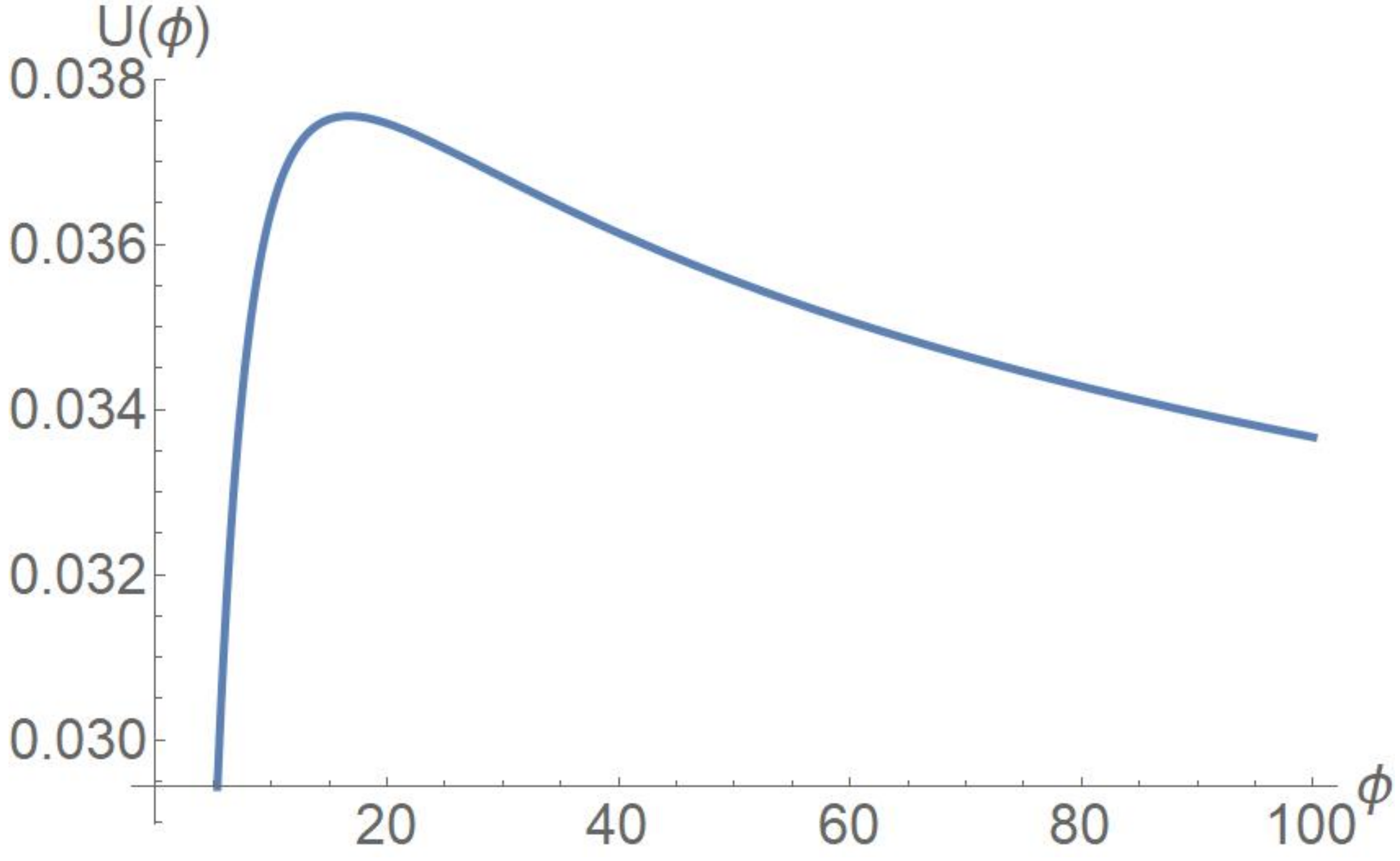
- $\phi_0 \propto \log(\alpha)$, since $V_+(\phi) \sim \alpha^4 e^{-\sqrt{\frac{2}{3}}\kappa\phi} m^2/\kappa^2$,
- No Λ dependence



Eternal Inflation

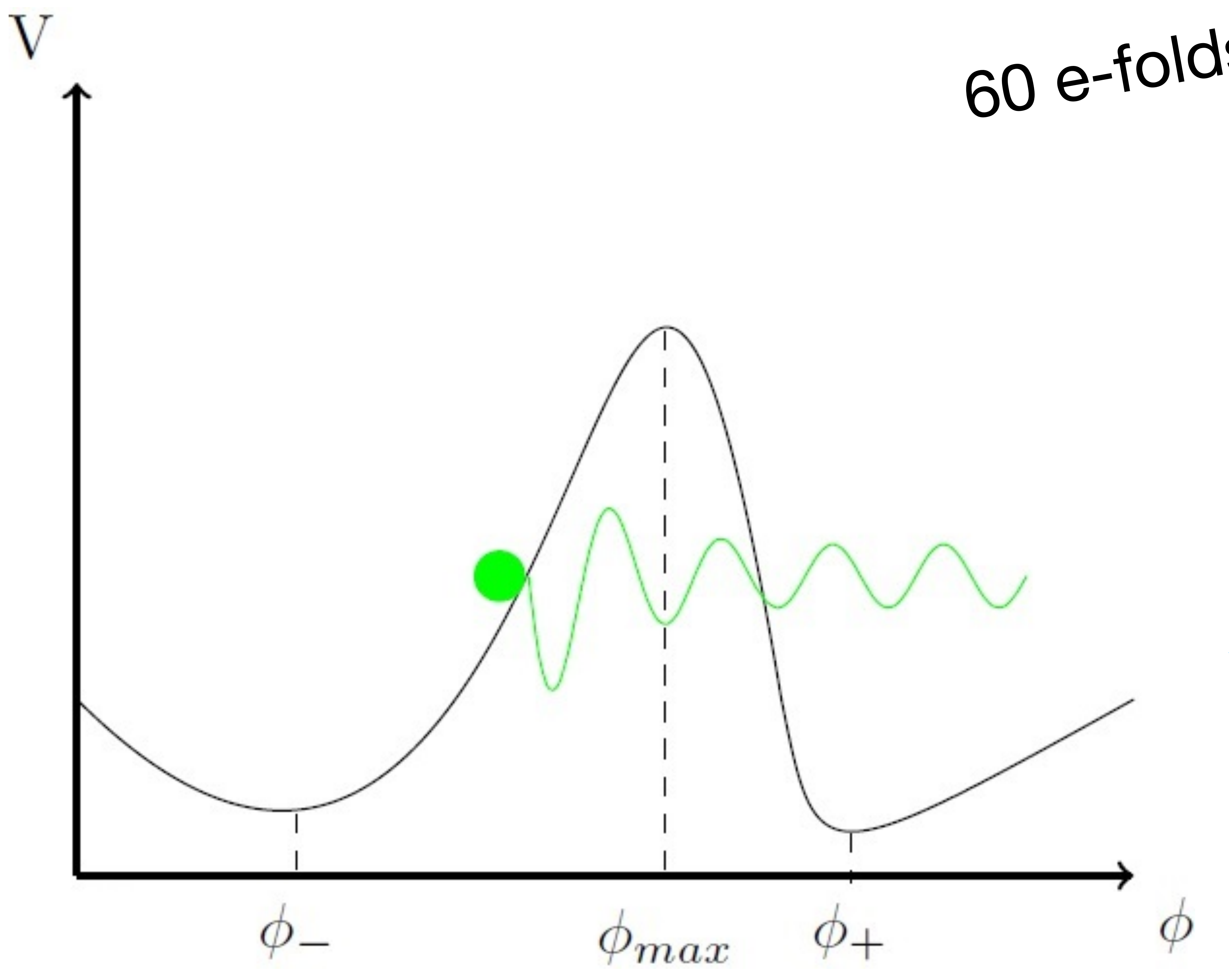
Asymptotic Safety in Veneziano limit of Yang Mills

- $\dfrac{N_F}{N_c} - \dfrac{11}{2} = \delta,\ N_F, N_c$ large
- $\mathscr{L} \ni \xi\phi^2 R$
- RG improvement gives the Lambert W function potential
- Inflation matching observables with $\xi \approx 1$ and $N_F \approx N_{SM}$
- Potential too steep for EI

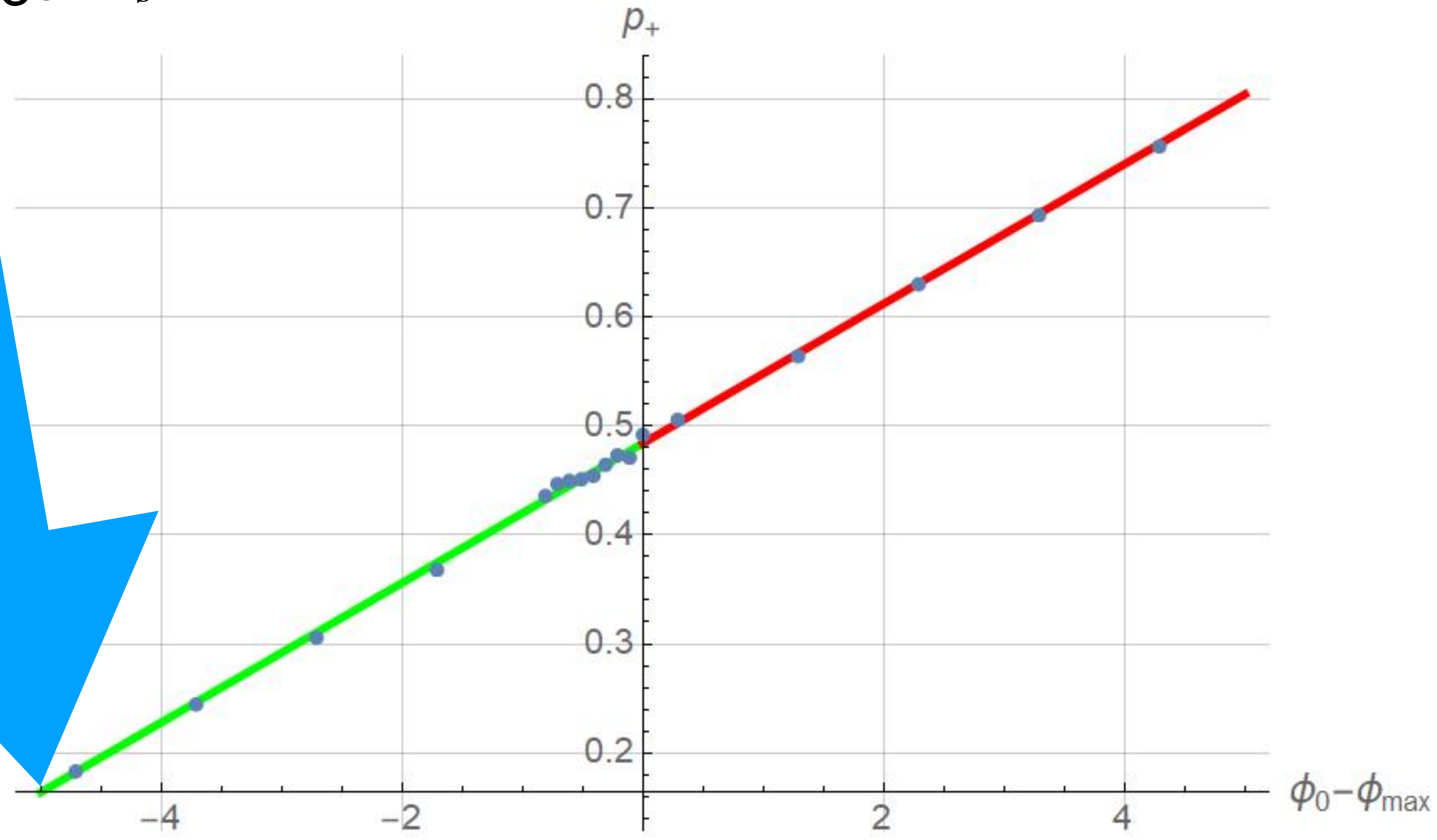


Eternal Inflation

Tunnelling effects



60 e-folds, correct n_s, r



$$N_F = 10, \quad \xi = \frac{1}{6}, \quad \delta = 0.1$$

Eternal inflation

Summary

- The existence of FP generically flattens the potential giving rise to eternal inflation
- Tunneling effects also can give rise to eternal inflation
- Finite action and eternal inflation: 2102.05550
- If you want to find out more see: 2101.00866 and 2102.13556

Eternal Inflation

Fokker Planck equation

- In infinitesimal time δt:

$$\delta\phi = -\frac{1}{3H}V'(\phi)\delta t + \delta\phi_q(\delta t), \quad \delta\phi_q \sim \mathcal{N}(0, H^3(\delta t)/(2\pi)^2),$$

- the probability distribution of ϕ at time t is given by Fokker Planck equation

$$\dot{P}[\phi, t] = \frac{1}{2}\frac{H^3}{4\pi^2}\partial_\phi\partial^\phi P[\phi, t] + \frac{1}{3H}\partial_\phi\left((\partial^\phi V)P[\phi, t]\right).$$

- For $V(\phi) \approx V_0$ solution is given by $P[\phi, t] = \dfrac{1}{\sigma(t)\sqrt{2\pi}}\exp\left[\dfrac{-\phi^2}{2\sigma(t)^2}\right].$